3 Nordhaus: EJ, 'To Slow or Not to Slow: The Economics of the Greenhouse Effect' 1991

Basic concepts for analyzing policies for slowing climate change.

Modeling the climate. When CO_2 is emitted, there is an instaneous effect on temperature, but also a delayed contribution. Convert the other GHGs into CO_2 equivalents. Model them together.

- $T(t)$ = temperature relative to 1860.
- $M(t)$ =stock of GHGs in atmospheric (billions of tons of CO_2).
- $E(t)$ =emissions of CO_2 equivalent GHGs.
- $g(\cdot)$ = equilibrium temperature relative to 1860 caused by increasing CO_2 concentration.
- α = delay parameter of temperature in response to radiative increase (per year). He cites 2 estimates of $\alpha = 0.0181, \alpha = 0.013$, but uses $\alpha = 0.02$.
- β = fraction of CO_2 equivalent emissions that enter the atmosphere. He uses $\beta = 0.5$.
- δ = rate of removal of GHGs from atmosphere (per year). Some goes through the sky, others sink into the ocean. He uses $\delta = 0.005$.

$$\dot{T}(t) = \alpha\left[g(M(t)) - T(t)\right] \tag{1}$$
$$\dot{M}(t) = \beta E(t) - \delta M(t) \tag{2}$$

3.1 Cost-Benefit Analysis (CBA)

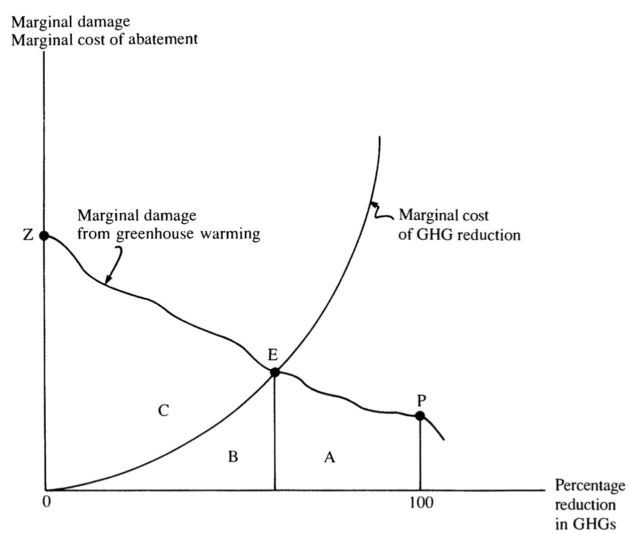

Fig. 1. Marginal costs of GHG reductions and marginal damage from GHG emissions.

3.2 Instruments

1. Pigouvian tax is a tax assessed against firms or individuals whose activities generate negative externalities. Here, a carbon tax. Tax the offending activity (GHG emissions). Redistribute the tax to those who pay the tax. Compensation calibrated to offset loss of welfare at the margin.
2. Command and control. Direct regulation of firms
3. Subsidize alternative energy (solar, ethanol)
4. Cap and trade.

3.3 Simplifications

3.3.1 Static efficiency and concept of cost effectiveness

1. Let there be n firms. R_i is firm $i's$ emissions reduction. R is the total desired emissions reduction. The least cost reduction (efficient cost reduction) in a static world is for every firm to reduce emissions by the same amount at the margin.
$$\frac{\partial C_i}{\partial R_i} = \lambda$$
All emitters face the same marginal cost of reduction (abatement). This is known as the cost-effective solution. Why? Suppose each firm faces quadratic costs of abatement
$$\begin{aligned}
C_i &= \alpha_i R_i + \beta_i R_i^2 \\
\min_{R_i} \sum_{i=1}^{n} C_i\left(R_i\right) &= \min_{R_i} \sum_{i=1}^{n} \left(\alpha_i R_i + \beta_i R_i^2\right) \quad \text{s.t. } R = \sum_{i=1}^{n} R_i \\
L &= \sum_{i=1}^{n} \left(\alpha_i R_i + \beta_i R_i^2\right) - \lambda \left(\sum_{i=1}^{n} R_i - R\right) \\
\frac{\partial L}{\partial R_i} &= \alpha_i + \beta_i R_i - \lambda = 0 \\
\lambda &= \alpha_i + \beta_i R_i = \frac{\partial C_i}{\partial R_i}
\end{aligned}$$
2. Now suppose we impose emissions tax τ per unit of emissions on firms. Each unit of emissions reduction R_i lowers the tax burden by τ. Here, it is efficient for the marginal cost of abatement to equal τ,
$$\frac{\partial C_i}{\partial R_i} = \tau$$
Why? For every unit of abatement, it costs the firm $\partial C_i / \partial R_i$, but it also reduces taxes by τ.The firm's problem is
$$\min_{R_i} C_i\left(R_i\right) - \tau R_i = \min_{R_i} \alpha_i R_i + \beta_i R_i^2 - \tau R_i$$
3. Now, suppose we have tradable permits (cap and trade). Let p be the price of permit. Replace τ with p in (2) above, because the firm's problem is
$$\min_{R_i} C_i\left(R_i\right) - p R_i$$

