Search Intensity, Job Advertising, and Efficiency

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This paper demonstrates that if both firms and workers search the other side of the market for job matches the equilibrium rate of unemployment is likely to be too high. Both sides ignore a positive externality of their search: when they establish a job match they remove from the market a job searcher, so they save society his search costs. I show that there is no feasible wage rate that can internalize this externality under fairly weak restrictions on the technology of search.

In this paper I look at efficiency within a two-sided model of job search. It is well established in the literature that if it is costly to bring together job vacancies and unemployed workers, some search unemployment may be socially efficient. Here I investigate whether the steady-state equilibrium unemployment rate is the socially efficient rate. I show that in general, if both firms and workers search the other side of the market for job matches, there will be "too much" unemployment in equilibrium. This is because both firms and workers ignore a positive externality of their search, so they search "too little." This externality arises from the fact that when one side of the market succeeds in establishing a match

with a member of the other side, the latter is also matched, and so it stops searching. Society saves the latter's search costs from this point on. Firms and workers ignore the other side's costs and returns when they search, so they ignore one of the benefits derived from their job search.

More formally, the positive externality arises because each participant's job-matching probability is an increasing function of the search intensity of participants on the other side of the market. In general, job-matching probabilities depend also on the number of searchers on each side of the market, and this dependence is also ignored by private agents. This introduces a second externality, which also contributes to the inefficiency of equilibrium outcomes. Unlike the first externality, the second could give rise to either too little or too much job search, depending on the technology of search, the conditions used to determine wages, and the level of equilibrium vacancies and unemployment (see Diamond and Maskin 1979; Diamond 1982; Pissarides 1982). However, in this paper I show that if the number of jobs is determined by competitive entry and exit, there will be too little search even when both externalities are present, under fairly weak restrictions on the technology of search. These restrictions amount to assuming that each participant's job-matching probability is an increasing function of the number of searchers on the other side of the market and a decreasing function of the number of searchers on his own side of the market.[1]

The argument of this paper is symmetric, and it can be made either for firms or for workers. It can also be made for any situation where two agents look for each other for a productive pairing—for instance, it could be made for a market like the one studied by Diamond and Maskin (1979) and for situations of social interaction, like Becker's (1973) theory of marriage. Before setting up the formal model, we consider briefly how the first externality highlighted in this paper influences workers, and why neither wages nor any other free-market variable can be adjusted to offset this influence.

Workers decide how much to spend on search on the basis of the wage they expect to receive and the leisure they derive during unemployment.

[1] Dale Mortensen (1982a, 1982b) has also noted, in two independently written papers, the positive externality discussed in this paper. In his model, the probability of a job match is the sum of the probabilities that each side finds the other, so it is possible to assign "property rights" to the job. His analysis is restricted to the study of the efficiency of the intensity of search by each side, with a fixed number of market participants. In this paper I follow the more traditional approach of labor-market analysis which does not assign property rights to the job (e.g., it would be difficult to say who finds whom in a market where both firms and workers look for each other randomly), and which assumes a variable supply of jobs in response to expected profit maximization. As a result, my analysis is better suited to the efficiency analysis of equilibrium unemployment and vacancies, which I shall carry out in later sections.

A socially efficient amount of resources will be devoted to search when the wage rate reflects the social benefits from job acceptance. The latter include the marginal product of the worker and, most important for the argument of this paper, the costs of maintaining a vacancy. Because by accepting a job the worker removes a vacancy from the market, he automatically saves the firm its vacancy costs—mainly advertising costs, and any other fixed costs that have to be paid, such as machine rentals and maintenance costs. Thus if firms also search for workers by advertising their vacancies, the social benefits from job acceptance exceed the marginal product of the worker. The wage which is required to induce the worker to spend a socially efficient amount of resources on job search must also exceed the marginal product of labor by the vacancy costs. But if wage rates exceeded marginal products, no firm would be willing to enter the market, because expected profits would always be negative. If firms are to enter the market the wage rate must be less than the marginal product of labor, and the difference must be sufficient to compensate them for the expected vacancy costs. It follows that the private payoffs from job search must necessarily be below the social, and so workers will search too little. By a perfectly symmetric argument firms advertise too little, and so equilibrium unemployment is above its socially efficient level.

