The Social Cost of Carbon with Economic and Climate Risks

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Uncertainty about future economic and climate conditions substantially affects the choice of policies for managing interactions between the climate and the economy. We develop a framework of dynamic stochastic integration of climate and economy, and show that the social cost of carbon is substantially affected by both economic and climate risks and is a stochastic process with significant variation. We examine a wide but plausible range of values for critical parameters with robust results and show that large-scale computing makes it possible to analyze policies in models substantially more complex and realistic than usually used in the literature.

I. Introduction

Global warming has been recognized as a growing potential threat to economic well-being. Determining which policies should be implemented requires analyses that incorporate models of both the climate and the economy and how they interact; this is the purpose of *integrated assessment*

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models (IAMs). This paper expands the scope of IAMs by adding uncertainties and risks to a canonical model of the economic and climate systems and shows that such risks and uncertainties significantly affect optimal climate policy. Almost all IAMs assume that both the climate and economic systems are deterministic and that economic agents are myopic. Studies of those IAMs focus on uncertainty about the key parameters. This study, instead, incorporates economic and climate risks into models where economic agents have rational expectations concerning the future of the economy and of the climate. This allows us to use ideas from economic growth, investment, and consumption capital asset pricing model theories to evaluate alternative climate change policies. As expected, explicit treatment of economic and climate risks significantly changes the results.

The impact of carbon emissions on society is measured by the social cost of carbon (SCC), defined as the marginal economic loss caused by an extra metric ton of atmospheric carbon.[1] The importance of SCC for policy decisions has led to a large effort to estimate its value. One prominent study was produced by the Interagency Working Group on Social Cost of Carbon (IWG)—a joint effort involving several US federal agencies. Their report (IWG 2010) was based on three well-known economic

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[1] We express the SCC in US dollars ($) per ton of carbon (tC). The SCC per ton of CO_2 equals 12/44 times the SCC per ton of carbon.

IAMs: FUND (climate framework for uncertainty, negotiation and distribution; Anthoff and Tol 2014), PAGE (policy analysis of the greenhouse effect; Hope 2011), and DICE (dynamic integrated climate-economy model; Nordhaus 2008). The IWG report gives a wide range of plausible values for the SCC, with a median of \$51/tC.[2] The Stern report (Stern 2007) presents an alternative analysis, focusing on parameter uncertainty. Using the probabilistic IAM PAGE (Hope 2011), the report addresses parametric uncertainty regarding the causes and impacts of climate change and argues for substantially higher SCC values.

The models behind the IWG and Stern reports use simplified descriptions of the climate. Many other analyses are based on far more complex climate models and much more detailed economic models and are included in the summary produced by the Intergovernmental Panel on Climate Change (IPCC). IPCC (2007) reports that estimates of the SCC vary across peer-reviewed studies, with an average estimate of \$43/tC but with a large range.

All of these studies are based on IAMs with deterministic models of economy and climate, with all economic agents knowing current and future output and climate conditions. Recent reviews, such as Pindyck (2013) and IPCC (2014), criticize the fact that SCC estimates are based on IAMs that ignore the considerable risk and uncertainty in both the economic system and the climate system and their interactions.

This study presents dynamic stochastic integration of climate and the economy (DSICE), a computational dynamic stochastic general equilibrium framework for studying global models of both the economy and the climate. We apply it to the specific issue of how the SCC depends on stochastic features of both the climate and the economy and use specifications of tastes consistent with the observed willingness to pay to reduce economic risk. This study focuses on the SCC as an example of the flexibility of the DSICE framework and its ability to examine a wide range of issues. DSICE builds on the canonical DICE model (Nordhaus 1992, 2008) by adding both economic and climate risks to the DICE framework. We choose to extend DICE because it is one of the few prominent IAM models based on dynamic models of agent decision-making.[3]

[2] The IWG report is based on solving FUND, PAGE, and DICE for thousands of values of the critical parameters, where those parameter choices are based on empirical data and expert opinion. The models express the SCC in terms of US dollars per ton of CO_2. After converting the SCC to \$/tC, they report that the 5th, 25th, 50th, 75th, and 95th percentile values are −\$33, \$15, \$51, \$102, and \$238/tC, respectively.

[3] Manne and Richels (2005) and Nordhaus and Yang (1996) are also based on dynamic models of agent decision-making. Many IAMs say that they have “recursive” dynamic models of the economy. This use of the term “recursive” was common in dynamic computable general equilibrium modeling in the 1970s. However, those models were recursive only in the sense that economic decisions today create the state of the economy tomorrow. Today we call those models “myopic,” because dynamic decisions, such as investment, were simple

We first examine how economic risks affect the SCC. Specifically, we assume that factor productivity growth implies consumption growth rates that display long-run risk as modeled by Bansal and Yaron (2004) and Beeler and Campbell (2011). We combine this with Epstein-Zin utility specifications of dynamic preferences (Epstein and Zin 1989). Current empirical analyses do not give us precise estimates of critical parameters, particularly risk aversion and the intertemporal elasticity of substitution (IES). We solve DSICE for parameter values covering the range of empirical estimates in the macroeconomic literature. We find that the 2005 SCC ranges from \$59 to \$99/tC over a plausible range of parameter values. These results demonstrate that we should be skeptical of studies that give one number relying on a single parameterization. In this case, sensitivity analysis does support the case for a significant carbon tax.

Of equal interest and greater novelty are our results on the dynamics of the future SCC. Factor productivity shocks create riskiness in future output and carbon emissions, which in turn makes the SCC a random process. A common interpretation of results in deterministic models is that they represent the expected path in stochastic models. In many cases, we find that the mean path for the SCC is close to that implied by deterministic models. DSICE, however, can also determine the stochastic features of the SCC process. The SCC is the shadow price of a state variable and, as we expect, is approximately a random walk. When we quantify the SCC process, we find that it displays substantial variance. For example, in our benchmark case the median SCC is \$286/tC in 2100, but with a 10 percent chance of exceeding \$700/tC and a 1 percent chance of exceeding \$1,200/tC. In general, the standard deviation of the SCC grows faster than its mean.

It is recognized that temperature increases may cause substantial irreversible damage to the climate.[4] Some studies (e.g., Scheffer et al. 2001; Weitzman 2009; Pindyck 2011) use the possibility of very high damage caused by a very low-probability catastrophic event to advocate aggressive mitigation policies. Moreover, the IAM literature has recently studied the importance of *climate tipping points*, which refer to "a critical threshold at which a tiny perturbation can qualitatively alter the state or development of [the climate] system," and *tipping elements*, which are defined as "large-scale components of the Earth system that may pass a tipping point" (both

functions of contemporaneous prices and states. Today, the description "recursive models" refers to models where agents have rational expectations; see, e.g., Stokey and Lucas (1989) and Ljungqvist and Sargent (2004). The antiquated use of the term "recursive" in the IAM literature (such as Babiker et al. 2009) is misleading to economists familiar with modern dynamic economic analysis.

[4] When quantifying temperature increases throughout this paper, we are referring to the year 1900 temperature level as the baseline level. Thus, our term "temperature change" is analogous to the term "temperature anomaly," which is often used in the climate literature and typically denotes the difference from some baseline temperature level.

definitions from Lenton et al. 2008, 1786). A key feature of a tipping element is that current temperature affects the likelihood of a tipping element experiencing a *tipping event*—that is, a transition to an irreversible climate process, called a *tipping process*. Examples of tipping processes include the irreversible melting of the Greenland ice sheet, the collapse of the West Antarctic ice sheet, and the weakening of the Atlantic thermohaline circulation. The incorporation of tipping elements in DSICE allows it to model a range of potentially irreversible climate processes, including but not limited to global catastrophes. These processes are uncertain and give DSICE a second source of risky damage to economic productivity.[5]

There is a consensus that the SCC specifically and optimal policy in general are sensitive to the choice of discount rate used by the social planner. Standard IAMs are deterministic, treat mitigation expenditures as a way of increasing consumption in the future, and use the same discount rate for all issues. Some have argued that mitigation has an insurance component. Schneider (1989), in his testimony to the Committee on Energy and Commerce in 1989, argued that investing in climate change mitigation is like "buying insurance against the real possibility of large and potentially catastrophic climate change." More recently, former Secretary of State George Shultz, who convinced President Reagan to support the Montreal Protocol, argued

> We all know there are those who have doubts about the problems presented by climate change. But if these doubters are wrong, the evidence is clear that the consequences, while varied, will be mostly bad, some catastrophic. So why don't we follow Reagan's example and take out an insurance policy? (Shultz 2015)

R&D is an example of insurance spending. The new technologies arising from today's R&D spending will arrive only with significant lags, and, if we are lucky, we may not need to use them, just as we do not file a claim

[5] Catastrophic climate change—in our case a climate tipping process—is just one of many possible types of critical events that might ultimately affect economic productivity. A more general concept of ecological thresholds of the Earth is the concept of *planetary boundaries* (Rockström et al. 2009; Steffen et al. 2015). Ocean acidification, loss of biodiversity, and ozone depletion are examples of Earth system processes that are believed to exhibit specific boundaries—that is to say, thresholds that lead to irreversible and abrupt environmental change. Furthermore, possible catastrophic events that affect the global economy are, of course, not limited ecological ones. Most recently, Martin and Pindyck (2015, 2017) consider a broad variety of potential catastrophes, such as a megavirus, a nuclear or bioterrorism attack, or a climate catastrophe. They study the willingness to pay to avoid these extreme events, which, interdependently, not only affect consumption dynamics but also might be fatal to the population.

to our insurer if we experience no bad events. Insurance has a negative expected rate of return, but we buy it because of its option value. Similarly, simple discounting rules motivated by a deterministic model are not valid in a dynamic stochastic context because they would ignore the real option aspect of R&D investments.

The results in this study demonstrate the importance of adding both economic and climate risk to IAMs. First, economic risks imply that there is great uncertainty about the future SCC and the value of future climate change policies. In particular, the possibility of climate change causing substantial damage is much higher in DSICE than in models that ignore economic risks. Second, while the SCC today is sensitive to parameter choices regarding preferences, we do arrive at a robust finding that the SCC is nontrivial, with \$40–\$100/tC being the range implied by the various opinions regarding dynamic preferences. Third, there is no simple discounting rule to apply to climate change policy decisions. The IAM literature argues over the right discount rate for valuing future damages from climate change (see, e.g., Nordhaus [2007] and Stern and Taylor [2007]). DSICE shows that there is no one discount rate. As standard finance theory teaches, the appropriate discount rate depends on covariance with consumption. In this study, we show that the damages arising from tipping events should be discounted at a much lower rate than damages from temperature increases. This may seem surprising, because we follow Nordhaus's approach and use the market equilibrium stochastic discount factor instead of some planner discount rate unrelated to the market pricing of risks. The intuition is clear: the damages from a permanent shock to productivity have little covariance with short-run fluctuations in consumption. Therefore, damages from tipping events should be discounted less than damages from temperature (which are proportional to output). Fourth, DSICE examines cases where both economic and climate risks are present and shows that the results differ significantly from a separate examination of these two sources of risk. More generally, this study shows that it is important to examine these issues in a model that incorporates multiple kinds of uncertainty.

Many studies express great skepticism about what is computationally feasible and use this as a reason to look at simple models. Traeger's claim that the curse of dimensionality limits what can be done is typical:

> The quantitative analysis of optimal mitigation policy under uncertainty requires a recursive dynamic programming implementation of IAMs. Such implementations are subject to the curse of dimensionality. Every increase in the dimension of the state space is paid for by a combination of (exponentially) increasing processor time, lower quality of the value or policy function

> approximations, and reductions of the uncertainty domain. Traeger (2014, 1)

The computational-mathematics literature has developed many methods that avoid the curse of dimensionality,[6] and DSICE uses them to solve problems generally considered intractable. This study solves nine-dimensional stochastic dynamic programming problems over multiple centuries. The nonstationary character of the problems makes value function iteration the only possible approach. The specifications of risks make these problems among the most computationally demanding ever solved in economics. However, DSICE is not limited by any curse of dimensionality because it uses efficient multivariate methods to approximate value functions. Even though DSICE solves millions (billions in some examples) of optimization problems, this is possible because it uses reliable optimization methods and parallelization to solve the Bellman equations (Bellman 1957; Cai and Judd 2010; Cai et al. 2015). The "curse of dimensionality" is not a valid excuse for oversimplified modeling.

Any numerical computation has numerical errors, and the fact that DSICE must solve billions of small numerical problems means that we need to subject any results to stringent accuracy checks. In recent years, the scientific-computing community has addressed this issue in its literature on verification, validation, and uncertainty quantification (VVUQ); see Oberkampf and Roy (2010) for a comprehensive discussion of this literature. The "verification" part of the VVUQ literature develops methods for verifying the accuracy of the numerical results. We develop such a test for each value function iteration and find that every value function computed by DSICE passes demanding verification tests. This gives us confidence that numerical errors do not affect our economic conclusions. Some of the cases we present below required tens of thousands of core hours, and sensitivity analysis demanded that we examine hundreds of cases to determine the robustness of results across empirically plausible parameter values. This study required the use of a few million core hours on Blue Waters, a modern supercomputer. DSICE is based on general-purpose numerical methods, implying that many economics problems with similar computational requirements can now be solved.

We present the climate model in Section II and the economic model in Section III. In Section IV, we formulate the dynamic programming problem and outline our solution method. Sections V–VII present and discuss the implications for SCC from stochastic specifications of factor productivity growth and a Markov chain process, first each in isolation and then combined. Section VIII concludes the paper.

[6] See Judd (1998) for an elementary introduction to some of those methods.

II. The Climate Model

Our climate model contains three modules. First is the carbon system, describing how carbon diffuses across the atmosphere, the upper ocean, and the deep ocean. Second is the temperature system, which describes how heat energy diffuses between the atmosphere, ocean, and space. The carbon and temperature systems are not closed, because economic and biological activity injects carbon into the atmosphere, carbon interacts with solar radiation to cause heating, and some heat is lost through radiation to space. We follow the specification for the carbon and temperature modules used in DICE-2007 (Nordhaus 2008).

We add a third system that models climate conditions other than carbon and temperature. A simple example of such a climate state is sea level. Prolonged periods of warm atmospheric temperatures will (likely) melt ice in glaciers (on land), which will then lead to higher sea levels. This makes sea levels and the condition of glaciers dependent on past temperatures. We focus on what are called tipping elements, which model irreversible changes. For example, if warming causes the melting of glaciers, even if temperatures fall back to preindustrial levels, glaciers reappear, if at all, only after millions of years (IPCC 2014).

Climate change affects economic productivity in different ways. Higher temperatures will likely reduce agricultural output, raise expenditures on cooling, and facilitate the spread of disease. There is evidence that higher temperatures will increase the likelihood of severe droughts and floods (Wuebbles 2016). Higher sea levels will worsen coastal flooding and may even cause some land to disappear. We provide details on the productivity effects below when we describe the economic model in DSICE.

