Papers due a week from Wednesday.

1. Issue is estimating and testing implications of the beta-risk model. In the single factor model, the key part is the cross-sectional relation that says the average excess return on asset i is proportional to it's exposure to the risk factor, measured by β_i
$$\bar{r}_i^e = \lambda \beta_i$$

2. This equation cries out for you to run the cross-sectional regression,
$$\bar{r}_i^e = \gamma + \lambda \beta_i + u_i$$
What restrictions does the theory impose on the data?
   (a) γ should be 0.
   (b) λ should be significantly different from 0.

3. Research design for a single factor model. Say you have assets with excess returns $r_{t,i}^e$, $i = 1, .., n$, and factor f_t.
   (a) Step 1. The time-series regressions. For each asset, estimate it's beta by regressing it's excess return on the factor.
$$\begin{aligned} r_{t,1}^e &= \alpha_1 + \beta_1 f_t + \epsilon_{t,1} \\ r_{t,2}^e &= \alpha_2 + \beta_2 f_t + \epsilon_{t,2} \\ &\vdots \\ r_{t,n}^e &= \alpha_n + \beta_n f_t + \epsilon_{t,n} \end{aligned}$$
   Save the betas, because they will become the independent variable in the cross-sectional regression. And compute the mean excess returns and save them too.
$$\begin{aligned} \bar{r}_1^e &= \frac{1}{T}\sum_{t=1}^{T} r_{t,1}^e \\ \bar{r}_2^e &= \frac{1}{T}\sum_{t=1}^{T} r_{t,2}^e \\ &\vdots \\ \bar{r}_n^e &= \frac{1}{T}\sum_{t=1}^{T} r_{t,n}^e \end{aligned}$$
   Now we have $\begin{pmatrix} \bar{r}_1^e & \hat{\beta}_1 \\ \bar{r}_2^e & \hat{\beta}_2 \\ \vdots & \vdots \\ \bar{r}_n^e & \hat{\beta}_n \end{pmatrix}$, use as 'data' in step 2.

(b) Step 2: run the cross-sectional regression. The $\hat{\beta}_i$ are the independent variable. The slope coefficient in the regression is λ

$$\bar{r}_i^e = \gamma + \lambda \hat{\beta}_i + u_i$$

However, there is a complication in computing the t-ratio on λ, because the $\hat{\beta}_i$ are not true data observations. They are estimates, and as such, are random variables. We call them 'generated regressors.'
In econometrics, when we run the regression (constant suppressed), where x_i are exogenous, we can treat them as constants.

$$\begin{aligned}
y_i &= b x_i + u_i \\
\hat{b} &= b + \frac{\sum x_i u_i}{\sum x_i^2} \\
Var\left(\hat{b} - b\right)^2 &= \left(\frac{1}{\sum x_i^2}\right)^2 \sum x_i^2 E\left(u_i^2\right) = \frac{\sigma_u^2 \sum x_i^2}{\left(\sum x_i^2\right)^2} = \frac{\sigma_u^2}{\sum x_i^2}
\end{aligned}$$

But in our case, the x_i are not constants. We cannot treat them as constants to compute the variance of the slope estimator.
The Fama-MacBeth method is a procedure to compute the standard error of $\hat{\lambda}$.

4. Fama-MacBeth: For $t = 1$, we have a cross-section of excess returns, $r_{t,1}^e, ... r_{t,n}^e$, and $\hat{\beta}_1, ..., \hat{\beta}_n$. Hence, run the cross-sectional regression

$$\begin{aligned}
r_{1,i}^e &= \gamma_1 + \lambda_1 \hat{\beta}_i + u_{1,i} \\
r_{2,i}^e &= \gamma_2 + \lambda_2 \hat{\beta}_i + u_{2,i} \\
r_{3,i}^e &= \gamma_3 + \lambda_3 \hat{\beta}_i + u_{3,i} \\
&\vdots \\
r_{T,i}^e &= \gamma_T + \lambda_T \hat{\beta}_i + u_{T,i}
\end{aligned}$$

At each step, save the estimated λ_t. Now you have a 'time-series' of λ_t.

(a) $\bar{\lambda} = \frac{1}{T} \sum_{t=1}^{T} \hat{\lambda}_t$ gives us another estimate of λ.
(b) To test if λ is signficant, regress λ_t on a constant and do Newey-West.

5. The time-series method we covered last time can only be used if the factor is a traded asset return, like the market in the CAPM. The reason is when we run the time-series regressions,

$$\begin{aligned}
r_{t,i}^e &= \alpha_i + \beta_i f_t + \epsilon_{t,i} \\
\hat{\lambda} &= \frac{1}{T} \sum f_t
\end{aligned}$$

Fama-MacBeth method can be used whether the factor is a traded asset return or if the factor is some economic quantity, such as consumption growth, or GDP growth, etc.