Veriship: Using Raw Shipment Data to Produce Forecasts by Region

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Math Problem Solving in Business

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I. Introduction

VeriShip and the Task

For this project, we worked in conjunction with a company called VeriShip, which is a shipment auditing company based out of Kansas City, Missouri. They are a consulting business hired by clients to conduct audits and provide insight and suggestions regarding best shipment practices. VeriShip not only helps recover the money their clients are owed, but also gives their clients valuable data to increase efficiency in their shipping processes, and engages them in their future shipping methods (Veriship). VeriShip provided us with partial raw data for two of its clients who ship out of the same center. Each of the two clients used the same shipping company for all of their shipments; Client A used FedEx while Client B used UPS. Our task was to transform this raw data into useful forecasts for key performance indicators, such as average shipping cost and the number of shipments being sent to different locations each year. In an early conference call, our liaison at VeriShip suggested that we first divide the country into regions and then perform our analysis for each region individually. This is because a company's key performance indicators may vary widely based on location, especially during expansion.

Key Terms

Below we define key terms that are relevant to understanding our study.

- *Base Charge:* The cost of a shipment based on the destination location and shipment type; weight is not included as a factor. This is the cost variable that VeriShip provided us.
- *Express Shipping:* A category of faster but significantly more expensive shipping options
- *Zone:* A commonly-used numeric indicator of general distance from the shipping center to a destination. An increase in zone number indicates an increase in shipping distance, with the closest zone to the shipment center being labeled "2" and the farthest being labeled "8."
- *Time Series:* Sequence of data points for a single variable equispaced over time
- *Time Plot:* Graphical representation of a time series
- *Seasonality:* Describes a time series that is affected by seasonal factors (time of year, day of the week, etc.) that is of a fixed and known frequency (Hyndman)

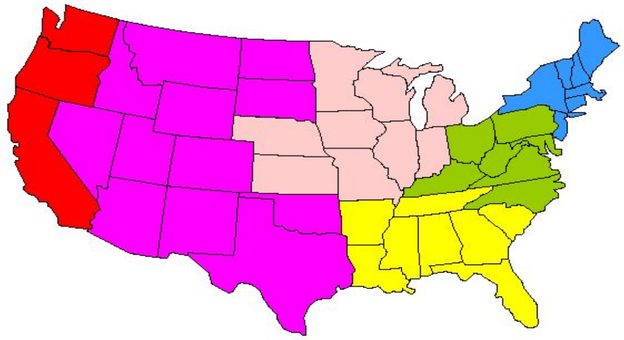
- *Interpolation:* Method of filling in missing data points in a time series
- *Stationary:* Describes a time series that lacks both seasonality and trend (Hyndman)
- *Loess:* "Method for estimating nonlinear relationships" (Hyndman)

The Data

VeriShip initially provided our group with raw data from two of its clients, resulting in data for approximately 140,000 shipments in total. For one of the clients, Client A, the shipment data spanned approximately two years, from early 2017 to early 2019. For the other, Client B, data spanned less than a year, from mid-early 2017 to early 2018. Client A exclusively used FedEx for their shipping services, while Client B exclusively used UPS. For each shipment, the relevant variables provided were the date, the destination state, the zone, the service type, and the base charge. For any given day there could be close to a hundred shipments or none at all. Almost all of the days where there were no shipments were due to weekends or holidays. Some of the data appeared to be inconsistent. Specifically, there were cases where shipments sent on the same day to the same location had significantly different zones listed. We believed this problematic data to be caused by shipment returns that were also recorded. We were able to scan through the data and identify and remove the shipments that appeared to be returns.

Regions

In order to make our analysis more useful, we divided the country into geographic regions and performed most of our analysis based on these separate regions rather than analyzing data for the United States as a whole. The logic behind this was that some regions may perform differently than others in respect to the amount of shipments these regions receive and the cost of those shipments. For instance, the west coast's consumer base might be growing more rapidly than the Southeast's, which could also affect the relative costs in these regions. Henceforth, analyzing these regions separately helped to minimize the otherwise skewed data that might result from a nationwide analysis. We divided the country into regions that had roughly similar populations while also ensuring that the states in each region were geographically close to each other. We used the population data from the 2017 U.S. Census Bureau to create six different regions: Southeast, Mid-Atlantic, Northeast, Midwest, West, and Pacific Coast. Below is a map highlighting the different regions along with a table of their populations and average zones.



Region	Population (millions)	Average Zone
Northeast	43.4	2.1
Mid-Atlantic	57.4	3.5
Midwest	55.0	5.0
Southeast	59.2	5.5
West	58.8	7.2
Pacific	51.3	8.0

Figure 1. Map of the U.S. with regions; table displaying populations and average zones (US Census Bureau).

Note that each region has a fairly distinct average zone, and the average zone increases as one moves farther away from the Northeast region. This implies that the distribution center for our data is located in the Northeast.

II. Methods

Time Series Construction

Because of the nature of the data and the analysis that we wanted to complete, we needed a powerful tool both for organizing and analyzing our data as well as performing complex statistical functions. Therefore, our team spent a significant amount of time at the beginning of our research learning the computing and statistical language R. In Appendix A we have included most of the code for R that we utilized in order to obtain our results. R is a free computing tool that can be downloaded online. Excel spreadsheets are able to be easily imported into R using the "Import Dataset" –> "From Excel…" tab in R Studio. In our use of R we consistently used the packages `dplyr`, `lubridate`, `fpp2`, `readxl`, and `ggplot`. We recommend loading all of these using the command `library()` at the beginning of each R session.

The first step in analyzing our data was creating time series for the different variables and regions. The main key performance indicators that we were interested in analyzing were the shipment count per day in each region and the daily average base charge for each region. In R, we first separated the data by state, and then combined the individual state data to create new data tables for each individual region. In Appendix A1 is an example of the code for separating the data for Iowa (IA) and then creating the dataset for the Mid-Atlantic.

Then we aggregated the data by day for the daily average base charge and the daily shipment count for each region. For the sake of efficiency we used acronyms for the different regions, i.e. MA=Mid-Atlantic. This code is found in A2.

Once the data was aggregated by date, in order to have equispaced time series we needed to interpolate data for the missing dates. We were able to perform this using the `zoo` package in R and chose linear approximations for the interpolation both for the base charge and the daily count. Once the values had been interpolated, we were able to store our data as time series objects (`ts`) in R. The code for interpolation and creating a `ts` object is in A3.

Now that we had equispaced time series, we needed to account for outliers in our data. This was especially relevant for the daily average base charge time series, as the average base charge for a given day could be misleading if only a few shipments had occurred on a given day and one of them was Express Shipping. We used an function in R called `tsclean` that automatically identified and replaced outliers with linear approximations. This code is found in A4. R identifies the outliers by fitting a Loess curve and then identifying any points that lay outside twice the range of the middle 80% of the residuals (Hyndman).

Time Series Analysis

Once our time series had been constructed, we were able to begin our actual analysis. The first step was to simply create time plots. In A5 is the code for creating the time plots for the daily average base charge; the code for the daily counts is similar. The time plots for daily average base charge and daily count are found below.

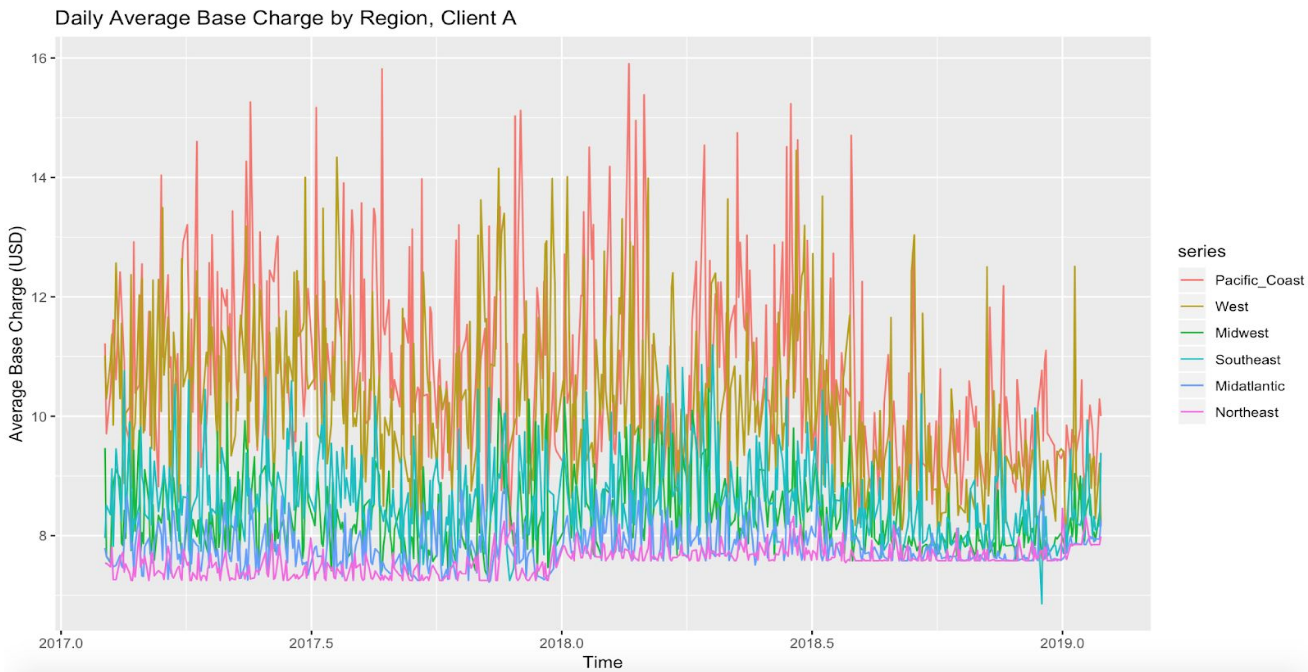


Figure 2. Time plot of Client A's daily average base charge for each region.

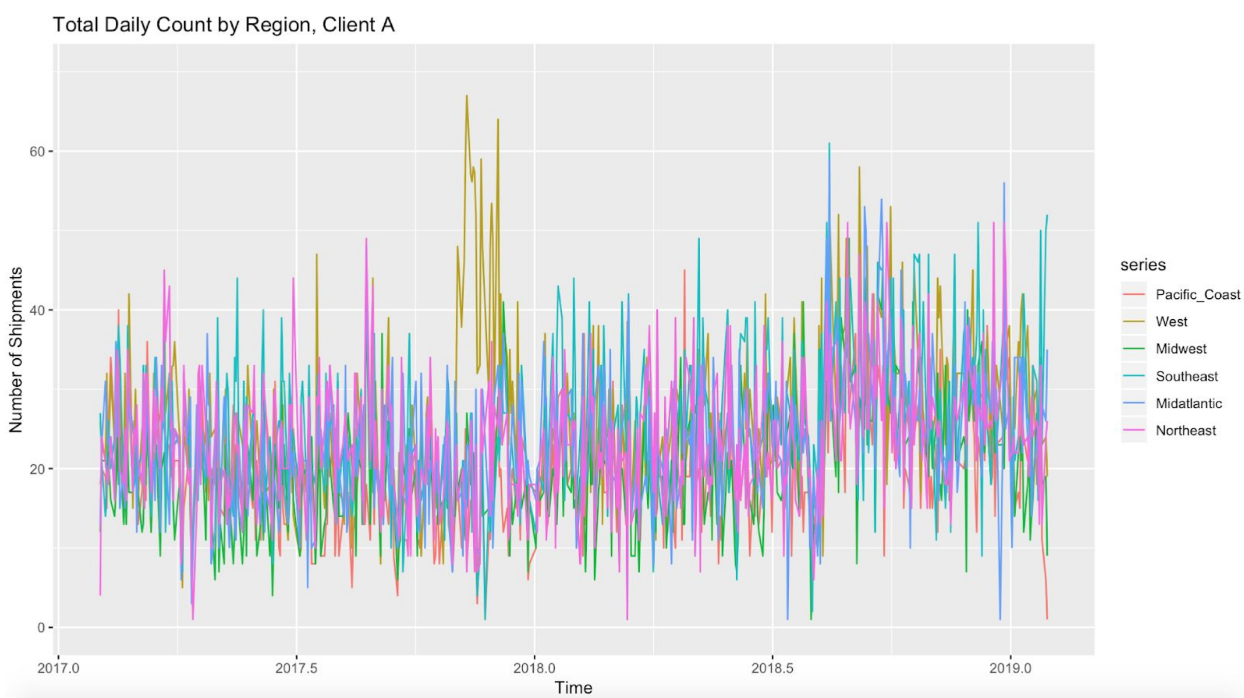


Figure 3. Time plot of Client A's total daily count for each region.

It was clear that the daily average base charge and the variance of each of these series increased by region directly with the increase in average zone per region. There did not appear to be any noticeable difference by region for the daily shipment count. The time plots for Client B were nearly identical, so we did not include them in this report, but they can be found in Appendix B. By a visual analysis most of the time series seemed to be approximately stationary; however, there were some interesting series, particularly the Mid-Atlantic and Northeast daily average base charge time series. These series appear to be stationary during a given year, but there is a sharp increase at the start of each year. This may indicate a yearly price increase that goes into effect at the beginning of each year in these two regions.

In order to present a more detailed description of our analysis, the remainder of the methods section will focus only on the time series for the daily average base charge for the Mid-Atlantic region from Client A.

Autocorrelation and Seasonality

We applied a number of methods to analyze our time series data before attempting to create forecasts, one of the most useful of which was the autocorrelation function (ACF). The ACF measures the correlation between a value and lagged values of that same variable. In other words, it finds the linear relationship between observations of a single variable at different relative times (Hyndman). The autocorrelation function can be written as:

$$r_k = \frac{\sum_{t=k+1}^{T} (y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{T} (y_t - \bar{y})^2} \tag{1}$$

where r_k is the autocorrelation at lag *k.* This formula can be thought of as being the covariance of the original value and the lagged value divided by the variance of the original value (Taboga). Autocorrelation is one of the main tools used for identifying seasonality in a time series. The code for the Mid-Atlantic Base Charge is in A6, and the resulting plot is found below.

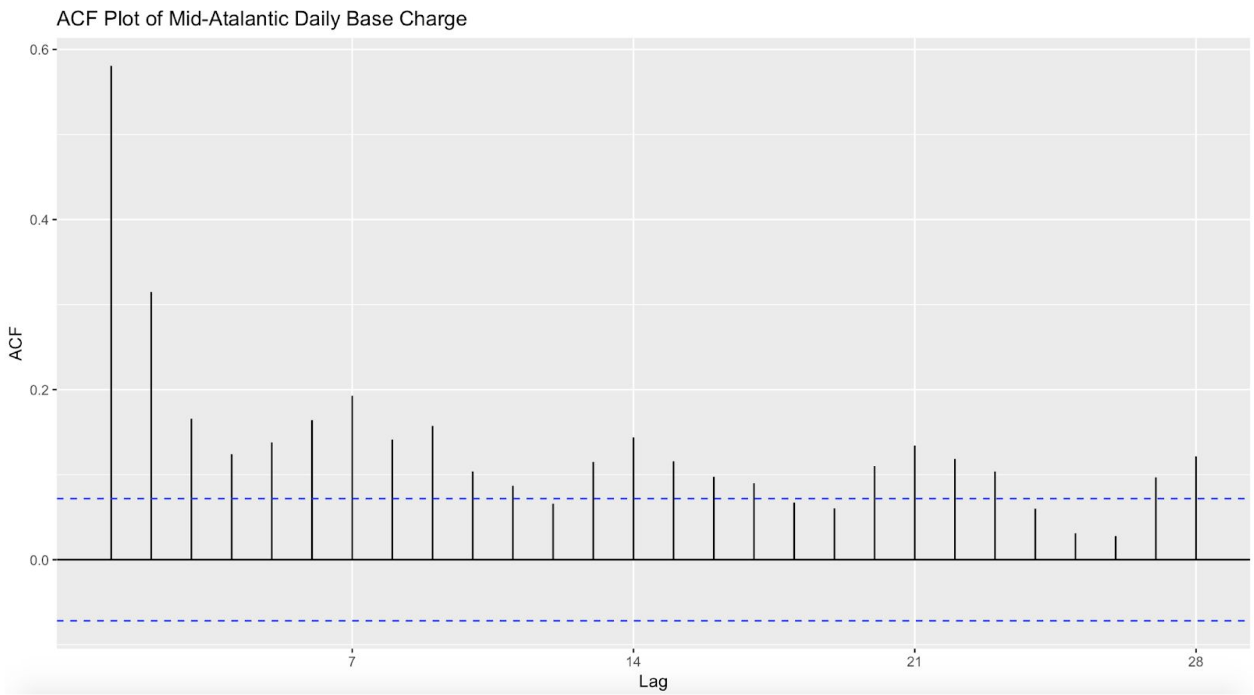


Figure 4. ACF plot of Client A's daily base charge in the Mid-Atlantic region.

The blue-dashed lines in the ACF plot represent that a value falling within them is insignificant with 95% confidence. As we can see in the ACF plot above, most of the values at least up to lag 28 are significant. We can also see that there is a recurring pattern with spikes occurring at lags multiples of 7; this implies that this time series has weekly seasonality, although the seasonality does not appear to be very strong based on the generally low ACF values.

