Money Demand

ECON 40364: Monetary Theory & Policy

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Readings

- Mishkin Ch. 19
- Friedman, Ch. 2 (section “The Demand for Money” through the end of the chapter)

Classical Monetary Theory

- We have now defined what money is and how the supply of money is set
- What determines the demand for money?
- How do the demand and supply of money determine the price level, interest rates, and inflation?
- We will focus on a framework in which money is neutral and the classical dichotomy holds: real variables (such as output and the real interest rate) are determined independently of nominal variables like money
- We can think of such a world as characterizing the "medium" or "long" runs (periods of time measured in several years)
- We will soon discuss the "short run" when money is not neutral

Velocity and the Equation of Exchange

- Let Y_t denote real output in period t, which we can take to be exogenous with respect to the money supply
- P_t is the dollar price of output, so $P_t Y_t$ is the dollar value of output (i.e. nominal GDP)
- $\frac{1}{P_t}$ is the "price" of money measured in terms of goods
- Define velocity as as the average number of times per year that the typical unit of money, M_t, is spent on goods and serves. Denote by V_t
- The "equation of exchange" or "quantity equation" is:

$$M_t V_t = P_t Y_t$$

- This equation is an identity and defines velocity as the ratio of nominal GDP to the money supply

From Equation of Exchange to Quantity Theory

- The quantity equation can be interpreted as a theory of money demand by making assumptions about velocity
- Can write:

$$M_t = \frac{1}{V_t} P_t Y_t$$

- Monetarists: velocity is determined primarily by payments technology (e.g. credit cards, ATMs, etc) and is therefore close to constant (or at least changes are low frequency and therefore predictable)
- Let $\kappa = V_t^{-1}$ and treat it as constant. Since money demand, M_t^d, equals money supply, M_t, our money demand function is:

$$M_t^d = \kappa P_t Y_t$$

- Money demand proportional to nominal income; κ does not depend on things like interest rates
- This is called the quantity theory of money

Velocity, Money Demand, and the Quantity Theory

- The terms "velocity" and "money demand" are often used interchangeably
- Re-write in terms of real balances (purchasing power of money):

$$\frac{M_t}{P_t} = \frac{1}{V_t} Y_t$$

- The demand for real balance is proportional to the real quantity of exchange
- $\frac{1}{V_t}$ is the demand "shifter" – demand for money goes up, means velocity goes down
- Quantity theory of money: assumes velocity is roughly constant (equivalently, demand for money is stable)

Money and Prices

- Take natural logs of equation of exchange:

$$\ln M_t + \ln V_t = \ln P_t + \ln Y_t$$

- If V_t is constant and Y_t is exogenous with respect to M_t, then:

$$d \ln M_t = d \ln P_t$$

- In other words, a change in the money supply results in a proportional change in the price level (i.e. if the money supply increases by 5 percent, the price level increases by 5 percent)

Money and Inflation

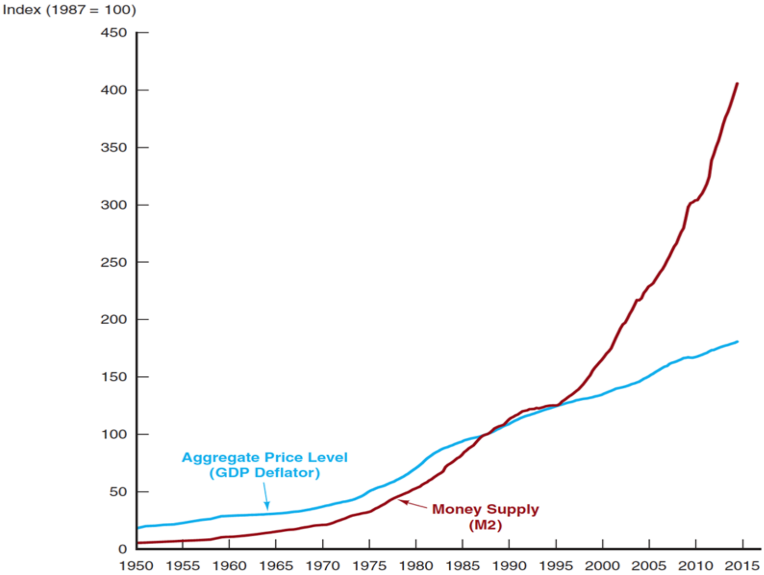
- Since the quantity equation holds in all periods, we can first difference it across time:

$$(\ln M_t - \ln M_{t-1}) + (\ln V_t - \ln V_{t-1}) = (\ln P_t - \ln P_{t-1}) + (\ln Y_t - \ln Y_{t-1})$$

