First Midterm

Ec142 – Spring 2018

Please read each question carefully. Start each question on a new bluebook page. The use of calculators and other computational aides is not allowed. Good luck!

[1] **[5 Points]** Please write your full name on this exam sheet and turn it in with your bluebook.

[2] **[25 Points]** Let W, X be a pair of regressors with the property that $\mathbb{C}(W, X) = 0$. In this problem you will consider the claim, for some outcome Y, that

$$\mathbb{E}^*[Y|W, X] = \mathbb{E}^*[Y|W] + \mathbb{E}^*[Y|X] - \mathbb{E}[Y]. \tag{1}$$

You may assume that all objects in the above expression are well-defined (i.e., all necessary moments exist and so on).

[a] **[5 Points]** Begin by showing that

$$\mathbb{E}^*[\mathbb{E}^*[Y|W]|X] = \mathbb{E}^*[\mathbb{E}^*[Y|X]|W] = \mathbb{E}[Y].$$

[b] **[5 Points]** Second verify (1) using the necessary and sufficient conditions of the Projection Theorem. That is show that

$$\begin{aligned}\mathbb{E}[U] &= 0\\ \mathbb{E}[UX] &= 0\\ \mathbb{E}[UW] &= 0\end{aligned}$$

for $U = Y - \mathbb{E}^*[Y|W] - \mathbb{E}^*[Y|X] + \mathbb{E}[Y]$.

[c] **[2 Points]** Finally show that

$$\mathbb{E}^*[Y|W, X] = \mathbb{E}[Y] + \frac{\mathbb{C}(Y, W)}{\mathbb{V}(W)}(W - \mathbb{E}[W]) + \frac{\mathbb{C}(Y, X)}{\mathbb{V}(X)}(X - \mathbb{E}[X]).$$

[d] **[3 Points]** The Vice Chancellor for Undergraduate Education is interested in boosting academic performance among first year students. She randomly divides first year students into two equal-sized groups. In the first group she randomly assigns half of students to receive a daily snack voucher worth \$5 dollars. In the second group she randomly assigns half of students to get two hours of structured advising each semester. At the end of the semester she records student grade point average. Explain how the Vice Chancellor can use her data to form an estimate of the best linear predictor of end-of-first year GPA given a

constant, a dummy variable for snack voucher receipt, and a dummy variable for receipt of extra advising.

[e] **[5 Points]** Under what circumstances is the linear regression computed in part [d] helpful for allocating resources across initiatives? Consider, and elaborate on, three cases: [i] snacks and advising are complements in the production of GPA, [ii] they are substitutes and [iii] they do not interact. **[~ 6 sentences]**

[f] **[5 Points]** Outline a more informative experiment for the Vice Chancellor. Explain why is it is "better" than the experiment described in part [d]. **[~ 6 sentences]**

[3] **[35 Points]** You've been hired by the Government of Honduras to assess the efficacy of treatment for decompression sickness among lobster divers in La Moskitia. In this region of Honduras, lobsters are harvested by divers who, on occasion, get decompression sickness. This can result in partial paralysis or worse. You are provided the following table of information about 300 diving accident victims.

		$Y = 0$ (No Limp)	$Y = 1$ (Limp)
$X = 0$ (Untreated)	$W = 0$ (Depth < 75')	90	10
	$W = 1$ (Depth ≥ 75')	10	40
$X = 1$ (Treated)	$W = 0$ (Depth < 75')	30	20
	$W = 1$ (Depth ≥ 75')	50	50

[a] **[2 Points]** What is the probability of a victim walking with a limp conditional on treatment ($X = 1$) and non-treatment ($X = 0$)?

[b] **[3 Points]** What is the probability of a victim receiving treatment conditional on having dived "deep" ($W = 1$) vs. "shallow" ($W = 0$)?