Section I of the paper describes the model, Section II derives the optimal job-searching strategy, and Section III describes the behavior of firms. Section IV carries the individual analysis to the aggregate level and shows how the externalities discussed in this paper arise. These four sections describe how the private economy works for any given level of the wage rate. The analysis is carried out for an economy in steady-state equilibrium, and agents are assumed to maximize the average steady-state flow of income. Since our concern is to show that *equilibrium* unemployment is inefficient, restricting the analysis to steady-state comparisons does not involve the loss of essential generality. The rate of discount, which is not important in this analysis, is omitted from the comparisons.

Section V introduces the social welfare function, defined as the steady-state flow of output, and demonstrates that there is no feasible wage rate that can equate the private job-search decisions with the socially efficient level.

I. Description of the Model

The main argument of this paper requires two critical assumptions. First, not all vacancies and unemployed workers become employed during a finite time interval. Second, the rate at which vacancies and unemployed workers meet can be raised either by firms or workers, but raising it is costly. We shall refer to the rate at which vacancies and unemployed workers meet as the rate of job matchings. Firms can raise this rate by spending more on job advertising, and workers by increasing their search

intensity. These activities are perfectly symmetric, and the difference in the terminology is merely a matter of convenience.

The assumption that not all unemployed workers and vacancies become employed during a finite time interval is common to all models of search, from Stigler (1962) down. The assumption that both firms and workers can raise the rate of job matchings, though implicit in most discussions of job search, is often suppressed in partial-equilibrium models (like, for instance, those in the Phelps et al. [1970] volume). Any other assumptions made here are auxiliary and are not crucial to the arguments of this paper.

The number of workers is assumed to be fixed and is used as the normalizing variable. The number of jobs is variable and determined by zero-profit conditions. Firms are assumed to be small and produce according to a constant returns to scale technology. They may open new jobs by joining the vacancy pool, so in equilibrium the expected profit from joining the vacancy pool must be equal to zero.

Each job-worker pair produces output y, which is shared between the firm and the worker according to the wage rate w. We assume for simplicity that there are no variations in productivities or wages.[2] This implies that neither workers nor firms reject any job offers, and there is no search on the job. Also, there are no quits and no layoffs. Job matchings may not materialize during a period, either because workers may not know where the vacancies are or because they may not know which vacancies other workers will visit. If a vacancy is searched by more than one worker simultaneously, the firm takes one worker at random and returns the others to the unemployment pool.

In order to avoid the asymptotic decline of unemployment to zero, we assume that each period a fraction s of randomly selected jobs are broken up. This creates an inflow into the unemployment pool which, when it is set equal to the outflow, gives the equilibrium rate of unemployment. The job-separation process is assumed to be exogenous, and to arise mainly from structural shifts of demand. A higher separation rate indicates faster structural change in the economy.

Unemployed workers spend c on job search and firms spend a on job advertising; in equilibrium both c and a are nonnegative. In addition, we assume for generality that unemployed workers enjoy leisure z and vacancies cost k to maintain, though the paper's main results do not depend on the existence of positive z and k.

Workers can raise the probability of finding a vacancy by increasing

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[2] A wage-determination model that implies that as long as productivities are the same for all jobs there will be no wage variability is the Nash model, recently applied to this context by Diamond (1982). Of course, there are other models that would imply wage variability, e.g., Butters's (1977) analysis of price dispersion in commodity markets where both sides of the market search could easily be extended to a labor market like the one described here.

their search expenditure, c, and firms can raise their probability of finding a worker by spending more on job advertising, a. Let these matching probabilities be denoted, respectively, by p and q, and write $p = p(c, \cdot)$ where $p_c > 0$ and $p_{cc} \leqslant 0$; and $q = q(a, \cdot)$ where $q_a > 0$ and $q_{aa} \leqslant 0$. Subscripts denote partial derivatives, and the dots indicate the fact that p and q will also depend on other variables which will not, in general, be under the control of the individual worker and firm.

There is a close connection between p, q, and the rate of job matchings, which suggests the other variables influencing p and q and which gives rise to the externalities discussed in this paper. We shall explore this connection in Section IV. First, we derive the optimal c and a chosen, respectively, by workers and firms.

II. Optimal Choice of Search Intensity

Workers choose the search intensity c to maximize the average steady-state flow of income, given the two states, employment and unemployment. The transition probability from employment to unemployment is s, and from unemployment to employment is p. Net income in each state is given by the wage rate w when in employment and by the value of leisure z, net of search costs c, when in unemployment. The problem faced by the unemployed worker is that by choosing a higher c he reduces net income during unemployment, but he also raises the transition probability from unemployment to employment. The optimal c is determined by a marginal condition that balances these two influences.

