Supplementary Information: Disrupting the Dichotomy between Compact and Low-Density Urbanism in the Archaeological Record

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W. Christopher Carleton[1], Sarah Klassen[2], John T. Murphy[3], Patrick Roberts[1]

[1] *Department of Coevolution of Land Use and Urbanisation, Max Planck Institute of Geoanthropology, Jena, Germany* [2] *Center for Collaborative Synthesis in Archaeology, University of Colorado, Boulder, Colorado, USA*
[3] *Department of Anthropology, Northern Illinois University, DeKalb, Illinois, USA*

emails: carleton@gea.mpg.de, sarah.klassen@colorado.edu, jmurphy7@niu.edu, roberts@gea.mpg.de

Overview

This computational notebook contains the code necessary to rerun the analyses described in the associated paper. In order to gather and prepare the data used in this notebook for the Hampshire case, run the other computational notebook in the same folder ("doomsday.ipynb") first. Then, run all of the following code in order.

Set Up

Libraries

```
# core
import numpy as np
import pandas as pd
from scipy.stats import norm
import warnings
```

```python
# Plotting
import matplotlib as mpl
import matplotlib.pyplot as plt
import seaborn as sns
import contextily as ctx

# chronocluster
from chronocluster import clustering
from chronocluster.utils import (
    clustering_heatmap,
    pdiff_heatmap,
    get_box,
    chrono_plot,
    chrono_plot2d,
    inclusion_legend
)
from chronocluster.distributions import ddelta
```

Plot Styling

```python
# basic styling
plt.style.use('ggplot')
sns.set_context("paper")

# matplotlib fonts
mpl.rcParams["font.size"] = 12
mpl.rcParams["legend.frameon"] = False
mpl.rcParams["legend.fontsize"] = 10
mpl.rcParams["axes.labelsize"] = 12
mpl.rcParams["axes.titlesize"] = 14
mpl.rcParams['figure.facecolor'] = 'white'
```

NOTE: Warnings

Some of the plotting workarounds have generated immaterial warnings below. In order to avoid these showing up in a PDF or HTML version of this notebook, like the one that will be subimitted as SI alongside the associated paper, run the next cell. In order to see the warnings, remove the cell or change plot_warnings to True.

```
plot_warnings = False
if not plot_warnings:
    warnings.filterwarnings("ignore")
```

Angkor

Data Wrangling

```
# data wrangling
df = pd.read_csv('../Data/temples_with_predicted_ages.csv')
df = df.dropna(subset=['xeast', 'ynorth', 'model_age_mean'])
```

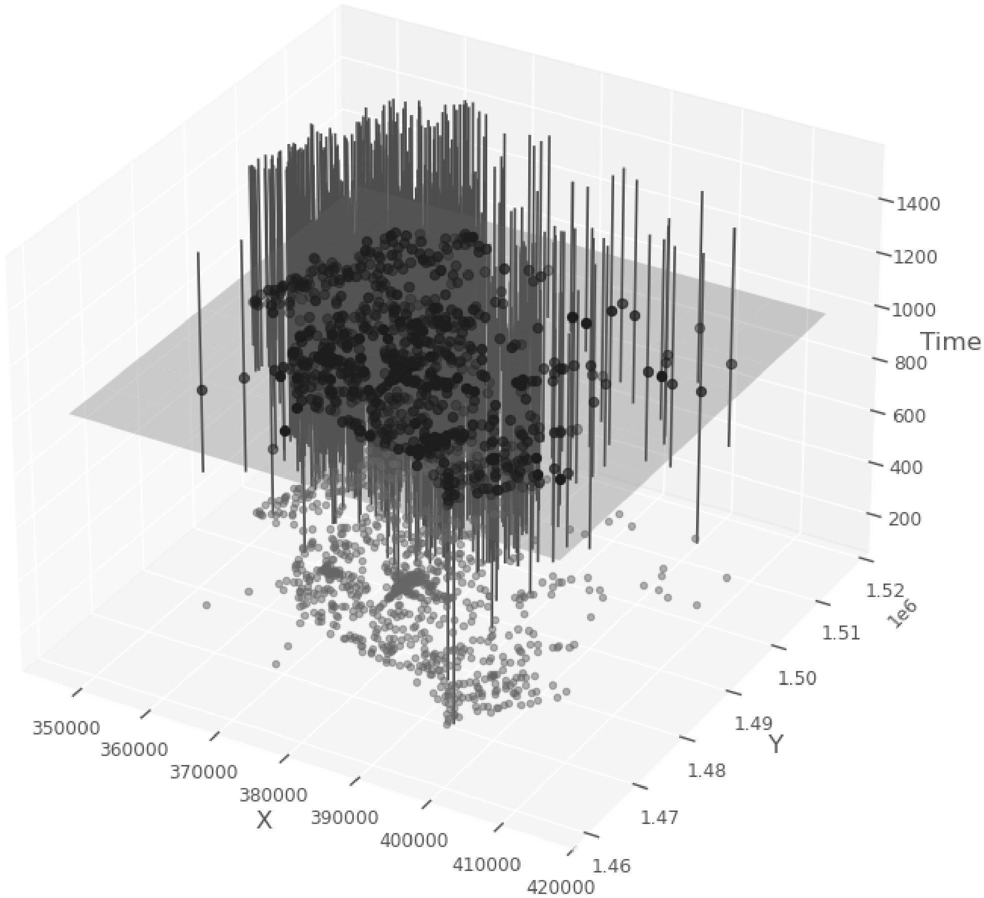
Create Points List

```
points = [
    clustering.Point(
        x=row['xeast'],
        y=row['ynorth'],
        start_distribution = (
            ddelta(d=row['model_age_mean'])
            if row['model_age_sd'] == 0
            else norm(loc=row['model_age_mean'], scale=row['model_age_sd'])
            ),
        end_distribution = ddelta(1500)
    )
    for _, row in df.iterrows()
]

# just double check the first ten look right
points[:10]
angkor_points = points
```

Figure S1: Spacetime Volume of Angkor's Temples

```
# Custom styling parameters
style_params = {
    'start_mean_color': None,  # Do not plot start mean points
    'end_mean_color': None, # Do not plot end mean points
```

```
    'mean_point_size': 10,
    'cylinder_color': (0.3, 0.3, 0.3),  # Dark grey
    'ppf_limits': (0.05, 0.95),  # Use different ppf limits
    'shadow_color': (0.4, 0.4, 0.4),  # grey
    'shadow_size': 10,
    'time_slice_color': (0.5, 0.5, 0.5),  # Grey
    'time_slice_alpha': 0.3,
    'time_slice_point_color': (0, 0, 0),  # Black
}

# Plot the points using the chrono_plot function with custom styling and a
# time slice plane
ax_stv_angkor, fig_stv_angkor = chrono_plot(points,
                                            style_params=style_params,
                                            time_slice=1000,
                                            title='Angkor')
ax_stv_angkor.set_box_aspect(None, zoom=0.85)
plt.savefig("../Output/spacetime_volume_Angkor.svg", bbox_inches='tight')
```



Define Time Slices

```
# Define the time slices
start_time = 800
end_time = 1200
time_interval = 50
time_slices = np.arange(start_time, end_time, time_interval)
time_slices
```

```
array([ 800,  850,  900,  950, 1000, 1050, 1100, 1150])
```

GPU Boosted Pairwise Distance Density KDEs

To speed up the production of the heatmap surfaces considerably, you can use GPU processing via the CUML library. If, however, you do not have access to a dedicated, compatiible GPU, set 'kde_custom=cuml_kde' to 'kde_custom=None' in all the subsequent cells where appropriate.

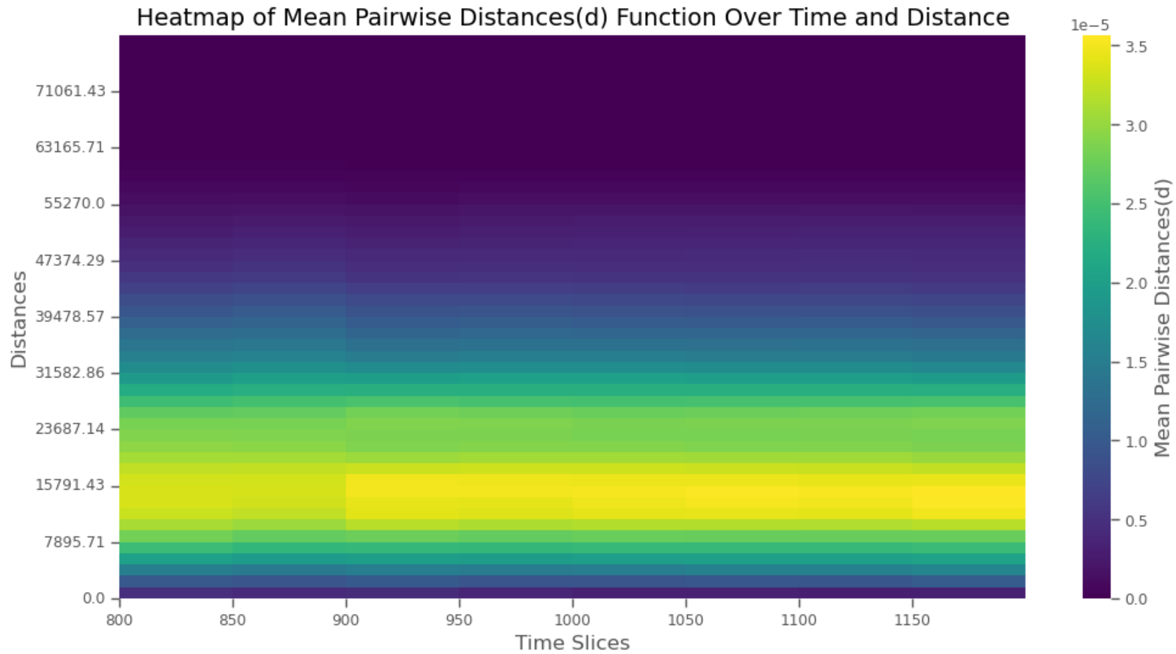
```
from cuml.neighbors import KernelDensity

def cuml_kde(distances, bandwidth, **kwargs):
    distances = np.array(distances).reshape(-1, 1)

    if bandwidth is None:
        n = len(distances)
        if n < 2:
            raise ValueError(
                "Data must contain at least 2 points ",
                "for bandwidth calculation."
            )
        std_dev = np.std(distances, ddof=1)
        bandwidth = std_dev * n ** (-1 / 5)

    kde = KernelDensity(kernel="gaussian", bandwidth=bandwidth, **kwargs)
    kde.fit(distances)

    def kde_function(points):
        points = np.array(points).reshape(-1, 1)
        # score_samples returns a cupy array; use .get() to convert to NumPy
        return np.exp(kde.score_samples(points).get())

    return kde_function
```

Figure S2: Heatmap of Pairwise Distance Density versus Time at Angkor

```
# Run the Monte Carlo simulation to get an ensemble of probable
# lists of points included in each time slice.
num_iterations = 500
simulations_angkor = clustering.mc_samples(
    points,
    time_slices=time_slices,
```

```
    num_iterations=num_iterations
)

# Get a bounding box for use later and to extract sensible distance limits
x_min, y_min, x_max, y_max = get_box(points)
max_distance = np.ceil(np.sqrt((x_max - x_min)**2 + (y_max - y_min)**2))

# set consistent pairwise bandwidth (binning of distances)
use_kde = True
pair_bw = None
kde_sample_n = 50
# here set kde_custom=None to use a CPU instead of GPU for processing.
kde_custom=cuml_kde

# Produce pairwise distances to explore clustering structure
pairwise_density_angkor, support_angkor = clustering.temporal_pairwise(
    simulations_angkor,
    time_slices,
    bw=pair_bw,
    use_kde=use_kde,
    kde_sample_n=kde_sample_n,
    max_distance=max_distance,
    kde_custom=kde_custom
)

# Visualize clustering with heatmap
clustering_heatmap(
    pairwise_density_angkor,
    support_angkor,
    time_slices,
    result_type='Pairwise Distances',
    save = "../Output/pdd_hm_angkor.png"
)
```

```
(<Figure size 1200x600 with 2 Axes>,
 <Axes: title={'center': 'Heatmap of Mean Pairwise Distances(d) Function Over Time and Dista
```



Complete Spatial Randomness

Figure S3: Heatmap of Pairwise Distance Density versus Time for CSR based on Angkor's Temples

```
# Get MC iterations for incorporating chronological uncertainty and CSR
csr_simulations_angkor = clustering.mc_samples(
    points,
    time_slices = time_slices,
    num_iterations = num_iterations,
    null_model=clustering.csr_sample,
    x_min=x_min,
    x_max=x_max,
    y_min=y_min,
    y_max=y_max
)

# Calulate the pairwise distances for the CSR sample
csr_pairwise_density_angkor, csr_support_angkor = clustering.temporal_pairwise(
    csr_simulations_angkor,
    time_slices,
    bw = pair_bw,
```

```
    use_kde = use_kde,
    kde_sample_n=kde_sample_n,
    max_distance = max_distance,
    kde_custom=kde_custom
)

# Visualize clustering with heatmap
clustering_heatmap(
    csr_pairwise_density_angkor,
    csr_support_angkor,
    time_slices,
    result_type='Pairwise Distances'
)
```

```
(<Figure size 1200x600 with 2 Axes>,
 <Axes: title={'center': 'Heatmap of Mean Pairwise Distances(d) Function Over Time and Distan
```