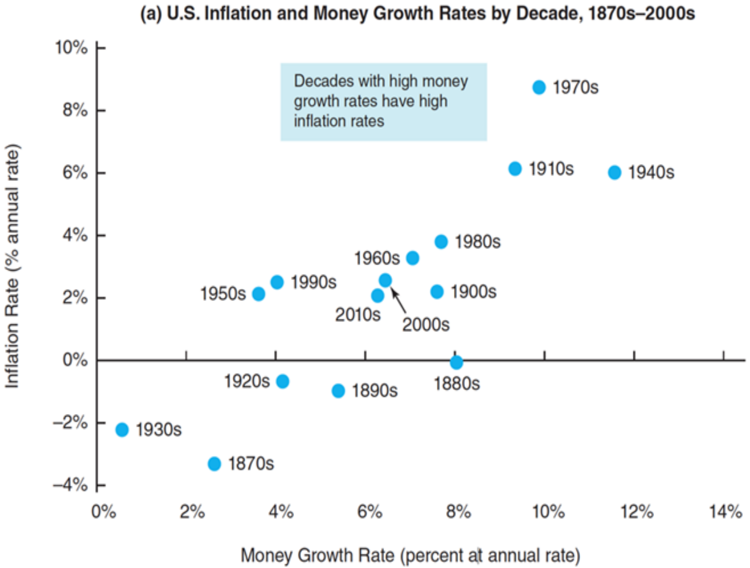
- The first difference of logs across time is approximately the growth rate
- Inflation, π_t, is the growth rate of the price level
- Constant velocity implies:

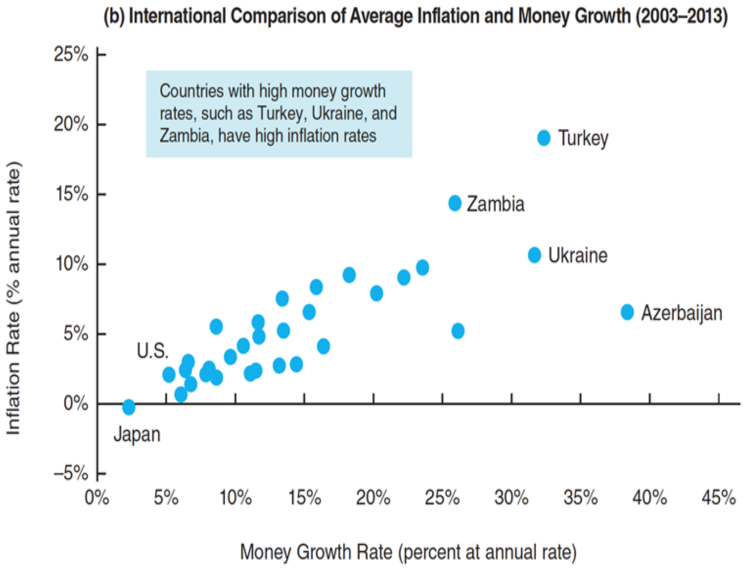
$$\pi_t = g_t^M - g_t^Y$$

- Inflation is the difference between the growth rate of money and the growth rate of output
- If output growth is independent of the money supply, then inflation and money growth ought to be perfectly correlated



Index (1987 = 100)
450
400
350
300
250
200
150
100
50
0
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
Aggregate Price Level
(GDP Deflator)
Money Supply
(M2)


(a) U.S. Inflation and Money Growth Rates by Decade, 1870s–2000s
Decades with high money growth rates have high inflation rates
Inflation Rate (% annual rate)
Money Growth Rate (percent at annual rate)
10%
8%
6%
4%
2%
0%
–2%
–4%
0%
2%
4%
6%
8%
10%
12%
14%
1970s
1910s
1940s
1980s
1960s
1990s
1950s
1900s
2010s
2000s
1920s
1890s
1880s
1930s
1870s


(b) International Comparison of Average Inflation and Money Growth (2003–2013)
Countries with high money growth rates, such as Turkey, Ukraine, and Zambia, have high inflation rates
Turkey
Zambia
Ukraine
Azerbaijan
U.S.
Japan
Inflation Rate (% annual rate)
25%
20%
15%
10%
5%
0%
–5%
0%
5%
10%
15%
20%
25%
30%
35%
40%
45%
Money Growth Rate (percent at annual rate)

