Problem 1:

(A) 12 is the identity

(B) the inverse of k is $\begin{cases} 12-k & \text{if } k \neq 12 \\ 12 & \text{if } k = 12 \end{cases}$

(C) Notice that

$$j * k = \begin{cases} j+k, & j+k \leq 12 \\ j+k-12, & j+k > 12 \end{cases}$$

If $j+k \leq 12$, $\quad j*k = j+k = k+j = k*j$

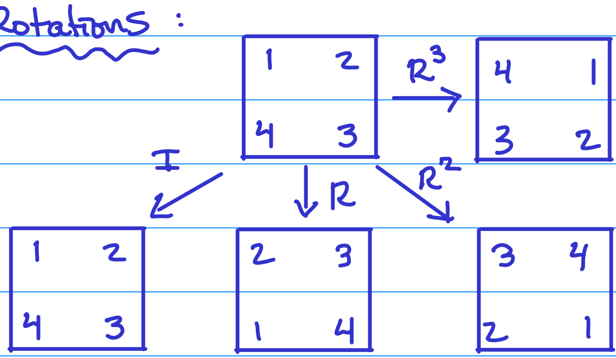
If $j+k > 12$, $\quad j*k = j+k-12 = k+j-12 = k*j$

In either case, $\quad j*k = k*j$.

Problem 2:

(A) There are two kinds of symmetries

Rotations:



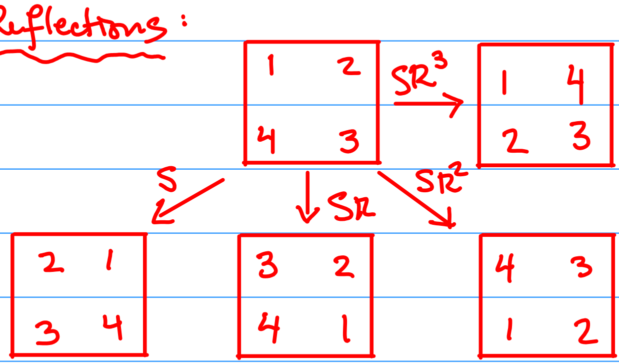
$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$R^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$R^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Reflections:



$$S = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$SR = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$SR^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$SR^3 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(B) Multiplication Table

	I	R	R^2	R^3	S	SR	SR^2	SR^3
I	I	R	R^2	R^3	S	SR	SR^2	SR^3
R	R	R^2	R^3	I	SR^3	S	SR	SR^2
R^2	R^2	R^3	I	R	SR^2	SR^3	S	SR
R^3	R^3	I	R	R^2	SR	SR^2	SR^3	S
S	S	SR	SR^2	SR^3	I	R	R^2	R^3
SR	SR	SR^2	SR^3	S	R^3	I	R	R^2
SR^2	SR^2	SR^3	S	SR	R^2	R^3	I	R
SR^3	SR^3	S	SR	SR^2	R	R^2	R^3	I

(C)
- Matrix multiplication is associative, so group is associative
- I is identity
- Inverse of R^j is R^{4-j}. Inverse of SR^j is $R^{4-j}S$

Thus it is a group.

The multiplication table is <u>not</u> symmetric, so the group is not Abelian.

Problem 3

(A) Let $e \in G$ be the identity.

$e * e = e$ so $e = e \times e$.

Since the cross product of a vector with itself is $\vec{0}$, we get $e = \vec{0}$.

(B) If $g \in G$, then $e * g = g$, so $g = \vec{0} \times g = \vec{0}$

Thus $g = \vec{0}$ $\forall g \in G$. $\therefore G = \{\vec{0}\}$