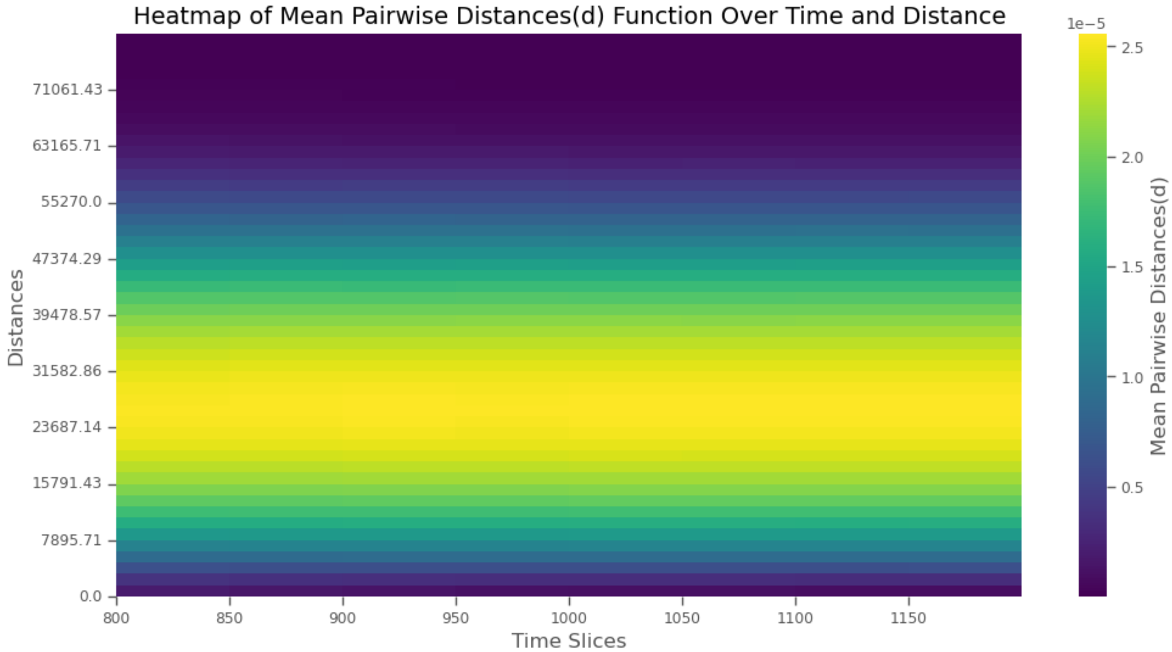
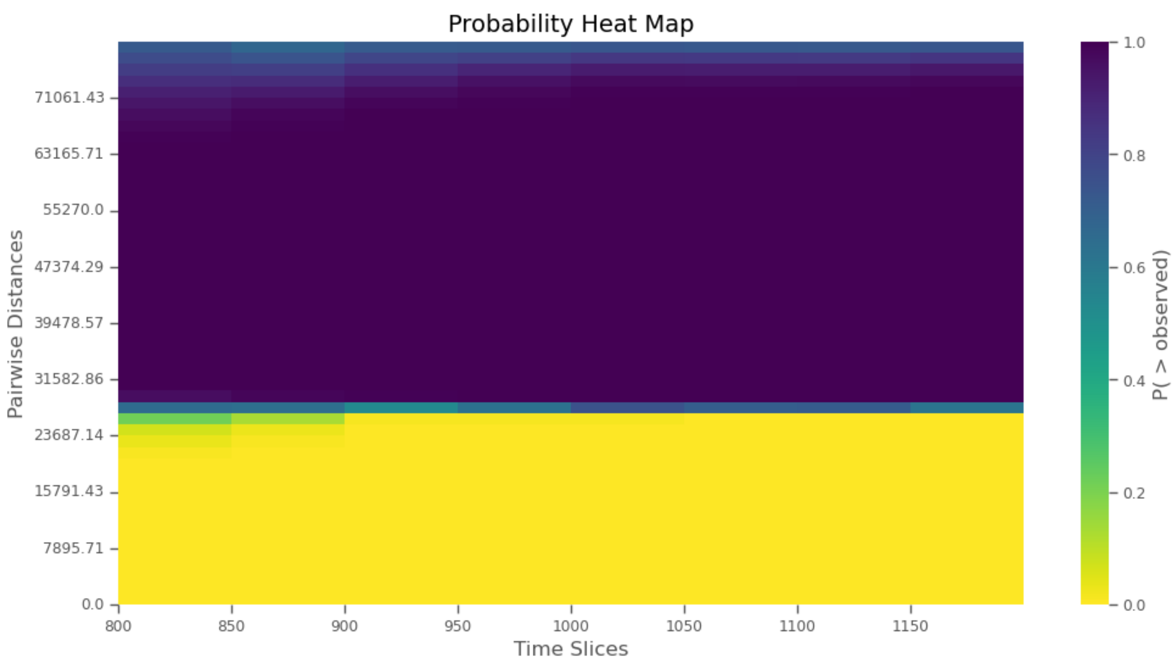


Figure S4: Heatmap of PDD Statistical Significance compared to CSR for Angkor's Temples

```
# Calculate the p-values for density differences between the observed points and
# the simulated CSR baseline per distance and temporal slice
p_diff_array_csr_angkor, diff_array_csr_angkor = clustering.p_diff(
    pairwise_density_angkor,
    csr_pairwise_density_angkor
)

# Plot the heatmap of probabilities
pdiff_heatmap(
    p_diff_array_csr_angkor,
    time_slices,
    csr_support_angkor
)
```

```
(<Figure size 1200x600 with 2 Axes>,
 <Axes: title={'center': 'Probability Heat Map'}, xlabel='Time Slices', ylabel='Pairwise Dis
```



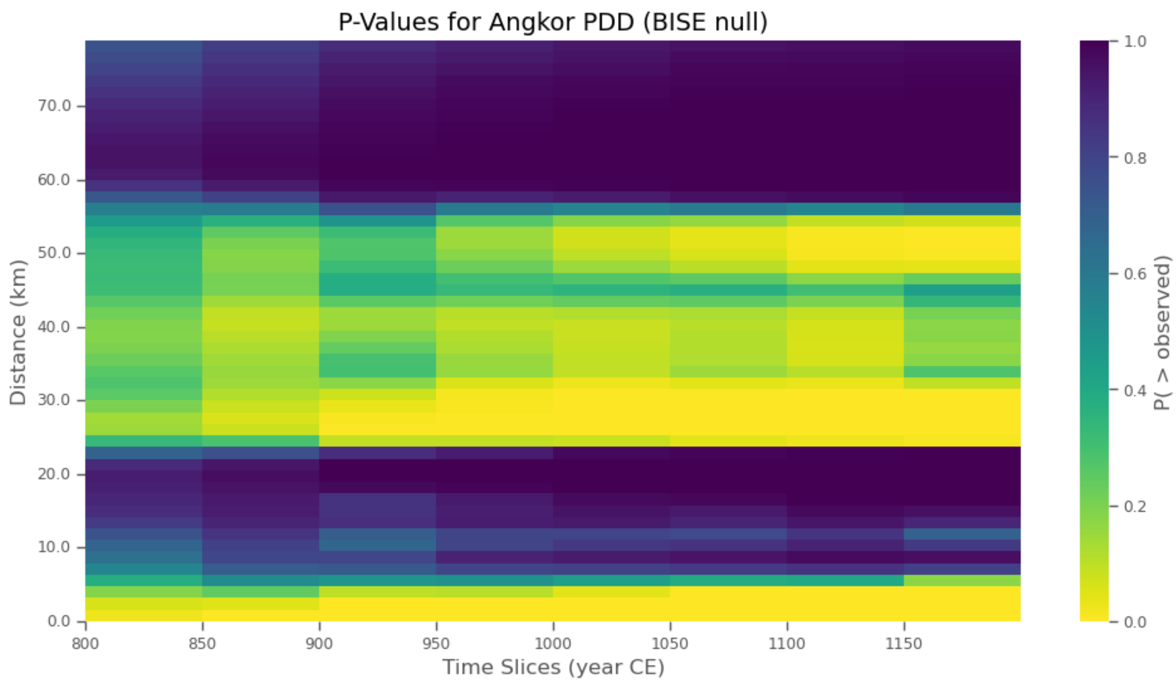
Baseline-Informed Spatial Expectation

Figure S5: Heatmap of PDD statistical significance compared to BISE based on Angkor's Temples

```
# Get MC iterations for incorporating chronological uncertainty with BISE
bise_simulations_angkor = clustering.mc_samples(points,
                                                time_slices,
                                                num_iterations = num_iterations,
                                                null_model = clustering.bise)

# Calulate the pairwise distances for the LISE sample
bise_pairwise_density_angkor, bise_support_angkor = clustering.temporal_pairwise(
    bise_simulations_angkor,
    time_slices,
    bw = pair_bw,
    use_kde = use_kde,
    kde_sample_n = kde_sample_n,
    max_distance = max_distance,
    kde_custom = kde_custom
)

# Calculate the p-values for density differences between the observed points and
# the simulated BISE baseline per distance and temporal slice
p_diff_array_bise_angkor, diff_array_bise_angkor = clustering.p_diff(
    pairwise_density_angkor,
    bise_pairwise_density_angkor
)
```

```
# Plot the heatmap of probabilities
fig, ax = pdiff_heatmap(
    p_diff_array_bise_angkor,
    time_slices,
    bise_support_angkor
)

# Custom ticks and labels here
tick_labels_km = np.arange(0, bise_support_angkor.max() / 1000, 10)
tick_labels_m = tick_labels_km * 1000
tick_positions = np.interp(
    tick_labels_m,
    bise_support_angkor,
    np.arange(len(bise_support_angkor))
)
```

```
ax.set_yticks(tick_positions)
ax.set_yticklabels(np.round(tick_labels_km, 1))
ax.set_ylabel("Distance (km)")

ax.set_xlabel("Time Slices (year CE)")
ax.set_title("P-Values for Angkor PDD (BISE null)")

plt.savefig("../Output/dpdd_hm_angkor.svg", bbox_inches='tight')
plt.savefig("../Output/dpdd_hm_angkor.png", bbox_inches='tight')
```



One Time Slice

Figure S6: Time Slice of PDD for Angkor compared to Null Models

```
from chronocluster.utils import plot_pdd

time_slice_idx = np.where(time_slices == 1000)[0][0]

# List of density arrays
density_arrays = [
```

```
    pairwise_density_angkor,
    csr_pairwise_density_angkor,
    bise_pairwise_density_angkor
]

# Generate the plot and get the figure and axis objects
fig, ax = plot_pdd(
    time_slices=time_slices,
    time_slice_idx=time_slice_idx,
    support=support_angkor,
    density_arrays=density_arrays,
    quantiles=[0.025, 0.975],
    density_names=["Empirical", "CSR", "BISE"],
    colors=["blue", "orange", "green"]
)

ax.set_title("PDD Angkor 1000 CE")

# Get current tick positions and convert labels to km
x_ticks = ax.get_xticks()
ax.set_xticklabels(np.round(x_ticks / 1000, 1))  # e.g. 1000 → 1.0 km

# Update axis label
ax.set_xlabel("Distance (km)")

# Show the plot
plt.show()

fig.savefig("../Output/pdd_null_angkor.png", dpi=300, bbox_inches="tight")
fig.savefig("../Output/pdd_null_angkor.svg", bbox_inches="tight")
```