Nominal and Real Interest Rates

- The nominal interest rate tells you what percentage of your nominal principal you get back (or have to pay back, in the case of borrowing) in exchange for saving your money. Denote by i_t
- There are many interest rates, differing by time to maturity and risk. Ignore this for now. Think about one period (riskless) interest rates – i.e. between t and $t+1$
- The real interest rate tells you what percentage of a good you get back (or have to pay back, in the case of borrowing) in exchange for saving a good. Denote by r_t
- Putting one good "in the bank" $\Rightarrow$ P_t dollars in bank $\Rightarrow$ $(1+i_t)P_t$ dollars tomorrow $\Rightarrow$ purchases $(1+i_t)\frac{P_t}{P_{t+1}}$ goods tomorrow

The Fisher Relationship

- The relationship between the real and nominal interest rate is then:

$$1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}}$$

- Since the inverse of the ratio of prices across time is the expected gross inflation rate, we have:

$$1 + r_t = \frac{1 + i_t}{1 + \pi^e_{t+1}}$$

- Here π^e_{t+1} is expected inflation between t and $t + 1$
- Approximately:

$$r_t = i_t - \pi^e_{t+1}$$

Classical Dichotomy

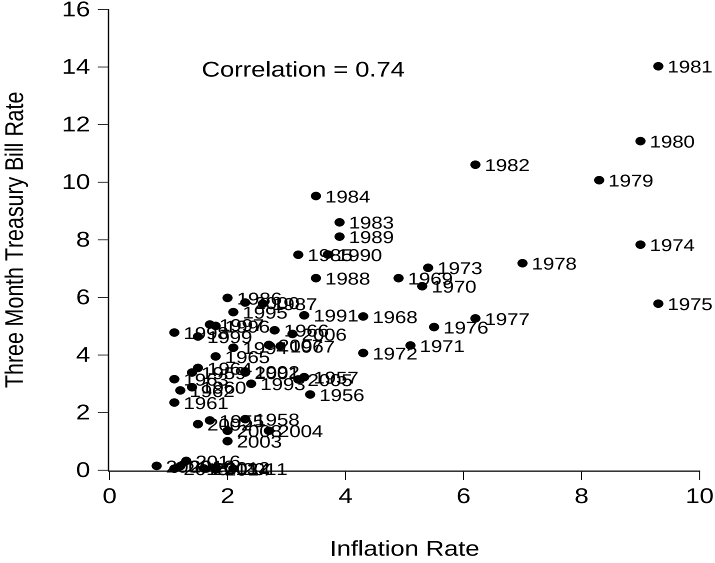
- In the classical dichotomy, r_t is independent of anything nominal
- So i_t moves one-for-one with π^e_{t+1}:

$$i_t = r_t + \pi^e_{t+1}$$

- What drives π^e_{t+1}? Plausible that it's realized inflation (adaptive expectations), so:

$$i_t = r_t + \pi_t$$

- So, there should be a tight connection between inflation and nominal interest rates
- To extent to which quantity theory holds (i.e. inflation driven by money growth), then also a tight connection between money growth and the level of nominal interest rates

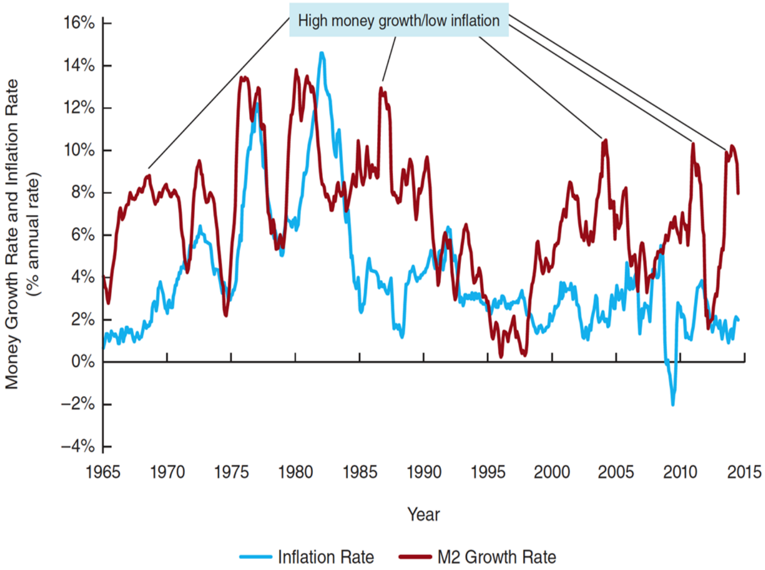

Correlation = 0.74
Three Month Treasury Bill Rate
Inflation Rate
0
2
4
6
8
10
12
14
16
1981
1980
1982
1979
1984
1983
1989
1974
1978
1973
1988
1970
1975
1968
1977
1976
1971
1972
1956
1961
2004
2003