A. *Carbon System*

We assume two sources for carbon emissions at each time t, an industrial source, $E_{\mathrm{Ind},t}$, related to economic activity, and an exogenous source, $E_{\mathrm{Land},t}$, arising from biological processes on the ground.[7] Total emissions are denoted by

$$E_t \equiv E_{\mathrm{Ind},t} + E_{\mathrm{Land},t}. \tag{1}$$

The details of $E_{\mathrm{Ind},t}$ are presented below, when we discuss the economic model. We follow DICE-2007 and aggregate the distribution of carbon in the world into three "boxes"—atmosphere, upper ocean, and lower ocean. The three-dimensional vector $\mathbf{M}_t = (M_{\mathrm{AT},t}, M_{\mathrm{UO},t}, M_{\mathrm{LO},t})^\top$ represents the masses of carbon in the atmosphere, upper levels of the ocean, and lower

[7] These emissions depend, e.g., on many biological processes and are also subject to uncertainty. However, here we adopt the DICE specification of land emissions that are assumed to be exogenous and deterministic.

levels of the ocean, respectively (in gigatons of carbon). The carbon concentrations evolve over time according to the physics of diffusion and are represented by the linear dynamical system

$$\mathbf{M}_{t+1} = \mathbf{\Phi}_{\mathrm{M}}\mathbf{M}_t + (E_t, 0, 0)^{\top}, \tag{2}$$

where

$$\mathbf{\Phi}_{\mathrm{M}} = \begin{bmatrix} 1-\phi_{12} & \phi_{21} & 0 \\ \phi_{12} & 1-\phi_{21}-\phi_{23} & \phi_{32} \\ 0 & \phi_{23} & 1-\phi_{32} \end{bmatrix}. \tag{3}$$

The coefficient ϕ_{ij} in $\mathbf{\Phi}_{\mathrm{M}}$ is the rate at which carbon diffuses from level i to level j, where $i, j = 1, 2, 3$ represent the atmosphere, upper ocean, and lower ocean, respectively. If $E_t = 0$, then this would be a closed system, implying that the column sums of $\mathbf{\Phi}_{\mathrm{M}}$ must be unity. Also note that there is no diffusion between the atmosphere and the lower ocean. Table A.4 (tables A.1–A.13 are available online) lists the parameter values for the carbon system. The exogenous processes and calibration of the module appear in appendixes A.1 and A.3 (apps. A.1–A.10 are available online).

B. Temperature System

The DICE family of models is based on the continuous-time differential equation system in Schneider and Thompson (1981). DSICE also uses 1-year time periods in all examples in this paper. The temperature system tracks the temperatures of the atmosphere (T_{AT}) and the ocean (T_{OC}), measured in degrees Celsius above the preindustrial level. That system is governed by the diffusion of heat, is represented by the vector $\mathbf{T}_t = (T_{\mathrm{AT},t}, T_{\mathrm{OC},t})^{\top}$, and evolves according to

$$\mathbf{T}_{t+1} = \mathbf{\Phi}_{\mathrm{T}}\mathbf{T}_t + (\xi_1 \mathcal{F}_t(M_{\mathrm{AT},t}), 0)^{\top}, \tag{4}$$

where

$$\mathbf{\Phi}_{\mathrm{T}} = \begin{bmatrix} 1-\varphi_{21}-\xi_2 & \varphi_{21} \\ \varphi_{12} & 1-\varphi_{12} \end{bmatrix}. \tag{5}$$

The coefficient φ_{ij} is the heat diffusion rate from level i to level j, where $i, j = 1, 2$ represent the atmosphere and the ocean, respectively. The coefficient ξ_2 is the rate of cooling arising from infrared radiation to space (Schneider and Thompson 1981), and ξ_1 represents heating due to radiative forcing. Atmospheric temperature rises through two sources of radiative forcing: an exogenous level, $F_{\mathrm{EX},t}$, and endogenous forcing due to carbon in the atmosphere. Total radiative forcing at t is

$$\mathcal{F}_t(M_{\mathrm{AT},t}) = \eta \log_2\left(\frac{M_{\mathrm{AT},t}}{M_{\mathrm{AT}}^*}\right) + F_{\mathrm{EX},t}, \tag{6}$$

where M_{AT}^* is the preindustrial atmospheric carbon concentration and η is the radiative-forcing parameter. Table A.5 lists the parameter values for the temperature system. The exogenous processes and calibration of the module appear in appendixes A.1 and A.3.

C. *Tipping-Element System*

The temperature and carbon systems in DSICE are deterministic and evolve smoothly over time. Recent work in the IAM literature has drawn attention to tipping points, which we defined in Section I. Tipping points are not necessarily irreversible, but they are essentially irreversible for the planning horizons related to the examples considered in this paper.

Let J represent some feature of the climate other than temperature or carbon. It has a finite set of possible values and represents the state of a tipping element; we refer to J as the *tipping state*. Changes in J are modeled by a Markov chain where transition probabilities depend on the vector of all climate states, $(\mathbf{T}, \mathbf{M}, J)$. The Markov transition process is denoted as

$$J_{t+1} = g_J(\mathbf{T}_t, \mathbf{M}_t, J_t, \omega_{J,t}),$$

where $\omega_{J,t}$ is one serially independent stochastic process.

The key properties of any tipping element include the likelihood of tipping events, the expected duration of the tipping process, the mean and variance of the long-run impacts on economic productivity, and how all of these depend on $(\mathbf{T}, \mathbf{M}, J)$. There is substantial uncertainty about all of these properties. Our examples are based on expert opinion expressed in the climate science literature on various tipping elements. We use Lenton's (2010) summary of the findings from Kriegler et al. (2009) and other expert elicitation studies to calibrate the likelihood of transitions in tipping elements. We also rely on damage estimates used in the IAM literature (e.g., Stern 2007; Nordhaus 2008; Smith et al. 2009; Hope 2011; IPCC 2014). The impact on economic productivity is included in our description of the economic system, below. Section VI examines some specific cases and precisely describes the Markov process for J. Our calibration of the tipping-element system appears in appendix A.4.

D. *Comparisons with Other IAM Climate Systems*

DSICE, as in DICE-2007, uses a five-dimensional system (two dimensions for temperature and three for the carbon cycle) to compute paths of world average carbon concentrations and temperature levels that are close to the results from much more complex models. DSICE uses a modified version of the temperature module in DICE-2007. Cai, Judd, and

Lontzek (2012b) point out that the computer code in Nordhaus (2008) has temperature increases in each period depend on carbon emissions 10 years in the future. Cai, Judd, and Lontzek (2012a) use differential-equation methods to solve the continuous-time physical model in Schneider and Thompson (1981) and find that applying the Euler method to the differential equations in Schneider and Thompson with 1-year time steps produces the discrete-time system defined above in equations (2) and (4), which successfully matches the MAGICC (model for the assessment of greenhouse-gas induced climate change) scenarios (Meinshausen, Raper, and Wigley 2011) that DICE-2007 aimed to match. The differential-equation approach produced results significantly different from those in DICE-2007. To avoid confusion about the different solution methods, "DICE-2007" refers to the model and results in Nordhaus (2008), and "DICE-CJL" refers to the model specified in Cai et al. (2012a). The mathematical details of this calibration method are presented in appendix A.3.

DICE is regarded as the canonical climate system in much of the IAM literature, but few studies use it, often citing tractability issues. Golosov et al. (2014) track only carbon concentration in the atmosphere to represent the whole climate system, implying that temperature is proportional to carbon concentrations. Jensen and Traeger (2014) also use a one-box climate model, with only atmospheric carbon concentration. The models in Golosov et al. (2014) and Jensen and Traeger (2014) ignore the lag between CO_2 emissions and their impact on temperature, a lag that climate scientists tell us is on the order of decades. Lemoine and Traeger (2014) use a two-box climate model, tracking only atmospheric carbon concentration and temperature, ignoring the impact of oceans on atmospheric temperature and CO_2 concentrations.

Others have studied tipping elements but make simplifying assumptions that do not conform with physical evidence. A common approach is to assume that there is a threshold (perhaps unknown to the planner) such that a tipping event will occur immediately when the temperature crosses that threshold (see, e.g., Keller, Bolker, and Bradford 2004). With economic uncertainty, temperatures can fall or rise. If the temperature were to cross a threshold, the tipping event would immediately happen even if the temperature quickly fell below that threshold.

More recent studies assume that the full impact of a tipping event is immediate and that its level is known (see, e.g., Lemoine and Traeger 2014). In our framework, that is equivalent to assuming that there are only two climate states related to tipping: pretipping and posttipping. Our tipping-element specification is also the first, and so far the only, one to address the recent critique by Kopits, Marten, and Wolverton (2014) of these assumptions. In DSICE, the timing of a tipping event is unknown, even conditional on knowing the full state of the climate system, the timing of the transitions after the tipping event is unknown, and—furthermore—the impact of the Markov chain is unknown before the tipping event

(Lenton et al. 2008; National Research Council 2013). DSICE is so far the only model that can handle these characteristics of Markov chain points. Other applications of DSICE have considered various transition times of Markov chain points (Cai et al. 2015; Cai, Lenton, and Lontzek 2016; Lontzek et al. 2015).

Earlier studies ignore expert opinion and climate science when they calibrate the process producing tipping events. Lemoine and Traeger (2014), for example, keep track of the historically maximum temperature level and assume that a tipping event cannot be triggered as long as the temperature is below that maximum level. This assumption ignores the inertia in the climate system, implying that long-lasting heating dynamics that might be present in the climate system could trigger climate tipping events. It also rules out the possibility that tipping might be triggered during periods of decreasing temperature trends. DSICE, instead, relies on expert opinion (Lenton et al. 2008; Kriegler et al. 2009) to calibrate its tipping elements and assumes that a tipping event is a random event with probability depending on the state of the climate system, assuming that the hazard rate of the tipping-point event is increasing with global warming. Lontzek et al. (2015) present a detailed discussion of studies on climate tipping in stochastic IAMs.

Several recent studies look at analytically tractable climate-economy formulations. Subsequent to our work, Anderson et al. (2014), for example, conduct a robustness analysis of model uncertainty in a simple IAM. Van der Ploeg and de Zeeuw (2016) decompose the SCC analytically in the face of catastrophic climate events. Gerlagh and Liski (2018) present an analytically tractable model of learning about impacts from climate change. Other numerical IAMs (e.g., Kelly and Kolstad 1999; Kelly and Tan 2015; Hwang et al. 2017) deal with uncertainty by incorporating Bayesian learning about the parameter describing climate sensitivity—that is to say, the equilibrium temperature change in response to changes of the radiative forcing (by carbon dioxide; see, e.g., Roe and Baker 2007). More generally, Bayesian methods could be applied to integrate model uncertainty into IAMs.[8] While this approach can generate insightful results regarding the evaluation of different policies, it is not the focus of our study. Instead, we apply an uncertainty quantification analysis to determine the robustness of our results across empirically plausible parameter values.

III. The Economic Model

DSICE merges a basic dynamic stochastic general equilibrium model with the commonly used box model for climate in DICE-2007. This allows us

[8] See, e.g., Brock, Durlauf, and West (2007) and Cogley et al. (2011) for an analysis in macroeconomics.

to explore how economic risks and climate risks interact and affect the evaluation of climate change policies.

A. *Predamage Output*

The economic side of DSICE is a simple stochastic growth model where production produces greenhouse gas emissions and output is affected by the state of the climate. We assume that time is discrete, with each period equal to 1 year. Let K_t be the world capital stock in trillions of dollars at time t, and let L_t be the world population in millions at time t.

The deterministic parts of our economic model are taken directly from the calibrated DICE-2007 model in Nordhaus (2008). This includes the production function, population growth, productivity growth, the carbon intensity of output, and the damage due to temperature levels.[9] In the absence of any climate damage, the gross world product is the Cobb-Douglas production function

$$f\left(K, L, \tilde{A}_t\right) = \tilde{A}_t K^{\alpha} L^{1-\alpha},$$

where $\alpha = 0.3$ (as in Nordhaus 2008) and $\tilde{A}_t$ is productivity at time t. Productivity is decomposed into two pieces: a deterministic trend A_t, and a stochastic productivity state ζ_t; that is to say, $\tilde{A}_t \equiv \zeta_t A_t$. The deterministic trend A_t is taken from Nordhaus (2008) and denoted as

$$A_t = A_0 \exp\left(\frac{\alpha_1 (1 - e^{-\alpha_2 t})}{\alpha_2}\right), \tag{7}$$

where α_1 is the 2005 growth rate and α_2 is the decline rate of the growth rate.

We add a stochastic component, ζ_t, to the productivity process so that we can examine how uncertainty about productivity affects climate change policies. Our specification of ζ_t uses ideas from the long-run-risk literature (e.g., Bansal and Yaron 2004; Hansen, Heaton, and Li 2008). We let χ_t represent the persistence of ζ_t and use the formulation introduced in Bansal and Yaron (2004):

$$\log(\zeta_{t+1}) = \log(\zeta_t) + \chi_t + \varrho\omega_{\zeta,t}, \tag{8}$$

$$\chi_{t+1} = r\chi_t + \varsigma\omega_{\chi,t}, \tag{9}$$

where $\omega_{\zeta,t}, \omega_{\chi,t} \sim$ i.i.d. $\mathcal{N}(0, 1)$ (i.i.d. means "independently and identically distributed") and ρ, r, and ς are parameters.

[9] The DSICE framework is flexible enough to allow for various functional forms of its components. However, to maximize comparability with the DICE model that is currently used in research for the design of US regulatory policy, we retain the assumptions of the DICE model.

For theoretical and computational reasons, we need to change some details of the Bansal and Yaron (2004) specifications. Bansal and Yaron (2004) assume that $\omega_{\zeta,t}, \omega_{\chi,t} \sim \text{i.i.d.}\,\mathcal{N}(0,1)$. Gaussian disturbances are unbounded; this implies the possibility of arbitrarily large growth rates and output. The unbounded character of the state space makes it difficult to even prove that expected utility exists. This creates computational challenges. Even if we could overcome these theoretical and computational challenges, our results for the SCC could be driven by highly unlikely tail events. Others have examined the impact of very bad outcomes that have very small probabilities. An example of this is the "dismal" theorem of Weitzman (2009), which states that the risk premium could be infinite for unboundedly distributed uncertainties. We want to avoid existence issues and to avoid repeating insights regarding extreme tail events. We also want to use numerically stable computational methods that allow us to verify our results. We achieve these goals by constructing a time-dependent, finite-state Markov chain for (ζ_t, χ_t) with parameter values implying conditional and unconditional moments of consumption processes observed in market data. The Markov transition processes are denoted

$$\begin{aligned} \zeta_{t+1} &= g_\zeta(\zeta_t, \chi_t, \omega_{\zeta,t}), \\ \chi_{t+1} &= g_\chi(\chi_t, \omega_{\chi,t}), \end{aligned}$$

where $\omega_{\zeta,t}$, and $\omega_{\chi,t}$ are two serially independent stochastic processes. This approach also makes it possible to directly apply reliable numerical methods for solving dynamic programming problems.

