Financial Econometrics, Econ 40357
Macro and Financial Time Series

N.C. Mark

University of Notre Dame and NBER

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Concepts to cover

- What a time series is
- Gross and rates of return
- Asset prices and returns
- Statistical independence and dependence
- Time series model
- Different characteristics of alternative assets
- The yield curve

What is a time series?

- It is a sequence of observations over time.
- Observations can be discrete or continuous.
- In this course we deal only with discrete time series.
  - Annual
  - Quarterly
  - Monthly
  - Weekly
  - Weekly
  - Every 5 minutes

Gross return and rate of return

- Rate of return r
- Gross return

$$R = 1 + r$$

- The **log** approximation:

$$\ln(1 + r) \simeq r$$

can be used if r is small (like 0.06, but not when $r = 0.4$).

Explanation ▸ Log approx explanation

Equity One period holding return

D_t is dividend paid between t to $t+1$. P_t is stock price

$$\text{Gross return} = (1 + r_{t+1}) = \frac{P_{t+1} + D_t}{P_t} = \frac{P_{t+1}}{P_t} + \underbrace{\frac{D_t}{P_t}}_{\text{dividend yield}}$$

Subtract 1 to get rate of return,

$$r_{t+1} = \frac{P_{t+1} + D_t}{P_t} - 1$$

If **asset is a coupon bond**, D_t is the coupon payment.

Multi-period holding return (equity)

We going to impound dividends into the price. is $P_{d,t+1} = P_{t+1} + D_t$.
Two-period holding return

$$r_{t,t+2} = \frac{P_{d,t+2}}{P_t} - 1 = \underbrace{\frac{P_{d,t+2}}{P_{d,t+1}}}_{(1+r_{t+2})}\underbrace{\frac{P_{d,t+1}}{P_t}}_{(1+r_{t+1})} - 1 = (1 + r_{t+1})(1 + r_{t+2}) - 1$$

Add 1 to both sides to get the two-period **gross return**.

$$(1 + r_{t,t+2}) = (1 + r_{t+1})(1 + r_{t+2})$$

For **small** returns, log approximation can be used

$$r_{t,t+2} \simeq r_{t+1} + r_{t+2}$$

$k-$**period** gross holding return,

$$(1 + r_{t,t+k}) = \prod_{j=1}^{k} (1 + r_{t+j})$$

Inflation adjusted real returns

Deflate by CPI inflation. Denote teh CPI by CPI_t.

$$P^{\text{real}} = \frac{P^{\text{nom}}}{CPI}$$

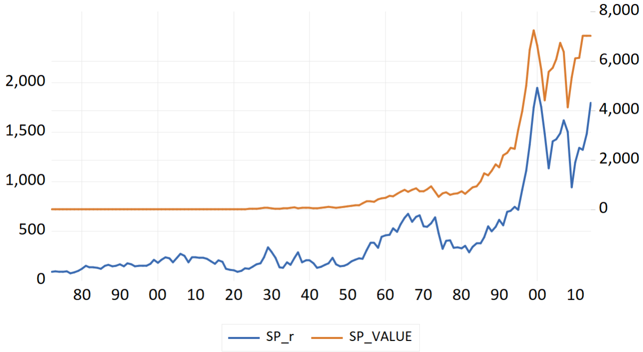
$$r_t^{\text{real}} = \frac{P_t^{\text{real}}}{P_{t-1}^{\text{real}}} - 1 = \frac{P_t^{\text{nom}}}{P_{t-1}^{\text{nom}}} \underbrace{\frac{CPI_{t-1}}{CPI_t}}_{(1+\pi_t)^{-1}} - 1$$

$$= \frac{1 + r_t^{\text{nom}}}{1 + \pi_t} - 1$$

The log approximation gives

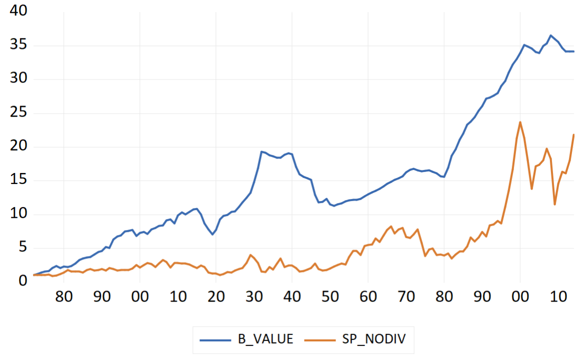
$$r_t^{\text{real}} = r_t^{\text{nom}} - \pi_t$$

