Public Economics (ECON 131)
Section #8: Savings and Corporate Taxation

March 17, 2021

Contents

1 Savings Taxation (Cont.)

1.1 Practice problems

1.1.1 Gruber, Ch. 22, Q.13 (Modified)

Consider again the model covered in Section 7 in which individuals live for two periods and have utility functions of the form $U(C_1, C_2) = \ln C_1 + \ln C_2$, but now we won't fix the parameters, but solve for a more general case: Individuals earn some income W in the first period and save with an interest rate r to finance consumption in the second period.

(a) Set up the individual's lifetime utility maximization problem. Solve for the optimal C_1 and C_2. (Hint: Rewrite C_2 in terms of income, C_1, and r.)

(b) Again, the government imposes a 20% tax on interest income. Solve for the new optimal levels of C_1 and C_2.

Solutions:

(a) Consumption in the second period is savings from the first period plus interest.

Savings is just income from the first period minus consumption during the first period:

$$C_2 = (W - C_1)(1 + r)$$

The utility maximization problem is

$$\max_{C_1, C_2} \ \ln C_1 + \ln C_2$$

$$\text{s.t. } C_2 = (W - C_1)(1 + r)$$

When the budget constraint is incorporated into the expression for C_2, as shown, the maximization problem is

$$\max_{C_1} \ \ln C_1 + \ln[(W - C_1)(1 + r)]$$

or

$$\max_{C_1} \ \ln C_1 + \ln(W - C_1) + \ln(1 + r)$$

Solving, the first-order condition is

$$\frac{1}{C_1} + \frac{-1}{(W - C_1)} = 0$$

From where we get

$$\frac{1}{C_1} = \frac{1}{(W - C_1)} \quad \text{or} \quad W - C_1 = C_1$$

Using the first-order condition, we get the following

$$C_1 = \frac{W}{2}$$

$$C_2 = (W - C_1)(1 + r) = \frac{W}{2}(1 + r)$$

(b) A tax on interest changes computation of C_2 to include a tax of 20% on interest earned on savings:

$$C_2 = (W - C_1)(1 + r) - 0.2(r(W - C_1))$$

The first component of this expression is just savings plus interest; the second component is 20% times the interest earned. This is equivalent to $C_2 = (W - C_1)(1 + 0.8r)$

The utility maximization problem here is

$$\max_{C_1, C_2} \ \ln C_1 + \ln C_2$$

$$\text{s.t. } C_2 = (W - C_1)(1 + 0.8r)$$

When the budget constraint is incorporated into the expression for C_2, as shown, the maximization problem is

$$\max_{C_1} \ \ln C_1 + \ln[(W - C_1)(1 + 0.8r)]$$

or

$$\max_{C_1} \ \ln C_1 + \ln(W - C_1) + \ln(1 + 0.8r)$$

Solving, the first-order condition is

$$\frac{1}{C_1} + \frac{-1}{(W - C_1)} = 0$$

Notice that this is the same FOC as the one we found in part (a). This comes as result of the properties of the log-log utility function (which is a monotonic transformation of a Cobb-Douglas function). Since the FOC is the same, the optimal value for C_1 is going to be the same:

$$C_1 = \frac{W}{2}$$

That is, the total effect of the tax on C_1 was 0, which implies that the income and substitution effect canceled out. Plugging the previous result into the budget constraint we get

$$C_2 = (W - C_1)(1 + 0.8r) = \frac{W}{2}(1 + 0.8r)$$

1.1.2 Adding labor decisions (Model with 3 variables)

Consider again the model covered in Section 7 in which individuals live for two periods, but now we will add labor supply to the individuals decisions. So let's assume that individuals have now utility functions of the form $U(C_1, C_2, l_1) = \ln C_1 + \ln C_2 + \ln(L - l)$ and earn a wage w for each unit of l supplied in period one. They again can save for their retirement with an interest rate r.

(a) Set up the individual's lifetime utility maximization problem. Solve for the optimal C_1, C_2 and l.

Solutions:

(a) Consumption in the second period is savings from the first period plus interest.

Savings is just income from the first period minus consumption during the first period:

$$C_2 = (wl - C_1)(1 + r)$$

The utility maximization problem is

$$\max_{C_1, C_2, l} \ \ln C_1 + \ln(L - l)$$

$$\text{s.t. } C_2 = (wl - C_1)(1 + r)$$

When the budget constraint is incorporated into the expression for C_2, as shown, the maximization problem is

$$\max_{C_1, l} \ \ln C_1 + \ln[(wl - C_1)(1 + r)] + \ln(L - l)$$

or

$$\max_{C_1} \ \ln C_1 + \ln(wl - C_1) + \ln(1 + r) + \ln(L - l)$$

Since we have 2 variables to solve for in the unconstrained maximization problem, we are going to have 2 different first order conditions, one for each variable. Solving, for the first-order condition with respect to C_1 we get

$$\frac{1}{C_1} + \frac{-1}{(wl - C_1)} = 0$$

From where we get

$$\frac{1}{C_1} = \frac{1}{(wl - C_1)} \quad \text{or} \quad wl - C_1 = C_1$$

$$C_1 = \frac{wl}{2} \tag{1}$$

Solving, for the first-order condition with respect to l we get

$$\frac{w}{wl - C_1} + \frac{-1}{(L - l)} = 0$$

From where we get

$$\frac{w}{wl - C_1} = \frac{1}{(L-l)} \quad \text{or} \quad wL - wl = wl - C_1$$

$$l = \frac{wL + C_1}{2w} \tag{2}$$

Plugging (2) into (1) and solving for C_1 we get

$$C_1 = \frac{wl}{2} = \frac{w\frac{wL+C_1}{2w}}{2} = \frac{L}{4}\frac{C_1}{4} \quad \text{or} \quad C_1 - \frac{C_1}{4} = \frac{L}{4}$$

