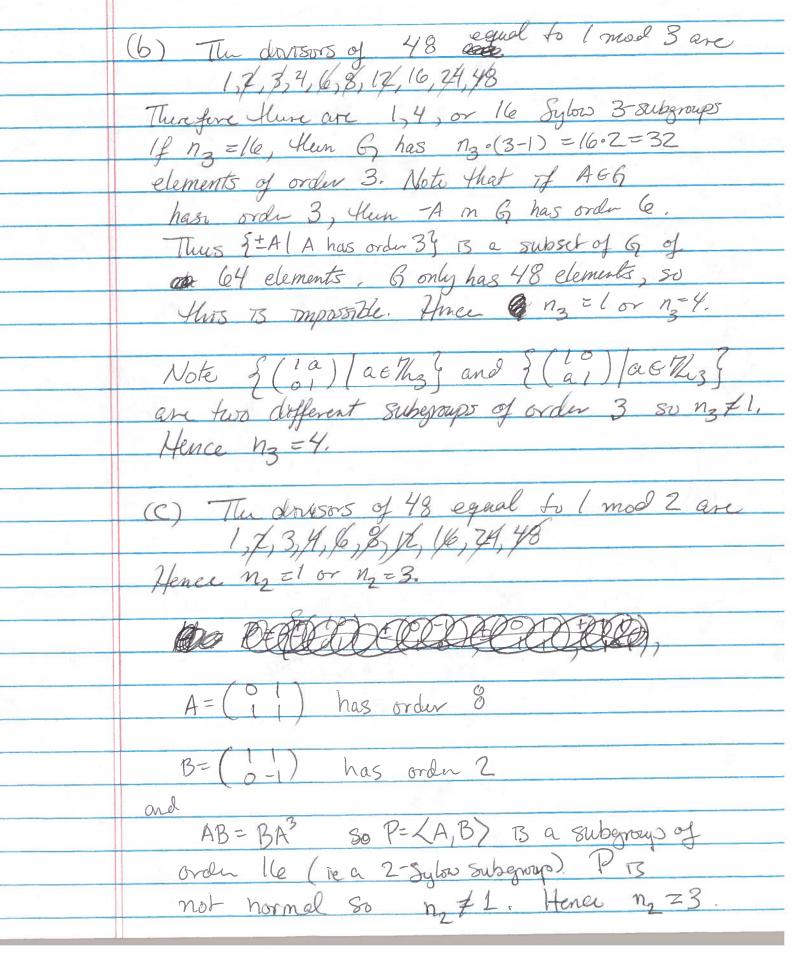
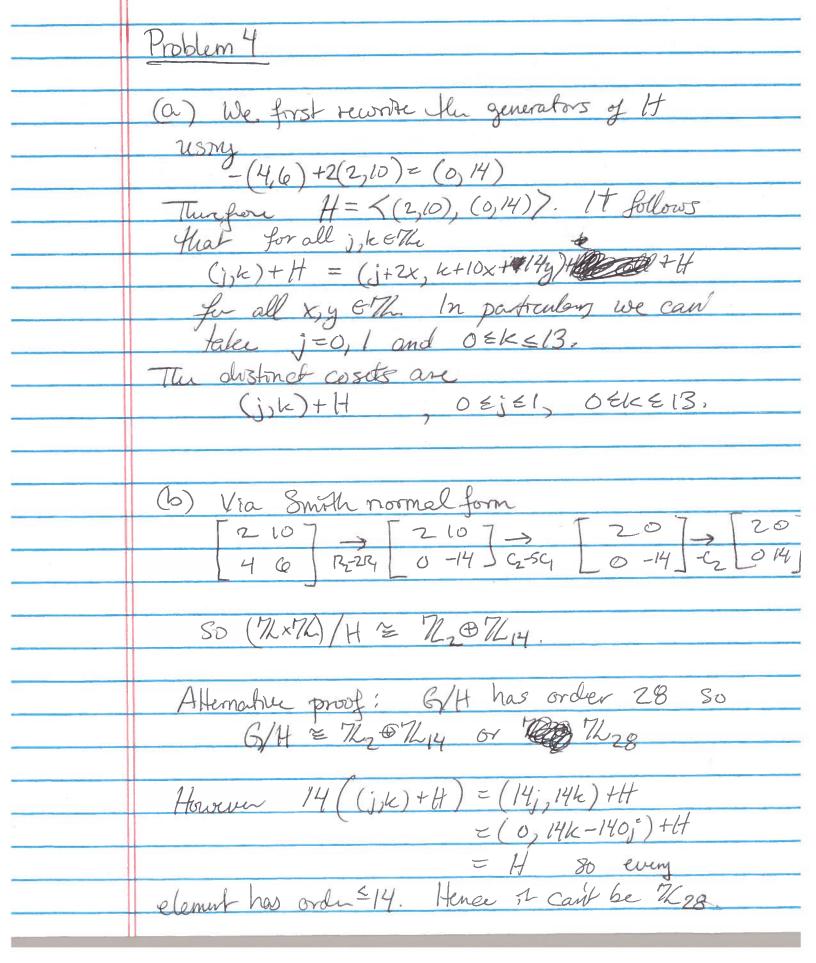