Stocks and bonds over the long run (Shiller annual data)



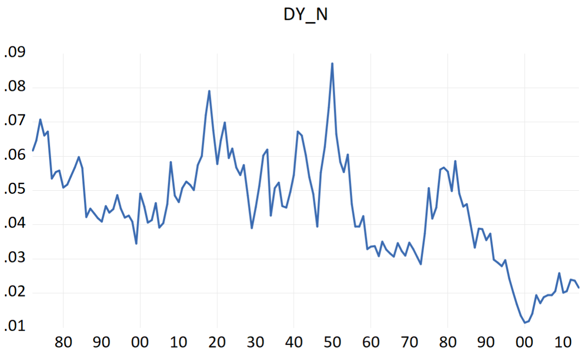
SP_r is real S&P price (left SP)
scale_VALUE is value of $1 invested in 1872 with dividend reinvestment. Note: Different scales.

Stocks and bonds: Accumulation of Value



B_VALUE: real value of $1 invested in 1 year TBill and reinvested
SP_NODIV: real value of $1 invested in S&P without dividend reinvestment

The Dividend Yield



	DY
Mean	0.044776
Maximum	0.087085
Minimum	0.011413
Std. Dev.	0.015137

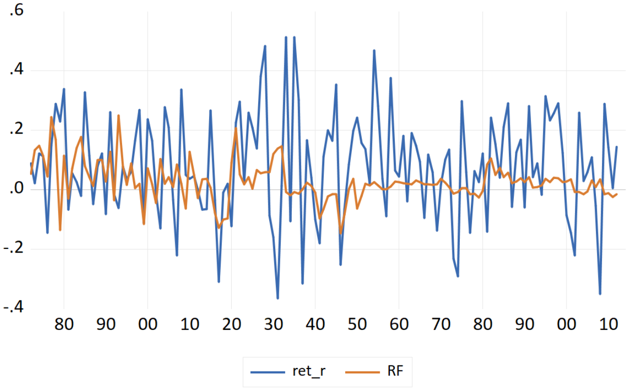
Independence, Dependence

- Statistical Independence
- Independent observations over time are
  - random shocks. Unforecastable. Unpredictable.
  - not particularly interesting in themselves
  - default model of ‘news’ or ‘innovations.’
- Time series are interesting to the extent that there is some **dependence** over time or across variables (assets).
- We want to **model** and **estimate** the dependence, to make sense of the macro and financial world.

Time-series models

- Why do we need models? What is the purpose?
- Testing a model. Testing the implications of a model.
- All models are false. Some are useful.

Stocks and bonds over the long run



Different assets have different return characterisitics

ret_r: Annual rate of return S&P

RF: Annual real T-bill rate (risk free return).

Stock and bond returns

	RET	RF
Mean	0.0799	0.02707
Median	0.0772	0.02029
Maximum	0.5144	0.2509
Minimum	-0.3654	-0.1470
Std. Dev.	0.1771	0.0656
Skewness	-0.0649	0.4850
Kurtosis	2.9152	4.7597
Jarque-Bera	0.1410	23.7136
Probability	0.9319	7.09E-06

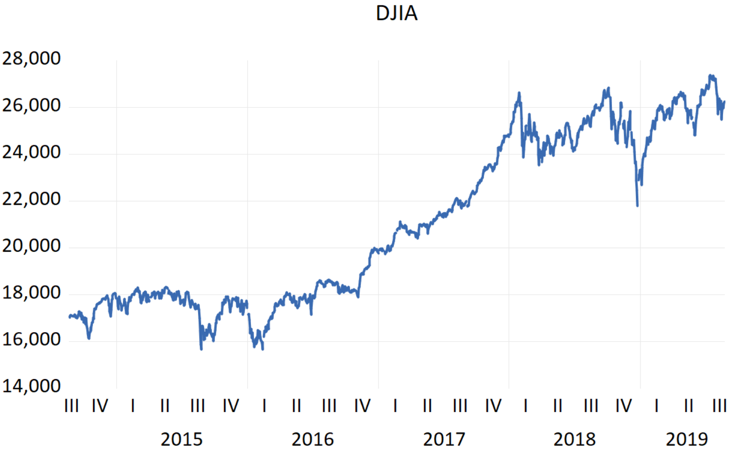
Excess returns: Subtract the risk-free rate from the return on an asset.

