Econ 131
Spring 2021
Emmanuel Saez

Problem Set 3 Solution

DUE DATE: April 21

Student Name:

Student ID:

GSI Name:

- **You must submit your solutions using this template.**
- **Although you may work in groups**, each student must submit individual sets of solutions. You must note the names other students that you worked with. Write their names here:

1. Essay (6 points)

Read the following recent New York Times article on the inequity of property tax assessment. Write a short essay [the essay has to fit in the page below] in light of property taxation in the Tiebout model that we discussed in class. Does it make sense in the context of the Tiebout model to complain about inequitable property taxes?

NYtimes article link:

`https://eml.berkeley.edu//~saez/course131/articleps3.pdf`

2. True/False Statements (10 points)

Determine whether each statement is true, false, or uncertain and explain why. Answers with no explanation will receive no points.

(a) Parks are an example of a pure public good.

UNCERTAIN. Most parks are large enough so that they are non-rival, and few public parks exclude people. It is, however, hypothetically possible to build a fence around a park to exclude people (think of a botanical garden).

(b) There is no reason for the government to impose social distancing to fight the epidemic because private agents can create markets to price the corresponding externality.

False: This is theoretically true based on the famous Coase theorem we discussed in class. However, in practice, this is an externality involving many agents and there is no way in practice that such a market could develop. In practice, the only way to get the proper level of social distancing is through mandated government orders.

(c) As CO2 emissions create a classic externality, the only policy needed to solve the problem is a tax on carbon that would apply in all countries at the same rate and set equal to the marginal damage created by CO2 emissions.

FALSE. This is the classic view of economists derived from the standard model of externalities. However, as we discussed in class, there are great difficulties in pricing the cost of the externality. Furthermore, carbon taxes might suffer from popular opposition as they are regressive (e.g., yellow vests in France, developing countries that want cheap sources of energy). Instead, the key goal should be decarbonization which can happen through a combination of policies: government mandated transition, subsidies to innovation in renewables, development of renewable infrastructure, and possibly a path of increased carbon taxes.

(d) A transfer of $1bn for road maintenance funding from the federal government to the State of California has the same impact on California road maintenance spending as a $1bn cut in federal taxes paid by California residents.

TRUE under the standard model (as long as the desired spending for road maintenance at the state level is above $1bn. Not true in the short-run as the state government takes time to adjust in response to Fed grants or Fed taxes changes (the so-called Flypaper effect that money sticks where it lands).

(e) According to the Tiebout model, local public good provision is efficient and tailored to the tastes of local residents. Hence, it is better to have a fully decentralized government.

FALSE: True if all public goods are local and society does not care about redistribution. False if there are global public goods (like national defense and expertise in solving problems that require coordination such as pandemic fighting). False if society cares about redistribution as local governments cannot do as much for redistribution given mobility threats (see class notes).

3. Externalities (8 points)

There are massive health concerns from cigarettes consumption. The harms are not restricted to those that choose to smoke, but to everyone that circulates in smoking environments. This is a case of negative externality on consumption. Environmental economists estimate that the marginal damage of cigarette consumption is \$2 per pack.

The aggregate demand function for cigarettes is given by $P = 10 - 5Q^D$, where Q is the quantity of pack of cigarettes per day in millions and P reflects the price per cigarette pack in dollars. The aggregate supply function is given by $P = 4 + 1Q^S$.

(a) Solve for the equilibrium private market price and quantity that will be generated without any government intervention.

Solution:

To determine the free market result, we want to set the private marginal benefit equal to the private marginal cost:

$$PMB = PMC$$

$$10 - 5Q = 4 + 1Q$$

$$P_{FM} = \$5/\text{pack}$$
$$Q_{FM} = 1\text{million packs}$$

(b) What is the socially optimal demand function taking into account externalities?

Solution:

We must subtract $MD = 2$ from PMB so that:

$$P = 10 - 5Q^D - MD$$

$$P = 8 - 5Q^D$$

(c) Solve for the socially optimal equilibrium price and quantity.

Solution:

$$SMB = SMC$$

$$8 - 5Q = 4 + 1Q$$

$$Q_{SO} = \frac{2}{3}\text{million packs}$$
$$P_{SO} = \frac{14}{3}/\text{pack}$$

(d) Calculate the dead-weight loss from the externality.

