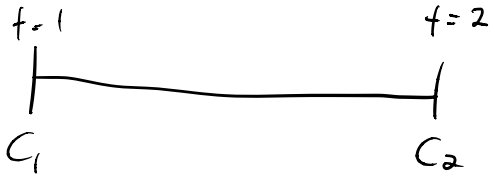


Y $\qquad$ $(Y - C_1)(1 + r)$

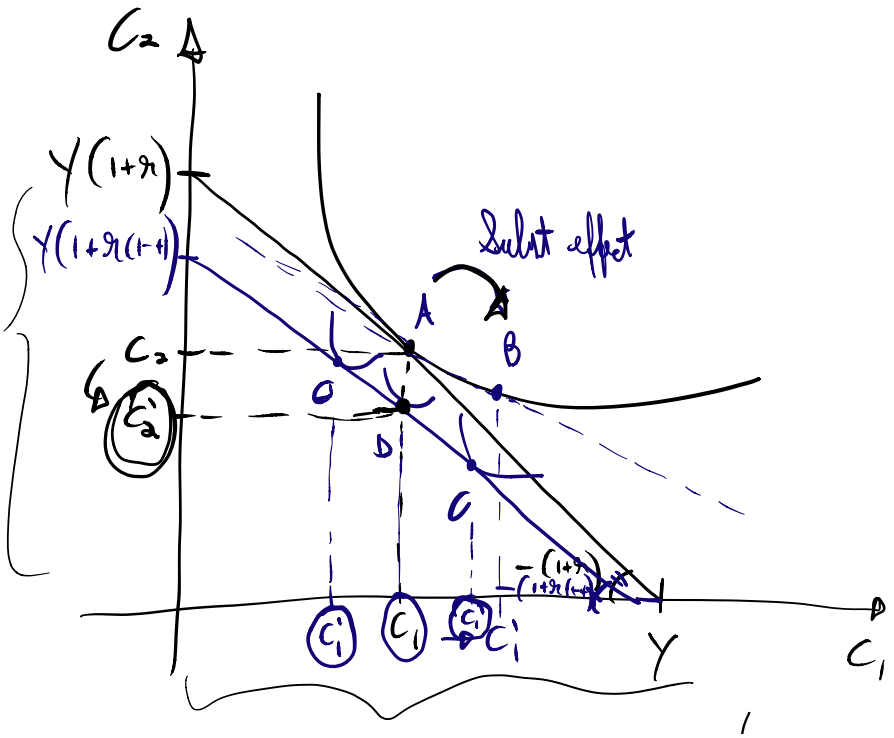
S

$\downarrow t$

$(Y - C_1)(1 + r(1 - t))$

IBC: $Y = C_1 + S = C_1 + \frac{C_2}{1 + r}$

IBC with capital/savings tax: $Y = C_1 + S = C_1 + \frac{C_2}{1 + r(1-t)}$



$$U(C_1, C_2) = \ln C_1 + \ln C_2$$

$Y = 100$, $r = 0.1$

$$C_2 = \underbrace{(100 - C_1)}_{S}(1 + 0.1)$$

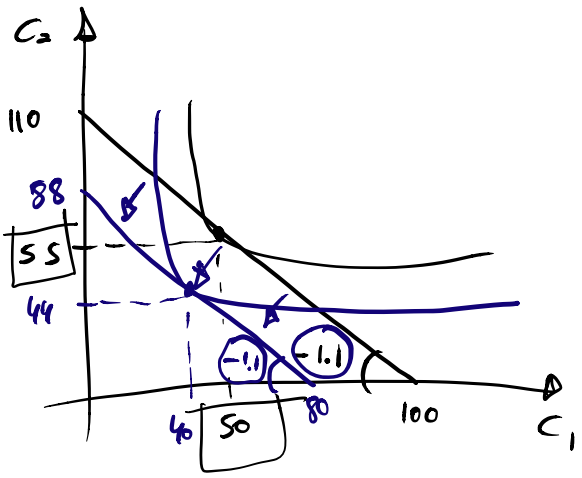
$\Rightarrow S = 100 - C_1 = 50$

$\Rightarrow C_2 = 50(1.1) = 55$

$$\max_{C_1} \ln C_1 + \ln\big((100 - C_1)(1.1)\big) \equiv \max_{C_1} \ln C_1 + \ln(110 - 1.1C_1)$$

$$[C_1]: \frac{1}{C_1} - \frac{1.1}{110 - 1.1C_1} = 0 \Rightarrow 1.1C_1 = 110 - 1.1C_1$$

$$2.2C_1 = 110 \Rightarrow C_1 = 50$$



b) $Y = 100 \Rightarrow Y' = 100\left(1 - \underbrace{t}_{=0.2}\right) = \boxed{80}$

$$C_2 = \underbrace{(80 - C_1)}_{S}\underbrace{(1+r)}_{=1.1} = 88 - 1.1C_1 \Rightarrow C_2 = 88 - 1.1(40)$$

$$C_2 = 88 - 44 = \boxed{44}$$

$S = 80 - C_1$

$= 80 - 40 = \boxed{40}$

$$\max_{C_1} \ln C_1 + \ln\left((88 - 1.1C_1)\right)$$

$$[C_1]: \frac{1}{C_1} = \frac{1.1}{88 - 1.1C_1} \Rightarrow 88 - 1.1C_1 = 1.1C_1$$

$$2.2C_1 = 88$$

$$\boxed{C_1 = 40}$$

c) $C_2 = (100 - C_1)(1+r) - \underbrace{0.2}_{t}\left(\underbrace{r}_{0.1}(100 - C_1)\right)$