The approximation of the stochastic growth process requires a careful formulation of the Markov chains for the productivity shock ζ_t and the rate of its growth persistence χ_t. Markov chains with only a few states cannot represent the kind of persistence properties observed in Bansal and Yaron (2004). After examining various possibilities, we choose $n_\zeta = 91$ values of ζ_t and $n_\chi = 19$ values of χ_t at each time t; the time dependence is required as a result of the fact that the variance of consumption levels grows over time. Appendix A.2 describes the discretization procedure in greater detail.

We calibrate the stochastic factor productivity growth so that the resulting consumption process is statistically close to empirical data. Calibrating the productivity process presents some computational challenges because we need to solve the economic model in DSICE for various values of ρ, r, and ς and to choose those values, which imply a consumption process that matches US data on per capita consumption growth.[10] For our

[10] We are grateful to Ravi Bansal for providing us with the annual per capita data on real consumption used in Bansal, Kiku, and Yaron (2012) and obtained from the Bureau of Economic Analysis website.

calibration, we solve versions of DSICE without stochastic climate impact because the consumption data we use come from the twentieth century, when climate damage to productivity was negligible. Appendix A.2 describes the calibration procedure in greater detail and shows that the statistics of simulation paths of our consumption growth from our calibrated parameters are close to those of the empirical data. The results of the calibration gave us the following values:

$$\varrho = 0.035,\ r = 0.775,\ \varsigma = 0.008.$$

B. Damage Function and Emissions

DSICE models two potential ways in which output is affected by the climate: global average temperature T_{AT} and the Markov chain state denoted by J. The function $\Omega(T_{\mathrm{AT},t}, J_t)$, referred to as the damage function, represents the impact of climate on output, in that gross world product equals

$$Y_t \equiv \Omega(T_{\mathrm{AT},t}, J_t) f(K_t, L_t, \zeta_t A_t),$$

where

$$\Omega(T_{\mathrm{AT},t}, J_t) = \Omega_{\mathrm{T}}(T_{\mathrm{AT},t})\Omega_J(J_t) = \frac{1}{1 + \pi_1 T_{\mathrm{AT},t} + \pi_2 (T_{\mathrm{AT},t})^2}(1 - D(J_t))$$

decomposes the damage function as the product of Ω_{T} (damage due to temperature rise) and Ω_J (damage related to the climate conditions implied by the Markov process, J). When $D(J_t) = 0$, our damage function reduces to $\Omega_{\mathrm{T}}(T_{\mathrm{AT},t})$, for which we use the damage function in Nordhaus (2008).[11] Here $D(J)$ equals the impact of state J on productivity. This study generalizes the damage function to include effects of the tipping state and associated past cumulative effects along with the impact of temperature on productivity.

We assume that industrial emissions are proportional to output, with the proportionality factor σ_t, which is referred to as the carbon intensity of output. The social planner can mitigate (i.e., reduce) emissions by a factor μ_t, $0 \le \mu_t \le 1$. Annual industrial carbon emissions (billions of metric tons of carbon) equal

[11] The damage function in DSICE is, of course, highly aggregated and to be considered only as an approximation of smooth damages from global warming. Recently, several studies have attempted to quantify climate-related damages on a disaggregated scale: e.g., Deschênes and Greenstone (2011) and Dell, Jones, and Olken (2014) make use of weather data to assess the impact of temperature on a local scale. While this approach certainly generates important insight, more research is needed, e.g., to quantify longer-term impacts of temperature increase. Here, we make these choices to keep the model simple and to facilitate comparisons with Nordhaus's canonical model. The only general requirement is that $\Omega(T_{\mathrm{AT},t}, J_t) > 0$.

$$E_{\mathrm{Ind},t} = \sigma_t(1-\mu_t)f(K_t, L_t, \zeta_t A_t). \tag{10}$$

We follow Nordhaus (2008) and assume that the cost of mitigation level μ_t is

$$\Psi_t = \theta_{1,t}\mu_t^{\theta_2} Y_t. \tag{11}$$

World output, net of damage, is allocated across total consumption C_t, mitigation expenditures Ψ_t, and gross capital investment I_t; that is to say,

$$Y_t = C_t + \Psi_t + I_t, \tag{12}$$

and the capital stock evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{13}$$

where $\delta = 0.1$ is the annual depreciation rate for capital. In all of our computations, $K_0 = 137$ trillion dollars.

C. *Epstein-Zin Preferences*

The additively separable utility functions commonly used in climate-economy models do not do well in explaining the willingness of people to pay to avoid risk. Epstein-Zin preferences (Epstein and Zin 1989) are used because they better explain observed equity premia. The equity premium tells us about society's willingness to pay to reduce consumption risk, which will in turn affect how much society is willing to pay to reduce the risk of economic damage from climate change.

We assume that agents care only about consumption.[12] Let C_t be the stochastic consumption process. Epstein-Zin preferences recursively define social welfare as

$$U_t = \left[(1-\beta)u(C_t, L_t) + \beta\left(\mathbb{E}_t\left\{U_{t+1}^{1-\gamma}\right\}\right)^{[1-(1/\psi)]/(1-\gamma)}\right]^{1/[1-(1/\psi)]}, \tag{14}$$

where $\mathbb{E}_t\{\cdot\}$ is the expectation conditional on the states at time t and β is the discount factor. Here,

$$u(C_t, L_t) = \frac{(C_t/L_t)^{1-1/\psi}}{1-1/\psi} L_t$$

[12] There may be other aspects of climate change that affect social welfare, but they are not included in DSICE or in the standard Nordhaus (2008) family of models. See, e.g., the Cai et al. (2015) version of DSICE including uncertain, nonmarket impacts such as ecosystem tipping points.

is the annual world utility function (assuming that each individual has the same power utility function), ψ is the IES,[13] and γ is the risk aversion parameter. Epstein-Zin preferences are flexible specifications of decision-makers' preferences regarding uncertainty and allow us to distinguish between risk preference and the desire for consumption smoothing. Even though we refer to γ as the risk aversion parameter, equilibrium risk premia will depend on interactions between ψ and γ.

We compare the results of DSICE with results of deterministic models such as DICE. Those models assume intertemporally separable preferences. It is important to note that in the absence of uncertainty, the risk aversion parameter, γ, disappears, and Epstein-Zin preferences become intertemporally separable. In deterministic models with time-separable utility, it is common to refer to $1/\psi$ as the "risk aversion" parameter, but that is misleading and implicitly assumes $\gamma = 1/\psi$. A better way to think about deterministic models is that they use Epstein-Zin preferences, with ψ representing the IES, but that the risk aversion parameter, γ, can take any value. This observation will be important when we discuss the impact of adding risks to DICE.

Empirical analyses of macroeconomic data have not given us definitive values for the parameters γ and ψ. We examine a range of parameter values, relying on the literature on long-run risk and asset pricing, in particular Bansal and Yaron (2004), Barro (2009), Bansal and Ochoa (2011), Pindyck and Wang (2013), Epstein, Farhi, and Strzalecki (2014), and Schorfheide, Song, and Yaron (2014). Most studies estimate or assume γ to be between 2 and 10 and argue for $\psi > 1$. Table 1 summarizes the range of values for ψ and γ in the literature. Our benchmark parameter specifications are $\psi = 1.5$ and $\gamma = 10$, choices consistent with the majority of opinion. We also solve DSICE for a broad range of values covering $2 \leq \gamma \leq 15$ and $0.5 \leq \psi \leq 2.0$.

D. *Relation to the Literature*

Some have used Epstein-Zin preferences to examine the SCC but use much simpler models. Bansal and Ochoa (2011) assume an endowment model, implying that their SCC tells one only how to price the exogenous shocks exerted by the climate on the endowment. DSICE, instead,

[13] Here we assume $\psi > 1$. When $0 < \psi < 1$, the utility function $u(C_t, L_t)$ is negative, and the formula becomes

$$U_t = -\left[-(1-\beta)u(C_t, L_t) + \beta\left(\mathbb{E}_t\left\{(-U_{t+1})^{1-\gamma} \mid C_t, L_t\right\}\right)^{(1-1/\psi)/(1-\gamma)}\right]^{1/(1-1/\psi)}.$$

While the standard formulation of Epstein-Zin preferences does not have a denominator of $1 - 1/\psi$ in the annual world utility function $u(C, L)$, here we use this formulation to ensure that it is consistent with our later reformulation of the Bellman equation (eq. [15]).

TABLE 1
Default or Estimated Values of IES and the Risk Aversion Parameter (RA) in the Literature

Reference	IES	RA
Bansal and Yaron 2004	1.5	10
Bansal and Ochoa 2011	1.5	10
Vissing-Jørgensen and Attanasio 2003	>1	5–10
Barro 2009	2	4
Pindyck and Wang 2013	1.5	3.066
Ackerman, Stanton, and Bueno 2013	1.5	10
Constantinides and Ghosh 2011	1.41	9.43
Schorfheide et al. 2014	1.7	10.8
Epstein et al. 2014	1.5	7.5
Jensen and Traeger 2014	1.5	10

builds on a Ramsey-type, representative-agent, stochastic growth model where the agent at each time must choose how to allocate output across consumption and savings. Savings are split between capital investment and mitigation expenditures, both of which are forms of spending that aim to improve future productivity. Our SCC represents the social trade-off between spending resources on investment and on mitigation.

IAMs have only recently used long-run risk and Epstein-Zin preferences. Jensen and Traeger (2014) assume levels of volatility much lower than those implied by empirical data. See appendix A.2 for a detailed comparison. The higher volatility in DSICE (which is calibrated to fit empirical data) presents computational challenges because the state variables cover a much larger region in the $(K, \mathbf{M}, \mathbf{T}, \zeta, \chi)$ space. DSICE is successful because it uses a flexible set of functions for approximations.

IV. The Dynamic Programming Problem

We formulate the nine-dimensional, social planner's dynamic optimization problem as a dynamic programming problem. The nine states include six continuous state variables (the capital stock K, the three-dimensional carbon system $\mathbf{M}$, and the two-dimensional temperature vector $\mathbf{T}$) and three discrete state variables (the climate shock J, the stochastic productivity state ζ, and the persistence of its growth rate χ). Let $\mathbf{S} \equiv (K, \mathbf{M}, \mathbf{T}, \zeta, \chi, J)$ denote the nine-dimensional state variable vector, and let $\mathbf{S}_+$ denote its next period's state vector.

The Epstein-Zin utility definition (eq. [14]) expressed utility in terms of consumption. We make a nonlinear change of variables[14] in terms of utils, $(U_t)^{1-1/\psi}$, and then get the following Bellman equation:

[14] That is, $V_t(\mathbf{S}) = (U_t(\mathbf{S}))^{1-1/\psi}/(1-\beta)$.

$$
\begin{aligned}
V_t(\mathbf{S}) &= \max_{C,\mu} u(C_t, L_t) + \beta\Big(\mathbb{E}_t\big\{(V_{t+1}(\mathbf{S}_+))^{(1-\gamma)/(1-1/\psi)}\big\}\Big)^{(1-1/\psi)/(1-\gamma)}, \\
\text{such that} \quad K^+ &= (1-\delta)K + Y_t - C_t - \Psi_t, \\
\mathbf{M}^+ &= \mathbf{\Phi}_{\mathrm{M}}\mathbf{M} + (E_t, 0, 0)^\top, \\
\mathbf{T}^+ &= \mathbf{\Phi}_{\mathrm{T}}\mathbf{T} + (\xi_1 \mathcal{F}_t(M_{\mathrm{AT}}), 0)^\top, \\
\zeta^+ &= g_\zeta(\zeta, \chi, \omega_\zeta), \\
\chi^+ &= g_\chi(\chi, \omega_\chi), \\
J^+ &= g_J\big(\mathbf{T}, \mathbf{M}, J, \omega_J\big),
\end{aligned} \tag{15}
$$

for $t = 0, 1, \ldots, 599$ and any $\psi > 1$.[15] The terminal value function V_{600} is given in appendix A.5.[16] In the model, consumption C and emission control rate μ are the two control variables.

A. *Numerical Solution Method*

We solve the nine-dimensional problem specified in equation (15) by using value function iteration. Three state variables (ζ, χ, and J) are discretized, and their movements are modeled as transitions of finite-state Markov chains. The productivity process states (ζ and χ) use Markov chains that have enough states to ensure that the resulting consumption processes match the conditional variance and autocorrelation observed in consumption data. Furthermore, J is calibrated to represent processes discussed in the climate literature on tipping points. At each discrete point in the (ζ, χ, J) space, the value function has six continuous states, (K, $\mathbf{M}$, $\mathbf{T}$), and is approximated by multivariate orthogonal polynomials after appropriate nonlinear changes of variables. The range of each continuous state variable is chosen so that all simulation paths stay in that range. This is a large problem, but the use of parallel programming methods and hardware makes it tractable. For example, the largest case has 366 billion optimization problems, but we solved it in less than 8 hours, using 110,688 cores in parallel on the Blue Waters supercomputer. See appendixes A.2 and A.5 of this paper and Cai et al. (2015) for an extended discussion of the mathematical and computational details.

[15] When $0 < \psi < 1$, the objective function of the optimization problem is

$$u(C_t, L_t) - \beta\Big(\mathbb{E}_t\big\{(-V_{t+1}(\mathbf{S}_+))^{(1-\gamma)/(1-1/\psi)}\big\}\Big)^{(1-1/\psi)/(1-\gamma)}.$$

[16] As in Nordhaus (2008), we assume a model time horizon of 600 periods. The terminal value function should be regarded as an approximation of the value function at the terminal period.

B. A Verification Test

One theme of the VVUQ literature (e.g., Oberkampf and Roy 2010) is the implementation of tests that check the correctness of the computer code; this is called *verification*. One common test is to apply the code to special cases where the solution is known. If all uncertainty is removed, then DSICE reduces to a deterministic optimal control problem that can be solved using nonlinear programming methods. We compare these optimal control solutions to our value function iteration results to see whether the value functions imply an optimal path in line with the results using nonlinear programming. Our tests show that paths implied by the value functions have at least 3-digit accuracy, often significantly more. See appendix A.7 for more details.

