

Financial Econometrics Econ 40357

Value at Risk (VaR)

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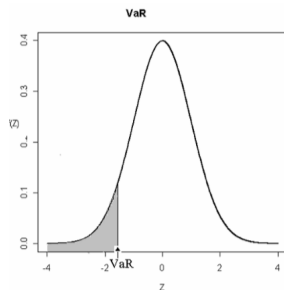
October 12, 2020

VaR

- Say you are in charge of a \$1M a portfolio. The $x\%$ VaR is the amount of money you stand to lose with probability $0.x$, within a certain time horizon. (The 10% VaR is the money you lose with probability 0.10).
- VaR is an **upper bound** on what you can lose, because you could lose it all.
- Who uses VaR?
 - External (e.g., regulators)—determine minimum required capital reserves for banks and other financial institutions.
 - Internal —control the risk undertaken by traders.
(e.g., the whale: Trader Bruno Iksil, nicknamed the London Whale, accumulated outsized CDS positions in the market, generated trading loss of US \$2 billion for JP Morgan Chase)

Normal VaR

- Assume conditional normality. All we need is the conditional mean and conditional variance of the portfolio rate of return.



- $\Delta W_{t+1} = r_{t+1} W_t$, where r_{t+1} is the portfolio's rate of return from t to $t + 1$, and $W_t = \$1M$.
- The 10% VaR is the answer to the question: What is

$$\text{Prob}_t(\Delta W_{t+1} = r_{t+1} W_t < -\$100K) = \text{Prob}_t(r_{t+1} < -0.10)$$

We want to find the 10% quantile of the return on the asset we've invested in r_t .

- Strategy is to map the problem into the standard normal variate.

The Conditional Normal VaR

- Normal probability density function (pdf). Let $\mu = E(r)$, $\sigma^2 = \text{Var}(r)$.

$$f(r) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(r-\mu)^2}$$

Once you know μ and σ , you know everything about the distribution. i.e., you can plot it exactly. μ is the mean, which sets location, and σ is the standard deviation, which modulates the scale.

- Conditional normal pdf. Let $\mu_t = E_t(r_{t+1})$, $\sigma_t^2 = \text{Var}_t(r_{t+1})$.

$$f(r_{t+1}|I_t) = \frac{1}{\sigma_t\sqrt{2\pi}} e^{-\frac{1}{2\sigma_t^2}(r_{t+1}-\mu_t)^2}$$

The Conditional Normal VaR

- Tabulations are for standardized normal random variable, $z_{t+1} \sim N(0, 1)$,

$$z_{t+1} = \frac{r_{t+1} - \mu_t}{\sigma_t} \quad (1)$$

$$\mu_t = E_t(r_{t+1}) \quad (2)$$

$$\sigma_t = \sqrt{\text{Var}_t(r_{t+1})} \quad (3)$$

- Express returns in terms of location and scale of $N(0, 1)$

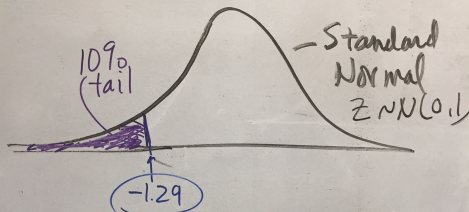
$$r_{t+1} = \mu_t + \sigma_t z_{t+1} \quad (4)$$

Quantiles of r_{t+1} now expressed in terms of μ_t , σ_t , and z_{t+1} .

The Conditional Normal VaR

Assume we know μ_t and σ_t (we'll estimate them with GARCH).

10% VaR is a dollar amount.
The upper bound on what you can lose
with probability 0.10



$$r_t = \mu_t + \sigma_t \cdot z$$

The Conditional Normal VaR

Say we've invested \$1m of our client's money in the market.

Daily market return (from Ken French). GARCH(1,1)-M model

Dependent Variable: MKT

Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)

Date: 10/11/20 Time: 13:59

Sample: 7/01/1926 9/03/2019

Included observations: 24560

Convergence achieved after 24 iterations

Coefficient covariance computed using outer product of gradients

Presample variance: backcast (parameter = 0.7)

