Ec141, Spring 2019

Professor Bryan Graham

Problem Set 5

Due: May 7th, 2019

Problem sets are due at 5PM in the GSIs mailbox. You may work in groups, but each student should turn in their own write-up (including a narrated/commented and executed Jupyter Notebook). Please use markdown boxes within your Jupyter notebook for narrative answers to the questions below. This is the last problem set of the semester, so I especially encourage you to work together with your classmates (you will learn more and enjoy it more)!

1 Production function: identification

Let Y_t, K_t, L_t and I_t respectively denote a firm's period t output, capital stock, labor and capital expenditure (i.e., investment). The econometrician observes $(Y_{it}, K_{it}, L_{it}, I_{it})$ for a random sample of $i = 1, \ldots, N$ firms in each of $t = 0, \ldots, T$ periods.

Output is produced according to

$$Y_t = F(K_t, L_t, A_t) = \exp(U_t + V_t) K_t^{\alpha} L_t^{\beta},$$

with U_t and V_t respectively equal to a beginning-of-period and end-of-period productivity shock. We assume that any period t decisions made by the firm occur *after* observing U_t, but *before* observing V_t.

We assume that U_t evolves according to the linear Markov process

$$U_t = \kappa + \lambda U_{t-1} + \epsilon_t$$

with $\mathbb{E}\left[\epsilon_t \middle| U_0^{t-1}\right] = 0$ for $U_0^{t-1} = (U_0, U_1, \ldots, U_{t-1})'$. The end-of-period productivity shock, V_t, is (mean-zero) white noise.

Capital evolves according to

$$K_t = (1 - \delta) K_{t-1} + I_{t-1}$$

with δ the rate of depreciation and I_t capital expenditure. An implication of the capital process is that a firm's capital stock is determined one period in advance.

The firm's information set, after making investment and hiring decisions for the current period, but prior to the realization of output, equals $\mathcal{I}_t = \left(K_0^{t+1}, L_0^t, Y_0^{t-1}, U_0^t\right)$.

[a] Let $\eta\left(Z_t,\alpha,\beta\right)=\ln Y_t-\alpha\ln K_t-\beta\ln L_t$ for $Z_t=\left(K_t,L_t,Y_t\right)'$ and, for $\theta=\left(\kappa,\lambda,\alpha,\beta\right)'$, let

$$\rho\left(Z_{t-1}^t,\theta\right)=\eta\left(Z_t,\alpha,\beta\right)-\kappa-\lambda\eta\left(Z_{t-1},\alpha,\beta\right).$$

Show that, for θ_0 the population parameter,

$$\mathbb{E}\left[\rho\left(Z_{t-1}^t,\theta_0\right)\middle|\mathcal{I}_{t-1}\right]=\mathbb{E}\left[\epsilon_t+V_t-\rho V_{t-1}\middle|\mathcal{I}_{t-1}\right]=0 \tag{1}$$

for $t=1,\ldots,T$.

[b] For $T=1$ show that

$$\mathbb{E}\left[\rho\left(Z_0^1,\theta_0\right)\begin{pmatrix}1\\K_0\\K_1\\L_0\end{pmatrix}\right]=0.$$

[c] Assume that $U_t=\sigma_t\left(I_t,K_t\right)$. Discuss this assumption with reference to Olley & Pakes (1996), Griliches & Mairesse (1998) and Ackerberg et al. (2015). Let $h_t\left(I_t,K_t,L_t\right)=\mathbb{E}\left[Y_t\middle|I_t,K_t,L_t\right]$. Show that

$$\mathbb{E}\left[\rho\left(Z_{t-1}^t,h_{t-1}^t,\theta_0\right)\middle|\mathcal{I}_{t-1}\right]=\mathbb{E}\left[\epsilon_t\middle|\mathcal{I}_{t-1}\right]=0$$

for

$$\rho\left(Z_{t-1}^t,h_{t-1}^t,\theta_0\right)=\eta\left(Z_t,h_t,\alpha,\beta\right)-\kappa-\lambda\eta\left(Z_{t-1},h_{t-1},\alpha,\beta\right) \tag{2}$$

with $\eta\left(Z_t,h_t,\alpha,\beta\right)=h_t\left(I_t,K_t,L_t\right)-\alpha\ln K_t-\beta\ln L_t$.

