Consumption

ECON 30020: Intermediate Macroeconomics

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Readings

- GLS Ch. 8

Microeconomics of Macro

- We now move from the *long run* (decades and longer) to the *medium run* (several years) and *short run* (months up to several years)
- In long run, we did not explicitly model most economic decision-making – just assumed rules (e.g. consume a constant fraction of income)
- Building blocks of the remainder of the course are *decision rules* of *optimizing agents* and a concept of *equilibrium*
- Will be studying optimal decision rules first
- Framework is dynamic but only two periods (t, the present, and $t+1$, the future)
- Consider *representative agents*: one household and one firm
- Unrealistic but useful abstraction and can be motivated in world with heterogeneity through insurance markets

Consumption

- Consumption the largest expenditure category in GDP (60-70 percent)
- Study problem of representative household
- Household receives exogenous amount of income in periods t and $t+1$
- Must decide how to divide its income in t between consumption and saving/borrowing
- Everything real – think about one good as "fruit"

Basics

- Representative household earns income of Y_t and Y_{t+1}. Future income known with certainty (allowing for uncertainty raises some interesting issues but does not fundamentally impact problem)
- Consumes C_t and C_{t+1}
- Begins life with no wealth, and can save $S_t = Y_t - C_t$ (can be negative, which is borrowing)
- Earns/pays real interest rate r_t on saving/borrowing
- Household a price-taker: takes r_t as given
- Do not model a financial intermediary (i.e. bank), but assume existence of option to borrow/save through this intermediary

Budget Constraints

- Two flow budget constraints in each period:

$$C_t + S_t \leq Y_t$$
$$C_{t+1} + S_{t+1} - S_t \leq Y_{t+1} + r_t S_t$$

- Saving vs. Savings: saving is a flow and savings is a stock. Saving is the change in the stock
- As written, S_t and S_{t+1} are stocks
- In period t, no distinction between stock and flow because no initial stock
- $S_{t+1} - S_t$ is flow saving in period $t+1$; S_t is the stock of savings household takes from t to $t+1$, and S_{t+1} is the stock it takes from $t+1$ to $t+2$
- $r_t S_t$: income earned on the stock of savings brought into $t+1$

Terminal Condition and the IBC

- Household would not want $S_{t+1} > 0$. Why? There is no $t+2$. Don't want to die with positive assets
- Household would like $S_{t+1} < 0$ – die in debt. Lender would not allow that
- Hence, $S_{t+1} = 0$ is a terminal condition (sometimes "no Ponzi")
- Assume budget constraints hold with equality (otherwise leaving income on the table), and eliminate S_t, leaving:

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

- This is called the *intertemporal budget constraint* (IBC). Says that present discounted value of stream of consumption equals present discounted value of stream of income.

Preferences

- Household gets utility from how much it consumes
- Utility function: $u(C_t)$. "Maps" consumption into utils
- Assume: $u'(C_t) > 0$ (positive marginal utility) and $u''(C_t) < 0$ (diminishing marginal utility)
- "More is better, but at a decreasing rate"
- Example utility function:

$$u(C_t) = \ln C_t$$
$$u'(C_t) = \frac{1}{C_t} > 0$$
$$u''(C_t) = -C_t^{-2} < 0$$

- Utility is completely ordinal – no meaning to magnitude of utility (it can be negative). Only useful to compare alternatives

Lifetime Utility

- Lifetime utility is a weighted sum of utility from period t and $t+1$ consumption:

$$U = u(C_t) + \beta u(C_{t+1})$$

- $0 < \beta < 1$ is the discount factor – it is a measure of how impatient the household is.

Household Problem

- Technically, household chooses C_t and S_t in first period. This effectively determines C_{t+1}
- Think instead about choosing C_t and C_{t+1} in period t

$$\max_{C_t, C_{t+1}} \quad U = u(C_t) + \beta u(C_{t+1})$$

s.t.

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

Euler Equation

- First order optimality condition is famous in economics – the "Euler equation" (pronounced "oiler")

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$$

- Intuition and example with log utility
- Necessary but not sufficient for optimality
- Doesn't determine *level* of consumption. To do that need to combine with IBC

Indifference Curve

- Think of C_t and C_{t+1} as different goods (different in time dimension)
- Indifference curve: combinations of C_t and C_{t+1} yielding fixed overall level of lifetime utility
- Different indifference curve for each different level of lifetime utility. Direction of increasing preference is northeast
- Slope of indifference curve at a point is the negative ratio of marginal utilities:

