Financial Econometrics Econ 40357
Topic 2: Exploratory data analysis . . .

N.C. Mark

University of Notre Dame and NBER

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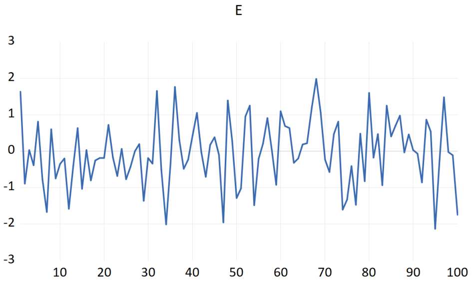
Concepts to cover

1. Time series and stochastic processes.
2. Our ‘models’ of the data generating process
3. Stationarity, and why it’s important
4. Exploratory data analysis. What do we learn?

Stochastic process, time series

- Stochastic $\Leftrightarrow$ random
- A time-series is a sequence of observations over time. We think of them as stochastic processes.
- The simplist stochastic process. $x_t \overset{iid}{\sim} N(\mu, \sigma^2)$



The driftless random walk

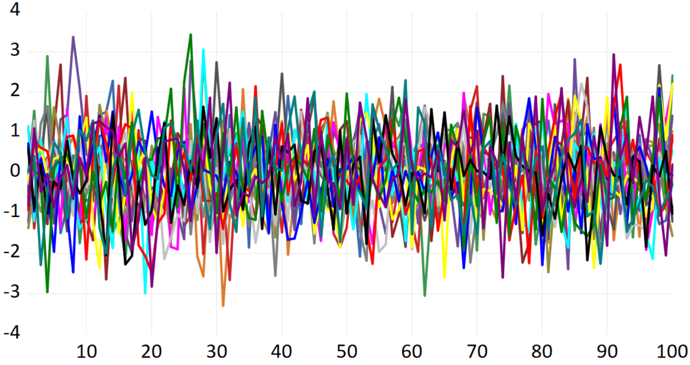
- Let $x_t \overset{iid}{\sim} N(\mu, \sigma^2)$ and $p_t = \ln(P_t)$ be the price of non-dividend paying stock (Netflix, Tesla, and for a long time, Apple).

$$p_t = p_{t-1} + x_t$$

$$\Delta p_t = x_t$$

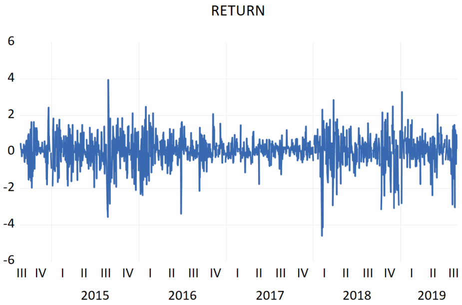
- Return is x_t. Sometimes referred to as **white noise**. A white noise process is i.i.d. (independent, identically distributed).
- This is a **powerful** statement. Independence of returns over time.

20 replications of a white noise process



Stationarity

- **Strict stationarity** says the distribution of X_t is the same for all t. So the distribution of X_t is the same as for X_{t+1}, etc.
- **Covariance stationarity** A **less restrictive** form says the covariance between X_t and X_{t-s} is the same, for all t.
- A time series can violate strict stationarity but still be covariance stationary. E.g., volatility clustering. Comparison of $\text{Cov}(x_t, x_{t-1})$ and $\text{Var}(x_t)$.



What do we mean?

- What do we mean by things like $\mathrm{E}(x_t)$ and $\mathrm{Var}(x_t)$?
- In observational time-series there is only one observation of x_t at t. One observation of x_{96}, where $t = 96$.
- What does the distribution of x_t have to do with (time-series) sample moments? (Sample mean, sample variance, etc.)

Ergodic Theorem

- $$lim_{T\rightarrow\infty}\frac{1}{T}\sum_{t=1}^{T}x_t = E\left(x_t\right)$$
  $$lim_{T\rightarrow\infty}\frac{1}{T}\sum_{t=1}^{T}x_t^2 = E\left(x_t^2\right)$$
  and so on
- If the conditions for the ergodic theorem hold, then we can use time-series sample moments to estimate the theoretical moments.

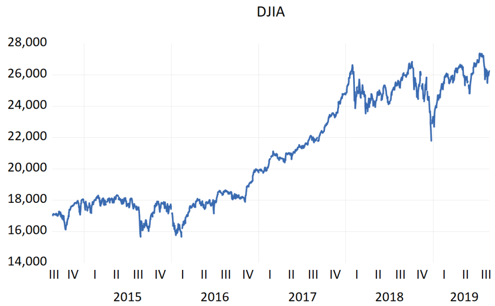
First thing you do

- Plot your data.
- Stare at it.
- Are there mistakes?
- What else are we looking for? **Properties** of the data
  - Are there **trends**? If so, trends need to be eliminated before doing econometric analysis. Why? Things like