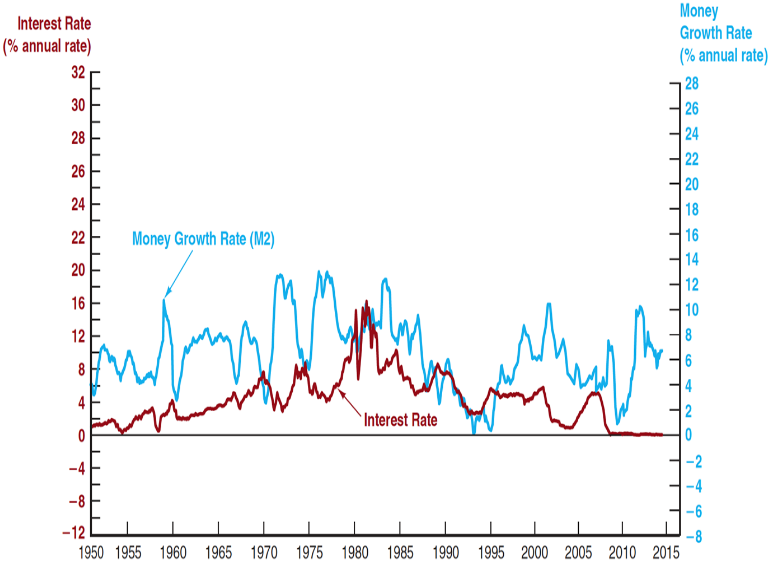
Theoretical Predictions

- The basic quantity theory in which the classical dichotomy holds (real output, real output growth, and the real interest rate independent of nominal things) makes a number of stark predictions
  1. The level of the money supply and the price level are closely linked
  2. The growth rate of the money supply and the inflation rate are closely linked
  3. The inflation rate and the nominal interest rate are closely linked
     - This is not an implication of quantity theory per se – follows from Fisher relationship plus classical dichotomy / monetary neutrality
     - But quantity theory goes a step further – nominal interest rates linked to money growth

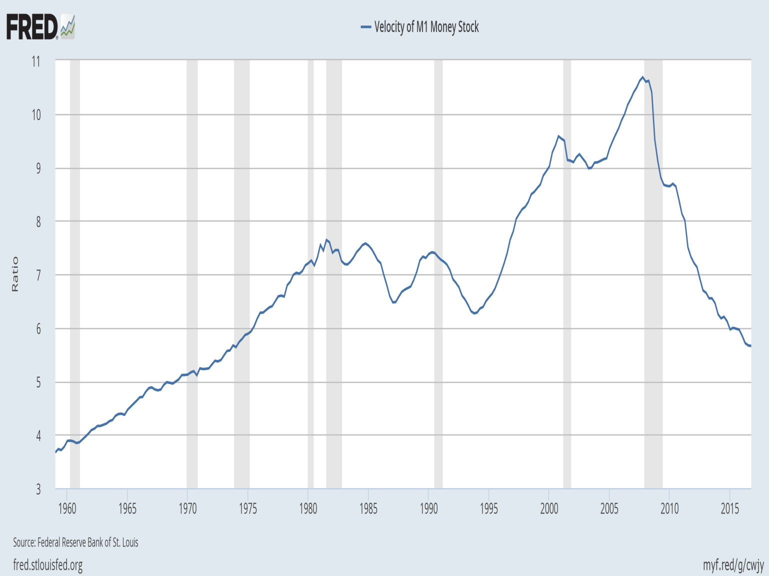
Problems with the Quantity Theory

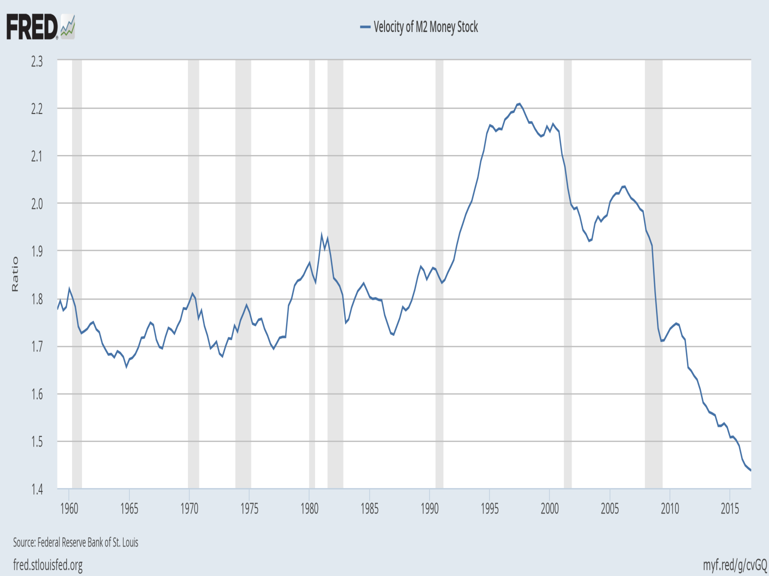
- The quantity theory seems to provide a pretty good theory of inflation and interest rates over the medium to long run as well as in a cross section of countries
- What about the short run?
- Problems with the quantity theory:
  - The shorter term relationships between money growth and both inflation and nominal interest rates are weak
  - Velocity is not constant and has become harder to predict, particularly since the early 1980s