3.3.2 Note to self:

1. Let $k(t)$ be the state and $x(t)$ be the control. Hamiltonian for the problem

$$\max_{x(t)} \int_t^T u(k(\tau), x(\tau), \tau)\, d\tau \quad \text{s.t. } \dot{k}(t) = f(k(t), x(t), t)$$

is

$$H = u(k(t), x(t), t) + \lambda(t) f(k(t), x(t), t)$$

with first-order conditions

$$\begin{aligned} \frac{\partial H}{\partial x} &= 0 \\ \frac{\partial H}{\partial k} &= \dot{\lambda}(t) \\ \frac{\partial H}{\partial \lambda} &= \dot{k}(t) \end{aligned}$$

2. Consider this problem:

$$\max_u \int_0^\infty e^{-\rho t} f(x, u, t) \quad \text{s.t. } \dot{x}(t) = g(x, u, t)$$

The (present value) Hamiltonian and first-order conditions

$$H = e^{-\rho t} f(x, u, t) + \lambda(t) g(x, u, t)$$

$$\frac{\partial H}{\partial u} = e^{-\rho t} f_u + \lambda g_u = 0 \tag{3}$$

$$\frac{\partial H}{\partial x} = e^{-\rho t} f_x + \lambda g_x = -\dot{\lambda}(t) \tag{4}$$

$$\dot{x}(t) = g(x, u, t) \tag{5}$$

$\lambda(t)$ is the shadow price of $x(t)$ discounted to time 0, and is called the present value multiplier. Rewrite in terms of the current value Hamiltonian. First, define a new costate variable

$$\mu(t) = e^{\rho t} \lambda(t)$$

which is the shadow price of $x(t)$ evaluated at t. It's the current value multiplier. Take the time derivative,

$$\dot{\lambda}(t) = \frac{d\left(e^{-\rho t} \mu(t)\right)}{dt} = e^{-\rho t} \dot{\mu}(t) - \rho e^{-\rho t} \mu(t) \tag{6}$$

Multiply the focs (3) and (4) by $e^{\rho t}$ and substitute $\dot{\lambda}(t)$ from (6). These are the focs when stated in terms of the current value multiplier.

$$\begin{aligned} f_u + \underbrace{e^{\rho t} \lambda}_{\mu} &= f_u + \mu g_u = 0 \\ f_x + \underbrace{e^{\rho t} \lambda}_{\mu} g_x &= f_x + \mu g_x = \underbrace{-\dot{\mu}(t) + \rho \mu(t)}_{-e^{\rho t} \dot{\lambda}(t)} \\ \dot{x}(t) &= g(x, u, t) \end{aligned}$$

They are associated with the current value Hamiltonian,

$$H = f(u, x, t) + \mu(t) g(u, x, t) \tag{7}$$

with focs

$$\frac{\partial H}{\partial u} = 0 \tag{8}$$

$$\frac{\partial H}{\partial x} = -\dot{\mu}(t) + \rho\mu(t) \tag{9}$$

$$\frac{\partial H}{\partial \mu} = \dot{x}(t) \tag{10}$$

3.3.3 Dynamic Efficiency

Modeling damage. Emissions add to GHG concentrations. Welfare should be reduced by higher GHG concentrations. Possible channels, as laid out in Nordhaus (A question of balance), drawing from existing empirical studies.

