International Macroeconomics
Lecture 3: Nominal Exchange Rates

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    1. Unit of account
    2. Medium of exchange
    3. Store of value
- Other, less discussed reasons
    1. Alternative source of government finance (seignorage)
    2. Ability to control nominal exchange rate/interest rate to
        - Smooth adverse domestic or foreign shocks
        - Stabilize prices

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- Set $M_t^s = M_t^d = M_t$ and take logs to get

$$m_t - p_t = -\eta \left[p_{t+1} - p_t\right]$$

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- Expression holds for p_{t+1} as well; substitute to get

$$p_t = \frac{1}{1+\eta} m_t + \frac{\eta}{1+\eta} \left[\frac{1}{1+\eta} m_{t+1} + \frac{\eta}{1+\eta} p_{t+2} \right]$$

Solving the Cagan Model

- Expand to the infinite horizon

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- If price growth less than exponential, second term $= 0$

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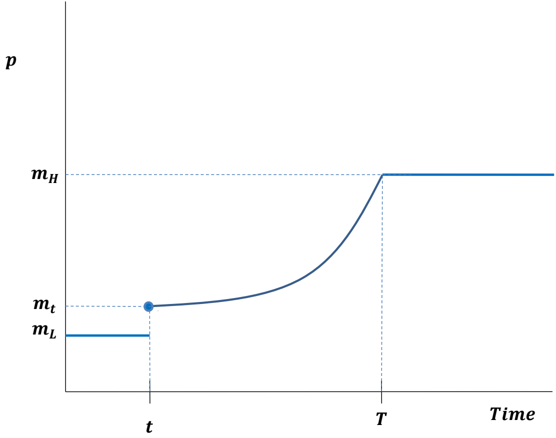
$$p_t = \begin{cases} m_L, & s < t \\ m_L + \left(\frac{\eta}{1+\eta}\right)^{T-t}(m_H - m_L), & t \leq s < T \\ m_H, & s \geq T \end{cases}$$

Anticipated Increase in Future Money Supply



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  - Implies growth rate of price level: $\frac{P_t}{P_{t-1}} = 1 + \mu$

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- Episodes of hyperinflation typically much greater than this
- Could be
    1. Speculation (crazy cases; seen before)
    2. Lack of credibility/commitment

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or in logs

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- NOTE: These are *nominal interest rates*; could bounce around with prices even if r constant
- In logs,

$$i_{t+1} = i^{\star}_{t+1} + e_{t+1} - e_t$$

Solving the Modified Cagan Model

- Use PPP and UIP to eliminate i_{t+1} and p_t from Cagan equation

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- Rearrange to solve for e_t just like p_t from original Cagan

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} [m_s - \phi y_s + \eta i_{s+1}^\star - p_s^\star]$$

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- Again, the weights imply full neutrality for the impact of *all variables* on the exchange rate

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  4. Decreases with foreign price
     - Mechanical! Foreign currency depreciates: Easier for home residents to purchase it

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- Fix foreign variables and home output at $\eta i^\star - \phi y - p^\star = 0$.
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$$\mu_t = \rho \mu_{t-1} + \epsilon_t$$

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- Implies that $E_t \mu_{t+1} = \rho \mu_t$, and $E_t \mu_s = \rho^{s-t} \mu_t$

Example: ER Volatility

- To solve, note that

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- Cagan model here boils down to: $m_t - e_t = -\eta(E_t e_{t+1} - e_t)$
  - Follows from setting $\eta i^\star - \phi y - p^\star = 0$

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- ER moves *more* than money initial shock: Expectations of future money growth

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    - Volatility in ER can cause large volatility in prices/production/income

Exchange Rate as a Policy Rule

- Government (central bank), can in principle control ε_t through M_t
- For small, open economies, movements in ER can be seriously damaging
  - Many small, open economies rely heavily on exports/imports
  - Even domestic goods produced often require traded intermediate inputs at some point in supply chain
  - Volatility in ER can cause large volatility in prices/production/income
- Government has interest in stabilizing ER: Called **Pegging the Exchange Rate**

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$$\rightarrow m_t = \bar{m} = \bar{e}$$

Need only fix money supply to constant level!