With transition probabilities s and p, the steady-state flow of income (which, in the limit, corresponds to the expected discounted income flow as the rate of discount tends to zero) is given by

$$U = \frac{p}{s + p} w + \frac{s}{s + p} (z - c). \tag{1}$$

Differentiating with respect to c and setting the partial derivative equal to zero, we derive the optimization condition

$$(w - z + c)p_c = s + p. \tag{2}$$

Condition (2) is then solved for the privately optimal search intensity c.

It can easily be shown that (2) implies $\partial c/\partial w > 0$, $\partial c/\partial z < 0$, $\partial c/\partial s < 0$. An increase in the wage rate, a decrease in the value of leisure, and a decrease in the separation rate increase the relative rewards from holding a job, so they raise the intensity of search.

Unemployed workers will participate and look for a job when the net returns from participation exceed the value of leisure. So in addition to

(2) equilibrium is characterized by $U \geq z$. Substituting from (1), this implies

$$w \geq z + \frac{s}{p} c. \tag{3}$$

Thus there is a lower limit to the feasible wage rate, given by the right-hand side of this inequality. If the wage rate were allowed to fall below this limit unemployed workers would leave the market, because they would not be compensated for the loss of leisure and their expected search costs.

III. Optimal Choice of Job Advertising

The problem facing firms with vacancies is anologous to that facing unemployed workers looking for a job. By spending more on advertising they raise the probability of moving from a state of unemployment (vacancy) to employment, where the rewards are higher. But they also raise their vacancy costs, since advertising expenditure is a component of costs.

The firm's income when the job position is filled is given by $y - w$. When it is vacant, the job position costs $k + a$. A firm moves from employment to vacancy with probability s, and from vacancy to employment with probability q, so its steady-state flow of income is given by

$$V = \frac{q}{s + q} (y - w) - \frac{s}{s + q} (k + a). \tag{4}$$

The firm chooses a to maximize this expression. Differentiating and setting the partial derivative equal to zero, we obtain

$$(y - w + k + a)q_a = s + q. \tag{5}$$

This condition is solved for the optimal level of advertising expenditure.

The comparative statics of a are derived from (5), and they are similar to those of c. An increase in $y - w$, or k, raises advertising expenditure, because it raises the relative rewards from employment. An increase in s reduces it, because it reduces the relative rewards.

Firms will participate only if the expected profits from a vacancy are nonnegative, that is, if $V \geq 0$. Substituting from (5), this may be stated as a condition on wages,

$$w \leq y - \frac{s}{q} (k + a). \tag{6}$$

This condition, combined with (3), gives the feasible range for wages. Unless wages are within the range bounded from below by $z + sc/p$ and from above by $y - s(k + a)/q$, either workers or firms will leave the market. We show below that there is no value of the wage within this range that can ensure that both c and a can be at a socially efficient level.

IV. Job Matchings and Steady-State Equilibrium

The worker and firm optimization conditions (2) and (5) jointly determine the private equilibrium values of search intensity and job advertising. There are two more unknowns in this economy, the equilibrium rates of unemployment and job vacancies. Equilibrium unemployment is determined by the equality of flows into and out of unemployment. Equilibrium vacancies are determined by zero-profit conditions for firms, given freedom of entry and exit. In order to derive these two conditions, we first need to discuss the relation between the individual matching probabilities p and q and the aggregate rate of job matchings.

Let u be the equilibrium rate of unemployment, v the equilibrium rate of job vacancies, and x the rate of job matchings taking place during a period (all variables normalized by the constant labor force). Since the matching probabilities for a typical unemployed worker and a typical vacancy are p and q, respectively, it follows that in equilibrium the number of workers taking jobs is up and the number of vacancies being matched is vq. But these numbers must be equal, because unemployed workers and vacancies meet in pairs. Moreover, since only the unemployed search, the number of pairs meeting each period constitutes the rate of job matchings, x. Thus, $x = up(c, \cdot) = vq(a, \cdot)$.

It follows that ceteris paribus the rate of job matchings increases when search intensity and job advertising increase, and also when the stocks of unemployment and vacancies increase. These results may be represented more generally by the matching function $x < \min(u, v)$, $x = x(c, a, u, v)$ with positive partial derivatives. The funcion $x(\cdot)$ describes the "search technology."