GARCH = $C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)$

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|--------------------|-------------|-----------------------|-------------|----------|
| @SQRT(GARCH) | 0.095860 | 0.016710 | 5.736624 | 0.0000 |
| C | 0.002417 | 0.011850 | 0.203949 | 0.8384 |
| Variance Equation | | | | |
| C | 0.013027 | 0.000547 | 23.82356 | 0.0000 |
| RESID(-1)^2 | 0.103378 | 0.002077 | 49.76361 | 0.0000 |
| GARCH(-1) | 0.885661 | 0.002403 | 368.5156 | 0.0000 |
| R-squared | -0.003614 | Mean dependent var | | 0.041567 |
| Adjusted R-squared | -0.003655 | S.D. dependent var | | 1.062239 |
| S.E. of regression | 1.064179 | Akaike info criterion | | 2.440337 |
| Sum squared resid | 27811.37 | Schwarz criterion | | 2.441988 |
| Log likelihood | -29962.34 | Hannan-Quinn criter. | | 2.440872 |
| Durbin-Watson stat | 1.858048 | | | |

Forecast mean and variance

Equation: UNTITLED Workfile: VOLATILITY::ps7_soln\

View Proc Object Print Name Freeze Estimate Forecast Stats Resids

Dependent Variable: MKT
Method: ML ARCH
Date: 10/11/20
Sample: 7/01/19
Included observations: 100
Convergence achieved
Coefficient covariance matrix
Resample variable: none
ARCH = C(3)

Variable
@SQRT(GARCH)

C
RESID(-1)^2
GARCH(-1)

Adjusted R-squared: 0.9999
S.E. of regression: 0.0000
Log likelihood: 100.0000
Ljung-Box Q-statistic: 1.858048

Forecast

Forecast of
Equation: UNTITLED Series: MKT

Series names
Forecast name: mktf
S.E. (optional):
GARCH(optional): garchf

Method
☒ Dynamic forecast
☐ Static forecast
☒ Coef uncertainty in S.E. calc

Forecast sample
09/03/2019 09/12/2019

☒ Insert actuals for out-of-sample observations

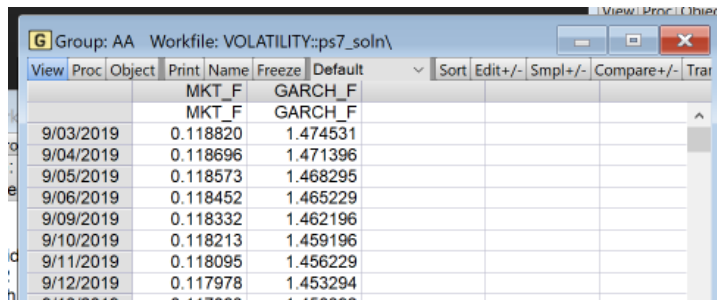
Output
Graph: Forecast

☒ Forecast evaluation

OK

Cancel

Forecast mean and variance



The screenshot shows the EViews software interface with a forecast table. The title bar indicates the group is 'AA' and the workfile is 'VOLATILITY::ps7_soln\'. The table has columns for dates, MKT_F, and GARCH_F. The data rows show values for dates from 9/03/2019 to 9/12/2019.

| | MKT_F | GARCH_F |
|-----------|----------|----------|
| 9/03/2019 | 0.118820 | 1.474531 |
| 9/04/2019 | 0.118696 | 1.471396 |
| 9/05/2019 | 0.118573 | 1.468295 |
| 9/06/2019 | 0.118452 | 1.465229 |
| 9/09/2019 | 0.118332 | 1.462196 |
| 9/10/2019 | 0.118213 | 1.459196 |
| 9/11/2019 | 0.118095 | 1.456229 |
| 9/12/2019 | 0.117978 | 1.453294 |

Note: Divide returns by 100 because Ken French states daily returns in percent.

VaR for 9/4/2019

- $\mu_t = 0.001186$, $\sigma_t = \sqrt{(1.474)}/100 = 0.0121$
- 10% quantile for z , the standard normal r.v. is $z^* = -1.29$
- $r^* = \mu_t + \sigma_t z^*$
- $r^* = 0.001186 + 0.0121(-1.29) = -0.0144$.
- $\text{VaR} = r^* W_t = -0.0144(\$1M) = -\$14,400$.

VaR for the next 2 days

Use forecasted values for μ_{t+1}, σ_{t+1}

$$z^* = -1.29$$

$$r_t^* = \mu_t + \sigma_t z^*$$

$$r_{t+1}^* = \mu_{t+1} + \sigma_{t+1} z^*$$

$$\begin{aligned} r_t^* + r_{t+1}^* &= \mu_t + \mu_{t+1} + (\sigma_t + \sigma_{t+1}) z^* \\ &= \frac{(0.118696 + 0.118573)}{100} - \frac{1.29(\sqrt{1.471396} + \sqrt{1.468295})}{100} = -0.028906 \end{aligned}$$

$$VaR = (\$1M)(r_t^* + r_{t+1}^*) = -28,906$$