High money growth/low inflation
Money Growth Rate and Inflation Rate
(% annual rate)
16%
14%
12%
10%
8%
6%
4%
2%
0%
−2%
−4%
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
Year
Inflation Rate
M2 Growth Rate



Interest Rate
(% annual rate)
Money
Growth Rate
(% annual rate)
Money Growth Rate (M2)
Interest Rate
32
30
28
26
24
22
20
16
12
8
4
0
−4
−8
−12
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
−2
−4
−6
−8
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015


FRED
Velocity of M1 Money Stock
Ratio
11
10
9
8
7
6
5
4
3
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
Source: Federal Reserve Bank of St. Louis
fred.stlouisfed.org
myf.red/g/cwjy



FRED
Velocity of M2 Money Stock
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
Ratio
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
Source: Federal Reserve Bank of St. Louis
fred.stlouisfed.org
myf.red/g/cvGQ


FRED
Velocity of M2 Money Stock
1.5
1.4
1.3
1.2
1.1
1.0
Ratio
Q3 2016
Q1 2017
Q3 2017
Q1 2018
Q3 2018
Q1 2019
Q3 2019
Q1 2020
Source: Federal Reserve Bank of St. Louis
fred.stlouisfed.org

Moving Beyond the Quantity Theory

- The key assumption in the quantity theory is that the demand for money (i.e. velocity) is stable (or at least predictable) – you hold money to buy stuff, and how much money you need is proportional to how much you buy
- Liquidity preference theory of money demand: money competes with other assets as a store of value. Money is more liquid (can be used in exchange), but how much you want to hold depends on return on other assets
- Demand for real money balances, $m_t = \frac{M_t}{P_t}$, is an increasing function of output, Y_t, but a decreasing function of the nominal interest rate, i_t:

$$\frac{M_t}{P_t} = L(\underset{-}{i_t}, \underset{+}{Y_t})$$

- But then velocity:

$$V_t = \frac{P_t Y_t}{M_t} = \frac{Y_t}{L(i_t, Y_t)}$$

Two Simple Models

- We can generate a liquidity preference theory of money demand via two different setups:
  1. Baumol-Tobin: this is an intratemporal portfolio allocation problem. Given desired spending, how to allocate wealth between money and bonds (which pay interest)
  2. Money in the Utility Function (MIU): this is an intertemporal problem with both a consumption-saving decision and a portfolio allocation problem
- Both generate something like: $m_t = L(i_t, Y_t)$

Baumol-Tobin

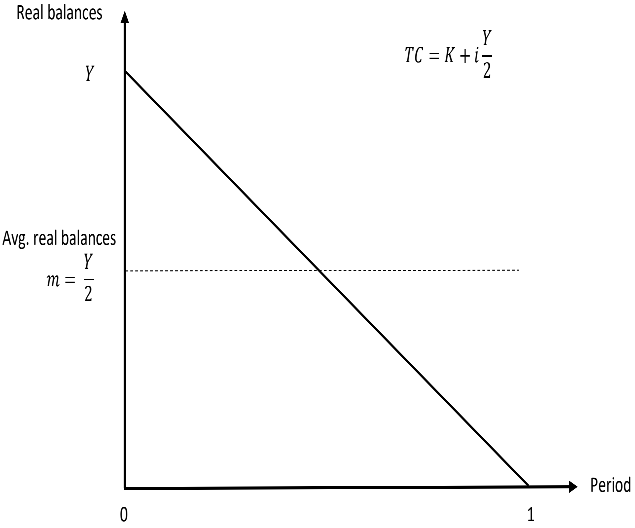
- You need to spend Y over the course of a period (say, a year). This is given.
- You have sufficient wealth to do this
- Average holdings of illiquid wealth earn nominal return i
- Need to determine how much real money balances to hold to hold, which earns nothing. Have to support transactions with real balances
- You can withdraw money as often as you please, but each withdrawal incurs a "shoeleather cost" of $K \geq 0$