Solution:

$$DWL = \frac{1}{2}(Q_M - Q_S)(MD) = \frac{1}{2}(1 - \frac{2}{3})(2) = \frac{1}{3}$$

(e) If the government uses a cigarette tax to address this externality, what is the optimal tax to offset the externality?

Solution:

The optimal tax for the externality is equal to the marginal damage at the socially optimal, i.e. \$2 per pack of cigarette.

(f) Calculate the revenue that would be raised by this tax.

Solution:

Revenue $= \tau \cdot Q_S =$ \$2/pack $\times\ \frac{2}{3}$million packs per day
Revenue = \$1.33m dollars per day

(g) Will there be deadweight loss associated with this tax? If yes, how much? If no, why not?

Solution:

No, there will be no DWL because this is a Pigouvian tax designed to offset a negative consumption externality. In contrast, there will be a net welfare benefit of the tax (rather than a deadweight loss) equal to the DWL of the externality that is eliminated by the tax.

(h) What are the distributional consequences of the cigarettes tax, is it a regressive, progressive or neutral tax?

Solution:

The answer to this question depends on how the consumption of cigarettes is distributed among individuals. In a society where cigarettes are luxury goods, the cigarettes tax paid as a share of income would probably increase with income, so a cigarette tax would be progressive; while in a society where cigarette use is more generalized, the amount that each individual would pay in cigarette tax would be somewhat constant across income brackets, so the cigarette tax paid as a proportion of income would be decreasing with income, and the cigarette tax would be regressive.

4. Public Goods (7 points)

Jamal and Leisha just had their first baby. They are planning on how much time to dedicate to their private leisure activities versus child caring, which can be thought as a common public good to the household. On their leisure times, Jamal appreciates watching sports on TV, while Leisha prefers to do yoga. After working, sleeping and eating, each spouse has 42 hours a month to devote to child caring (C_i) or leisure activities (L_i).

Jamal's utility over the time spent watching sports games and child caring is given by $U_J = 2\ln L_J + \ln C$ while Leisha's utility over the time spent doing yoga and child caring is given by $U_L = 2\ln L_L + 2\ln C$, where C is the total amount of time spent by both on child caring, given by the sum of each individual's contribution: $C = C_J + C_L$. For this problem we are assuming that both Jamal and Leisha benefit from the increases in the total amount of time they both spend on child caring C but they don't derive utility from the time the other spends on their own leisure activities.

(a) Write down Jamal's utility maximization problem.

Solution:

Jamal's personal leisure time, L_J, can be rewritten as $42 - C_J$ because all the time not spent on child caring (C_J) can be spent on leisure. The public good enjoyed by Jamal can be rewritten as $C_J + C_L$ because the child caring done by either benefits both of them. If Jamal optimizes his own function, he will choose the amount of child caring that maximizes his own utility, taking into consideration the amount of child caring done by the other spouse. Therefore Jamal's utility maximization problem is given by:

$$\max_{L_J, C_J} \; 2 \cdot \ln(L_J) + \ln(C_J + C_L)$$
$$\text{s.t. } L_J = 42 - C_J$$

Which can be rewritten as:

$$\max_{C_J} U_J = 2 \cdot \ln(42 - C_J) + \ln(C_J + C_L)$$

(b) Find Jamal's optimal number of hours devoted to child caring (C_J) as a function of the time on the same task spent by Leisha (C_L).

Solution:

Set $\partial U_J / \partial C_J$ equal to zero:

$$-2/(42 - C_J) + 1/(C_J + C_L) = 0$$
$$2/(42 - C_J) = 1/(C_J + C_L)$$

Cross-multiply:

$$2(C_J + C_L) = 42 - C_J$$

Solving for C_J yields:

$$C_J = (42 - 2C_L)/3 = 14 - \frac{2}{3}C_L$$

This is a response function which allows Jamal to calculate his optimal C_J as a function of the contribution to C made by Leisha.

(c) Now, write down Leisha's utility maximization problem.

Solution:

Leisha's utility maximization problem is given by:

$$\max_{L_L, C_L} \; 2 \cdot \ln(L_L) + 2 \cdot \ln(C_L + C_J)$$
$$\text{s.t. } L_L = 42 - C_L$$

Which can be rewritten as:

$$\max_{C_L} U_L = 2 \cdot \ln(42 - C_L) + 2 \cdot \ln(C_J + C_L)$$

(d) Find Leisha's optimal number of hours devoted to child caring (C_L) as a function of the time on the same task spent by Jamal (C_J).

