1.1.) $U(c_1, c_2) = \ln(c_1) + \ln(c_2)$

W : income , r is interest rate

a) $c_1, c_2 = ?$

$$\boxed{c_2 = (W - c_1)(1+r)} \Rightarrow c_2 = \left(W - \frac{W}{2}\right)(1+r) = \boxed{\frac{W}{2}(1+r)}$$

$$\max_{c_1} \ln c_1 + \ln\left((W - c_1)(1+r)\right)$$

FOC: $[c_1]$: $\frac{1}{c_1} = \frac{\cancel{1+r}}{\cancel{(1+r)}(W - c_1)}$

$$c_1 = W - c_1$$

$$\boxed{c_1 = \frac{W}{2}}$$

b) $c_2 = (W - c_1)(1+r) - 0.2\left(\overbrace{r\,\underbrace{(W - c_1)}_{=S}}^{\text{interest income}}\right)$

$$\boxed{c_2 = (W - c_1)(1 + 0.8r)} \Rightarrow \boxed{c_2 = \frac{W}{2}(1 + 0.8r)}$$

$$\max_{c_1} \ln c_1 + \ln\left((W - c_1)(1 + 0.8r)\right)$$

$$\max_{c_1} \ln c_1 + \ln(W - c_1) + \ln(1 + 0.8r)$$

$\underline{FOC}$: $[C_1]$: $\frac{1}{C_1} = \frac{\cancel{1.08r}}{(W-C_1)(\cancel{1.08r})}$

$W - C_1 = C_1$

$$\boxed{C_1 = \frac{W}{2}}$$

1.1.2) $U(C_1, C_2, l_1) = \ln C_1 + \ln C_2 + \ln(\underbrace{L}_{\text{total time}} - l_1)$

leisure

labor

given income

interest is also given

$$\boxed{C_2 = (w\,l - C_1)(1+r)}$$

savings

labor

consumption

$$\max_{C_1, l} \ \ln C_1 + \ln\Big((w\,l - C_1)(1+r)\Big) + \ln(L - l)$$

$[C_1]$: $\frac{1}{C_1} = \frac{1}{w\,l - C_1} \Rightarrow \boxed{C_1 = \frac{w\,l}{2}}$ (1)

$[l]$: $\frac{w}{w\,l - C_1} - \frac{1}{L - l} = 0$

$$\frac{w}{wl - c_1} = \frac{1}{L - l}$$

$$w(L - l) = wl - c_1$$

$$2wl = c_1 + wL$$

$$\boxed{l = \frac{c_1}{2w} + \frac{L}{2}} \quad (2)$$

$$(1) + (2): \quad l = \frac{\cancel{w}\,l}{4\cancel{w}} + \frac{L}{2}$$

$$\frac{3}{2\cancel{4}}\, l = \frac{L}{\cancel{2}} \Rightarrow \boxed{l^* = \frac{2L}{3}} \quad (3)$$

$$\boxed{c_1 = \frac{wl}{2}} \quad (1)$$

$$(3) \text{ in } (1): \quad c_1 = \frac{w}{\cancel{2}}\left(\frac{\cancel{2}L}{3}\right) \Rightarrow \boxed{c_1^* = \frac{wL}{3}}$$

$$\boxed{c_2 = (wl - c_1)(1+r)} \quad (4)$$

$$c_2 = \left(w \underbrace{\left(\frac{2L}{3}\right)}_{= l^*} - \underbrace{\left(\frac{wL}{3}\right)}_{c_1^*} \right)(1+r)$$

$$\boxed{c_2^* = \frac{wL}{3}(1+r)}$$

+ l

2.2) $\max_{K} \pi = \max_{K} \overbrace{F(K)}^{\text{output}} - \underbrace{rK}_{\text{cost}}$

$F(K)$: inputs

2.3) $Y = L^{1/2} K^{1/2} \Rightarrow$ Production function

input 1: L, input 2: K

$F(K) = \underbrace{pY}_{\text{Revenue}} - \underbrace{wL}_{\text{cost of input 1}}$, $L = 1$, $w = 2$, $p = 2$, $r = 0.25$

$\max_{K} \underbrace{F(K) - rK}_{\pi}$

$\max_{K} \underbrace{p}_{=2}(\cancel{L}^{1/2} K^{1/2}) - \underbrace{w}_{2}\cancel{L}^{1} - \underbrace{r}_{0.25}K$

