Section 11

- Expected Utility Models

$$E[U] = (1-\underset{\substack{\uparrow \\ \text{adverse event occurs}}}{q})\,U(\underbrace{\text{good state}}) + q\,U(\underbrace{\text{bad state}})$$

$$EU = (1-q)\,U[\underset{\substack{\uparrow \\ \text{wealth}}}{W} - \underset{\substack{\uparrow \\ \text{price}}}{p}] + q\,U[W - p - \underset{\substack{\uparrow \\ \text{damage}}}{d} + \overset{\substack{\text{benefit} \\ \uparrow}}{b}]$$

$$b = \frac{p}{q} \Rightarrow \boxed{p = bq}$$

2.3) I : $q_{I} = 2 + 3 = \boxed{5\%}$

II : $q_{II} = 2 + 8 = 10\%$

III : $q_{III} = 2 + 58 = 60\%$

$$u(c) = \log(c)$$

$$W = \begin{cases} 500 & \text{if healthy} \\ \boxed{10 \quad \text{if sick}} \end{cases}$$

a)

$$\max_{b} E[U] = \max (1-q) \log\left(500 - \overset{qb}{\cancel{p}}\right) + q \log\left(10 - \underset{qb}{\cancel{p}} + b\right)$$

$$[b] : \frac{-q(1-q)}{500 - qb} + \frac{q(1-q)}{10 - qb + b} = 0$$

$$\frac{\cancel{q(1-q)}}{10-qb+b} = \frac{\cancel{q(1-q)}}{500-qb}$$

$$10 - \cancel{qb} + b = 500 - \cancel{qb}$$

$$\boxed{b = 490}$$

$$W^{\text{NO INSURANCE}} = \begin{cases} 500 & (\text{good state}) \\ 10 & (\text{bad state}) \end{cases}$$

<u>WITH INSURANCE:</u>

$$W = \begin{cases} \boxed{500 - p} & \text{(good)} \\ \qquad \| & \\ \boxed{10 + \underbrace{490}_{= b} - p} & \text{(bad)} \end{cases}$$

b) $p = b\,q$

$$\Rightarrow \begin{cases} p_{I} = b\,q_{I} = 490(0.05) = \boxed{24.5} \\ p_{II} = b\,q_{II} = 490(0.1) = 49 \\ p_{III} = b\,q_{III} = 490(0.6) = 294 \end{cases}$$

$$E[U] = (1 - q_i)\log(500 - p_i) + q_i \log\left(10 - p_i + \frac{p_i}{q_i}\right)$$