C. The SCC

The climate literature interprets the SCC as a marginal concept—that is to say, the monetized economic loss caused by a 1-metric-ton increase in atmospheric carbon. We follow that notation: in DSICE, the SCC is the marginal cost of atmospheric carbon expressed in terms of the numeraire good, which can be either consumption or capital, as there are no adjustment costs. We define the SCC as the marginal rate of substitution between atmospheric carbon concentration and capital, as in

$$\mathrm{SCC}_t = \frac{-1{,}000(\partial V_t/\partial M_{\mathrm{AT},t})}{\partial V_t/\partial M_{\mathrm{AT},t}}. \tag{16}$$

It is important to remember that the SCC is a relative shadow price—that is to say, a ratio of two marginal values—and does not express the total social cost of climate damage.[17] For example, as we change economic and/or climate risks, the SCC may go up or down because that change in risks will affect the marginal value of carbon, the marginal value of consumption, and the marginal value of investment. DSICE is a general equilibrium model where the results arise from assumptions about tastes and technology as well as their equilibrium interactions.

The SCC will often be the *optimal carbon tax*. The optimal carbon tax is the tax on carbon that would equate the private and social costs of carbon. In DSICE we also examine the optimal carbon tax, which is the Pigovian tax policy because the externality from carbon emissions can be directly dealt with by a carbon tax and because there are no other market imperfections. The social planner in DSICE chooses mitigation μ_t, which is equivalent to choosing a carbon tax equal to

[17] Because K is measured in trillions of dollars and M_{AT} is measured in billions of tons of carbon, the factor 1,000 is needed to express the SCC in units of dollars per ton of carbon.

$$\frac{1{,}000\theta_{1,t}\theta_2\mu_t^{\theta_2-1}}{\sigma_t} \tag{17}$$

in units of dollars per ton of carbon. If $\mu_t < 1$, then the carbon tax equals SCC. However, if $\mu_t = 1$, its maximum value, then the carbon tax equals only that level that will drive emissions to zero and may be far less than the SCC. In such cases, mitigation policies have reached their limit of effectiveness. Alternative policies may reduce carbon concentrations directly, as would carbon removal and storage technologies, or reduce temperature directly, as would certain solar geoengineering technologies. We do not explicitly include those technologies in DSICE, but our SCC numbers will identify equilibrium paths along which the SCC is so high that these more direct technologies may be competitive.[18] We leave a quantitative analysis of these issues to future studies.

The business-as-usual (BAU) case is the solution to the dynamic programming problem (15) but with the restriction that $\mu = 0$, that is, no mitigation. It is used in the IAM literature as an approximation of the competitive equilibrium in the absence of any carbon tax. We generally do not report any BAU results because they differ little from the optimal mitigation results.

D. *Simulation Procedures*

In Sections V, VI, and VII, respectively, we analyze the implications for climate policy of only stochastic growth, only stochastic climate, and stochastic growth and stochastic climate combined. In each of these sections, we define a benchmark parameter specification and show its implications for the SCC and other economic variables. We then perform a sensitivity analysis on some parameters, using tables to report how today's optimal level of the SCC is affected by different parameter choices.

All three sections follow the same procedure. First, we solve the dynamic programming problem, computing the value function and policy decision rules at each time t. We then use the decision rules to simulate some paths of the key economic variables. We use 2005 as the first year in order to be comparable with Nordhaus (2008) and similar studies. These paths are affected by shocks to productivity; we simulate 10,000 paths that differ only in the realized shocks. This set of paths allows us not only to

[18] Robock et al. (2009) critically assess geoengineering options in the context of decision-making on climate change and point out severe side effects of geoengineering, as well as the lack of knowledge associated with this technology. Recently, Heutel, Moreno-Cruz, and Shayegh (2016) present one of the first studies to address geoengineering in a stochastic IAM. Their study focuses on the effectiveness of geoengineering to deal with different types of damage from a tipping point. Further research is needed to improve understanding of how different sources of uncertainty affect the effectiveness of geoengineering as an option to combat climate change.

compute the expected value of the SCC and economic variables but also to compute their distribution at any time t. The large number of simulations allows us to compute even the 99th percentile and gives us a good measure of the important tail events. Each simulation begins with the stochastic growth states equal to $\chi_0 = 0$ and $\zeta_0 = 1$. Other initial values are given in tables A.4 and A.5. In cases with a tipping element, we assume that a tipping point has not yet been triggered in the year 2005.

V. The SCC with Stochastic Growth

This section analyzes the impact of stochastic growth and risk preferences on the SCC and on the combined climate and economic system. This section excludes tipping elements so that it can focus on the impact of productivity risk. We first present a detailed analysis of a benchmark example based on parameter specifications; we call this our *stochastic growth benchmark* case. The stochastic growth benchmark case assumes Epstein-Zin parameter values $\psi = 1.5$ and $\gamma = 10$. This stochastic growth benchmark case allows us to exposit key features of the resulting dynamic processes, such as consumption, output, productivity, climate states, and the SCC. We then perform a sensitivity analysis for empirically plausible alternative preference parameters, focusing on how preference assumptions affect the SCC in 2005.

A. *The Stochastic Growth Benchmark*

Figures 1 and 2 display features of the solution to the Bellman equation (15) for the stochastic growth benchmark case. Each panel summarizes the results of our 10,000 simulations. We also display results from two deterministic cases of DICE-CJL. DICE-CJL with $\psi = 0.5$ (solid red line) represents the DICE-2007 choice of ψ, and DICE-CJL with $\psi = 1.5$ (dashed red line) represents the choice in our stochastic growth benchmark. The gray area represents the 1st–99th percentiles of the SCC paths, and the other lines represent various quantiles.

Figure 1 displays the SCC process for 2005–50. First, note that moving from the DICE-2007 choice of $\psi = 0.5$ to our benchmark choice of $\psi = 1.5$ substantially increases the SCC. However, when we add uncertainty with $\gamma = 10$, we see a decline in the SCC to \$61/tC, which is still higher than that implied by DICE-2007 preferences. During 2005–50, we see that the no-risk line (dashed red line) exceeds the average SCC with long-run risk (black line) by about \$50/tC. Therefore, in the next half-century, uncertainty reduces the SCC. We discuss the intuition for this below when we carry out sensitivity analysis. The productivity risk does substantially increase the range of the SCC. By 2050, there is a 25 percent chance of the SCC being almost \$200/tC or greater and a 10 percent chance of it

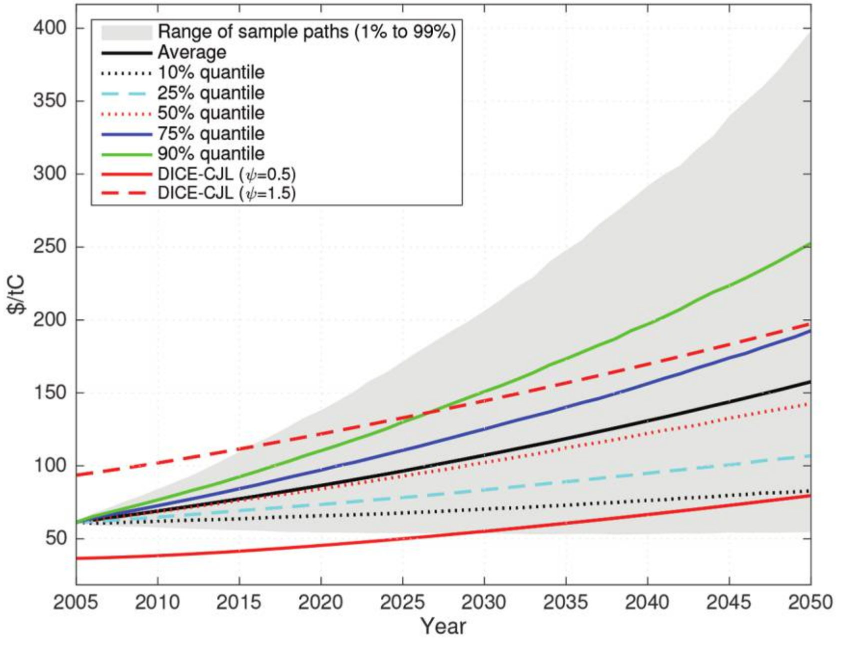

FIG. 1.—SCC ($/tC): 2005–50

exceeding $250/tC. The large range is easily explained. If there is a sustained sequence of positive shocks to productivity, then output will rise, which will have two effects: the damage due to temperature will increase proportionately, and emissions will also rise and increase the risk of even greater damage from high temperatures in the future. Recall that the SCC results in figure 1 represent the solution to the optimal tax and mitigation policy.

Figure 2 describes several variables over the twenty-first century. Panel *A* describes the SCC process. After 2050, the average SCC in DSICE moves closer to the SCC in DICE-CJL with $\psi = 1.5$. By 2100, the impact of uncertainty is small. Therefore, while uncertainty substantially lowers the SCC before 2050, that effect nearly disappears over the following 50 years. After 2050, the variation in the SCC in DSICE continues to grow rapidly. The 1st–99th percentile range is $67–$1,282/tC, and even the 10 percent and 90 percent quantiles in 2100 show a range of $127–$667/tC.

Figure 2*B* displays the optimal carbon tax, and figure 2*C* displays the optimal rate of mitigation. If mitigation is less than 100 percent, then a Pigovian tax policy would be to impose a tax equal to the SCC, thereby equating the private and social costs of carbon. Panels *A*–*C* show that this is always the case before 2085. After 2085, the emission control rate may

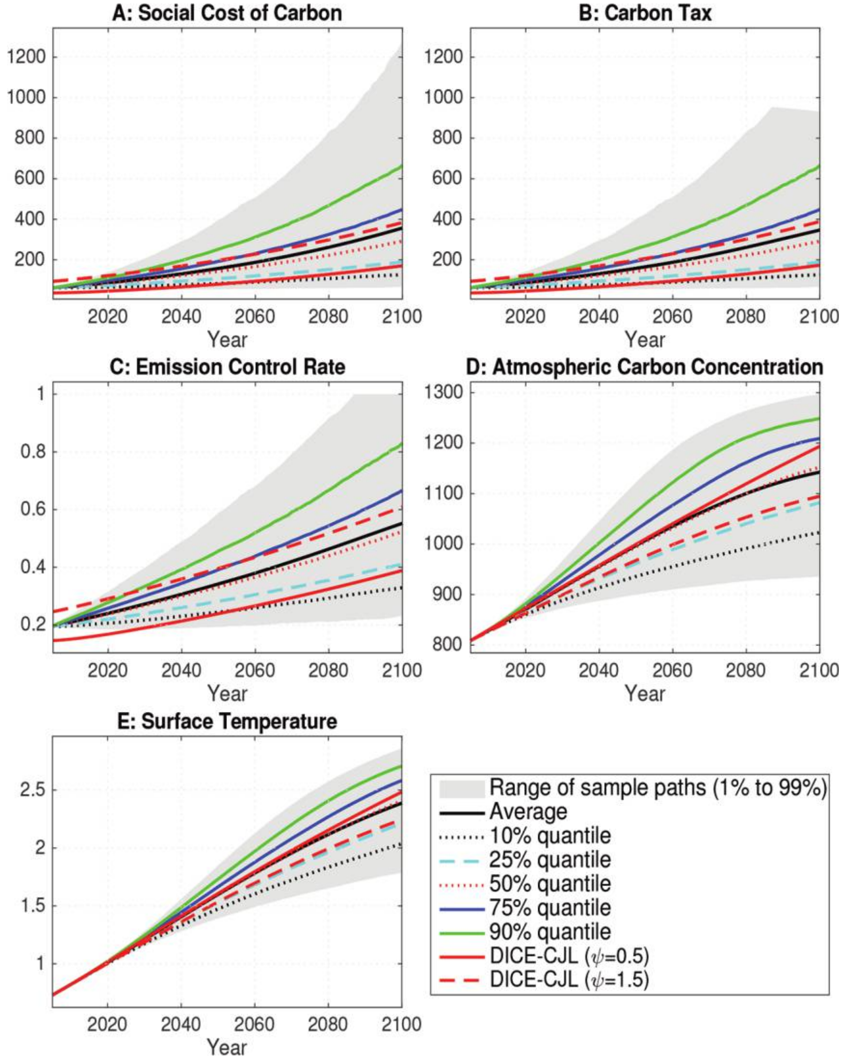

FIG. 2.—Simulation results of the stochastic growth benchmark—climate system and policies.

hit its limit of $\mu = 1$, and optimal policy requires only that the carbon tax be large enough to eliminate all emissions. In that case, the optimal tax can be much less than the SCC, which is true at 2100 in about 3.5 percent of simulation runs. When the optimal tax is less than the SCC, the benefits of other carbon policies, such as carbon removal and storage or solar geoengineering technologies, would be valued using the SCC.

Table 2 lists the mean and standard deviation of SCC_t and their statistics on a $\log_{10}$ scale.[19] Table 2 tells us that both the mean and the standard deviation of $\log_{10}(\mathrm{SCC}_t)$ are increasing over time. In fact, the standard deviation of $\log_{10}(\mathrm{SCC}_t)$ is increasing much faster than its mean.

Figures 2*D* and 2*E* both indicate that long-run risk will, on average, lead to more accumulation of carbon in the atmosphere and higher temperature, a natural consequence of the reduced mitigation efforts. However, the economic risk implies uncertainty in atmospheric temperature in 2100, with the 10th–90th percentile range being 0.6°C.

Our findings point to one very important fact: there is great uncertainty about all aspects of the combined economic and climate system. For many variables, the mean value at each point in time is close to the solution of the purely deterministic model. Tracking the mean is all one can ask of any deterministic model, and in that sense deterministic models can be successful. However, there is great uncertainty about the future value of each key variable. This fact is of particular importance for understanding the SCC. The SCC is the marginal cost of extra carbon in terms of wealth, making it the marginal rate of substitution between mitigation expenditures and investment expenditures in physical capital. At the margin, these two uses of savings have different impacts on future economic variables, making allocation decisions between mitigation and investment essentially a portfolio choice problem; a large SCC represents the amount of investment in new capital that one is willing to sacrifice to reduce carbon emissions by a gigaton.

In comparison to a model with purely deterministic growth, DSICE implies a lower ratio of consumption to gross world output and higher ratios of investment in capital accumulation and abatement. Figure 3 presents the details, displaying the optimal dynamic distribution of the four ratios for the first 100 years with various quantiles of 10,000 simulations of the solution to the dynamic programming problem (15).

For example, the black dotted lines represent the 10 percent quantiles for each ratio. Similarly, the cyan dashed lines, red dotted lines, blue solid lines, and green solid lines represent the 25, 50, 75, and 90 percent quantiles at each time, respectively. The black solid lines represent the sample mean path. As explained above, two cases of DICE-CJL ($\psi = 0.5$ or 1.5) make it comparable with DICE-2007. We denote these two special deterministic cases by red solid and red dashed lines, respectively. The lower (upper) edge of the gray areas represent the 1 percent (99 percent) quantile; the gray areas represent the 98 percent probability range of each ratio.[20]

[19] We include the $\log_{10}$ scale because the SCC distribution is more like a lognormal distribution.

[20] We use the same graphical exposition for all plots describing the distributions of simulation results.

