Financial Econometrics Econ 40357
Value at Risk (VaR)

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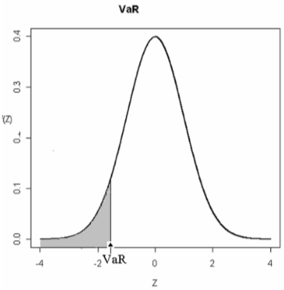
VaR

- Say you are in charge of a \$1M a portfolio. The x% VaR is the amount of money you stand to lose with probability 0.x, within a certain time horizon. (The 10% VaR is the money you lose with probability 0.10).
- VaR is an upper bound on what you can lose, because you could lose it all.
- Who uses VaR?
  - External (e.g., regulators)–determine minimum required capital reserves for banks and other financial institutions.
  - Internal –control the risk undertaken by traders.
    (e.g., the whale: Trader Bruno Iksil, nicknamed the London Whale, accumulated outsized CDS positions in the market, generated trading loss of US \$2 billion for JP Morgan Chase)

Normal VaR

- Assume conditional normality. All we need is the conditional mean and conditional variance of the portfolio rate of return.



- $\Delta W_{t+1} = r_{t+1} W_t$, where r_{t+1} is the portfolio's rate of return from t to $t+1$, and $W_t = \$1M$.
- The 10% VaR is the answer to the question: What is

$$\text{Prob}_t \left(\Delta W_{t+1} = r_{t+1} W_t < -\$100K \right) = \text{Prob}_t \left(r_{t+1} < -0.10 \right)$$

  We want to find the 10% quantile of the return on the asset we've invested in r_t.
- Strategy is to map the problem into the standard normal variate.

The Conditional Normal VaR

- Normal probability density function (pdf). Let $\mu = E(r), \sigma^2 = Var(r)$.

$$f(r) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(r-\mu)^2}$$

  Once you know μ and σ, you know everything about the distribution. i.e., you can plot it exactly. μ is the mean, which sets location, and σ is the standard deviation, which modulates the scale.

- Conditional normal pdf. Let $\mu_t = E_t(r_{t+1}), \sigma_t^2 = Var_t(r_{t+1})$.

$$f(r_{t+1}|I_t) = \frac{1}{\sigma_t\sqrt{2\pi}} e^{-\frac{1}{2\sigma_t^2}(r_{t+1}-\mu_t)^2}$$

The Conditional Normal VaR

- Tabulations are for standardized normal random variable, $z_{t+1} \sim N(0,1)$,

$$z_{t+1} = \frac{r_{t+1} - \mu_t}{\sigma_t} \tag{1}$$

$$\mu_t = E_t(r_{t+1}) \tag{2}$$

$$\sigma_t = \sqrt{\text{Var}_t(r_{t+1})} \tag{3}$$

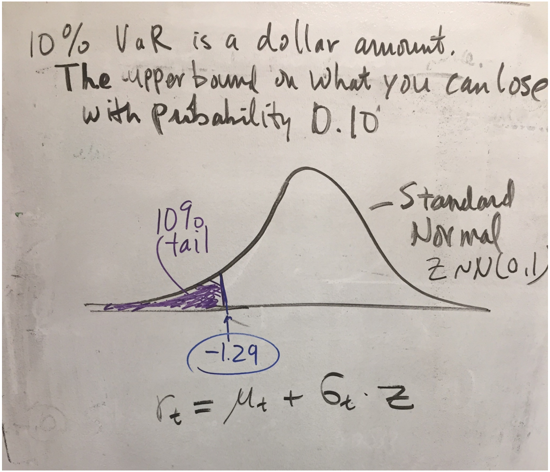
- Express returns in terms of location and scale of $N(0,1)$

$$r_{t+1} = \mu_t + \sigma_t z_{t+1} \tag{4}$$

Quantiles of r_{t+1} now expressed in terms of μ_t, σ_t, and z_{t+1}.

The Conditional Normal VaR

Assume we know μ_t and σ_t (we'll estimate them with GARCH).



