Problem Set 4

ECON 40364: Monetary Theory and Policy
Prof. Sims
Fall 2020

Instructions: Please answer all questions to the best of your ability. You may consult with other members of the class, but each student is expected to turn in his or her own assignment. This problem set is due on Sakai by 2:20 pm on Wednesday, October 21.

1. **Yield Curves:** Please go to the following website to access historical daily yields on US government debt of different maturities. Please use Microsoft Excel for all calculations which follow.

   (a) Create a table showing the yields on Treasury securities with maturities of 1, 2, 3, 5, 7, 10, and 20 year maturities on the following specific dates:

      i. December 31, 1993
      ii. April 20, 1995
      iii. December 27, 2000
      iv. April 23, 2004
      v. January 3, 2007
      vi. January 3, 2014

   (b) Create a plot of the yield curve for each of these dates. What does the yield curve "normally" look like? What are a couple of dates where the yield curve looks "different"?

   (c) For each of the given dates, use data on the 1, 2, and 3 year maturity yields to infer market expectations of one year interest rates 1 and 2 years ahead (i.e. back out the one year "forward rates" for each date). In which years was the market expecting rates to decline versus increase? Is your answer consistent with your plots of the yield curves?

2. **The Term Premium in a Micro-Founded Model:** Suppose that you have a representative household that lives for 31 years (t through $t+30$). In period t, it can purchase risk-free (i.e. no chance of default) discount bonds with face value of 1 with maturities ranging from 1 to 30 years. Denote these quantities by $B_{t,t,t+n}$, for $n = 1,\dots,30$ (the first subscript is the date of observation, the second the date of issuance, and the third the date of maturity). These bonds have market prices of $P_{t,t+n}$ for $n = 1,\dots,30$ (we need not keep track of the date of issuance when calculating prices, as all that matters is the time until maturity). The household has an exogenous and known income endowment of Y_t in period t.

   In period t, the household's budget constraint is that its consumption plus its purchases of bonds of all these different maturities add up to its income:

   $$C_t + \sum_{n=1}^{30} P_{t,t+n} B_{t,t,t+n} = Y_t$$

   The household's expected lifetime utility is:

   $$\mathbb{E}[U] = \ln C_t + \beta\,\mathbb{E}\ln C_{t+1} + \beta^2\,\mathbb{E}\ln C_{t+2} + \dots \beta^{30}\,\mathbb{E}\ln C_{t+30} = \mathbb{E}\sum_{n=0}^{30} \beta^n\,\mathbb{E}\ln C_{t+n}$$

The household knows its current income. Future income is uncertain. In particular, with probability p future income in any period is $Y_{t+n} = Y^l$ and with probability $1-p$ it is $Y_{t+n} = Y^h$, for $n \geq 1$ and $Y^h > Y^l$. Income draws across time are independent – e.g. income in period $t+5$ will be low with probability p and high with probability $1-p$ regardless of what income was in periods t through $t+4$.

Bonds are in zero net supply – that is, the household has to simply consume its income each period, regardless of whether income is high or low. We can nevertheless still price all of the bonds in consideration. This means that $C_{t+n} = Y_{t+n}$ for all n.

One can show that, in period t, the price of any bound satisfies:

$$P_{t,t+n} = \mathbb{E}\left[\frac{\beta^n Y_t}{Y_{t+n}}\right]$$

This is just the generic expression that the price of an asset is the expected value of the stochastic discount factor, $\frac{\beta^n u'(C_{t+n})}{u'(C_t)}$, with the payout on the asset, which in the case of these discount bonds is just 1 in period $t+n$ and 0 everywhere else. In writing the above, I have simply imposed bonds being in zero supply and have used the log specification for utility.

(a) Suppose that $Y^h = 1.1$ and $Y^l = 0.9$, with $p = 1/2$ and $Y_t = 1$. Suppose further that $\beta = 0.95$. Create an Excel sheet to solve for the prices of all thirty bonds in period t (hint: this should be fairly straightforward – you type in one formula and then fill it).

(b) Use your sequence of bond prices to solve for the yields on all thirty bonds in period t. Print out a plot of the yield curve.

(c) According to the expectations hypothesis, the price of a long bond ought to equal the product of expected prices of one period bonds. In other words, for a two period bond:

$$P^{eh}_{t,t+2} = P_{t,t+1}\,\mathbb{E}[P_{t+1,t+2}]$$

For an n period bond, we would have:

$$P^{eh}_{t,t+n} = P_{t,t+1}\,\mathbb{E}[P_{t+1,t+2}] \times \mathbb{E}[P_{t+2,t+3}] \times \cdots \times \mathbb{E}[P_{t+n-1,t+n}]$$

Given the information in the problem, the expected future one period bond prices should all be the same, and in fact equal to the current one period bond price. Use this information to calculate the yield curve implied by the expectations hypothesis. Plot this along with your yield curve from (b) in the same graph.

(d) Define the term premium as the difference, for each maturity, between the actual yield to maturity and the yield to maturity implied by the expectations hypothesis. Print out a plot of the term premium.

(e) Consider a *mean-preserving spread*, wherein $Y^h = 1.2$ and $Y^l = 0.8$ (instead of $Y^h = 1.1$ and $Y^l = 0.9$). Re-do previous parts to compute the term premium as a function of the time to maturity, and compare it to your answer from (d).

3. **The Gordon Growth Model:** Suppose that there is a stock which currently pays a dividend of 1 ($D_t = 1$). It is expected that dividends will grow into the future at a constant rate of $g = 0.02$ and will do so forever (i.e. $D_{t+h} = (1+g)^h D_t$ for $h \geq 0$. The discount rate for equity is constant $\kappa^e = 0.05$.

(a) Imposing a no-bubble condition, solve for the price of the stock in period t.

(b) What is the expected price of the stock in $t+1$? What is the expected return? Decompose the expected return into dividend and capital gain.

(c) For the general case (i.e. use symbols, not actual numbers), derive an expression for the dividend component of the expected return (as a function of κ^e and g) and the capital gain component of the expected return (as a function of κ^e and g). As g gets bigger, which term – dividends or capital gains – drives a bigger component of the total expected return?

4. **Bubbles:** Suppose that you have a stock that currently pays a dividend of $D_t = 1$. Future dividends are discounted at a constant rate of $\kappa^e = 0.07$, and dividends are expected to grow at a constant rate forever of $g = 0.02$. The current price of the stock is $P_t = 25$.

(a) What is the magnitude of the bubble term for this stock?

(b) Given your answer on (a), what would you expect the price of the stock to be in $t+1$?

(c) Given your answers on (a)-(b), what is your expected return from holding this stock from t to $t+1$?

(d) Suppose that the bubble bursts in period $t+1$. What is your realized return (not your expected return) from holding this stock from t to $t+1$?