TABLE 2
SCC STATISTICS FROM 10,000 SIMULATION PATHS

	SCC_t (\$/tC)		$\log_{10}(SCC_t)$	
	Mean	Standard Deviation	Mean	Standard Deviation
2020	87	18	1.924	.087
2050	158	71	2.153	.184
2100	357	247	2.457	.279

From figure 3*A*, we see that, with more than 90 percent probability, I_t/Y_t will be greater than in the case of a purely deterministic model (the red solid line is below the black dotted line). Furthermore, we find that in 2005, I_t/Y_t is at 32 percent, about 8 percentage points higher than under the deterministic growth assumption, and that the expected difference is

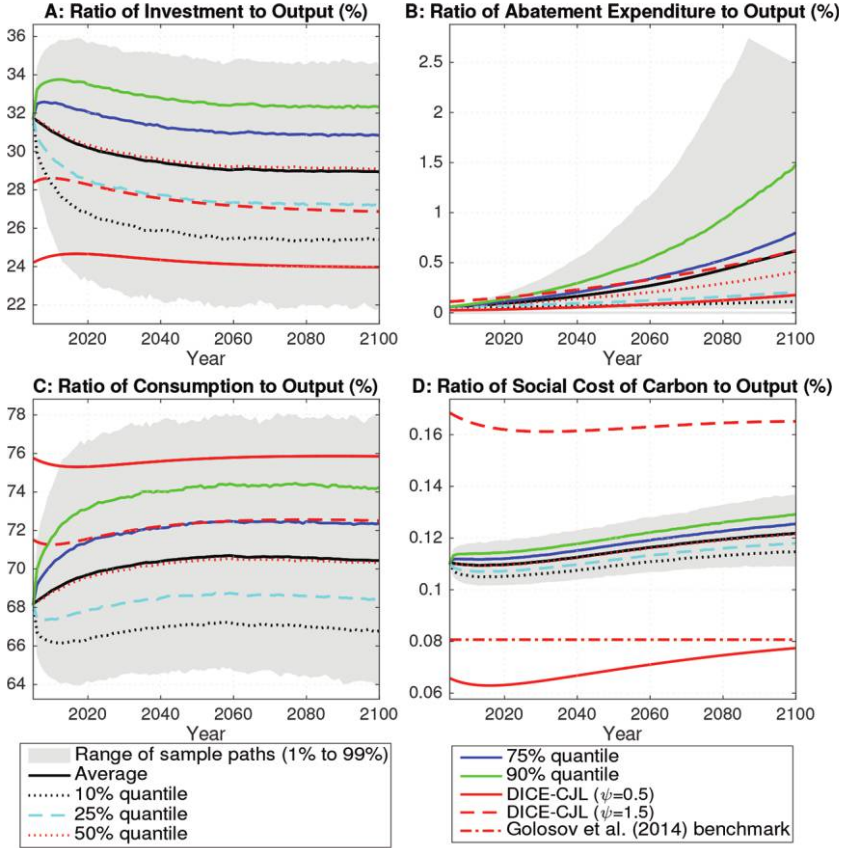

FIG. 3.—Simulation results of the stochastic growth benchmark—ratios to gross world output.

about 5 percentage points toward the end of this century. Overall, the assumption of stochastic factor productivity growth with persistence leads to a significant expected increase in capital investments and thus to a precautionary buildup of the capital stock.[21] Also, throughout the 10,000 simulations of DSICE, we found I_t/Y_t to range roughly between 22 and 36 percent, expanding the distributional results reported in table 2.

Figure 3*C* presents quite the opposite statistical picture for C_t/Y_t. We see that, with more than 90 percent probability, C_t/Y_t will be lower than in a purely deterministic model and that toward the end of this century, C_t/Y_t appears to stabilize at about 70 percent—a reduction of about 6 percentage points from the deterministic model. Overall, this reduction is not fully offset by higher capital investments, and, as figure 3*B* indicates, that difference is allocated to expenditures on the abatement of emissions. We find that the latter, which is denoted by Ψ_t/Y_t, is generally quite low and does not exceed 0.2 percent in this century in the deterministic case. Yet, as the black solid line in *B* indicates, there is a 50 percent probability that the expenditures on emissions abatement will be at least three times higher by the year 2100 when growth is modeled stochastically. Furthermore, with more than 20 percent probability, more than 1 percent of gross world output should be devoted to mitigation by the year 2100.

Some recent research has argued for simple rules of thumb for the SCC. In particular, Golosov et al. (2014) set up a dynamic, forward-looking climate-economy model with logarithmic utility and full capital depreciation to argue that the SCC is proportional to output. Barrage (2014) shows that the benchmark in Golosov et al. (2014) implies that the ratio of the SCC to decadal gross world output is 8.07×10^{-5} (i.e., $\mathrm{SCC}_t/Y_t = 8.07 \times 10^{-4}$ for annual gross world output Y_t) and constant over time with constant productivity growth but that it increases over time for the productivity process in Nordhaus (2008), approaching 8.07×10^{-4} from below.[22]

Figure 3*D* uses red dash-dotted lines to represent the results of Golosov et al. (2014). This plot shows that the SCC_t/Y_t ratio is stochastic and that its mean or quantile paths have an upward trend, while its volatility is also expanding. Its mean path increases from 0.11 percent in 2005 to 0.12 percent in 2100. This implies that long-run risk shows that the constant SCC_t/Y_t in Golosov et al. (2014) does not hold true in general.

B. Sensitivity Analysis for Preference Parameters

Empirical work suggests plausible values for ψ and γ, but the data do not give us precise values for the key parameters. We next examine how the

[21] The simulation paths of gross world output, capital, and per capita consumption growth are shown in fig. A.2 in app. A.8.

[22] More precisely, we report the ratio $(\mathrm{SCC}_t/1{,}000)/Y_t$ in order to have the same units used in Golosov et al. (2014).

TABLE 3
Initial Consumption-Output Ratio in Stochastic Growth Cases

		Risk Aversion Parameter (γ)				
IES (ψ)	DICE-CJL	2	5	10	15	∞
.50	.76	.75	.73	.71	.69	.58
1.25	.72	.71	.70	.69	.68	.64
1.50	.72	.71	.69	.68	.68	.65
1.75	.71	.70	.69	.68	.67	.66
2.00	.71	.69	.68	.67	.67	.67

SCC varies across values of ψ and γ. Each example differs from the stochastic growth benchmark only in the preference specification, with no change in the stochastic growth productivity process.

Our sensitivity analysis of IES (i.e., ψ) and risk aversion (i.e., γ) in stochastic growth cases will look at consumption, capital investment, and the SCC in 2005. Tables 3–6 report the sensitivity of the initial ratios of consumption to gross world output (C_t/Y_t), capital investment to gross world output (I_t/Y_t), and abatement expenditure to output and of the SCC, respectively, assuming the values of the elasticity of intertemporal substitution to be $\psi = 1.25$, 1.5, 1.75, and 2.0, and the risk aversion parameter to be $\gamma = 2$, 5, 10, and 15 (we also include the cases with $\psi = 0.5$ and/or $\gamma = \infty$ for comparison and cases with $\psi = 0.7$, 0.9, and 1.1 for SCC in table 6). For example, in our benchmark example with $\psi = 1.5$ and $\gamma = 10$, the 2005 C_t/Y_t is 0.68, the 2005 I_t/Y_t is 0.32, and the optimal 2005 SCC is \$61/tC. We see that the SCC ranges from \$57 to \$99/tC for these combinations of $1.25 \leq \psi \leq 2$ and $2 \leq \gamma \leq 15$, about 60–168 percent larger than the 2005 SCC of the baseline deterministic model with $\psi = 0.5$.

Tables 3–6 show that the patterns of the impact of IES and risk aversion in the stochastic growth cases are similar to those of IES and productivity growth in the deterministic growth cases. We use the extreme case of $\gamma = \infty$ to provide some intuition. In that case, the planner focuses on the worst-case scenario, which in our discretization of the productivity process means

TABLE 4
Initial Investment-Output Ratio in Stochastic Growth Cases

		Risk Aversion Parameter (γ)				
IES (ψ)	DICE-CJL	2	5	10	15	∞
.50	.24	.29	.27	.29	.31	.42
1.25	.28	.29	.30	.31	.32	.36
1.50	.28	.29	.31	.32	.32	.35
1.75	.29	.30	.31	.32	.33	.34
2.00	.29	.31	.32	.33	.33	.33

TABLE 5
Initial Abatement-Expenditure-to-Output Ratio in Stochastic Growth Cases

		Risk Aversion Parameter (γ)				
IES (ψ)	DICE-CJL	2	5	10	15	∞
.5	2.6(−4)	2.9(−4)	4.1(−4)	5.7(−4)	6.6(−4)	8.0(−4)
1.25	9.1(−4)	8.4(−4)	7.0(−4)	5.9(−4)	5.5(−4)	3.9(−4)
1.5	1.1(−3)	9.9(−4)	7.6(−4)	5.9(−4)	5.5(−4)	3.5(−4)
1.75	1.3(−3)	1.1(−3)	8.1(−4)	6.3(−4)	5.5(−4)	3.3(−4)
2.0	1.4(−3)	1.2(−3)	8.6(−4)	6.4(−4)	5.6(−4)	3.1(−4)

Note.—$a(-n)$ represents $a \times 10^{-n}$.

that decisions are the same as if productivity were deterministic and constantly declining, allowing us to use deterministic optimal control methods for that case; see appendix A.9 for details. The last column of table 6 displays how IES affects economic quantities when $\gamma = \infty$. In this case, DSICE is essentially optimal growth with declining productivity. A small IES implies a strong desire to smooth per capita consumption and save more today to reduce a future decline in consumption. As IES increases, smoothing consumption is less of a priority, implying less savings in both forms, capital and mitigation, and a smaller SCC.

When there is no risk (the "DICE-CJL" column of table 6) or if risk aversion is small, then the patterns are reversed. In these cases, the positive future growth in productivity implies that as IES increases, there is less desire for smoothing and more willingness to save for the future, when productivity is higher. The SCC and the share of savings going to mitigation rise as IES increases, indicating that the relative importance of avoiding climate damage increases.

Table 6 shows that the SCC is decreasing over risk aversion when $\psi \geq 0.9$ and increasing over risk aversion when $\psi \leq 0.7$. From the above discussion, we see that risk aversion in DSICE is related to productivity growth in DICE-CJL: $\gamma = \infty$ corresponds to DICE-CJL with negative productivity

TABLE 6
Initial Social Cost of Carbon (\$/tC) in Stochastic Growth Cases

		Risk Aversion Parameter (γ)				
IES (ψ)	DICE-CJL	2	5	10	15	∞
.5	37	39	49	60	66	76
.7	51	52	56	60	62	62
.9	64	63	61	60	60	55
1.1	75	71	65	60	58	50
1.25	82	77	68	61	58	48
1.5	94	86	71	61	57	45
1.75	103	93	74	62	57	43
2	111	99	76	62	57	41

growth, and a smaller risk aversion corresponds to DICE-CJL with a higher productivity growth. Appendix A.10 shows that the SCC from DICE-CJL with a range of ψ and 2005 productivity growth rate has a pattern similar to that of table 6.

C. *Comparisons with Others' SCC Estimates*

Our results differ from those of other studies on SCC estimates, but usually for understandable reasons. Golosov et al. (2014) obtain a relatively high SCC as a result of the assumption that all global warming effects from carbon emissions occur instantaneously, ignoring the lags in the climate system. Anthoff and Tol (2014) use their model, FUND, to argue for lower estimates for the SCC in the early twenty-first century because their disaggregated damage function takes into account the fact that increases in CO_2 plus mild warming can improve agricultural productivity in the upper latitudes. However, FUND does not allow agents to optimize consumption decisions dynamically. Future versions of DSICE will aim to include more sophisticated damage functions.

The IWG (2010) report finds that lowering the rate of discounting for damages from 3 percent to 2 percent increases the SCC by a factor of 1.64. Another major component that affects the SCC is the dynamic structure of damages caused by global warming. For example, Dietz and Stern (2015) assume much higher damages due to high temperature than does DICE (when the temperature increase is 4°C, the damage is 50 percent of output in Dietz and Stern [2015] but only 4 percent in DICE-2007). We do not examine these questions extensively in this study, but the limited experimentation we did indicates that DSICE responds to these changes in the same way.

VI. The SCC with Stochastic Climate Tipping

We next study how a tipping element in the climate system may affect the SCC in the absence of any economic uncertainty. First, we present a Markov chain specification of a representative climate tipping element. We then specify benchmark parameters calibrated to a "representative" scenario and study the optimal climate policy; we call this our *climate tipping benchmark*. In a final step, we present a multidimensional sensitivity analysis. In light of the numbers provided in the few studies available, we conclude that the impact of potential tipping-point events should be carefully assessed.

A. *A Markov Chain Specification of the Climate Tipping Process*

We study a simple example of tipping, with uncertainty about the time of tipping and the long-run damage. The initial state, which we call the *pretipping state*, is denoted $\mathcal{J}_0$, and $D(\mathcal{J}_0) = 0$. "Tipping" is the time and event when the climate leaves state $\mathcal{J}_0$ and moves to some other state in the tipping process. We follow the common assumption that warming alone causes tipping (Smith et al. 2009; IPCC 2014) and assume that the probability that a tipping event occurs at time t is a function of temperature,

$$p_{\mathrm{tip},t} = 1 - \exp\left(-\lambda \max\left\{0,\ T_{\mathrm{AT},t} - \underline{T_{\mathrm{AT}}}\right\}\right), \tag{18}$$

where λ is the hazard rate parameter and $\underline{T_{\mathrm{AT}}}$ is the temperature for which $p_{\mathrm{tip},t} = 0$. The hazard rate parameter, λ, together with the temperature process, determines the duration of the initial state $\mathcal{J}_0$.

In this study, we examine a simple tipping process that models the gradual nature of the tipping process and uncertainty about the ultimate damage caused. After a tipping event, the state J follows one of three possible processes, each one also modeled by a Markov chain. Define $\mathcal{M}_i$, for $i = 1$, 2, or 3, to be a Markov chain with states $\{\mathcal{J}_{i,1}, \mathcal{J}_{i,2}, \mathcal{J}_{i,3}, \mathcal{J}_{i,4}, \mathcal{J}_{i,5}\}$. Each process $\mathcal{M}_i$ moves in sequence through transient tipping states $\mathcal{J}_{i,2}$, $\mathcal{J}_{i,3}$, and $\mathcal{J}_{i,4}$ and ultimately arrives at the absorbing state $\mathcal{J}_{i,5}$.

