UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2019
D. Romer

Problem Set 2
Due at the start of lecture Tuesday, September 17

1. In analyzing the Solow model in reading and lecture, we linearized the equations of motion for k and y around k^* and y^*. In many contexts, however, it is more helpful to work with loglinearized than with linearized systems. Thus: Linearize the equation of motion for $\ln k$ around $\ln k^*$, and simplify the resulting expression as much as possible.

2. Romer, Problem 2.3.

3. Romer, Problem 9.4. (Note: "9" is not a typo.)

4. (From last year's midterm.) Consider the optimization problem of a representative household in the Ramsey-Cass-Koopmans model. For simplicity, population growth and technological progress are both absent (so $n = g = 0$). As usual, the consumer supplies 1 unit of labor inelastically at each point in time, and so labor income at time t is $W(t)$. Thus, in the absence of any complications, the consumer's objective function would be

(1) $$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta}}{1-\theta} dt, \quad \rho > 0, \quad \theta > 0,$$

and the consumer's flow budget constraint would be

(2) $$\dot{D}(t) = r(t)D(t) + W(t) - C(t).$$

In addition, $D(0)$ (the consumer's initial assets) would be given, and the consumer would have to satisfy its lifetime budget constraint, which would imply

(3) $$\lim_{t\to\infty} e^{-R(t)}D(t) \geq 0,$$

where, as usual,

(4) $$R(t) \equiv \int_{\tau=0}^{t} r(\tau)d\tau.$$

This problem asks you to consider a variation on this set-up: <u>the price that the consumer pays for consumption goods at time t, $P(t)$, rather than always being equal to 1, is an increasing function of the consumer's consumption</u>: $P(t) = P\big(C(t)\big), P(\cdot) > 0, P'(\cdot) \geq 0$. Thus the total amount the consumer pays at time t is $P\big(C(t)\big)C(t)$ rather than just $C(t)$. (Intuitively, imagine that when the consumer is consuming more, they have less time to shop, and so end up paying more on average for what they buy.)

a. How, if at all, does this change in assumptions affect equations (1)–(4)?

b. What is the Hamiltonian? (You are welcome to use either the current-value or the present-value Hamiltonian, but please state which you are using.)

c. Find the conditions for optimality.

d. Use your results to find an expression for $\dot{C}(t)/C(t)$ analogous to our usual expression, which would be (since $n = g = 0$) $\dot{C}(t)/C(t) = [r(t) - \rho]/\theta$. (Hint: Your answer should simplify to $\dot{C}(t)/C(t) = [r(t) - \rho]/\theta$ in the special case $P(C(t)) = 1$ for all $C(t)$.)

EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

5. Show that on the balanced growth path of the Solow model, $K/Y = s/(n + g + \delta)$.

6. Consider an economy described by the Solow model that is on its balanced growth path. Assume that the saving rate is s_0. Now suppose that from time t_0 to time t_1, the saving rate rises gradually from s_0 to s_1 (where $s_1 > s_0$), and then remains at s_1.

Sketch the resulting path over time of log output per worker. For comparison, also sketch on the same graph: (i) the path that log output per worker would have followed if the saving rate had remained at s_0; (ii) the path that log output per worker would have followed if the saving rate had jumped discontinuously from s_0 to s_1 at time t_0 (and remained at s_1).

Explain your answer.

7. Romer, Problem 1.12.

8. (From an old final exam.) Consider an infinitely-lived household. The household's initial wealth, $A(0)$ is zero; its labor income is constant and equal to $\bar{Y}, \bar{Y} > 0$; and the real interest rate is constant and equal to $\bar{r} > 0$. The household's flow budget constraint is therefore $\dot{A}(t) = \bar{r}A(t) + \bar{Y} - C(t)$, and, as usual, the present discounted value of the household's consumption cannot exceed the present discounted value of the its lifetime resources.

In contrast to our usual model, however, the household obtains utility not only from consumption, but also from holding wealth. Specifically, its objective function is

$$\int_{t=0}^{\infty} e^{-\rho t}[u(C(t)) + v(A(t))]dt,$$

where $u'(\bullet) > 0$, $u''(\bullet) < 0$, $v'(\bullet) > 0$, $v''(\bullet) < 0$, and $\rho > 0$.

a. For this part only, assume $\rho = \bar{r}$. Without doing any math, explain whether $C(0)$ will be less than, equal to, or greater than $\bar{Y}$, or whether it is not possible to tell.

b. What is the current value Hamiltonian?

c. Find the conditions that characterize the solution to the household's maximization problem. Use them to find an expression for $\dot{C}(t)/C(t)$ that does not involve the costate variable.

9. Consider an infinitely-lived household maximizing the utility function $\int_{t=0}^{\infty} e^{-\rho t}U(C(t))dt$ subject to the usual intertemporal budget constraint. Let $r(t)$ denote the real interest rate at t and let $R(t) \equiv \int_{\tau=0}^{t} r(\tau)d\tau$. Then the Euler equation relating consumption at two dates, A and B ($B > A$) is:

A. $\frac{\dot{C}(A)}{C(A)} = \frac{[r(t)-\rho]}{\theta}$.

B. $\frac{\dot{C}(A)}{C(A)} = \frac{\dot{C}(B)}{C(B)}$.

C. $U'(C(A)) = \left[\frac{e^{R(B)-R(A)}}{e^{\rho(B-A)}}\right] U'(C(B))$.

D. $U'(C(A)) = \left[\frac{[r(B)-r(A)]}{[\rho(B-A)]}\right] U'(C(B))$.

10. Romer, Problem 2.2.

11. Romer, Problem 2.4.

12. (Note: This problem is challenging.) Romer, Problem 1.14.