The condition $up = vq$ (which is an identity) implies that in addition to c, p must depend positively on v/u and a; and in addition to a, q must depend negatively on v/u and positively on c. These dependencies give rise to the externalities discussed in this paper. The matching probability of a typical job searcher rises when firms advertise more and when there are more job vacancies. Similarly, the matching probability of a typical vacancy rises when workers search more intensely and when there are more job searchers. Under these conditions, and if $p(\cdot)$ and $q(\cdot)$ are not generalized further through the introduction of new arguments, the search technology $x(\cdot)$ is homogeneous of degree 1 in u and v, and its partial derivatives with respect to c and a satisfy $x_c = up_c, x_a = vq_a$.

There are two generalizations of $p(\cdot)$ and $q(\cdot)$ that are potentially of interest, one of which we shall adopt. First, the matching probabilities may depend on the number of searchers and vacancies, independently of the dependence that results from the identity $up = vq$. If this is the case, $x(\cdot)$ may not be homogeneous of degree 1 in u and v, though it is still the case that $p = x/u$ and $q = x/v$. We adopt this generalization by assuming that x is concave and increasing in all its arguments.

Second, the matching probabilities may depend (negatively) on the intensities with which agents on the same side of the market look for job matches. This would introduce a negative externality which would work in a direction opposite to that of the positive externality discussed in this paper. It would also imply that there is no direct connection between the partial derivatives of $x(\cdot)$ and those of $p(\cdot)$ and $q(\cdot)$, since by searching more intensely a typical searcher would raise his own matching probability by more than the aggregate rate of job matchings. I do not adopt this generalization in this paper; as far as the effects of c and a are concerned, I assume that the only externalities that exist are the ones implied by the identity $up = vq$. In some sense, these externalities are more basic because they are derived from an identity and not assumed a priori. But to evaluate the potential quantitative significance of each externality, we would need to assume explicit functional forms for the search technology, and there may as well be search technologies which imply that the negative externality ignored here is quantitatively important.

Thus, under the assumptions in this paper the partial derivatives of $x(\cdot)$ bear a simple relation to those of $p(\cdot)$ and $q(\cdot)$. We shall make use of this property to compare the private equilibrium of this economy with the socially efficient state. A worker raises his own matching probability at the rate x_c/u when he searches more intensely, and he raises aggregate job matchings at the rate x_c. Similarly, a firm raises its own matching probability at the rate x_a/v and aggregate job matchings at the rate x_a.

With knowledge of the x function we can state the two remaining conditions needed for a full characterization of the private equilibrium. First, steady-state equilibrium requires equality between flows out of and into unemployment. Flows out of unemployment during a period are given by x, and flows into unemployment are given by $s(1 - u)$. Hence in equilibrium

$$x = s(1 - u). \tag{7}$$

Second, firms are free to open new jobs via the vacancy pool. Recall that the expected profit from a vacancy is given by V, and that if (6) is satisfied, $V \geq 0$. If (6) holds with strict inequality, $V > 0$, and new jobs will be created. In steady-state equilibrium the expected profits from a

vacancy must be zero, giving (6) with equality as one of the equilibrium conditions. We rewrite this by replacing q by the equivalent expression x/v:

$$\frac{x}{v}(y - w) = s(k + a); \tag{8}$$

the expected profits from a filled job position must be equal to the expected costs of a vacant position.

The equilibrium of the economy is characterized by the steady-state relation (7), by the optimization condition for workers (2), by the optimization condition for firms (5), and by the zero-profit condition (8). These four conditions are then solved simultaneously for the four unknowns u, v, c, and a.

This leaves undetermined the distribution of output between firms and workers, that is, wages. However, a theory of wages is not essential for the results of this paper. We show that there is *no* wage that can lead to a socially efficient outcome for vacancies and unemployment, so whichever method is used to determine wages the inefficiency will be present.[3]

V. Social Efficiency

Social efficiency is defined in terms of the flow of social output along the steady-state path. Employed workers produce output y, unemployed workers enjoy leisure $z - c$, and vacancies cost $k + a$. The flow of social output therefore is

$$Y = (1 - u)y + u(z - c) - v(k + a). \tag{9}$$

A socially efficient combination of c, a, u, and v is one that maximizes (9) subject to the steady-state condition (7). In order to derive this we set up the Lagrangian

$$L = (1 - u)y + u(z - c) - v(k + a) + \lambda[x - (1 - u)s]$$

and maximize with respect to the four variables. The first-order conditions are

$$\frac{\partial L}{\partial c} = -u + \lambda x_c = 0, \tag{10}$$

$$\frac{\partial L}{\partial a} = -v + \lambda x_a = 0, \tag{11}$$

[3] Provided that no property rights can be assigned to jobs (Mortensen 1982*b*) and that there is a unique wage for all jobs.