The damage factor in state $\mathcal{J}_{i,j}$ is $\mathfrak{D}_{i,j} = D(\mathcal{J}_{i,j})$. The damages at the terminal states,$\{\mathfrak{D}_{1,5}, \mathfrak{D}_{2,5}, \mathfrak{D}_{3,5}\}$ represent three different levels of long-run permanent damages. Let J_∞ denote the random long-run state, and let $\mathfrak{D}_\infty$ denote the random long-run damage from tipping (i.e., $\mathfrak{D}_\infty = D(J_\infty)$).

Therefore, we begin in state $\mathcal{J}_0$ and leave $\mathcal{J}_0$ at time t with probability $p_{\mathrm{tip},t}$, and, after a tipping event, J jumps to one of $\{\mathcal{J}_{1,1}, \mathcal{J}_{2,1}, \mathcal{J}_{3,1}\}$ with equal probability. Before the tipping event, we do not know which $\mathcal{M}_i$ process will be followed after tipping, and we do not know the final level of damage. However, at the tipping time, we learn which $\mathcal{M}_i$ governs the posttipping process and the long-run damage.

This is a stark simplification, but it does allow us to distinguish the expected duration of the transient posttipping process from the uncertainty about the ultimate damage level. The fact that the uncertainty about $\mathfrak{D}_\infty$ is resolved when the climate tipping process is triggered allows us to make statements about the relative impact of the hazard rate of the tipping-point event, the expected duration of the whole transient posttipping process, and the mean and variance of $\mathfrak{D}_\infty$. The unconditional mean of $\mathfrak{D}_\infty$ is denoted $\bar{\mathfrak{D}}_\infty$, and the variance of $\mathfrak{D}_\infty$ is $q\bar{\mathfrak{D}}_\infty^2$, where q is called the "mean squared-variance ratio." The ratio q is analogous to the square of the Sharpe ratio, a concept used in portfolio theory, and arises naturally in our discussion of results.

In our examples, for each posttipping Markov chain $\mathcal{M}_i$, we assume that its expected duration from the time when the tipping event occurs to the time when the tipping state reaches the absorbing state is known, and it is denoted $\bar{\Gamma}$. We also refer to it as the *expected duration of the posttipping process*. There are four transient stages in the posttipping process, and we assume that each transient stage from $\mathcal{J}_{i,j}$ to $\mathcal{J}_{i,j+1}$ has the same expected duration[23] and that all transitions have an exponential distribution, implying that the transition probability is $p = 1 - \exp(-4/\bar{\Gamma})$.[24] The complete mathematical description of J and its calibration are contained in appendix A.4.

B. The SCC with Stochastic Climate Tipping

We choose parameter values that are roughly the average of the range of opinions in the literature.[25] More precisely, the climate tipping benchmark case assumes $\lambda = 0.0035$, $\bar{\mathfrak{D}}_\infty = 0.05$, $q = 0.2$, and $\bar{\Gamma} = 50$. The choice of $\lambda = 0.0035$ implies that the conditional annual probability of tipping increases by 0.35 percent for a warming of 1°C. The Epstein-Zin preference parameters in the climate tipping benchmark case are again $\psi = 1.5$ and $\gamma = 10$.

First, we use figure 4 to show two sample paths of damage to output, $1 - \Omega(T_{\mathrm{AT},t}, J_t)$, in percentage terms (solid lines) and their corresponding paths for the SCC (dashed lines). The two lines in panel *A* represent one sample realization of a tipping process that tips in 2146 with the largest long-run damage level, and the two lines in panel *B* represent one sample realization of a tipping process that tips in 2102 with the smallest long-run damage level. The sample paths of damage to output clearly show the four transient posttipping stages (jumps).

The realized duration of the whole transient posttipping process is 38 years for the sample path in panel *A* (from 2146 to 2184) and 69 years for the sample path in panel *B* (from 2102 to 2171), while the expected duration of the posttipping process, $\bar{\Gamma}$, is 50 years. Moreover, the realized final damage level $\mathfrak{D}_\infty$ is 7.74 percent for the sample path in panel *A* and 2.26 percent for the sample path in panel *B*, while its expectation is 5 percent. Therefore, the jumps in SCC paths just represent the fact that the decision with regard to emissions mitigation strongly depends on whether the tipping process has been triggered or not. This is due to the fact that once the tipping process is triggered, we cannot prevent or delay the sequential damages that occur during its multiple posttipping stages, and thus there is no strong incentive to change mitigation policy, such as strong discontinuous changes in the SCC.

It is obvious that adding a tipping element to DICE will increase the SCC. The question is how much the SCC is increased, given the magnitude of the tipping-point damages. Figure 5 helps address that issue. It is

[23] Technically, DSICE could easily handle cases where the duration of each stage depends on the entire state space.

[24] Experimentation indicated that five posttipping stages is adequate to approximate processes with more states.

[25] A discussion of that literature is contained in app. A.4.

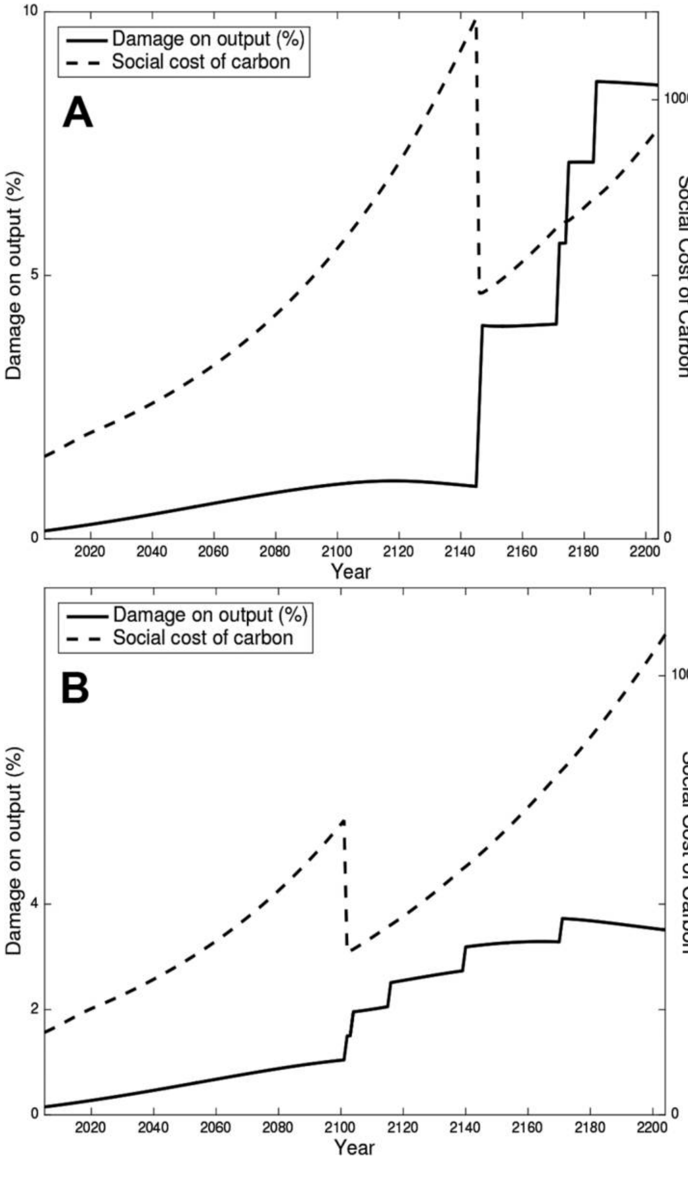

FIG. 4.—Sample paths of damage on output and the social cost of carbon for the stochastic climate tipping benchmark.

based on computing the BAU case with and without tipping. The dashed line shows the relative increase in damage when we add the tipping element. The increase is negligible until 2050 and peaks in around 2200 at 53 percent. Reducing carbon emissions in 2005 would delay any tipping

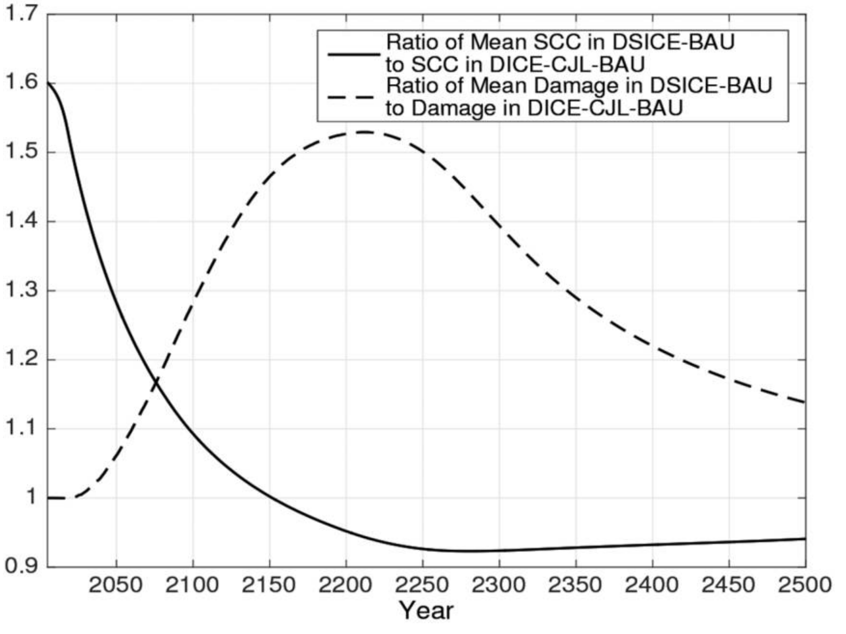

FIG. 5.—Comparison of DSICE and DICE-CJL with $\psi = 1.5$ under business as usual (BAU).

and shift the dashed curve to the right, but most of those changes would occur after 2100. The solid line shows the impact of tipping on the marginal SCC and shows that the marginal damage of carbon rises by 60 percent in 2005. Even though the tipping element only moderately increases damages 200 years from now, the impact on the SCC today is substantial.

We next consider the implications of a climate tipping point for the dynamics of the SCC, the carbon tax, the emissions control rate, and the two most important climate states (atmospheric carbon concentration and surface temperature). Figure 6 shows the results of 10,000 simulation paths over the first 200 years for these variables.

Adding climate tipping risk is expected to result in a more intense mitigation, compared to that of a deterministic model. Throughout this century it is optimal to more than double mitigation efforts in response to the threat of a tipping point in the climate. Emissions reductions imply a strict reduction of atmospheric carbon concentrations (fig. 6*D*), compared to the result from the deterministic model. The resulting path of surface temperature (fig. 6*E*) corresponds to the temperature paths from the lowest of the most recent emissions scenarios used by the IPCC (2013),

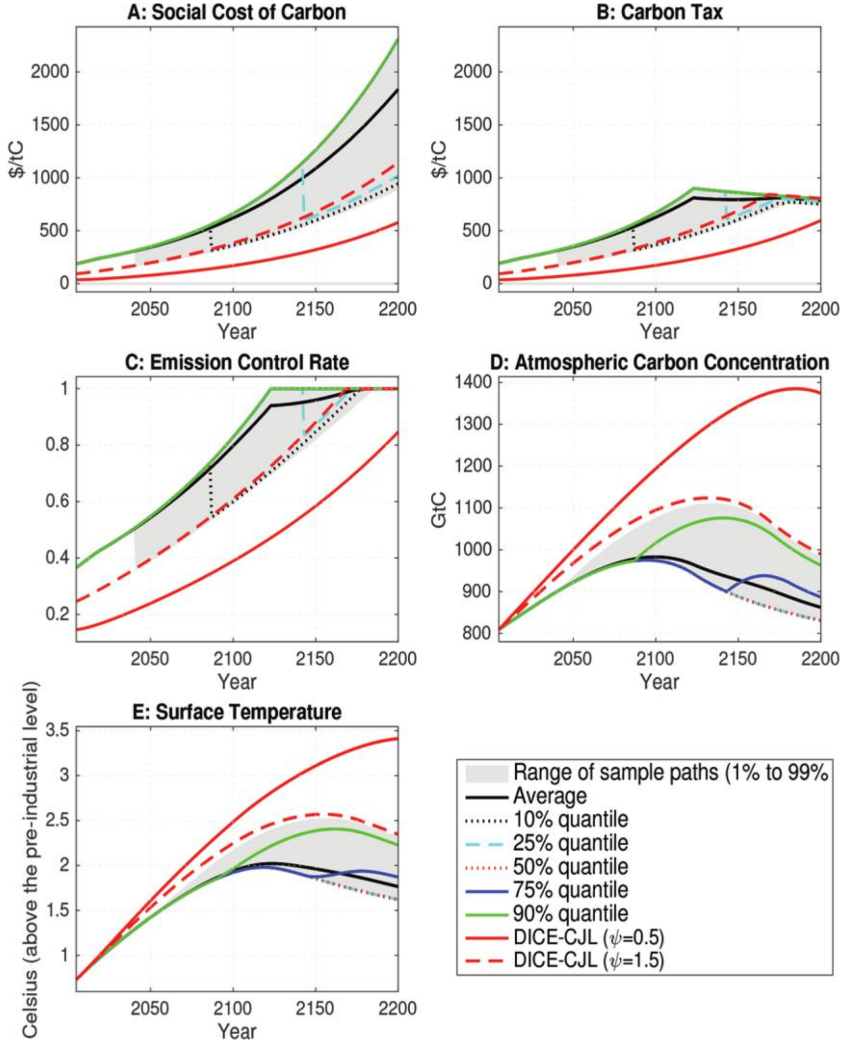

FIG. 6.—Simulation results for the climate tipping benchmark—climate system and policies.

implying a peak temperature increase before 2100 of around 2°C and a decline afterward.

A striking result in figure 6*A* is the 2005 SCC of $188/tC, a large increase over the $94/tC resulting from DICE-CJL (where $\psi = 1.5$) in the absence of any tipping risk. To underline the significance of this major increase in the SCC, recall our rather conservative assumptions on the nature of the tipping-point processes: we assume an expected duration of the tipping process of 50 years and expect posttipping damage of 5 percent and a mean squared-variance ratio of 0.2. As can be seen from the blue dashed line in figure 6*A*, these assumptions indicate that there is a 75 percent probability that a tipping process will not be triggered before 2150. Yet today's optimal SCC is \$188/tC, twice the value obtained from the same model when the tipping point is ignored. This strongly suggests that analyses of climate policy that simply ignore the potential of abrupt changes to the climate system—as does the current US government study (IWG 2010)—are significantly underestimating the SCC.

Related to the analysis in the previous section, we also note here that by the year 2125, some of our 10,000 simulated paths will produce a carbon tax that is less than the SCC, indicating that the benefits of mitigation may be exhausted. In fact, it appears that, with a slightly higher than 75 percent probability, mitigation policies will reach the limits of their effectiveness by 2125 and that alternative carbon management options might prove useful.

