# Stock Prices and the Stock Market

## ECON 40364: Monetary Theory & Policy


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Fall 2020

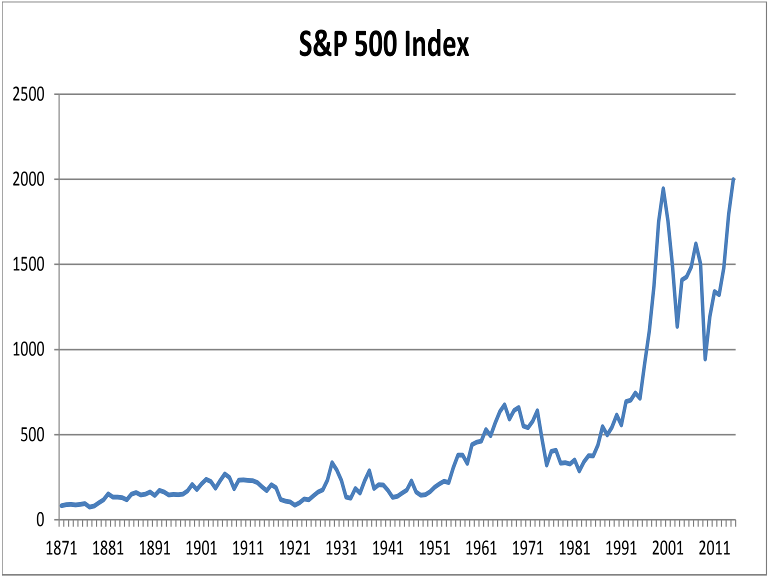
# Readings

- Text:
  - Mishkin Ch. 7
  - GLS Ch. 35

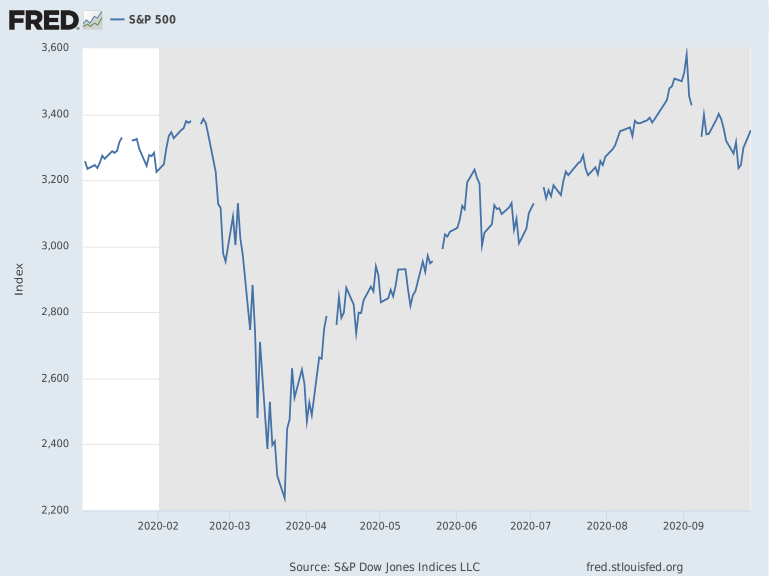
# Stock Market

- The stock market is the subject of major news coverage and is obviously of interest to monetary policymakers
- Several different indexes – S&P 500, Dow Jones Industrial Average, NASDAQ, Russell 2000, Wilshire 5000
- What is a stock?
- How are stocks priced?
- How do returns on stocks compare to alternative investments?
- Are there bubbles?
- Should monetary policymakers care?

# Stock Prices Over Time



# Stock Prices in 2020



# What is a Stock?

- A stock, or sometimes an equity, is a share of ownership in a firm
- A stockholder has an ownership share equal to his/her share of ownership in total stock outstanding
- Gives owner voting rights
- Stockholder is also a residual claimant on firm's assets (in event of bankruptcy/liquidation, debt claimants are "senior" to equity holders)
- May get periodic dividend payments (distributed profits)
- Can also earn money from capital gains (changes in price of shares)

# How is a Stock Priced?

- Just like for bonds, the price of a stock (or any asset) is equal to the present discounted value of cash flows (dividends plus capital gains)
- Let $\kappa_e$ be the discount rate for equity (the $e$ is for equity)
- Let $D_{t+1}$ be the dividend payout in period $t+1$ and $P_{t+1}$ the share price in $t+1$. Period $t$ is the present and $P_t$ is the price
- Share price ought to equal:

$$P_t = \mathbb{E}\left[\frac{D_{t+1}}{1+\kappa_e} + \frac{P_{t+1}}{1+\kappa_e}\right]$$

- The expectation operator reflects fact that future dividends and price are unknown
- Price composed of two components – dividend, $\frac{D_{t+1}}{1+\kappa_e}$, and capital gain, $\frac{P_{t+1}}{1+\kappa_e}$
- Fundamentally same idea as for bonds. Issue is what is appropriate $\kappa_e$?