$$\frac{\partial L}{\partial u} = -y + z - c + \lambda(x_u + s) = 0,$$ (12)

$$\frac{\partial L}{\partial v} = -k - a + \lambda x_v = 0.$$ (13)

The socially efficient values of c, a, u, and v, obtained as the solutions of (10)–(13) and (7), can now be compared with the privately optimizing values obtained from (2), (5), (8), and (7). The former are independent of the wage rate, because firms and workers are treated equally in the social welfare function. Distributional considerations do not influence social efficiency. The private solutions depend on the wage rate, so the comparison of the two sets of solutions is carried out by asking whether there is a feasible value of the wage rate which can equate the two sets. Feasibility of the wage rate is defined by the participation conditions (3) and (6).

Conditions (10) and (11) imply that at the social optimum

$$\frac{x_c}{u} = \frac{x_a}{v}.$$ (14)

The left-hand side shows the effect of a small rise in expenditure on search on the probability that an unemployed person becomes employed. The right-hand side shows the effect of a small rise in advertising expenditure on the probabilty that a vacancy becomes filled. It is socially efficient to allocate expenditure between search and advertising until these two effects are equalized. If this condition is satisfied, the marginal effect of each kind of expenditure on employment is, in general, different. Employment is given by x/s, so expenditure on search and advertising have the same marginal effect on employment only if $u = v$. If $u > v$ expenditure on search has a greater marginal effect, and conversely if $u < v$. Since the second partial derivatives of x with respect to c and a are negative, this result implies that compared with the case $u = v$, relatively more should be spent on job advertising when the efficient rate of unemployment exceeds the efficient rate of vacancies. Conversely, relatively more should be spent on job search when the efficient unemployment rate falls below the vacancy rate.

Next, we rewrite (10) and (11) in a way directly comparable with (2) and (5), by substituting away λ. Solving (12) for λ and substituting in (10) yields

$$(y - z + c)x_c = ux_u + us.$$ (15)

Solving also (13) for λ and substituting in (11) we obtain

$$(k + a)x_a = vx_v. \tag{16}$$

Conditions (2) and (5) may also be expressed in terms of x by using the fact that $p = x/u$ and $q = x/v$. Making the substitutions, we obtain

$$(w - z + c)x_c = su + x, \tag{2'}$$

$$(y - w + k + a)x_a = sv + x. \tag{5'}$$

Conditions (15) and (16), which give, respectively, the socially efficient conditions for search intensity and job advertising, can be compared directly with the corresponding private conditions (2') and (5'). There is also a third condition for the social efficiency of the rate of vacancies, and this may be obtained by substituting λ from (12) into (13):

$$(y - z + c)x_v = (k + a)(x_u + s). \tag{17}$$

In the private solutions the equilibrium rate of vacancies is given by the zero-profit condition (8).

It follows immediately by comparing (15), (16), and (17) with (2'), (5'), and (8) that there cannot be a wage rate which can equate the private with the social solutions. At best, the wage rate can be chosen in such a way as to make one of the private conditions equivalent to the corresponding social condition. But this value (even if it exists) cannot, in general, make the other two conditions equivalent as well. The wage rate cannot provide simultaneously efficient incentives for both workers and firms, who make the three choices of search intensity, job advertising, and potential entry and exit.

A stronger result can be derived by imposing a fairly weak restriction on the technology of search. Thus, we can show that if a, u, and v are arbitrarily fixed at their socially efficient solutions, and the wage rate is used to regulate c, then if $x \geq ux_u$ the value of the wage rate that is needed to bring c to its socially efficient solution violates the firm participation condition (6). The restriction $x \geq ux_u$ implies that the typical worker's job-matching probability, x/u, declines as the number of workers searching, u, rises. If this is satisfied, all feasible values of the wage rate are below the value needed for the social efficiency of c. As a result, workers will search "too little," regardless of the method used to fix wages.

