Ec141, Spring 2019

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Review Sheet 2

This review sheet is designed to assist you in your exam preparations. I suggest preparing written answers to each question. You may find it useful to study with your classmates (indeed I encourage you to do so and also to be generous with one another as you prepare). In the exam you may bring in a single 8.5 x 11 sheet of notes. No calculators or other aides will be permitted. Please bring blue books to the exam. The midterm exam will occur in class on Thursday, May 2nd.

[1] For $s \in \mathbb{S}$, a hypothetical years-of-schooling level, let an individual's potential earnings be given by $\log Y(s) = \alpha_0 + \beta_0 s + U$. Here U captures unobserved heterogeneity in labor market ability and other non-school determinants of earnings. Let the total cost of s years of schooling be given by $(\delta_0^* W + V^*) s + \frac{\kappa}{2} s^2$. Here W is an observable variable which shifts the marginal cost of schooling and V^* is unobserved heterogeneity. You may assume that both U and V^* are conditionally mean zero given W. Agents choose years of completed schooling to maximize expected utility

$$S = \arg\max_{s \in \mathbb{S}} \mathbb{E}\left[\log Y(s) - (\delta_0^* W + V^*) s - \frac{\kappa}{2} s^2 \middle| W, V\right].$$

[a] Show that observed schooling is given by

$$S = \gamma_0 + \delta_0 W + V, \quad \mathbb{E}[V|W] = 0$$

for $\gamma_0 = \beta_0/\kappa$, $\delta_0 = -\delta^*/\kappa$, and $V = -V^*/\kappa$.

[b] Assume that W measures commute time to the closest four year college from a respondent's home during adolescence. What sign do you expect δ_0 to have? Explain.

[c] Assume that $\mathbb{E}[U|W, V] = \mathbb{E}[U|V] = \lambda V$. Restate this assumption in words (HINT: Think about V as a latent variable/attribute). What sign do you expect λ to have? Briefly argue for and against this assumption?

[d] Let $\log Y = \log Y(S)$ denote actual earnings. Show that

$$\mathbb{E}^*[\log Y| S, V] = \alpha_0 + \beta_0 S + \lambda V. \tag{1}$$

[e] What determines variation in S conditional on $V = v$? What is the relationship between this variation and the unobserved determinants of log earnings? Use your answers to provide an intuitive explanation (i.e., use words) for why the coefficient on schooling in (1) equals β_0.

[f] The random sample $\{(Y_i, S_i, W_i)\}_{i=1}^N$ is available. Suggest a procedure for consistently estimating β_0.

[g] Let

$$\mathbb{E}^*[\log Y| S] = a_0 + b_0 S.$$

From you analysis in part [f] you learn that $\lambda \approx 0$. Guess what value b_0 takes. Justify your answer.

[2] Let Y equal tons of banana's harvested in a given season for a randomly sampled Honduran banana planation. Output is produced using labor and land according to $Y = AL^{\alpha_0} D^{1-\alpha_0}$, where L is the number

of employed workers and D is the size of the plantation in acres and we assume that $0 < \alpha_0 < 1$. The price of a unit of output is P, while that of a unit of labor is W. These prices may vary across plantations (e.g., due to transportation costs, labor market segmentation etc.). We will treat D as a fixed factor; A captures sources of plantation-level differences in farm productivity due to unobserved differences in, for example, soil quality and managerial capacity. Plantation owners choose the level of employed labor to maximize profits. The observed values of L are therefore solutions to the optimization problem:

$$L = \arg\max_{l} P \cdot A l^{\alpha_0} D^{1-\alpha_0} - W \cdot l.$$

[a] Show that the amount of employed labor is given by

$$L = \left\{ \alpha_0 \frac{P}{W} A \right\}^{\frac{1}{1-\alpha_0}} D. \tag{2}$$

[b] Let $a_0 = \frac{1}{1-\alpha_0} \ln \alpha_0 + \frac{1}{1-\alpha_0} \mathbb{E}[\ln A]$, $b_0 = \frac{1}{1-\alpha_0}$, and $V = \frac{1}{1-\alpha_0} \{\ln A - \mathbb{E}[\ln A]\}$. Show that the log of the labor-land ratio is given by

$$\ln\left(\frac{L}{D}\right) = a_0 + b_0 \ln\left(\frac{P}{W}\right) + V \tag{3}$$

and that, letting $c_0 = \mathbb{E}[\ln A]$ and $U = \ln A - \mathbb{E}[\ln A]$, the log of planation yield (output per unit of land) is given by

$$\ln\left(\frac{Y}{D}\right) = c_0 + \alpha_0 \ln\left(\frac{L}{D}\right) + U. \tag{4}$$

[c] Briefly discuss the content and plausibility of the restriction

$$\mathbb{E}[\ln A | \ln(P/W)] = \mathbb{E}[\ln A]. \tag{5}$$

[d] Using (3), (4) and (5) show that the coefficient on $\ln(L/D)$ in $\mathbb{E}^*[\ln(Y/D) | \ln(L/D)]$ equals

$$\alpha_0 + (1-\alpha_0) \frac{\mathbb{V}(\ln A)}{\mathbb{V}(\ln A) + \mathbb{V}(\ln(P/W))}.$$

Provide some economic intuition for this result.

[e] Using (3), (4) and (5) show that the coefficient on $\ln(L/D)$ in $\mathbb{E}^*[\ln(Y/D) | \ln(L/D), V]$ equals α_0. Provide some economic intuition for this result.

[f] Assume that all plantations face the same output price (P) and labor cost (W). What value does the coefficient on $\ln(L/D)$ in $\mathbb{E}^*[\ln(Y/D) | \ln(L/D)]$ equal now? Why?

[3] Consider the following model of supply and demand:

$$\begin{aligned} \ln Q_i^D(p) &= \alpha_1 + \alpha_2 \ln(p) + U_i^D \\ \ln Q_i^S(p) &= \beta_1 + \beta_2 \ln(p) + U_i^S, \end{aligned}$$

with i indexing a generic random draw from a population of 'markets' ; U_i^D and U_i^S are market-specific demand and supply shocks. We assume that $\left(U_i^S, U_i^D\right) \overset{i.i.d}{\sim} F$ for $i = 1, 2, \ldots, N$. In each market the observed price and quantity pair (P_i, Q_i) coincides with the solution to market clearing condition

$$Q_i^D\left(P_i\right) = Q_i^S\left(P_i\right) = Q_i.$$

[a] Provide an economic interpretation of the parameters α_2 and β_2. What signs do you expect them to take? Why?

[b] Depict the market equilibrium graphically. Solve for the equilibrium values of $\ln Q_i$ and $\ln P_i$ algebraically. How is the market price and quantity related to the demand and supply shocks, U_i^D and U_i^S? Provide some economic content for your answer. Can you use a figure to illustrate it?

[c] Calculate $\mathbb{E}^*\left[\ln Q \middle| \ln P\right]$. You may assume that $\mathbb{C}\left(U^D, U^S\right) = 0$. Evaluate the coefficient on $\ln\left(P\right)$, does it coincide with an economically interpretable parameter? Assume that $\mathbb{V}\left(U_i^S\right) / \left(\mathbb{V}\left(U_i^S\right) + \mathbb{V}\left(U_i^D\right)\right) \approx 1$, does your answer change? Why?