Public Economics (ECON 131)
Review Session

Contents

1 My Personal Tips for True/False/Uncertain

- The T/F/U usually reference a specific topic or concept from the course – your first step is to recognize the relevant topic, so you can identify what was discussed in class, the relevant empirical literature, etc. For example, if you are asked what the effects might be on labor supply if the government "provided a $0.25 subsidy per dollar earned on the first $10,000 that a worker makes each year, which phases out gradually after some threshold," recognize that this is asking you to think of something similar to the EITC. Thus, you should be thinking about substitution and income effects; extensive and intensive margins of labor supply; and the empirical research on the EITC's effects.

- Before jumping in and answering, think for a second about arguments on both sides. In the above subsidy example, the substitution and income effects might push individuals in different directions, and those incentives will change depending on whether you are on the phase-in, plateau, or phase-out portion of the subsidy. Before immediately writing the first answer that comes to mind, you should definitely think about whether the answer is straightforward (usually not!)

- There are sometimes key concepts from the course that are especially relevant to the questions. In the above example, the substitution effect, income effect, and intensive and extensive margins are likely relevant key concepts. If there are key concepts/terms, you may consider underlining them in your answer to help them stand out.
- Using material *from the course* is paramount to answering these questions! Your answer does not need to be long and complicated; the very best answers often succinctly answer these questions with appropriate references to theoretical and empirical concepts covered in class.

2 Review of Concepts

2.1 Business Tax

2.1.1 Tax Rates

Assume that the personal tax rate $\tau_p = .37$, that the corporate tax rate is $\tau_c = .21$, and the dividend tax rate is $\tau_d = .2$. Let's say an entrepreneur sets up a business that makes \$100 in its first year.

(a) If the entrepreneur sets up the business as a corporation, what tax is paid when the business earns \$100, if none of that income is distributed to the owner?

$\tau_c \cdot \$100 = \$21$

(b) How much tax is then paid if the corporation distributes the after-tax profits to the owner?

$\tau_d \cdot (1 - \tau_c) \cdot \$100 = \tau_d \cdot \$79 = \$15.80$

(c) What if the business is instead an S-corporation or a partnership? How much tax is paid when it makes \$100 in the first year?

$\tau_p \cdot \$100 = \$37$. *Note that it does not matter if the money is distributed by the business to the owner or not – the owner must pay this tax regardless!*

2.1.2 Optimal K

Assume that Mel starts a corporation. The gross profit function is $F(K) = pY - wL$ with $Y = L^{1/2}K^{1/2}$. Assume she'll hire just one employee and will have a fixed labor input $L = 1$ with $w = 1$. The market price of her goods she sells is $p = 8$. Her cost of capital is $r = 1$.

(a) Ignoring all tax, solve for her optimal investment K.

$$\max_K \pi = F(K) - rK = p \cdot L^{1/2}K^{1/2} - wL - rK = 8K^{1/2} - 1 - 1K$$

With first order condition:

$$\frac{\partial \pi}{\partial K} = \frac{4}{K^{1/2}} - 1 = 0$$
$$4 = K^{1/2}$$
$$K^* = 16$$

(b) Is there any pure profit?

$\pi = 8(16)^{1/2} - 1 - 1(16) = 15$. *So yes, there is pure profit.*

(c) Now assume that the government imposes a corporate tax $\tau_c = .25$, and allows a deduction for labor expenses but not for capital expenses. What is the new optimal K?

$$\max_K \pi = (1-\tau_c)F(K) - rK = (.75)(p \cdot L^{1/2}K^{1/2} - wL) - rK = 6K^{1/2} - .75 - 1K$$

With first order condition:

$$\frac{\partial \pi}{\partial K} = \frac{3}{K^{1/2}} - 1 = 0$$
$$3 = K^{1/2}$$
$$K^* = 9$$

(d) How much tax revenue is collected?

$$\tau_c \cdot F(K) = (.25)(p \cdot L^{1/2}K^{1/2} - wL) = .25(8(9)^{1/2} - 1) = 5.75$$

(e) Now assumed the government allows a full deduction of capital expenses. How much tax revenue does the government collect now?