# Returns and Price

- The realized return on equity is defined as the cash flow divided by purchase price. This equals dividend plus capital gain (share price appreciation):

$$R_{t+1} = \frac{D_{t+1} + P_{t+1} - P_t}{P_t}$$

- The expected return is:

$$R^e_{t+1} = \mathbb{E}\left[\frac{D_{t+1} + P_{t+1} - P_t}{P_t}\right] = \kappa_e$$

- These will in general not be equal due to unexpected fluctuations in dividend payments and unexpected share price movements
- You may demand compensation for this uncertainty, or risk, in the form of a higher expected return, $\kappa_e$
- For example, for a relatively safe stock, you may use a low $\kappa_e$ to price it and a higher $\kappa_e$ to price a riskier stock
- But what exactly do we mean by risk and how do you determine $\kappa_e$?

# A Micro-Founded Asset Pricing Model

- Suppose that an agent lives for two periods, $t$ and $t+1$
- Agent can save via one of two assets:
  1. Risk-free discount bond, $B_t$. Purchase price of $P_t^B$ in period $t$ and which pays out 1 with certainty in $t+1$
  2. Risky stock, $S_t$. Purchase price of $P_t^S$ in period $t$, pays an unknown dividend per share, $D_{t+1}$, in period $t+1$. Price in period $t+1$ is 0 (since world ends after $t+1$)
- Earns income, $Y_t$ and $Y_{t+1}$. Period $t+1$ income is uncertain, period $t$ income is known
- Flow budget constraints:

$$C_t + P_t^S S_t + P_t^B B_t \leq Y_t$$

$$C_{t+1} \leq Y_{t+1} + B_t + D_{t+1} S_t$$

# Preferences

- Household wants to maximize expected lifetime utility:

$$U = u(C_t) + \beta\, \mathbb{E}\left[u(C_{t+1})\right]$$

- Future consumption is uncertain because future income and the dividend payout on the risky stock are uncertain

# Optimality Conditions

- Assuming constraints hold with equality, plugging in to lifetime utility to eliminate $C_t$ and $C_{t+1}$, and taking derivatives with respect to $B_t$ and $S_t$ and setting equal to zero yields the following first order conditions:

$$P_t^B = \mathbb{E}\left(\frac{\beta u'(C_{t+1})}{u'(C_t)}\right)$$
$$P_t^S = \mathbb{E}\left(\frac{\beta u'(C_{t+1})}{u'(C_t)} D_{t+1}\right)$$

- We call $\frac{\beta u'(C_{t+1})}{u'(C_t)}$ the stochastic discount factor (also sometimes called the "pricing kernel"). Called stochastic because future marginal utility is unknown at time $t$
- These FOC look similar – price ($P_t^B$ or $P_t^S$) equals product of SDF and cash flow in period $t+1$ (1 or $D_{t+1}$)
- Again, the same as for pricing bonds

# If There Were No Uncertainty over Stock Payout

- The returns on each asset are just future cash flows divided by current price.
- Endowment economy structure: consumption equals endowment each period, assets in zero net supply
- Since the future cash flow on the bond is known with certainty, its (gross) return is known:

$$R^e_{B,t+1} = \frac{1}{P^B_t} = \left[\mathbb{E}\left(\frac{\beta u'(Y_{t+1})}{u'(Y_t)}\right)\right]^{-1}$$

- If $D_{t+1}$ were known, then we could write:

$$R^e_{S,t+1} = \frac{D_{t+1}}{P^S_t} = \left[\mathbb{E}\left(\frac{\beta u'(Y_{t+1})}{u'(Y_t)}\right)\right]^{-1}$$