ARIMA Modeling

Now that we had created and analyzed our time series, we were ready to decide on and implement a forecasting model. We decided on the ARIMA(*p,d,q*) model, as it is able to handle time series both with and without seasonality, and it seems to be one of the more effective models for extracting useful information about a time series and using it to create forecasts (Roopam). The ARIMA (*p,d,q*) model is made up of three distinct parts: **A**uto **R**egression, **I**ntegration, and **M**oving **A**verage. The order of each of these parts corresponds to *p, d,* and *q* respectively. The intended goal of creating an ARIMA model is to create a best fit model of our actual time series, where the residuals from our best fit model and the original are white noise. In other words, we want to extract all the useful information from the original time series to make our forecast, and this is done by effectively choosing the parameters *p, d,* and *q* (Roopam).

The first of the three steps in the ARIMA model is integration of order *d.* Integration is intended to extract seasonality and trend from the time series, so an integrated series should be stationary. The most common method of integration is differencing, which is performed by subtracting each value by the previous value:

$$y'_t = y_t - y_{t-1} \tag{2}$$

Usually, differencing is necessary only once, and therefore *d*=1, but occasionally second-order differencing is required, in which case *d*=2 (Hyndman).

In comparing the original Mid-Atlantic daily average base charge time plot to the differenced time series, we can see why differencing is often necessary for the ARIMA model to be more effective. The code for this is found in A7.

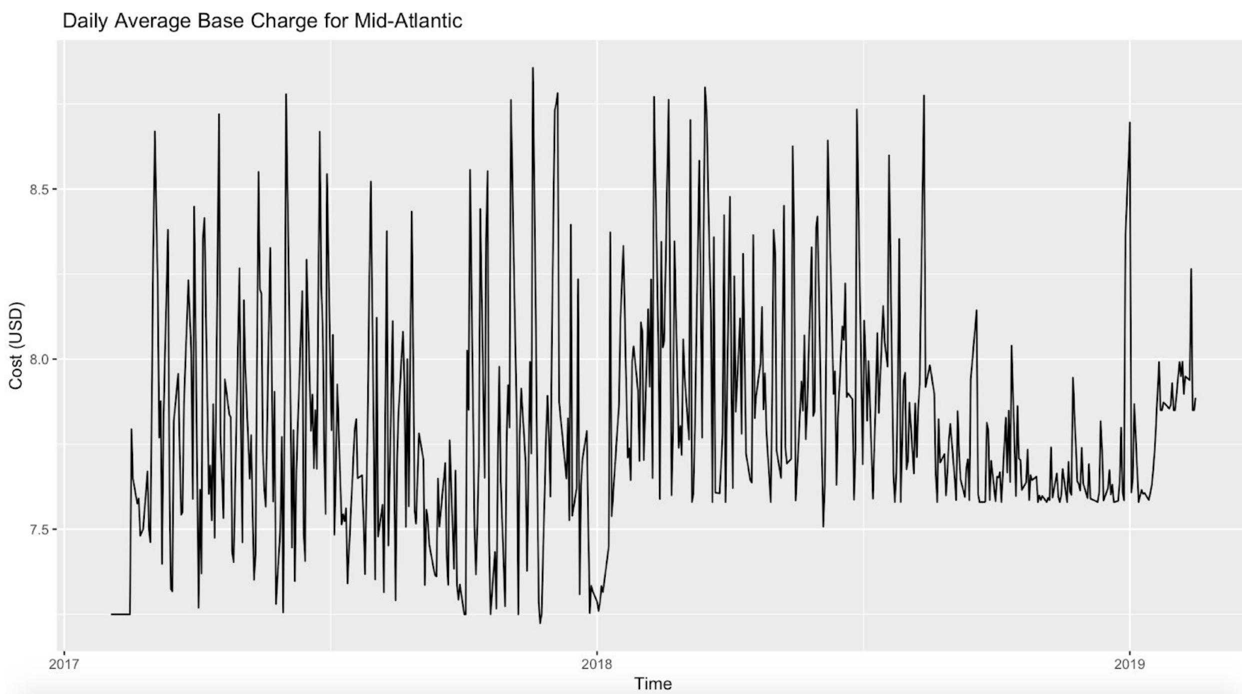


Figure 5. Time plot of Client A's daily average base charge in the Mid-Atlantic region.

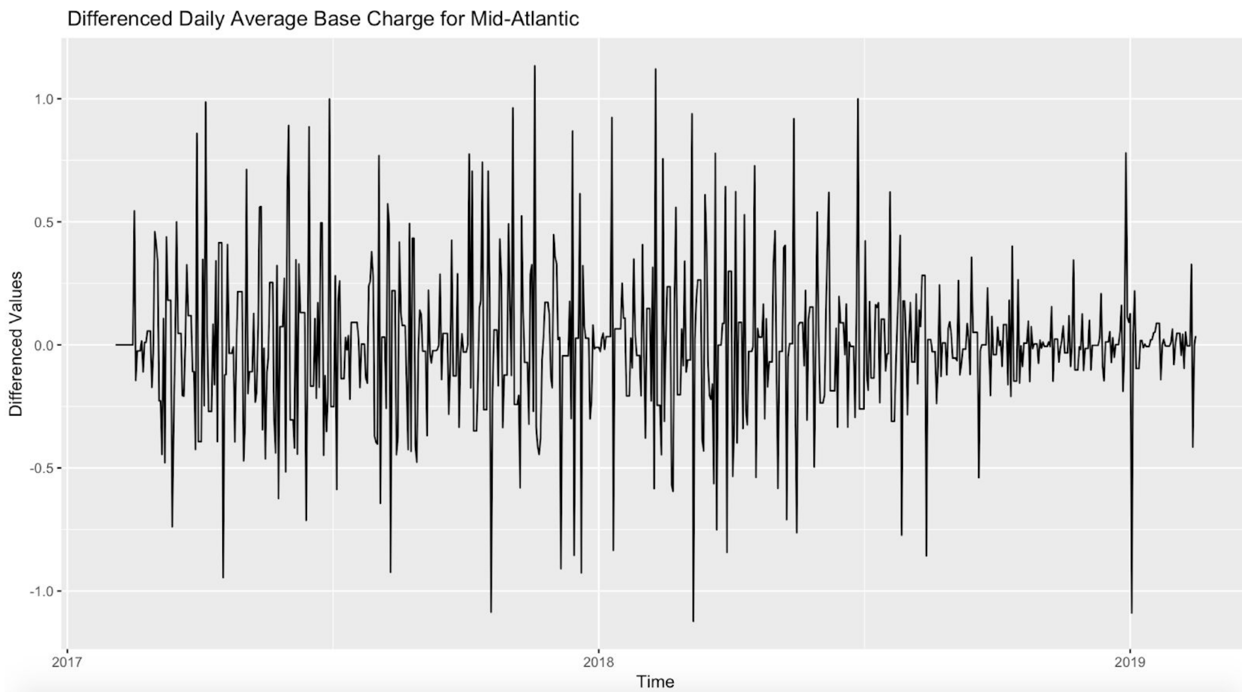


Figure 6. Time plot of Client A's differenced daily average base charge for the Mid-Atlantic region.

As stated before, in the original time series at the beginning of each year there is a sharp increase in the daily average base charge that affects the base charge throughout the whole year. So if we were using the original series as seen in Figure 5 to create a forecast, there would be additional variance from 2017 and 2018 affecting the forecast that would likely not be representative of what will actually happen in 2019. However, when we difference the series as shown in Figure 6, the additional and unnecessary variance is removed, and this allows us to focus on only the relevant relationships in the data when creating our best fit model.

Once the series has been made stationary, the next step is autoregression of order *p* on the differenced series. The model performs an autoregression on *p* previous terms in order to extract the influence of previous values on the current value (Roopam). In other words, we are taking "a regression of the variable against itself" (Hyndman).

$$y'_t = c + \varphi_1 y'_{t-1} + \varphi_2 y'_{t-2} + \ldots + \varphi_p y'_{t-p} + \varepsilon_t \tag{3}$$

With c=mean, φ_i = coefficient, and ε_t =error term.

Lastly, the model performs a moving average of order *q.* A moving average model assumes the current value is dependent on the previous and current error terms, so we use this to extract the influence of the errors on the current term (Hyndman). It is created using "multiple linear regression values with the lagged error values as independent or predictor variables" (Roopam).

$$y'_t = c + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \ldots + \theta_q\varepsilon_{t-q} \tag{4}$$

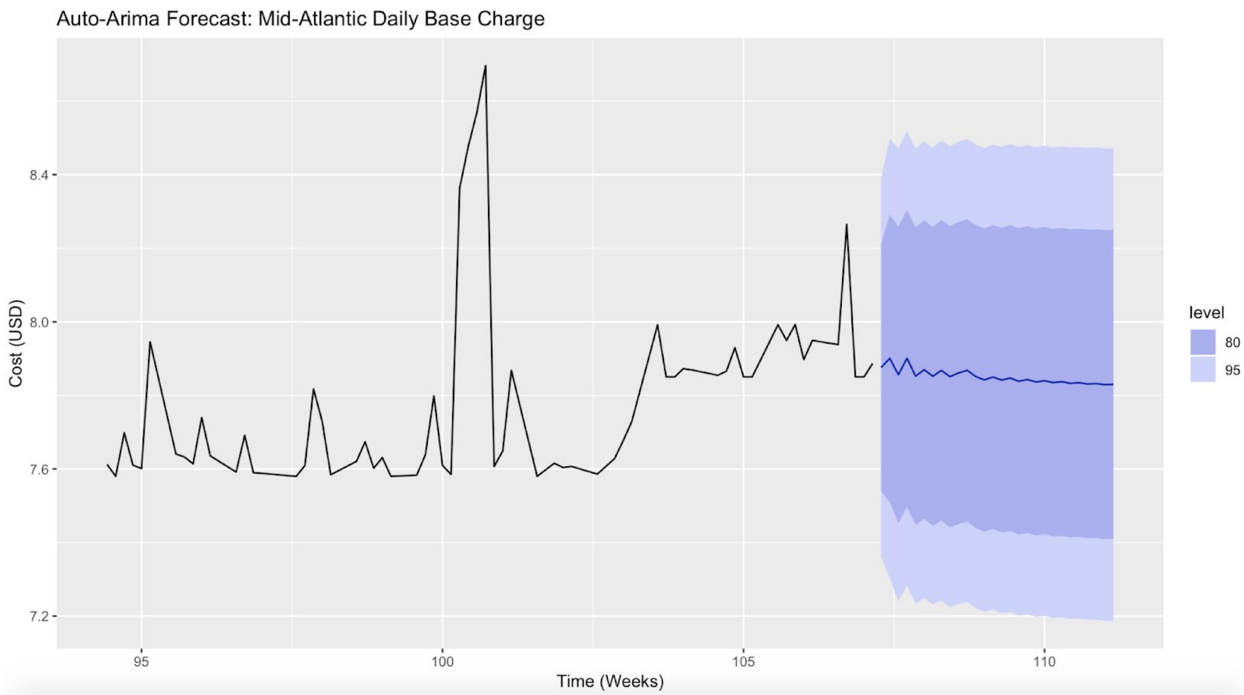
Combining equation 1, 2, and 3 we obtain the full ARIMA(*p,d,q*) model:

$$y'_t = c + \varphi_1 y'_{t-1} + \ldots + \varphi_p y'_{t-p} + \theta_1\varepsilon_{t-1} + \ldots + \theta_q\varepsilon_{t-q} + \varepsilon_t \tag{5}$$

The model described above is for the non-seasonal ARIMA model for non-seasonal data; however, the ARIMA model is capable of handling seasonal data as well, which it does by including additional parameters, ARIMA(*p,d,q*)(*P,D,Q*). The *P, D,* and *Q* correspond to the seasonal parameters and act similarly to their non-seasonal counter-parts, but involve backshifts for the seasonal period (Hyndman). For example, if a time series with weekly seasonality had *P=2,* this means that an autoregression would be performed using lags 7 and 14.

III. Results

R has a built in function called "Auto-ARIMA" that runs a number of ARIMA models with different *(p,d,q)(P,D,Q)* parameters and chooses what it determines to be the best out of these models. R chooses the parameters based on what combination yields the lowest Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC). The mathematical details of these are beyond the scope of this paper, but both criteria are intended to be indicators of the best-fit model. Lower AIC and BIC values are preferable when choosing an ARIMA model. In general, we found the Auto-ARIMA function to be useful and reliable. Below is the results of the Auto-ARIMA Mid-Atlantic base charge 4-week forecast. The code is in A8.



ARIMA(2,0,4)(0,0,2)[7]
AIC=133.65 BIC=179.77

Figure 7. Auto-ARIMA forecast of Client A's average daily base charge for the Mid-Atlantic region.

In this graphic, the blue line is the actual forecasted points, while the darker blue zone represents the 80% confidence interval, and the light blue represents the 95% confidence interval. The Auto-ARIMA function picked a model with autoregression of order 2, differencing of order 0, and moving average of order 4, with a weekly seasonal component (as indicated by the "[7]") and a weekly moving average of order 2. The AIC and BIC are fairly low in this given model; however, we were not very pleased with this model. We believed that this should be a d=1 model. We felt that the lack of differencing resulted in the downward skew seen in the forecast, which seems unlikely to actually occur based on previous visual analysis. In addition, we felt that some of the other parameters could potentially be changed as well.

In order to determine new parameters, we plotted the ACF for the differenced series. The code for doing so can be found in A9.

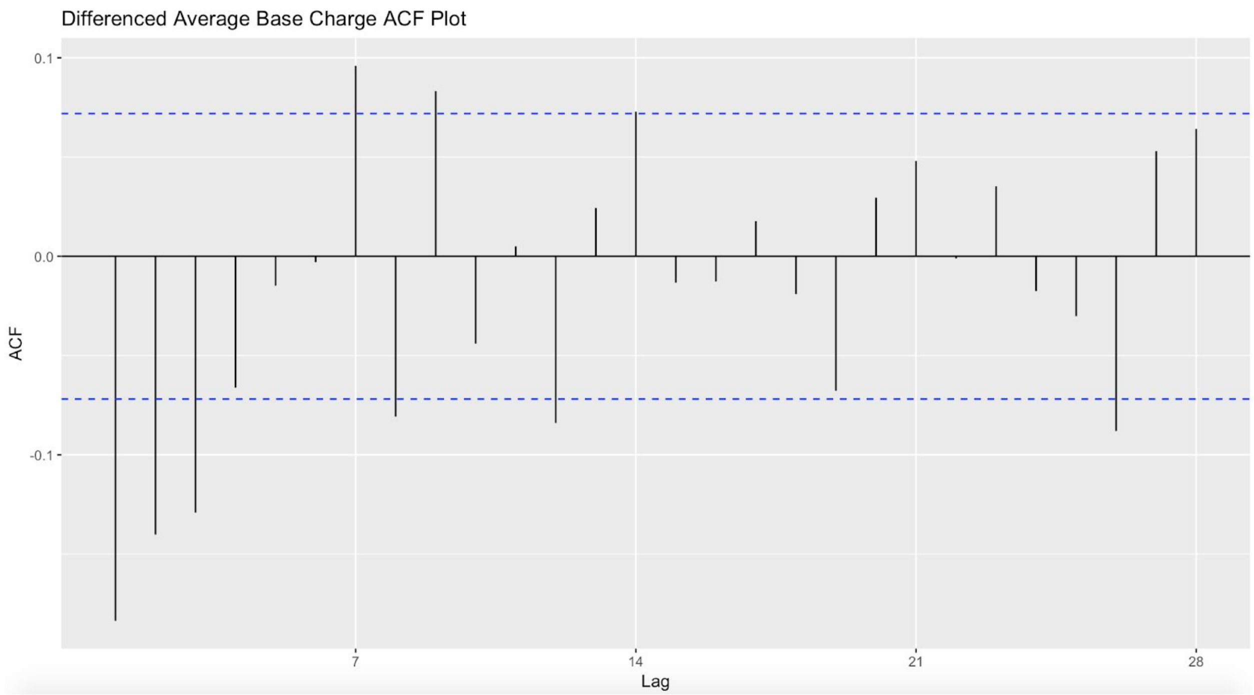
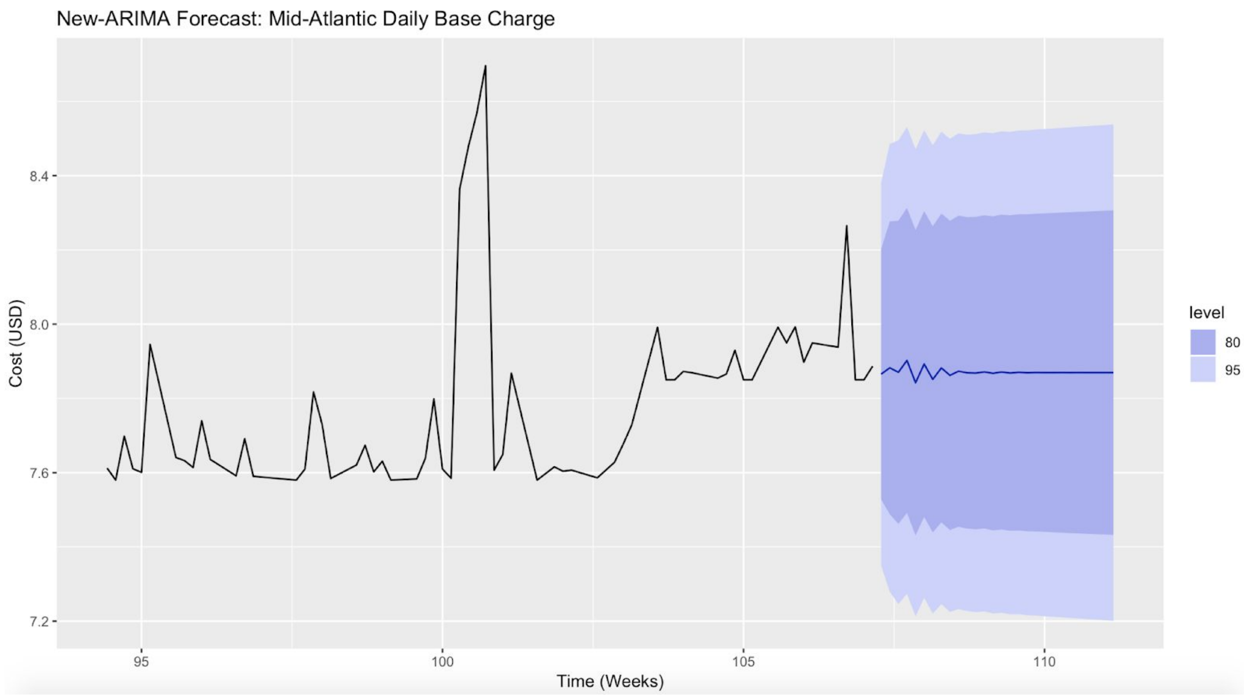


Figure 8. ACF plot of Client A's differenced average base charge for the Mid-Atlantic region.

