International Macroeconomics
Lecture 5: Growth in Open Economies

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April 7th, 2016

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- Start here, then think about implications of opening borders

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  5. Constant, exogenous growth
     - Labor Force: $L_{t+1} = (1 + n)L_t$
     - Labor Productivity: $E_{t+1} = (1 + g)E_t$

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$$\rightarrow k_{t+1}^E (1+n)(1+g) = (1-\delta)k_t^E + sf(k_t^E)$$

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- If we define $1 + z = (1 + n)(1 + g)$,

$$k^E_{t+1} = \frac{1-\delta}{1+z} k^E_t + \frac{sf(k^E_t)}{1+z}$$

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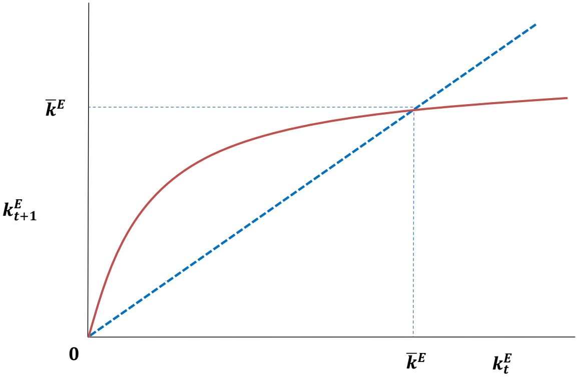
- When $F(K_t,E_tLT)=K_t^\alpha(E_tL_t)^{1-\alpha}$, then

$$\bar{k}^E=\left(\frac{s}{z+\delta}\right)^{\frac{1}{1-\alpha}}$$

Solow Model Graph



Blue dashed: 45-degree line
Red line: Solow Law of Motion

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4. Whole economy grows at $\approx g + n$

$$Y_t = E_t L_t \left(\frac{s}{z + \delta} \right)^{\frac{\alpha}{1-\alpha}}$$

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- Back out consumption in SS

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- FOC(s) tells optimal saving to maximize $\bar{c}^E$

$$s^\star = \alpha$$

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- For simplicity, set $\delta = 0$. Per-period resource constraint

$$K_{t+1} + C_t = F(K_t, E_t L_t) + K_t$$

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- Dynamic system for (k_t, c_t)!

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- With this assumption, we can normalize the system by E_t as well: System becomes

$$(1) \quad \frac{c^E_{t+1}}{c^E_t} = \frac{\beta^\sigma [1 + f'(k^E_{t+1})]^\sigma}{1+g}$$

$$(2) \quad k^E_{t+1} - k^E_t = \frac{f(k^e_t) - c^E_t}{1+z} - \frac{z}{1+z} k^e_t$$

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- Saddle-path dynamics to SS

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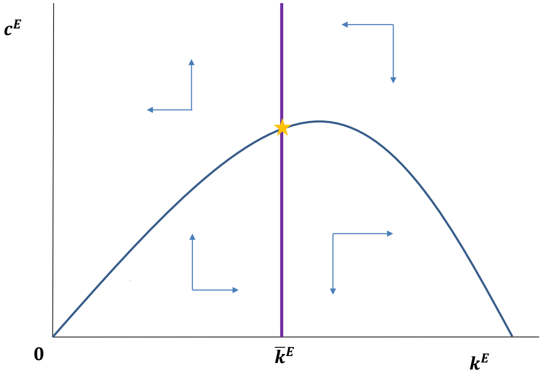

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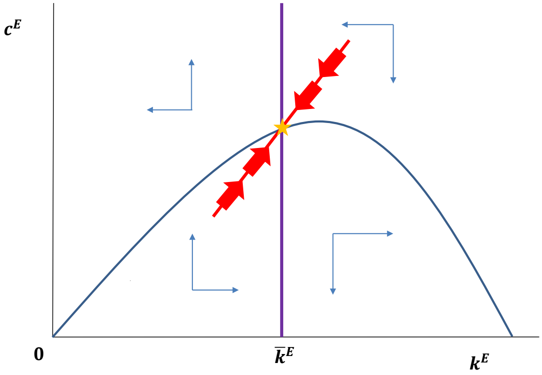

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   - In LR, converge to $\bar{c} < c^\star$

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- Underscores that immigration generally must have adverse effect on some groups, but not whole economy

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- $F(H, L)$ assumed to be CRTS
- Human capital analogous to physical capital
  1. Must forgo current consumption to accumulate it in following period (education)
  2. Undergoes some depreciation δ; requires constant replenishing