C. *Discounting Damages*

To understand the impact of a tipping element on the SCC, we compute the discount rate of damages from carbon emissions, denoted by ρ_Δ, which is implicitly used to value the marginal damage; that is, we solve

$$\mathrm{SCC}_0 = \sum_{t=0}^{\infty}(1+\rho_\Delta)^{-t}\Delta_t$$

for the unknown ρ_Δ, where SCC_0 is given by the DSICE solution and Δ_t is the expected extra damage to consumption at time t caused by one extra unit of carbon emission in 2005. We do this for DICE-CJL with $\psi = 1.5$, which has no tipping element, and the benchmark tipping case described above (with $\psi = 1.5$ and $\gamma = 10$), but we use their BAU versions to cancel out the impact of mitigation on damages by following the method of IWG (2010) for computing the discount rate of damages. For comparison, we also compute the internal rate of return on capital investment, which is the rate used to discount the additional consumption caused by one extra unit of capital in 2005.

Table 7 displays ρ_Δ and the internal rate of return on capital investment for both cases. We first see that marginal damages in DICE-CJL BAU are discounted at 3.7 percent and capital investment at 2.7 percent, a difference that is natural, given that BAU sets mitigation to zero. These results are also similar to those of other deterministic models. In contrast, we see that ρ_Δ is only 2.4 percent in the tipping benchmark DSICE BAU case. But the internal rates of return on capital investment are close to each other between DICE-CJL BAU and DSICE BAU. This tells us that

TABLE 7
Discount Rate of Damages and Internal Rate of Return on Capital Investment (%)

	DSICE BAU with the Benchmark Tipping Setting	DSICE BAU with the Benchmark Tipping Setting but $\Omega_{\mathrm{T}}(T_{\mathrm{AT}}) \equiv 1$	DICE-CJL BAU with $\psi = 1.5$
Discount rate of damages	2.4	2.4	3.7
Internal rate of return on capital investment	2.6	2.7	2.7

our higher SCC is not simply implied by higher potential damage from climate tipping events; otherwise the discount rate of damage would be the same as with DICE-CJL. This lower discount rate of damages from carbon emissions indicates that part of the justification of the higher SCC is demand for insurance. To illustrate this justification more clearly, table 7 also reports ρ_{Δ} and the internal rate of return on capital investment for the case of DSICE BAU with the benchmark tipping setting but $\Omega_{\mathrm{T}}(T_{\mathrm{AT}}) = 1$ (i.e., damages are only from tipping events). Both rates are, respectively, close to those in the case of DSICE BAU with the benchmark tipping setting and the default damage factor for temperature. This implies that the lower discount rate of damages in DSICE BAU with the benchmark tipping setting is not arising from the potentially nonlinear effect of additional damages from tipping events. The intuition is that the damages from a permanent shock to productivity have little covariance with short-run fluctuations in consumption. Therefore, damages from tipping events should be discounted less than damages from temperature (which are proportional to output).

D. *Sensitivity Analysis for the Climate Tipping Process*

We next examine how the 2005 SCC is affected by parameter uncertainty in the climate tipping process by recomputing the SCC over a range of parameter choices that reflect scientific opinions. We examine the six-dimensional collection of parameter values defined by the tensor product of the following finite sets:

$$\begin{aligned} &\lambda \in \{0.0025, 0.0035, 0.0045\}, \quad \bar{\mathfrak{D}}_{\infty} \in \{0.025, 0.05, 0.10\}, \\ &q \in \{0, 0.2, 0.4\}, \quad \psi \in \{0.5, 1.25, 1.5, 1.75, 2.0\}, \\ &\gamma \in \{2, 5, 10, 15\}, \quad \bar{\Gamma} \in \{5, 50, 200\}. \end{aligned}$$

We compute the SCC in 2005 for all the cases, and table 8 presents the 2005 SCC for some of the representative cases. For example, when $\lambda = 0.0025$, $\bar{\Gamma} = 5$, $\bar{\mathfrak{D}}_{\infty} = 0.025$, and $q = 0$ (i.e., the first row in table 8), the 2005 SCC is \$132/tC for $\psi = 1.5$ and $\gamma = 10$. The value of the SCC with the climate tipping process is always greater than that in the deterministic

TABLE 8
Initial Social Cost of Carbon ($/tC) with Stochastic Climate Tipping

	$\psi = .5$		$\psi = 1.5$		$\psi = 2$	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
	$\lambda = .0025$, $\bar{\Gamma} = 5$, $\bar{\mathfrak{D}}_\infty = .025$					
$q = 0$	61	61	128	132	160	164
$q = .4$	61	61	129	135	161	169
	$\lambda = .0025$, $\bar{\Gamma} = 5$, $\bar{\mathfrak{D}}_\infty = .100$					
$q = 0$	74	87	275	349	348	412
$q = .4$	75	107	285	413	357	458
	$\lambda = .0025$, $\bar{\Gamma} = 200$, $\bar{\mathfrak{D}}_\infty = .025$					
$q = 0$	59	59	111	112	136	138
$q = .4$	59	59	111	112	136	139
	$\lambda = .0025$, $\bar{\Gamma} = 200$, $\bar{\mathfrak{D}}_\infty = .100$					
$q = 0$	62	64	174	195	237	276
$q = .4$	63	65	177	224	241	318
	$\lambda = .0045$, $\bar{\Gamma} = 5$, $\bar{\mathfrak{D}}_\infty = .025$					
$q = 0$	63	64	148	151	188	192
$q = .4$	63	64	149	155	189	200
	$\lambda = .0045$, $\bar{\Gamma} = 5$, $\bar{\mathfrak{D}}_\infty = .100$					
$q = 0$	91	113	365	418	422	461
$q = .4$	94	144	375	462	429	498
	$\lambda = .0045$, $\bar{\Gamma} = 200$, $\bar{\mathfrak{D}}_\infty = .025$					
$q = 0$	59	59	120	121	149	151
$q = .4$	59	59	120	122	150	153
	$\lambda = .0045$, $\bar{\Gamma} = 200$, $\bar{\mathfrak{D}}_\infty = .100$					
$q = 0$	66	69	227	259	318	356
$q = .4$	67	71	233	304	324	394

Note.—λ: hazard rate; $\bar{\Gamma}$: mean duration of tipping process, $\bar{\mathfrak{D}}_\infty$: mean long-run damage level; q: mean squared-variance ratio.

case, in which the climate tipping process is ignored. This is expected, since the tipping element increases possible future damage.

Table 8 also shows that the SCC is larger for a higher IES (i.e., ψ) and a higher value of the risk aversion parameter γ. Furthermore, we observe that the initial-time SCC increases with higher (present-discounted) expected damage from the climate tipping process, which can be caused by a higher hazard rate parameter (λ), a shorter expected duration of the tipping process ($\bar{\Gamma}$), a higher mean long-run damage level ($\bar{\mathfrak{D}}_\infty$), or a higher mean squared-variance ratio of the expected damage level (q). As mentioned above, our specification of a climate tipping point in an economic growth model is unique by the standards of how climate scientists view the nature of climate tipping points (e.g., Lenton and Ciscar 2013).

Table 8 shows that with climate risk only, a higher IES (i.e., ψ) implies a higher SCC. Given the positive productivity growth in our cases with climate risk only, a higher IES implies a lower desire to smooth per capita consumption, leading to a higher investment in mitigation for higher future consumptions. This effect occurs because the discount rate of damages from tipping events is less than the internal rate of return on capital investment, as shown in table 7.

Table 8 also shows that a higher risk aversion will imply a higher SCC too. Since our tipping probability $p_{\mathrm{tip},t}$ can be reduced by slowing the temperature increase through mitigation and the variance of output at time t is proportional to $p_{\mathrm{tip},t}(1-p_{\mathrm{tip},t})$, we see that the variance of output can be diminished by reducing $p_{\mathrm{tip},t}$ (note that $p_{\mathrm{tip},t}$ is always smaller than 0.5 in our cases), and so can the variance of consumption be reduced. Thus, a higher risk aversion implies a greater value for mitigation and therefore also a higher SCC for the cases with climate risk only.

Table 8 shows that when $\bar{\mathfrak{D}}_\infty$, the mean long-run damage level, is small, the SCC is insensitive to risk aversion γ or to the mean squared-variance ratio q (see rows with $\bar{\mathfrak{D}}_\infty = 0.025$), as a small $\bar{\mathfrak{D}}_\infty$ implies a much lower variance of the uncertain, final, long-run damage level, which is equal to $q\bar{\mathfrak{D}}_\infty^2$. For example, when $\lambda = 0.025$, $\bar{\Gamma} = 5$, $\bar{\mathfrak{D}}_\infty = 0.025$, and $\psi = 0.5$, the SCC is \$61/tC for all $\gamma \in \{2, 10\}$ and $q \in \{0, 0.4\}$. However, when $\bar{\mathfrak{D}}_\infty$ is large (e.g., $\bar{\mathfrak{D}}_\infty = 0.1$), the SCC increases significantly when γ increases from 2 to 10, since a large $\bar{\mathfrak{D}}_\infty$ implies a much larger variance of the uncertain, final, long-run damage level. This reflects the nonlinear effect of climate tipping damage on the SCC. In models assuming separable preferences, it is only the mean of the uncertain damage, and not its variance, that affects the SCC.

In addition, from the results of all the cases we find that one common pattern exists: the 2005 SCC is linear in q. Figure 7 shows the numbers for the SCC for various values of γ and q when $\psi = 1.5$, $\lambda = 0.0035$, $\bar{\Gamma} = 50$, and $\bar{\mathfrak{D}}_\infty = 0.05$.[26] Other cases have the same qualitative pattern, so we omit them here. In figure 7, the four lines represent, from bottom to top, the cases of $\gamma = 2$, 5, 10, and 15. We see that all these lines are straight and that a higher γ implies a greater slope, meaning that it is more sensitive to the variance of the uncertain damage level.

This is not surprising, since it fits into the logic of the basic consumption-based capital asset pricing model (Lucas 1978), which tells us that the price of risk is related to its covariance with the aggregate endowment. Since the magnitude of the damage is proportional to output, the damage

[26] In fig. 7, the horizontal axis is the variance of the uncertain damage level at the final absorbing stage—namely, $q\bar{\mathcal{D}}_\infty^2$—and it is scaled by 10,000.

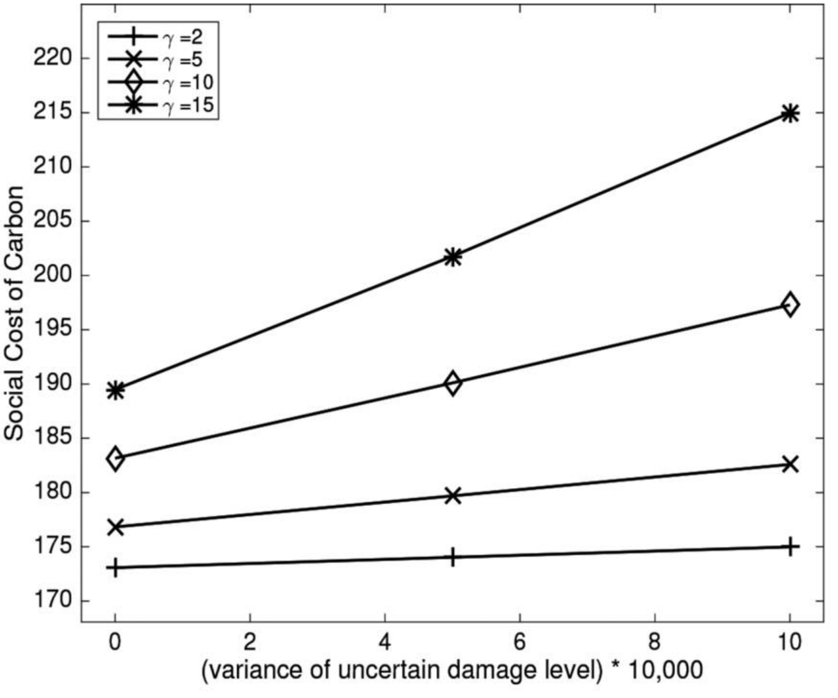

FIG. 7.—Sensitivity of the social cost of carbon to the risk aversion parameter and uncertainty regarding posttipping damage ($\psi = 1.5$, $\lambda = 0.0035$, $\bar{\Gamma} = 50$, and $\bar{\mathfrak{D}}_{\infty} = 0.05$).

is strongly related to output; in fact, in this case climate damage is the only stochastic element of output conditional on the tipping event. Therefore, the correlation is unity, and the SCC has a price-of-risk component that is linear in the variance of the uncertain damage level.

Note that the SCC increases as the variance of the uncertain damage level increases. One interpretation of variance is that it represents our ignorance of the consequences of an unfolding tipping process. With that interpretation, the horizontal difference represents the decline in the SCC that would result if we carried out more scientific research and reduced the uncertainty regarding the posttipping damage level. This observation shows that DSICE could be used to identify the value of reducing uncertainty and to indicate which kinds of scientific studies would be the most valuable to pursue. We leave this point for further research.

E. *Other Approaches to Damage Functions with Tipping Events*

A few studies attempt to integrate stochastic tipping into their deterministic models (e.g., Lempert et al. 2000; Nordhaus 2009; Ackerman, Stanton, and Bueno 2010; Ackerman and Stanton 2012). They do this by altering the damage function in an ad hoc manner, such as increasing the convexity of the damage function. Similarly, Mastrandrea and Schneider (2001) increase the exponent of the DICE-type damage function with the purpose of quantifying the effects of an abrupt nonlinear climatic change: the shutdown of the thermohaline circulation. The reasoning for increasing the exponent is to deal with abrupt nonlinear changes by creating a more nonlinear damage function. Lontzek et al. (2015) discuss this approach, showing with the DSICE model that this approach fails to capture the implications of stochastic tipping points. It is, furthermore, incompatible with the real-world task of decision-making under uncertainty. Therefore, DSICE shows that there is no need for such ad hoc approximations.

Several IAMs have carried out analyses of a climate catastrophe event that directly affects utility. Early studies include Gjerde, Grepperud, and Kverndokk (1999) and Castelnuovo, Moretto, and Vergalli (2003). These studies assume that an uncertain level of global temperature triggers an irreversible drop in utility and find that large reductions in emissions are optimal. Nevertheless, these studies apply a certainty-equivalent approach to uncertainty and do not address stochasticity. Most recently, Cai et al. (2015) use the DSICE framework to include the possibility of a catastrophic event with nonmarket impacts, such as impacts on the ecosystem. Most cost-benefit analyses rarely take account of environmental tipping points leading to abrupt and irreversible impacts on market and nonmarket goods and services, including those provided by the climate and by ecosystems. Cai et al. (2015) shows that including environmental tipping-point impacts in a stochastic dynamic IAM profoundly alters cost-benefit assessment of global climate policy. The risk of a tipping point, even if it has only nonmarket impacts, could substantially increase the present optimal carbon tax.

