Financial Econometrics Econ 40357
Topic 10: The Fama-MacBeth Method

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Brooks, pp. 588.

Background

- The theory is about the cross section,

$$\bar{r}_i^e = \lambda \beta_i + \alpha_i$$

- Theory says we should run a cross-sectional regression of the mean excess return on asset *i* on its beta and a constant.
  - Test if Jensen's alpha is zero.
  - Test to see if λ is significant.
- Estimation is straightforward. Challenge is computing standard errors on the lambdas (to form t-ratios), because the regressors (the betas) are generated regressors. They are not data but are estimated. Fama and MacBeth came up with a strategy for getting the standard errors on the lambdas.

Apply the Method on CAPM

- We have n assets over T time periods with excess returns $r_{t,i}^e$. The CAPM is a single-factor model. $f_t = r_{t,m}^e$ is the excess return on the market portfolio.
- Stage 1: Run n individual time-series regressions.

$$
\begin{aligned}
r_{t,1}^e &= \alpha_1 + \beta_1 f_t + \epsilon_{t,1} \\
r_{t,2}^e &= \alpha_2 + \beta_2 f_t + \epsilon_{t,2} \\
&\vdots \\
r_{t,n}^e &= \alpha_n + \beta_n f_t + \epsilon_{t,n}
\end{aligned}
$$

This gives us n estimated betas $\hat{\beta}_i$. Sometimes people call the β's the 'factor loadings.'

- Stage 2 : Run a single cross-sectional regression of the (time-series) average excess returns on betas.

$$\bar{r}_i^e = \lambda \beta_i + \alpha_i$$

The betas are the independent variable here. The estimated slope coefficient is λ, is called the price of risk.
You can run this regression without constant, and test the alphas–this is the only time you are permitted to run without constant. OR, you can run with constant, and test if constant (γ) is zero,

$$\bar{r}_i^e = \gamma_i + \lambda \beta_i + \alpha_i$$

If you include constant, the α_i will be zero on average.

Test the significance of λ

- Recall, the problem: betas not data. They are estimated. We call them 'generated regressors.' Cannot use the t-ratio produced by the regression package. Fama-MacBeth does the following
- For each time period, $t = 1, \dots T$, run a cross-sectional regressions of returns (at time t) on the β_i, $i = 1, \dots, n$.

$$\begin{aligned}
r^e_{1,i} &= \lambda_1 \beta_i + \alpha_{1,i}(+\gamma_1) \\
r^e_{2,i} &= \lambda_2 \beta_i + \alpha_{2,i}(+\gamma_2) \\
&\vdots \\
r^e_{T,i} &= \lambda_T \beta_i + \alpha_{T,i}(+\gamma_T)
\end{aligned}$$

  - The $(+\gamma_t)$ is in case we are running with constant.
  - Note: regressors in every regression is the same. Only the dependent variable changes from one regression to the other.
  - The λ_t are slope coefficients. γ_t is the constant in regression t and $\alpha_{t,i}$ is the error term.

- Fama-MacBeth assumes the λ_t are i.i.d. The assumption is often justified by noting that returns are (almost) uncorrelated over time.
- Run the time-series regression of λ_t on a constant

$$\lambda_t = c + u_t$$

  The t-ratio on the constant is the t-ratio for $\hat{\lambda}$.
- If you ran the cross-sectional regression with constant γ,repeat the process for γ_t to test if constant is zero.

Testing if $\alpha = 0$

Estimate the alpha of return or portfolio i as

$$\bar{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}_{t,i} \tag{1}$$

Construct $T \times n$ matrix of residuals,

$$A = \begin{pmatrix} \hat{\alpha}_{1,1} & \hat{\alpha}_{1,2} & \cdots & \hat{\alpha}_{1,n} \\ \hat{\alpha}_{2,1} & \hat{\alpha}_{2,2} & \cdots & \hat{\alpha}_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\alpha}_{T,1} & \hat{\alpha}_{T,2} & \cdots & \hat{\alpha}_{T,n} \end{pmatrix}$$

Get covariance matrix of A', call it Σ_α.It will be an $n \times n$ matrix.

$$\begin{aligned} \Sigma_\alpha &= Cov(A') \\ &= \begin{pmatrix} Var(\hat{\alpha}_1) & Cov(\hat{\alpha}_1, \hat{\alpha}_2) & \cdots & Cov(\hat{\alpha}_1, \hat{\alpha}_N) \\ \vdots & & & \\ Cov(\hat{\alpha}_N, \hat{\alpha}_1) & Cov(\hat{\alpha}_N, \hat{\alpha}_2) & \cdots & Var(\hat{\alpha}_N) \end{pmatrix} \end{aligned}$$

where

$$Var(\hat{\alpha}_i) = \frac{1}{T}\sum_{t=1}^{T}(\hat{\alpha}_{t,i} - \bar{\hat{\alpha}}_i)^2$$

$$Cov(\hat{\alpha}_i, \hat{\alpha}_j) = \frac{1}{T}\sum_{t=1}^{T}(\hat{\alpha}_{t,i} - \bar{\hat{\alpha}}_i)(\hat{\alpha}_{t,j} - \bar{\hat{\alpha}}_j)$$

Form the test statistic

$$T\left(\bar{\alpha}_1, \bar{\alpha}_{2,} \cdots \bar{\alpha}_n\right) \Sigma_{\alpha}^{-1} \begin{pmatrix} \bar{\alpha}_1 \\ \bar{\alpha}_2 \\ \vdots \\ \bar{\alpha}_n \end{pmatrix} \sim \chi^2_{n-1}$$

where the $\bar{\alpha}_i$ are the time-series means from (1).

Time Series or Fama-MacBeth?

- You need to run the cross-section regression when the factor is not a return.
- For time-series approach, factor must be a return because you estimate the factor risk premium by the mean return.

$$\hat{\lambda} = \frac{1}{T}\sum_{t=1}^{T} f_t$$

  (If f_t isn't a return–say it's consumption growth, then mean consumption growth isn't the factor risk premium!)
- When the factor is a return, you can compare the time-series and the cross-section results. They are not necessarily the same.