HH Income

- In each period, *each* household supplies one unit of unskilled labor and brings h_t^N units of human capital to production process
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- Euler Equation in this environment

$$\frac{c_{t+1}^N}{c_t^N} = \beta(1 + w_{s,t+1} - \delta)$$

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- Assume competitive markets: Wages equal to marginal product
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- Exactly like NCG model: Goes to SS along saddle path

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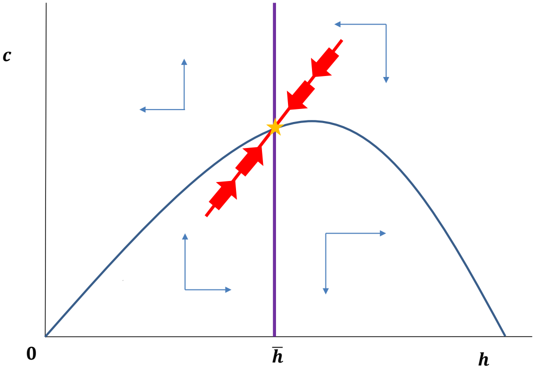

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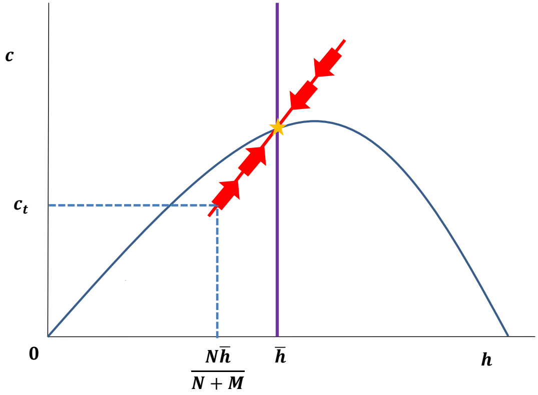
- On graph, implies that h_t drops from $\bar{h}$

'Immigration Shock'



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    3. Since F is concave, it must be that

$$\frac{F(N\bar{h}, N+M) - F(N\bar{h}, N)}{M} > F_L(N\bar{h}, N+M)$$

i.e. Gains are bigger than the losses

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- Under 'Golden Rule' saving ($s = \alpha$) and if $z = r$, then SS is same
  - Even here, though, the *speed of convergence* is very different

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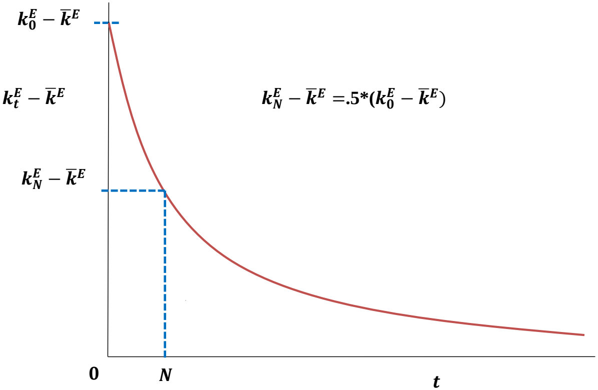
  - Rate of convergence: $1 - \mu$
  - Half-life: N such that $x_{t+1} - \bar{x} = \frac{1}{2}(x_t - \bar{x})$

$$\to N = \frac{\ln(2)}{-\ln(\mu)} \approx \frac{.7}{1-\mu}$$

Convergence with Exponential Decay



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  3. Across all countries: No chance

Convergence: US States

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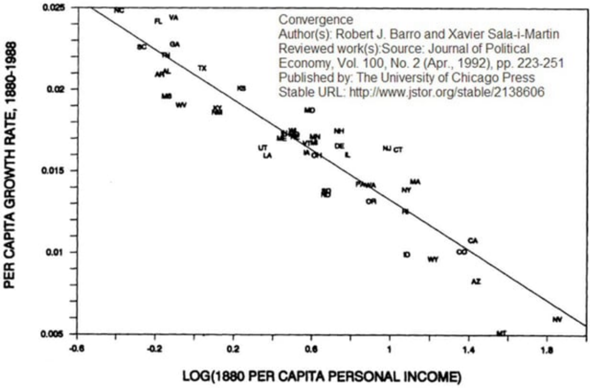
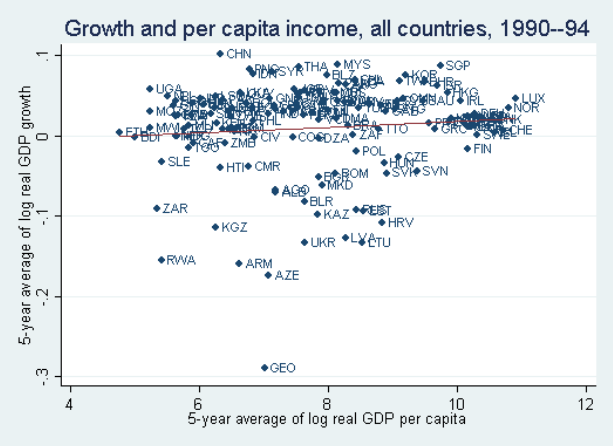