[d] Compare restrictions (1) and (2).

2 Production function: estimation

The file `mf_firms.out` contains 29,836 firm-by-year observations for a sample of publicly traded manufacturing firms drawn from the *S&P Capital IQ - Compustat* database. The following firm attributes, measured from 1998 to 2014 inclusive, are included:

`gvkey` – Compustat firm identification code

`year` – calendar year

`Y` – total real sales by the firm (in millions of 2009 US$)

`K` – capital stock (in millions of 2009 US$)

`L` – employees (in thousands)

`M` – materials expenditures (in millions of 2009 US$)

`VA` - total real valued added by the firm (in millions of 2009 US$)

`w` - annual wage rate (in 2009 US$)

`i` – real investment (in millions of 2009 US$)

`naics_4digits` – NAICS four digit sector code for the firm

This is the same dataset you used in Problem Set 1. For this assignment, keep only those observations corresponding to 2013 ($t = 0$) and 2014 ($t = 1$). Further only retain "complete cases"; that is firms with information on `VA,K,L` and `i`. Additionally drop any firms where valued added and/or investment, in either 2013 and/or 2014, is *not* strictly positive. What is left will be our estimation sample.

[a] Construct a table of summary statistics for the estimation sample. How many firms are in the sample?

[b] Consider the model outlined above with $U_t = \sigma_t (I_t, K_t)$ for

$$\sigma_t (I_t, K_t) = \pi_{0t} + \pi_{1t} \ln I_t + \pi_{2t} \ln K_t + \pi_{3t} (\ln I_t)^2 + \pi_{4t} (\ln K_t)^2 + \pi_{5t} (\ln I_t) (\ln K_t) .$$

Let $W_t = \left(1, \ln I_t, \ln K_t, (\ln I_t)^2, (\ln K_t)^2, (\ln I_t)(\ln K_t), \ln L_t\right)'$. Write a computer program the implements the following procedure:

[c] Explain why (3) should be small when $\alpha^{(s)} \approx \alpha_0$ (here α_0 denotes the true capital elasticity). More generally give a verbal justification for the estimation procedure with reference to the theoretical analysis in the first part of the problem set. Can you think of any modifications you might like to make to your procedure? Speculate on any advantages or disadvantages of these modifications.

[d] Compute $\hat{U}_{i1}$ as $\hat{h}_{i1} - \hat{\alpha} \ln K_{i1} - \hat{\beta}_1 \ln L_{i1}$. Plot a histogram of $\hat{U}_{i1}$. Compare your analysis with the productivity analysis you undertook in Problem Set 1. Compute the average, standard deviation and 5th, 25th, 50th, 75th and 95th percentiles of the sample distribution of $\hat{U}_{i1}$.

[e] Use the bootstrap procedure described in lecture in the context of the inverse probability weighting (IPW) estimator for the average treatment effect (ATE) to construct confidence intervals for α, β, $\alpha + \beta$ and the moments of the distribution of U_{i1} estimated in part (d) above. Specifically each bootstrap sample should consist of a random sample of firms drawn from the original sample *with replacement.* Note that firms, not firm-years, are sampled.

Algorithm 1 Olley-Pakes Estimation Procedure

1. Compute the OLS fit of Y_{i0} onto W_{i0} for $i = 1, \ldots, N$. Let $\hat{h}_{i0}$ denote the fitted values associated with this OLS fit; construct these fitted values. Let $\hat{\beta}_0$ be the estimated coefficient on $\ln L_0$ in this fit.

2. Compute the OLS fit of Y_{i1} onto W_{i1} for $i = 1, \ldots, N$. Let $\hat{h}_{i1}$ denote the fitted values associated with this OLS fit; construct these fitted values. Let $\hat{\beta}_1$ be the estimated coefficient on $\ln L_1$ in this fit.