The relatively large spike at lag 7 in the differenced ACF plot implies that a moving average of order 7 may be a useful parameter (Hyndman; Roopam). In addition, we replaced the seasonal moving average with order 1, because this resulted in lower AIC and BIC values when *d*=1. We created our new model using the code in A10, producing the graph below.



ARIMA(2,1,7)(0,0,1)[7]
AIC=139.18 BIC=189.89

Figure 9. New-ARIMA forecast of Client A's average daily base charge in the Mid-Atlantic Region.

The New-ARIMA model did have somewhat higher AIC and BIC values; however, we determined that this was necessary in order to create a more realistic forecast. In this model the downward skew of the Auto-ARIMA model has been removed.

Finally, in order to ensure that the New-ARIMA model had accomplished its purpose and had removed any useful information from the original time series, we plotted the residuals and the ACF of the residuals (Roopam).

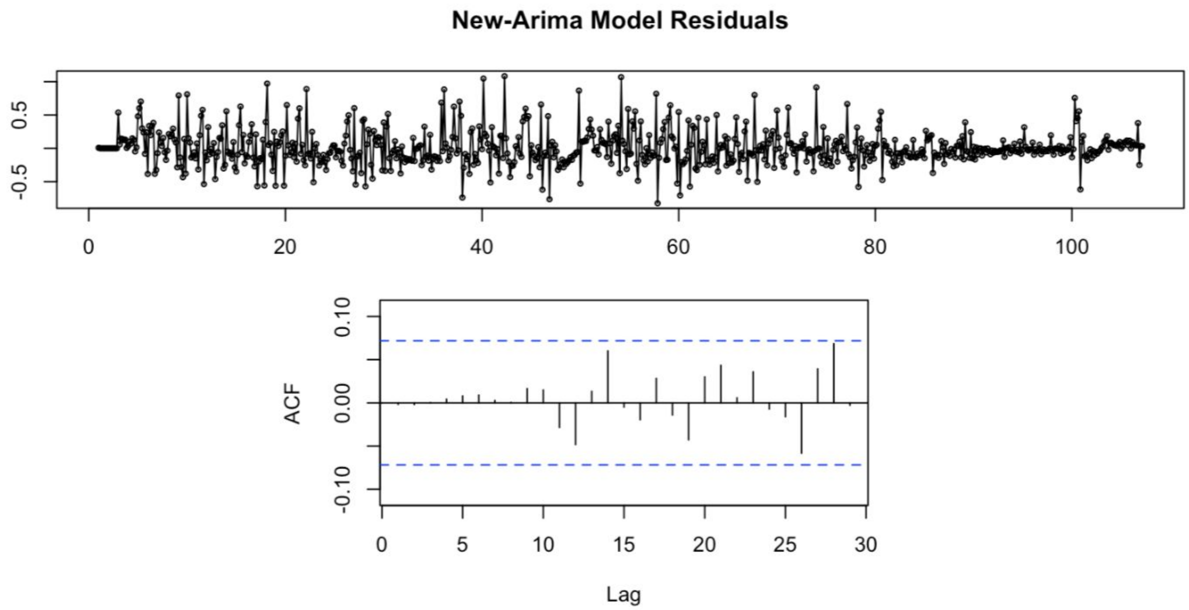


Figure 10. Plot of the residuals of the New-ARIMA model along with ACF plot of the residuals.

The residuals appear to be white noise based on the graph, and this is confirmed by there being no significant ACF values; thus this would be considered an effective ARIMA model.

NOTE: Forecast graphs and 2-week numeric forecast data for all other ARIMA forecasts are located in Appendix C-Appendix F.

IV. Conclusion

The ARIMA model is an effective and useful tool for forecasting many different types of data, including data that may or may not display seasonality and trend. While the Auto-ARIMA function in R is useful and seems to be reliable, sometimes one must use his or her own judgement to create a new model that he or she believes to be a more accurate representation of the data, as our team did in the daily average base charge for the Mid-Atlantic region. Although the AIC and BIC were higher for our New-ARIMA model, based on our interpretation and analysis of the data, our opinion is that it is a better model than the Auto-ARIMA chosen by R. Unfortunately, due to time constraints we were not able to perform as rigorous an analysis on the other 23 time series that we created, but rather relied on the Auto-ARIMA function in R to create these models. Hopefully this paper can serve as an effective guide for anyone who wants to more rigorously analyze and forecast these remaining or similar other data.

In addition, there are many other forecasting models that could be potentially used with this data and that also have useful functions in R; a description of some additional forecasting tools in R can be found in Hyndman's textbook, *Forecasting: Principles and Practice*, which we relied on heavily throughout our research both for data analysis theory and for instruction on R code. It may also be useful to experiment with alternative regions, or even analyze the data at a state level, as there may be useful information in the data that was not captured due to the size of our chosen regions.

We believe that attempting to forecast key performance indicators for any given business is a beneficial practice, and it is our hope that others may be able to use this paper as a reference for useful forecasting practices.

Works Cited

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Taboga, Marco. "Autocorrelation." *StatLect*, www.statlect.com/fundamentals-of-statistics/autocorrelation.

US Census Bureau. Census.gov. Census.gov, www.census.gov/.

VeriShip. "Parcel Audit & Carrier Contract Negotiation | VeriShip Home." *VeriShip*, veriship.com/.

APPENDIX A: R Codes

[1] > F_IA <- subset(F_veriship_CSV, F_veriship_CSV.DestinationStateProvince == "IA")
> #Here the "F_" is an indicator that we are using Client A's data, which only used FedEx
> midwest_total <- data.frame(rbind(F_IA, F_IL, F_IN, F_KS, F_MI, F_MN, F_MO, F_NE, F_WI))

[2] > MA_BCavg <- aggregate(base_charges ~ Date, data = midatlantic_total, mean)
> #For the daily count we first had to add a new row of all 1s to the data frame to
> midatlantic_total$newcol <- rep(1,nrow(midwest_total))
> MA_daily_count <- aggregate(newcol ~ F_toofull_dates, data = midwest_total, sum)

[3] > z_MA_BCavg <- read.zoo(MW_BCavg)
> zz_MA_BCavg <- na.approx(z_MW_BCavg, xout = seq(start(z_MW_BCavg), end(z_MA_BCavg), "day"))
> MA_BCavg_interp <- fortify.zoo(zz_MW_BCavg)
> MA_BCavg_ts <- ts(MW_BCavg_interp[,2], start = c(2017, 32), frequency = 365)

[4] > MA_BC_clean_ts <- tsclean(MW_BCavg_ts)

[5] > total_clean_BC <- data.frame(PC_BC_clean_ts, W_BC_clean_ts, MW_BC_clean_ts, SE_BC_clean_ts, MA_BC_clean_ts, NE_BC_clean_ts)
> total_clean_BC_ts <- ts(total_clean_BC_ts, frequency = 365, start = c(2017, 33))
> autoplot(total_clean_BC_ts) +ggtitle("Daily Average Base Charge by Region, Client A") + xlab("Time") + ylab("Average Base Charge (USD)")

[6] > ggAcf(MA_BC_clean_ts) + ggtitle("ACF Plot of Mid-Atlantic Daily Base Charge")

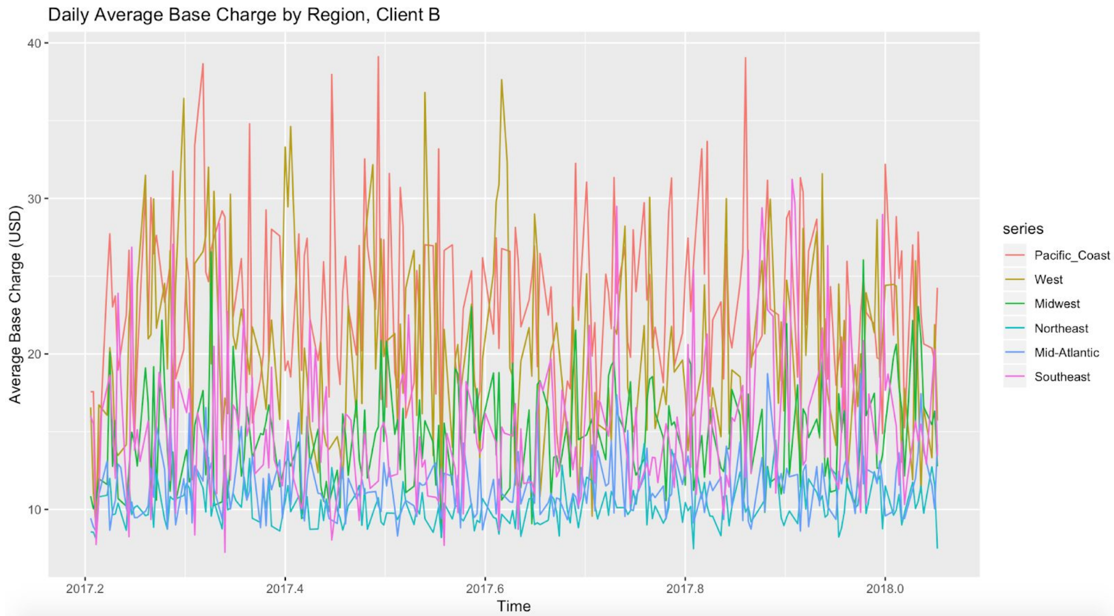
[7] > autoplot(MA_BC_clean_ts) +ggtitle(" Daily Average Base Charge for Mid-Atlantic") + ylab("Cost (USD)") + xlab("Time")
> autoplot(diff(MA_BC_clean_ts)) +ggtitle(" Differenced Daily Average Base Charge for Mid-Atlantic") + ylab("Differenced Values") + xlab("Time")

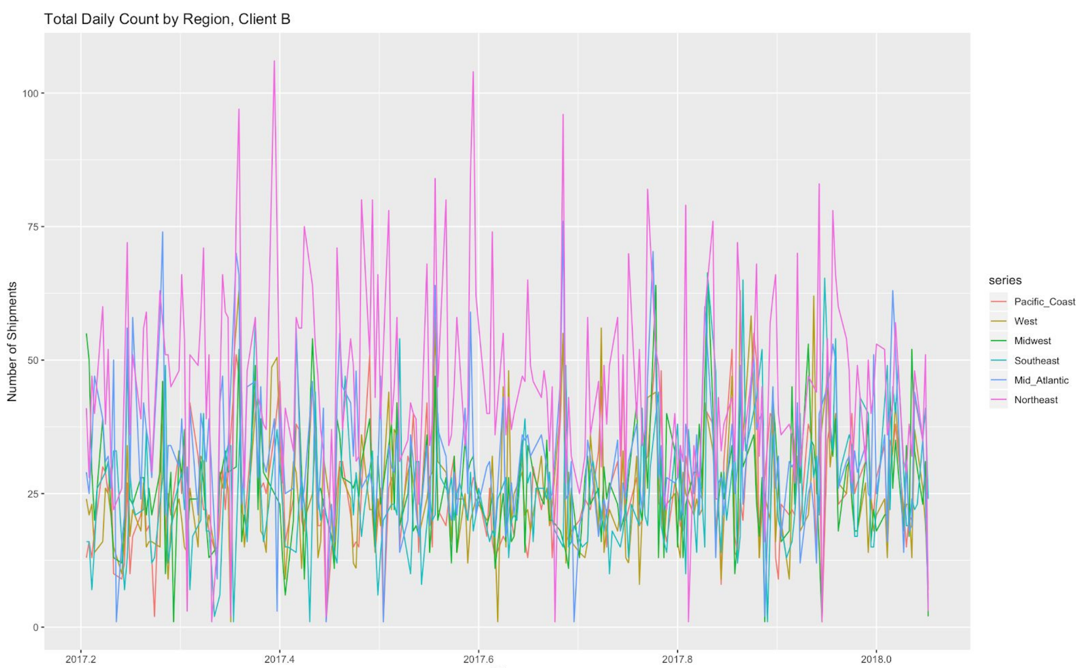
[8] > MA_BC_clean <- auto.arima(MA_BC_clean_, stepwise = FALSE)
> #Note: the "stepwise = FALSE" command is not always necessary, but it forces R to search significantly more potential models, which also substantially increases the amount of time for Auto-ARIMA to work. "stepwise=FALSE" was not used for any of the additional ARIMA forecasts in Appendix C.
> MA_BC_clean %>% forecast(h=28) %>% autoplot(include=90) + ggtitle("Auto-Arima Forecast: Mid-Atlantic Daily Base Charge") + ylab("Cost (USD)") + xlab("Time (Weeks)")

[9]
```
> MA_BC_diff <- diff(MA_BC_clean_ts)
> ggAcf(MA_BC_diff, main="Differenced Average Base Charge ACF Plot")
```

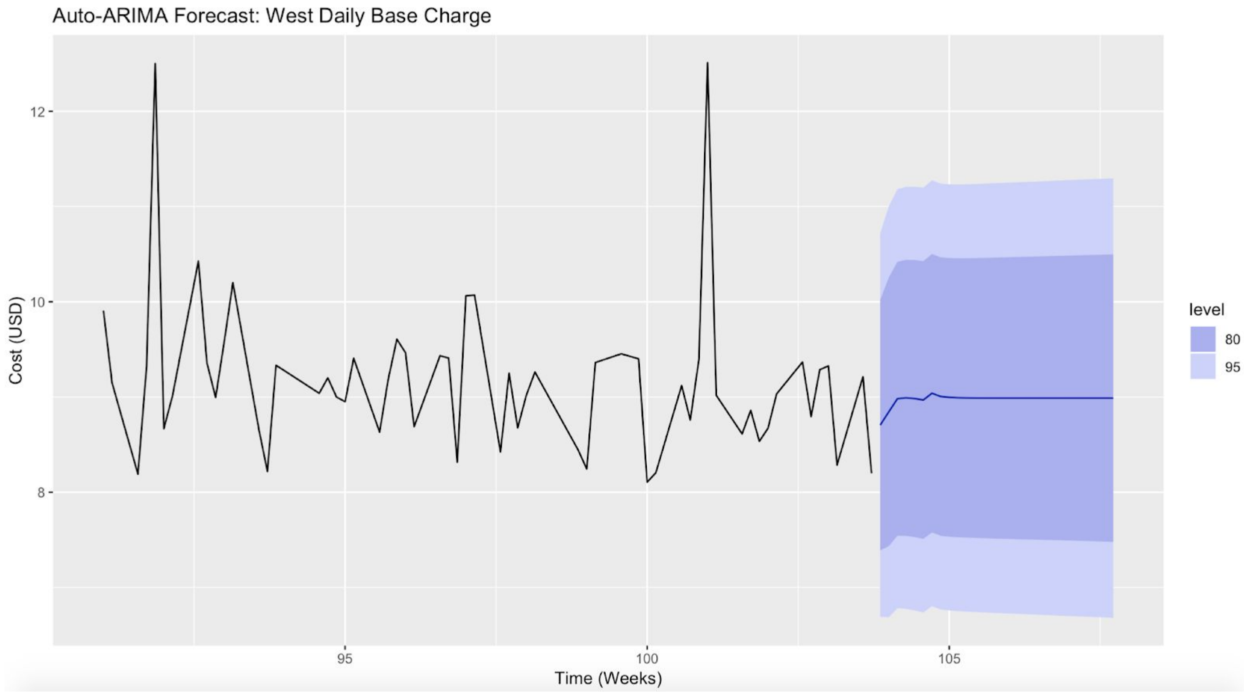
[10]
```
> new_MA_BC_clean <- MA_BC_clean_ts %>% Arima(order=c(2,1,7),
seasonal=c(0,0,1))
> new_MA_BC_clean %>% forecast(h=28) %>% autoplot(include=90) +
ggtitle("New-ARIMA Forecast: Mid-Atlantic Daily Base Charge") + ylab("Cost (USD)")
+ xlab("Time (Weeks)")
```

