Estimation by maximum likelihood.

u_t is normally distributed with mean 0 and conditional variance σ_t^2. So we know how to write down the joint pdf of $(u_1, u_2, ..., u_T)$. Substitute $u_t = r_t - a - \beta x_t$, and now the joint pdf turns into the likelihood function. Take logarithm of the likelihood function and ask Eviews to maximize it with respect to a, β, and the $\alpha's$.

Lagrange multiplier test $TR^2 = 24576 * 0.1216 = R^2T = 2988.4$

Solution to highly persistent volatility is not adding more and more lags in the ARCH model, because of the usual arguments. Instead, the solution is GARCH

Oh, one more thing. The $\alpha_j's$ have to be positive to keep the variance positive. And $0 < \beta < 1$ in GARCH. If $\beta = 1$,

$$\sigma_t^2 = \sigma_{t-1}^2 + \alpha_0 + \alpha_1 u_{t-1}^2$$

This is like a random walk in the variance. Then the variance is unbounded. We rule this out with $0 < \beta < 1$.

GARCH(p,q): p is the number of ARCH terms (lagged values of the process squared, q is the number of lagged variances.GARCH(2,1)

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \beta \sigma_{t-1}^2$$

GARCH(1,2)

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2$$

Why does GARCH take care of the problem of too many parameters?

Let's look at at GARCH(1,1) model

$$\begin{aligned}
\sigma_{t-1}^2 &= \alpha_0 + \alpha_1 u_{t-2}^2 + \beta \sigma_{t-2}^2 \\
\sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \\
\sigma_{t-2}^2 &= \alpha_0 + \alpha_1 u_{t-3}^2 + \beta \sigma_{t-3}^2
\end{aligned}$$

Substitute the first equation into the second

$$\begin{aligned}
\sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \left(\alpha_0 + \alpha_1 u_{t-2}^2 + \beta \sigma_{t-2}^2\right) \\
&= \alpha_0 + \beta \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_1 \beta u_{t-2}^2 + \beta^2 \sigma_{t-2}^2
\end{aligned}$$

Substitute expression for σ_{t-2}^2 into the last equation

$$\begin{aligned}
\sigma_t^2 &= \alpha_0 + \beta \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_1 \beta u_{t-2}^2 + \beta^2 \left(\alpha_0 + \alpha_1 u_{t-3}^2 + \beta \sigma_{t-3}^2\right) \\
&= \left(\alpha_0 + \beta \alpha_0 + \beta^2 \alpha_0\right) + \left(\alpha_1 u_{t-1}^2 + \alpha_1 \beta u_{t-2}^2 + \alpha_1 \beta^2 u_{t-3}^2\right) + \beta^3 \sigma_{t-3}^2 \\
&= \frac{\alpha_0}{1-\beta} + \frac{\alpha_1}{\beta} \left(\beta u_{t-1}^2 + \beta^2 u_{t-2}^2 + \beta^3 u_{t-3}^2 + \cdots\right)
\end{aligned}$$

This is a controlled ARCH(∞). Controlled because coefficients on lagged u_t^2 are powers of β

What I did in the LM test was

$$r_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \cdots + \alpha_p r_{t-p}^2 + v_t$$

ARCH or GARCH in the mean

$$\begin{aligned} r_t &= a + b\sigma_t^2 + u_t \\ E_{t-1}\left(r_t\right) &= a + b\sigma_t^2 \end{aligned}$$

$$\begin{aligned} r_t &= a + b\sigma_t + u_t \\ u_t &\tilde{} N\left(0, \sigma_t^2\right) \\ \sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 \end{aligned}$$