    $$lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} x_t$$

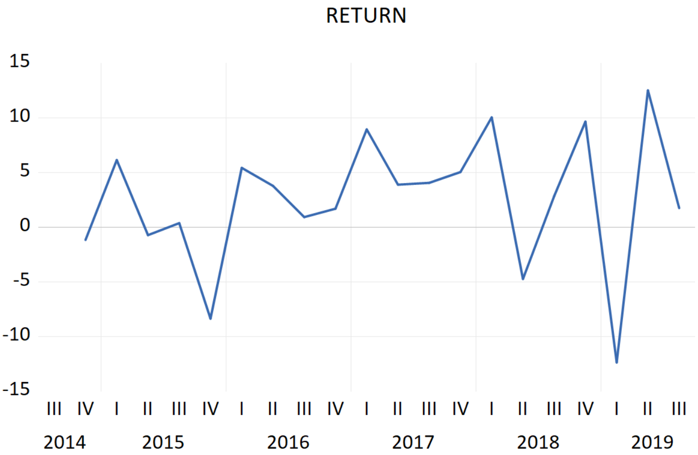
    doesn't exist if there is a trend.
  - Are there structrual **breaks**?
  - Is there **volatility clustering**?

Let’s look at plots of DJIA price and retunrs

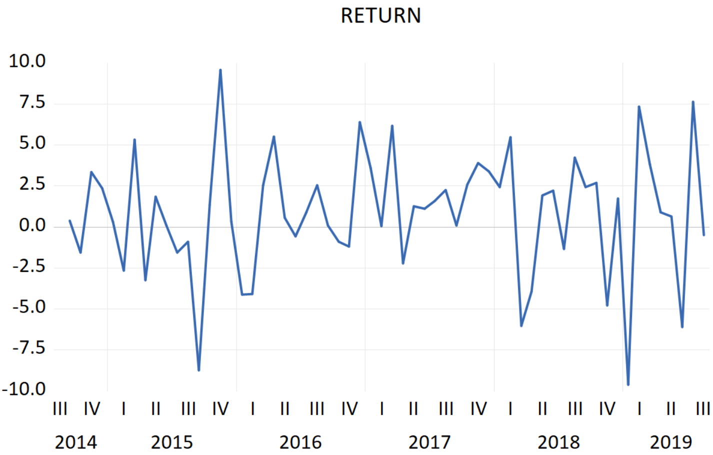


- There is a **trend**. We say these data are **unit-root nonstationary**. Will explain the term **unit-root** later.
- Contrast with non identicalness of distribution, under volatility clustering, also not strictly stationary, could be covariance stationary not unit-root nonstationary.
- To analyze these data, transform the observations to induce stationarity. i.e., Dont look at price P or log price $\ln(P)$, but look at returns $\Delta \ln(P)$ (ignoring dividends here).

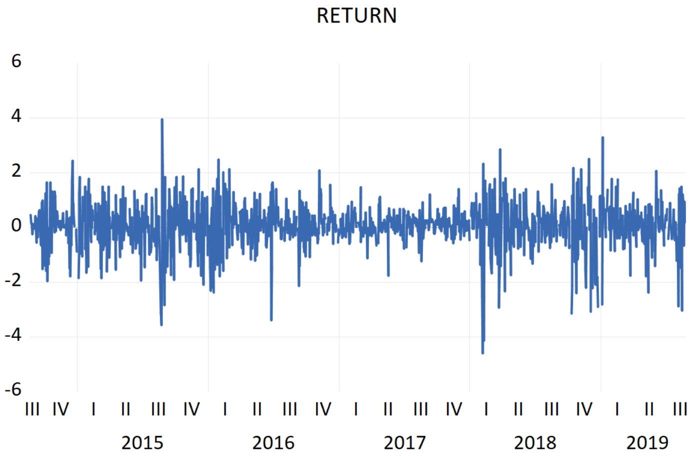
Quarterly returns



Monthy returns



Daily returns



Questions

1. Do monthly DJIA returns look strictly stationary? Covariance stationary?
2. Do daily DJIA returns look strictly stationary? Covariance stationary?

The Normal (Gaussian) benchmark

- We usually take the normal distribution as a benchmark. Why? B/C properties are well understood, the normal is a good model for many natural phenomena, it has good mathematical properties, especially on the **asymptotics**, through the **central limit theorem**.
- We typically focus on symmetry and tail thickness. We know what these are for the normal. Is the data well described thusly, or are there significant deviations?
- First, we must be familiar with the following concepts

Moments of a Distribution

- The k-th theoretical moment of a distribution or of the random variable x)

$$E\left(x^k\right)$$

  The k-th central moment

$$E\left(x-\mu\right)^k$$

  where the first moment is $\mu = E\left(x\right)$.