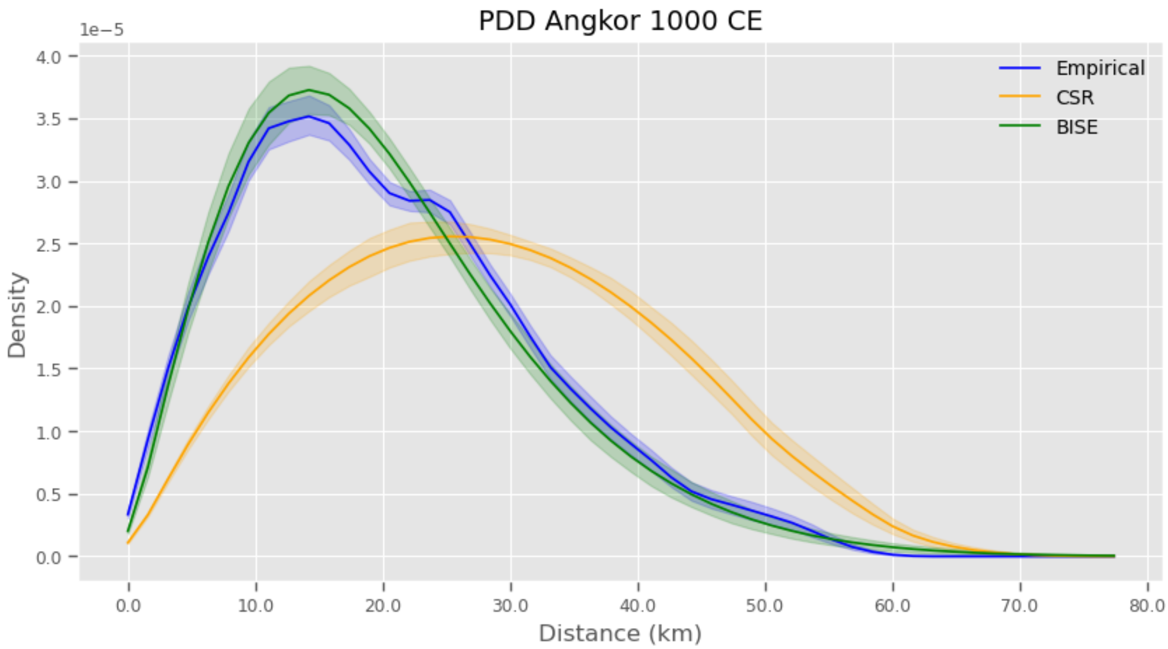
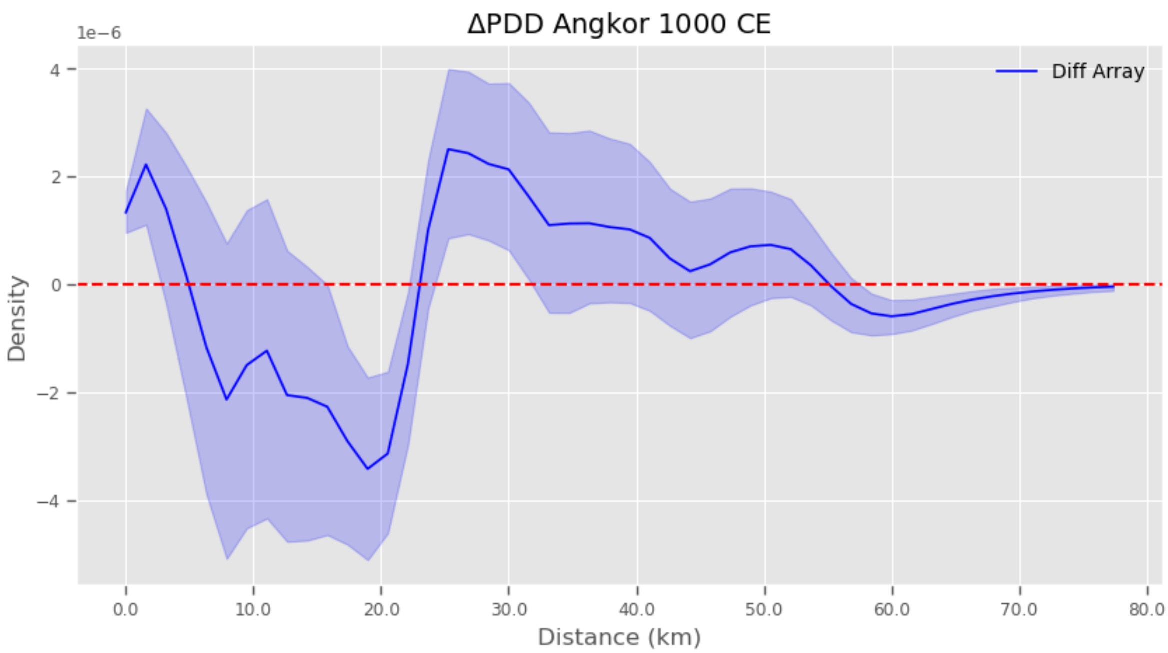


Figure S7: Difference between Angkor PDD and BISE Null at 1000 CE

```
# List of density arrays
density_arrays = [diff_array_bise_angkor]

time_slice_idx = np.where(time_slices == 1000)[0][0]

# Generate the plot and get the figure and axis objects
fig1, ax1 = plot_pdd(
    time_slices=time_slices,
    time_slice_idx=time_slice_idx,
    support=support_angkor,
    density_arrays=density_arrays,
    quantiles=[0.025, 0.975],
    density_names=["Diff Array"],
    colors=["blue"]
)

# Add a horizontal line at y=0
ax1.axhline(y=0, color='red', linestyle='--', linewidth=1.5)

ax1.set_title("$\Delta$PDD Angkor 1000 CE")
```

```python
#ax1.set_xlabel("Distance (m)")

# Get current tick positions and convert labels to km
x_ticks = ax1.get_xticks()
ax1.set_xticklabels(np.round(x_ticks / 1000, 1))  # e.g. 1000 → 1.0 km

# Update axis label
ax1.set_xlabel("Distance (km)")

# Show the plot
plt.show()

fig.savefig("../Output/dpdd_t1000_angkor.png", dpi=300, bbox_inches="tight")
fig.savefig("../Output/dpdd_t1000_angkor.svg", bbox_inches="tight")
```

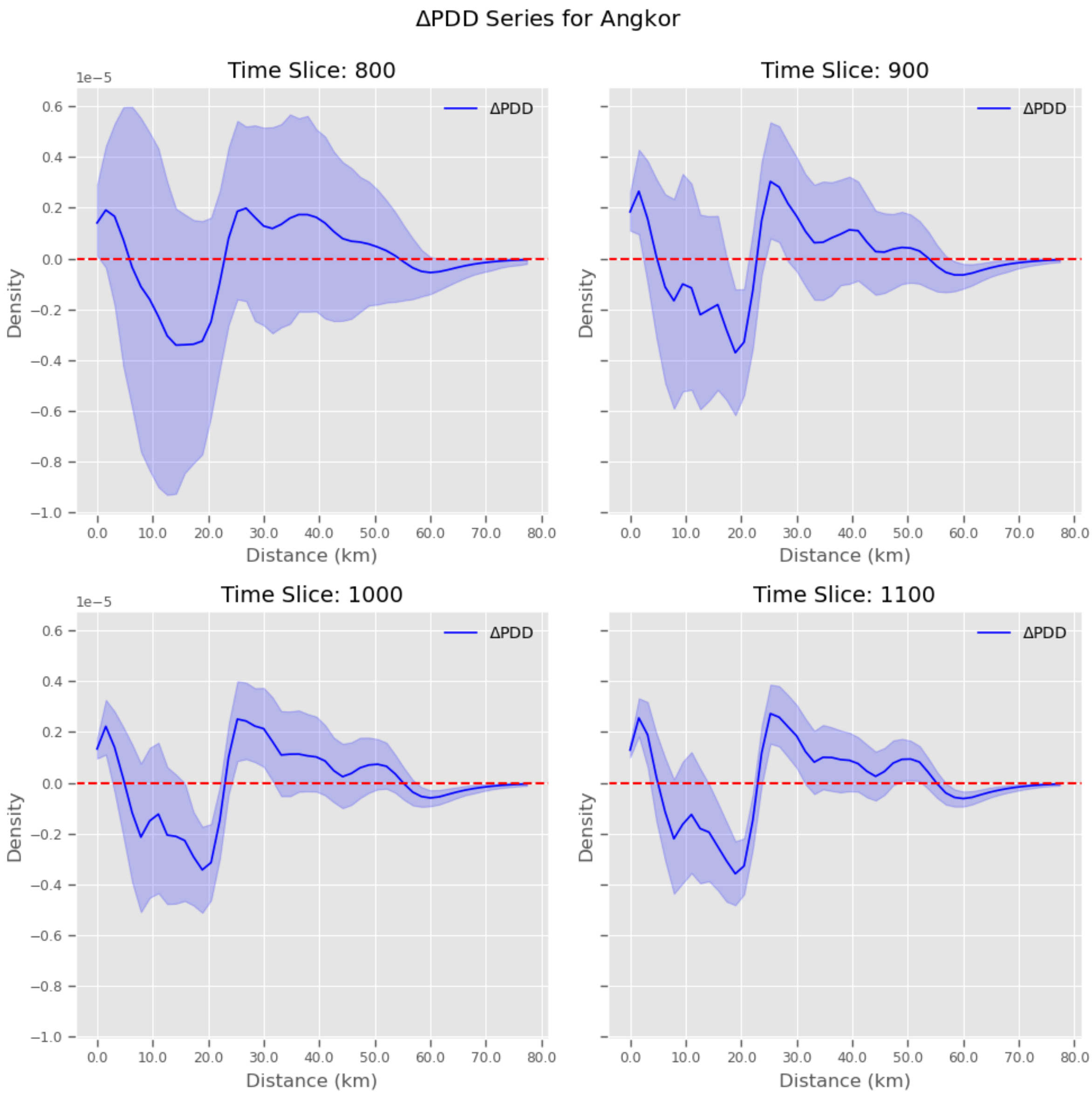


Series of Slices

Figure S8: Difference between Angkor PDD and BISE Null at 4 Time Slices

```python
# List of time_slice_idx values
time_slice_indices = [0, 2, 4, 6]

# Create a figure and axes for subplots
# Create a 2x2 grid of subplots
fig, axes = plt.subplots(2, 2, figsize=(10, 10), sharey=True)  # 2 rows, 2 columns

axes_flat = axes.flatten()

# Loop through each time_slice_idx and generate the plots
for idx, (ax, time_slice_idx) in enumerate(zip(axes_flat, time_slice_indices)):
    # Generate the plot for the current time_slice_idx
    fig, _ = plot_pdd(
        time_slices=time_slices,
        time_slice_idx=time_slice_idx,
        support=support_angkor,
        density_arrays=density_arrays,
        quantiles=[0.025, 0.975],
        density_names=["$\Delta$PDD"],
        colors=["blue"],
        ax=ax
    )

    # Add a horizontal line (optional)
    ax.axhline(y=0, color='red', linestyle='--', linewidth=1.5)

    # Add a title for each panel
    ax.set_title(f"Time Slice: {time_slices[time_slice_idx]}")
    ax.set_xlabel("Distance (m)")
    # Get current tick positions and convert labels to km
    x_ticks = ax.get_xticks()
    ax.set_xticklabels(np.round(x_ticks / 1000, 1))  # e.g. 1000 → 1.0 km

    # Update axis label
    ax.set_xlabel("Distance (km)")

# Adjust layout and show the stitched plot
plt.tight_layout(rect=[0, 0, 1, 0.95])
fig.suptitle("$\Delta$PDD Series for Angkor")
plt.show()

fig.savefig("../Output/dpdd_series_angkor.png", dpi=300, bbox_inches="tight")
```

```
fig.savefig("../Output/dpdd_series_angkor.svg", bbox_inches="tight")
```



Hampshire County at Domesday

This section processes the Domesday data for Hampshire. The data were originally taken from an online repository and processed with another notebook in this repo (doomsday.ipynb). The processed data were then saved to the repo to reduce redundant processing for use in this notebook, but for full replication, you would need to run the cells in the other notebook before proceeding further here.

Data Wrangling

```
# data wrangling
doomsday_places = pd.read_csv('../Data/doomsday_places.csv')
doomsday_places = doomsday_places.dropna(subset=['easting', 'northing'])
doomsday_places
```

	PlacesIdx	County	Phillimore	Hundred	Vill	Area	XRefs
0	1	WOR	15,8	'Doddingtree'	Abberley	NaN	NaN
1	6	ESS	20,20. 24,51. 34,16	'Winstree'	Abberton	NaN	NaN
2	11	WOR	9,1a	Pershore	Abberton	NaN	NaN
3	16	DOR	13,1	'Uggescombe'	Abbotsbury	NaN	NaN
4	21	DEV	5,6	Merton	Abbotsham	NaN	NaN
...	...	...	...	...	...	...	...
13453	73861	STS	2,22	Offlow	Yoxall	NaN	NaN
13454	73866	SUF	7,18. 44,4	'Blything'	Yoxford	NaN	NaN
13455	73871	CHS	FT1,4	Ati's Cross	Ysceifiog	Ati's Cross	NaN
13456	73876	DEV	6,3	North Tawton	Zeal Monachorum	NaN	NaN
13457	73881	WIL	64,1. 67,32	Mere	Zeals	NaN	NaN

Includes removing two problematic points in the data with a likely incorrect county labels.

```
# isolating Hampshire for comparison with Angkor
counties = ['HAM']
doomsday_df = doomsday_places[doomsday_places['County'].isin(counties)]

# I know there is a probable county designation error for the following point
# (observed in QGIS as an kind of spatial outlier surrounded by points  with a
# different designation and appears to be a duplicate point where the alternate
# one has the same county designation as the other surrounding points)

# PlacesIdx of the mislabelled point is 10221 while the alternate is 10226
drop_idx = doomsday_df[doomsday_df['PlacesIdx'].isin([10221, 30086])].index
doomsday_df = doomsday_df.drop(drop_idx)
```

Create Points List

```
doomsday_points = [
    clustering.Point(
        x=row['easting'],
        y=row['northing'],
        start_distribution = ddelta(1066),
        end_distribution = ddelta(1086)
    )
    for _, row in doomsday_df.iterrows()
]

# just double check the first ten look right
doomsday_points[:10]
```

```
[Point(x=456000.0, y=134000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=
 Point(x=458000.0, y=88000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=10
 Point(x=459000.0, y=86000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=10
 Point(x=435000.0, y=86000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=10
 Point(x=456000.0, y=83000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=10
 Point(x=447000.0, y=117000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=
 Point(x=427000.0, y=107000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=
 Point(x=458000.0, y=133000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=
 Point(x=471000.0, y=139000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=
 Point(x=460000.0, y=98000.0, start_distribution=ddelta(d=1066), end_distribution=ddelta(d=10
```

Define Time Slices and Spatial Limits

```
# Define the time slices
start_time = 1066
end_time = 1086
time_interval = 5
time_slices_ham = np.arange(start_time, end_time, time_interval)

# Get a bounding box for use later and to extract sensible distance limits
x_min, y_min, x_max, y_max = get_box(doomsday_points)
max_distance = np.ceil(np.sqrt((x_max - x_min)**2 + (y_max - y_min)**2))
```

