Problem Set 5

ECON 30020: Intermediate Macroeconomics
Professor Sims
University of Notre Dame, Spring 2018

Instructions: You may work on this problem set in groups of up to four people. Should you choose to do so, please make sure to legibly write each group member's name on the first page of your solutions. This problem set is due in class on Thursday March 1.

1. **The MPC and IS Curve:** Write down the generic definition of the IS curve for an endowment economy. Graphically show to how derive the IS curve. How will the MPC affect the slope of the IS curve (i.e. if the MPC is bigger, will the IS curve be flatter or steeper)? For the purposes of this exercise, assume that the sensitivity of consumption to the interest rate is independent of the value of the MPC. Show graphically and briefly provide some written intuition.

2. **Uncertainty and Equilibrium Interest Rates:** Suppose that there is uncertainty and that the third derivative of the utility function is positive. In this case, a generic consumption function for an optimizing household can be written:

$$C_t = C^d(Y_t, E[Y_{t+1}], r_t, unc_t)$$

Here consumption today depends on the expected value of income tomorrow, $E[Y_{t+1}]$, instead of just income tomorrow. The term unc_t measures uncertainty about future income – the bigger unc_t, the more uncertain future income is. Since the third derivative of the utility function is positive, $\frac{\partial C^d(Y_t, E[Y_{t+1}], r_t, unc_t)}{\partial unc_t} < 0$.

   (a) For a generic functional form, use the $IS - Y^s$ diagram for an endowment economy to show how r_t ought to react to an increase in uncertainty. Provide some basic (written) economic intuition for why this happens.

   (b) The Euler equation (which underpins the generic consumption function) when the future is uncertain is characterized by:

$$u'(C_t) = \beta(1 + r_t)E\left[u'(C_{t+1})\right]$$

Suppose that flow utility function is the natural log – i.e. $u(C_t) = \ln C_t$. Suppose that $\beta = 0.9$. Current income is $Y_t = 1$ and the market-clearing condition in period t is $C_t = Y_t$. There are two states of the world in period $t+1$. In state h, we have $Y^h_{t+1} = 1.6$. This state occurs with probability $p = \frac{1}{2}$. In state l, we have $Y^l_{t+1} = 0.7$. This state occurs with probability $(1 - p) = \frac{1}{2}$. Market-clearing requires that consumption simply equal income in both states of the world – $C^h_{t+1} = Y^h_{t+1}$ and $C^l_{t+1} = Y^l_{t+1}$. Use the provided information to solve for a numeric value of the equilibrium real interest rate.

(c) Continue with the setup from the previous part. Suppose that the good state tomorrow gets better, with $Y^h_{t+1} = 1.65$, and the bad state gets worse, with $Y^l_{t+1} = 0.65$. Show that this does not affect the expected value of future income, $E[Y_{t+1}]$.

(d) Solve for the a numeric value of the new equilibrium real interest rate when the future is more uncertain as in the previous part. How does it compare to the previous numeric value of the real interest rate you found?

3. **Yield Curves as Recession Predictors:** Suppose that you have a representative household who lives for three periods – t, $t+1$, and $t+2$. There is no uncertainty over the future. Lifetime utility is:

$$U = \ln C_t + \beta \ln C_{t+1} + \beta^2 \ln C_{t+2}$$

The household has access to two different savings vehicles – one period savings bonds (equivalently debt) and two period bonds. The one period bonds pay out interest in the subsequent period. The two period bonds must be held for two periods. Imposing terminal conditions and that flow budget constraints must hold with equality, the household faces the following sequence of flow budget constraints:

$$C_t + S_{1,t} + S_{2,t} = Y_t$$
$$C_{t+1} + S_{1,t+1} = Y_{t+1} + (1 + r_{1,t})S_{1,t}$$
$$C_{t+2} = Y_{t+2} + (1 + r_{1,t+1})S_{1,t+1} + (1 + r_{2,t})^2 S_{2,t}$$

1 subscripts denote one period bonds, while 2 subscripts refer to two period bonds. In period t, the household can save/borrow in either one period bonds, $S_{1,t}$, or two period bonds, $S_{2,t}$. One period bonds pay interest, $r_{1,t}$, plus principal in period $t+1$. In period $t+1$, the household can save/borrow again in one period bonds, $S_{1,t+1}$, which pay interest, $r_{1,t+1}$, plus principal back in period $t+2$. The household will not choose to save/borrow through two period bonds in period $t+1$ because there is no period $t+3$. In period $t+2$, the household earns income, gets interest plus principal on one period bonds brought from $t+1$ into $t+2$ (i.e. $(1+r_{1,t+1})S_{1,t+1}$), and gets interest plus principal on two period bonds brought from period t (i.e. $(1+r_{2,t})^2 S_{2,t}$). For the interest plus principal on two period bonds the gross interest rate is squared because the bonds are held for two periods, and there is thus compounding.

(a) One way to think about the household's optimization problem is to choose a savings *plan* in period t. That is, in period t, choose $S_{1,t}$, $S_{2,t}$, and $S_{1,t+1}$ to maximize lifetime utility subject to the three flow budget constraints. Eliminate the consumption terms by writing them in terms of the savings variables in each period and write down the unconstrained optimization problem faced by the household.

(b) Derive three first order conditions characterizing an optimal savings plan.

(c) Use your first order conditions to derive a condition that must hold between $r_{2,t}$, $r_{1,t}$, and $r_{1,t+1}$.

(d) Use your previous answers to argue that $r_{2,t} \approx \frac{1}{2}\left[r_{1,t} + r_{1,t+1}\right]$.

(e) Suppose that the representative household is living in an endowment economy in which $S_{1,t} = S_{2,t} = S_{1,t+1} = 0$ in equilibrium, which implies that $C_t = Y_t$, $C_{t+1} = Y_{t+1}$, and $C_{t+2} = Y_{t+2}$. Derive expressions for the equilibrium values of $r_{1,t}$, $r_{2,t}$, and $r_{1,t+1}$.

(f) A yield curve is a plot of interest rates at a point in time as a function of the time to maturity associated with the interest rate. The yield curve in period t would simply be a plot of $r_{1,t}$ and $r_{2,t}$ against time to maturity – 1 and 2 periods. Suppose that $Y_t = Y_{t+1} = Y_{t+2}$. What will the yield curve look like?

(g) Now suppose that, at period t, the household expects Y_{t+2} to fall relative to Y_{t+1} and Y_t (which are unaffected relative to the previous problem). In particular, suppose that $\frac{Y_{t+2}}{Y_{t+1}} = 0.9$. What will this due to the shape of the yield curve?

(h) An empirical regularity in the data is that "inverted yield curves" (downward-sloping yield curves) are often predictive of recessions. Does this empirical regularity make sense in light of your answers on this problem? Explain briefly.