One "Trip to the Bank"

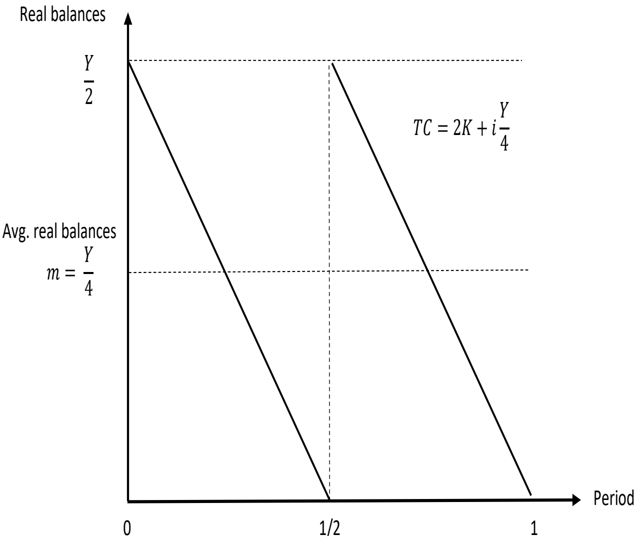
- Suppose you withdraw all the funds you need at the beginning of a period. So you make one trip
- Then your average real balance holdings over the period are $Y/2$
- You forego $iY/2$ in interest by holding money instead of bonds
- And pay a shoeleather cost of K
- Total cost is:

$$TC = K + \frac{iY}{2}$$

One Trip

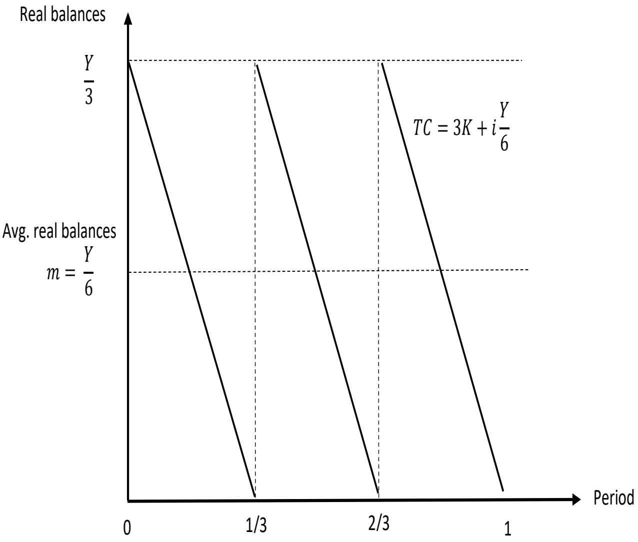

Real balances
$TC = K + i\frac{Y}{2}$
Y
Avg. real balances
$m = \frac{Y}{2}$
Period
0
1

Two Trips



Real balances
$\frac{Y}{2}$
$TC = 2K + i\frac{Y}{4}$
Avg. real balances
$m = \frac{Y}{4}$
Period
0
1/2
1

Three Trips



Real balances
$\frac{Y}{3}$
$TC = 3K + i\frac{Y}{6}$
Avg. real balances
$m = \frac{Y}{6}$
Period
0
1/3
2/3
1

General Case

- Total cost as a function of trips, T, is:

$$TC = TK + i\frac{Y}{2T}$$

- Average real balance holdings:

$$m = \frac{Y}{2T}$$

- Re-write total cost in terms of m instead of T:

$$TC = \frac{KY}{2m} + im$$

Money Demand Function

- Use calculus to get first order condition:

$$m = \sqrt{\frac{KY}{2i}}$$

- Or re-arranging:

$$m = \left(\frac{KY}{2}\right)^{\frac{1}{2}} i^{-\frac{1}{2}}$$

- Demand for real balances increasing in Y and decreasing in i

Money in the Utility Function

- Suppose that there is a representative household who receives utility from consuming goods and holding real money balances, $m_t = \frac{M_t}{P_t}$. Flow utility:

$$U\left(C_t, \frac{M_t}{P_t}\right) = \ln C_t + \psi \ln\left(\frac{M_t}{P_t}\right)$$

- Flow budget constraint:

$$P_t C_t + B_t - B_{t-1} + M_t - M_{t-1} \leq P_t Y_t - P_t T_t + i_{t-1} B_{t-1}$$

- B_{t-1} and M_{t-1}: stocks of bonds and money household enters t with
- Both enter as stores of value. Difference being that bonds pay interest

Problem

- Household discounts future utility flows by $\beta \in [0, 1)$
- Both a dynamic aspect to the problem and a portfolio allocation aspect to problem
    1. Dynamic: how much to consume today vs future?
    2. Portfolio allocation: how much to save in money (no interest) vs bonds (interest-bearing)?