Figure S9: Spacetime Volume of Hampshire's Estates

```
# Custom styling parameters
style_params = {
    'start_mean_color': None,  # Do not plot start mean points
    'end_mean_color': None, # Do not plot end mean points
    'mean_point_size': 10,
    'cylinder_color': (0.3, 0.3, 0.3),  # Dark grey
    'ppf_limits': (0.05, 0.95),  # Use different ppf limits
    'shadow_color': (0.4, 0.4, 0.4),  # grey
    'shadow_size': 10,
    'time_slice_color': (0.5, 0.5, 0.5),  # Grey
    'time_slice_alpha': 0.3,
    'time_slice_point_color': (0, 0, 0),  # Black
}

# Plot the points using the chrono_plot function with
# custom styling and a time slice plane
ax_stv_doomsday, fig_stv_doomsday = chrono_plot(
    doomsday_points,
    style_params=style_params,
    time_slice=1076,
    title='Hamphsire'
)
ax_stv_doomsday.set_box_aspect(None, zoom=0.85)
plt.savefig("../Output/spacetime_volume_Hampshire.svg", bbox_inches='tight')
```

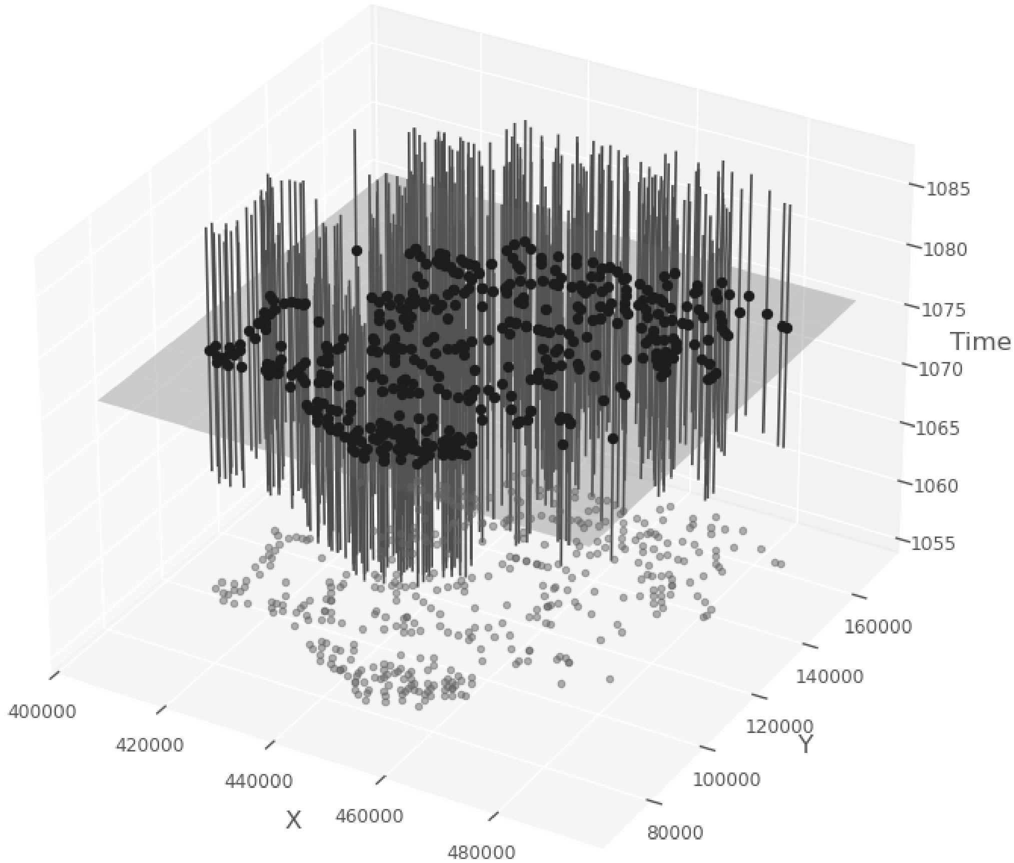


Figure S10: 2D Spatiotemporal Point Scatters of Angkor's Temples and Hampshire's Estates

```
# Style
scatter_style = {
    "point_color": (0, 0, 0),
    "point_size": 10,
}

width_mm = 90  # Example width in mm
```

```
width_inch = width_mm / 25.4  # Convert mm to inches

# Create a figure with two subplots side by side (1 row, 2 columns)
fig, (ax_2d_angkor, ax_2d_hampshire) = plt.subplots(
    1,
    2,
    figsize = (2 * width_inch, 1.15 * width_inch)
)

y_delta = 0.18e6

# Plot for Angkor
ax_2d_angkor, fig_2d_angkor = chrono_plot2d(
    angkor_points,
    time = 1000,
    style_params = scatter_style,
    crs = "EPSG:32648",
    basemap_provider = ctx.providers.CartoDB.Voyager,
    ax = ax_2d_angkor,
)

ax_2d_angkor.set_title("Angkor 1000 CE", fontsize = 10)
ax_2d_angkor.set_xlabel("Easting", fontsize = 8)
ax_2d_angkor.set_ylabel("Northing", fontsize = 8)
ax_2d_angkor.tick_params(axis = 'both', labelsize = 8)

# Plot for Hampshire
ax_2d_hampshire, fig_2d_hampshire = chrono_plot2d(
    doomsday_points,
    time = 1076,
    style_params = scatter_style,
    crs = "EPSG:27700",
    basemap_provider = ctx.providers.CartoDB.Voyager,
    ax = ax_2d_hampshire,
)

ax_2d_hampshire.ticklabel_format(style = 'sci', scilimits = (0, 0))
ax_2d_hampshire.set_title("Hampshire 1076 CE", fontsize = 10)
ax_2d_hampshire.set_xlabel("Easting", fontsize = 8)
ax_2d_hampshire.set_ylabel("Northing", fontsize = 8)
ax_2d_hampshire.tick_params(axis = 'both', labelsize = 8)
```

```
inclusion_legend(
    ax = None,
    shared = True,
    fig = fig,
    alphas = [0.2, 0.5, 0.8, 1.0]
)

# Adjust layout
fig.tight_layout(rect = [0, 1, 0, 1])

# Save figure
plt.savefig(
    "../Output/combined_inclusion_scatter.svg",
    bbox_inches = "tight",
    dpi = 300
)
plt.savefig(
    "../Output/combined_inclusion_scatter.png",
    bbox_inches = "tight",
    dpi = 300)
```

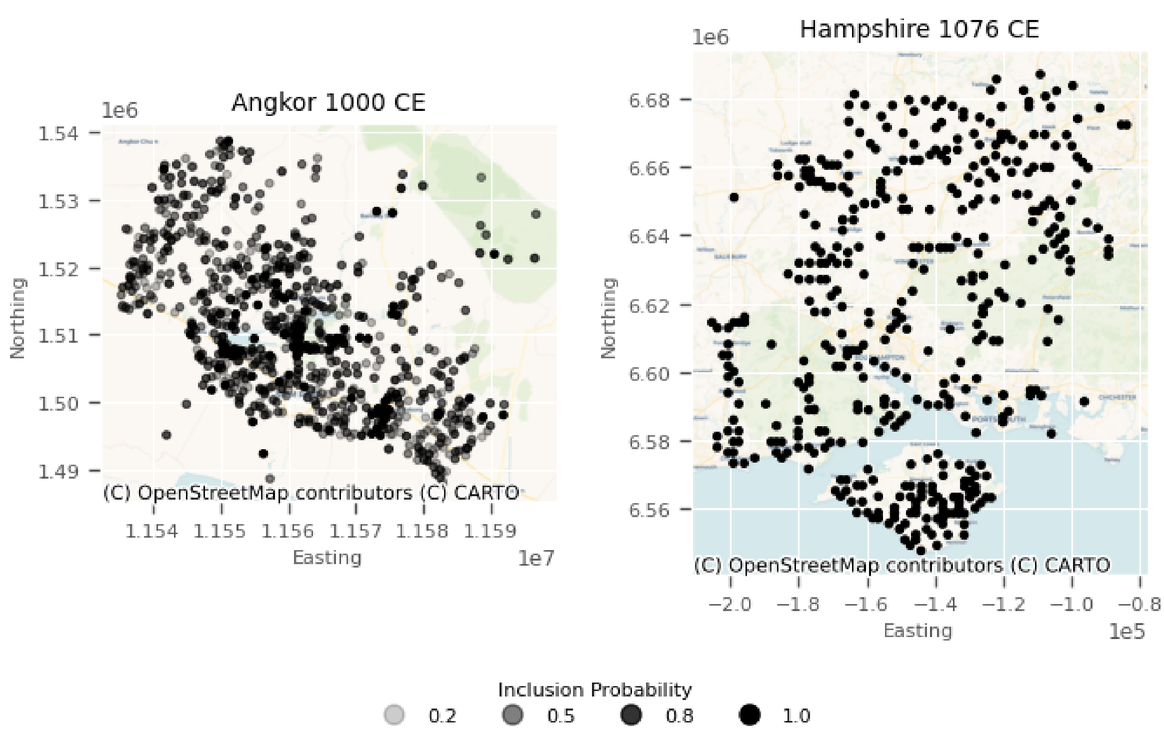
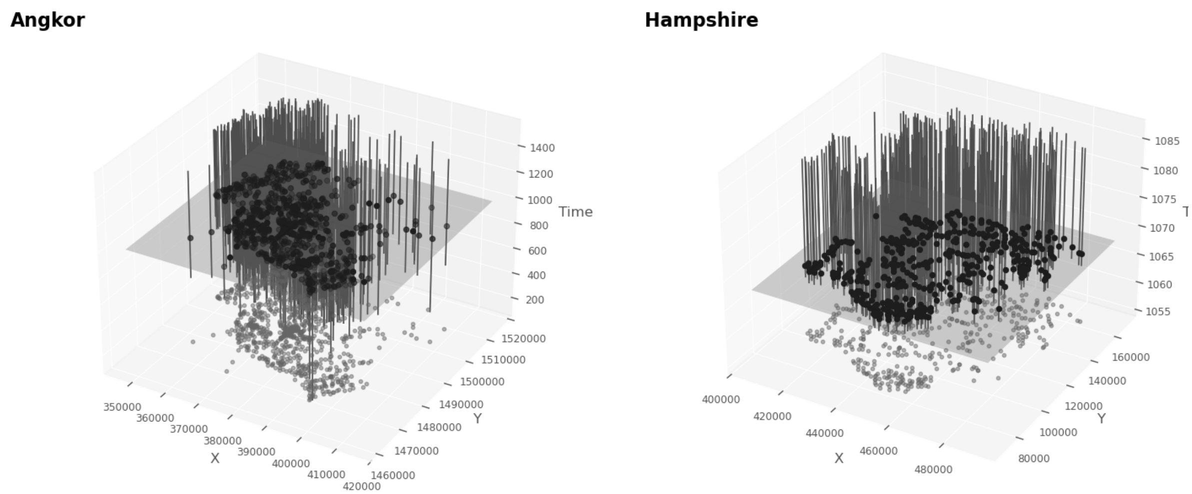


Figure S11: Spacetime Volumes for Angkor and Hampshire Combined

```
fig, axs = plt.subplots(
    1,
    2,
    figsize = (16, 6),
    subplot_kw = {"projection": "3d"}
)

# Plot both datasets into subplots
chrono_plot(
    points,
    ax = axs[0],
    style_params = style_params,
    time_slice = 1000
)
chrono_plot(
    doomsday_points,
    ax = axs[1],
    style_params = style_params,
    time_slice = 1068
)

# Add panel labels
fig.text(
    0.05,
    0.95,
    "Angkor",
    fontsize = 16,
    weight = "bold",
    transform = fig.transFigure
)
fig.text(
    0.52,
    0.95,
    "Hampshire",
    fontsize = 16,
    weight = "bold",
    transform = fig.transFigure
)

# Save combined figure
fig.tight_layout()
```

```python
# Z-axis labels (use labelpad to bring them in from the edge)
axs[0].set_zlabel("Time", labelpad = 10)
axs[0].ticklabel_format(style = 'plain', axis = 'both')  # or 'y' or 'both'
axs[1].set_zlabel("Time", labelpad = 10)
axs[1].ticklabel_format(style = 'plain', axis = 'both')  # or 'y' or 'both'


fig.savefig(
    "../Output/spacetime_volume_combined.png",
    dpi = 300,
    bbox_inches = "tight",
    pad_inches = 1.0
)
fig.savefig(
    "../Output/spacetime_volume_combined.svg",
    bbox_inches = "tight",
    pad_inches = 1.0
)
```




```python
# Run the Monte Carlo simulation to get an ensemble of probable
# lists of points included in each time slice.
simulations_ham = clustering.mc_samples(
    doomsday_points,
    time_slices = time_slices_ham,
    num_iterations = num_iterations
)
```