$C_2 = (100 - C_1)(1 + r(1-t))$ (after the interest rate)

$C_2 = (100 - C_1)(1 + 0.1(0.8)) = (100 - C_1)(1.08)$

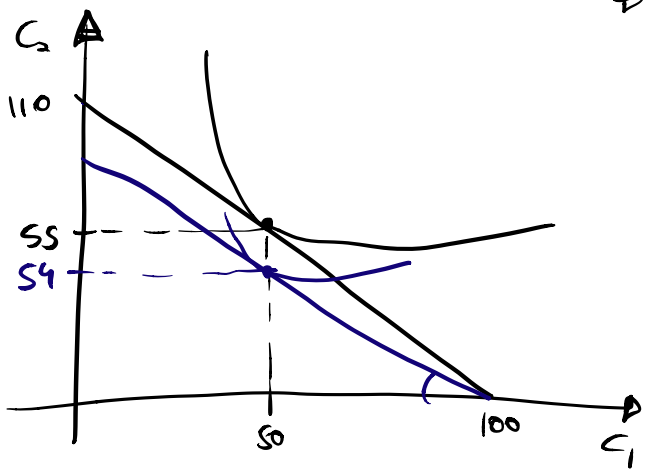
$C_2 = 108 - 1.08C_1 \Rightarrow C_2 = 108 - 1.08(50) = 108 - 54 = \boxed{54}$

$$\max_{C_1} \ln C_1 + \ln\left(108 - 1.08C_1\right)$$

$$[C_1]: \quad \frac{1}{C_1} = \frac{1.08}{108 - 1.08C_1} \quad \Rightarrow \quad 108 - 1.08C_1 = 1.08C_1$$

$$\Rightarrow \quad 2.16\,C_1 = 108$$

$$\Rightarrow \quad \boxed{C_1 = 50}$$



a) $$(1+r)C_1 + \frac{(1+r)C_2}{1.1} = \boxed{100} \qquad (I)$$

Income tax: $C_2 = (80 - C_1)\,1.1$

$$C_1 + \frac{C_2}{1.1} = 80 = 100(1 - 0.2)$$

$$C_1 + \frac{C_2}{1.1} = 100(0.8)$$

$$1.25 = \frac{1\,C_1}{0.8} + \frac{C_2 \cdot 1}{1.1(0.8)} = \boxed{100} \qquad (II)$$

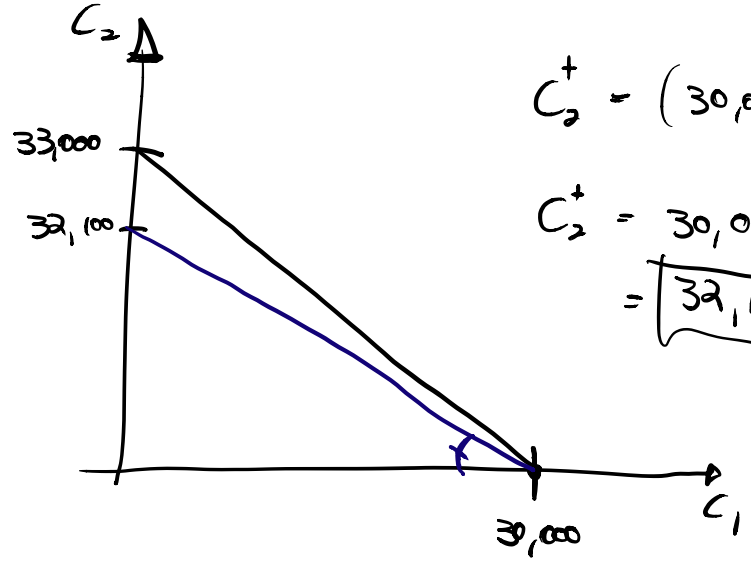
($1 = 1.25$)

(I) + (II) :

$$(1+r)C_1 + (1+r)\frac{C_2}{1.1} = 1.25C_1 + \frac{1.25C_2}{1.1}$$

$$\Rightarrow \boxed{r = 0.25 = 25\%}$$

2)



$$C_2^0 = (30{,}000 - 0)\underbrace{(1+r)}_{1.1} = 33{,}000$$

$$C_2^+ = (30{,}000)\underbrace{(1+r}_{0.1}\underbrace{(1-t)}_{0.7})$$

$$C_2^+ = 30{,}000(1.07)$$

$$= \boxed{32{,}100}$$