Problem 4: To do Hos problem, it helps to first rewrite H. Note (6,9) -3(2,2) € = (0,3) € H $H = \langle (2,2), (0,3) \rangle$. and in fact Then for any (j,k) & The x The, (j,k)+H=(j+2a,k+2a+3b)+tfor all a, b = The Who we may take j=0,1 and k=0,1,2 So the only dishnet cosits are (0,0)+H, (0,1)+H, (0,2)+H (1,0)+H , (1,1)+H , (1,2)+H. (b) To do this we can reduce the matrix [22] > [22] 20] Therefore (7/0 1/H & 1/2 0 1/3 -Finally, we put this in invariant factor form to get These. Alternative approach: G/H T3 an Abelian group of order Ce and Herre 13 only one of shore up to 130morphism, namely The Problem 5: Let XEG and consider XNEG/N. The order of XN must drande [G/N]=[G:N]=n and therefore $(XN)^n = eN$. Since $(XN)^n = x^nN$ if follows $x^nN = N$ and therefore $x^n \in N$. Problem 6: To show a & Then is a unit we must show there exists XEThin with ax = 1 m 7/m Note gcd (a,m) = 1 (3) 7 x,y & with = 1 mod n = 3x EThn with ax=1 m 7hn. Thus ged (a,n)=1 (2) a 13 a unit Problem 7: If a EN(R) and bER Ilun Mrn exists no with a"=0 and therefore $(ab)^n = a^nb^n = 0.b^n = 0.50$ aben(R) Likewise, if a, b EN(tr) are both nilpotent then there exist moo, no with a^m=0 and b"=0- It follows (a+b) = Zu (mtn k mtn-k = 20 = 0. So atheN(r)

Practice Exam 2 Version 2
Problem
a) False
(b) False
(c) True
(d) True (e) False
(G) Faisc
Prodem 2
(a) Let 4: G → G be a group homomorphism.
Then 4 induces an isomorphism
4: G/ku(4) -> mg(4)
(b) Let 6 be a finite group. If p is a prime
divisor of 191 Hus & has an element of order p.
C) GL2(tR)
Problem 3
(a) 191=48 = 24.3 so the Sylow Z-subgroups
must have 24 = 16 elements and the Sylow 3-subgroups must have 3 elements
3- Subgroups must have 3 elements





-	Problem 5
	G = La) for some a6 g.
	Therefor
	$G/H = 9 \times H \times 66$
	= SakH heth? = (att).
	In particular G/H 13 cyclic, generated by att
	Problemle
	Since R 13 a rong, mult. 13 associative.
	Also LERX is an identity. Formally,
	Also Lett is an identity. Formally, if next, the usa unit so FVER
	with rev=vre=12 Hence re has con-
	muse. Thus 12x is a group,
	Problem 7
	First note the binomial coefficial
	P = P(p-1)(p-2)(p-k+1)
	13 an meger ju OEKEP and when
	OKK < P the numerator is dovisible by
	13 an integer for OEKEP and whom OKKEP the numerator is dovisible by p but the denominator isn't.
	Thus
	Thus (P) = 0 mod p for OCKEP
	Hence (

$$(a+b)^{p} = \sum_{n=0}^{p} (k) a b^{p} + k$$

$$= (p) a^{0} b^{p} + (p) a^{0} = b^{p} + a^{p}$$

$$= (p) b^{p} + (p) a^{p} = b^{p} + a^{p}$$

$$(b) F_{p}(x+y) = (x+y)^{p} = x^{p} + y^{p} = f_{p}(x) + F_{p}(y)$$

$$by part (a)$$

$$Also F_{p}(xy) = (xy)^{p} = x^{p}y^{p} = F_{p}(xy)$$

$$30 f_{p} is a rmg homomorphism.$$