$$r_t^e = r_t - r_t^f$$

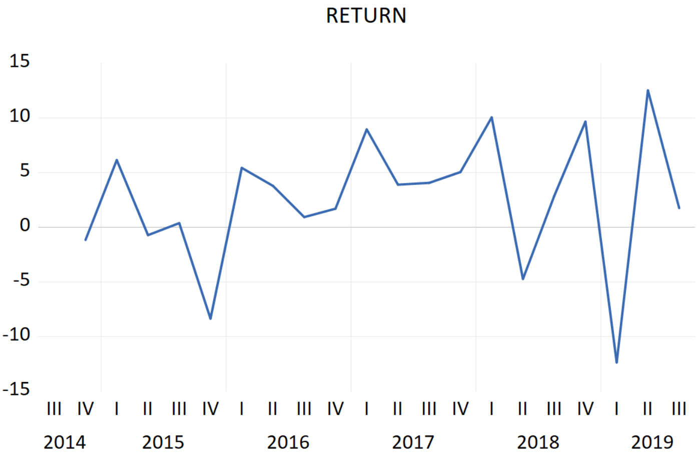
Why do people like to do this?
What is the S&P mean excess return?

Characteristics Change with Sampling Frequency

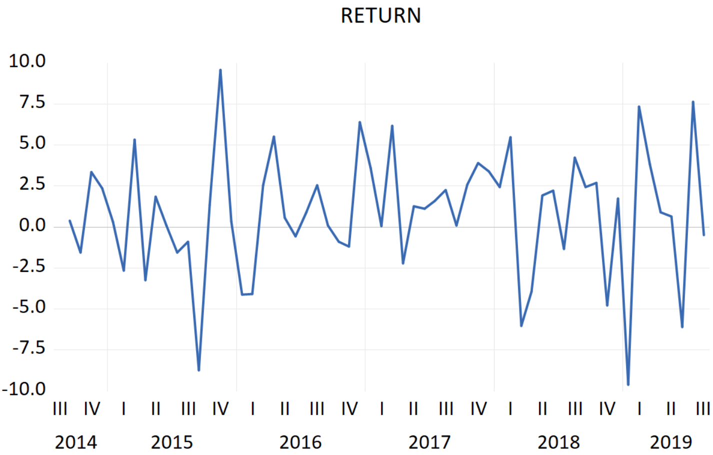
Let's look at plots of DJIA price and retunrs