Optimality Conditions

- Plugging constraints in and taking derivatives yields:

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} (1 + i_t) \frac{P_t}{P_{t+1}}$$

$$\psi \frac{P_t}{M_t} = \frac{1}{C_t} - \beta \frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}}$$

- First is the standard consumption Euler equation taking Fisher relationship into account
- Second is the portfolio allocation part
- Combine to get:

$$\psi \frac{P_t}{M_t} = \frac{1}{C_t} \left(1 - \frac{1}{1 + i_t} \right)$$

"Closing" Model

- Government sets money supply, taxes, and issues bonds. Does no spending:

$$i_{t-1}B_{t-1} = P_t T_t + B_t - B_{t-1} + M_t - M_{t-1}$$

- Finances interest expenses with (i) taxes, (ii) new debt issuance, (iii) "printing" new money (no banking sector so no distinction between M and MB)
- Market-clearing: households holds all debt and money issued by government
- End up with equilibrium market-clearing condition: $C_t = Y_t$

Money Demand Function

- Making use of market-clearing and combining the FOC yields:

$$\psi m_t^{-1} = \frac{1}{Y_t} \frac{i_t}{1 + i_t}$$

- Re-arranging:

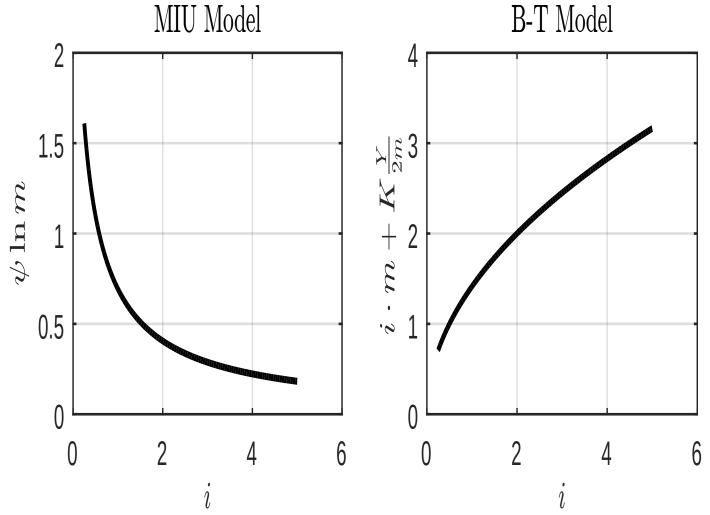
$$m_t = \psi Y_t \frac{1 + i_t}{i_t}$$

- Demand for real balances: (i) increasing in Y_t and ψ (i.e. in the "usefulness" of money, as put by Friedman), (ii) decreasing in i_t (i.e. the "cost" of money, as put by Friedman)
- Zero lower bound: must have $i_t \geq 0$ to get non-negative real balances. At $i_t \to 0$, demand for real balances goes to infinity

Friedman Rule

- Milton Friedman argued that optimal monetary policy in the medium to long run would target a nominal interest rate of zero
- With a positive real rate of interest, this would require deflation
- Basic intuition: a positive nominal interest rates dissuades people from holding money by increasing the opportunity cost of liquidity relative to bonds, whereas the marginal cost of producing (fiat) money is essentially zero
- At a social optimum, want to equate private cost of holding money (interest rate) to the public cost of producing money (zero)
- Holds in both the MIU model ($i = 0$ maximizes utility) and the B-T model ($i = 0$ minimizes the cost of holding money)
- Why don't central banks follow Friedman rule? Because of the zero lower bound and short run stabilization policy
- Does help us understand desire for low interest rates, however

Optimality of $i = 0$



Money and Inflation: The Case of Hyperinflations

- Milton Friedman famously said that "inflation is everywhere and always a monetary phenomenon"
- Simple logic based on the quantity equation. Works pretty well in the medium to long run
- What about extreme situations of inflation, or what are called "hyperinflations"?
- Monetary phenomena triggered by fiscal problems