Note that we must solve the maximization problem given the new deduction. The deduction means that the taxpayer saves the amount $\tau_c \cdot rK$, so the maximization is now:

$$\max_K \pi = (1-\tau_c)F(K) - rK + \tau_c \cdot rK = (1-\tau_c)(F(K) - rK)$$

We know that the maximization will give us the same result as the case without tax, $K = 16$, because we are maximizing the exact same function as before $F(K) - rK$, but just scaled by the coefficient $(1-\tau_c)$.

You can do the math to check, but we'll proceed using $K = 16$.

$$\tau_c(F(K) - rK) = .25(8(16)^{1/2} - 1 - 1(16)) = .25 \cdot 15 = 3.75$$

Note the revenue is lower even though there is more output, because the deduction removes rK from the tax base.

2.2 Externalities and Optimal Tax

Consider a town with 2 residents, Abby and Ben. Abby plays her music really loud, and Ben prefers the quiet. Consider a model where Q represents the hours of music played. Abby's marginal benefit and marginal cost curves are given by:

$$MC_A = 2Q + 3$$

$$MB_A = 18 - Q$$

Ben gets 0 benefit from any quantity of music, and gets annoyed quickly, so his marginal cost curve is:

$$MC_B = 3Q + 3$$

Assume that all the parties are self-interested.

(a) Without any government intervention, how many hours of music will be played?

Abby will maximize her own utility without regard to Ben:

$$MC_A = MB_A$$

$$2Q + 3 = 18 - Q$$

$$Q_{FM} = 5$$

(b) What number of hours of music is socially optimal?

We must solve for the social equilibrium, aggregating the individuals' marginal benefit and marginal cost curves vertically to arrive at the SMB and SMC curves

$$SMC = MC_A + MC_B = (2Q + 3) + (3Q + 3)$$

$$SMB = MB_A + MB_B = (18 - Q) + (0)$$

$$SMC = SMB$$

$$5Q + 6 = 18 - Q$$

$$Q_{SO} = 2$$

(c) If we wanted to have Abby perfectly internalize her externality, so that a tax imposed by the government causes her private marginal curves to perfectly reflect the social curves, what is that tax we should charge? (*Hint: It's not a flat tax, but instead is a function of Q!*). What do you notice about the relationship between optimal tax t and Ben's marginal cost curve?

Note that $SMB = MB_A$ because $MB_B = 0$. So the curve we need to shift is Abby's private marginal cost curve. We want to impose a tax so that $MC_B = SMC$ for all Q

$$MC_A + t = SMC$$

$$2Q + 3 + t = 5Q + 6$$

$$t = 3Q + 3$$

The optimal tax t is the same as Ben's marginal cost curve! This is intuitive – to internalize the externality fully, Abby must take into account the full burden she's imposing on Ben.

(d) Let's say the town council is politically gridlocked and will only agree to a flat tax, and not one that varies with Q. Is there a *flat* tax t (i.e. it does not vary with Q) that could still achieve the socially optimal Q_{SO} you calculated in part (b)?

Yes! We must find a constant value t that solves

$$MC_A + t = SMC$$

$$2Q + 3 + t = 5Q + 6$$

$$2(2) + 3 + t = 5(2) + 6$$

$$t = 9$$

3 Misc. Old Exam Questions

3.1 T/F (g) Spring 2019

Disability insurance in the United States does not discourage labor supply because rejected applicants are very unlikely to work.

True: This is the analysis by Bound AER which provides an upper bound on the labor supply looking at rejected applicants. 1/3 of rejected applicants work so the upper bound is not zero. Most recently, Maestas-Mullen-Strand AER obtain causal effect of DI on LFP using natural variation in DI examiners stringency and large SSA admin data linking DI applicants and examiners and find relatively small but significant effects.