Then the optimal C_1 is $\frac{wL}{3}$. Plugging this result into (2) we get

$$l = \frac{wL + C_1}{2w} = \frac{wL + \frac{wL}{3}}{2w} = \frac{L + \frac{L}{3}}{2} \quad \text{or} \quad l = \frac{2L}{3}$$

Finally, plugging the optimal C_1 and l into the budget constraint delivers the optimal C_2

$$C_2 = (wl - C_1)(1+r) = \left(w\frac{2L}{3} - \frac{wL}{3}\right)(1+r) \quad \text{or} \quad C_2 = \frac{wL}{3}(1+r)$$

2 Business Taxation

2.1 Key Concepts

- Choice of entity: Business owners can choose between forming their business as a C-corporation, or as a pass-through (S-Corporations and Partnerships).
- C-corporations and pass-throughs are taxed in different ways:

	C-Corporation	Pass-through
Tax on \$1 of business income	Corporate tax τ_c	Personal tax τ_p
Tax on \$1 distributed to the owners	Dividend tax τ_d	none
Total after-tax income to owner	$(1-\tau_c)(1-\tau_d)$	$(1-\tau_p)$

2.2 Business Taxation and Investment

- An important question is how business taxation and the existence of deductions influences investment.
- In lecture, we assume a simple functional form based on Hall & Jorgenson (1967), where a firm earns profits as a function of capital, $F(K)$, and also faces a cost for using capital r, resulting in capital cost rK.
- Thus, the firm chooses capital to maximize $F(K) - rK$, yielding the first order condition $F'(K) = r$.
- A tax will distort the investment decision if it causes the firm to choose capital K such that $F'(K) \neq r$.

2.3 Practice Problem

(a) An entrepreneur decides to start a new business. The gross profit function is $F(K) = pY - wL$ with $Y = L^{1/2}K^{1/2}$. Assume she'll hire just one employee and will have a fixed labor input $L = 1$ with $w = 2$. The market price is $p = 2$. Her cost of capital is $r = .25$.

(b) Solve for her optimal investment K.

$$\max \pi = F(K) - rK = (2) \cdot L^{1/2}K^{1/2} - 2L - 0.25K = 2K^{1/2} - 2 - 0.25K$$

With first order condition:

$$\frac{\partial \pi}{\partial K} = \frac{1}{K^{1/2}} - 0.25 = 0$$
$$4 = K^{1/2}$$
$$K^* = 16$$

(c) Is there any pure profit?

$\pi = (2) \cdot (1)^{1/2}(16)^{1/2} - 2(1) - 0.25(16) = 2$. So yes, there is pure profit $= 2$.

(d) Let's say she forms the business as an S-corporation. The personal tax rate is $\tau_p = 25\%$, and *no deduction is allowed* for capital expenditures. Solve for her optimal investment K. Is there pure profit?

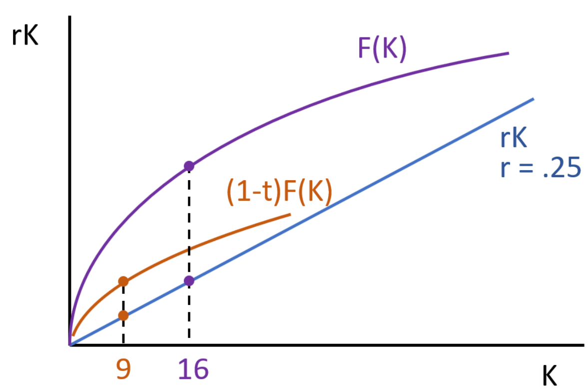
$$\max \pi = (1-\tau_p)F(K) - rK = (1-.25)[(2) \cdot L^{1/2}K^{1/2} - 2L] - 0.25K = (.75)[2K^{1/2} - 2] - 0.25K$$

With first order condition:

$$\frac{\partial \pi}{\partial K} = \frac{.75}{K^{1/2}} - 0.25 = 0$$
$$3 = K^{1/2}$$
$$K^* = 9$$

$\pi = (.75)[(2) \cdot (1)^{1/2}(9)^{1/2} - 2(1)] - 0.25(9) = 3 - 2.25 = 0.75$.

(e) Graph the two solutions (with x-axis K and y-axis rK) on the same graph.



(f) Now assume that a deduction is allowed for capital expenditures (i.e. that rK can be deducted from the firm's tax liability). What is the optimal K now?

$$\max \pi = (1-\tau_p)[F(K)-rK] = (1-.25)[(2)\cdot L^{1/2}K^{1/2}-2L-0.25K] = (.75)[2K^{1/2}-2-0.25K]$$

With first order condition:

$$\frac{\partial \pi}{\partial K} = \frac{1}{K^{1/2}} - .25 = 0$$
$$4 = K^{1/2}$$
$$K^* = 16$$

So with the full deduction for capital expenditure, the optimal K is no longer distorted by the tax.