The Conditional Normal VaR

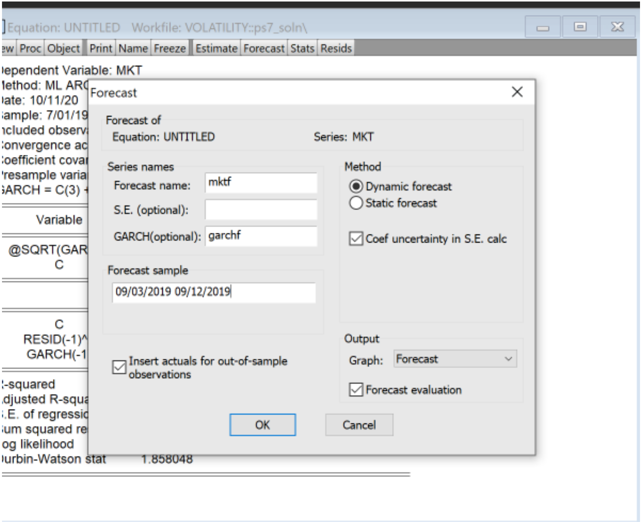
Say we've invested \$1m of our client's money in the market.
Daily market return (from Ken French). GARCH(1,1)-M model

Dependent Variable: MKT
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 10/11/20 Time: 13:59
Sample: 7/01/1926 9/03/2019
Included observations: 24560
Convergence achieved after 24 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)ˆ2 + C(5)*GARCH(-1)

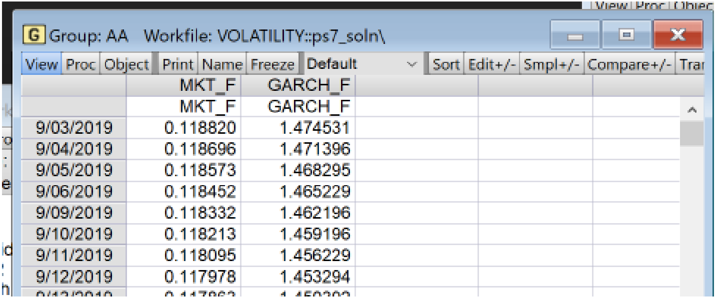
Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.095860	0.016710	5.736624	0.0000
C	0.002417	0.011850	0.203949	0.8384
Variance Equation				
C	0.013027	0.000547	23.82356	0.0000
RESID(-1)ˆ2	0.103378	0.002077	49.76361	0.0000
GARCH(-1)	0.885661	0.002403	368.5156	0.0000

R-squared	−0.003614	Mean dependent var	0.041567
Adjusted R-squared	−0.003655	S.D. dependent var	1.062239
S.E. of regression	1.064179	Akaike info criterion	2.440337
Sum squared resid	27811.37	Schwarz criterion	2.441988
Log likelihood	−29962.34	Hannan-Quinn criter.	2.440872
Durbin-Watson stat	1.858048		

Forecast mean and variance



Forecast mean and variance



	MKT_F	GARCH_F
	MKT_F	GARCH_F
9/03/2019	0.118820	1.474531
9/04/2019	0.118696	1.471396
9/05/2019	0.118573	1.468295
9/06/2019	0.118452	1.465229
9/09/2019	0.118332	1.462196
9/10/2019	0.118213	1.459196
9/11/2019	0.118095	1.456229
9/12/2019	0.117978	1.453294

Note: Divide returns by 100 because Ken French states daily returns in percent.

VaR for 9/4/2019

- $\mu_t = 0.001186$, $\sigma_t = \sqrt{(1.474)}/100 = 0.0121$
- 10% quantile for z, the standard normal r.v. is $z^* = -1.29$
- $r^* = \mu_t + \sigma_t z^*$
- $r^* = 0.001186 + 0.0121(-1.29) = -0.0144.$
- VaR = $r^* W_t = -0.0144(\$1M) = -\$14,400.$

VaR for the next 2 days

Use forecasted values for μ_{t+1}, σ_{t+1}

$$\begin{aligned}
z^* &= -1.29 \\
r_t^* &= \mu_t + \sigma_t z^* \\
r_{t+1}^* &= \mu_{t+1} + \sigma_{t+1} z^* \\
r_t^* + r_{t+1}^* &= \mu_t + \mu_{t+1} + (\sigma_t + \sigma_{t+1})\, z^* \\
&= \frac{(0.118696 + 0.118573)}{100} - \frac{1.29(\sqrt{1.471396} + \sqrt{1.468295}}{100} = -0.028906 \\
VaR &= (\$1M)(r_t^* + r_{t+1}^*) = -28,906
\end{aligned}$$