APPENDIX B: Client B Time Plots



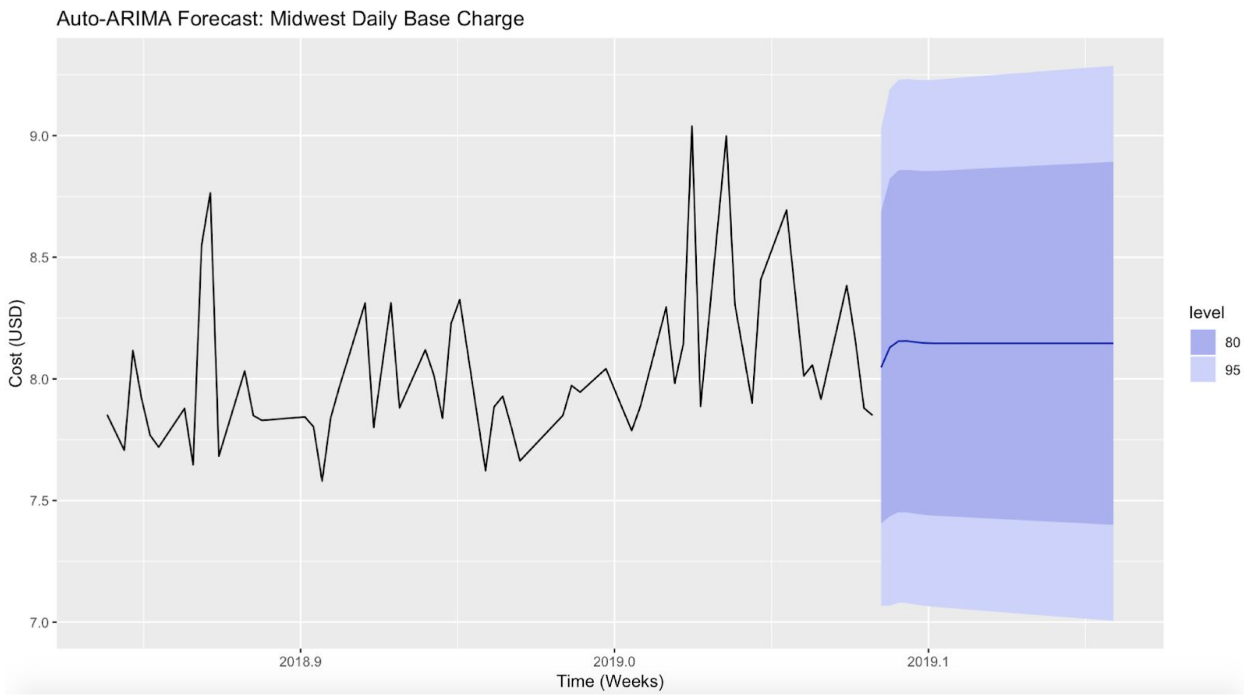


APPENDIX C: Client A Daily Average Base Charge



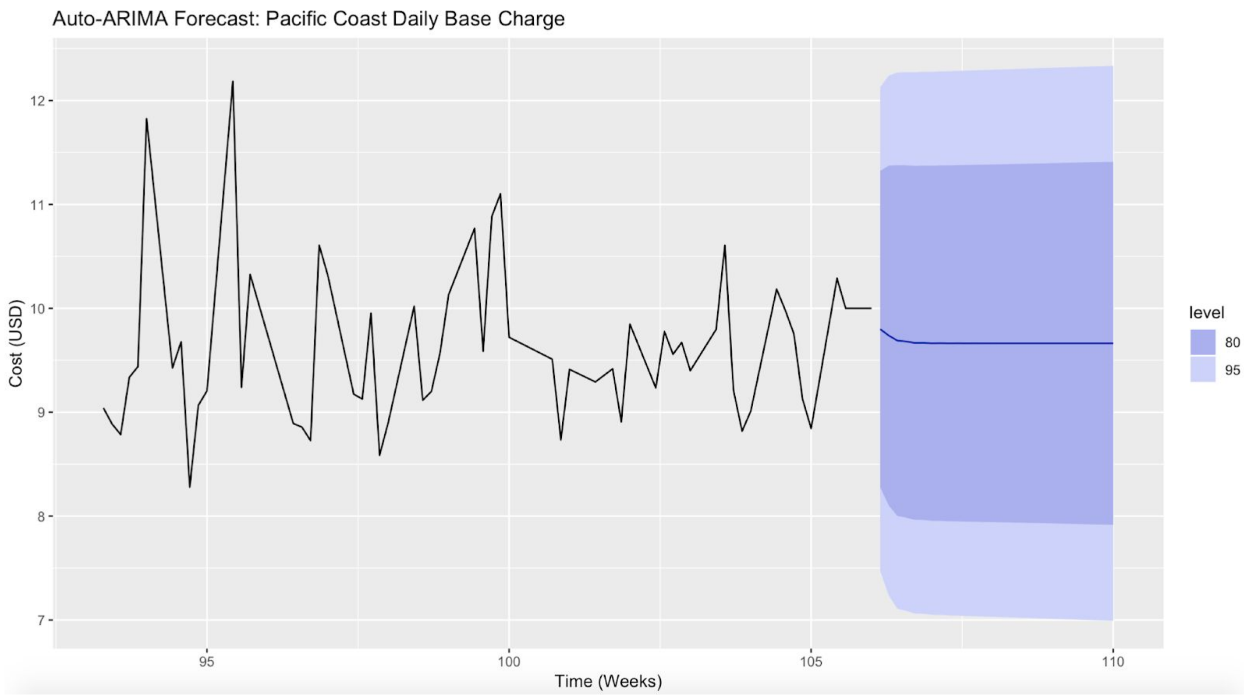
ARIMA(2,1,1)(0,0,1)[7]
AIC=2106.65 BIC=2129.58

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
8.704620	7.388614	10.02063	6.691962	10.71728
8.844775	7.433857	10.25569	6.686962	11.00259
8.982213	7.543432	10.42099	6.781788	11.18264
8.991215	7.542078	10.44035	6.774951	11.20748
8.983476	7.528796	10.43816	6.758736	11.20822
8.968437	7.509837	10.42704	6.737701	11.19917
9.039283	7.577378	10.50119	6.803493	11.27507
9.005296	7.543366	10.46723	6.769467	11.24113
8.996332	7.533183	10.45948	6.758639	11.23402
8.991952	7.526943	10.45696	6.751414	11.23249
8.990105	7.522879	10.45733	6.746176	11.23403
8.989302	7.519703	10.45890	6.741744	11.23686
8.988956	7.516915	10.46100	6.737664	11.24025
8.988807	7.514297	10.46332	6.733739	11.24387



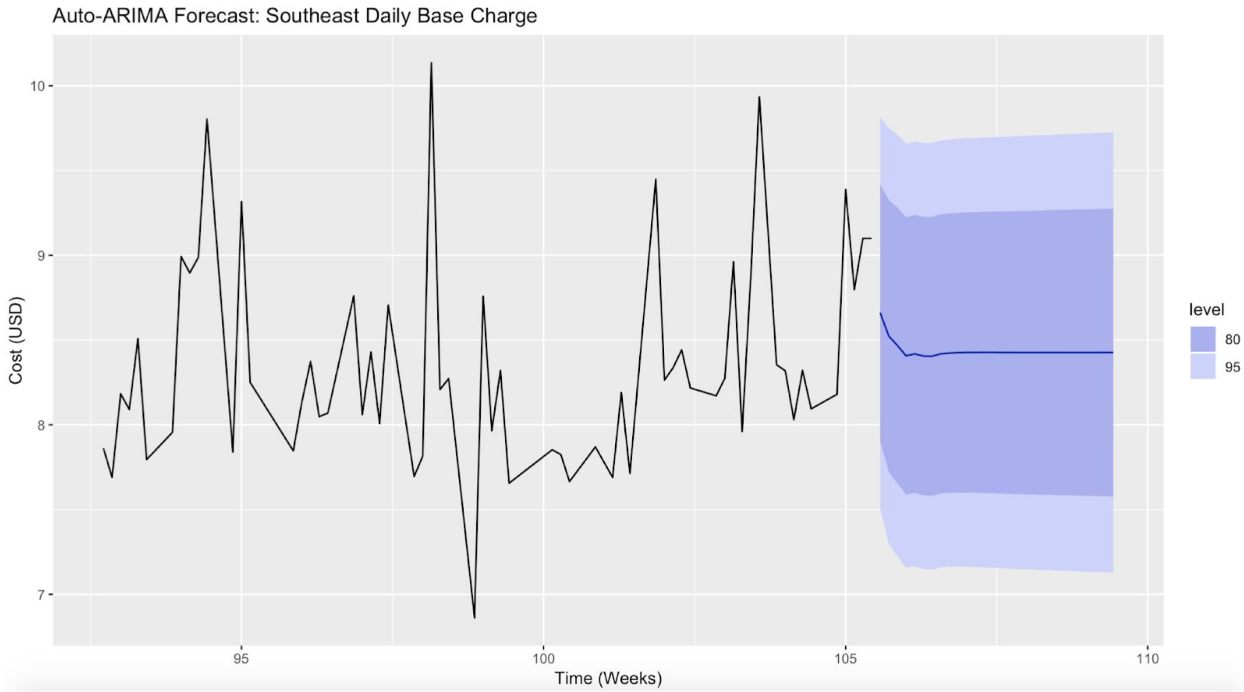
ARIMA(3,1,2)
AIC=1068.05 BIC=1095.6

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
8.047448	7.406157	8.688738	7.066678	9.028217
8.129088	7.435068	8.823108	7.067676	9.190500
8.154668	7.452008	8.857329	7.080042	9.229295
8.155291	7.451112	8.859471	7.078342	9.232241
8.150820	7.445687	8.855952	7.072413	9.229227
8.147583	7.441199	8.853967	7.067262	9.227904
8.146223	7.438230	8.854217	7.063440	9.229006
8.145919	7.436120	8.855717	7.060376	9.231461
8.145975	7.434315	8.857635	7.057585	9.234366
8.146074	7.432554	8.859595	7.054839	9.237309
8.146132	7.430767	8.861497	7.052076	9.240188
8.146152	7.428955	8.863349	7.049294	9.243011
8.146155	7.427134	8.865176	7.046507	9.245803
8.146153	7.425313	8.866993	7.043723	9.248583



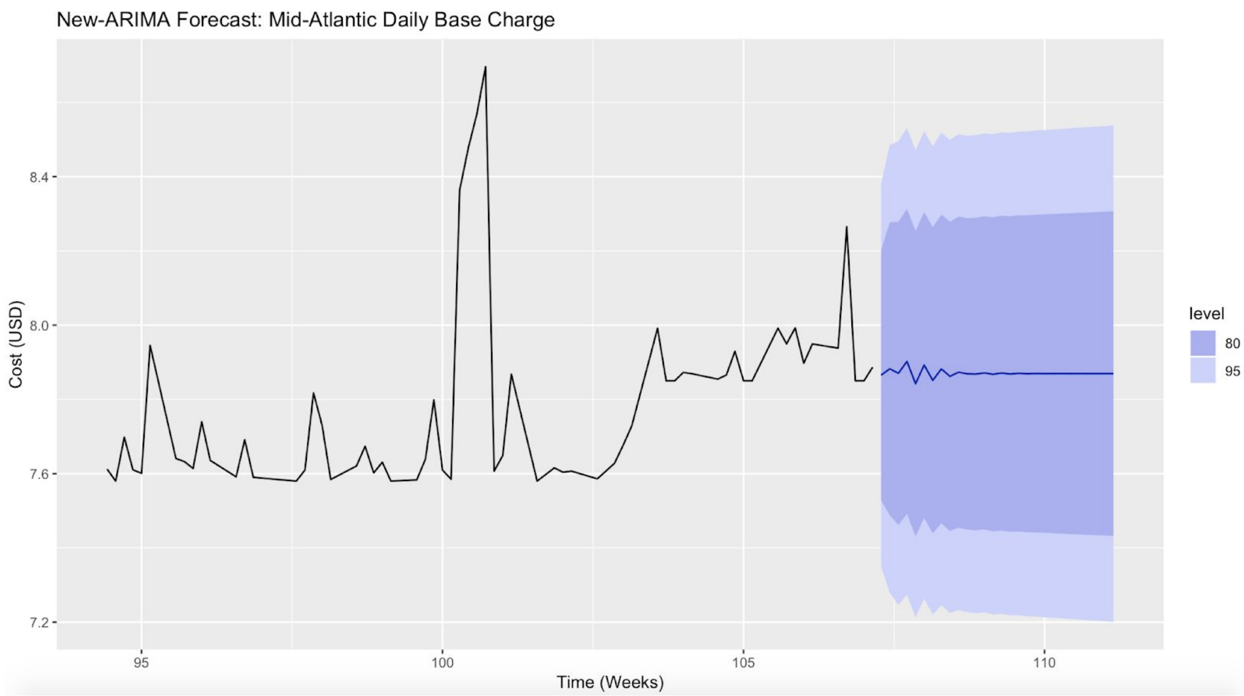
ARIMA(2,1,2)
AIC=2347.02 BIC=2370.02

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
9.799794	8.277333	11.32226	7.471390	12.12820
9.737332	8.100762	11.37390	7.234414	12.24025
9.688829	8.001315	11.37634	7.108000	12.26966
9.680622	7.985105	11.37614	7.087553	12.27369
9.667218	7.963821	11.37062	7.062097	12.27234
9.667818	7.962268	11.37337	7.059405	12.27623
9.663429	7.954713	11.37215	7.050173	12.27669
9.664667	7.954309	11.37502	7.048900	12.28043
9.662981	7.950379	11.37558	7.043782	12.28218
9.663775	7.949514	11.37804	7.042039	12.28551
9.663052	7.946813	11.37929	7.038291	12.28781
9.663476	7.945519	11.38143	7.036087	12.29087
9.663146	7.943310	11.38298	7.032885	12.29341
9.663359	7.941767	11.38495	7.030411	12.29631



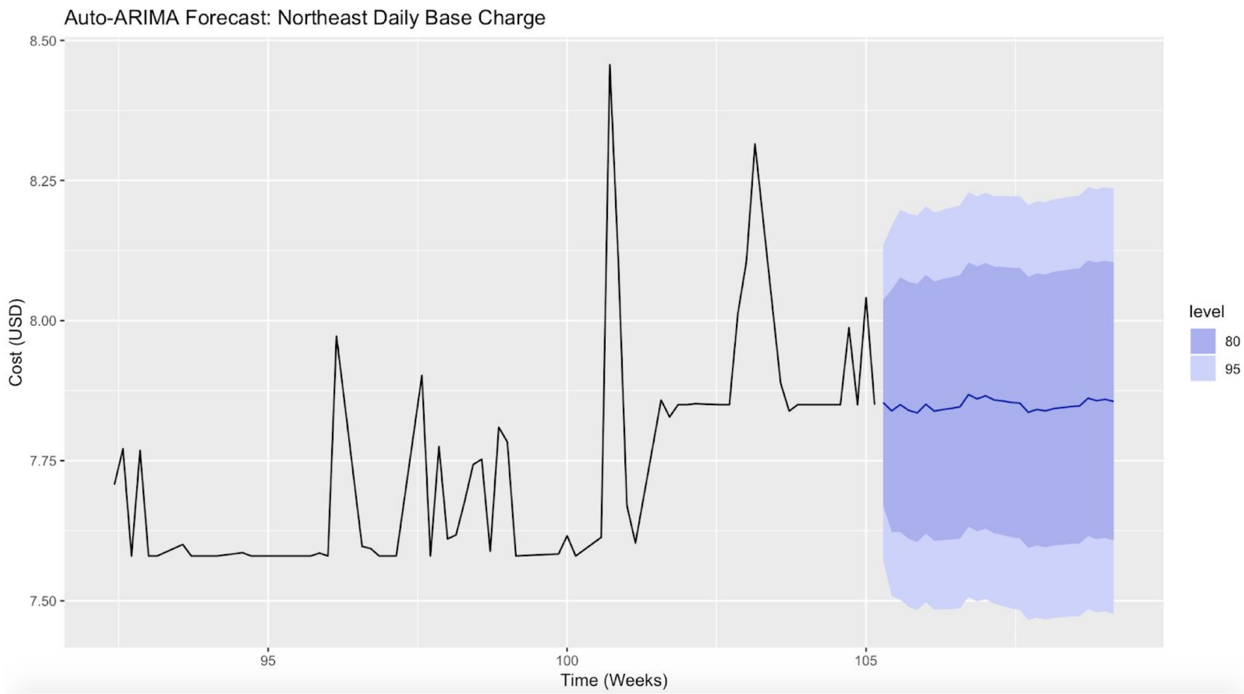
ARIMA(2,1,1)(1,0,0)[7]
AIC=1309.53 BIC=1332.51