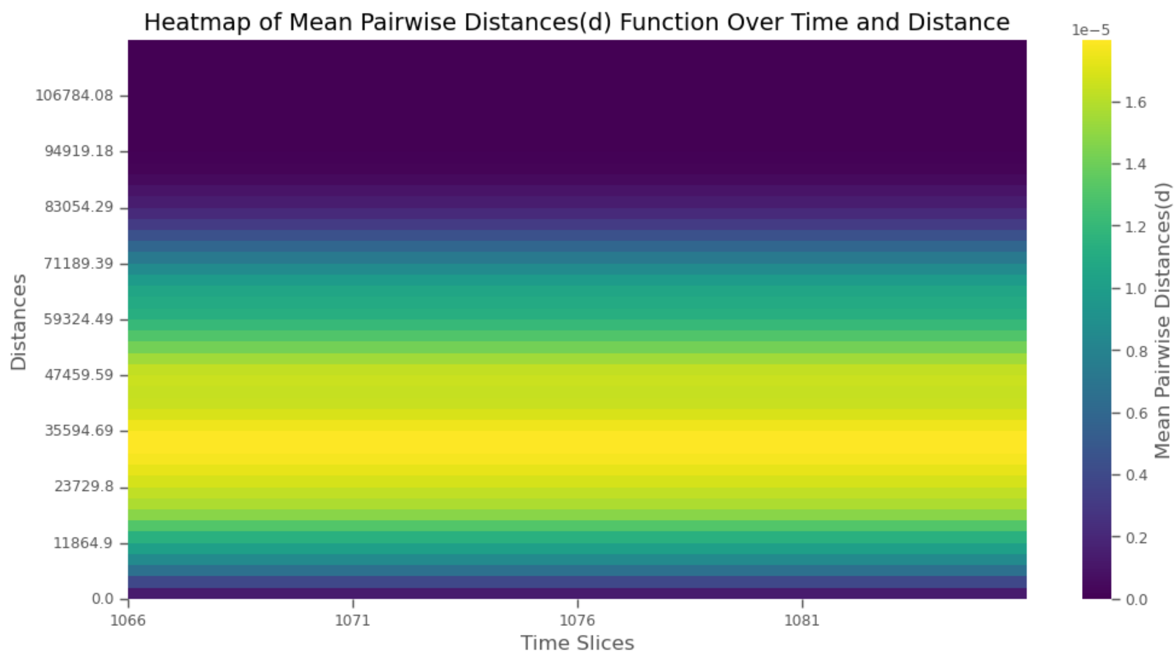
Sampling and Pairwise Distance Density Estimation

```python
# Get a bounding box for use later and to extract sensible distance limits
x_min, y_min, x_max, y_max = get_box(doomsday_points)
max_distance = np.ceil(np.sqrt((x_max - x_min)**2 + (y_max - y_min)**2))

# set consistent pairwise bandwidth (binning of distances)
# same as before with Angkor data
```

Figure S12: Heatmap of Pairwise Distance Density versus Time for Hampshire's Estates

```python
# Produce pairwise distances to explore clustering structure
pairwise_density_ham, support_ham = clustering.temporal_pairwise(
    simulations_ham,
    time_slices_ham,
    bw=pair_bw,
    use_kde = use_kde,
    kde_sample_n = kde_sample_n,
    max_distance = max_distance,
    kde_custom = kde_custom
)

# Visualize clustering with heatmap
clustering_heatmap(
    pairwise_density_ham,
    support_ham,
    time_slices_ham,
    result_type = 'Pairwise Distances'
)
```

```
(<Figure size 1200x600 with 2 Axes>,
 <Axes: title={'center': 'Heatmap of Mean Pairwise Distances(d) Function Over Time and Dista
```



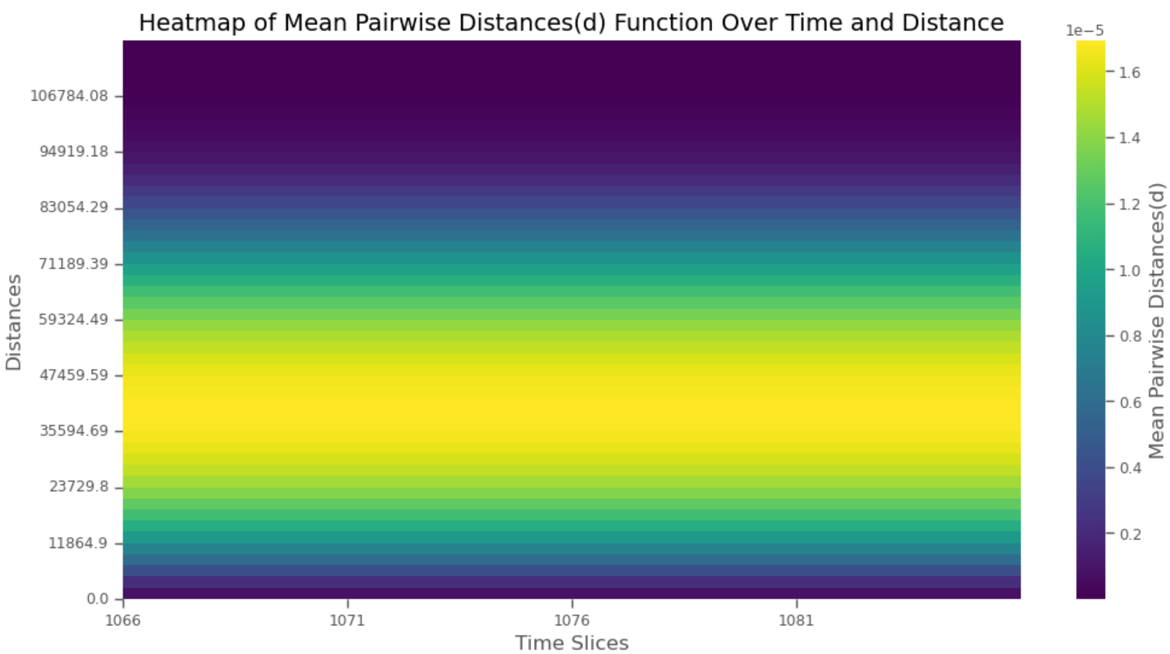
Complete Spatial Randomness

Figure S13: Heatmap of PDD versus Time of CSR for Hampshire

```
# Get MC iterations for incorporating chronological uncertainty and CSR
csr_simulations_ham = clustering.mc_samples(
    doomsday_points,
    time_slices = time_slices_ham,
    num_iterations = num_iterations,
    null_model = clustering.csr_sample,
    x_min = x_min,
    x_max = x_max,
    y_min = y_min,
    y_max = y_max
)

# Calulate the pairwise distances for the CSR sample
csr_pairwise_density_ham, csr_support_ham = clustering.temporal_pairwise(
    csr_simulations_ham,
    time_slices_ham,
    bw = pair_bw,
    use_kde = use_kde,
```

```
    kde_sample_n = kde_sample_n,
    max_distance = max_distance,
    kde_custom = kde_custom
)

# Visualize clustering with heatmap
clustering_heatmap(
    csr_pairwise_density_ham,
    csr_support_ham,
    time_slices_ham,
    result_type = 'Pairwise Distances'
)
```

```
(<Figure size 1200x600 with 2 Axes>,
 <Axes: title={'center': 'Heatmap of Mean Pairwise Distances(d) Function Over Time and Distan
```

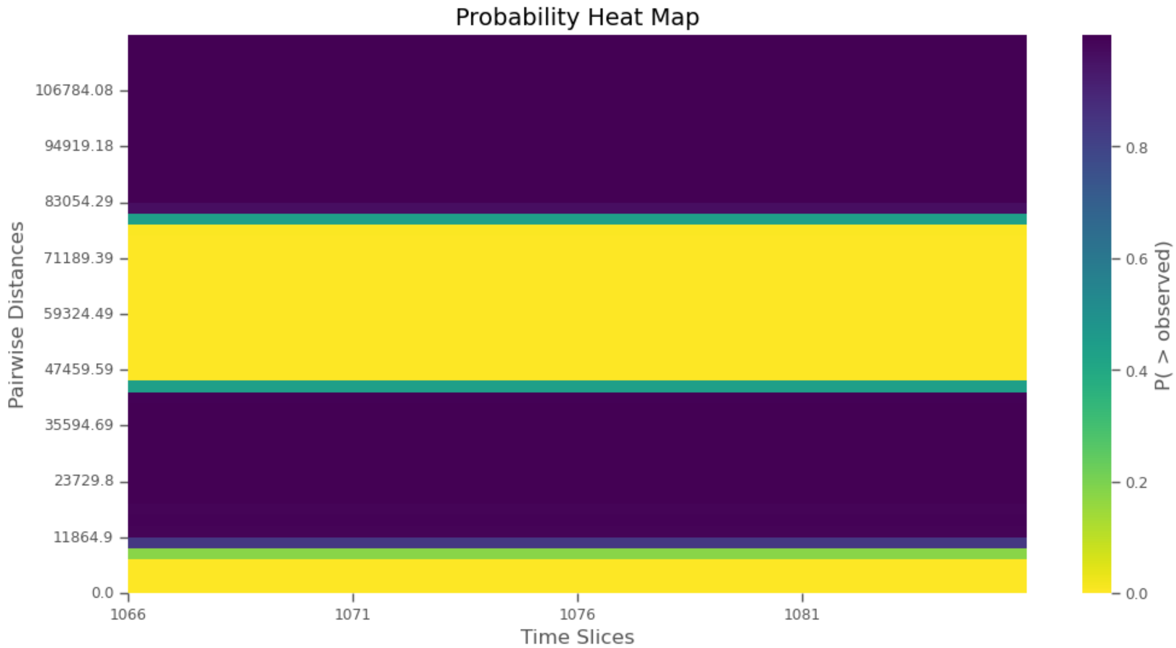


Baseline-Informed Spatial Expectation

Figure S14: Heatmap of PDD Statistical Significance compred to BISE for Hampshire

```
# Get MC iterations for incorporating chronological uncertainty with BISE
bise_simulations_ham = clustering.mc_samples(
    doomsday_points,
    time_slices_ham,
    num_iterations=num_iterations,
    null_model=clustering.bise
)

# Calulate the pairwise distances for the LISE sample
bise_pairwise_density_ham, bise_support_ham = clustering.temporal_pairwise(
    bise_simulations_ham,
    time_slices_ham,
    bw = pair_bw,
    use_kde = use_kde,
    kde_sample_n=kde_sample_n,
    max_distance = max_distance,
    kde_custom=kde_custom
)

# Calculate the p-values for density differences between
# the observed points and the simulated CSR baseline per
# distance and temporal slice
p_diff_array_bise_ham, diff_array_bise_ham = clustering.p_diff(
    pairwise_density_ham,
    bise_pairwise_density_ham
)

# Plot the heatmap of probabilities
pdiff_heatmap(
    p_diff_array_bise_ham,
    time_slices_ham,
    bise_support_ham
)
```

```
(<Figure size 1200x600 with 2 Axes>,
 <Axes: title={'center': 'Probability Heat Map'}, xlabel='Time Slices', ylabel='Pairwise Dis
```



One Time Slice

Figure S15: PDD of Hampshire Estates compared to CSR and BISE Null Models at 1066 CE

```
#from chronocluster.utils import plot_pdd
time_slice_idx = np.where(time_slices_ham == 1066)[0][0]

# List of density arrays
density_arrays = [
    pairwise_density_ham,
    csr_pairwise_density_ham,
    bise_pairwise_density_ham]

# Generate the plot and get the figure and axis objects
fig, ax = plot_pdd(
    time_slices=time_slices_ham,
    time_slice_idx=time_slice_idx,
    support=support_ham,
    density_arrays=density_arrays,
    quantiles=[0.025, 0.975],
    density_names=["Empirical", "CSR", "BISE"],
```

```
    colors=["blue", "orange", "green"]
)

ax.set_title("PDD Hampshire 1066 CE")

# Get current tick positions and convert labels to km
x_ticks = ax.get_xticks()
ax.set_xticklabels(np.round(x_ticks / 1000, 1))  # e.g. 1000 → 1.0 km

# Update axis label
ax.set_xlabel("Distance (km)")

# Show the plot
plt.show()

fig.savefig(
    "../Output/pdd_null_hampshire.png",
    dpi = 300,
    bbox_inches = "tight"
)
fig.savefig(
    "../Output/pdd_null_hampshire.svg",
    bbox_inches = "tight"
)
```

