Problem Set 3

ECON 30020: Intermediate Macroeconomics

Professor Sims

University of Notre Dame, Spring 2018

Instructions: You may work on this problem set in groups of up to four people. Should you choose to do so, please make sure to legibly write each group member's name on the first page of your solutions. This problem set is due in class on Thursday February 8.

1. **The Black Death:** The Black Death was a pandemic that wiped out somewhere between 30 and 60 percent of the population of Europe in the middle of the 14th century. This problem asks you to analyze what the Solow model would predict after a large, one-time reduction in the population.

   Suppose that the economy is characterized by the Solow Model with labor augmenting productivity. Assume that there is no trend population growth. The period t value of the population/total labor input is N_t and is exogenously given. It is expected that future values of population/labor will be equal to the current value. The key equations of the model for the purposes of this problem are:

   $$K_{t+1} = sA_t K_t^\alpha \left(Z_t N_t\right)^{1-\alpha} + (1-\delta)K_t$$

   $$Y_t = A_t K_t^\alpha \left(Z_t N_t\right)^{1-\alpha}$$

   $$Z_t = (1+z)^t$$

   $$N_{t+h} = N_t$$

   $$N_t \text{ exogenously given}$$

   (a) Define $\widehat{k}_t = \frac{K_t}{Z_t N_t}$ as capital per efficiency unit of labor (and similarly for output, $\widehat{y}_t$). Re-write the capital accumulation equation in terms of $\widehat{k}_{t+1}$ and $\widehat{k}_t$ and similarly for the production function to derive an expression for $\widehat{y}_t$ in terms of $\widehat{k}_t$.

   (b) Solve for algebraic expressions for steady state capital per efficiency unit of labor and output per efficiency unit of labor assuming that A_t is constant at some A^*.

   (c) The real wage is equal to the marginal product of labor, $\frac{\partial Y_t}{\partial N_t}$. Derive an expression for the real wage and write it in terms of the capital stock per efficiency unit of labor. Argue (using math) that when $\widehat{k}_t \to \widehat{k}^*$, the real wage grows at rate z.

   (d) Draw the main Solow diagram for this model and assume that the economy initially sets in the steady state. Suppose that the Black Death happens and reduces the population by $\frac{1}{2}$ (so that N_t declines and is expected to remain at this lower level). Graphically

show what should happen to the capital stock per efficiency unit of labor both in the period of the Black Death as well as dynamically.

(e) Use your analysis from the previous part to draw an impulse response diagram for the log level of output per capita after the Black Death.

(f) What would the model predict would happen to the real wage both in the short run (i.e. the period of the Black Death) and the long run (after the economy has had time to transition to steady state after the shock)? Does your answer conform with what a simple supply-demand for labor analysis would indicate? Explain briefly.

2. GLS, Chapter 6, Exercise 3.

3. GLS, Chapter 7, Exercise 1. Note that there is a small typo in part (d). Instead of saying "i.e. you should create a column with 1, 1.01, 1.02, 1.03, and so on" it should read "i.e. you should create a column with 1, 1.1, 1.2, 1.3, and so on."