International Macroeconomics
Lecture 2: Real Exchange Rates

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  - **The Real Exchange Rate:** The relative cost of a common reference basket of goods between two countries
  - Expressed in a common, numeraire good
- Remember that prices are not necessarily money!
  - In this lecture, we develop a wholly consistent theory of exchange rates before ever introducing money

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- Logic: No-Arbitrage
  - If it *didn't hold*, then a bunch of consumers could save money by converting currency and purchasing the same basket in another country
  - If such opportunities *did exist*, they would be seized immediately and market forces would quickly drive price wedges together until PPP held

PPP in the Data

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- Does PPP hold in the data? Nope
- Big-Mac Example (2015)

Country	Big Mac Price (US $)
Venezuela	0.67
South Korea	3.76
Norway	5.65
Russia	1.88
Switzerland	6.82
US	4.79

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  - NTB almost never change

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- Capital can cross borders and sectors, but labor can only cross sectors

$$L_{T,s} + L_{N,s} = L_s$$

A Firm's Problem

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- Firms maximize profits (discount future at interest rate):

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Solving the Firm's Problem

- Firms choose capital and labor

$$\text{FOC}(K_{T,s}) : 0 = -1 + \frac{1}{1+r} A_{T,s} F_K(K_{T,s}, L_{T,s}) + \frac{1}{1+r}$$

$$\text{FOC}(L_{T,s}) : 0 = -w_s + A_{T,s} F_L(K_{T,s}, L_{T,s})$$

Symmetric conditions for nontradable sector

Solving the Firm's Problem

- Assume *Constant Returns to Scale* in production

$$\rightarrow y_T = A_T F(k_T, 1) = A_T f(k_T)$$

where $y_T = Y_T / L_T$ and $k_T = K_T / L_T$

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- Using the equality: $F(K, L) = Lf(k)$, we totally differentiate to derive

$$F_K(K, L) = f'(k)$$

$$F_L(K, L) = f(k) - f'(k)k$$

Determining the Relative Price

- In any period, 4 FOCs from firms in the two sectors
  1. $A_T f'(k_T) = r$
  2. $A_T[f(k_T) - f'(k_T)k_T] = w$
  3. $pA_N g'(k_N) = r$
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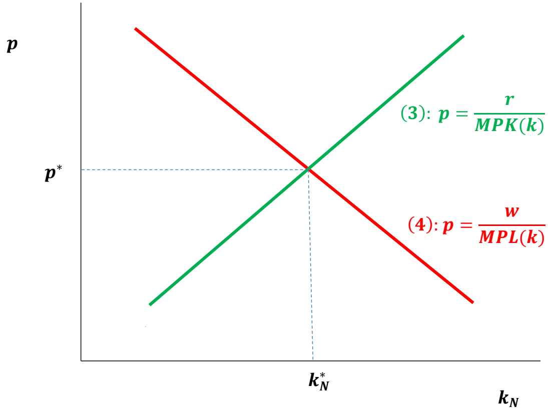
     - Eqn (3): p increasing in k_N
     - Eqn (4): p decreasing in k_N
- p entirely determined by world prices/technologies (no demand-side impact)

Relative Price and Market for Nontradables



Deriving the Exchange Rate

- Price level geometric average of different prices (why later)

$$P = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_N^{\alpha_N}$$

where $\sum_i \alpha_i = 1$ and $\alpha_i \geq 0$

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- Two goods in our economy: Assume $\alpha_T = \gamma$ and $\alpha_N = 1 - \gamma$

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- Exchange rate:

$$E = \frac{P}{P^\star} = \left(\frac{p}{p^\star}\right)^{1-\gamma}$$

  - Determined by relative prices of nontradables!