To show this result we use (2') and (15). The wage rate needed to equate the marginal effect x_c in the two cases can be obtained from

$$\frac{us + x}{w - z + x} = \frac{us + ux_u}{y - z + c}.$$

Solving for w, we obtain

$$w = \frac{us + x}{us + ux_u}(y - z + c) + z - c. \tag{18}$$

But firms will participate if (6) is satisfied. Using (18) this requires (noting $q = x/v$)

$$\frac{us + x}{us + ux_u}(y - z + c) + z - c \leqslant y - \frac{sv}{x}(k + a).$$

Rearranging terms, we obtain

$$\frac{x - ux_u}{us + ux_u}(y - z + c) \leqslant -\frac{sv}{x}(k + a). \tag{19}$$

Now, $y - z + c > 0$ and $k + a > 0$, so a sufficient condition for a contradiction in (19) is $x \geqslant ux_u$.

Similar problems arise when we compare (16), the condition for the social efficiency of a, with (5′). Holding c, u, and v fixed at their socially efficient solutions, we can again show that if the typical firm's job-matching probability falls as the number of vacancies arises (i.e., if $x \geqslant vx_v$), the privately chosen level of advertising will be too low. We could obtain this result from the other side's participation condition, as in the case of search intensity, because the typical firm's problem is symmetric to that of the typical worker. But we could also obtain it directly from the zero-profit condition (8).

Thus, using (8) to substitute $y - w$ out of (5′) we obtain

$$(k + a)x_a = x. \tag{20}$$

Comparing now (20) with the social condition (16), and noting that x is a concave function of a, we obtain that a sufficient condition for the private solution of a, obtained from (20), to lie below the social solution, is $x \geqslant vx_v$. So, as with workers, if the job-matching probability x/v is a decreasing function of v, competitive creation and closure of jobs will ensure that firms advertise too little at all feasible values of the wage rate.

The externalities which give rise to the inefficiency of equilibrium outcomes arise because each side's matching probability depends on what the other side does and how many job searchers and vacancies exist. In order to appreciate the relative contribution of each externality, we can eliminate artificially the second externality by imposing enough restrictions on the technology of search to ensure that the job-matching prob-

abilities are independent of numbers. One set of sufficient restrictions is (i) x is homogeneous of degree 1 in u and v, and (ii) in equilibrium, $u = v$. The first restriction ensures that the matching probabilities depend only on the u/v ratio and the second ensures that this ratio is equal to unity. Hence now,

$$\frac{x}{u} = \frac{x}{v} = x(c, a, 1, 1). \tag{21}$$

The wage rate required for the social efficiency of c under (21) may then be obtained by noting that (14) implies $x_c = x_a$, and so, adding (15) and (16),

$$(y - z + c + k + a)x_c = ux_u + vx_v + us. \tag{22}$$

Under homogeneity of x the right-hand side of (22) is equal to the right-hand side of (2'), so social efficiency is achieved when

$$w = y + k + a. \tag{23}$$

Output y is equal to the marginal product of labor (by the assumption of constant marginal productivity), and $k + a$ is equal to the costs of maintaining a vacancy. When a worker accepts a job, he removes a vacancy from the market, and he saves society costs equal to $k + a$. Hence $y + k + a$ corresponds to the social marginal product of labor. Condition (23) states that workers make socially efficient decisions only if wages are equal to the full value of their marginal product. But workers cannot be compensated to such an extent, because firms will then be making losses.[4]

Similarly, the same argument could be made for firms. Since $x_a = x_c$, (22) also holds for x_a, and under homogeneity and equality between u and v, the right-hand side is also equal to the right-hand side of (5'). Hence firms make socially efficient decisions (even when the zero-profit condition is ignored) when

$$y - w = y - z + c. \tag{24}$$

As before, output y is the output society enjoys when a vacancy is matched,

[4] This condition for wages, and also condition (24) below for profits, correspond to Mortensen's (1982b) conditions for the efficiency of search by workers and firms, respectively. In his model (23) and (24) were feasible because of the existence of property rights: the side that succeeds in discovering the match receives all the surplus and compensates the other side sufficiently to induce it to accept the offer. Participation is then ensured because there is a positive probability that either side will establish a job match and thus make a positive profit.

and $z - c$ is the net leisure which the worker who takes the vacancy forgoes. Hence $y - z + c$ corresponds to the social marginal product of a job. Firms advertise efficiently when the profit rate is equal to their marginal product. But for this to be the case we require $w = z - c$, and this violates the worker's participation condition (3).

