Algebraic Extensions

Simple Extensions:

· x algebraic of degree de

$$F(x) = \{a_0 + a_1x + a_2x^2 + ... + a_3x^{d-1} | a_0, ..., a_3 \in F\}$$

· a transcendental

$$F(x) = \left\{ f(x) / g(x) \middle| f(x), g(x) \in F[x], g \neq 0 \right\}$$

Observation ~ if x is algebraic, then F(x) is finite dimensional as an F-vector space

Det: The degree of an extension field E

Theorem: If [E:F] is finite, then every element $\alpha \in E$ is algebraic over E.

Duf: An extension field is <u>algebraic</u> if every element a EE is algebraic over F.

The algebraic closure of F in E is

F = { X E | x is algebraic over F}

Theorem: FE is a field.
There is a largest algebraic extension of a field F, called the algebraic closure.
field F called the algebraic closure.
Def: A field E is called algebraically closed if every pohnomial fix EEXT has a root on E. An algebraically closed algebraic extension E of F is called an algebraic closure of F.
if every polynomial f(x) EE(x) has a root on E.
An algebraicable closed algeborase extension E
of F To called our almotories character of F.
J. Caralle Ma and Market Con son
Notation: F is the alaphoraic chosine of F.
The state of the s
Theorem: D= Exec/x is algebraicle is a proper subset of C.
is a proper subset of P.
Proof:
B is countable, but C 3 uncountable
T
Theorem: If XEER algebraic over F
Theorem: If $\alpha \in E$ algebraic over F and decentral $(\alpha,F) = d$, then a basis for $F(\alpha)$ as an F -vector space is
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for 1 (h) as on 1. vector space is
$\{1, \alpha, \alpha^2, \dots, \alpha^{d-1}\}$
In a die la
in particular,

 $[F(\alpha):F] = \deg irr(\alpha,F)$

and

$$Q(3) = \{a_0 + a_1 + a_2 + a_2 + a_2 + a_2 + a_3 + a_4 + a_5 + a_6 + a_$$

Ex:

Thronem:



Remonds us of Subgroups,