Stabilizing the ER: II

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- Money supply must respond 1:1 to movements in endogenous variables to offset their impact
    - *Relinquish control of money supply/monetary policy!* (More on this later)

Stabilizing the ER: III

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- Further difficulty: Prone to *Speculative Attacks*

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  - Purchases home assets (asset): $B_{H,t}$ e.g. government bonds
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- $B_{F,t} \geq 0$, since one can't issue foreign debt, only purchase foreign assets

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  7. Typically abandon peg to not lose reserves

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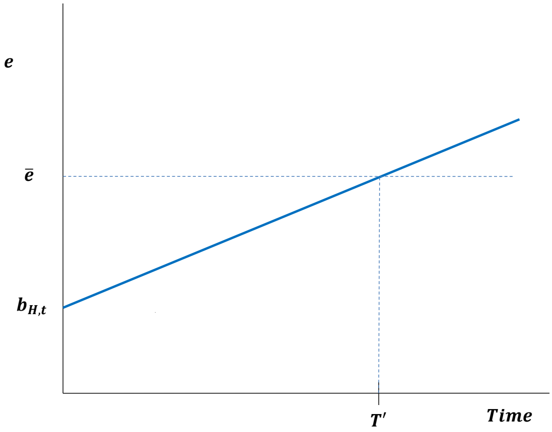
Speculative Attack



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- If foreign investors waited until T, they would suffer a loss in trying to sell off assets in home, since currency will collapse i.e. $\varepsilon \uparrow$
    1. Investors know that things will get really bad in time T: Get out before time T
    2. In doing so, cause the collapse to happen earlier: $T' < T$
    3. Implies *massive* and *rapid* loss of foreign reserves for home country ('Sudden Stop')

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1. Contrary to popular belief, currency attacks can be rational and foreseen
2. Less likely with large stockpiles of foreign reserves
3. Less likely when fiscal authority behaves i.e. runs surpluses and has no need of seignorage revenue/debt deflation
4. Won't happen with floating currency (no need to fix M_t)

Real and Nominal ER

- Can derive real ER movements from
  1. Inflation in two countries
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- In log-differences, can derive real-exchange rate movements:

$$\hat{R} = \hat{P} - \hat{P}^\star - \hat{E}$$

- $\hat{P}$: Domestic inflation
- $\hat{P}^\star$: Foreign inflation

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- What are the implications of this stickiness?
  - Shocks to economy tend to be absorbed by ER rather than prices
  - Mundell-Fleming-Dornbusch Model: Exchange rate *overshoots* in response to shocks since prices move little

Mundell-Fleming-Dornbusch Model

- Start with Cagan model (keep foreign objects constant)
  1. UIP: $i_{t+1} = i^{\star} + e_{t+1} - e_t$
  2. Money Demand: $m_t - p_t = -\eta i_{t+1} + \phi y_t$

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- Note that $q_t = -\log(R_t)$ i.e. currency weak when q_t is large

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- Assume that demand for home production is *increasing* in q_t

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where $\delta > 0$

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- Currency weak when q_t high: More world demand for exports
- In short-run though, q_t will be tied to e_t (why in second)

$$y_t^d = \bar{y} + \delta(e_t + p^\star - p_t - \bar{q})$$

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- Plug in to derive

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Solving the MFD Model: Money Demand

- Assume $p^\star = \bar{y} = i^\star = 0$
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$$\Delta e_{t+1} = e_{t+1} - e_t = \frac{e_t}{\eta} - \frac{1 - \phi\delta}{\eta} q_t - \frac{\phi\delta\bar{q} + m_t}{\eta}$$

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Dynamic System

- Assume constant money supply, $\bar{m}$
- Two equations govern dynamics
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     - $e_t < (1 - \phi\delta)q_t + (\phi\delta\bar{q} + \bar{m})$ implies $\Delta e_{t+1} < 0$
     - $e_t > (1 - \phi\delta)q_t + (\phi\delta\bar{q} + \bar{m})$ implies $\Delta e_{t+1} > 0$