Experiment 1: Rise in A_T

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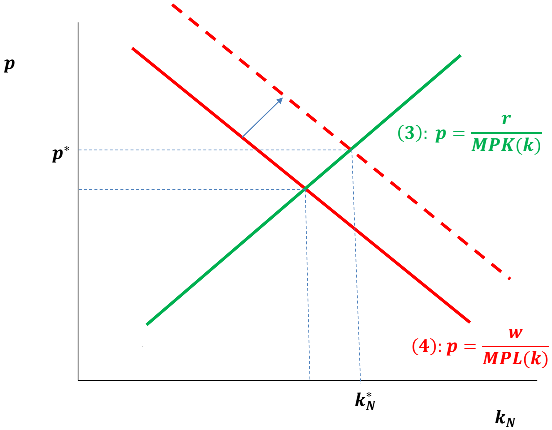
- **Intuition:** Both firms must break even (perfect competition/zero-profit)
  1. A_T increases T output/revenue/profit
  2. Tradable price cannot adjust, so to meet zero-profit condition in T, wages rise
  3. Rise in wages pushes up costs in N-sector
  4. To meet zero-profit condition in N-sector, p rises to increase revenues

Experiment 1: Rise in A_T



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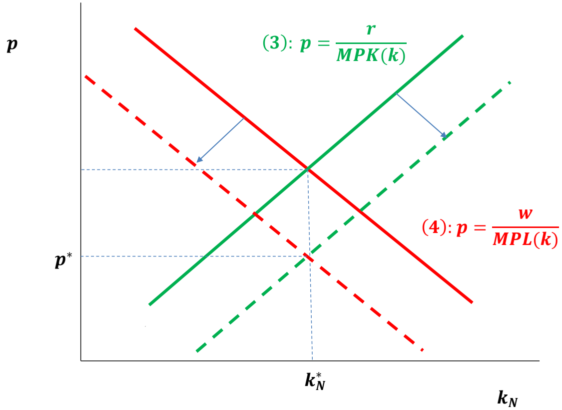
  - Capital-Labor Ratio (N) may rise or fall
- **Intuition:** Again zero-profit
  1. A_N increase raises quantity/revenue/profit in N
  2. Wages/r cannot rise to offset this (both determined in T)
  3. p must fall to lower revenues $\rightarrow$ Back to zero-profit

Experiment 2: Rise in A_N



The Exchange Rate: The HBS Effect

- *Harrod-Balassa-Samuelson Effect*: Countries with higher productivity in tradables relative to non-tradables have higher price levels ('stronger currencies')
  - Highly productive tradables push wages up (meet zero-profit)
  - Prices of non-tradables must rise as wages rise (meet zero-profit)
  - Price of tradables same everywhere; price of non-tradables higher in home

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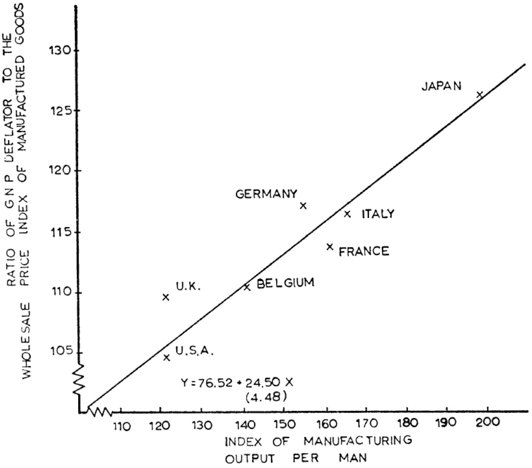
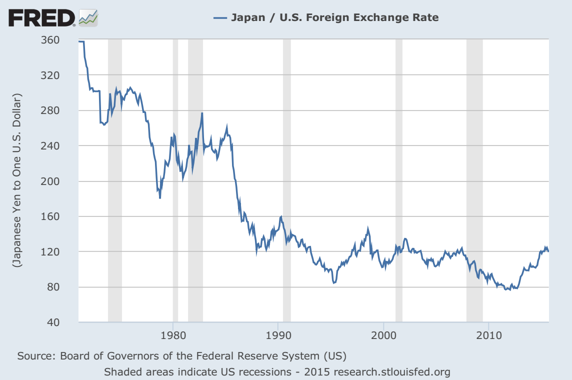
  - Price of tradables same everywhere; price of non-tradables higher in home
- Opposite happens in countries with productive advantage in non-tradables
  - Wages rigid/set in T sector: Prices must fall to prevent positive profits
  - Tradable prices same everywhere; price of non-tradables lower in home

The HBS Effect in the Data: Cross-Country



Year: 1993

The HBS Effect in the Data: Japan



The Eurozone

- Paradox: Peripheral Eurozone (Spain, Italy, Greece, etc.) in 2000's
  - Booming non-tradable sectors (housing/construction)
  - Same time, exchange rates seemed to be *too strong*
    - Lack of **Competitiveness**
  - How can we reconcile this?

