Problem Set 3
Econ 40357 Financial Econometrics
University of Notre Dame
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FALL 2020

1. Let y_t be a 3×1 vector of variables. We model it as a VAR(2),

$$y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + \epsilon_t$$

   (a) Write out all the equations of the VAR in full, carefully defining any new notation that you use.

   Let $y_t = \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix}, A_j = \begin{pmatrix} a_{j,11} & a_{j,12} \\ a_{j,21} & a_{j,22} \end{pmatrix}$ for $j = 1, 2, A_0 = \begin{pmatrix} a_{0,1} \\ a_{0,2} \end{pmatrix}$ and $\epsilon_t = \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}$. Then

$$y_{1t} = a_{0,1} + a_{1,11} y_{1t-1} + a_{1,12} y_{2t-1} + a_{2,11} y_{1t-2} + a_{2,12} y_{2t-2} + \epsilon_{1t}$$
$$y_{2t} = a_{0,2} + a_{1,21} y_{1t-1} + a_{1,22} y_{2,t-1} + a_{2,12} y_{1t-2} + a_{2,22} y_{2t-2} + \epsilon_{2t}$$

   (b) Louie and Kai use the same set of data but work independently. They arrive at different lag lengths for the VAR. Describe and evaluate two methods for determining the appropriate lag length.

   Use the information criteria method. For AIC, the chosen lag length p is that which minimizes $AIC = 2\ln|\hat{\Sigma}_p| + 2k/T$, where k is the number of slope coefficient in the system, $2\ln|\hat{\Sigma}_p|$ is the negative of the log likelihood function, and T is the sample size. For BIC, the chosen lag length p is that which minimizes $BIC = 2\ln|\hat{\Sigma}_p| + k\ln(T)/T$.

2. Define the following terms and describe the processes they represent

   (a) Weak (covariance) stationarity Let $\{y_t\}$ be the stochastic process in question. It will be weakly stationary if it has finite first and second moments (mean and variance), and if $cov(y_t, y_{t-k})$ is constant and depends only on k

   (b) Strict stationarity

   $\{y_t\}$ is strictly stationary if each observation is drawn from the same distribution.

   (c) Deterministic trend

   There is a part of $\{y_t\}$ that evolves as a deterministic function of time. For example, z_t, the deviation of y_t from a time trend,

$$y_t - (\alpha_0 + \alpha_1 t) = z_t$$

   is stationary.

(d) Stochastic trend

`This arises when the original time series has a unit root, and the mean change is not zero. The mean change is called the drift, and it creates a stochastic trend in the original series.`

$$\begin{aligned} y_t &= \mu + y_{t-1} + \epsilon_t \\ &= y_0 + \mu t + \epsilon_t + \epsilon_{t-1} + \cdots + \epsilon_1 \end{aligned}$$

3. Louie wants to test for a unit root in some time-series data. He uses the ADF test and runs the regression

$$\Delta y_t = a_0 + \beta y_{t-1} + \gamma_1 \Delta y_{t-1} + \gamma_2 \Delta y_{t-2} + \epsilon_t$$

He obtains regression output $\hat{\beta} = -0.24$,$\text{se}\left(\hat{\beta}\right) = 0.12, \hat{\gamma}_1 = 0.04$ $\text{se}(\hat{\gamma}_1) = 0.22, \hat{\gamma}_2 = 0.10$, $\text{se}(\hat{\gamma}_2) = 0.10$.

(a) Which coefficient should he use to form the t-ratio to do the test? `Louie should look at` $\hat{\beta}$

(b) The 5% critical value for the test is -2.86. What is the conclusion from this test? $t = -0.24/0.12 = -2.0$. `Cannot reject the hypothesis that` y_t `has a unit root.`

(c) What should be the next step?

`Consider a data transformation to induce stationarity. Usually, we would consider differencing the observatons. Check to see if` Δy_t `has a unit root. Louie would then run the ADF test on the regression,`

$$\Delta^2 y_t = \alpha_0 + \beta \Delta y_{t-1} + \gamma_1 \Delta^2 y_{t-1} + \gamma_2 \Delta^2 y_{t-2} + \epsilon_t$$

4. Use the Eviews workfile ps03.wf1, sheet CPI_Venezuela. The variable P is the CPI. For $\ln(P)$,use the ADF test to answer the following.

(a) Is $\ln(P)$ stationary or not? Explain your answer.

`Testing for unit root in` $\ln(P)$ `gives ADF test statistic` 2.432. `Cannot reject`

(b) Is $\Delta \ln(P)$ stationary or not? Explain. What do we typically call $\Delta \ln(P)$?

`Testing for unit root in` $\Delta \ln(P)$ `gives ADF test statistic 4.095. Cannot reject`

(c) Is $\Delta^2 \ln(P) = \Delta(\Delta \ln(P))$ stationary or not? Explain

`Testing for unit root in` $\Delta^2 \ln(P)$ `gives ADF test statistic of 0.364. Cannot reject`

5. Use the Eviews workfile ps03.wf1, sheet vars_lps_climateexra. q_cyp is the logarithm of the real U.S. dollar price of the Cypriot pound, spliced to the euro in 1999. clim_factor is the global temperature shock. dq_cyp is the change in q_cyp

Estimate a VAR(2) for dq_cyp and clim_factor.

(a) Does dq_cyp Granger cause clim_factor? Does clim_factor Granger cause dq_cyp? Which variable is econometrically exogenous to the other?

