UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2019
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Problem Set 3
Due at the start of lecture, Thursday, September 26

1. a. Explain in a few sentences (with or without math) what is wrong with the following argument: "In the planner's problem in the Ramsey-Cass-Koopmans model, if capital exceeds the golden-rule level, the value of capital (that is, the amount at the margin that an increase in capital contributes to the planner's objective function) is negative. We can see this from the equation of motion for the costate variable: $\dot{\mu}(t) = \mu(t)[f'(k(t)) - (n+g)] + \beta\mu(t)$. If capital exceeds its golden-rule level, $f'(k)$ is less than $n + g$, and so the contribution of capital to social welfare at t is negative."

b. Explain in one sentence what is wrong with the following argument: "The premise of the argument in part (a) makes no sense, because one of the central results of the model is that capital can never be greater than its golden-rule level."

2. (Growth with forced saving. From the 2018 midterm.) Consider a Ramsey-Cass-Koopmans economy that is on its balanced growth path.

Suppose that (perhaps because of some type of national emergency) at some time, t_0, the government imposes an unexpected forced saving plan of fixed, known duration. Specifically, for $t_0 < t < t_1$ (where $t_1 > t_0$), the government requires households to set $c(t) = \bar{c}$ (where, as usual, c is consumption per unit of effective labor, and where $\bar{c}$ is strictly positive, and strictly less than the value of c on the balanced growth path).

Describe the behavior of k and c both from time t_0 to time t_1 and after time t_1. (Note: Phase diagrams, sketches of k and c as functions of time, and qualitative answers are all that is expected or desired.)

3. (Originally from the 2015 midterm.) Romer, Problem 2.13.

4. Consider the Diamond overlapping-generations model with the depreciation rate, δ, strictly less than 1. Let K denote capital, W the wage per worker (not per unit of effective labor), C_{1t} the consumption of a representative individual born in period t in the first period of their life, L_t the number of people born in period t, r_{t+1} the real interest rate from period t to period $t + 1$, and n the rate of population growth. Then the capital stock in period $t + 1$ is given by:

A. $K_{t+1} = (W_t - C_{1t})L_t$.
B. $K_{t+1} = (1-\delta)K_t + (W_t - C_{1t})L_t$.
C. $K_{t+1} = (1 + r_{t+1})[(1-\delta)K_t + (W_t - C_{1t})L_t]$.
D. $K_{t+1} = (1 + r_{t+1})[(1-\delta)[\frac{K_t}{1+n}] + (W_t - C_{1t})L_t]$.

5. (The Diamond model with labor supply in both periods of life.) Consider the Diamond overlapping-generations model. Assume, however, that each individual supplies one unit of labor in <u>each</u> period of life. For simplicity, assume no population growth; thus total labor supply is $2L$, where L is the number of individuals born each period.

In addition, assume that there is no technological progress, and that production is Cobb-Douglas. Thus, $Y_t = BK_t^{\alpha}[2L]^{1-\alpha},\ B > 0, 0 < \alpha < 1$. Factors are paid their marginal products.

The utility function of an individual born at time t is $U_t = \ln C_{1,t} + \ln C_{2,t+1}$.

Finally, there is 100 percent depreciation, so $K_{t+1} = Y_t - [LC_{1,t} + LC_{2,t}]$.

a. Consider an individual born in period t who receives a wage of w_t in the first period of life and a wage of w_{t+1} in the second period, and who faces an interest rate of r_{t+1}. What is the individual's first-period consumption and saving as a function of w_t, w_{t+1}, and r_{t+1}?

b. What will be the wage at t as a function of K_t? What will be the interest rate at t as a function of K_t? (Hint: Don't forget that the depreciation rate is not assumed to be zero.)

c. Explain intuitively why $K_{t+1} = (w_t - C_{1,t})L$.

d. Derive an equation showing the evolution of the capital stock from one period to the next.

6. Consider the Diamond overlapping-generations model where k is converging to a balanced-growth-path value from above. Then:

A. The real interest rate is rising over time.
B. The real interest rate is falling over time.
C. The real interest rate is constant over time.
D. The behavior of the real interest rate is not monotonic.
E. It is not possible to tell.

7. In a Diamond economy, the balanced growth path <u>cannot</u> be dynamically inefficient if:

A. Utility is logarithmic and production is Cobb-Douglas.
B. Individuals' discount rate (ρ) exceeds the economy's growth rate ($n + g$).
C. The initial capital stock is less than the golden rule capital stock.
D. None of the above.

EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

8. Romer, Problem 2.7.

9. Romer, Problem 2.14.

10. In the Ramsey-Cass-Koopmans model, the one-time destruction of half of the economy's capital stock:

A. Shifts the $\dot{c} = 0$ locus to the left and does not affect the $\dot{k} = 0$ locus.
B. Shifts the $\dot{k} = 0$ locus down and does not affect the $\dot{c} = 0$ locus.
C. Shifts the $\dot{c} = 0$ locus to the left and shifts the $\dot{k} = 0$ locus down.
D. Does not affect either the $\dot{c} = 0$ locus or the $\dot{k} = 0$ locus.

11. Romer, Problem 2.6, parts (a)–(e).

12. Romer, Problem 2.10, parts (a)–(c).

13. Romer, Problem 2.11.

14. Romer, Problem 2.12.

15. Romer, Problem 2.6, parts (f)–(g).

16. Romer, Problem 2.10, parts (d)–(f).

17. (Problem 2, continued.) Suppose that, as in Problem 2, initially the economy is a standard Ramsey-Cass-Koopmans economy, and that it is on its balanced growth path. Now suppose that at time t_0, the government (unexpectedly) announces that it will impose a forced saving plan at time t_1 ($t_1 > t_0$) that will last until t_2 ($t_2 > t_1$). Specifically, from time t_1 to time t_2, $c(t)$ is given by $c(t) = \bar{c}$ (where, as before $\bar{c}$ is strictly positive, and strictly less than the value of c on the balanced growth path).

Describe the behavior of k and c: at time t_0; from t_0 to t_1; at t_1; from t_1 to t_2; at t_2; after t_2. (Again, phase diagrams and qualitative answers are all that is expected or desired.)

(Note: The version posted online has one additional extra problem.)

18. (Zero discounting. Note: This problem is challenging.) In his famous 1928 *Economic Journal* article on optimal saving, Frank Ramsey argued that it is morally indefensible to discount the welfare of future generations. He therefore argued that a benevolent economic planner should:

$$\text{Maximize } V = \int_{t=0}^{\infty} u[c(t)]dt$$

$$\text{subject to } \dot{k}(t) = f[k(t)] - c(t),\ k(0) \text{ given.}$$

(Ramsey assumed zero population growth.) You may be able to see right away the problem with this formulation: <u>any</u> path that approaches a constant steady-state consumption level will yield an <u>infinite</u> value of V. Thus, it is not clear how to compare such paths and identify one as "optimal".

Ramsey finessed the problem in the follow way. He defined $\bar{c}$ to be the "bliss" or maximal steady-state consumption level (the existence of which presupposes that $f(k)$ eventually becomes decreasing in k or asymptotes to a finite maximum as k goes to ∞). He then redefined his problem as that of <u>minimizing</u> $\int_{t=0}^{\infty}\{u(\bar{c}) - u[c(t)]\}dt$, society's cumulative distance from "bliss", subject to the above constraints. Note that this integral can be finite if $c(t) \rightarrow \bar{c}$ as $t \rightarrow \infty$ (and if it isn't, it's not the optimum we seek in any case).

A. Use the Maximum Principle to derive necessary conditions for a solution to the Ramsey problem. (You can assume a depreciation rate of 0 for capital.) The resulting Euler condition is sometimes called the Keynes-Ramsey rule because J. M. Keynes, a friend of Ramsey's and editor of *Economic Journal*, helped him to interpret it intuitively. Show that the economy indeed should converge to "bliss" (also known as the "golden rule" in growth theory). Interpret the model's intertemporal Euler condition. You can do so by addressing the follow question: Suppose the economy starts with $k(0)$ less than the level that maximizes $f(k)$. Since Ramsey believed in intergenerational equality, why isn't it optimal in his view for each generation simply to consume $f[k(0)]$?

B. Let $\{c^*(t)\}_{t=0}^{\infty}$ denote the Ramsey consumption path starting from an initial capital stock $k(0)$, and let $\{c(t)\}_{t=0}^{\infty}$ be any other consumption path. Show that the Ramsey path *overtakes* any other feasible consumption path starting from $k(0)$, in the following sense: there exists a finite time J such that for all $T > J$, $\int_0^T u[c^*(t)]dt > \int_0^T u[c(t)]dt$.

C. Ramsey states the Keynes-Ramsey rule as: "*rate of saving multiplied by marginal utility of consumption should always equal bliss minus actual rate of utility enjoyed.*" (By "bliss" he meant $u(\bar{c})$.) Can you derive this rule?