4 2016 Final – Math Exercises

2. Exercise - Insurance (20 points)

An insurance company offers an individual an insurance plan that insures against getting injured. The individual earns a wage $w = \$10$ if not injured, but earns \$0 if injured. The insurance company offers a plan with benefit b and premium p. The individual can choose the probability that she gets injured, denoted by s. She can choose s to be anywhere between 1% and 99% (which implies she can't choose s to be less than 1% or greater than 99%). Intuitively, this captures the extent to which she can choose to engage in risky activities like bicycling without a helmet, sky diving, and crossing the street without looking both ways. The individual faces an expected utility function of the following form:

$$E[U] = (1 - s)\ln(w - p) + s\ln(b - p) - (1 - s)^2$$

In this equation, $w - p$ is consumption in the good state and $b - p$ is consumption in the bad state. The last term, $(1 - s)^2$, captures the fact that the individual gains utility from engaging in risky activities (higher levels of $(1 - s)$ imply higher levels of safety).

For this problem, the following values may be useful:

$\ln(0.01) \approx -4.61$	$\ln(0.1) \approx -2.30$	$\ln(1) = 0$	$\ln(2) \approx 0.69$	$\ln(3) \approx 1.10$	$\ln(4) \approx 1.39$
$\ln(5) \approx 1.61$	$\ln(6) \approx 1.79$	$\ln(7) \approx 1.95$	$\ln(8) \approx 2.80$	$\ln(9) \approx 2.20$	$\ln(10) \approx 2.30$

(a) The insurance company decides to offer an insurance plan (Plan #1) that offers full insurance. Suppose that the insurance company believes that the probability that the individual gets injured is 10%. What is the actuarially fair benefit and premium? (2 points)

Solution: for full insurance, we would set $b = 10$. The actuarially fair premium is given by $p = (s)b$, so $p = 1$

(b) Write out the individual's optimization problem under Plan #1 and solve for the individual's choice of s under Plan #1. (2 points)

Solution: The worker maximizes

$$E[U] = (1 - s)\ln(9) + s\ln(9) - (1 - s)^2 = \ln(9) - (1 - s)^2$$

Clearly, the worker chooses s to be as large as possible, and lets $s = 0.99$.

(c) What is the worker's level of utility under Plan #1? (1 point)

Solution: The worker's utility is given by $\ln(9) - 0.0001 \approx 2.20$.

(d) The insurance company realizes that the probability that the individual will get injured is not 10%, and recalculates the premium to provide actuarially fair insurance (but still provides full insurance). What is the new premium level? Call this Plan #2. (2 points)

Solution: Now, $p = sb = 0.99 * 10 = 9.9$.

(e) Calculate the individual's utility under Plan #2. **HINT:** Your solution should be less than 0. (1 point)

Solution: Now, the worker's utility is given by $\ln(0.1) - 0.0001 \approx -2.30$.

(f) Would the individual still take up Plan #2 if that were the only plan available to her? (2 points)

Solution: Yes. The individual has concave utility and is risk averse. She faces infinitely worse expected utility if she does not take up the insurance.

3. Exercise - Public Goods (20 points)

Suppose two houses owned by Alice and Ben have access to a private park. As a private park, maintenance is provided by the owners of each house. Each has the ability to pay a maintenance company w dollars per hour to clean the park. Let the total number of hours paid for each month be equal to $H^{TOT} = H_A + H_B$, with H_A the number of hours paid for by Alice, and H_B the number of hours paid for by Ben.

Suppose both Alice and Ben each have the same amount of spending money equal to I per month, which they can spend on cleaning or on a private consumption good X. The price of the private consumption good is \$1 per unit. Thus, the budget constraint for Alice is $X_A + wH_A = I$, for example. Suppose both Alice and Ben have the same utility for cleaning the park:

$$U_A = ln(X_A) + ln(H_A + H_B)$$
$$U_B = ln(X_B) + ln(H_A + H_B)$$

(a) How many hours will Alice and Ben each hire the maintenance company for in the private optimum as a function of w and I? How many total hours? (Hint: After solving Alice's optimization problem, you can set $H_A = H_B$ since the problem is symmetric, just like in the Fireworks example.) (4 points)

Solution: The Lagrangian for Alice is $L = ln(X_A) + ln(H_A + H_B) + \lambda(I - X_A - wH_A)$. Then, first order conditions are:

(a) $\frac{\partial L}{\partial X_A} = \frac{1}{X_A} - \lambda = 0$

(b) $\frac{\partial L}{\partial H_A} = \frac{1}{H_A + H_B} - w\lambda = 0$

(c) $I - X_A - wH_A = 0$

From 1+2 we get that $w(H_A + H_B) = X_A$. Subbing into the budget constraint, we get $2wH_A + wH_B = I$. By symmetry, $H_A = H_B$, so $H_A^* = H_B^* = \frac{I}{3w}$, or $H^{TOT^*} = H_A^* + H_B^* = \frac{2I}{3w}$. (Note: it is also correct to solve for best-response function: $H_A = \frac{I - wH_B}{2w}$

(b) Suppose a social planner chose X_A, X_B, H_A, and H_B. What is the socially optimal amount of maintenance provided H^{TOT})? (3 points)

Solution: Set up a global Lagrangian: $L = ln(X_A) + ln(X_B) + 2ln(H_A + H_B) + \lambda_1(I - X_A - wH_A) + \lambda_2(I - X_B - wH_B)$.

FOC:

(a) $\frac{\partial L}{\partial X_A} = \frac{1}{X_A} - \lambda_1 = 0$
(b) $\frac{\partial L}{\partial X_B} = \frac{1}{X_B} - \lambda_2 = 0$
(c) $\frac{\partial L}{\partial H_A} = \frac{2}{H_A+H_B} - w\lambda_1 = 0$
(d) $\frac{\partial L}{\partial H_A} = \frac{2}{H_A+H_B} - w\lambda_2 = 0$
(e) $I - X_A - wH_A = 0$
(f) $I - X_B - wH_B = 0$

From (1), (2), (3), and (4) we see that $\frac{2}{H_A+H_B} = w\frac{1}{X_A} = w\frac{1}{X_B}$. This implies $X_A = X_B$ and $H_A = H_B$. Thus, $X_A = X_B = wH_A = wH_B$. Sub into the budget constraint to get $H_A^* = \frac{I}{2w}$ and $H^{TOT*} = \frac{I}{w}$.

(c) Why is the result you found in (a) different than what you found in (b)? (2 points)

Solution: Because neither Alice nor Ben takes into account the external benefits of their actions cleaning on the other party when choosing how many cleaning services to buy.

Suppose a third resident, Cathy, moves into the neighborhood. Suppose also for the rest of this question that each resident has a different income, with $I_A = 300$, $I_B = 600$, and $I_C = 900$. Also, for the rest of this question, suppose that the demand for total hours of cleaning by each resident i is equal to: $H_i^{TOT} = \frac{I_i}{2w}$ (so ignore whatever demand estimates you calculated in (a) and (b)). [Note: Each person gets utility from *total* cleaning expenditure H^{TOT}, which is why they each can be thought of as having demand over H^{TOT}.]

(d) What is the social demand for cleaning $H^{TOT} = H_A^{TOT} + H_B^{TOT} + H_C^{TOT}$ as a function of the wage? What is the socially optimal level of cleaning (i.e. social demand for cleaning) when wages are $w = 30$? (3 points)

Solution: Total demand is just the sum of the individual demand curves: $H^{TOT} = H_A^{TOT} + H_B^{TOT} + H_C^{TOT} = \frac{300}{2w} + \frac{600}{2w} + \frac{900}{2w} = \frac{900}{w}$. When $w = 30$, then optimal $H^{TOT} = 30$.

Now, suppose that Alice, Ben, and Cathy create a home-owner's association that decides how many cleaners to hire. Each resident is responsible to pay for a share of the hourly wages of the cleaners a, b, and c, where a is the fraction of the wage paid by Alice, b is the share paid by Ben, c is the share paid for by Cathy, and where $a + b + c = 1$. For the rest of the question, continue to assume that $w = 30$.