[c] **[5 Points]** A government official worries that treatment is harming the divers and thinks it would be better to do nothing. Present a counter-argument to this official. **[~ 4 sentences]**

[d] **[5 Points]** Let $Y(0)$ and $Y(1)$ denote a divers potential outcome given non-treatment and treatment respectively. Discuss the conditional independence assumption assumption

$$(Y(0), Y(1)) \perp X | W = 0, 1.$$

Make a positive and negative argument for this assumption. **[~ 4 sentences]**

[e] **[10 Points]** Using the assumption in part [d] construct the IPW estimate of the average treatment effect on the outcome. Report your result to the government official. Your

report should include an explanation for why and how your are adjusting for accident depth. Is treatment effective? **[~ 6 sentences]**

[f] **[5 Points]** Say instead you were given the table:

		$Y = 0$ (No Limp)	$Y = 1$ (Limp)
$X = 0$ (Untreated)	$W = 0$ (Depth < 75')	90	10
	$W = 1$ (Depth ≥ 75')	0	0
$X = 1$ (Treated)	$W = 0$ (Depth < 75')	30	20
	$W = 1$ (Depth ≥ 75')	75	75

Can you compute the ATE is this case? Why or why not? **[~ 4 sentences]**

[4] **[35 Points]** Let Y equal tons of banana's harvested in a given season for a randomly sampled Honduran banana planation. Output is produced using labor and land according to $Y = AL^{\alpha_0}D^{1-\alpha_0}$, where L is the number of employed workers and D is the size of the plantation in acres and we assume that $0 < \alpha_0 < 1$. The price of a unit of output is P, while that of a unit of labor is W. These prices may vary across plantations (e.g., due to transportation costs, labor market segmentation etc.). We will treat D as a fixed factor; A captures sources of plantation-level differences in farm productivity due to unobserved differences in, for example, soil quality and managerial capacity. Plantation owners choose the level of employed labor to maximize profits. The observed values of L are therefore solutions to the optimization problem:

$$L = \arg\max_{l} P \cdot Al^{\alpha_0}D^{1-\alpha_0} - W \cdot l.$$

[a] **[3 Points]** Show that the amount of employed labor is given by

$$L = \left\{\alpha_0 \frac{P}{W} A\right\}^{\frac{1}{1-\alpha_0}} D. \tag{2}$$

[b] **[7 Points]** Let $a_0 = \frac{1}{1-\alpha_0}\ln\alpha_0 + \frac{1}{1-\alpha_0}\mathbb{E}[\ln A]$, $b_0 = \frac{1}{1-\alpha_0}$, and $V = \frac{1}{1-\alpha_0}\{\ln A - \mathbb{E}[\ln A]\}$. Show that the log of the labor-land ratio is given by

$$\ln\left(\frac{L}{D}\right) = a_0 + b_0 \ln\left(\frac{P}{W}\right) + V \tag{3}$$

and that, letting $c_0 = \mathbb{E}[\ln A]$ and $U = \ln A - \mathbb{E}[\ln A]$, the log of planation yield (output

per unit of land) is given by

$$\ln\left(\frac{Y}{D}\right) = c_0 + \alpha_0 \ln\left(\frac{L}{D}\right) + U. \tag{4}$$

[c] **[5 Points]** Briefly discuss the content and plausibility of the restriction **[~ 6 sentences]**

$$\mathbb{E}\left[\ln A | \ln\left(P/W\right)\right] = \mathbb{E}\left[\ln A\right]. \tag{5}$$

[d] **[10 Points]** Using (3), (4) and (5) show that the coefficient on $\ln\left(L/D\right)$ in $\mathbb{E}^*\left[\ln\left(Y/D\right)|\ln\left(L/D\right)\right]$ equals

$$\alpha_0 + \left(1-\alpha_0\right)\frac{\mathbb{V}\left(\ln A\right)}{\mathbb{V}\left(\ln A\right) + \mathbb{V}\left(\ln\left(P/W\right)\right)}.$$

Provide some economic intuition for this result. **[~ 6 sentences]**

[e] **[5 Points]** Using (3), (4) and (5) show that the coefficient on $\ln\left(L/D\right)$ in $\mathbb{E}^*\left[\ln\left(Y/D\right)|\ln\left(L/D\right), V\right.$ equals α_0. Provide some economic intuition for this result. **[~4 sentences]**

[f] **[5 Points]** Assume that all plantations face the same output price (P) and labor cost (W). What value does the coefficient on $\ln\left(L/D\right)$ in $\mathbb{E}^*\left[\ln\left(Y/D\right)|\ln\left(L/D\right)\right]$ equal now? Why? **[~ 6 sentences]**