- We would have $R^e_{B,t+1} = R^e_{S,t+1}$ – no equity risk premium

# Numerical Example

- Suppose that $Y_t = 1$, $\beta = 0.95$, and that the utility function is natural log
- Allow for uncertainty over endowment, but not on payout from stock
- Suppose that endowment can take on two values in $t+1$ – $Y_{t+1}^l$ or $Y_{t+1}^h$, where $Y_{t+1}^h \geq Y_{t+1}^l$. $p$ is the probability of the low state, and $1-p$ is the probability of the high state
- Then we have:

$$\mathbb{E}\left(\frac{\beta u'(Y_{t+1})}{u'(Y_t)}\right) = p \times \beta \frac{Y_t}{Y_{t+1}^l} + (1-p) \times \beta \frac{Y_t}{Y_{t+1}^h}$$

- Suppose we have $p = 0.5$, $Y_{t+1}^l = 0.9$, and $Y_{t+1}^h = 1.1$. Then the expected value of the SDF is 0.9596, so this is the bond price, $P_t^B$
- Suppose $D_{t+1} = 1.1$ with certainty. Then the price of the stock is 1.0556

# Price and Yields

- The yield is just the expected return, which in both cases here are known. The yield on the riskless bond is:

$$1 + i_B = \frac{1}{P_t^B} = 1.0421$$

- The yield on the stock is then:

$$1 + \kappa_e = \frac{D_{t+1}}{P_t^S} = 1.0421$$

- If there is no uncertainty over asset payouts, you use the same discount rate to price different assets:

$$\frac{1}{1.0421} = 0.9596 = P_t^B$$
$$\frac{1.1}{1.0421} = 1.0556 = P_t^S$$

# Now Enter Uncertainty over Future Dividend

- Now suppose that the future dividend takes on two values, $D^l_{t+1}$ and $D^h_{t+1}$, where $D^h_{t+1} > D^l_{t+1}$. Suppose that these different values materialize in the same high/low state for the endowment with the same probabilities ($p$ and $1-p$)
- Suppose that $D^l_{t+1} = 1$ and $D^h_{t+1} = 1.2$. With $p = 0.5$ we have $\mathbb{E}[D_{t+1}] = 1.1$, just like before.
- But what is price of stock? Key insight here is that, in general:

$$\mathbb{E}\left(\frac{\beta u'(Y_{t+1})}{u'(Y_t)} D_{t+1}\right) \neq \mathbb{E}\left(\frac{\beta u'(Y_{t+1})}{u'(Y_t)}\right) \mathbb{E}\left(D_{t+1}\right)$$

- Plugging in our numbers from above, we get a stock price of $P^s_t = 1.0460$, which is less than the case where there was no uncertainty
- This means that the expected yield on the stock is:

$$1 + \kappa_e = \frac{E[D_{t+1}]}{P^s_t} = \frac{1.1}{1.0460} = 1.0516$$

- This is higher than the yield on the bond

# Equity Risk Premium

- Define the equity risk premium as the difference between the discounts rates on equity and the riskless bond:

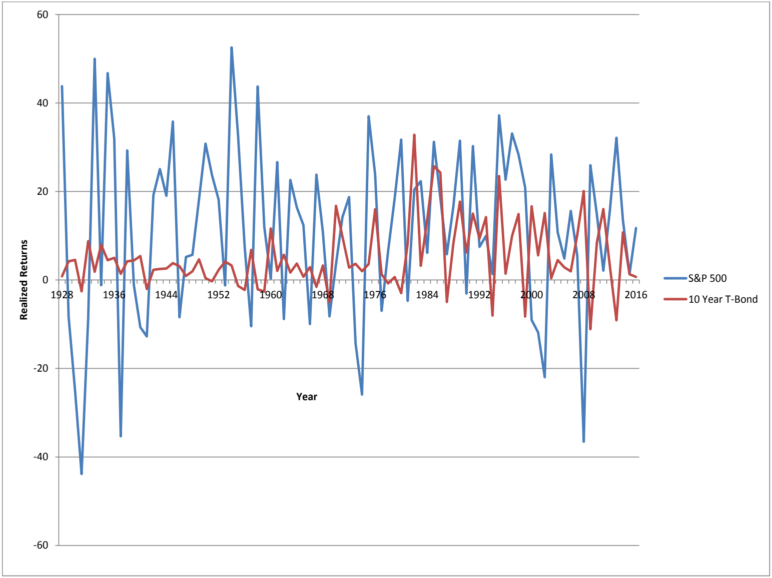
$$\psi = \kappa_e - i_B$$

- In our numerical example, this works out to be 0.0096
- As when thinking about the risk and term structures of interest rates, it is covariance of stock payouts with marginal utility that drives an equity premium, not variance per se