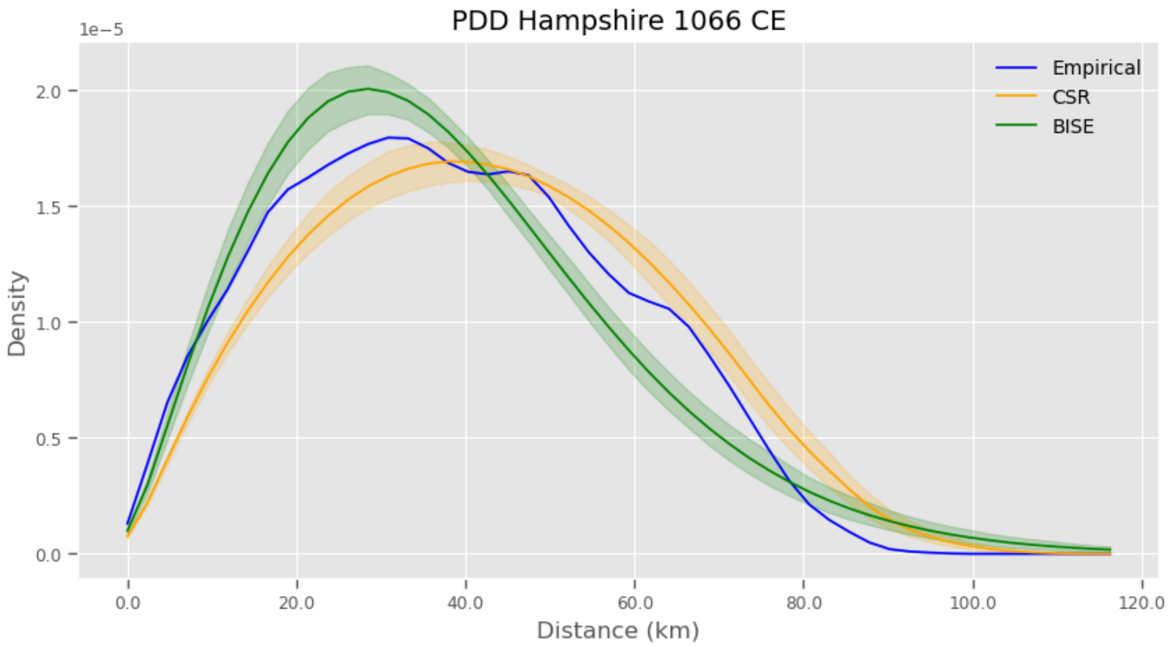


Figure S16: Difference between PDD and BISE Null Model for Hampshire's Estates at 1066 CE

```
# List of density arrays
density_arrays = [diff_array_bise_ham]

# Generate the plot and get the figure and axis objects
fig, ax = plot_pdd(
    time_slices=time_slices_ham,
    time_slice_idx=time_slice_idx,
    support=support_ham,
    density_arrays=density_arrays,
    quantiles=[0.025, 0.975],
    density_names=["$\Delta$PDD"],
    colors=["blue"]
)

# Add a horizontal line at y=0
ax.axhline(y=0, color='red', linestyle='--', linewidth=1.5)

ax.set_title("$\Delta$PDD Hampshire 1066 CE")
#ax.set_xlabel("Distance (m)")
```

```
# Get current tick positions and convert labels to km
x_ticks = ax.get_xticks()
ax.set_xticklabels(np.round(x_ticks / 1000, 1))  # e.g. 1000 → 1.0 km

# Update axis label
ax.set_xlabel("Distance (km)")

# Show the plot
plt.show()
```

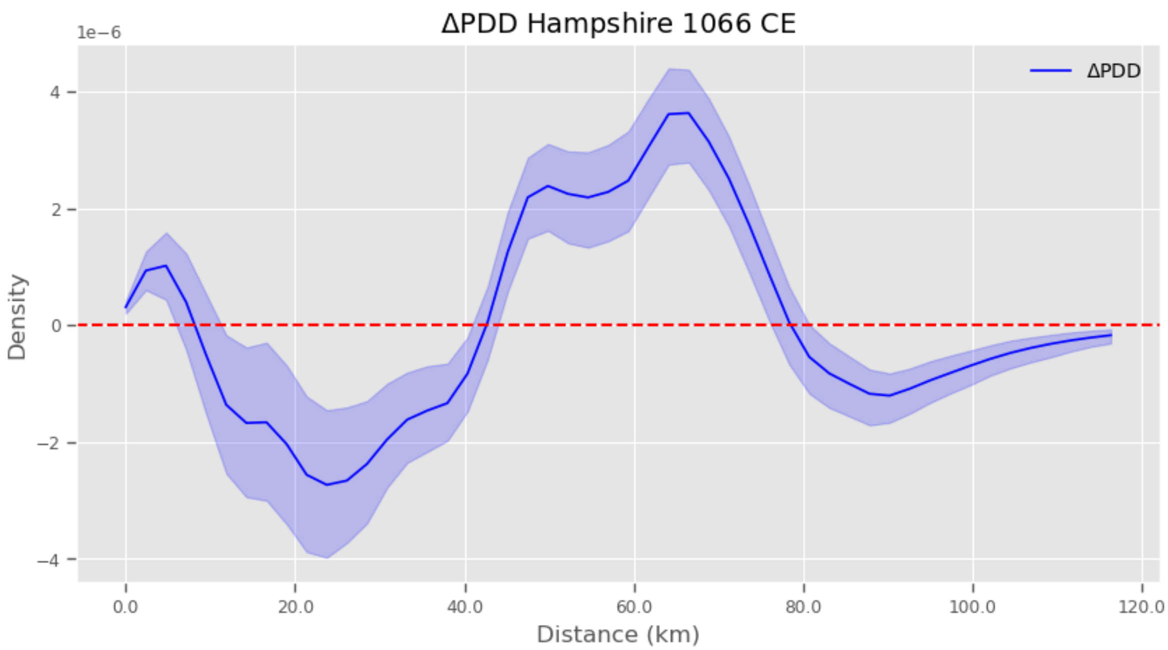


Figure S17: Δ PDDs of Angkor and Hampshire Side by Side

```
# Create a figure with two side-by-side subplots
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 5), sharex=False, sharey=True)

# First plot, Angkor
time_slice_idx = np.where(time_slices == 1000)[0][0]
plot_pdd(
    time_slices = time_slices,
    time_slice_idx = time_slice_idx,
    support = support_angkor,
    density_arrays = [diff_array_bise_angkor],
```

```
    quantiles = [0.025, 0.975],
    density_names = ["$\Delta$PDD Angkor"],
    colors = ["blue"],
    ax=ax1
)
ax1.axhline(y=0, color='red', linestyle='--', linewidth=1.5)
ax1.set_title("$\Delta$PDD Angkor")
#ax1.set_xlabel("Distance (m)")

# Get current tick positions and convert labels to km
x_ticks = ax1.get_xticks()
ax1.set_xticklabels(np.round(x_ticks / 1000, 1))  # e.g. 1000 → 1.0 km

# Update axis label
ax1.set_xlabel("Distance (km)")

# Second plot, Hampshire
time_slice_idx = np.where(time_slices_ham == 1066)[0][0]
plot_pdd(
    time_slices = time_slices_ham,
    time_slice_idx = time_slice_idx,
    support = support_ham,
    density_arrays = [diff_array_bise_ham],
    quantiles = [0.025, 0.975],
    density_names = ["$\Delta$PDD Hampshire"],
    colors = ["green"],
    ax=ax2
)
ax2.axhline(y=0, color='red', linestyle='--', linewidth=1.5)
ax2.set_title("$\Delta$PDD Hampshire")
#ax2.set_xlabel("Distance (m)")

# Get current tick positions and convert labels to km
x_ticks = ax2.get_xticks()
ax2.set_xticklabels(np.round(x_ticks / 1000, 1))  # e.g. 1000 → 1.0 km

# Update axis label
ax2.set_xlabel("Distance (km)")

# Adjust layout and show the combined plot
plt.tight_layout()
plt.show()
```

```
fig.savefig(
    "../Output/dpdd_compared.png",
    dpi = 300,
    bbox_inches = "tight"
)
fig.savefig(
    "../Output/dpdd_compared.svg",
    bbox_inches = "tight"
)
```

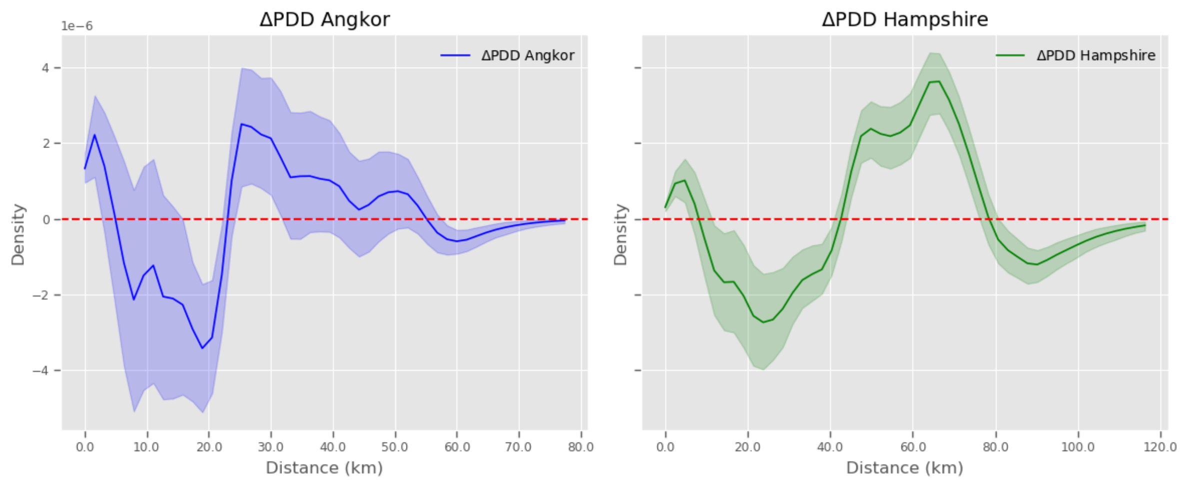
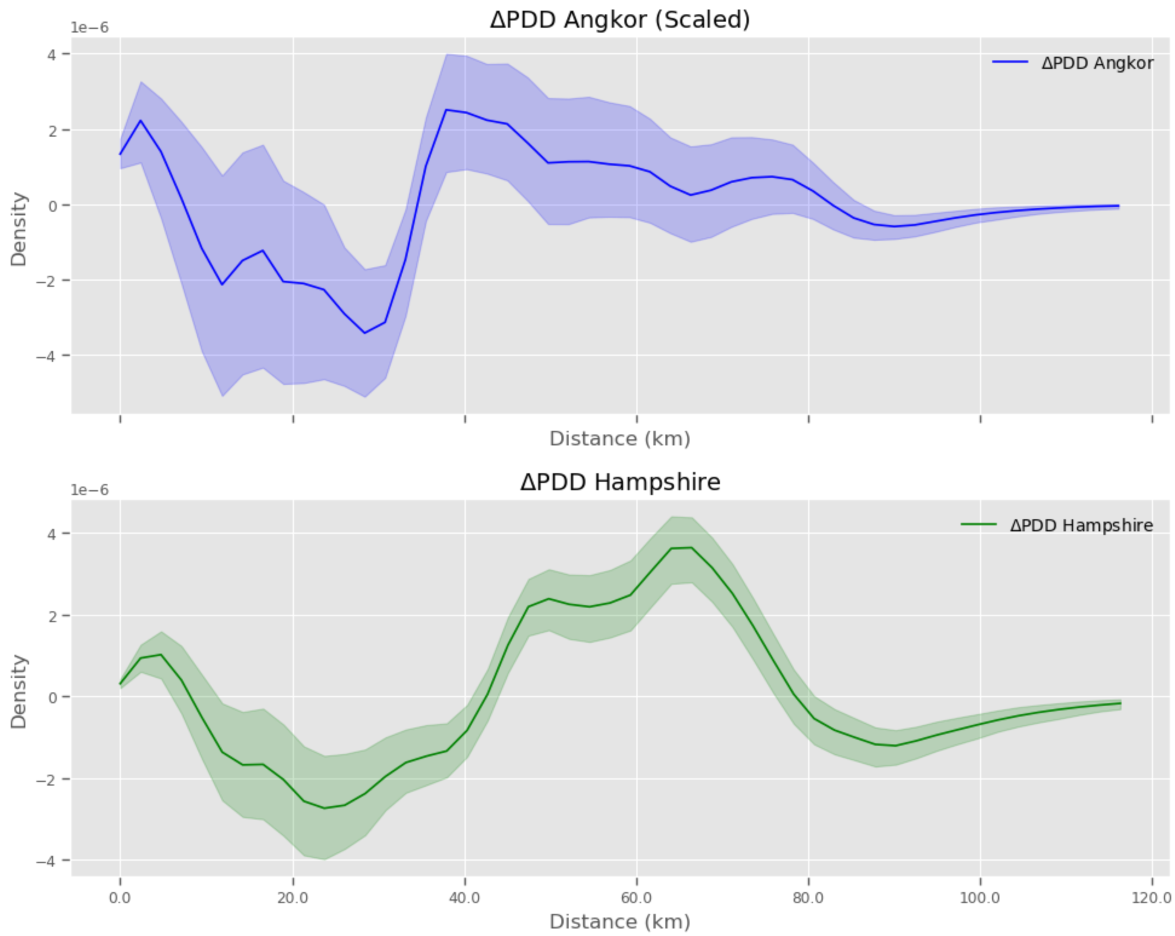


Figure S18: △ PDDs compared with Angkor's PDD Scaled

```
import numpy as np
import matplotlib.pyplot as plt

# Scaling ratio for Angkor x-axis
scaling_ratio = 1.5
scaled_support_angkor = np.array(support_angkor) * scaling_ratio

# Create a vertically stacked figure with shared x-axis
fig, (ax1, ax2) = plt.subplots(
    2, 1,
    figsize=(10, 8),
    sharex=True,
    gridspec_kw={'height_ratios': [1, 1]}
)
```

```
# First plot: Angkor (scaled x-axis)
time_slice_idx_angkor = np.where(time_slices == 1000)[0][0]
plot_pdd(
    time_slices=time_slices,
    time_slice_idx=time_slice_idx_angkor,
    support=scaled_support_angkor,
    density_arrays=[diff_array_bise_angkor],
    quantiles=[0.025, 0.975],
    density_names=["$\Delta$PDD Angkor"],
    colors=["blue"],
    ax=ax1
)
ax1.set_title("$\Delta$PDD Angkor (Scaled)")
ax1.tick_params(labelbottom=False)
#ax1.set_xlabel("Distance (m)")

# Get current tick positions and convert labels to km
x_ticks = ax1.get_xticks()
ax1.set_xticklabels(np.round(x_ticks / 1000, 1))  # e.g. 1000 → 1.0 km

# Update axis label
ax1.set_xlabel("Distance (km)")

# Second plot: Hampshire
time_slice_idx_ham = np.where(time_slices_ham == 1066)[0][0]
plot_pdd(
    time_slices = time_slices_ham,
    time_slice_idx = time_slice_idx_ham,
    support = support_ham,
    density_arrays = [diff_array_bise_ham],
    quantiles = [0.025, 0.975],
    density_names = ["$\Delta$PDD Hampshire"],
    colors = ["green"],
    ax=ax2
)
ax2.set_title("$\Delta$PDD Hampshire")
#ax2.set_xlabel("Distance (m)")

# Get current tick positions and convert labels to km
x_ticks = ax2.get_xticks()
ax2.set_xticklabels(np.round(x_ticks / 1000, 1))  # e.g. 1000 → 1.0 km
```

```python
# Update axis label
ax2.set_xlabel("Distance (km)")

# Final layout tweaks
plt.tight_layout()
plt.show()

fig.savefig(
    "../Output/dpdd_scaled.png",
    dpi=300,
    bbox_inches="tight"
)
fig.savefig(
    "../Output/dpdd_scaled.svg",
    bbox_inches="tight"
)
```



