

Financial Econometrics Econ 40357

Introduction to Forecasting

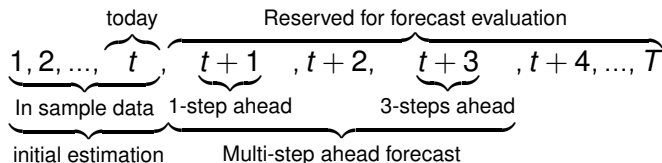
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Forecast evaluation

Time map



- In EViews, Static forecasts give a sequence of one-step ahead forecasts.
- Dynamic forecasts give the multi-period ahead forecasts.
- The estimated parameters are not automatically updated

Forecasts (conditional expectation) and forecast errors

- Illustrate ideas with ARMA(2,1) model (deviation from mean)

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \theta \epsilon_{t-1} + \epsilon_t$$

- One-step ahead forecast (conditional expectation)

$$\hat{y}_{t+1|t} = \hat{\rho}_1 y_t + \hat{\rho}_2 y_{t-1} + \hat{\theta} \hat{\epsilon}_t$$

- One-step ahead forecast error

$$fe_{t+1|t} = y_{t+1} - \hat{y}_{t+1|t}$$

- Two-step ahead forecast

$$\hat{y}_{t+2|t} = \hat{\rho}_1 \hat{y}_{t+1|t} + \hat{\rho}_2 y_t + \underbrace{\hat{\theta} \hat{\epsilon}_{t+1|t}}_0 = \hat{\rho}_1 \hat{y}_{t+1|t} + \hat{\rho}_2 y_t$$

- Two-step ahead forecast error

$$fe_{t+2|t} = y_{t+2} - \hat{y}_{t+2|t}$$

- Similarly, for k -step ahead forecasts and forecast errors.

Random Walk Forecast

Random walk with drift

$$\begin{aligned}y_{t+1} &= \mu + y_t + \epsilon_t \\ \hat{y}_{t+1|t} &= \hat{\mu} + y_t \\ \hat{y}_{t+2|t} &= \hat{\mu} + \hat{y}_{t+1|t} = 2\hat{\mu} + y_t \\ &\vdots \\ \hat{y}_{t+k|t} &= k\hat{\mu} + y_t\end{aligned}$$

Driftless Random Walk (no change) Forecast

Driftless random walk, set $\mu = 0$.

$$\begin{aligned}y_{t+1} &= y_t + \epsilon_t \\ \hat{y}_{t+1|t} &= y_t \\ \hat{y}_{t+2|t} &= \hat{y}_{t+1|t} = y_t \\ &\vdots \\ \hat{y}_{t+k|t} &= y_t\end{aligned}$$

The no-change forecast.

$$\hat{y}_{t+1|t} - y_t = 0$$

This model requires **no estimation**. Can your model beat the no-change forecast?

Estimation windows

- Estimate over fixed sub-sample. $t = 1, \dots, t_0$. Forecast $\hat{y}_{t_0+1|t_0}, \hat{y}_{t_0+2|t_0+1}, \dots, \hat{y}_{T|T-1}$
- Estimate over full sample. $t = 1, \dots, T$. This produces 'in-sample' forecasts. These are pseudo forecasts because they use out-of-sample information. Cheating
- Recursive updated estimates—'Out-of-sample' forecasts.
 - Estimate from $t = 1, \dots, t_0$, forecast $\hat{y}_{t_0+1|t_0}$.
 - Re-estimate from $t = 1, \dots, t_0 + 1$, forecast $\hat{y}_{t_0+2|t_0+1}$, and so on until,
 - Estimate from $t = 1, \dots, T - 1$, forecast $\hat{y}_{T|T-1}$.
- Rolling sample estimates.
 - Estimate $t = 1, \dots, t_0$, forecast $\hat{y}_{t_0+1|t_0}$.
 - Estimate $t = 2, \dots, t_0 + 1$, forecast $\hat{y}_{t_0+2|t_0+1}$, and so on until
 - Estimate $t = T - t_0 + 1, \dots, T - 1$, forecast $\hat{y}_{T|T-1}$ Do this if you suspect substantial parameter instability

Evaluate forecast accuracy

- You have $T - t_1 + 1$ forecast errors, generated from your preferred model and the random walk item.
- Compute accuracy measures for both models and compare.
- Root-mean-square forecast errors (RMSFE).

$$\text{RMSFE} = \sqrt{\frac{1}{T - t_1 + 1} \sum_{t=t_1}^T fe_{t,t-1}^2} \quad (1)$$

- Mean absolute forecast errors (MAFE).

$$\text{MAFE} = \frac{1}{T - t_1 + 1} \sum_{t=t_1}^T |fe_{t,t-1}| \quad (2)$$

Evaluate forecast accuracy

Theil's U statistic: Compare your forecasts to the 'no-change' forecast.

$$U = \frac{RMSFE_{\text{Model}}}{RMSFE_{\text{NC}}} \quad (3)$$

If $U < 1$, you are beating the no-change forecast. Your model has some predictive content.

Evaluate forecast accuracy

Money making ability (economic significance) Sign prediction (buy-sell signals). For returns, let z_t be such that

$$z_t = \begin{cases} 1 & \text{if } (y_{t+1}\hat{y}_{t+1}) > 0 \text{ (same sign)} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

$$\begin{array}{l} \text{Percentage forecasts} \\ \text{with same sign} \end{array} = \frac{100}{T - t_1} \sum_{t=t_1}^{T-1} z_t$$

Your model will be 'successful' if it exceeds 50%

Eviews example

200 observations. Estimate AR(1) on observations 3 to 180. Ask for static forecasts of observations 181 to 200

Equation: UNTITLED Workfile: ARIMA_MODELS:sim_impulse\

View Proc Object Print Name Freeze Estimate Forecast Stats Resids

Dependent Variable: Y
Method: Least Squares
Date: 09/21/19 Time: 16:35
Sample: 3 180
Included observations: 178

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.044413	0.040535	1.095663	0.2747
Y(-1)	0.908610	0.031480	28.86323	0.0000

R-squared	0.825585	Mean dependent var	0.487592
Adjusted R-squared	0.824594	S.D. dependent var	1.195054
S.E. of regression	0.500507	Akaike info criterion	1.464781
Sum squared resid	44.08923	Schwarz criterion	1.500531
Log likelihood	-128.3655	Hannan-Quinn criter.	1.479279
F-statistic	833.0862	Durbin-Watson stat	1.894360
Prob(F-statistic)	0.000000		

Forecast

Forecast of
Equation: UNTITLED Series: Y

Series names
Forecast name: yf
S.E. (optional):
GARCH(optional):

Method
☐ Dynamic forecast
☒ Static forecast
☒ Coef uncertainty in S.E. calc

Forecast sample
181 200

☒ Insert actuals for out-of-sample observations

Output
Graph: Forecast & Actuals
☒ Forecast evaluation

OK Cancel

Eviews example

