Review from last time:

- Set - collection of stuff.
- Relation R from A to B is just a subset of $A \times B$

Relation R on A is a subset of $A \times A$

- reflexive, symmetric, transitive!

Functions:

Def: A <u>function</u> from A to B is a relation R from A to B satisfying

- for any $a \in A$ $\exists! \, b \in B$ satisfying aRb

Function notation: if f is a function from A to B we write "$f: A \to B$". Moreover, instead of saying afb we will write $f(a) = b$

<u>Ex</u>: Define a relation R from $\mathbb{Z}$ to $[0, \infty)$ by setting nRx if and only if $n = x^2$.

Is this a function???

Need to check for every $n \in \mathbb{Z}$ $\exists! \, x \in [0, \infty)$ satisfying nRx $(n = x^2)$

FAILS for $n = -1$!!!

Sara's Q: non-perfect squares? $2R\sqrt{2}$

<u>Ex</u>: Define a relation R from $\mathbb{N}$ to $\mathbb{R}$ by setting nRx if and only if $n = x^2$

Is this a function?

NO! $2R\sqrt{2}$ and $2R-\sqrt{2}$

Ex: The relation R from $\mathbb{N}$ to $[0,\infty)$ defined by saying nRx iff $n = x^2$ is a function!

Function notation $R(n) = \sqrt{n}$

Properties of functions: $f: A \to B$

- f is injective (one-to-one) if for any $a_1, a_2 \in A$
  $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$
- f is surjective (onto) if for all $b \in B$ there exists $a \in A$ w/ $f(a) = b$
- f is bijective if its injective and surjective

Ex: Consider the function $f: \mathbb{R} \to \mathbb{R}^2$ defined by $f(x) = (x^2, x^3)$.

Q: injective? S'pose $x_1, x_2 \in \mathbb{R}$ and $f(x_1) = f(x_2)$

Then $\overbrace{(x_1^2, x_1^3)}^{f(x_1)} = \overbrace{(x_2^2, x_2^3)}^{f(x_2)}$

$x_1^2 = x_2^2$

$x_1 = \pm x_2$

$\sqrt[3]{x_1^3} = \sqrt[3]{x_2^3} \Rightarrow x_1 = x_2$

YES!

Q: surjective? $(-1, 0)$ is not in the image!

because if $f(x) = (-1, 0)$

$(x^2, x^3) = (-1, 0)$

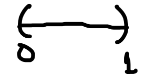
NO!

x is a pineapple

$x = 0$

Ex: $f(x): (0,\infty) \to (0,1)$ $f(x) = \frac{1}{x+1}$

This is a bijection



Groups – Intuitive notion:

A group is a set of ways of transforming a given object which is sufficiently nice.

Deck of 52 playing cards
- cut the deck
- riffle shuffle
- etc.

Each transformation rearranges the cards in the deck

$$G = \{ g \mid g \text{ is a way of rearranging the cards} \}$$

This is an example of a group!
- there is a way of taking two transformations and making a new one   Composition!

  This is called multiplying
- Multipication is associative

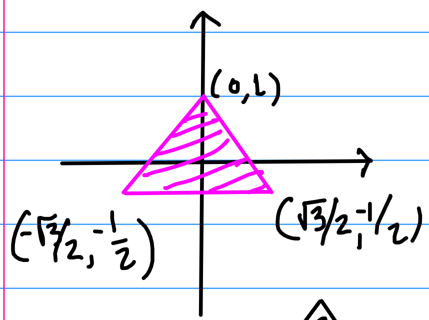
  $$g_1 \cdot (g_2 \cdot g_3) = (g_1 \cdot g_2) \cdot g_3$$
- every transformation is reversible

This is the symmetric group S_{52}.

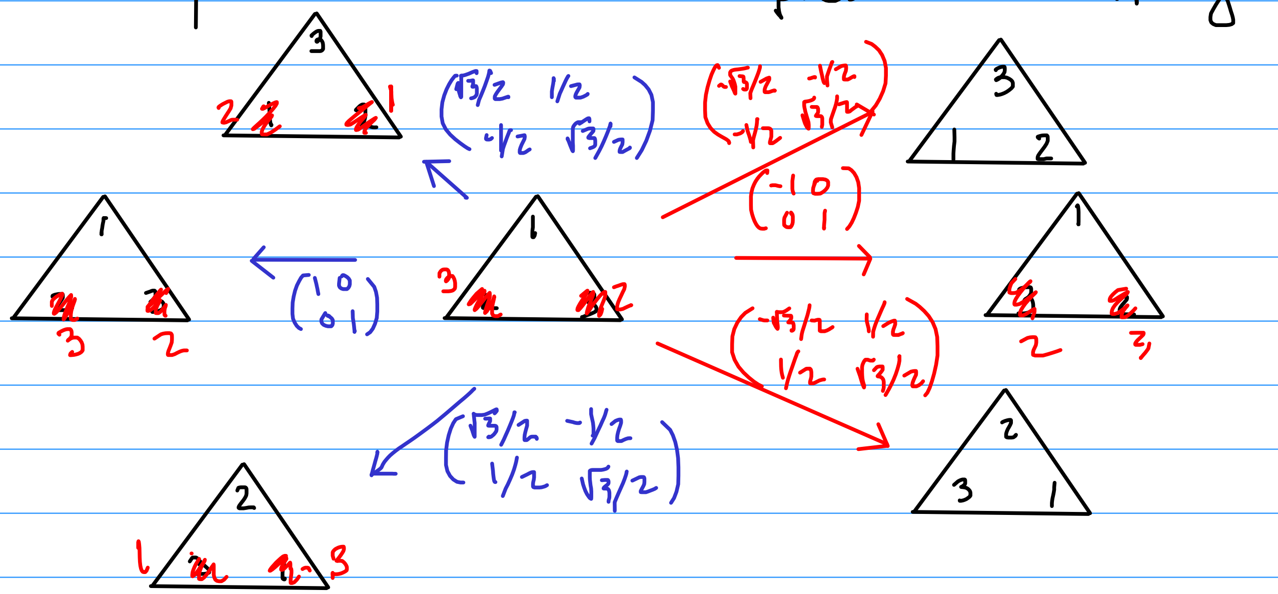
More examples of groups
- symmetries in geometry
- algebraic equations
- number theory

examples used to decide what a group should be

Groups of Symmetries:



Group of (linear) symmetries:
all the ways we can do a linear transformation which preserves the triangle



$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$: $(x,y) \longmapsto (ax+by, cx+dy)$

This is a group with six elements

dihedral group D_3

- dihedral group D_n.

Problem 3:

Equivalence classes:

$\{x \mid x$ is a playing card which is a heart$\}$

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