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
8.661114	7.905730	9.416499	7.505854	9.816375
8.522926	7.721227	9.324625	7.296832	9.749020
8.467691	7.654578	9.280804	7.224143	9.711240
8.406919	7.589722	9.224116	7.157124	9.656714
8.418738	7.599251	9.238224	7.165441	9.672034
8.405687	7.584460	9.226914	7.149729	9.661645
8.404551	7.581775	9.227327	7.146223	9.662878
8.418739	7.595607	9.241870	7.159868	9.677609
8.423177	7.599076	9.247278	7.162824	9.683531
8.424955	7.599668	9.250241	7.162787	9.687122
8.426955	7.600389	9.253521	7.162831	9.691079
8.426552	7.598672	9.254432	7.160419	9.692685
8.426984	7.597778	9.256189	7.158823	9.695145
8.427020	7.596485	9.257555	7.156826	9.697214



ARIMA(2,1,7)(0,0,1)[7]
AIC=139.18 BIC=189.89

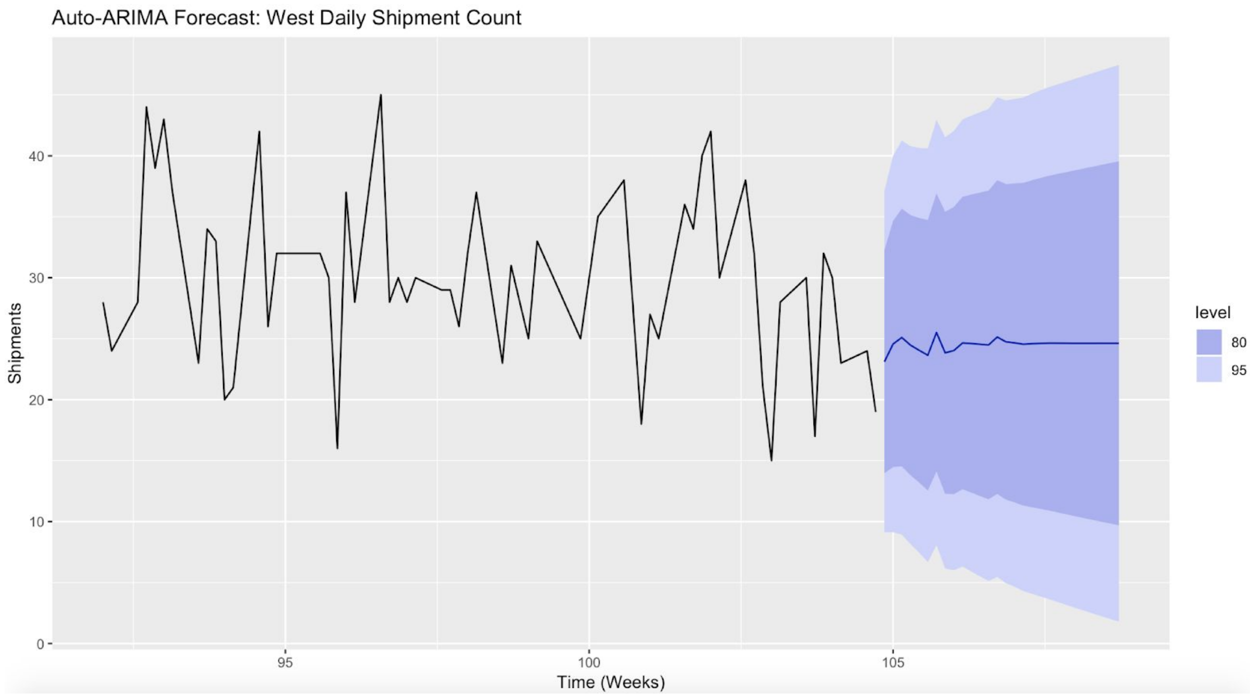
Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
7.865403	7.527973	8.202834	7.349348	8.381459
7.882450	7.488150	8.276750	7.279420	8.485480
7.870364	7.462063	8.278665	7.245922	8.494806
7.902385	7.491494	8.313276	7.273981	8.530788
7.842178	7.430870	8.253485	7.213137	8.471218
7.892638	7.480882	8.304394	7.262912	8.522365
7.851225	7.438866	8.263585	7.220576	8.481875
7.882089	7.466155	8.298024	7.245972	8.518206
7.862027	7.445481	8.278573	7.224975	8.499079
7.873296	7.454363	8.292229	7.232593	8.513998
7.868810	7.449343	8.288276	7.227290	8.510329
7.868191	7.447248	8.289135	7.224414	8.511969
7.871312	7.449585	8.293038	7.226336	8.516287
7.867562	7.444700	8.290423	7.220851	8.514272



ARIMA(1,1,3)(2,0,1)[7]
AIC=-755.37 BIC=-718.64

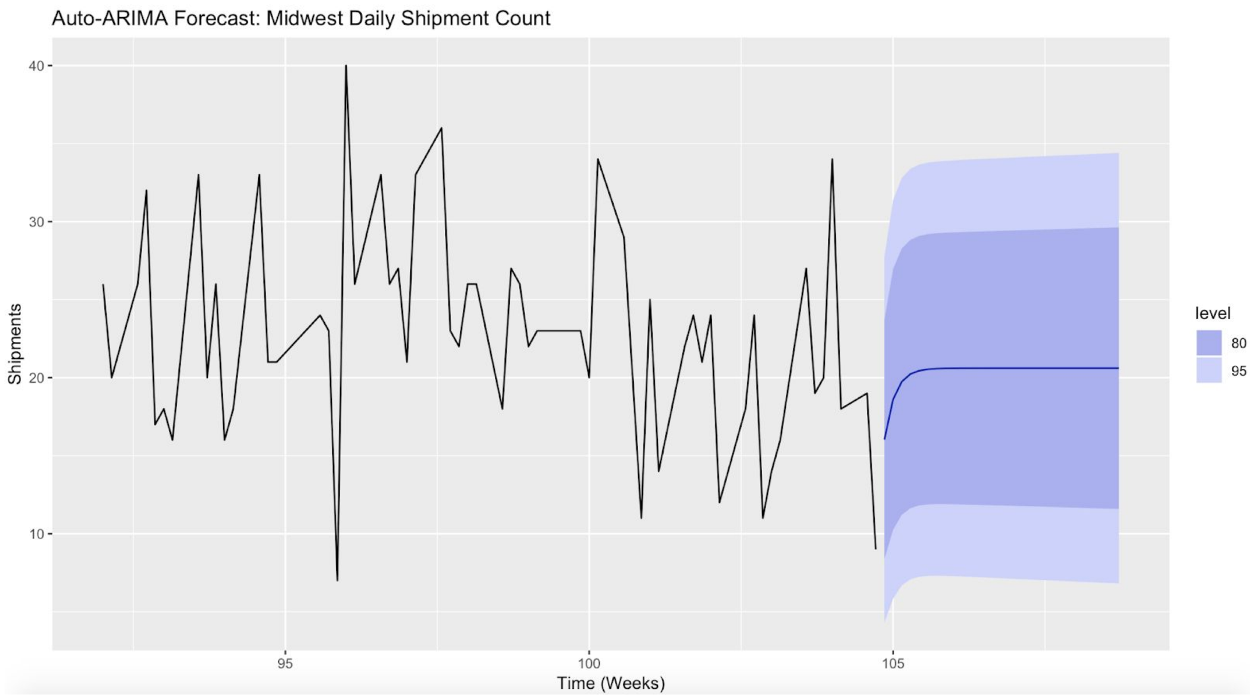
Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
7.853743	7.670353	8.037132	7.573273	8.134213
7.838908	7.622795	8.055020	7.508392	8.169424
7.850039	7.622823	8.077255	7.502543	8.197535
7.839788	7.610460	8.069117	7.489060	8.190516
7.835409	7.605156	8.065662	7.483268	8.187550
7.850655	7.619713	8.081596	7.497460	8.203849
7.838459	7.606888	8.070030	7.484301	8.192617
7.841364	7.608307	8.074422	7.484934	8.197795
7.843360	7.609202	8.077518	7.485246	8.201474
7.846169	7.611124	8.081214	7.486699	8.205640
7.868077	7.632313	8.103840	7.507507	8.228646
7.860326	7.623887	8.096765	7.498724	8.221928
7.865805	7.628704	8.102907	7.503190	8.228421
7.858398	7.620639	8.096158	7.494777	8.222020

APPENDIX D: Client A Daily Shipment Count



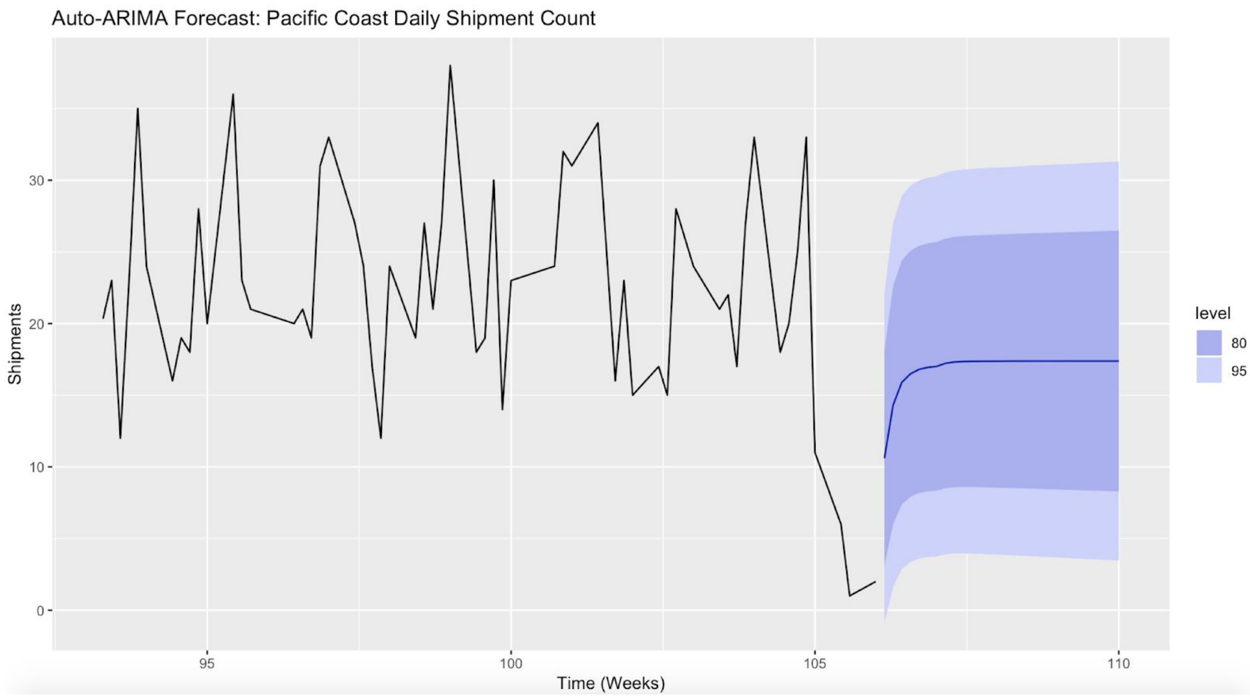
ARIMA(3,1,1)(0,0,2)[7]
AIC=4922.89 BIC=4955

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
23.11408	13.96724	32.26093	9.125188	37.10298
24.56813	14.47235	34.66392	9.127966	40.00830
25.09810	14.53527	35.66093	8.943645	41.25255
24.47494	13.80961	35.14027	8.163731	40.78615
24.05150	13.20357	34.89942	7.461024	40.64197
23.64650	12.55517	34.73784	6.683767	40.60924
25.50707	14.10733	36.90681	8.072676	42.94147
23.84410	12.28019	35.40800	6.158626	41.52957
24.03058	12.25377	35.80740	6.019498	42.04167
24.65057	12.66108	36.64007	6.314224	42.98692
24.60508	12.38557	36.82458	5.916950	43.29320
24.54450	12.10409	36.98491	5.518539	43.57046
24.49028	11.83480	37.14576	5.135389	43.84517
25.13959	12.27622	38.00297	5.466758	44.81243



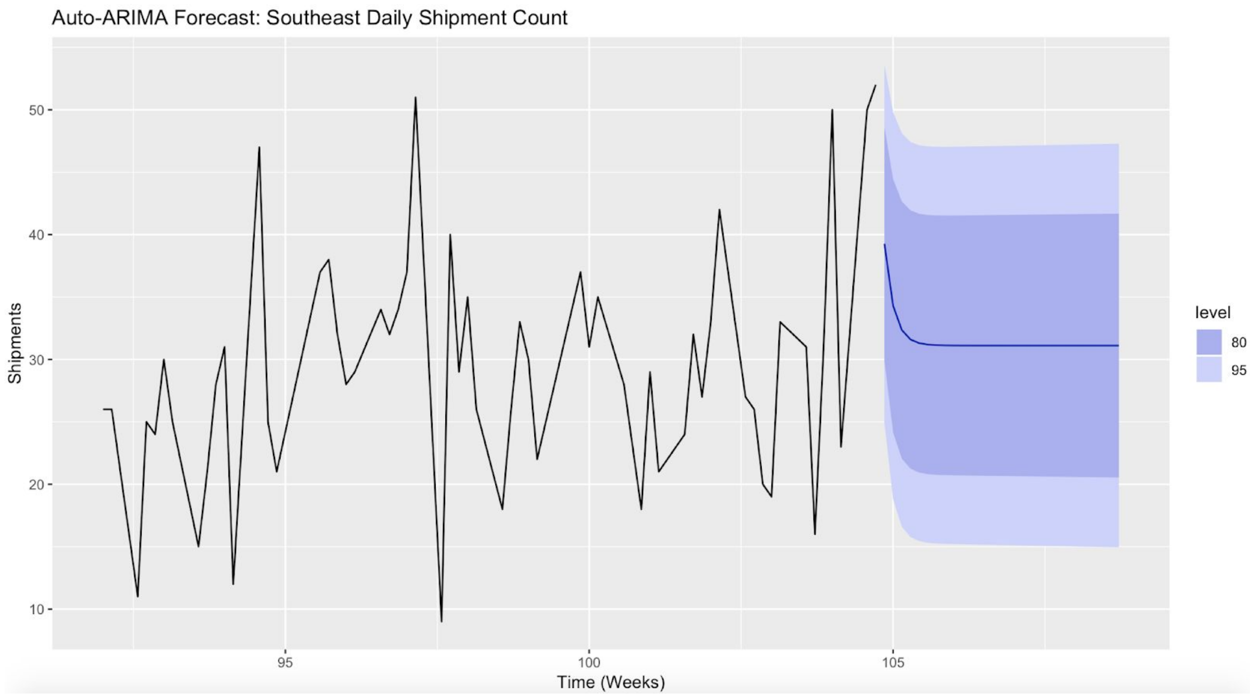
ARIMA(2,1,1)
AIC=4663.5 BIC=4681.85

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
16.02977	8.367549	23.69199	4.311414	27.74813
18.60914	10.258403	26.95988	5.837788	31.38050
19.74418	11.211320	28.27704	6.694297	32.79406
20.23459	11.635797	28.83337	7.083873	33.38530
20.44683	11.814139	29.07953	7.244266	33.64940
20.53868	11.882646	29.19471	7.300419	33.77694
20.57842	11.903027	29.25382	7.310549	33.84630
20.59562	11.902483	29.28876	7.300613	33.89063
20.60307	11.892879	29.31325	7.281985	33.92415
20.60629	11.879368	29.33321	7.259617	33.95296
20.60768	11.864183	29.35118	7.235655	33.97971
20.60828	11.848291	29.36828	7.211031	34.00554
20.60854	11.832110	29.38498	7.186146	34.03094
20.60866	11.815821	29.40149	7.161175	34.05614



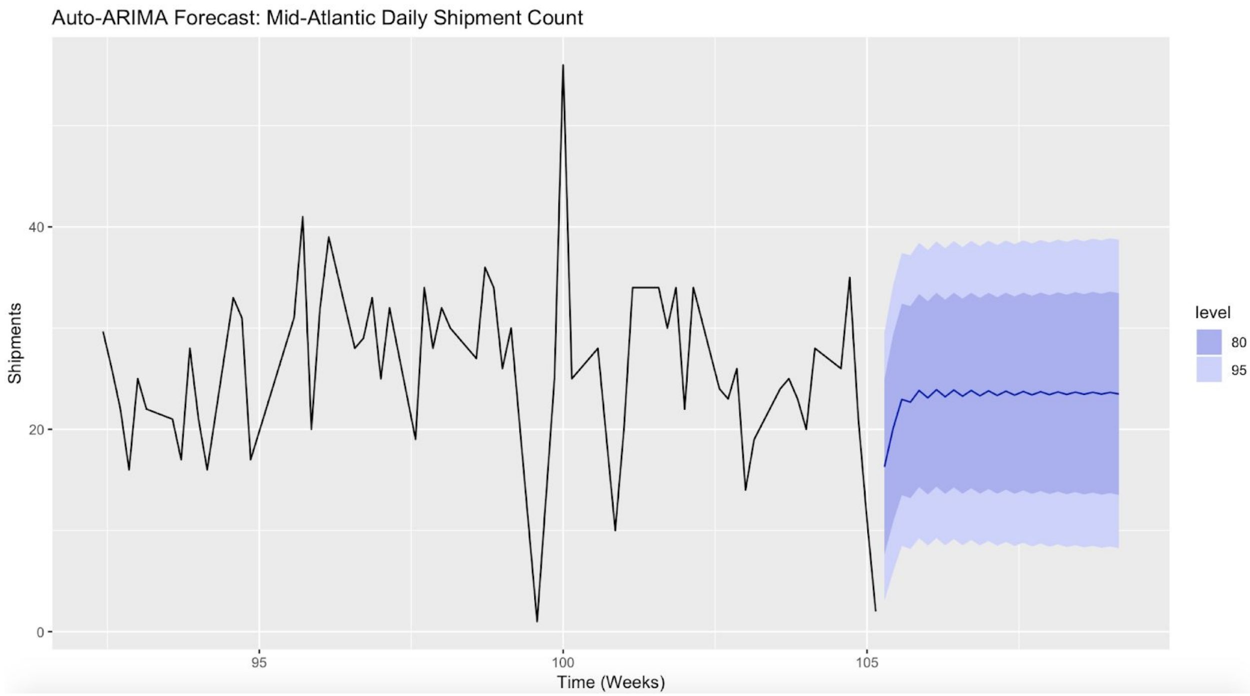
ARIMA(1,1,1)(1,0,0)[7]
AIC=4687.93 BIC=4706.33