$[K]: \frac{1}{K^{1/2}} - 0.25 = 0 \Rightarrow \boxed{K^* = 16}$

c) $\pi = 2(\sqrt{16}) - 2 - 0.25(16)$

$$\pi = 8 - 2 - 4 = \boxed{2}$$

d) $\tau_p = 0.25$

$$\max \pi = \underbrace{(1 - \tau_p)}^{0.75} F(K) - rK$$

$$\max_K \pi = 0.75 \left[\underset{2}{p} K^{1/2} - \underset{2}{w} L \right] - 0.25 K$$

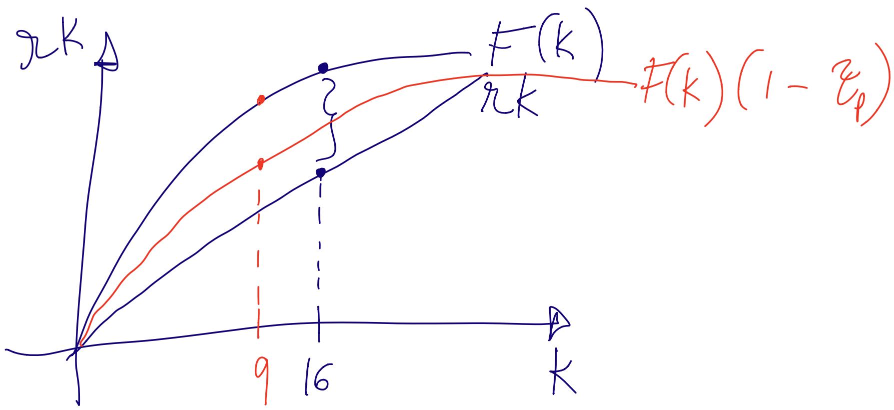
$$\max_K \pi = \frac{3}{4} \left[2\sqrt{K} - 2L \right] - \frac{1}{4} K$$

$$[K]: \frac{0.75}{K^{1/2}} - 0.25 = 0$$

$$K^{1/2} = 3 \Rightarrow \boxed{K^* = 9}$$

$$\pi = \frac{3}{4} \left[2\sqrt{9} - \cancel{2L} \right] - \frac{1}{4} 9$$

$$\pi = 3 - 2.25 = \boxed{0.75}$$



f) $\max \pi = (1 - \tau_f)\left[F(K) - rK\right]$

$$\max \pi = (1 - 0.25)\left[2L^{1/2}K^{1/2} - 2L - 0.25K\right]$$

$$\max_K \pi = (0.75)\left[2K^{1/2} - 2 - 0.25K\right]$$

$$[K]: \frac{1}{K^{1/2}} - 0.25 = 0 \Rightarrow K^{1/2} = 4$$

$$\boxed{K^* = 16}$$

1.1) $U(c_1, c_2) = \ln c_1 + \ln c_2$

W = income , r = interest rate

a) c_1, c_2 ?

$$c_2 = \underbrace{(W - c_1)}_{S}(1+r) = \left(W - \frac{W}{2}\right)(1+r) = \boxed{\frac{W}{2}(1+r)}$$

$$\max_{c_1} \ln c_1 + \ln\left((W-c_1)(1+r)\right)$$

FOC: $[c_1]$: $\frac{1}{c_1} = \frac{\cancel{1+r}}{(W-c_1)\cancel{(1+r)}}$

$$\boxed{c_1 = \frac{W}{2}}$$

b) $C_2 = (W - C_1)(1 + r) - \underbrace{0.2(r(W - C_1))}_{\text{tax on interest earned}}$

$$\boxed{C_2 = (W - C_1)(1 + 0.8r)} \Rightarrow \boxed{C_2 = \frac{W}{2}(1.08r)}$$

$$\max_{C_1} \ln C_1 + \ln\left((W - C_1)(1 + 0.8r)\right)$$

$$\max_{C_1} \ln C_1 + \ln(W - C_1) + \ln(1 + 0.8r)$$

$$[C_1]: \frac{1}{C_1} = \frac{1}{W - C_1} \Rightarrow \boxed{C_1 = \frac{W}{2}}$$

