Public Economics (ECON 131)
Section #7: Capital Income and Savings Taxation

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Contents

Page

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1 Capital Income and Savings Taxation

1.1 Key concepts

- Intertemporal choice model: The choice individuals make about how to allocate their consumption over time.

- Savings: The excess of current income over current consumption.

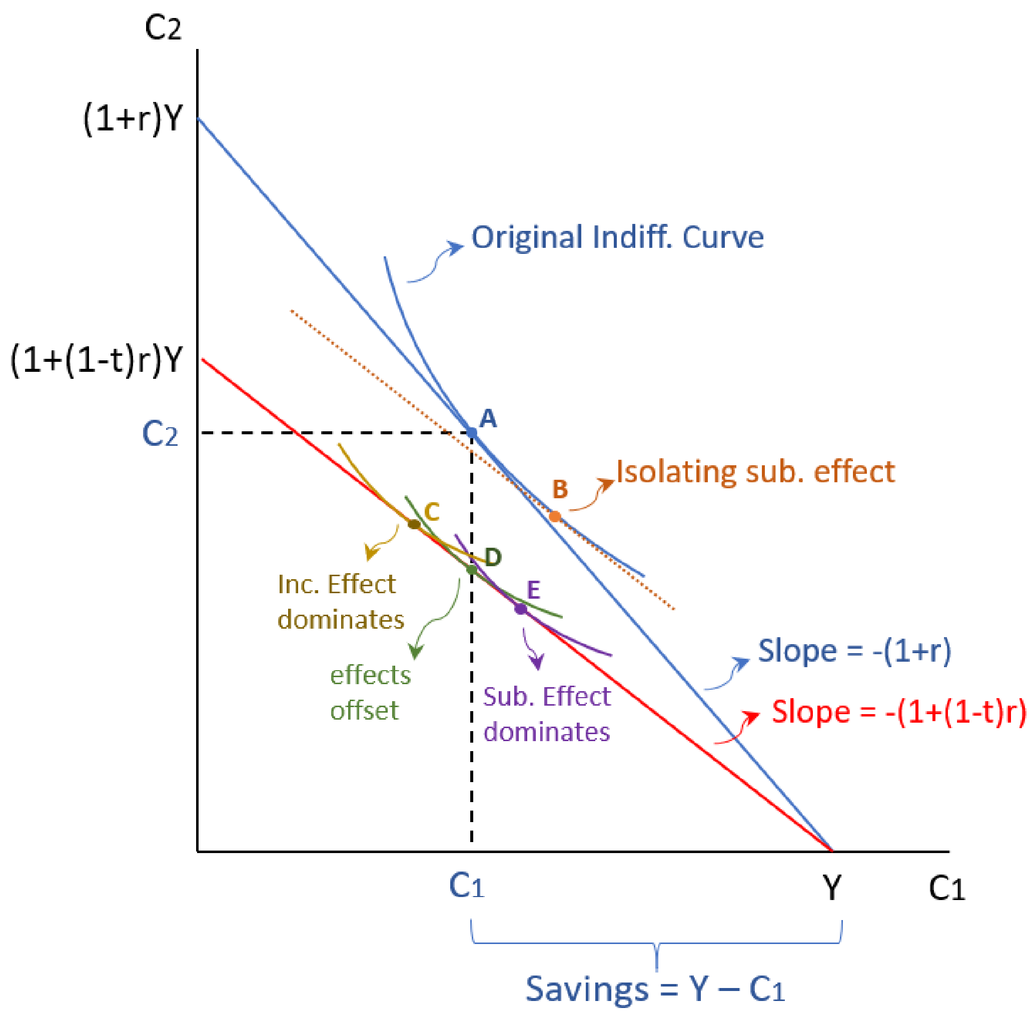
- Intertemporal budget constraint (IBC): The measure of the rate at which individuals can trade off consumption in one period for consumption in another period. Note, the opportunity cost of first-period consumption is the interest income not earned on savings for second-period.