First Peaks

```
from scipy.signal import find_peaks
import warnings

def find_first_peak(pdd_slice, support):
    """
    Finds the first peak in a PDD slice.

    Parameters:
    -----------
    pdd_slice : np.ndarray
        A 1D array of PDD values for a single realization.
```

```
    support : np.ndarray
        Array of distance values (x-axis).

    Returns:
    --------
    float
        Distance (x-coordinate) of the first peak.
    """
    # Find all peaks in the PDD slice
    peaks, _ = find_peaks(pdd_slice)

    # If peaks exist, return the first one
    if len(peaks) > 0:
        return support[peaks[0]]

    # If no peaks are found, return NaN
    return np.nan

def find_all_first_peaks(diff_array, support, time_slice_idx):
    """
    Finds the first peak for all realizations in a PDD difference array and returns
    both the peak locations and their corresponding densities.

    Parameters:
    -----------
    diff_array : np.ndarray
        3D array of PDD difference values (distances x time_slices x realizations).
    support : np.ndarray
        Array of distance values (x-axis).
    time_slice_idx : int
        Index of the time slice to analyze.

    Returns:
    --------
    peaks : list
        List of first peak locations for all realizations.
    densities : list
        List of density values at the first peak for all realizations.
    """
    peaks = []
    densities = []
    num_realizations = diff_array.shape[2]
```

```
    for realization_idx in range(num_realizations):
        # Extract the PDD slice for the current realization
        pdd_slice = diff_array[:, time_slice_idx, realization_idx]

        # Find the first peak location
        peak_location = find_first_peak(pdd_slice, support)

        # if no peak, just return nan
        if np.isnan(peak_location):
            warnings.warn("No peak found.", UserWarning)
            peaks.append(np.nan)
            densities.append(np.nan)
        else:
            # Get the density value at the peak
            peak_density = pdd_slice[support == peak_location][0]

            # Append results
            peaks.append(peak_location)
            densities.append(peak_density)

    return np.array(peaks), np.array(densities)
```

Set Common Parameters

```
num_iterations = 500
use_kde = True
pair_bw = None
kde_sample_n = 100
kde_custom=cuml_kde
max_distance = 15000
```

Angkor First Peak

```
time_slice = 1100

# Run the Monte Carlo simulation to get an ensemble of probable
# lists of points included in each time slice.
simulations = clustering.mc_samples(
```

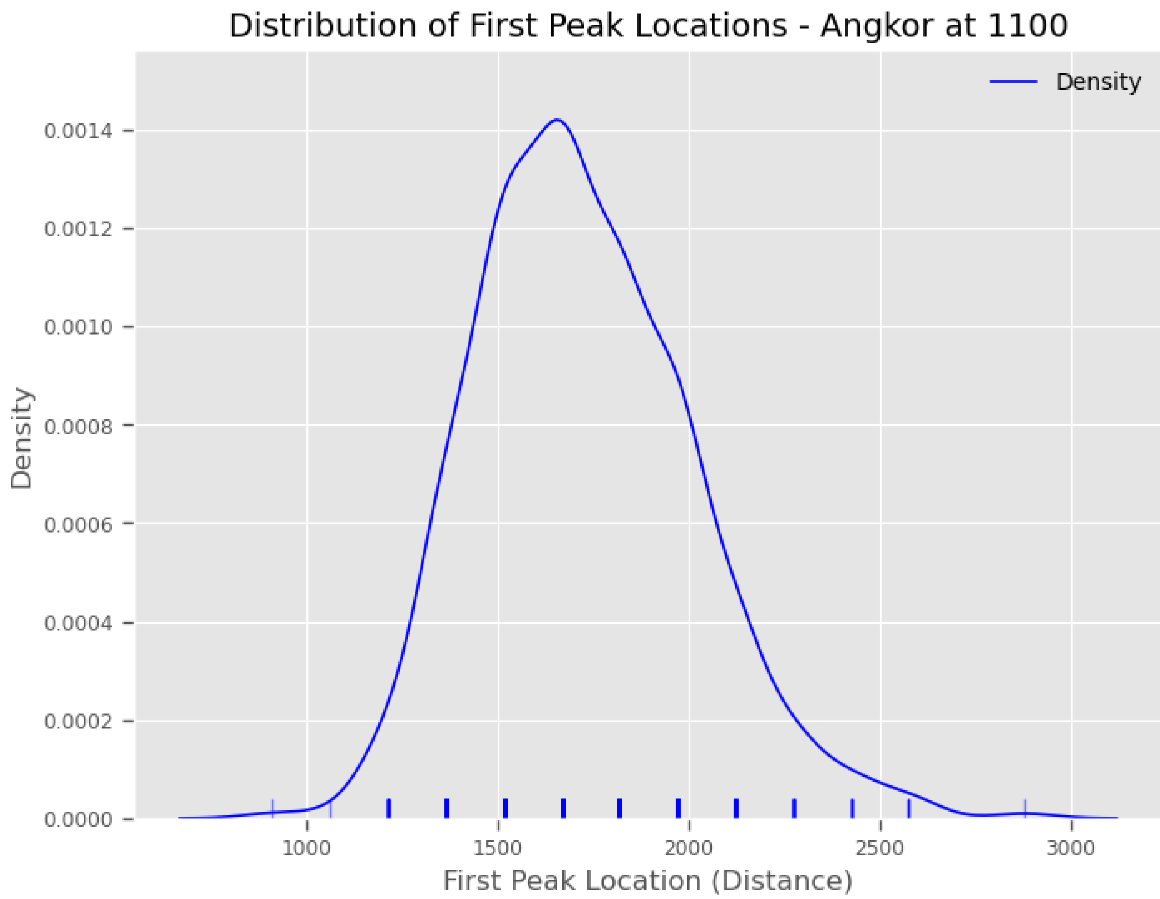
```
    points,
    time_slices=[time_slice],
    num_iterations=num_iterations
)

# Produce pairwise distances to explore clustering structure
pairwise_density_angkor, support_angkor = clustering.temporal_pairwise(
    simulations,
    [time_slice],
    bw=pair_bw,
    use_kde=use_kde,
    kde_sample_n=kde_sample_n,
    max_distance=max_distance,
    kde_custom=kde_custom
)

# Get MC iterations for incorporating chronological uncertainty with BISE
bise_simulations = clustering.mc_samples(
    points,
    [time_slice],
    num_iterations=num_iterations,
    null_model=clustering.bise
)

# Calulate the pairwise distances for the LISE sample
bise_pairwise_density_angkor, bise_support_angkor = clustering.temporal_pairwise(
    bise_simulations,
    [time_slice],
    bw = pair_bw,
    use_kde = use_kde,
    kde_sample_n=kde_sample_n,
    max_distance = max_distance,
    kde_custom=kde_custom
)

# Calculate the p-values for density differences between the observed points and
# the simulated CSR baseline per distance and temporal slice
p_diff_array_angkor, diff_array_angkor = clustering.p_diff(
    pairwise_density_angkor,
    bise_pairwise_density_angkor
)
```

```
p_pdd_peaks_angkor, _ = find_all_first_peaks(
    diff_array_angkor,
    support_angkor,
    0
)

# Convert to a Pandas DataFrame and use describe()
summary_stats = pd.DataFrame(p_pdd_peaks_angkor, columns=["Values"]).describe()

# Display the summary statistics
summary_stats
```

	Values
count	500.000000
mean	1723.939394
std	279.623577
min	909.090909
25%	1515.151515
50%	1666.666667
75%	1969.696970
max	2878.787879

Figure S19: Distribution of First Peak Locations for Angkor at 1000 CE

```
# Assuming `peaks` is your data
# Plot density and rug plot
plt.figure(figsize=(8, 6))
sns.kdeplot(p_pdd_peaks_angkor, color='blue', label="Density")
sns.rugplot(p_pdd_peaks_angkor, color='blue', alpha=0.5)

# Add labels and title
plt.xlabel("First Peak Location (Distance)")
plt.ylabel("Density")
plt.title(f"Distribution of First Peak Locations - Angkor at {time_slice}")
plt.legend()

# Show the plot
plt.show()
```



Hampshire First Peak

```
time_slice = 1066

# Run the Monte Carlo simulation to get an ensemble of probable
# lists of points included in each time slice.
simulations = clustering.mc_samples(
    doomsday_points,
    time_slices=[time_slice],
    num_iterations=num_iterations
)

# Produce pairwise distances to explore clustering structure
pairwise_density_hampshire, support_hampshire = clustering.temporal_pairwise(
```

```
    simulations,
    [time_slice],
    bw=pair_bw,
    use_kde=use_kde,
    kde_sample_n=kde_sample_n,
    max_distance=max_distance,
    kde_custom=kde_custom
)

# Get MC iterations for incorporating chronological uncertainty with BISE
bise_simulations = clustering.mc_samples(
    doomsday_points,
    [time_slice],
    num_iterations=num_iterations,
    null_model=clustering.bise
)

# Calulate the pairwise distances for the BISE sample
bise_pairwise_density_hampshire, bise_support_hampshire = clustering.temporal_pairwise(
    bise_simulations,
    [time_slice],
    bw = pair_bw,
    use_kde = use_kde,
    kde_sample_n=kde_sample_n,
    max_distance = max_distance,
    kde_custom=kde_custom
)

# Calculate the p-values for density differences between the observed points
# and the simulated CSR baseline per distance and temporal slice
p_diff_array_hampshire, diff_array_hampshire = clustering.p_diff(
    pairwise_density_hampshire,
    bise_pairwise_density_hampshire
)
```

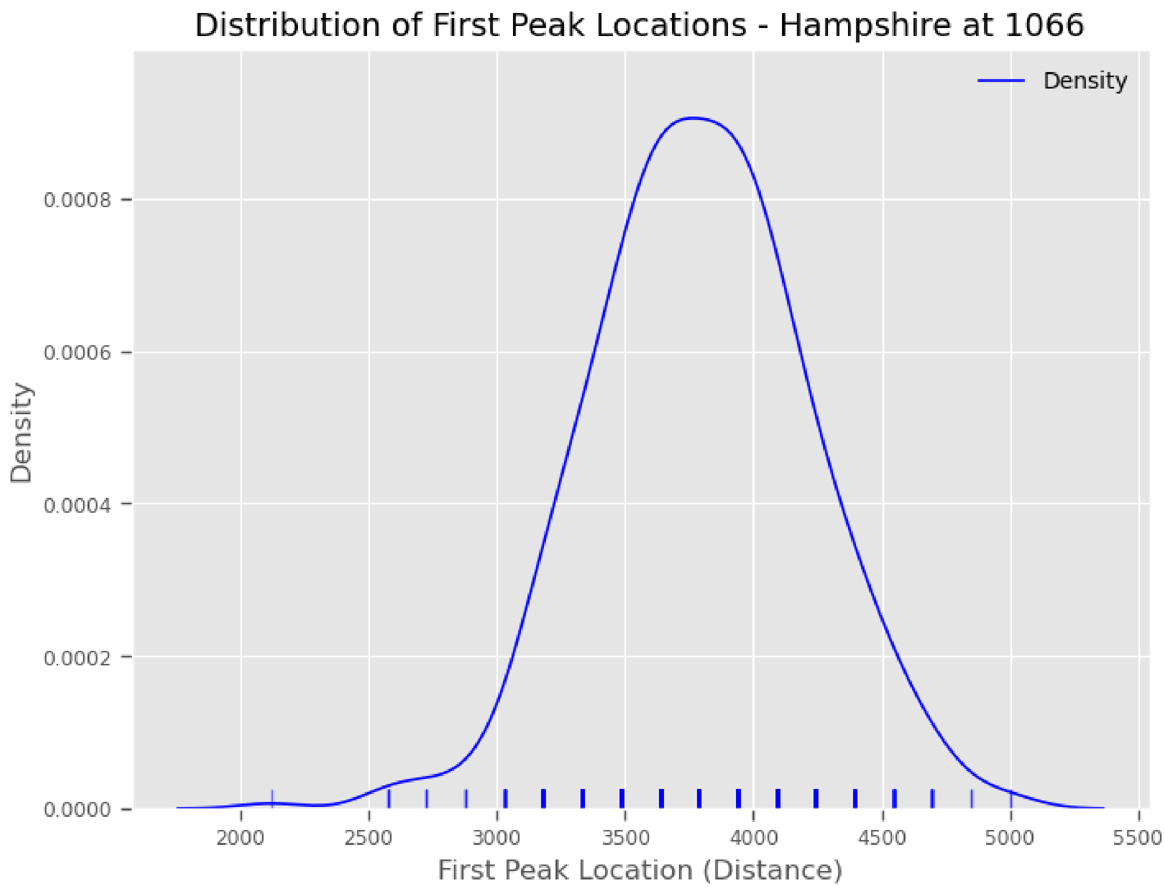
```
p_pdd_peaks_hampshire, _ = find_all_first_peaks(
    diff_array_hampshire,
    support_hampshire,
    0
)

# Convert to a Pandas DataFrame and use describe()
```

```
summary_stats = pd.DataFrame(
    p_pdd_peaks_hampshire,
    columns = ["Values"]
).describe()

# Display the summary statistics
summary_stats
```