I) $E[U] = \log(500 - 24.5) = 6.16$

II) $E[U] = \log(500 - 49) = 6.11$

III) $E[U] = \log(500 - 294) = 5.33$

$\Rightarrow$ **NOT** insured

$$E[U] = (1 - q)\log(500) + q\log(10)$$

$$\mathrm{I}) \; E[U] = 0.95 \log(500) + 0.05 \log(10) = 6.02$$

$$\mathrm{II}) \; E[U] = 0.9 \log(500) + 0.1 \log(10) = 5.82$$

$$\mathrm{III}) \; E[U] = 0.4 \log(500) + 0.6 \log(10) = 3.87$$

$$\Rightarrow \; U_{INS} \geq \boxed{U_{\text{NO INSURANCE}}}$$

$$\log(500 - p) \geq (1-q) \log(500) + q \log(10)$$

$$\log(500 - p) \geq \log\left(500^{1-q} \cdot 10^{q}\right)$$

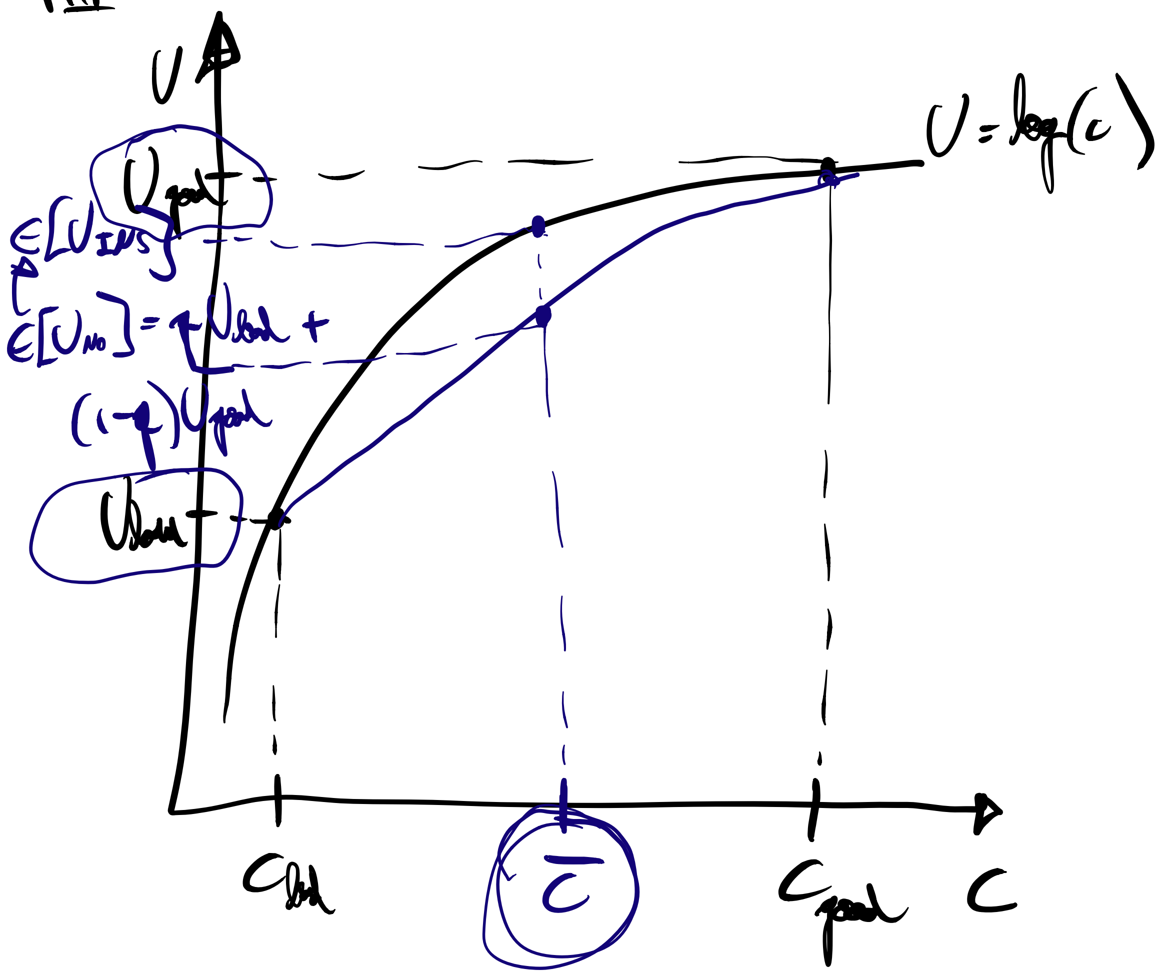
$$500 - p \geq 500^{1-q} \cdot 10^{q}$$

$$\boxed{p = \underbrace{500 - 500^{1-q} \cdot 10^{q}}}$$

$$P_{I} = 500 - 500^{0.95}\,10^{0.05} = \boxed{88.83}$$

$$P_{II} = 500 - 500^{0.9}\,10^{0.1} = \boxed{161.88}$$

$$P_{III} = 500 - 500^{0.4}\,10^{0.6} = 452.1$$



d) $\boxed{p = \bar{p}\, q}$

$$\boxed{q = \frac{0.05 + 0.1 + 0.6}{3}} = \frac{.75}{3} = .25$$

$$p = 490\,(.25) = \boxed{122.5}$$

e) $p' > p_{max}$ for type I

$\Rightarrow$ So I opts out

f) $$q = \frac{0.1 + 0.6}{2} = \frac{0.7}{2} = \boxed{0.35}$$

$$P = q\,\ell = (490)\,0.35 = \boxed{171.5}$$

$\Rightarrow$ Type II drops out

f) $W = \begin{cases} 500, & \text{if healthy} \\ 10, & \text{if not} \end{cases}$

$\Rightarrow W = \begin{cases} 500, & \text{if healthy} \\ \boxed{0}, & \text{if not} \end{cases}$

$U = \log(c)$