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
10.59793	3.106729	18.08912	-0.8588715	22.05472
14.30638	6.006632	22.60612	1.6130125	26.99974
15.89423	7.385564	24.40289	2.8813485	28.90711
16.48987	7.905575	25.07416	3.3613264	29.61841
16.80261	8.179500	25.42572	3.6147009	29.99052
16.94236	8.292579	25.59214	3.7136625	30.17105
17.00707	8.335203	25.67894	3.7445940	30.26955
17.21899	8.514286	25.92370	3.9062917	30.53170
17.31046	8.580850	26.04007	3.9596740	30.66125
17.34968	8.598294	26.10107	3.9655880	30.73378
17.36467	8.592815	26.13653	3.9492741	30.78007
17.37235	8.580612	26.16410	3.9265439	30.81817
17.37578	8.564420	26.18713	3.8999697	30.85158
17.37735	8.546522	26.20818	3.8717641	30.88294



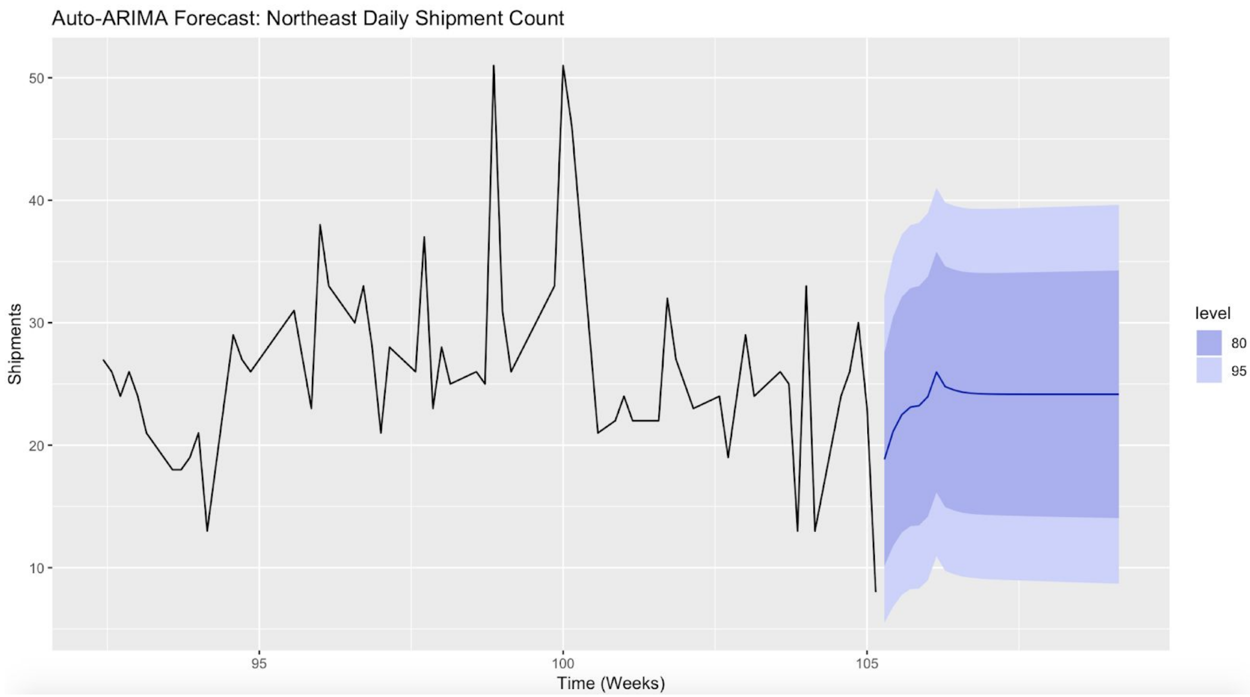
ARIMA(1,1,1)
AIC=4953.69 BIC=4967.46

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
39.27274	29.91174	48.63374	24.95633	53.58916
34.30167	24.15531	44.44802	18.78416	49.81918
32.36004	22.05759	42.66250	16.60380	48.11629
31.60167	21.25499	41.94836	15.77779	47.42556
31.30547	20.93955	41.67138	15.45217	47.15876
31.18977	20.81160	41.56794	15.31772	47.06182
31.14458	20.75645	41.53272	15.25730	47.03186
31.12693	20.72967	41.52419	15.22570	47.02817
31.12004	20.71398	41.52610	15.20534	47.03474
31.11735	20.70261	41.53208	15.18938	47.04531
31.11629	20.69294	41.53965	15.17515	47.05744
31.11588	20.68393	41.54783	15.16159	47.07018
31.11572	20.67519	41.55625	15.14831	47.08314
31.11566	20.66656	41.56476	15.13514	47.09618



ARIMA(2,1,2)
AIC=4861.43 BIC=4884.39

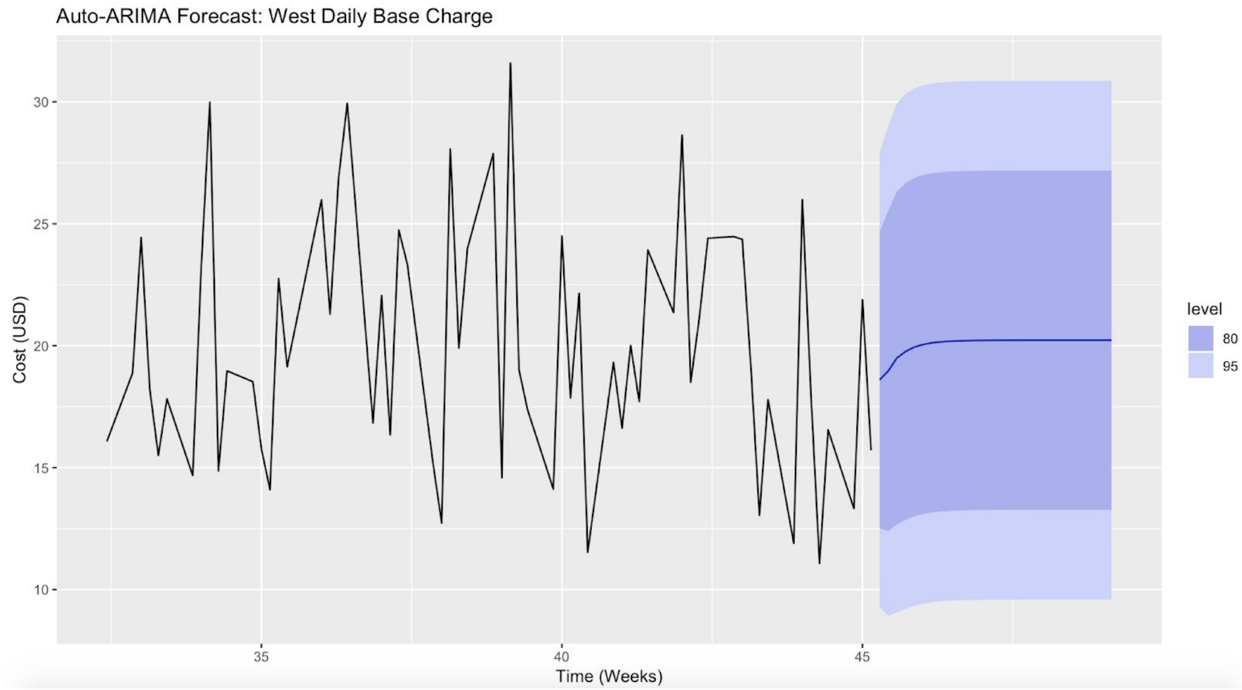
Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
16.27529	7.620452	24.93012	3.038860	29.51171
20.07557	10.817273	29.33387	5.916225	34.23492
22.95075	13.495476	32.40603	8.490155	37.41135
22.68624	13.196734	32.17576	8.173291	37.19920
23.83456	14.298150	33.37096	9.249881	38.41923
23.10771	13.559274	32.65614	8.504638	37.71078
23.90966	14.329020	33.49030	9.257334	38.56199
23.21310	13.621720	32.80448	8.544351	37.88184
23.87759	14.257454	33.49773	9.164861	38.59032
23.26755	13.636049	32.89906	8.537438	37.99767
23.83632	14.177861	33.49477	9.064983	38.60765
23.30935	13.638718	32.97997	8.519397	38.09930
23.79883	14.102634	33.49503	8.969776	38.62789
23.34463	13.635515	33.05374	8.495821	38.19344



ARIMA(2,1,1)(0,0,1)[7]
AIC=4870.47 BIC=4893.43

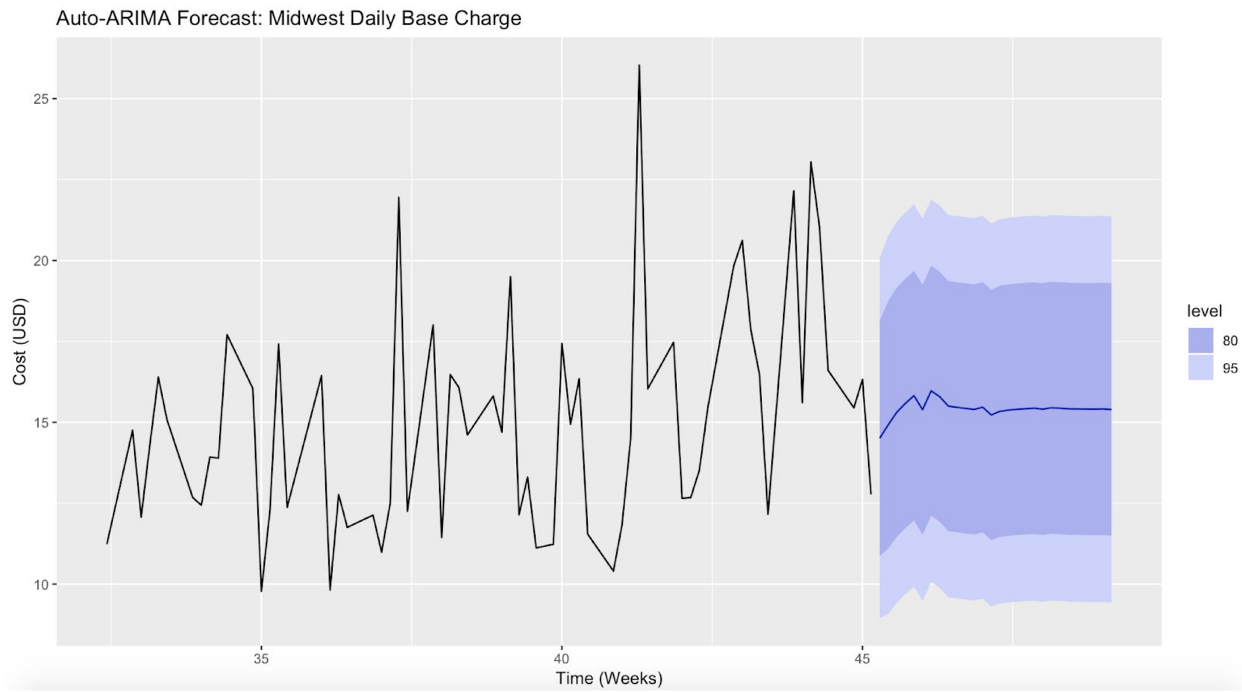
Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
16.27529	7.620452	24.93012	3.038860	29.51171
20.07557	10.817273	29.33387	5.916225	34.23492
22.95075	13.495476	32.40603	8.490155	37.41135
22.68624	13.196734	32.17576	8.173291	37.19920
23.83456	14.298150	33.37096	9.249881	38.41923
23.10771	13.559274	32.65614	8.504638	37.71078
23.90966	14.329020	33.49030	9.257334	38.56199
23.21310	13.621720	32.80448	8.544351	37.88184
23.87759	14.257454	33.49773	9.164861	38.59032
23.26755	13.636049	32.89906	8.537438	37.99767
23.83632	14.177861	33.49477	9.064983	38.60765
23.30935	13.638718	32.97997	8.519397	38.09930
23.79883	14.102634	33.49503	8.969776	38.62789
23.34463	13.635515	33.05374	8.495821	38.19344

APPENDIX E: Client B Daily Average Base Charge



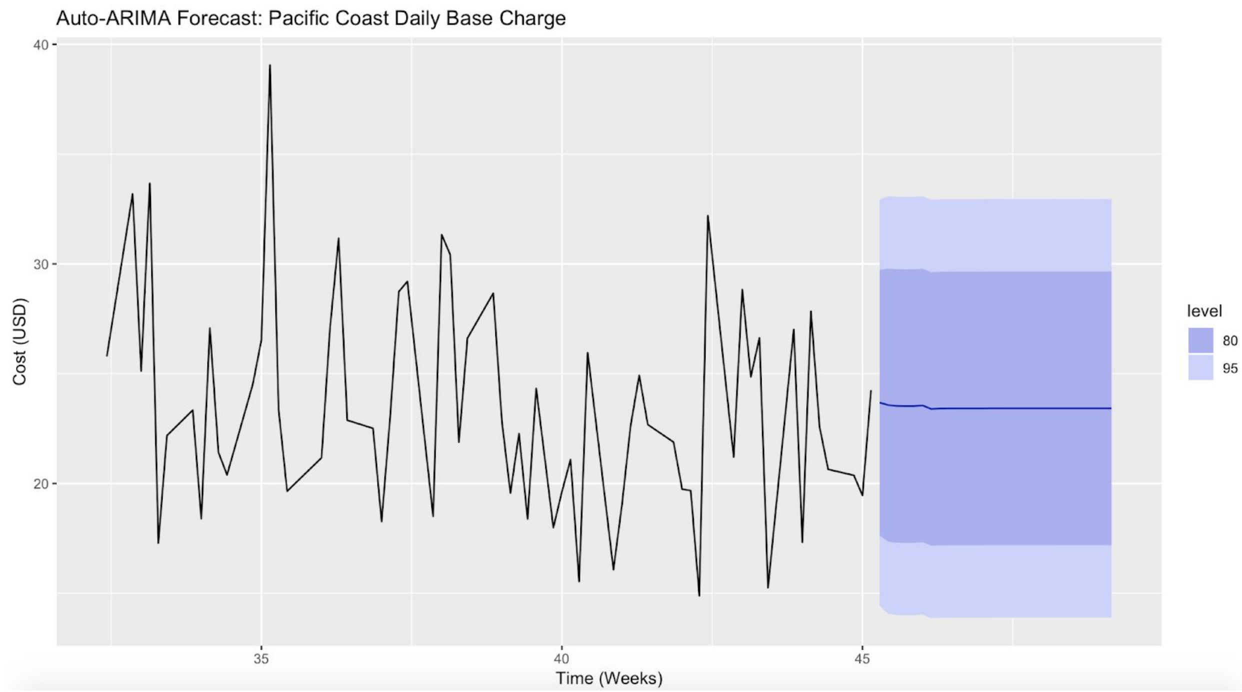
ARIMA(2,0,0)
AIC=1850.47   AICc=1850.6   BIC=1865.41

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
18.59817	12.51663	24.67970	9.297265	27.89907
18.96470	12.39376	25.53563	8.915322	29.01407
19.49415	12.67603	26.31227	9.066738	29.92157
19.75907	12.85717	26.66097	9.203521	30.31462
19.93720	13.00205	26.87236	9.330801	30.54360
20.04498	13.09720	26.99277	9.419264	30.67070
20.11255	13.15988	27.06522	9.479352	30.74575
20.15439	13.19983	27.10895	9.518314	30.79047
20.18041	13.22513	27.13570	9.543223	30.81760
20.19657	13.24101	27.15213	9.558952	30.83419
20.20661	13.25094	27.16228	9.568825	30.84439
20.21284	13.25713	27.16856	9.574997	30.85069
20.21672	13.26099	27.17245	9.578846	30.85459
20.21912	13.26339	27.17486	9.581243	30.85701



