Econ 204 – Problem Set 3

Due Friday, August 7, 2015

1. Take any mapping f from a metric space X into a metric space Y. Prove that f is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$. (Hint: use the closed set characterization of continuity).

2. A function $f : X \to Y$ is *open* if for every open set $A \subset X$, its image $f(A)$ is also open. Show that any continuous open function from $\mathbb{R}$ into $\mathbb{R}$ (with the usual metric) is strictly monotonic.

3. Suppose f, g are continuous functions from metric spaces (X, d) into (Y, ρ). Let E be a dense subset of X (in a metric space, a set A is dense in B if $\overline{A} \supset B$). Show that $f(E)$ is dense in $f(X)$. Further, if $f(x) = g(x)$ for every $x \in E$, then $f(x) = g(x)$ for every $x \in X$.

4. Show that in a metric space, a set is closed if and only if its intersection with any compact set is closed.

5. Show that a metric space X is connected if and only if every continuous function $f : X \to \{0, 1\}$ is constant.

6. Let (X, d) be a compact metric space and let $\Phi(x) : X \to 2^X$ be a upper-hemicontinuous, compact-valued correspondence, such that $\Phi(x)$ is non-empty for every $x \in X$. Prove that there exists a compact non-empty subset K of X, such that $\Phi(K) \equiv \bigcup_{x \in K} \Phi(x) = K$.