VII. The SCC with Stochastic Growth and Climate Tipping

The previous two sections have examined the impacts on SCC from stochastic growth and stochastic climate tipping, both in isolation. The real-world system includes both uncertainties, and this section presents the results of DSICE in the presence of long-run risk in both economic growth and the climate tipping process. The optimal policy will now have to balance the need to delay the triggering of the tipping-point process with the accumulation of additional capital in the face of stochastic growth and with the desire to smooth our consumption patterns. We study a stochastic growth and climate tipping benchmark case of parameter specification and carry out a sensitivity analysis.

A. *The Stochastic Growth and Climate Tipping Benchmark*

We use $\psi = 1.5$, $\gamma = 10$, $\lambda = 0.0035$, $\bar{\mathfrak{D}}_\infty = 0.05$, $q = 0.2$, and $\bar{\Gamma} = 50$ for the stochastic growth and climate tipping benchmark. Figure 8 shows the results of 10,000 simulation paths over the first 100 years with regard to the dynamics of the SCC, the carbon tax, and the ratio of the SCC to gross world output. Other variables, such as capital, consumption and its growth, atmospheric carbon concentration, and surface temperature, have pictures visually similar to the corresponding pictures in figures 2 and 6, so we omit them. We use the same line and color types as in previous figures.

We first study the SCC. Its initial-time level is \$124/tC, and at 2100 the average (or expected) SCC is \$461/tC. Thus, the path of the expected SCC is situated between its paths obtained from our analyses of each risk component in isolation. In 2005, the SCC of \$124/tC is even exactly the average of the numbers obtained for the two cases from the previous sections (\$61 and \$188/tC). Compared to that in a deterministic model, which ignores both risk components and has $\psi = 0.5$, we find that the

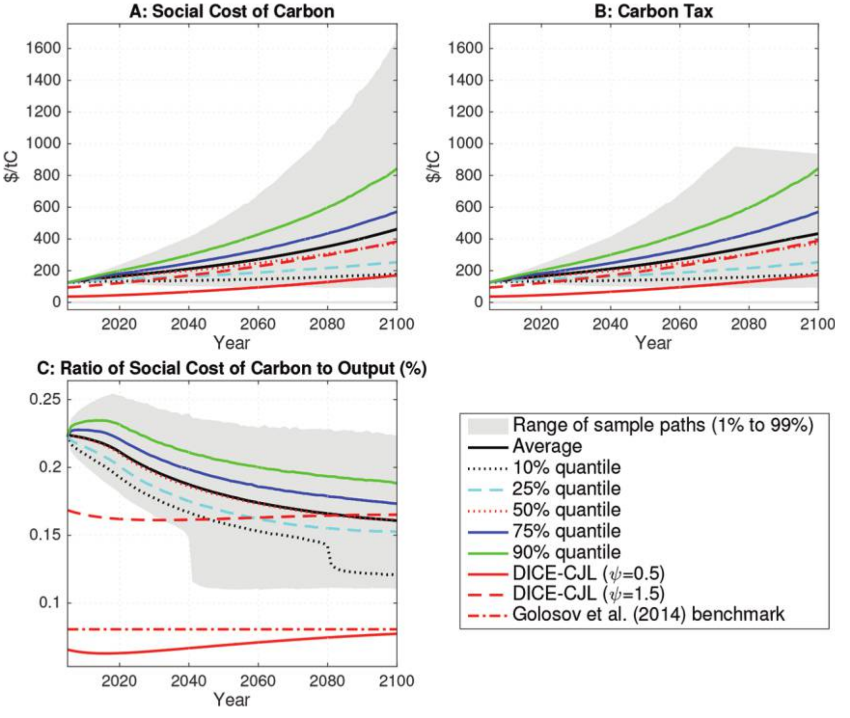

FIG. 8.—Simulation results for the stochastic growth and climate tipping benchmark

2005 SCC increases by a factor of more than 3 and that with 90 percent probability the SCC will be significantly higher throughout this century.

The presence of both stochastic growth and climate tipping risk also increases the variance of the future SCC relative to the separate stochastic growth and climate tipping benchmarks. For example, the SCC in 2100 ranges from \$100/tC (the 1 percent quantile) to \$1,700/tC (the 99 percent quantile). The carbon tax, which we present in figure 8*B*, is also more likely to hit its upper bound after 2072 than in either of the single-risk benchmarks. The combination of these risks implies that there is a probability of about 7.5 percent that mitigation policies will have reached the limit of their effectiveness by 2100.

We also revisit the analysis of the SCC_t/Y_t ratio of Section V.A, here in the case of stochastic growth and climate tipping. Figure 8*C* shows that the SCC_t/Y_t ratio is quite stochastic. First, we note that, compared to that in the deterministic version of the model, SCC_t/Y_t is about three times larger in 2005, while at 2100 it is expected to be about twice as large. Second, we find that the expected SCC_t/Y_t is decreasing over time by about 50 percent and is thus not constant. Third, and most importantly, we find that the ratio of the SCC to gross world output is not close to any simple path but is rather a stochastic process varying over the interval [0.00108, 0.00215] at year 2100 and over an even larger interval for the second half of this century. When contrasting our results with the constant SCC_t/Y_t ratio postulated by Golosov et al. (2014), we find that the inclusion of a long-run risk and a tipping process with Epstein-Zin preference makes SCC_t/Y_t significantly uncertain and leads to substantially different qualitative and quantitative results.[27]

Table 9 shows the SCC in 2005, the mean SCC in 2100, and the 90 percent quantile SCC in 2100 for our three benchmark examples. We see that the 2005 SCC and the mean SCC in 2100 of the stochastic growth and tipping benchmark are, respectively, between those of the stochastic growth benchmark and the climate tipping benchmark but that the 90 percent quantile SCC in 2100 of the stochastic growth and tipping benchmark is larger than the corresponding ones of both the stochastic growth benchmark and the climate tipping benchmark.

Furthermore, when comparing our results to those for DICE-CJL with $\psi = 0.5$, we find that the interaction between multiple uncertainties, such as long-run risk and a tipping process, could be nontrivial: for example, while either long-run risk or a tipping process leads to a higher

[27] The Golosov et al. (2014) model produces—because of its simplicity—elegant results. Our approach in this paper is to use scientifically valid model features rather than constructing simplified models. We see the additional complexity in our model as justified in the sense that we come closer to being consistent with the best science and also provide very different results.

TABLE 9
SCC ($/tC) for Three Benchmark Examples

	DICE-CJL		DSICE (ψ = 1.5, γ = 10)		
	ψ = .5	ψ = 1.5	Stochastic Growth	Climate Tipping	Stochastic Growth with Tipping
2005 SCC	37	94	61	188	124
Mean SCC in 2100	180	389	357	620	461
Standard deviation of SCC in 2100	0	0	247	105	316
90% quantile SCC in 2100	180	389	667	662	844

2005 SCC than does the deterministic DICE-CJL (with $\psi = 0.5$), their combination does not imply a further increase in the 2005 SCC when compared to cases with only one type of uncertainty. Thus, it is necessary to carry out a systematic analysis rather than a stylized one. For the case of DICE-CJL with $\psi = 1.5$, we have argued in Section V.B that, as there is no risk, the positive future growth in productivity and a higher ψ (here 1.5) lead to larger incentives to avoid climate damage increases. The SCC will be larger in that case, when compared to the case of stochastic growth only. Nevertheless, the combination of stochastic growth and climate tipping will produce an SCC substantially larger than that in the deterministic case.

B. *Sensitivity of the Stochastic Growth and Climate Tipping Benchmark*

We compute the sensitivity of the stochastic growth and climate tipping benchmark case to several parameters. Table 10 lists the 2005 SCC for selected combinations of the parameter values for the sensitivity analysis. These parameters are the hazard rate λ, the mean duration time of the tipping process $\bar{\mathcal{D}}$, the mean long-run damage level $\bar{\mathfrak{D}}_\infty$, the mean squared-variance ratio q, the IES ψ, and the risk aversion parameter γ.

We find that some qualitative properties found in previous examples with only climate risk still hold—that is, a higher hazard rate parameter λ, a shorter expected duration $\bar{\Gamma}$, a higher mean damage level $\bar{\mathfrak{D}}_\infty$, or a larger mean squared-variance ratio q will lead to a higher SCC, although their quantitative values differ substantially. For example, by comparing the cases of $\lambda = 0.0035$ and $\lambda = 0.0045$ from table 10 with the other values, $\bar{\mathfrak{D}}_\infty = 10$ percent, $\bar{\Gamma} = 50$, and $q = 0$, we see that the case with $\lambda = 0.0045$ has a larger SCC than the case with $\lambda = 0.0035$. Moreover, the range of the initial SCC is wide, from \$43/tC (the case with $\lambda = 0.0025$, $\bar{\Gamma} = 200$, $\bar{\mathfrak{D}}_\infty = 0.025$, $q = 0$, $\psi = 0.5$, and $\gamma = 2$) to \$477/tC (the case with $\lambda = 0.0045$, $\bar{\Gamma} = 5$, $\bar{\mathfrak{D}}_\infty = 0.10$, $q = 0.4$, $\psi = 2$, and $\gamma = 2$).

TABLE 10
Initial Social Cost of Carbon ($/tC) under Stochastic Growth and Climate Tipping

	$\psi = .5$		$\psi = 1.5$		$\psi = 2$	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
	$\lambda = .0025, \bar{\Gamma} = 200, \bar{\mathfrak{D}}_\infty = .025$					
$q = 0$	43	70	102	76	120	78
	$\lambda = .0035, \bar{\Gamma} = 5$					
$\bar{\mathfrak{D}}_\infty = .05, q = 0$	63	133	177	144	217	149
$\bar{\mathfrak{D}}_\infty = .10, q = .4$	93	337	313	373	400	384
	$\lambda = .0035, \bar{\Gamma} = 50, \bar{\mathfrak{D}}_\infty = .05$					
$q = 0$	55	111	156	124	192	128
$q = .2$	55	114	157	124	193	132
	$\lambda = .0035, \bar{\Gamma} = 50, \bar{\mathfrak{D}}_\infty = .10$					
$q = 0$	73	187	247	212	317	220
	$\lambda = .0045, \bar{\Gamma} = 5, \bar{\mathfrak{D}}_\infty = .10$					
$q = 0$	102	294	356	314	455	324
$q = .2$	104	339	365	372	466	383
$q = .4$	106	394	374	438	477	446
	$\lambda = .0045, \bar{\Gamma} = 50, \bar{\mathfrak{D}}_\infty = .05$					
$q = 0$	59	122	171	136	212	141

Note.—λ: hazard rate; $\bar{\Gamma}$: mean duration of tipping process, $\bar{\mathfrak{D}}_\infty$: mean long-run damage level; q: mean squared-variance ratio.

Furthermore, by comparing the columns with $\gamma = 2$ or comparing the columns with $\gamma = 10$, we see that a higher IES always implies a higher SCC for the cases with $\gamma \leq 10$. This is consistent with our findings in earlier examples with stochastic growth only or climate risk only. However, the qualitative properties of the risk aversion parameter γ are now nontrivial: table 10 shows that the effect of a higher γ on the SCC can be positive or negative. This nontriviality comes from the combination of economic risk and climate tipping risk: with economic risk only, if ψ is small (i.e., $\psi < 0.7$), then a higher risk aversion implies a higher SCC; but with climate tipping risk only, a higher risk aversion always implies a higher SCC for any IES. For the cases with $\psi = 0.5$ in table 10, a higher risk aversion implies a higher SCC, but in most cases in table 10 with $\psi = 1.5$ and all cases with $\psi = 2$, a higher γ will result in a smaller SCC, implying that for lower levels of ψ the effect of higher γ is more likely to be positive. This complicated pattern originates from the interaction between the nontrivial pattern of the impact of ψ and γ on the SCC in the stochastic growth cases and the pattern in the climate tipping cases.

VIII. Summary and Conclusion

This study has presented DSICE, a computational framework for the stochastic integrated assessment of issues related to the joint evolution of the economy and the climate. We analyzed the optimal level and dynamic properties of the SCC and the associated optimal carbon tax in the face of stochastic and irreversible climate change and its interaction with economic factors including growth uncertainty and preferences regarding risk. We did this in a manner that allows us to compare our results to those of the deterministic model in Nordhaus (2008), an influential and well-known IAM. The specific examples in this study show three basic points.

First, we use Epstein-Zin preferences in order to specify tastes that are more compatible with the evidence on risk aversion and the IES. This also allows us to separate the impact of risk aversion from the impact of intertemporal substitution. The impact of risk aversion depended on the nature of the risk, with the SCC rising as risk aversion increases for the tipping case, but with more ambiguous implications for the case of stochastic growth.

Second, the incorporation of long-run risk shows that the SCC is itself a stochastic process with considerable uncertainty. Climate change issues are not just about the expected state of the climate in the future but also about avoiding disasters. Climate change policy has to recognize the uncertainty about the future SCC and be prepared to consider policies, such as geoengineering and carbon capture, that are currently considered to have costs that would never be justified by deterministic models that essentially focus on the expected SCC. An examination of parameter uncertainty also shows that the range of plausible SCC values is much larger than implied by other integrated assessment analyses. We found this great uncertainty for single parameterizations; when we also include uncertainty regarding economic parameters, that uncertainty is even greater.

Third, climate scientists have recently argued that tipping elements in the climate system contribute to uncertainty regarding future climate conditions. We incorporate the concept of tipping elements into DSICE and find that the threat of a tipping element leads to significant and immediate increases in the SCC, where the increase in the SCC is disproportionately large relative to the damage caused by tipping. This is true even for moderate assumptions regarding the likelihood and impacts of climate tipping events. The SCC can be very high, even without assuming catastrophic climate change events but rather by merely assuming plausibly parameterized examples of uncertain and irreversible climate change. An internal rate-of-return analysis showed that one should use a smaller discount rate when valuing damage from tipping events, a natural result because the damage caused by tipping is less correlated to total consumption than is damage to output.

Finally, we have also shown that it is possible to solve empirically plausible nine-dimensional models of the climate and the economy that include (1) productivity shocks of the kind studied in macroeconomics, (2) dynamically nonseparable preferences consistent with observed prices of risk, and (3) stochastic tipping elements in the climate system. Our examination of examples including both the usual Nordhaus-style damage function for output and damages arising from tipping events shows that these separate sources of damages must be examined together. The usual approach is to study them separately or to create ad hoc approximations. This study shows that analyses that avoid such shortcuts are both feasible and necessary if we are to obtain reliable answers. This study has ignored many features that might be important, but this weakness is unavoidable when modeling a complex system, particularly one that is a merger of two enormously complex systems. This study has shown that it is possible to look at models of much greater complexity than before and that, in the case of the SCC, incorporating that complexity significantly alters the implications for economic policy. The fact that we solve a nine-dimensional dynamic programming problem with long-run risk over a span of 600 years implies that it is straightforward to use those nine dimensions to model other topics, such as geoengineering, robust optimization, and learning.

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