- We find the optimal consumption level at each time by solving a lifetime utility maximization problem:

- Using the information from the IBC we can (1) graph the opportunity set (2) show how a tax on savings modifies the intertemporal consumption choice, and (3) decompose the substitution and income effects.

IBC is given by: $Y = C_1 + S = C_1 + \frac{C_2}{1+r}$

IBC with capital income/savings tax is given by $Y = C_1 + S = C_1 + \frac{C_2}{1+(1-\tau)r}$



To isolate the substitution effect, move along the original indifference curve from A to B, the point where the indifference curve is parallel to the *new* budget constraint. The vertical difference from A to B is the substitution effect on C_2, while the horizontal distance from A to B is the substitution effect on C_1.
The total effect is the movement from A to the new equilibrium on the new budget line (which could be point C, D, or E). The vertical change from A is the total effect on C_2, while the horizontal change from A is the total effect on C_1.
The income effect is the net of the total effect minus the substitution effect. So if point C is the new equilibrium, the income effect is greater than the substitution effect for C_1. If point

E is the new equilibrium, the substitution effect is greater than the income effect for C_1. If the two effects are equal for C_1, then the old and new C_1 will be the same, and point D will be the new equilibrium.

1.2 Practice problem

1.2.1 Gruber, Ch. 22, Q.13

Consider a model in which individuals live for two periods and have utility functions of the form $U(C_1, C_2) = \ln C_1 + \ln C_2$. They earn income of \$100 in the first period and save S to finance consumption in the second period. The interest rate, r, is 10%.

(a) Set up the individual's lifetime utility maximization problem. Solve for the optimal C_1, C_2, and S. (Hint: Rewrite C_2 in terms of income, C_1, and r.) Draw a graph showing the opportunity set.

(b) The government imposes a 20% tax on labor income. Solve for the new optimal levels of C_1, C_2, and S. Explain any differences between the new level of savings and the level in part (a), paying attention to any income and substitution effects.

(c) Instead of the labor income tax, the government imposes a 20% tax on interest income. Solve for the new optimal levels of C_1, C_2, and S. (Hint: What is the new after-tax interest rate?) Explain any differences between the new level of savings and the level in (a), paying attention to any income and substitution effects.

(d) Returning to the labor income tax in part (a): What consumption tax rate would result in the same effects as the 20% labor income tax rate?

Solutions:

(a) Consumption in the second period is savings from the first period plus interest.

Savings is just income from the first period minus consumption during the first period:

$$C_2 = (100 - C_1)(1 + 0.1)$$

The utility maximization problem is $\max \ln C_1 + \ln C_2$ subject to the budget constraint.

When the budget constraint is incorporated into the expression for C_2, as shown, the maximization problem is

$$\max \ln C_1 + \ln((100 - C_1)(1.1)) = \max \ln C_1 + \ln(110 - 1.1C_1).$$

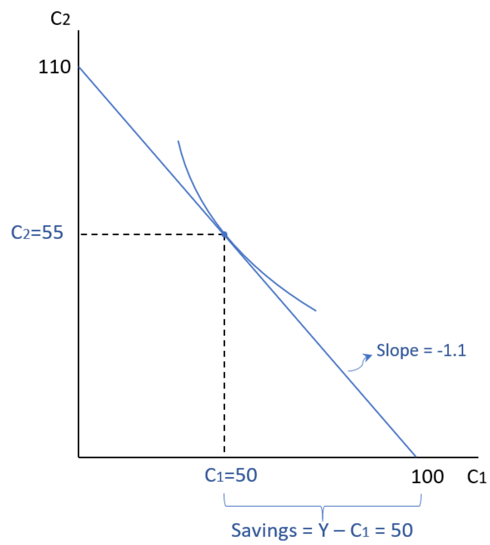
Solving, the first-order condition is

$$\frac{1}{C_1} = \frac{1.1}{(110 - 1.1C_1)} \quad \text{or} \quad 110 - 1.1C_1 = 1.1C_1$$

Using the first-order condition, we get the following:

$$C_1 = 110/2.2 = 50.$$
$$C_1 = 50, \text{ so savings is } 100 - 50 = 50.$$
$$C_2 = S(1 + r) = 50(1.1) = 55.$$

Graphically:



(b) The 20% tax is imposed on the entire \$100 earned in the first period.