(e) Suppose Alice, Ben and Cathy agree to share the cost of cleaners equally, so that $a = b = c = \frac{1}{3}$. Under this arrangement, how many hours of cleaning would Alice want the home-owner's association to buy? How many would Ben want? How many would Cathy want? (2 points)

Solution: Under this regime, each person pays $10 per cleaner. Plugging 10 into their demand functions, we calculate that Alice chooses 15, Ben chooses 30, and Cathy chooses 45.

(f) The home-owner's association would like to create a policy that all three residents would approve, and which also achieves the socially optimal number of cleaning hours. Show this is possible by calculating the individual shares a, b, and c. Assume the association knows each individual's demand function. (3 points)

Solution: For Alice to want the socially optimal solution, she needs $H_A = \frac{300}{30*2*a} = 30$, or $a = 1/6$ (this is the maximum she is willing to pay). Using similar calculations, $b = 1/3$ and $c = 1/2$.

(g) *Challenge question:* In part (e), you solved for Alice's optimal number of cleaners, Ben's optimal number of cleaners, and Cathy's optimal number of cleaners. Suppose the residents vote on these three different policies using majority voting. Assume two rounds of voting: First, residents vote on whether they prefer Alice or Ben's policy. Then, residents vote on whether they prefer Cathy's policy from the winner of the first vote. The winner of this second vote is the policy that is chosen. Which policy will get chosen by the vote? Would your answer change if a different pair of policies were chosen in the first round? (3 points)

Solution: Note that Alice, Ben and Cathy each have single-peaked preferences. As a result, the median voter theorem applies, and the choice of the median voter (Ben) always wins the election, regardless of which pair of policies are chosen in the first round.

4. Exercise - Tiebout sorting (10 points)

Suppose there are two cities, A and B. Each city offers a level of public goods, denoted by g_A and g_B, respectively. There are 10 individuals in the world, numbered 1,2,...,10. Each individual chooses which city to live in, and each has a different utility function. In particular, the utility of individual i living in city c is given by:

$$U_i = 10 - |g_c - i|$$

where g_c is the level of public goods in city c, and where $|.|$ is the absolute-value function (Recall: $|x| = x$ if x is positive, and $|x| = -x$ if x is negative). For example, the utility of person 3 living in city B is given by $U_3 = 10 - |g_B - 3|$, and the utility of person 7 living in city A is given by $U_7 = 10 - |g_A - 7|$. Intuitively, one way to think about this is that person 7's ideal level of public goods is 7, and deviations from that level of public goods yield increasing levels of disutility. Assume that individuals choose to live in the city which gives them the most utility.

(a) Suppose the two cities have levels of public goods $g_A = 1$ and $g_B = 10$. Which city does each individual choose to live in? What is the population of each city? (2 points)

Solution: Individuals sort into cities closest to their ideal level of public goods. Thus, individuals 1-5 live in city A and individuals 6-10 choose to live in city B. The population of each city is 5.

(b) Now, suppose a new city is built, denoted by city C. Let $g_c = 5.5$. Now, which city does each individual choose to live in? What is the population of each city? (2 points)

Solution: Once again, individuals sort into cities that best fit their preferences. Individuals 1, 2, and 3 live in city A. Individuals 4, 5, 6, and 7 live in city C. Individuals 8, 9, and 10 live in city B. The population of cities A and B is 3, and the population of city C is 4.

(c) Suppose it costs nothing to build new cities. What does the Tiebout model say will happen? In particular, how many cities are necessary to achieve Tiebout efficiency? (3 points)

Solution: "Entrepreneurial" cities would be created, one for each type. Each person would have their own city, and the population of each city would be one.

(d) *Challenge question*: Suppose again that there are only two cities, A and B, and suppose each city is led by a politician that chooses g_c and wants as many people in their city as possible.

Suppose that there are now an infinite number of individuals with the same utility function as above, with i distributed uniformly between 1 and 10. What is the equilibrium amount of public goods provision for these two cities? (3 points)

Solution: The equilibrium will be $g_A = g_B = 5.5$. If $g_A \neq g_B$, then one politician could capture a larger share of individuals by choosing g_c to be closer to the other city's level of public goods. If $g_A = g_B \neq 5.5$, either politician could capture a larger share of individuals by choosing g_c closer to 5.5.