Hyperinflations

Table 8.1 Hyperinflations in History

Country	Year	Highest Inflation per Month %	Country	Year	Highest Inflation per Month %
Argentina	1989/90	196	Hungary	1945/46	$1.295 * 10^{16}$
Armenia	1993/94	438	Kazakhstan	1994	57
Austria	1921/22	124	Kyrgyzstan	1992	157
Azerbaijan	1991/94	118	Nicaragua	1986/89	127
Belarus	1994	53	Peru	1921/24	114
Bolivia	1984/86	120	Poland	1989/90	188
Brazil	1989/93	84	Poland	1992/94	77
Bulgaria	1997	242	Serbia	1922/24	309,000,000
China	1947/49	4,209	Soviet Union	1945/49	279
Congo (Zaire)	1991/94	225	Taiwan	1995	399
France	1789/96	143	Tajikistan	1993/96	78
Georgia	1993/94	197	Turkmenistan	1992/94	63
Germany	1920/23	29,500	Ukraine	1990	249
Greece	1942/45	11,288	Yugoslavia		59
Hungary	1923/24	82			

SOURCE: Peter Bernholz, *Monetary Regimes and Inflation: History, Economic and Political Relationships* (Edward Elgar Publishing, March 27, 2006).

Hyperinflations Usually a Fiscal Phenomenon

- Most hyperinflations in history are associated with fiscal mischief
- Government's budget constraint:

$$P_t G_t + i_{t-1} B_{G,t-1} = P_t T_t + M_t - M_{t-1} + B_{G,t} - B_{G,t-1}$$

- Here P_t is the nominal price of goods (i.e. the price level), $B_{G,t-1}$ is the stock of debt with which a government enters period t, $B_{G,t}$ is the stock of debt the government takes from t to $t+1$, i_{t-1} is the nominal interest rate on that debt, T_t is tax revenue (real), and M_t is the money supply
- Deficit equals change in money supply plus change in debt:

$$P_t G_t + i_{t-1} B_{G,t-1} - P_t T_t = M_t - M_{t-1} + B_{G,t} - B_{G,t-1}$$

Monetizing the Debt

- If tax revenue doesn't cover expenditure (spending plus interest on debt), then government either has to issue more debt or "print more money"
- In some cases printing more money is explicit, in others implicit
- Monetizing the debt: fiscal authority issues debt to finance deficit, but monetary authority buys the debt by doing open market operations, which creates base money

Application: Seigniorage and the Inflation Tax

- Recall from the government's budget constraint above when talking about hyperinflations that *nominal* revenue from printing money is simply: $M_t - M_{t-1}$
- *Real* revenue from printing money is $\frac{M_t - M_{t-1}}{P_t}$
- We call the real revenue from printing money seigniorage
- This can be written:

$$\text{Seigniorage} = \frac{M_t - M_{t-1}}{P_t}$$

- This can equivalently be written:

$$\text{Seigniorage} = \frac{M_t - M_{t-1}}{M_{t-1}} \frac{M_{t-1}}{M_t} \frac{M_t}{P_t}$$

More Seigniorage

- Define the growth rate of money as:

$$g_t^M = \frac{M_t - M_{t-1}}{M_{t-1}}$$

- Then the expression for seigniorage can be written:

$$\text{Seigniorage} = \frac{g_t^M}{1 + g_t^M} m_t$$

- This is approximately:

$$\text{Seigniorage} = g_t^M m_t$$

- Seigniorage is tax revenue from printing more money – g_t^M is effectively the "tax rate" and m_t is the "tax base"

Seigniorage in the Medium to Long Run

- Drop time subscripts
- Suppose that the real interest rate is constant and invariant to nominal variables (classical dichotomy)
- Fisher relationship:

$$i = r + \pi$$

- Suppose that the inflation rate equals the money growth rate, so:

$$i = r + g^M$$

- If demand for real balances is generically given by: $m = L(i, Y)$, then we can write demand for real balances as:

$$m = L(r + g^M, Y)$$

"Optimal" Inflation Tax

- Suppose that a central bank wants to pick g^M to maximize seigniorage. Problem is:

$$\max_{g^M} \quad g^M L(r + g^M, Y)$$

- Provided money demand is decreasing in nominal interest rate (i.e. $L_i(\cdot) < 0$), then two competing effects of higher g^M:
  1. Tax rate: higher $g^M \Rightarrow$ higher tax rate
  2. Base: higher $g^M \Rightarrow$ lower tax base
- First order condition:

$$g^M = -\frac{L(r + g^M, Y)}{L_i(r + g^M, Y)}$$

- Revenue-maximizing growth rate of money inversely related to interest sensitivity of money demand
- If money demand interest insensitive (e.g. quantity theory), then revenue-maximizing $g^M = \infty$!
- Desire for seigniorage another reason to move away from Friedman rule