Practice Problem Set 4.1

ECON 30020: Intermediate Macroeconomics
Professor Sims
University of Notre Dame, Spring 2018

Instructions: This problem set covers material covered in class the week with class meetings of February 13 and February 15. It is only for practice in preparation for the upcoming midterm. You need not turn anything in. Solutions will be made available by the end of the business day on Friday February 16.

1. **Precautionary Saving with Different Utility Functions:** Suppose that a household is uncertain about its future income. Income can take on two different values in the future, with $Y_{t+1}^h \geq Y_{t+1}^l$. It takes on the h state with probability p (between 0 and 1) and the l state with probability $1-p$. The flow budget constraints must hold (with equality) in both states of the world. The now three flow budget constraints imposing the terminal condition are:

$$C_t + S_t = Y_t$$

$$C_{t+1}^h = Y_{t+1}^h + (1+r_t)S_t$$

$$C_{t+1}^l = Y_{t+1}^l + (1+r_t)S_t$$

The household's objective is to maximize expected lifetime utility, or:

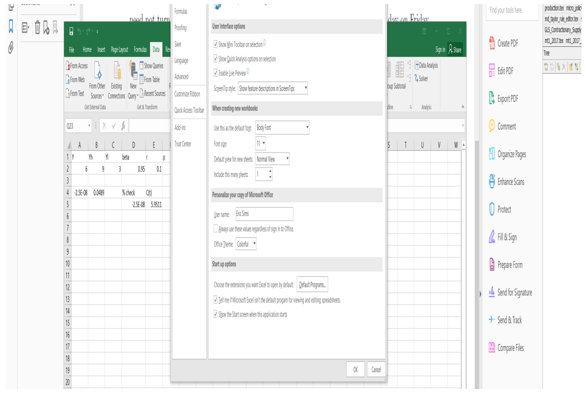
$$E[U] = u(C_t) + \beta \times p \times u(C_{t+1}^h) + \beta \times (1-p) \times u(C_{t+1}^l)$$

(a) Write the optimization problem as one of choosing S_t and turn it into an unconstrained problem. That is, write C_t, C_{t+1}^h, and C_{t+1}^l in terms of S_t, and then take a derivative with respect to S_t and set it equal to zero. Derive the Euler equation. Argue that if $Y_{t+1}^h = Y_{t+1}^l$, $p = 1$, or $p = 0$, this is exactly the same Euler equation we derived in the case where future income is known in period t.

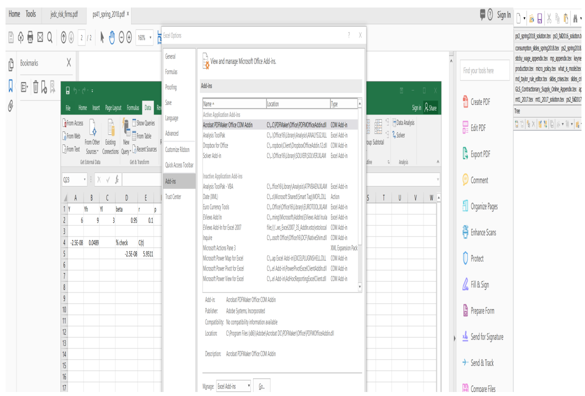
(b) Now suppose that the utility function is the natural log – $u(C_t) = \ln C_t$. Suppose further that $r_t = 0.1$ and $\beta = 0.95$. Suppose that $Y_t = 6$, $Y_{t+1}^h = 9$, and $Y_{t+1}^l = 3$. Assume $p = 0.5$. What is the expected value of Y_{t+1}?