The introduction of variable u and v, of the restrictions implied by the zero-profit condition, and of a nonhomogeneous search technology alters conditions (23) and (24). In order to check whether the additional externalities introduced in this way lead to more or less search and advertising, we derive again the wage rate required for efficiency in each case, in a way directly comparable to (23) and (24). Thus, adding again (15) and (16), and using (14) to eliminate x_a, we obtain

$$\left[y - z + c + \frac{v}{u} (k + a) \right] x_c = u x_u + v x_v + u s. \qquad (25)$$

Hence, now the wage rate needed to equate (25) with (2') is

$$w = \frac{x + us}{u x_u + v x_v + us} \left[y + \frac{v}{u} (k + a) \right]$$

$$+ \frac{x - (u x_u + v x_v)}{u x_u + v x_v + us} (-z + c). \qquad (26)$$

Comparing with (23), the wage needs to be higher now if

$$(u x_u + v x_v + us)(y + k + a)$$

$$\leqslant (x + us) \left[y + \frac{v}{u} (k + a) \right] + [x - (u x_u + v x_v)](-z + c).$$

After some straightforward manipulation this becomes

$$[x - (u x_u + v x_v)](y - z + c + k + a)$$

$$+ (x + us)\left(\frac{v}{u} - 1\right)(k + a) \geqslant 0. \qquad (27)$$

Sufficient conditions for the satisfaction of (27) are

$$x \geqslant u x_u + v x_v \qquad (28a)$$

$$v \geqslant u. \qquad (28b)$$

Under these circumstances, the second externality (which arises because the matching probability depends on u and v) increases the gap between the socially efficient search intensity and the private outcome. Condition (28a) relates to the "returns to scale" of the search technology with respect to u and v. If there are constant returns, whether the second externality reinforces or not the first externality depends solely on the relative magnitude of u and v in equilibrium: if $v \geq u$ it reinforces it, if $v \leq u$ it partially offsets it.

A similar condition may be derived for job advertising. Working in the same way as in the derivation of (28a) and (28b), and using the zero-profit condition to substitute out $k + a$, we now find that the second externality reinforces the first if $ux_u + vx_v + us > x > ux_u + vx_v$; otherwise it partially offsets it. If x is subject to constant returns to scale, the second externality does not introduce further distortions in the choice of job advertising. This is the main difference between the distortions in advertising and search intensity, and it is due to the zero-profit condition holding for firms but not for workers. For search intensity, there remained some additional distortions even under constant returns to scale. Apart from this difference, the general rule is that increasing returns to scale imply that the two externalities are more likely to work in the same direction, whereas decreasing returns imply that the second externality partially offsets the first. Of course, as long as $x \geq ux_u$ and $x \geq vx_v$, the first externality always dominates the second, as I have proved.

VI. Conclusions

The question addressed in this paper is related to the social efficiency of the equilibrium rate of search unemployment. I have demonstrated that if both sides of the market, firms and workers, search for each other the equilibrium rate of unemployment is likely to be too high, because both sides will search too little. The reason for this is that both sides ignore an important positive externality of their job search: when they succeed in establishing a job match they remove from the other side of the market an unemployed participant, and so they save society his search costs. I have shown that there is no feasible wage rate that can induce either firms or workers to devote the socially efficient amount of resources to job search. The wage rate required to induce workers to search efficiently is above the feasible range (firms would make negative profits), and the wage rate required to induce firms to search efficiently is below the feasible range (workers would earn less than the value of leisure).

I have demonstrated these results within a model of competitive entry and exit of firms, by using a fairly general function to represent the technology of search. Under the search technology used, there is a second externality which might, in general, induce more or less job search—the job-matching probability of each participant depends on the number of

searchers in the market. One of the major results of this paper has been the demonstration that if each participant's job-matching probability is a decreasing function of the number of searchers on his own side of the market, the second externality cannot offset entirely the effects of the first externality. The level of search unemployment will be too high in all cases, though if the technology of search is subject to diminishing returns to scale it will not be as high as when there are constant or increasing returns.

Since there is no feasible wage rate that could internalize these externalities, a solution to the problem could be direct government intervention. The state could help bring about the efficient rate of unemployment either by subsidizing job search by workers and firms or by setting up its own subsidized employment agencies. I have not worked out the optimal level of search subsidies, but this is straightforward. Employment agencies are more problematical because of the uncertain effects they are likely to have on private job-search activities (Pissarides 1979).

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