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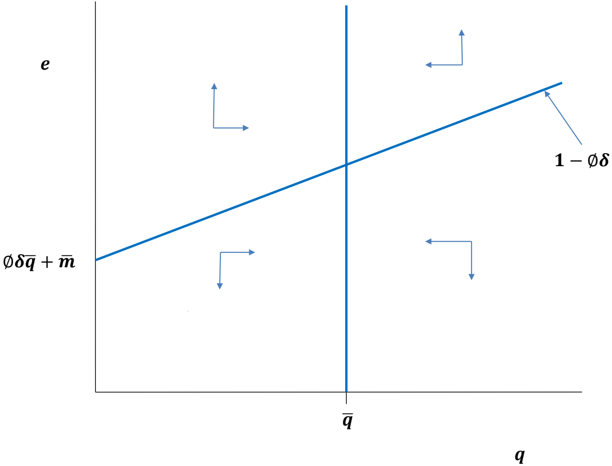
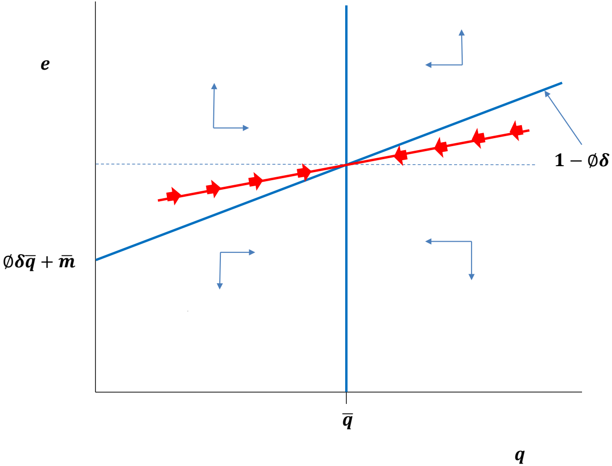
  1. $\Delta q_{t+1} = -\psi\delta[q_t - \bar{q}]$
  2. $\Delta e_{t+1} = \frac{e_t}{\eta} - \frac{1-\phi\delta}{\eta} q_t - \frac{\phi\delta\bar{q}+\bar{m}}{\eta}$
- Steady state implications
  1. $q_{ss} = \bar{q}$
     - $q_t < \bar{q}$ implies $\Delta q_{t+1} > 0$
     - $q_t > \bar{q}$ implies $\Delta q_{t+1} > 0$
  2. $e_t = (1-\phi\delta)q_t + (\phi\delta\bar{q} + \bar{m})$
     - $e_t < (1-\phi\delta)q_t + (\phi\delta\bar{q} + \bar{m})$ implies $\Delta e_{t+1} < 0$
     - $e_t > (1-\phi\delta)q_t + (\phi\delta\bar{q} + \bar{m})$ implies $\Delta e_{t+1} > 0$

Steady-States and Trajectories in MFD Model



Saddle-Path Trajectory in MFD Model



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- Initial nominal/real ER at shock, (q_0, e_0) must
  1. Lie on 45^o line through old SS
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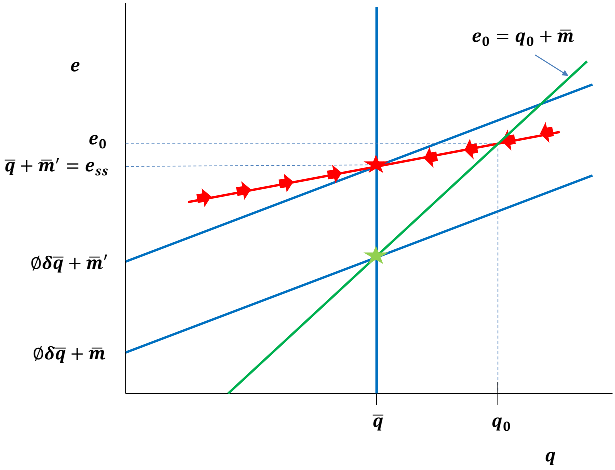
Overshooting in MFD Model



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- ER must *depreciate excessively initially* such that they can appreciate from then on out

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    - If e_t can move, then q_t responds immediately and stimulates output growth