$$\text{slope} = -\frac{u'(C_t)}{\beta u'(C_{t+1})}$$

- Given assumption of $u''(\cdot) < 0$, steep near origin and flat away from it

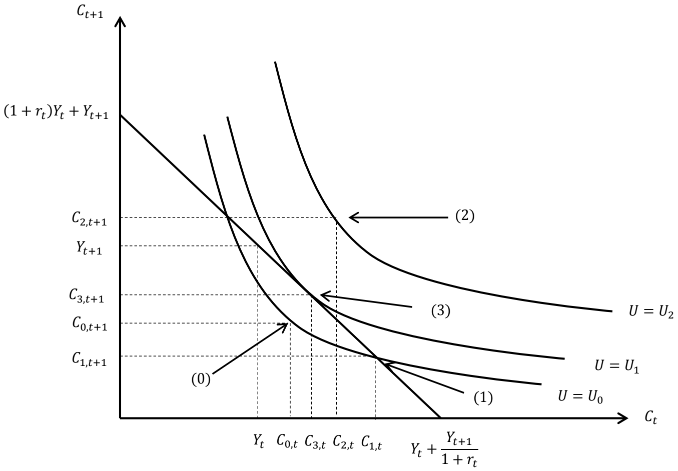
Budget Line

- Graphical representation of IBC
- Shows combinations of C_t and C_{t+1} consistent with IBC holding, given Y_t, Y_{t+1}, and r_t
- Points inside budget line: do not exhaust resources
- Points outside budget line: infeasible
- By construction, must pass through point $C_t = Y_t$ and $C_{t+1} = Y_{t+1}$ ("endowment point")
- Slope of budget line is negative gross real interest rate:

$$\text{slope} = -(1 + r_t)$$

Optimality Graphically

- Objective is to choose a consumption bundle on highest possible indifference curve
- At this point, indifference curve and budget line are tangent (which is same condition as Euler equation)



Consumption Function

- What we want is a *decision rule* that determines C_t as a function of things which the household takes as given – Y_t, Y_{t+1}, and r_t
- Consumption function:

$$C_t = C^d(Y_t, Y_{t+1}, r_t)$$

- Can use indifference curve - budget line diagram to qualitatively figure out how changes in Y_t, Y_{t+1}, and r_t affect C_t

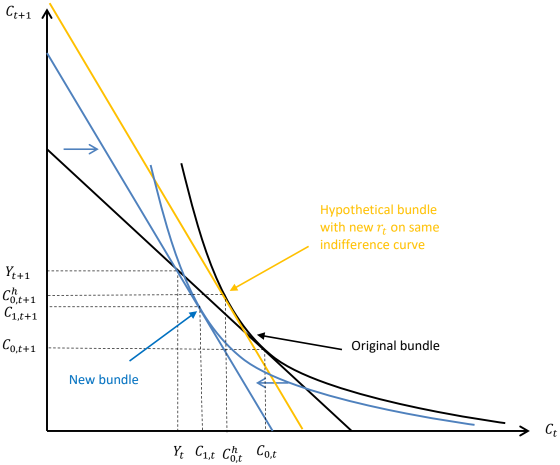
Increases in Y_t and Y_{t+1}

- An increase in Y_t or Y_{t+1} causes the budget line to shift out horizontally to the right
- In new optimum, household will locate on a higher indifference curve with higher C_t and C_{t+1}
- Important result: wants to increase consumption in *both* periods when income increases in *either* period
- Wants its consumption to be "smooth" relative to its income
- Achieves smoothing its consumption relative to income by adjusting saving behavior: increases S_t when Y_t goes up, reduces S_t when Y_{t+1} goes up
- Can conclude that $\frac{\partial C^d}{\partial Y_t} > 0$ and $\frac{\partial C^d}{\partial Y_{t+1}} > 0$
- Further, $\frac{\partial C^d}{\partial Y_t} < 1$. Call this the marginal propensity to consume, MPC

Increase in r_t

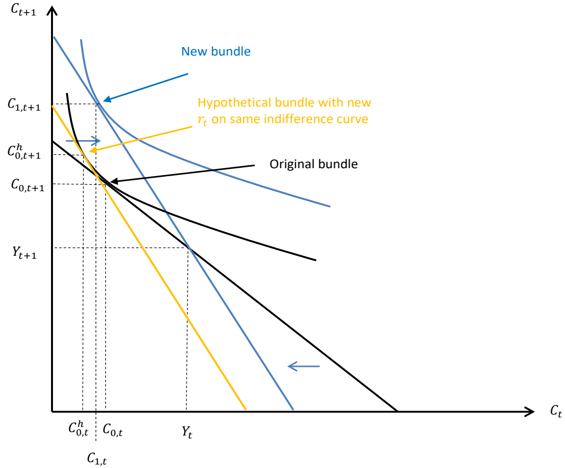
- A little trickier
- Causes budget line to become steeper, pivoting through endowment point
- Competing income and substitution effects:
  - Substitution effect: how would consumption bundle change when r_t increases and income is adjusted so that household would locate on unchanged indifference curve?
  - Income effect: how does change in r_t allow household to locate on a higher/lower indifference curve?
- Substitution effect always to reduce C_t, increase S_t
- Income effect depends on whether initially a borrower ($C_t > Y_t$, income effect to reduce C_t) or saver ($C_t < Y_t$, income effect to increase C_t)