- Accelerated capital depreciation
- Damages to the agricultural sector
- Costs of sea-level rise
- Adverse impacts on health
- Potential costs of catastrophic damages

Nordhaus (EJ article) models the welfare effect by explicitly modeling effective consumption C as decreasing in temperature T, which is increasing in the stock of atmospheric GHGs M. We going to simplify a bit. Here, the planner's problem is to choose consumption $C(t)$ and emissions $E(t)$. A higher stock of GHGs M hurts welfare. There is economic damage in the sense that emissions lowers output $F_E < 0$.

1. Write the planner's problem as

$$\max_{C(t),E(t)} \int_t e^{-\rho t} U \left[C(t), \overset{(-)}{M}(t) \right] dt$$

   subject to

$$\begin{aligned} \dot{K}(t) &= Y(t) - C(t) = F[K(t), E(t)] - C(t) \\ \dot{M}(t) &= E(t) - \delta M(t) \end{aligned}$$

2. $Y(t)$ is net output, after accounting for economic damages of emissions. The state variables are M and K. The depressing effect of GHGs on welfare is written in general functional form. In this fomulation, $\kappa \geq 0$ and $\mu \leq 0$.
3. κ is the shadow value of consumption. If I get another unit of consumption, I can consume it or invest it. In equilibrium, the marginal benefit of investing equals the marginal cost of not consuming today.
4. μ is the shadow value of emissions. What would I pay for additional emissions? Not positive. You should pay me something to put up with additional emissions, because they are bads, not goods. Think of μ as the SCC in this model.
5. The current value Hamiltonian

$$\begin{aligned} H &= U[C(t), M(t)] + \kappa \dot{K}(t) + \mu \dot{M}(t) \\ &= U[C(t), M(t)] + \kappa [F[K(t), E(t)] - C(t)] + \mu [E(t) - \delta M(t)] \end{aligned}$$

6. First-order conditions

$$\begin{aligned}
\frac{\partial H}{\partial C} &= \frac{\partial U}{\partial C} - \kappa = 0 \\
&\rightarrow \underbrace{\kappa = \frac{\partial U}{\partial C}}_{(a)} \\
\frac{\partial H}{\partial E} &= \kappa \frac{\partial F}{\partial E} + \mu = 0 \\
&\rightarrow \underbrace{\frac{\partial F}{\partial E} = -\frac{\mu}{\kappa}}_{(b)} \\
\frac{\partial H}{\partial M} &= \rho \mu \left(t \right) - \dot{\mu} \\
&\rightarrow \underbrace{\frac{\dot{\mu}}{\mu} = \rho + \delta - \frac{1}{\mu} \frac{\partial U}{\partial M}}_{(c)} \\
\frac{\partial H}{\partial K} &= \rho \kappa \left(t \right) - \dot{\kappa} \\
&\rightarrow \underbrace{\frac{\dot{\kappa}}{\kappa} = \rho + \frac{\partial F}{\partial K}}_{(d)}
\end{aligned}$$

7. $\mu < 0$ is the shadow price of emission. Think of it as what we want to be paid as compensation for allowing another unit of emission. $\dot{\mu}/\mu > 0$ means $\dot{\mu} < 0$. Shadow price grows in magnitude over time. Why? Because the stock of GHGs, is getting bigger over time so additional emissions cause more harm. If we go back in time, a marginal emission in 1860 has no effect so $|\mu| \simeq 0$, but when temperature has increased $4^o C$, marginal emissions are very harmful.

8. Term (b) says the marginal cost of abatement $\partial F/\partial E$ equals the shadow price of abatement μ normalized by marginal utility κ, which converts the marginal cost of abatement $\partial F/\partial E$ into units of money.

9. The planner sets emissions such that the marginal cost of emissions reduction equals the shadow price of emissions. Term (c) says shadow price of emissions grows at discount rate ρ plus absorption rate δ minus the marginal damage $\partial U/\partial M$ divided by the shadow price μ. This makes the last term unit-less, just like ρ and δ. This last term $\frac{1}{\mu}\frac{\partial U}{\partial M}$ measures how fast the worsening climate hurts welfare.

10. $\downarrow \rho$, $\downarrow |\partial U/\partial M| \Longrightarrow \downarrow \dot{\mu}/\mu \Longrightarrow \uparrow \mu$ (becomes less negative). That is, if the future becomes less important, or GHG concentrations are less damaging to welfare, the shadow price increases (the amount of compensation for emissions decreases).