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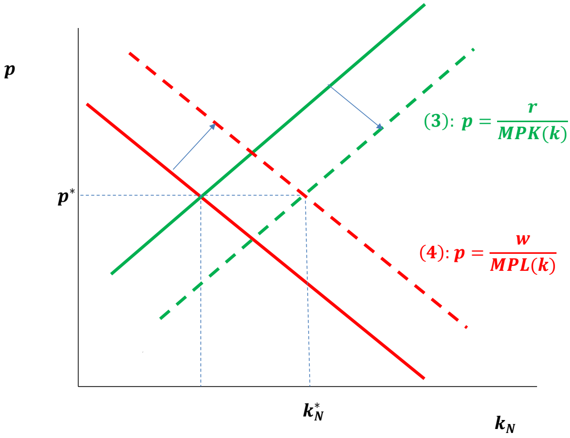
  1. $k_{T,s} \uparrow$ (borrowing costs fall)
  2. Implies $w_s \uparrow$
  3. Impact on p? Hard to tell
     - Borrowing costs fall, but wages rise
     - Need closer look...

Drop in r

Unambiguous rise in $k_N^\star$
Impact on $p^\star$ not obvious



Drop in r: Impact on p

- Consider impact a small change on r has on p
- Under the assumption of Constant-Returns-To-Scale, easy to show that firms make zero profits

$$\rightarrow A_T f(k_T) = rk_T + w$$

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- Totally differentiate

$$\frac{dA_T}{A_T} + \frac{f'(k_T)}{f(k_T)} dk_T = \frac{rdk_T + k_T dr + dw}{rk_T + w}$$

Drop in r: Impact on p

- Since $r = A_T f'(k_T)$, we get

$$\frac{dA_T}{A_T} + \frac{r}{A_T f(k_T)} dk_T \times \frac{k_T}{k_T} = \frac{r dk_T \times \frac{k_T}{k_T} + k_T dr \times \frac{r}{r} + dw \times \frac{w}{w}}{rk_T + w}$$

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- Call $\hat{x} = dx/x$. Since $rk_T + w = A_T f(k_T)$, we get

$$\hat{A}_T + \frac{rk_T}{A_T f(k_T)} \hat{k}_T = \frac{rk_T}{A_T f(k_T)} \hat{k}_T + \frac{rk_T}{A_T f(k_T)} \hat{r} + \frac{w}{A_T f(k_T)} \hat{w}$$

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- Denote the labor-share of income in T: $\mu_{LT} = \frac{w}{A_T f(k_T)} \hat{w}$

$$\hat{A}_T = \mu_{LT} \hat{w} + (1 - \mu_{LT}) \hat{r}$$

Drop in r: Impact on p

- Same approach for N implies

$$\hat{p} + \hat{A}_N = \mu_{LN}\hat{w} + (1 - \mu_{LN})\hat{r}$$

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- Impose $\hat{A}_N = \hat{A}_T = 0$. Zero-profit in T-sector implies

$$\hat{w} = -\left[\frac{1 - \mu_{LT}}{\mu_{LT}}\right]\hat{r}$$

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- Zero-profit in N sector implies

$$\hat{p} = \hat{r}\left[1 - \frac{\mu_{LN}}{\mu_{LT}}\right]$$

Drop in r: Impact on p

$$\hat{p} = \hat{r}\left[1 - \frac{\mu_{LN}}{\mu_{LT}}\right]$$

- Non-tradables tend to be labor intensive $\rightarrow \mu_{LN} > \mu_{LT}$

Drop in r: Impact on p

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- **Interest rate convergence strengthens the exchange rate and reduces competitiveness**
- Captures many features of Eurozone in 2000s
  1. Lack of competitiveness
  2. Boom in investment ($k_T, k_N \uparrow$) $\rightarrow$ Increase in capital inflows
  3. Non-tradable boom with rise in p e.g. housing, services

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  3. Constant labor supply, $L = \bar{L}_T + \bar{L}_N$
- Decomposition of labor not predetermined