	Values
count	500.000000
mean	3793.636364
std	419.669614
min	2121.212121
25%	3484.848485
50%	3787.878788
75%	4090.909091
max	5000.000000

Figure S20: Distribution of First Peak Locations for Hampshire at 1066 CE

```
# Assuming `peaks` is your data
# Plot density and rug plot
plt.figure(figsize=(8, 6))
sns.kdeplot(p_pdd_peaks_hampshire, color='blue', label="Density")
sns.rugplot(p_pdd_peaks_hampshire, color='blue', alpha=0.5)

# Add labels and title
plt.xlabel("First Peak Location (Distance)")
plt.ylabel("Density")
plt.title(f"Distribution of First Peak Locations - Hampshire at {time_slice}")
plt.legend()

# Show the plot
plt.show()
```

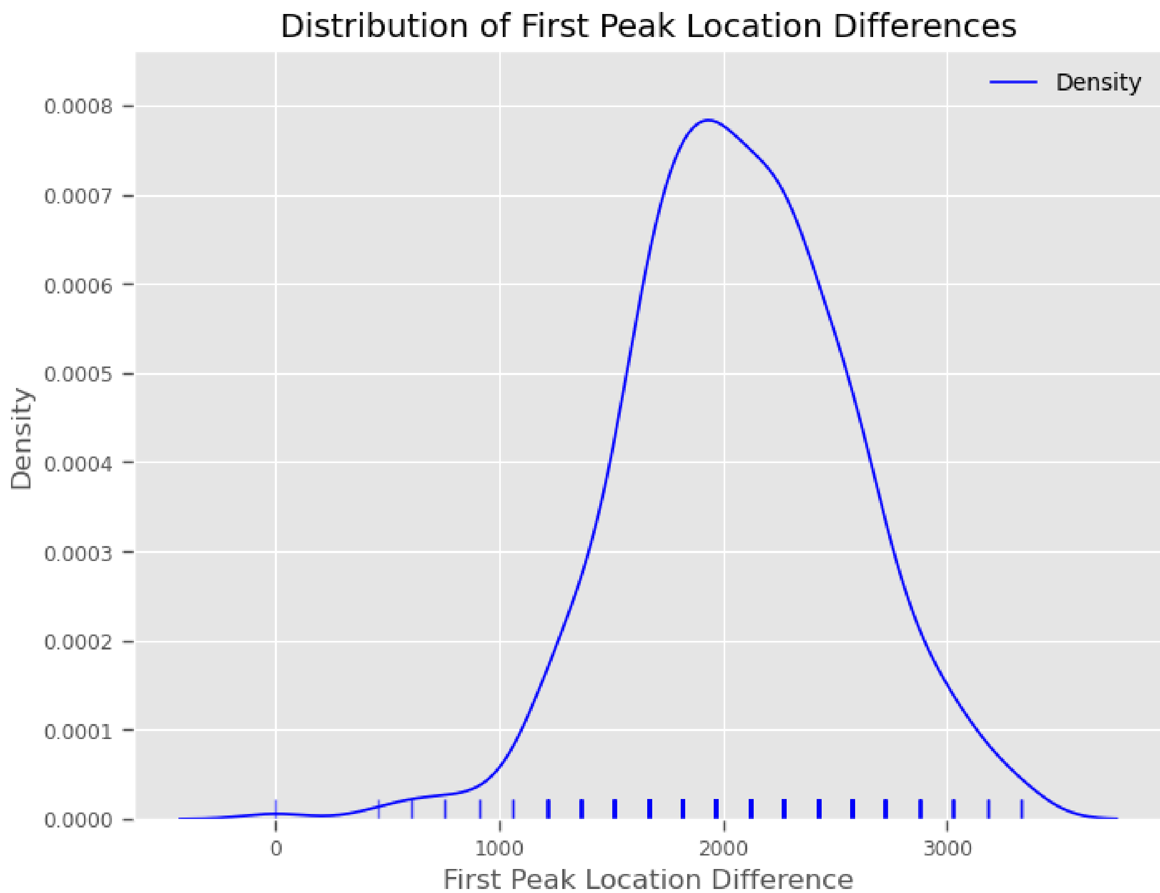


Difference Distribution

Figure S21: Distribution of First Peak Location Differences between Angkor and Hampshire

```
# Assuming `peaks` is your data
# Plot density and rug plot
plt.figure(figsize=(8, 6))
sns.kdeplot(
    p_pdd_peaks_hampshire - p_pdd_peaks_angkor,
    color = 'blue',
    label = "Density")
sns.rugplot(
    p_pdd_peaks_hampshire - p_pdd_peaks_angkor,
    color = 'blue',
    alpha = 0.5)
```

```python
# Add labels and title
plt.xlabel("First Peak Location Difference")
plt.ylabel("Density")
plt.title("Distribution of First Peak Location Differences")
plt.legend()

# Show the plot
plt.show()
```



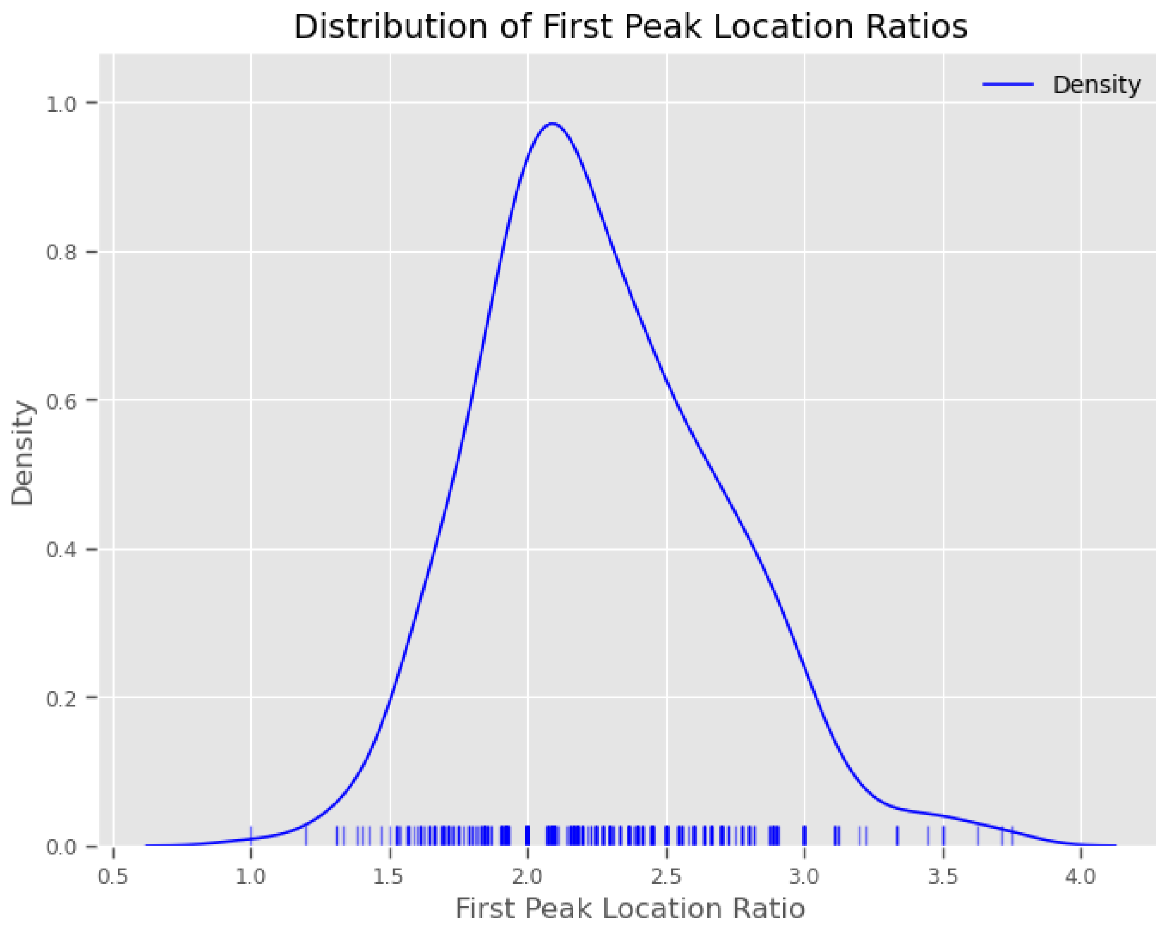

```python
# Convert to a Pandas DataFrame and use describe()
summary_stats = pd.DataFrame(
    p_pdd_peaks_hampshire / p_pdd_peaks_angkor,
    columns = ["Values"]
).describe()
```

```
# Display the summary statistics
summary_stats
```

	Values
count	500.000000
mean	2.256439
std	0.434235
min	1.000000
25%	1.927198
50%	2.181818
75%	2.545455
max	3.750000

Figure S22: Distribution of First Peal Location Ratios between Angkor and Hampshire

```
# Assuming `peaks` is your data
# Plot density and rug plot
plt.figure(figsize=(8, 6))
sns.kdeplot(
    p_pdd_peaks_hampshire / p_pdd_peaks_angkor,
    color = 'blue',
    label = "Density"
)
sns.rugplot(
    p_pdd_peaks_hampshire / p_pdd_peaks_angkor,
    color = 'blue',
    alpha = 0.5
)

# Add labels and title
plt.xlabel("First Peak Location Ratio")
plt.ylabel("Density")
plt.title("Distribution of First Peak Location Ratios")
plt.legend()

# Show the plot
plt.show()
```



Extended Analyses

```
# get unique list of counties
counties = doomsday_places['County'].unique()

# Ensure the Output directory exists
output_dir = "../Output"
os.makedirs(output_dir, exist_ok=True)

# loop over it
for j in tqdm(counties, desc="Processing Counties"):
    # isolate j
    doomsday_df = doomsday_places[doomsday_places['County'] == j]
```

```python
# create list of points
doomsday_points = [
clustering.Point(
    x=row['easting'],
    y=row['northing'],
    start_distribution = ddelta(1066),
    end_distribution = ddelta(1086)
)
for _, row in doomsday_df.iterrows()
]

# Define the time slices
start_time = 1066
end_time = 1086
time_interval = 5
time_slices = np.arange(start_time, end_time, time_interval)
time_slices

# Get a bounding box for use later and to extract sensible distance limits
x_min, y_min, x_max, y_max = get_box(doomsday_points)
max_distance = np.ceil(np.sqrt((x_max - x_min)**2 + (y_max - y_min)**2))

# Run the Monte Carlo simulation to get an ensemble of probable
# lists of points included in each time slice.
num_iterations = 500


# set consistent pairwise bandwidth (binning of distances)
use_kde = True
pair_bw = None
kde_sample_n = 50
kde_custom=cuml_kde

# Run the Monte Carlo simulation to get an ensemble of probable
# lists of points included in each time slice.
simulations = clustering.mc_samples(doomsday_points,
                                    time_slices=time_slices,
                                    num_iterations=num_iterations)

# set consistent pairwise bandwidth (binning of distances)
# same as before with Angkor data
# Produce pairwise distances to explore clustering structure
```

```
pairwise_density, support = clustering.temporal_pairwise(
    simulations,
    time_slices,
    bw = pair_bw,
    use_kde = use_kde,
    kde_sample_n = kde_sample_n,
    max_distance = max_distance,
    kde_custom = kde_custom
)

# Get MC iterations for incorporating chronological uncertainty with BISE
bise_simulations = clustering.mc_samples(
    doomsday_points,
    time_slices,
    num_iterations=num_iterations,
    null_model=clustering.bise
)

# Calulate the pairwise distances for the LISE sample
bise_pairwise_density, bise_support = clustering.temporal_pairwise(
    bise_simulations,
    time_slices,
    bw = pair_bw,
    use_kde = use_kde,
    kde_sample_n=kde_sample_n,
    max_distance = max_distance,
    kde_custom=kde_custom
)

# Calculate the p-values for density differences between the observed points and
# the simulated CSR baseline per distance and temporal slice
p_diff_array, diff_array = clustering.p_diff(
    pairwise_density,
    bise_pairwise_density
)

#from chronocluster.utils import plot_pdd
time_slice_idx = np.where(time_slices == 1066)[0][0]

# List of density arrays
density_arrays = [diff_array]
```

```
# Generate the plot and get the figure and axis objects
fig, ax = plot_pdd(
    time_slices=time_slices,
    time_slice_idx=time_slice_idx,
    support=support,
    density_arrays=density_arrays,
    quantiles=[0.025, 0.975],
    density_names=["Diff Array"],
    colors=["blue"]
)

# Add a horizontal line at y=0
ax.axhline(y=0, color='red', linestyle='--', linewidth=1.5)

# Save the plot to the Output directory
output_path = os.path.join(output_dir, f"pdd_{j}.png")

plt.savefig(output_path, dpi=300, bbox_inches='tight')

# Close the plot to free memory
plt.close(fig)
```