1.1.2) $U(C_1, C_2, l) = \ln C_1 + \ln C_2 + \ln(\overset{\text{total hours}}{L} - \overset{\text{work}}{l})$

leisure

$$\boxed{C_2 = (wl - C_1)(1 + r)} \quad (S)$$

$$\max \ln C_1 + \ln\left((wl - C_1)(1 + r)\right) + \ln(L - l)$$

FOC: $[c_1]$: $\frac{1}{c_1} = \frac{1}{wl - c_1} \Rightarrow c_1 = wl - c_1$

$$\boxed{c_1 = \frac{wl}{2}} \quad (1)$$

$[l]$: $\frac{w}{wl - c_1} + \frac{-1}{L - l} = 0$

$$\frac{w}{wl - c_1} = \frac{1}{L - l} \Rightarrow L - l = \frac{wl - c_1}{w}$$

$$l(2w) = wL + c_1$$

$$\boxed{l = \frac{wL}{2w} + \frac{c_1}{2w}} \quad (2)$$

$(1) + (2)$: $l = \frac{wL}{2w} + \underbrace{\frac{wl}{2}}_{\overset{(1)}{=} c_1}\left(\frac{1}{2w}\right)$

$$l = \frac{2wL + wl}{4w}$$

$$4wl = 2wL + wl$$

$$3\cancel{w}\, l = 2\cancel{w}\, L$$

$$\boxed{l^* = \frac{2L}{3}} \qquad (3)$$

$$(3) + (1): \quad C_1 = \frac{w\,l}{2} = \frac{w}{\cancel{2}}\left(\frac{\cancel{2}L}{3}\right)$$

$$\boxed{C_1^* = \frac{wL}{3}} \qquad (4)$$

From (5), $C_2 = \left(w\overset{(3)}{l} - \overset{(4)}{C_1}\right)(1+r)$

$$C_2 = \left(w\overset{(3)}{\frac{2L}{3}} - \overset{(4)}{\frac{wL}{3}}\right)(1+r)$$

$$\boxed{C_2^* = \frac{wL}{3}(1+r)}$$

2.3) $F(K) = \underbrace{p}_{\text{revenue}} \underbrace{Y}_{\text{output}} - \underbrace{wL}_{\text{cost of labor}}$

$L = 1$

$w = 2$

$p = 2$

$r = 0.25$

$Y = L^{1/2} \; K^{1/2}$

$\max \pi = pY - wL - rK$

$\max_K \; 2\left(\cancel{L^{1/2}} K^{1/2}\right) - 2\cancel{L} - 0.25K$

$[K]: \; \dfrac{1}{K^{1/2}} - 0.25 = 0 \Rightarrow K^{1/2} = 4$

$\boxed{K^* = 16}$

c) $\pi = \cancel{2}\left(\overset{8}{\sqrt{16}}\right) - 2 - \cancel{\tfrac{1}{4}}\,\overset{4}{\cancel{16}}$

$\boxed{\pi = 2}$

d) $\max_K \; (1-\tau_p) F(K) - rK$

$\max_K \; 0.75\left[2 L^{1/2} K^{1/2} - 2L\right] - 0.25K$

$$[K]:\ \frac{0.75}{K^{1/2}} - 0.25 = 0$$

$$K^{1/2} = 3 \Rightarrow \boxed{K^{*} = 9}$$

$$\pi = 0.75\left[2\left(\sqrt{9}\right) - 2\right] - \frac{1}{4}9$$

$$\pi = 0.75(4) - \frac{1}{4}9 = 3 - 2.25 = \boxed{0.75}$$

Profits before taxes

f) $\text{Max } \pi = (1-\tau_p)(F(K) - rK)$

$$\text{Max } (1-0.25)\left(2\cancel{L^{1/2}}K^{1/2} - 2\cancel{L} - 0.25K\right)$$

$$\max_{K}\ 0.75\left(2K^{1/2} - 2 - \frac{1}{4}K\right)$$

$$[K]: \cancel{0.75}^{\frac{3}{4}}\left(\frac{1}{K^{1/2}} - \frac{1}{4}\right) = 0$$

$$\Rightarrow K^{1/2} = 4 \Rightarrow \boxed{K^* = 16}$$

$$[K]: \frac{0.75}{K^{1/2}} = \frac{1}{4} \quad \cancel{\frac{3}{4}}$$

$$K^{1/2} = 4 \Rightarrow \boxed{K^* = 16}$$