- Sample moments are the sample counterparts. Let $\{x_t\}_{t=1}^{T}$ be a sequence of time-series observations (e.g., returns).
- Mean and variance. First two moments

$$\mu = E\left(x_t\right); \quad \bar{x}_T = \frac{1}{T}\sum_{t=1}^{T} x_t$$

$$E\left(x_t-\mu\right)^2; \quad \hat{\sigma}_T^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(x_t-\bar{x}_T\right)^2$$

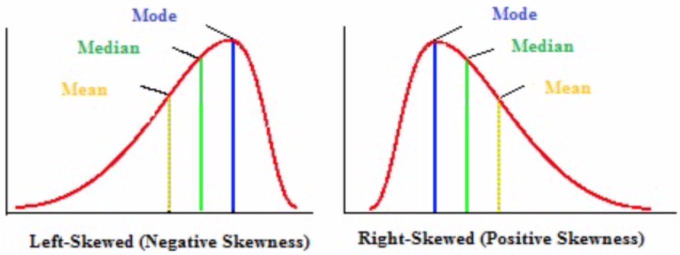
Moments of a distribution

- Third moment for symmetry/asymmetry. Skewness measure

$$\frac{E\left(x_t-\mu\right)^3}{\sigma^3}; \quad sk_T=\frac{\frac{1}{T-1}\sum_{t=1}^{T}\left(x_t-\bar{x}_T\right)^3}{\hat{\sigma}_T^3}$$

- For **normal** distribution, skewness measure is **0**. Skewness is **0** for all symmetric distributions.
- If x_t is a return, might want to know if it has a heavy left tail (propensity to crash) or heavy right tail (propensity to boom).

Skewed Left Skewed Right



Moments of a distribution

- Fourth moment measures tail thickness. The theoretical measure is kurtosis

$$\frac{E\left(X_t-\mu\right)^4}{\sigma^4}; \quad kurt_T = \frac{\frac{1}{T-1}\sum_{t=1}^{T}\left(x_t-\bar{X}_T\right)^4}{\hat{\sigma}_T^4}$$

  For **normal** distribution, kurtosis is **3**.

- Distribution has **excess kurtosis** if the measure **exceeds 3**. These are **fat-tailed** distributions and **peaked**. There is a higher probability of extreme events than predicted by the normal. Called **leptokurtotic. Pay attention** to whether the software computes kurtosis or **excess** kurtosis.

Properties of the normal distribution

- Standard normal. For $-\infty \leq x \leq \infty <$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2} \tag{1}$$

- (General) Normal

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2} \tag{2}$$

1. Distribution is **symmetric** around μ (mean, location)
2. Dispersion regulated by σ (scale). How is σ a measure of scale? If X is household income in dollars, then $100X$ is household income in cents.

$$\sqrt{\mathrm{Var}(100X)} = 10\sqrt{\mathrm{Var}(X)} \tag{3}$$

3. In finance, **standard deviation** $\Leftrightarrow$ **volatility**

Properties of the normal distribution

Tail probabilities converge to 0 at a well defined rate. Loosely speaking normal tail probabilities converge to 0 quickly (even though it's possible to have realizations that are arbitraily large or small).

Conclusion: Assessments of normality involve checking for distributional **symmetry** and appropriate **tail thickness**. How do we do that? Through examination of **sample moments**.

Varying kurtosis

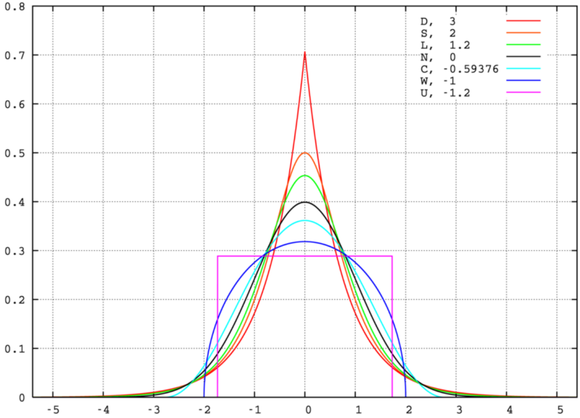


Figure: Distributions with differing kurtosis

Jarque-Bera test for normality

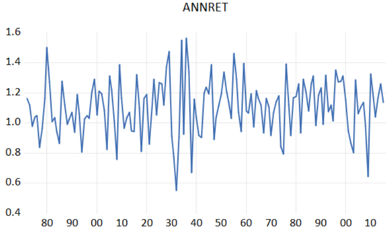
1. The Jarque-Bera statistic measures the difference between skewness and kurtosis in the data and the normal distribution.
2. Let sk_T be sample skewness, and $kurt_T$ be sample kurtosis. Jarque and Bera showed that their statistic JB

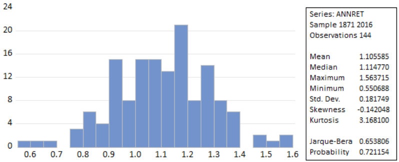
$$\mathrm{JB} = \frac{T}{6}\left(sk_T^2 + \frac{(kurt_T - 3)^2}{4} \right) \sim \chi_2^2$$

   under the **null** hypothesis of normality.
3. Eviews produces JB test and p-values when asking for descriptive statistics.

Returns

S&P Annual Returns





Daily Returns