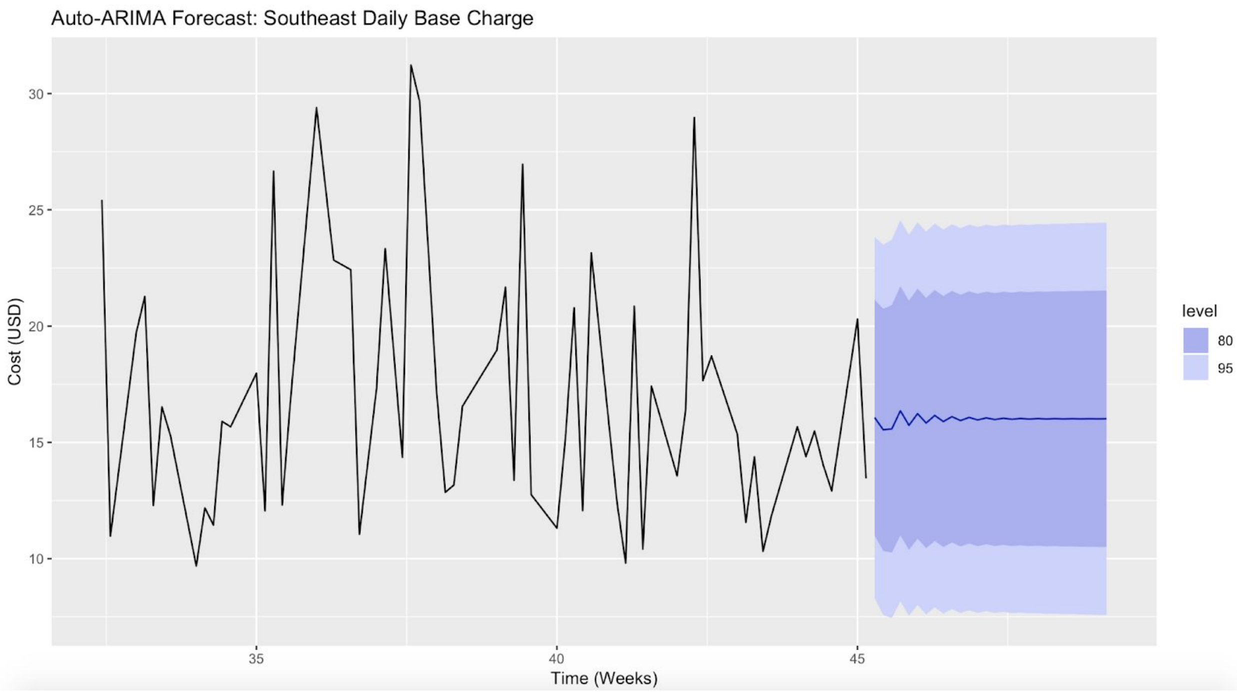
ARIMA(1,1,1)(2,0,0)[7]
AIC=1529.41 BIC=1548.08

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
14.50750	10.87501	18.14000	8.952081	20.06292
14.92389	11.09953	18.74826	9.075036	20.77275
15.31316	11.46349	19.16283	9.425600	21.20072
15.58622	11.73102	19.44142	9.690201	21.48225
15.82325	11.96573	19.68078	9.923679	21.72283
15.39052	11.53139	19.24965	9.488494	21.29255
15.97297	12.11243	19.83351	10.068788	21.87716
15.79057	11.92999	19.65114	9.886328	21.69480
15.50078	11.63970	19.36186	9.595765	21.40579
15.46399	11.60194	19.32605	9.557485	21.37050
15.43097	11.56775	19.29418	9.522692	21.33924
15.39911	11.53468	19.26353	9.488975	21.30924
15.46807	11.60241	19.33373	9.556056	21.38008
15.22704	11.36015	19.09394	9.313140	21.14095



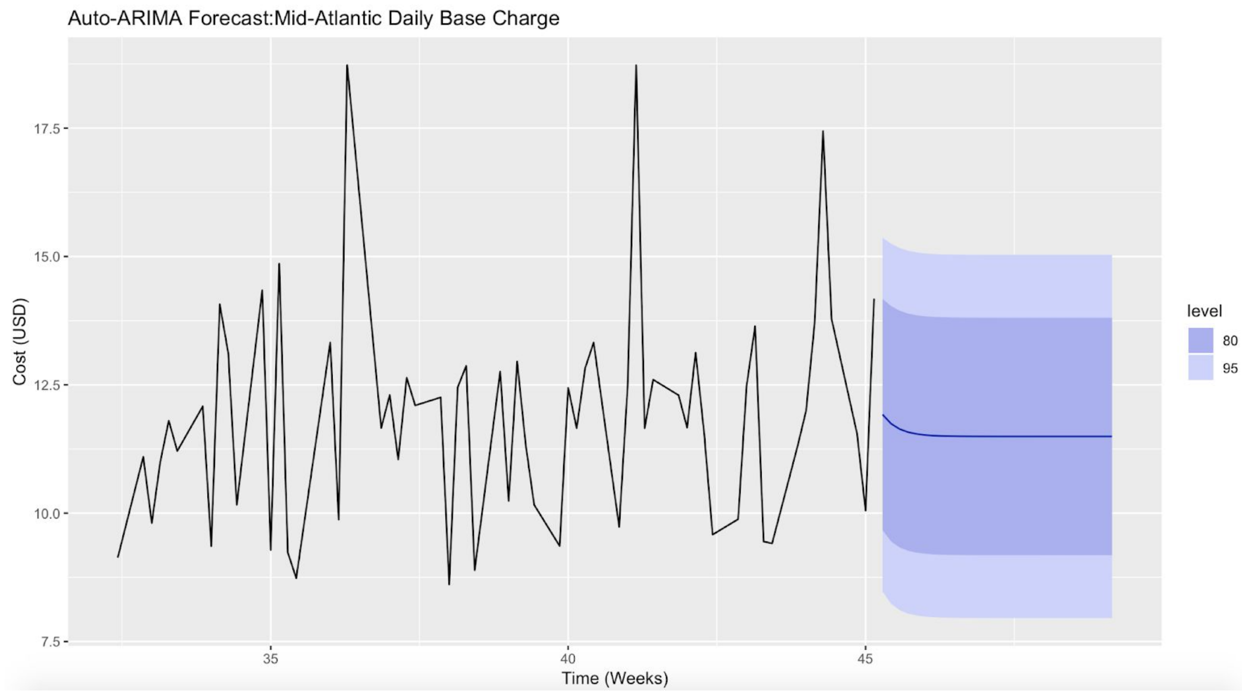
ARIMA(1,0,0)(1,0,0)[7]
AIC=1846.79   BIC=1861.74

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
23.68255	17.63482	29.73028	14.43334	32.93175
23.56985	17.35340	29.78631	14.06260	33.07710
23.53076	17.30489	29.75662	14.00912	33.05240
23.52379	17.29739	29.75019	14.00133	33.04624
23.52446	17.29804	29.75089	14.00196	33.04696
23.55433	17.32790	29.78076	14.03183	33.07684
23.39435	17.16792	29.62078	13.87185	32.91685
23.41331	17.18363	29.64300	13.88584	32.94 32.9
23.41706	17.18719	29.64693	13.88930	32.94482
23.41836	17.18849	29.64824	13.89059	32.94614
23.41859	17.18872	29.64847	13.89082	32.94637
23.41857	17.18869	29.64845	13.89080	32.94635
23.41758	17.18770	29.64745	13.88980	32.94535
23.42291	17.19303	29.65279	13.89513	32.95068



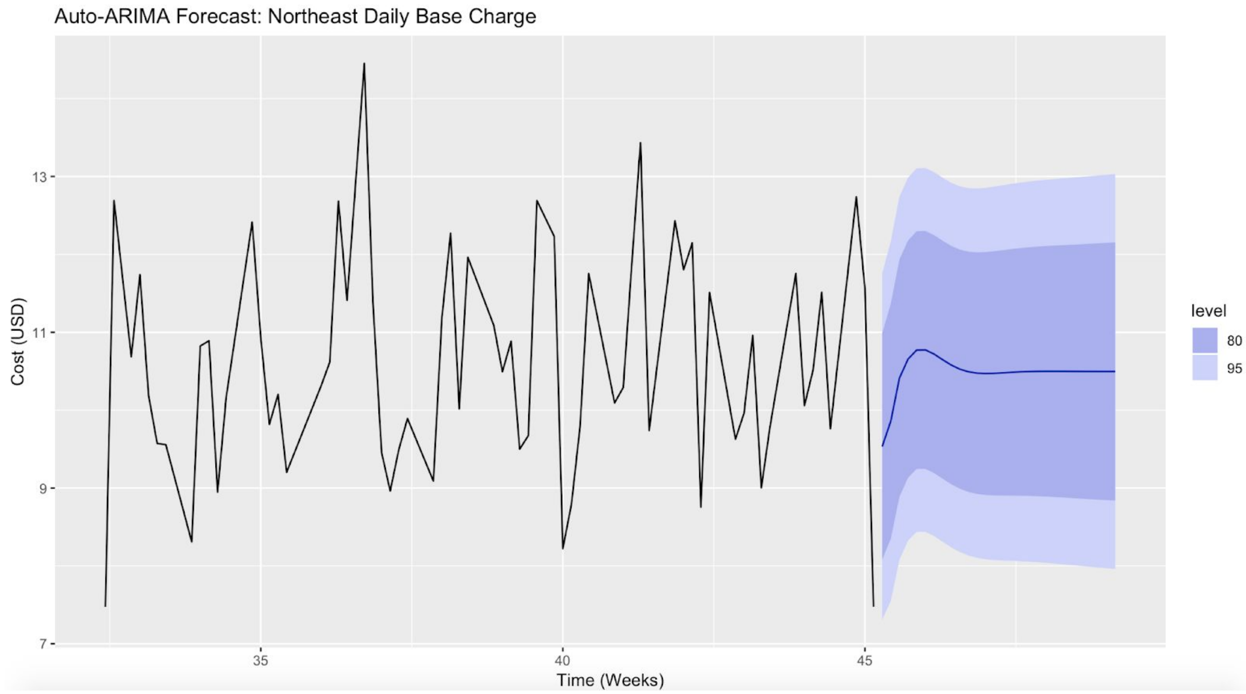
ARIMA(1,1,4)
AIC=1736.27 BIC=1758.67

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
16.06675	10.99426	21.13925	8.309037	23.82447
15.54101	10.33457	20.74746	7.578444	23.50358
15.58194	10.26212	20.90175	7.445984	23.71789
16.35873	11.00221	21.71526	8.166631	24.55083
15.73416	10.37747	21.09086	7.541811	23.92652
16.23634	10.85681	21.61587	8.009058	24.46362
15.83258	10.45268	21.21248	7.604728	24.06042
16.15721	10.76167	21.55275	7.905447	24.40898
15.89619	10.49906	21.29333	7.641986	24.15040
16.10606	10.69721	21.51491	7.833937	24.37818
15.93732	10.52563	21.34901	7.660855	24.21379
16.07299	10.65182	21.49416	7.782032	24.36395
15.96391	10.53891	21.38890	7.667096	24.26072
16.05162	10.61847	21.48476	7.742332	24.36090



ARIMA(1,0,1)
AIC=1234.41 BIC=1249.36

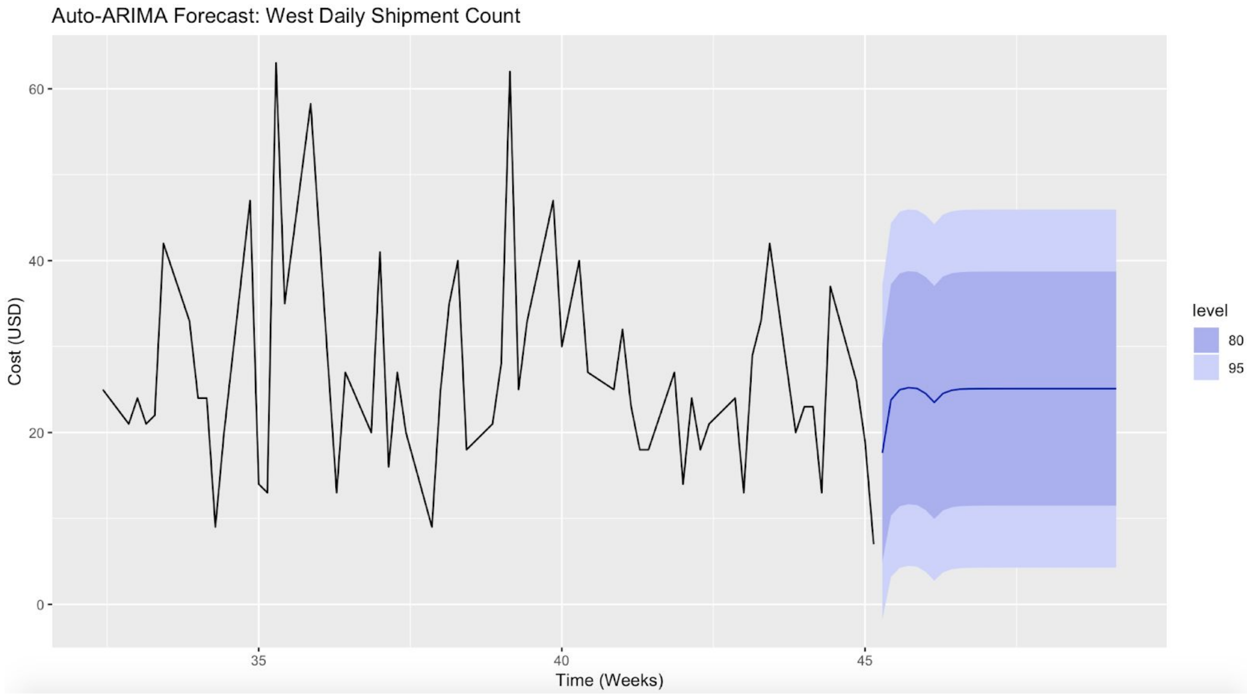
Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
11.92153	9.669177	14.17389	8.476851	15.36622
11.74287	9.450511	14.03522	8.237011	15.24872
11.63890	9.333164	13.94464	8.112579	15.16523
11.57841	9.268157	13.88866	8.045183	15.11164
11.54321	9.231432	13.85499	8.007649	15.07877
11.52273	9.210434	13.83503	7.986378	15.05908
11.51081	9.198342	13.82328	7.974193	15.04743
11.50388	9.191348	13.81641	7.967168	15.04059
11.49984	9.187293	13.81239	7.963103	15.03658
11.49750	9.184939	13.81005	7.960745	15.03425
11.49613	9.183570	13.80869	7.959375	15.03288
11.49533	9.182775	13.80790	7.958579	15.03209
11.49487	9.182312	13.80743	7.958116	15.03163
11.49460	9.182043	13.80716	7.957847	15.03136



ARIMA(3,1,2)
AIC=963.56 BIC=985.96

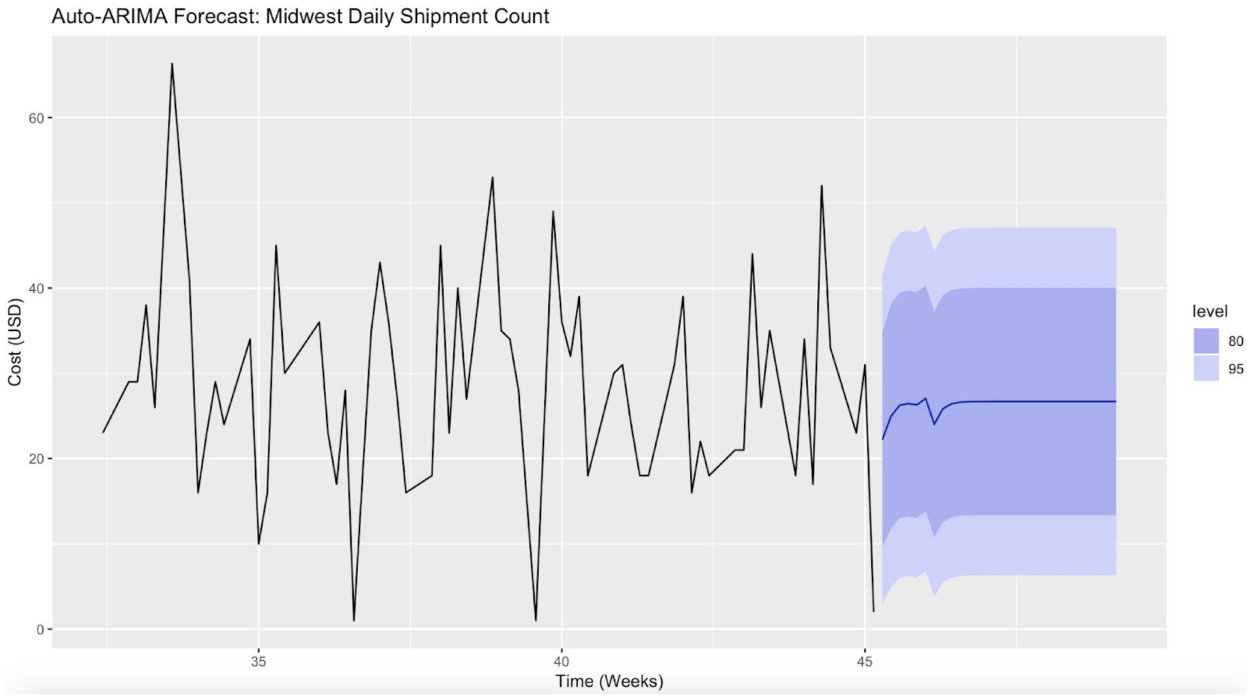
Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
9.532580	8.079917	10.98524	7.310923	11.75424
9.859434	8.350636	11.36823	7.551926	12.16694
10.411909	8.888526	11.93529	8.082096	12.74172
10.655808	9.132338	12.17928	8.325861	12.98575
10.771378	9.245981	12.29678	8.438485	13.10427
10.773647	9.246517	12.30078	8.438103	13.10919
10.721090	9.193740	12.24844	8.385210	13.05697
10.648280	9.120597	12.17596	8.311891	12.98467
10.579558	9.049528	12.10959	8.239578	12.91954
10.526538	8.991468	12.06161	8.178851	12.87422
10.492410	8.950130	12.03469	8.133696	12.85112
10.475088	8.924444	12.02573	8.103583	12.84659
10.470137	8.910893	12.02938	8.085479	12.85480
10.472743	8.905225	12.04026	8.075432	12.87005