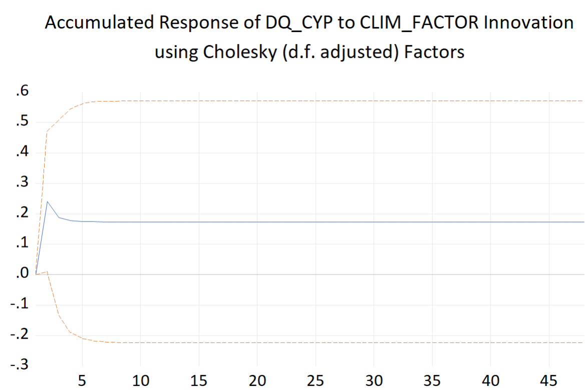
`It is fine with me if you simply looked at the t-ratios on lagged dq_cyp in the clim_factor equation (0.469 and 0.745) and said they are not significant concluding dq_cyp does not Granger cause climate factor. If you were enterprising and curious, and clicked view, lag structure, Granger causality, you would have found the F-test results.`

`dq_cyp does not Granger cause clim_factor.`

`clim_factor does Granger cause dq_cyp.`

`clim_factor is econometrically exogenous to dq_cyp`

(b) Generate cumulated impulse responses of dq_cyp to a shock in clim_factor at horizons 1-48. Report the graph of the responses. Comment on what you find.



(c) Generate the impulse response for q_cyp(p)-q_cyp for $p = 1, 12, 24, 36, 48$, by local projection. Comment on what you find, and contrast with your results in part b.

	coeff	t-ratio
R1	2.503	2.149
R12	-0.303	-0.047
R24	-1.484	-0.173
R36	6.536	0.613
R48	20.108	1.752

`The VAR doesn't capture all of the longer-horizon dynamics of the impulse responses.`

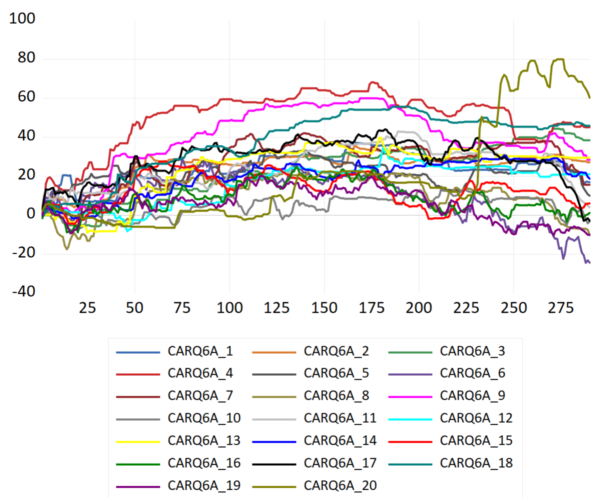
6. Event study. Use the Eviews workfile ps03.wf1, sheet Event. The event in question is the date at which a firm was included in an index. The Eviews workfile contains the $\hat{\epsilon}_{t,i}$, in event time, from the regression

$$r_{t,i} = \hat{\alpha}_i + \hat{\beta}_i r_{t,m} + \hat{\epsilon}_{t,i}$$

for firms $i = 1, ..., 20$. α and β are estimated with observations $1 - 259$ (the pre-event window). Obseveration 260 is the event date. We will use observations $260 - 290$ as the event window. Treat $\hat{\epsilon}_{t,i}$ as the abnormal return $AR_{t,i}$.

From the pre-event window, assume $\epsilon_{t,i} = AR_{t,i} \sim N\left(0, \sigma^2_{ar(i)}\right)$

(a) Plot the cumulated abnormal returns for all 20 firms over the pre-event and the event window. Put them all in one graph.

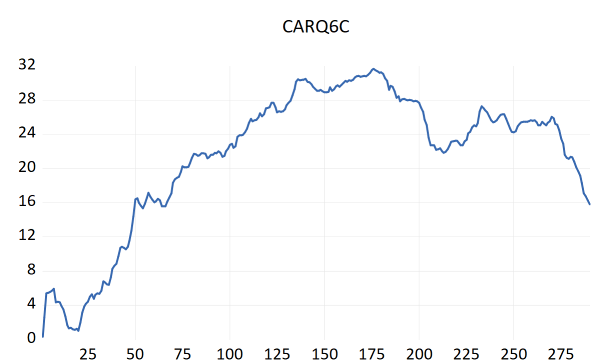


(b) For each firm, individually test the hypothesis that the event had no effect on its cumulated abnormal return.

	CAR	VAR	SCAR
R1	-9.313	1.699	-7.145
R2	-2.335	0.716	-2.759
R3	-1.531	0.927	-1.590
R4	6.303	2.499	3.987
R5	-12.426	1.558	-9.954
R6	-18.565	3.598	-9.787
R7	-21.472	1.556	-17.216
R8	-19.060	1.903	-13.816
R9	-6.295	1.139	-5.899
R10	-11.597	1.658	-9.006
R11	-23.640	1.187	-21.701
R12	-0.898	1.183	-0.826
R13	-0.952	1.457	-0.789
R14	-9.991	0.842	-10.889
R15	-6.475	1.400	-5.472
R16	-3.756	2.590	-2.334
R17	-30.731	1.832	-22.705
R18	0.426	0.798	0.477
R19	-5.349	2.471	-3.403
R20	-18.901	5.313	-8.200
FOR EQ(10)	-9.828	0.091	-32.612

We are calculating the formula in equation (4) from the lecture slides.

(c) Plot the cumulated abnormal returns averaged over all 20 firms, over the pre-event and the event window.



(d) Test the null hypothesis that the event had no effect on the cumulated abnormal return, averaged across all firms.

`We are calculating the formula in equation (10) from the lecture slides.`