Fig. 1.—Growth rate from 1880 to 1988 vs. 1880 per capita income

Convergence: Heterogeneous Countries



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$$(1-\tau_i)f'(\bar{k}^E) = r + \delta$$

$$\rightarrow \bar{k}_t = E_t \left(\frac{(1-\tau_i)\alpha}{r+\delta} \right)^{\frac{1}{1-\alpha}}$$

   - Variation in τ_i will imply different levels of capital and output-per-worker

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- Implies a steady-state

$$\bar{k}^E = \left(\frac{s}{z+\delta}\right)^{\frac{1}{1-\alpha}}$$

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- We will linearize below around $\bar{k}^E$

$$k_{t+1}^E - k_t^E = \frac{s(k_t^E)^\alpha}{1+z} - \frac{\delta + z}{1+z} k_t^E$$

Linearizing

$$k_{t+1}^E - k_t^E = \underbrace{\left[\frac{s(\bar{k}^E)^\alpha}{1+z} - \frac{\delta+z}{1+z}\bar{k}^E\right]}_{=0 \ \ By \ \ Definition} + \left[\frac{s\alpha(\bar{k}^E)^{\alpha-1}}{1+z} - \frac{\delta+z}{1+z}\right] \times \left[k_t^E - \bar{k}^E\right]$$

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- Move k_t^E to RHS and subtract $\bar{k}^E$ from both sides

$$k_{t+1}^E - \bar{k}^E = \underbrace{\left[1 + \frac{s\alpha(\bar{k}^E)^{\alpha-1}}{1+z} - \frac{\delta+z}{1+z}\right]}_{\mu} \times \left[k_t^E - \bar{k}^E\right]$$

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- Substitue in $\bar{k}^E$ to get

$$\mu = \frac{1+\alpha z + (\alpha-1)\delta}{1+z} \approx \underbrace{.96}_{In \;\; Data}$$

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  - If we started from k_t^E far away from $\bar{k}^E$, it should be much faster
- Even closed economy (no capital flows) converges too fast!
  - Need something else...(next time)

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- Study one example: Human and physical capital
  - Capital inflows can only finance physical capital
  - Human and physical capital complementary: Both important for convergence
  - Barriers to human capital $\rightarrow$ Physical capital less productive $\rightarrow$ Less physical capital investment $\rightarrow$ Slower accumulation of physical capital

Environment

- Production involves labor, physical capital, and human capital
- Continue to assume Cobb-Douglas

$$Y_t = K_t^{\alpha} H_t^{\phi} (E_t L_t)^{1-\alpha-\phi}$$

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- Assume only human capital depreciates: δ
- Can borrow abroad to finance *physical capital alone*

$$-B_t \leq K_t$$

- Limited-Commitment: If country/agents default, creditors can seize K_t (collateral)
- Won't let them borrow at risk-free rate more than can be seized

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- Resource constraint (again, solving SPP with no prices)

$$I_{H,t} + K_{t+1} + C_t + B_{t+1} = Y_t + K_t + (1+r)B_t$$

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- Further, can substitute into production function: $y_t^E = \chi(h_t^E)^{\nu}$, where

$$\chi = \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}, \quad \nu = \frac{\phi}{1-\alpha}$$

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- One more simplifying assumption: Constant Solow saving rule on human capital accumulation:

$$H_{t+1} = (1-\delta)H_t + s \times \underbrace{(Y_t - rK_t)}_{GNP}$$

- Same as Solow model! Normalize to get

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$$h^E_{t+1} = \frac{1-\delta}{1+z}h^E_t + \frac{s(1-\alpha)\chi}{1+z}\left(h^E_t\right)^\nu$$

Convergence

- Repeat Solow convergence exercise on new system

$$h^E_{t+1} - h^E_t = \frac{s'(h^E_t)^{\nu}}{1+z} - \frac{z+\delta}{1+z} h^E_t$$

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$$\bar{h}^E = \left(\frac{s'}{z+\delta}\right)^{\frac{1}{1-\nu}}$$

- Linearize around $\bar{h}^E$ to get

$$h_{t+1}^E - \bar{h}^E = \mu'(h_t^E - \bar{h}^E)$$

where

$$\mu' = \frac{1+\nu z + (\nu-1)\delta}{1+z}$$

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- Intuition: Even though *can* accumulate capital from abroad, do so at a slow rate
    - Capital and human capital complementary
    - Each slow down accumulation of other