3. Let $\alpha^{(s)}$ be the current value of α(a good starting value might be $\alpha^{(0)} = 0.05$). Create the variables $\eta\left(Z_{i0}, \hat{h}_{i0}, \alpha^{(s)}, \hat{\beta}_0\right) = \hat{h}_{i0} - \alpha^{(s)} \ln K_{i0} - \hat{\beta}_0 \ln L_{i0}$ and $\eta\left(Z_{i1}, \hat{h}_{i1}, \alpha^{(s)}, \hat{\beta}_1\right) = \hat{h}_{i1} - \alpha^{(s)} \ln K_{i1} - \hat{\beta}_1 \ln L_{i1}$.

4. Compute the OLS fit of $\eta\left(Z_{i1}, \hat{h}_{i1}, \alpha^{(s)}, \hat{\beta}_1\right)$ onto a constant and $\eta\left(Z_{i0}, \hat{h}_{i0}, \alpha^{(s)}, \hat{\beta}_0\right)$. Let the intercept and slope coefficient from this fit be $\hat{\kappa}^{(s)}$ and $\hat{\lambda}^{(s)}$ respectively.

5. Form the sample moment 4×1 vector

$$\begin{aligned}\psi_N\left(\hat{h}_0^1, \alpha^{(s)}, \hat{\beta}_0, \hat{\beta}_1, \hat{\kappa}^{(s)}, \hat{\lambda}^{(s)}\right) = & \frac{1}{N}\sum_{i=1}^{N}\Bigg(\eta\left(Z_{i1}, \hat{h}_{i1}, \alpha^{(s)}, \hat{\beta}_1\right) - \hat{\kappa}^{(s)} \\ & - \hat{\lambda}^{(s)}\eta\left(Z_{i0}, \hat{h}_{i0}, \alpha^{(s)}, \hat{\beta}_0\right)\Bigg)\begin{pmatrix} 1 \\ \ln K_{i0} \\ \ln K_{i1} \\ \ln L_{i0} \end{pmatrix}\end{aligned}$$

   and the associated quadratic form

$$\psi_N\left(\hat{h}_0^1, \alpha^{(s)}, \hat{\beta}_0, \hat{\beta}_1, \hat{\kappa}^{(s)}, \hat{\lambda}^{(s)}\right)' \psi_N\left(\hat{h}_0^1, \alpha^{(s)}, \hat{\beta}_0, \hat{\beta}_1, \hat{\kappa}^{(s)}, \hat{\lambda}^{(s)}\right). \tag{3}$$

6. Repeat steps 3 to 5 for $\alpha^{(s)} = 0.05, 0.06, \ldots, 0.95$. Let $\hat{\alpha}$ be the $\alpha^{(s)}$ which minimizes (3). Let $\hat{\kappa}$ and $\hat{\lambda}$ be the associated $\hat{\kappa}^{(s)}$ and $\hat{\lambda}^{(s)}$ estimates from step 4.

If firm i is included in a given bootstrap sample, then both years of data for that firm are included. For each bootstrap sample implement the Olley-Pakes procedure (i.e., repeat your analysis from parts (b) to (d)). Use the resulting bootstrap distribution to form confidence intervals. Can you accept the null hypothesis of constant returns to scale?

[f] What have you learned about the distribution of productivity across large U.S. manufacturing firms? What else are you interested in learning? What data/methods might help you do so?

References

Ackerberg, D. A., Caves, K., & Frazer, G. (2015). Identification properties of recent production function estimators. *Econometrica*, 83(6), 2411 – 2451.

Griliches, Z. & Mairesse, J. (1998). *Econometrics and Economic Theory in the 20th Century*, chapter Production functions: the search for identification, (pp. 169 – 203). Cambridge University Press: Cambridge.

Olley, G. S. & Pakes, A. (1996). The dynamics of productivity in the telecommunications equipment industry. *Econometrica*, 64(6), 1263 – 1297.