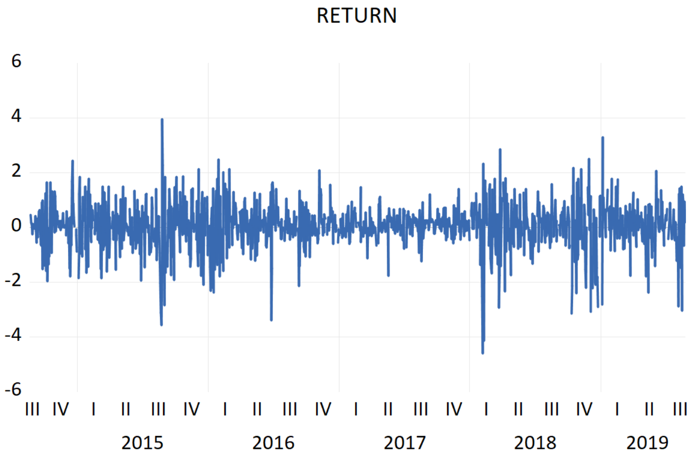
Quarterly returns



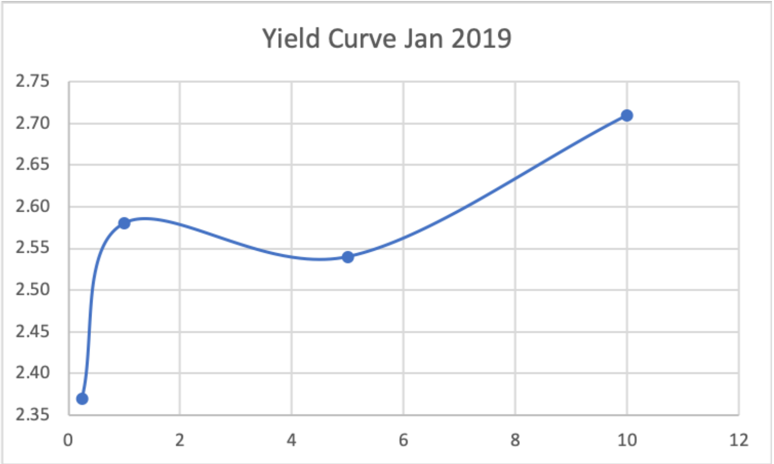
Monthy returns



Daily returns



The yield curve in January



Yields on 3Mo, 1, 5, and 10 year Treasuries. Upward sloping is the normal state. What's the interpretation?

Interpretation of yield spread

$$P_{1,t}u'\left(c_t\right) = \beta E_t u'\left(c_{t+1}\right)$$
$$P_{10,t}u'\left(c_t\right) = \beta^{10} E_t u'\left(c_{t+10}\right)$$

$P_{1,t} = \frac{1}{1+r_{1,t}}, P_{10,t} = \frac{1}{\left(1+r_{10,t}\right)^{10}}$.

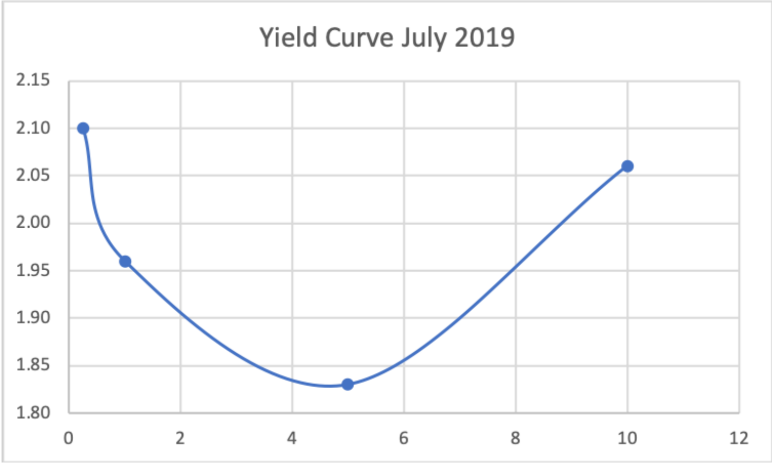
Assume deterministic world, $u\left(c\right) = \ln\left(c\right), u'\left(c\right) = \frac{1}{c}$. Make substitutions

$$\frac{1}{1+r_{1,t}} = \beta\left(\frac{c_t}{c_{t+1}}\right), \quad \frac{1}{\left(1+r_{10,t}\right)^{10}} = \beta^{10}\left(\frac{c_t}{c_{t+10}}\right)$$

Take logs and multiply through by -1

$$r_{1,t} = \left(\ln\left(c_{t+1}\right) - \ln\left(c_t\right)\right) - \ln\left(\beta\right)$$
$$r_{10,t} = \frac{1}{10}\left(\ln\left(c_{t+10}\right) - \ln\left(c_t\right)\right) - \ln\left(\beta\right)$$

The yield curve in July



Log approx explanation

First-order taylor approximation of an arbitrary, continuously differentiable function about the point x_0,

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + R \tag{1}$$

where R is the approximation error. Let $x = 1 + r$, and f be the log function, and expand about $x_0 = 1$ (i.e., $r_0 = 0$), then $\ln(1 + r) \simeq r$ for 'small' r. Back to Gross return and rate of return

Rule of 70 explanation

Start with 1. Find the value of n^* that makes this true:

$$\underbrace{1}_{\text{Starting}} (1 + r)^{n^*} = 2 \tag{2}$$

Solve for n^*,

$$n^* = \frac{\ln(2)}{\ln(1 + r)} \simeq \frac{0.693}{r} = \frac{69.3}{r(100)} \simeq \frac{70}{r(100)} \tag{3}$$

Back Rule of 70