Solution:

Set $\partial U_L/\partial C_L$ equal to zero:

$$
\begin{aligned}
-2/(42 - C_L) + 2/(C_J + C_L) &= 0 \\
2/(42 - C_L) &= 2/(C_J + C_L)
\end{aligned}
$$

Cross-multiply:

$$C_J + C_L = 42 - C_L$$

Solving for C_L yields:

$$C_L = (42 - C_J)/2 = 21 - \frac{1}{2}C_J$$

This is a response function which allows Leisha to calculate her optimal C_L as a function of the contribution to C made by Jamal.

(e) Use the response functions found in (b) and (d) to find the amount of time Jamal and Leisha spend on child caring and on leisure if they optimize their own functions.

Solution:

Plugging Leisha's response functions into Jamal's response function yields

$$
\begin{aligned}
C_J &= 14 - \frac{2}{3}C_L \\
C_J &= 14 - \frac{2}{3} \cdot \left(21 - \frac{1}{2}C_J\right) \\
C_J &= 14 - 14 + \frac{1}{3}C_J \\
\frac{2}{3}C_J &= 0 \\
C_J &= 0
\end{aligned}
$$

Then Leisha will spend $C_L = 21 - \frac{1}{2}(0) = 21$ hours child caring and $L_L = 42 - 21 = 21$ hours doing yoga, while Jamal will spend 42 hours watching sports and won't contribute to the child care at all.

(f) From a utilitarian perspective (maximizing aggregate utility), what is the socially optimal amount of time they should spend on each task? Assume that the child's utility doesn't enter the planner's social welfare function.

Solution:

The social planner maximizes $U_J + U_L$ by choosing $\{L_L, L_J, C_L, C_J\}$ subject to the budget constraints $42 = L_J + C_J$ and $42 = L_L + C_L$. This is equivalent to maximizing the following Lagrangian:

$$
\begin{aligned}
max_{\{L_L,L_J,C_J,C_L\}} L = {} & [2\ln(L_J) + \ln(C_L + C_J)] + [2\ln(L_L) + 2\ln(C_L + C_J)] \\
& + \lambda_1(42 - L_J - C_J) + \lambda_2(42 - L_L - C_L)
\end{aligned}
$$

or

$$
\begin{aligned}
max_{\{L_L,L_J,C_J,C_L\}} L = {} & 2\ln(L_J) + 2\ln(L_L) + 3\ln(C_L + C_J) \\
& + \lambda_1(42 - L_J - C_J) + \lambda_2(42 - L_L - C_L)
\end{aligned}
$$

Which gives first-order conditions:

(i) $\frac{2}{L_J} - \lambda_1 = 0$

(ii) $\frac{3}{C_L + C_J} - \lambda_1 = 0$

(iii) $\frac{2}{L_L} - \lambda_2 = 0$

(iv) $\frac{3}{C_L + C_J} - \lambda_2 = 0$

(v) $42 - L_J - C_J = 0$

(vi) $42 - L_L - C_L = 0$

There are a bunch of constraints, but they simplify quickly. Notice from (ii) and (iv) that $\lambda_1 = \lambda_2 = \frac{3}{C_L + C_J}$. Then using $\lambda_1 = \lambda_2$, we know from (i) and (iii) that $\frac{2}{L_J} = \frac{2}{L_L}$ or $L_J = L_L$ which given the time constraints in (v) and (vi) imply in turn that $C_J = C_L$ or $C = 2C_J = 2C_L$. Then, from (i) and things we have derived we know that $\frac{3}{C_L + C_J} = \frac{2}{L_J}$, or $\frac{3}{2C_J} = \frac{2}{42 - C_J}$ which delivers $4C_J = 126 - 3C_J$. Solving, we get $L_i = 18$ or $C = C_J + C_L = 18 + 18 = 36$ and $L_J = L_L = 24$.

(g) Is the answer for (f) different than (e)? If so, why? If not, why not?

Solution: Intuitively, in the computation in part (e), we set the marginal utility of the last hour of child caring to each spouse equal to the marginal utility of personal leisure for that spouse. In part (f), we set the sum of the marginal utilities of the last hour of child caring the social marginal utility - equal to the marginal utility of leisure for either spouse. Since the social marginal utility of child caring exceeds the individual marginal utilities of that hour of child caring, a central planner optimally chooses more time on child caring than individuals would if they were acting alone.