APPENDIX F: Client B Daily Shipment Count



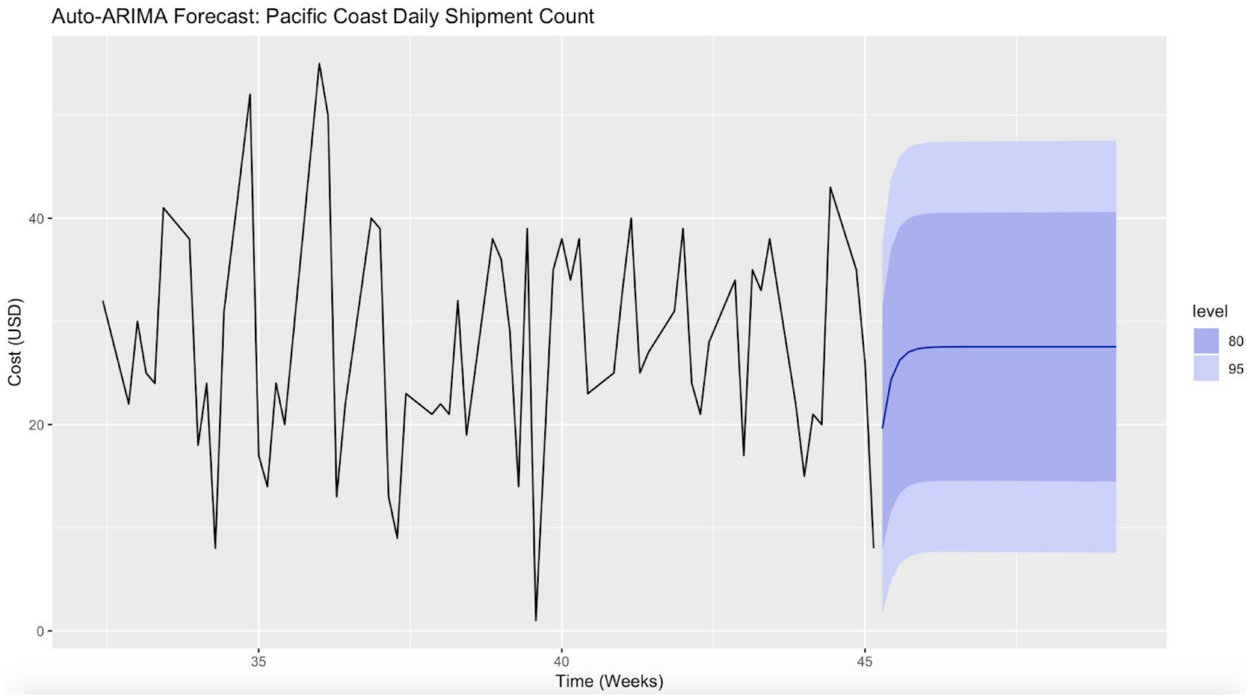
ARIMA(1,0,0)(0,0,1)[7]
AIC=2306.1 BIC=2321.04

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
17.62752	4.944089	30.31094	-1.770112	37.02514
23.78986	10.333623	37.24610	3.210321	44.36940
24.97283	11.422662	38.52300	4.249637	45.69602
25.21908	11.657163	38.78100	4.477918	45.96024
25.13339	11.569999	38.69678	4.389973	45.87681
24.54382	10.980248	38.10740	3.800124	45.28753
23.50859	9.944994	37.07219	2.764858	44.25233
24.54569	10.934389	38.15698	3.729004	45.36237
24.91320	11.295920	38.53047	4.087370	45.73902
25.04343	11.425401	38.66145	4.216454	45.87040
25.08958	11.471457	38.70770	4.262459	45.91669
25.10593	11.487799	38.72406	4.278795	45.93307
25.11173	11.493592	38.72986	4.284588	45.93886
25.11378	11.495646	38.73191	4.286641	45.94092



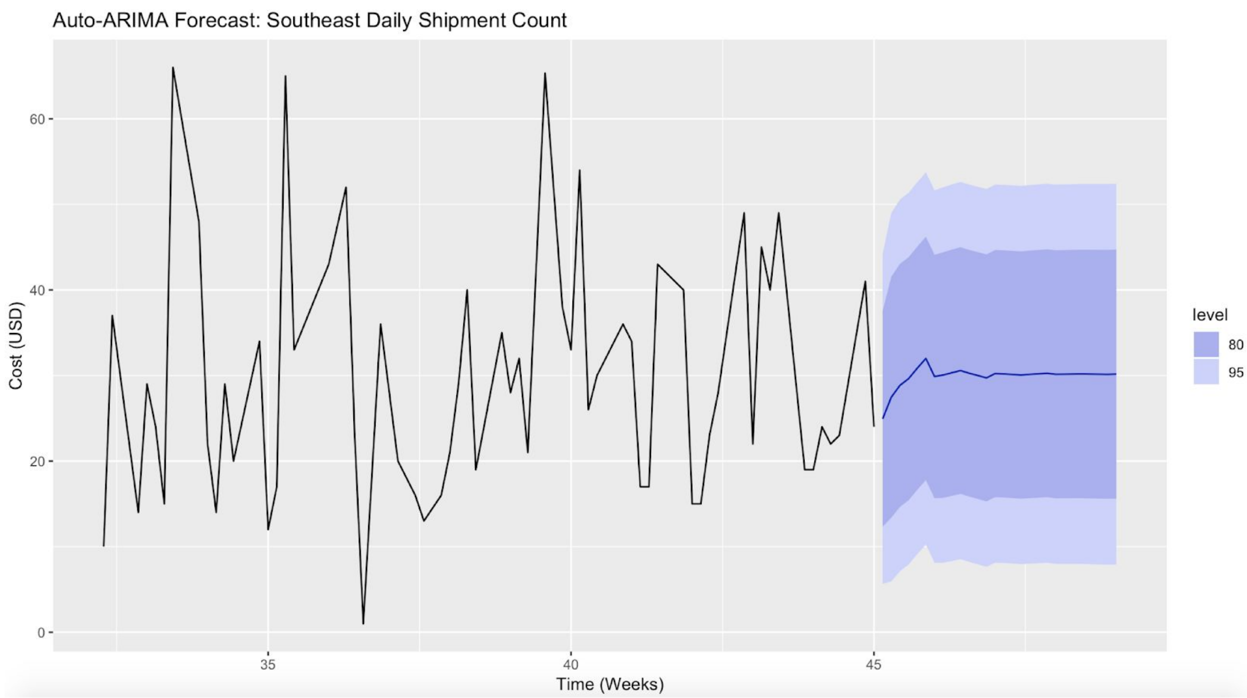
ARIMA(1,0,0)(0,0,1)[7]
AIC=2300.17 BIC=2315.11

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
22.20981	9.647213	34.77241	2.996975	41.42265
24.96533	11.791138	38.13953	4.817139	45.11353
26.25313	13.019485	39.48678	6.014016	46.49225
26.45071	13.211150	39.69027	6.202550	46.69887
26.30401	13.063860	39.54416	6.054948	46.55307
27.05703	13.816820	40.29724	6.807877	47.30618
24.02522	10.785007	37.26544	3.776061	44.27438
25.85167	12.533783	39.16957	5.483717	46.21963
26.42849	13.102879	39.75411	6.048725	46.80826
26.61066	13.284276	39.93704	6.229714	46.99161
26.66819	13.341730	39.99465	6.287127	47.04925
26.68636	13.359891	40.01283	6.305284	47.06743
26.69210	13.365628	40.01857	6.311021	47.07317
26.69391	13.367440	40.02038	6.312833	47.07499



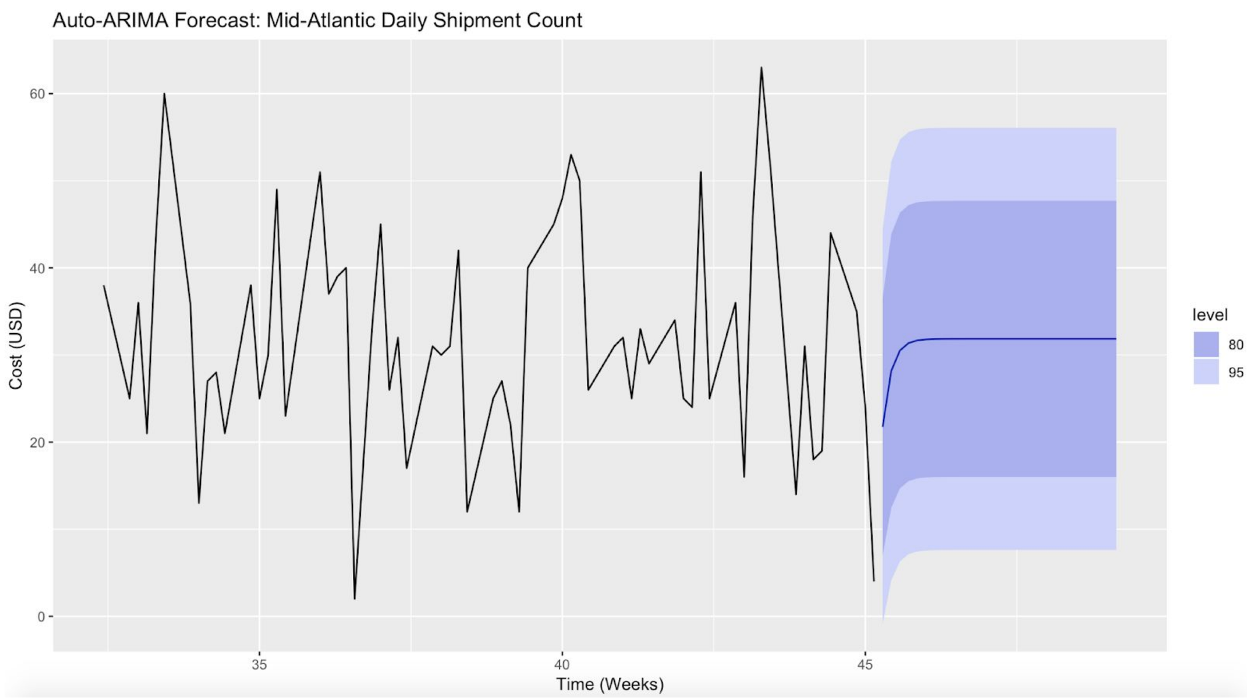
ARIMA(1,1,1)
AIC=2252.43 BIC=2263.63

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
19.60949	7.868875	31.35010	1.653771	37.56520
24.32300	11.584067	37.06193	4.840483	43.80552
26.23671	13.311940	39.16148	6.469980	46.00344
27.01369	14.045242	39.98213	7.180162	46.84722
27.32915	14.346430	40.31186	7.473796	47.18449
27.45722	14.467592	40.44685	7.591296	47.32315
27.50922	14.514909	40.50354	7.636135	47.38231
27.53033	14.532121	40.52855	7.651283	47.40939
27.53891	14.537091	40.54072	7.654346	47.42347
27.54239	14.537088	40.54769	7.652499	47.43227
27.54380	14.535065	40.55253	7.648658	47.43894
27.54437	14.532223	40.55652	7.644007	47.44474
27.54461	14.529049	40.56016	7.639029	47.45018
27.54470	14.525741	40.56366	7.633919	47.45548



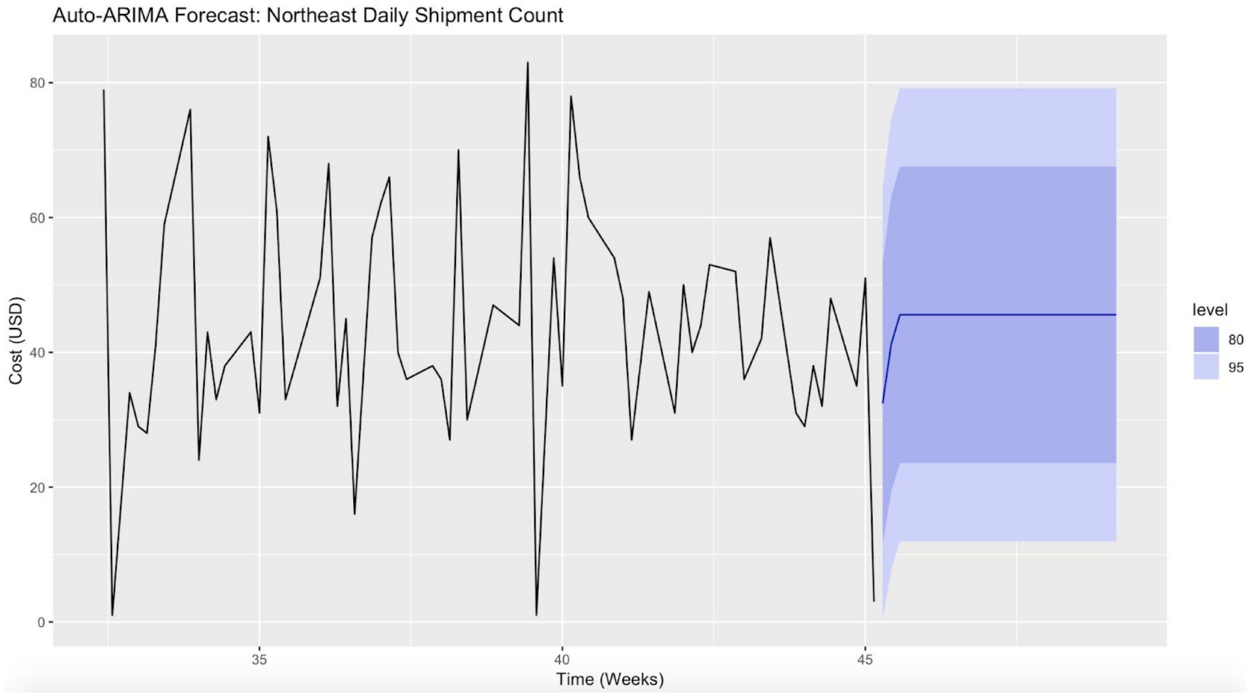
ARIMA(1,1,4)(1,0,1)[7]
AIC=2293.39 BIC=2323.23

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
24.93050	12.32883	37.53218	5.657909	44.20310
27.46274	13.38460	41.54087	5.932091	48.99339
28.84944	14.64943	43.04945	7.132396	50.56649
29.62772	15.42689	43.82855	7.909429	51.34602
30.85071	16.64034	45.06108	9.117825	52.58360
31.99527	17.77770	46.21284	10.251375	53.73917
29.87947	15.65429	44.10464	8.123942	51.63500
30.05881	15.71403	44.40358	8.120362	51.99725
30.32238	15.93132	44.71345	8.313144	52.33162
30.58099	16.17446	44.98752	8.548106	52.61387
30.26889	15.85438	44.68341	8.223794	52.31399
29.99819	15.57429	44.42210	7.938733	52.05765
29.71886	15.28585	44.15188	7.645467	51.79226
30.22808	15.78590	44.67025	8.140676	52.31548



ARIMA(1,0,0)
AIC=2399.17 BIC=2410.38

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
21.73626	6.973797	36.49873	-0.8409811	44.31351
28.17729	12.471509	43.88307	4.1573703	52.19721
30.51638	14.690393	46.34238	6.3126196	54.72015
31.36584	15.524063	47.20762	7.1379334	55.59375
31.67432	15.830467	47.51818	7.4432359	55.90541
31.78635	15.942221	47.63048	7.5548442	56.01786
31.82704	15.982868	47.67120	7.5954724	56.05860
31.84181	15.997638	47.68598	7.6102396	56.07338
31.84718	16.003003	47.69135	7.6156041	56.07875
31.84912	16.004951	47.69330	7.6175524	56.08070
31.84983	16.005659	47.69401	7.6182600	56.08140
31.85009	16.005916	47.69426	7.6185170	56.08166
31.85018	16.006009	47.69436	7.6186103	56.08175
31.85022	16.006043	47.69439	7.6186442	56.08179



ARIMA(0,0,2)
AIC=2614.93 BIC=2629.87

Point.Forecast	Lo.80	Hi.80	Lo.95	Hi.95
32.41194	11.53703	53.28685	0.4865202	64.33737
41.25569	19.37102	63.14037	7.7859666	74.72542
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606
45.57319	23.59491	67.55146	11.9603162	79.18606