(c) Now numerically solve for the optimal level of S_t using Excel's solver command. To get this in Excel 2016 you to do an add-in. Do the following steps:

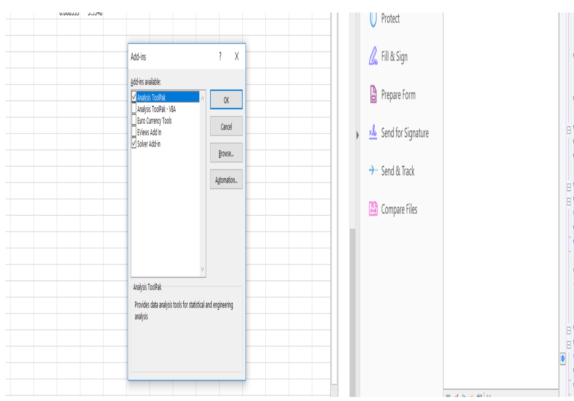
- Click on "file" and go down to "options"
- In the left panel of the box that opens there is a "Add-ins" option. (see screen shot)



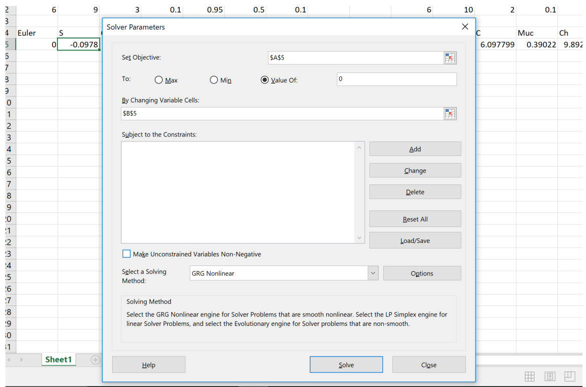
- Click on “Add-ins” and a new box will appear. If you don’t have “Solver add-in” in the “Active” applications select it under the “Inactive” heading. Then click “go” after the “Manage” dialogue



- Make sure “Solve Add-in” is checked in the following dialogue box.



Now you have the solver add-in. Following the instructions here to find the value of S_t which makes the Euler equation hold, given Y_t, Y_{t+1}, r_t, β, and p. Use this to determine the optimal level of C_t. Effectively what the numerical solver is doing is this: it is guessing a value of S_t and seeing whether the Euler equation you derived holds given that guess. It is then iterating on that guess until it finds the value of S_t where the Euler equation holds exactly. Note that when using the solver you *do not* want to impose that "unconstrained variables be non-negative" (i.e. you want to allow for the possibility that S_t is negative – make sure the relevant button in the screen shot below is not clicked):



(d) Now suppose that $Y_{t+1}^h = 10$ and $Y_{t+1}^l = 2$, with everything else the same. Repeat this process. What happens to the optimal C_t and S_t? Is there a precautionary savings motive here?

(e) Instead suppose that the utility function is given by the following:

$$u(C_t) = C_t - \frac{a}{2}C_t^2$$

What is the marginal utility of consumption? What must be true about the value of a for this to be positive?

(f) Suppose that this is the utility function. Assume a value of $a = 0.1$. Repeat steps (c) - (d) with this utility function. What happens to the optimal S_t and C_t when future income becomes more uncertain?

(g) What explains the differences in your answers on (d) compared to (f)?

2. In the 1950s and 1960s, it was common practice to estimate regressions of the following sort:

$$C_t = \alpha + \gamma Y_t + u_t$$

The hope would be that $\gamma = \frac{\partial C_t}{\partial Y_t}$, the marginal propensity to consume (MPC). If changes in income observed in the data are persistent in the sense that changes in Y_t are positively correlated with changes in future income (i.e. Y_{t+1}), will this regression identify the MPC? Why or why not?

3. Suppose that you are a policymaker and are interested in stimulating consumption spending. You want to give households a tax cut (effectively more income). On the basis of the permanent income hypothesis (PIH), how will the stimulative effect of the tax cut depend upon the persistence of the tax cuts? Explain briefly.

4. Evaluate the following statement. “Since the drop in income in retirement is (to a large degree) predictable far in advance, according to the PIH we should not observe much of a drop in consumption when people actually do retire.”