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  - r, w, and p only determine capital-to-labor ratios

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$$p(r)\bar{Y}_N = [rk_N(r) + w(r)](L - \bar{L}_T)$$

- Substitute out $\bar{L}_T$ to get a PPF for $(\bar{Y}_N, \bar{Y}_T)$:

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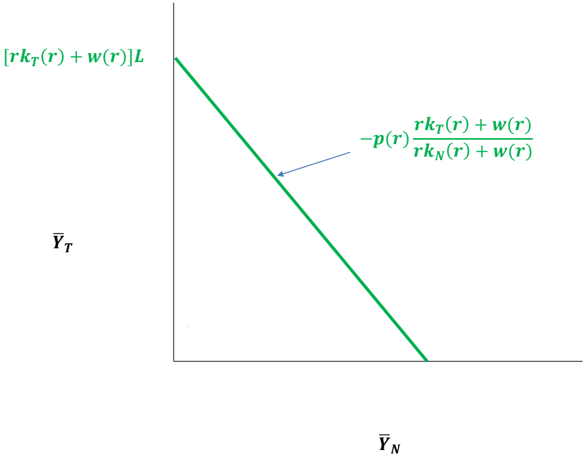
- This is the **GDP Line**, since it denotes 'stuff' produced domestically
- Slope greater than $p(r)$ when N-sector more labor-intensive

$$k_T(r) > k_N(r)$$

GDP Line



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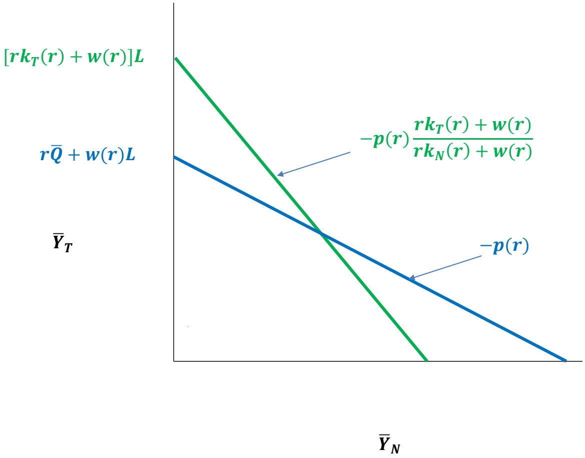
- Consumption is GNP since we can write

$$\underbrace{\bar{C}_T + p(r)\bar{C}_N}_{\text{SS Expenditures}} = \underbrace{w(r)L + r\bar{K}}_{\text{SS GDP}} + \underbrace{r\bar{B}}_{\text{SS NFFP}}$$

GDP and GNP



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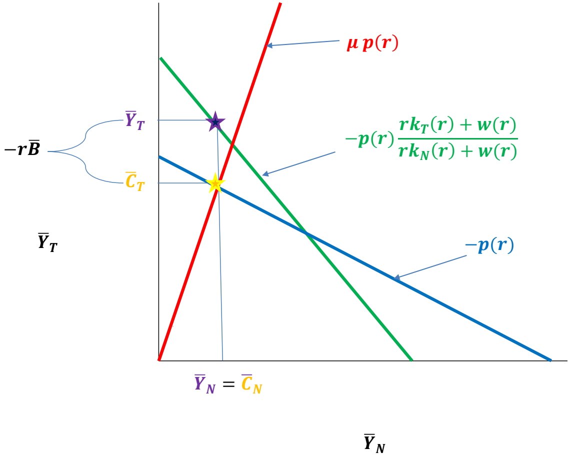
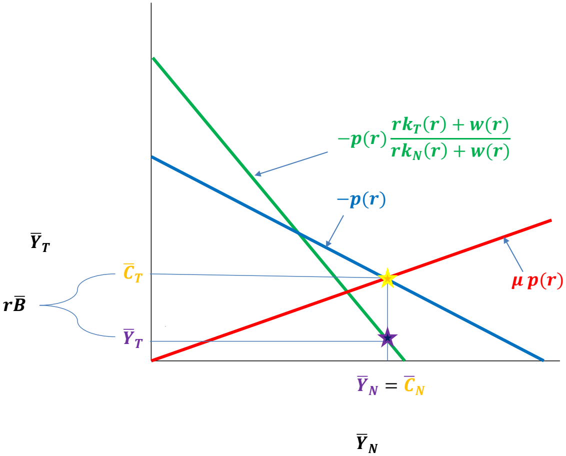
  for some constant $\mu > 0$
- Can't trade N-goods, so $\bar{Y}_N = \bar{C}_N$
- Implies gap in tradables financed by foreign assets