Borrower



- Sub effect: $\downarrow C_t$. Income effect: $\downarrow C_t$
- Total effect: $\downarrow C_t$

Saver



- Sub effect: $\downarrow C_t$. Income effect: $\uparrow C_t$
- Total effect: ambiguous

The Consumption Function

- We will assume that the substitution effect always dominates for the interest rate
- Qualitative consumption function (with signs of partial derivatives)

$$C_t = C(\underset{+}{Y_t}, \underset{+}{Y_{t+1}}, \underset{-}{r_t}).$$

- Technically, partial derivative itself is a function
- However, we will mostly treat the partial with respect to first argument as a parameter we call the MPC

Algebraic Example with Log Utility

- Suppose $u(C_t) = \ln C_t$
- Euler equation is:

$$C_{t+1} = \beta(1 + r_t)C_t$$

- Consumption function is:

$$C_t = \frac{1}{1+\beta}\left[Y_t + \frac{Y_{t+1}}{1+r_t}\right]$$

- MPC: $\frac{1}{1+\beta}$. Go through other partials

Permanent Income Hypothesis (PIH)

- Our analysis consistent with Friedman (1957) and the PIH
- Consumption ought to be a function of "permanent income"
- Permanent income: present value of lifetime income
- Special case: $r_t = 0$ and $\beta = 1$: consumption equal to *average* lifetime income
- Implications:
  1. Consumption forward-looking. Consumption should not react to changes in income that were predictable in the past
  2. MPC less than 1
  3. Longer you live, the lower is the MPC
- Important empirical implications for econometric practice of the day. Regression of C_t on Y_t will not identify MPC (which is relevant for things like fiscal multiplier) if in historical data changes in Y_t are persistent

Applications and Extensions

- Book considers several applications / extensions:
- You are responsible for this material though we will only *briefly* discuss these in class
  1. Wealth (GLS Ch. 8.4.1):
     - Can assume household begins life with some assets other than strict savings (e.g. housing, stocks) and potentially allow household to accumulate more wealth
     - Unsurprising implication: increases in value of wealth (e.g. increase in house prices) can result in more consumption/less saving
  2. Permanent vs. transitory changes in income (GLS Ch. 8.4.2)
     - Household will adjust consumption more (and saving less) to shocks to income the more *persistent* these are (persistent in sense of change in Y_t being correlated with change in Y_{t+1} of same sign)

Consumption Under Uncertainty

- GLS Ch. 8.4.4-8.4.5
- Suppose that future income is *uncertain*
- Suppose it can take on two values: $Y_{t+1}^h \geq Y_{t+1}^l$. Let $p \in [0,1]$ be the probability of the high state and $1-p$ the probability of the low state. Expected value of income is: $E(Y_{t+1}) = pY_{t+1}^h + (1-p)Y_{t+1}^l$
- Everything dated t is known
- Period $t+1$ budget constraint must hold in both states of the world:

$$
\begin{aligned}
C_{t+1}^h &\leq Y_{t+1}^h + (1+r_t)S_t \\
C_{t+1}^l &\leq Y_{t+1}^l + (1+r_t)S_t
\end{aligned}
$$

- Uncertainty of future income translates into uncertainty over future consumption

Expected Utility

- Expected lifetime utility:

$$E(U) = u(C_t) + \beta \times \left[pu(C_{t+1}^h) + (1-p)u(C_{t+1}^l) \right]$$

- This is equivalent to:

$$E(U) = u(C_t) + \beta E\left[u(C_{t+1})\right]$$

- Key insight: expected value of a function is *not* equal to the function of expected value (unless the function is linear)

Euler Equation

- Euler equation looks almost same under uncertainty but has expectation operator:

$$u'(C_t) = \beta(1 + r_t)E\left[u'(C_{t+1})\right]$$

- With log utility:

$$\frac{1}{C_t} = \beta(1 + r_t)\left[p\frac{1}{C_{t+1}^h} + (1 - p)\frac{1}{C_{t+1}^l}\right]$$

- Precautionary saving: if $u'''(\cdot) > 0$, then $\uparrow$ uncertainty over future income results in $\downarrow C_t$

Random Walk Hypothesis

- Continue to allow future income to be uncertain
- But instead assume that $u'''(\cdot) = 0$ (no precautionary saving). Further assume that $\beta(1 + r_t) = 1$. Then Euler equation implies:

$$E\left[C_{t+1}\right] = C_t$$

- Consumption *expected* to be *constant* – simple implication of desire to smooth consumption applied to model with uncertainty
- Consumption ought not react to changes in Y_{t+1} which were *predictable* from perspective of period t:
  - e.g. retirement, Social Security withholding throughout year
  - After Hall (1978), this is one of the most tested implications in macroeconomics
  - Generally fails – potential evidence of liquidity constraints (GLS Ch. 8.4.6)