After-tax income is $\$100(1 - tax)$, but since it is only a tax on labor income, interest earned is not subject to additional tax.

The new optimization problem is

$$\max \ln C_1 + \ln(80 - C_1)(1.1) = \max \ln C_1 + \ln(88 - 1.1C_1)$$

The first-order condition is $1/C_1 = 1.1/(88 - 1.1C_1)$

Using the first-order condition, we get

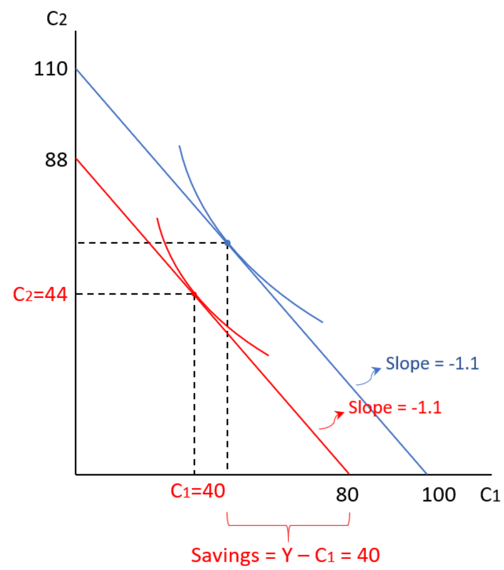
$$C_1 = 40$$
$$S = 80 - 40 = 40$$
$$C_2 = S(1 + r) = 40(1.1) = 44$$

Income and substitution effects

- The tax reduced current consumption and savings by reducing income; this is an income effect.

- Because it is a tax only on labor income, not on interest earned, the tax represents a parallel shift of the budget constraint, a pure income effect.
- The relative prices of C_1 and C_2 did not change, so there is no substitution effect.

Graphically:



(c) A tax on interest rather than on labor income changes computation of C_2 to include a tax of 20% on interest earned on savings:

$$C_2 = (100 - C_1)(1 + r) - 0.2(r(100 - C_1))$$

The first component of this expression is just savings plus interest; the second component is 20% times the interest earned. This is equivalent to $C_2 = (100 - C_1)(1 + .8r)$ or $(100 - C_1)(1.08)$

The maximization problem here is

$$\max \ln C_1 + \ln(108 - 1.08C_1)$$

The first-order condition is $1/C_1 = 1.08/(108 - 1.08C_1)$

Using the first-order condition, we get

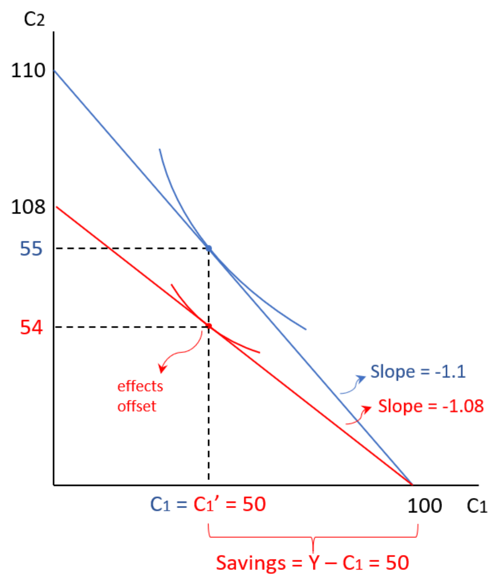
$$C_1 = 108/2.16 = 50.$$
$$S = 100 - 50 = 50.$$
$$C_2 = 50(1.08) = 54$$

Income and substitution effects

- In this case, the lower return to savings (because interest is taxed) changed the relative prices of C_1 and C_2

- That is, sacrificing C_1 yields less increase in C_2 than before the tax.
- However, this substitution effect (which would be expected to decrease savings) is exactly offset by the income effect.
- The income effect: with the lower rate of return, the individual is worse off in period 2 for any given level of savings, leading him to reduce his consumption in period 1, thereby increasing saving.