$$\bar{C}_T - \bar{Y}_T = r\bar{B}$$

Example 1: High Demand for Tradables



Example 1: High Demand for Nontradables



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- Tradables are relatively capital-intensive e.g. manufacturing
  1. When economy wants more tradables, it borrows capital from abroad, since capital can cross borders: High $\bar{K}$ and low (negative) $\bar{B}$
  2. When economy wants more nontradables, domestic capital not as helpful $\rightarrow$ Store wealth abroad and use to purchase tradables: Low $\bar{K}$ and high $\bar{B}$

Zooming Out from Steady State

- May want to know how consumption/prices respond to fluctuations
- Specify preferences and maximization problem:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

Use CES to allow for multiple goods i.e. in any period

$$C_s = \Omega(C_{T,s}, C_{N,s}) = \left[\gamma^{\frac{1}{\theta}} C_{T,s}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{N,s}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

- Consumer solves two problems
  1. *Intra*period problem: Optimal composition of C_s in any s
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- Already solved the second before. Turn to the first

The Intraperiod Problem

- Definition: The **Consumption-Based Price Index**, P, is the minimum expenditure $C_T + pC_N$ required to set $C = \Omega(C_T, C_N) = 1$, given p i.e.

$$P = \min_{C_T, C_N} C_T + pC_N$$

$$\text{s.t. } \Omega(C_T, C_N) \geq 1$$

Finding P

- Recall the demand functions implied by CES when total expenditure/income is Z

$$C_T = \frac{\gamma Z}{\gamma + (1-\gamma)p^{1-\theta}}, \quad C_N = \frac{p^{-\theta}(1-\gamma)Z}{\gamma + (1-\gamma)p^{1-\theta}}$$

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- Demand functions come from utility-max $\iff$ cost-minimization
- If $Z = P$, then we know that $C = 1$ i.e.

$$1 = \left[\gamma^{\frac{1}{\theta}} \left(\frac{\gamma P}{\gamma + (1-\gamma)p^{1-\theta}}\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} \left(\frac{p^{-\theta}(1-\gamma)P}{\gamma + (1-\gamma)p^{1-\theta}}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

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- Problem shrinks to one good (C) in one price (P)!
- Intertemporal problem easy now!

Implications

- Plug $Z = C[\gamma + (1-\gamma)p^{1-\theta}]^{\frac{1}{1-\theta}}$ into demand to see how consumption shares depend on prices and price-index

$$\frac{C_T}{C} = \gamma \left(\frac{1}{P}\right)^{-\theta}, \quad \frac{C_N}{C} = (1-\gamma)\left(\frac{p}{P}\right)^{-\theta}$$

- When $\theta \to 1$ and we go to Cobb-Douglas utility, we get

$$P = (1)^{\gamma} p^{1-\gamma}$$

which we used in our analysis of exchange rates

The Intertemporal Problem

- Consumer wants to maximize $\sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$ s.t. a lifetime budget constraint:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} P_s C_s = (1+r)Q_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (w_s L_s - G_s)$$

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- The NPV of all investment and returns on capital/foreign assets is reflected in Q_t
- Alternatively, write period by period BC into utility direction

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u\left(\frac{(1+r)Q_s - Q_{s+1} + w_s L_s - G_s}{P_s}\right)$$

- Recall that both K and B yield return r in equilibrium: Decomposition of Q doesn't matter for individual consumer

Solving

- FOC(Q_{s+1}) reveals Euler equation

$$\frac{u'(C_s)}{P_s} = \beta(1+r)\frac{u'(C_{s+1})}{P_{s+1}}$$

  scaled by prices in each period
- Remember, P_s determined by p_s, in turn determined by r, $A_{T,s}$, and $A_{N,s}$

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- At end of day, not any harder to solve than standard model with one sector!