Graphically:



(d) Since only relative prices matter, a consumption tax which has the same effects as a labor income tax must not change relative prices.

This implies that the consumption tax must be uniform (the same across both goods, since otherwise the relative prices between goods would change) and be set so that the budget constraint is unchanged.

Hence, we must find τ such that

$$(1+\tau)C_1 + (1+\tau)\frac{1}{1.1}C_2 = 100$$

is the same as the budget constraint in part (b),

$$\begin{aligned} C_2 &= (100 \cdot (1 - .2) - C_1)(1 + 0.1) \\ C_1 + \frac{1}{1.1}C_2 &= 100 \cdot (1 - .2) \\ \frac{1}{0.8}C_1 + \frac{1}{0.8}\frac{1}{1.1}C_2 &= 100 \\ 1.25C_1 + 1.25\frac{1}{1.1}C_2 &= 100 \end{aligned}$$

This means that $(1 + \tau) = 1.25$, i.e. $\tau = .25$, so a 25% consumption tax is equivalent to the 20% labor income tax.

2 Additional problems for practice

2.1 Gruber Ch. 22, Q.1

Suppose that a person lives for two periods, earning \$30,000 in income in period 1, which she consumes or saves for period 2. What is saved earns interest of 10% per year.

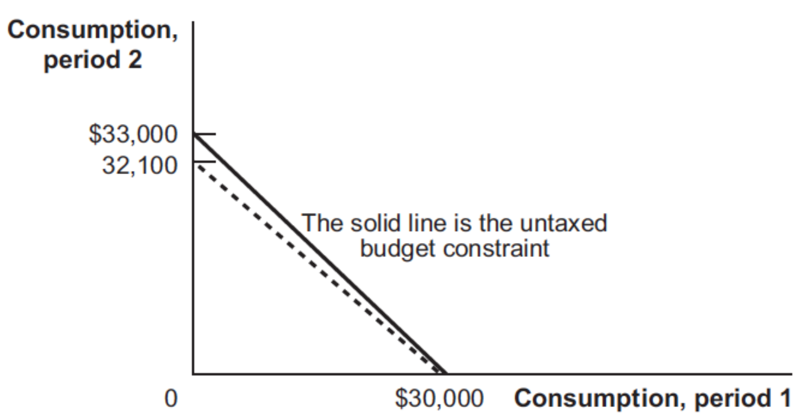
(a) Draw that person's intertemporal budget constraint.

(b) Draw that person's intertemporal budget constraint if the government taxes interest at the rate of 30%.

Solutions:

(a) The x-axis intercept is consumption out of the entire earnings of period 1: \$30,000.

The y-axis intercept is income available in period 2 if the entire period 1 earnings were saved, earning interest at the rate of 10%. This amount is 30,000(1 + r) = \$33,000.



(b) When interest is taxed at 30%, the y-axis intercept becomes $30,000(1 + 10\%[1 - 30\%])$, or $30,000 \cdot 1.07 = 32,100$. This is shown as a dashed line in the figure in a.

2.2 Gruber Ch. 22, Q.2

Suppose that the government increases its tax rate on interest earned. Afterward, savings increase. Which effect dominates, the income effect or the substitution effect? Explain.

Solutions:

The subsititution effect of an increase in the tax on interest earned makes savings relatively less attractive to consuming in period 1. If the exercise says that savings increase after the taxes increase it must be the case that the income effect goes in the opposite direction of the substitution effect, and the income dominates the substitution effect.