# Equity Premium

- Simple theory would suggest that stocks offer high expected yields if returns co-vary negatively with the stochastic discount factor
- Stands to reason that we should observe a negative covariance, and hence positive equity premium, in the data. Why?
  - In periods of recession, we are likely to observe both low endowment and low stock returns (dividends in the two period example)
  - Vice-versa for expansion
  - With log utility, SDF in two period endowment example is $\beta \frac{Y_t}{Y_{t+1}}$. When $Y_{t+1}$ is low (recession), SDF is high and dividend rate, $D_{t+1}$ is likely to be low
  - You most like assets which give high returns when output is low, not high. Hence demand a premium to hold such assets (stocks)

# Realized Returns on S&P 500 and 10 Yr Treasury Bond



# Asset Pricing for Dummies

- In real world, time lasts for more than two periods and stocks have no maturity date (i.e. they in principal provide cash flows in perpetuity)
- This means that they are most comparable not to short term riskless bond but longer maturity bonds
- Asset pricing for dummies:
    1. All assets are substitutable ways to transfer resources intertemporally, but have different risk characteristics
    2. There is a risk free yield/rate that is an equilibrium construct that depends on demand and supply forces (short term gov. debt)
    3. Longer term government debt is priced off of short term debt (term premium)
    4. Privately-issued debt is priced off of government debt of comparable maturity (risk premium)
    5. Stocks are priced off of longer maturity private debt (equity premium)
    6. These premia depend on covariances

# Moving Beyond Two Periods

- We started with the pricing equation:

$$P_t = \mathbb{E}\left[\frac{D_{t+1}}{1+\kappa_e} + \frac{P_{t+1}}{1+\kappa_e}\right]$$

- "Solve forward":

$$P_t = \mathbb{E}\left[\frac{D_{t+1}}{1+\kappa_e} + \frac{1}{1+\kappa_e}\left(\frac{D_{t+2}}{1+\kappa_e} + \frac{P_{t+2}}{1+\kappa_e}\right)\right]$$

- Keep going. Since stock never matures (unlike a bond), we get:

$$P_t = \mathbb{E}\left[\sum_{j=1}^{\infty} \frac{D_{t+j}}{(1+\kappa_e)^j}\right] + \mathbb{E}\left[\lim_{T\to\infty} \frac{P_{t+T}}{(1+\kappa_e)^T}\right]$$

- No bubble condition: last term drops out (PDV of price in infinite future is zero), so stock price is just PDV of future dividends

# Gordon Growth Model

- Impose the no bubble condition. Assume that dividends grow at a constant rate across time, $D_{t+1} = (1+g)D_t$ and so on for any two adjacent periods. Hence, no uncertainty over future dividends
- After some math, you get the following condition:

$$P_t = \frac{(1+g)D_t}{\kappa_e - g}$$

- Note that this is essentially a price-earnings ratio, if you take $D_t$ to be earnings. Then you get the PE ratio of:

$$PE = \frac{1+g}{\kappa_e - g}$$

- The PE ratio will be higher the:
  1. Lower is $\kappa_e$ (i.e. the less risky the stock is)
  2. The higher is $g$ (the more dividends are expected to grow)

# Monetary Policy and the Stock Market

- Gordon growth model provides a simple and intuitive way to understand how monetary policy might affect the stock market
- Expansionary monetary shock (increase in the money supply resulting in lower short term interest rates):
  - Likely lowers $\kappa^e$ because of lower short term bond rates (which influence longer term bond rates, off of which stocks are "priced")
  - Likely raises $g$ because an expanding economy is good for dividends
  - Both ought to raise stock prices
- At onset of COVID-19, stock prices plummeted, but have since recovered
  - Massive monetary stimulus has likely lowered discount rate and reduced probability of low future cash flows

# Rational Expectations and Efficient Markets

- Rational expectations: agents form optimal, model-consistent expectations using all available information
- Intuition: if you make choices optimally, and choices depend on expectations, it makes sense to use all available information to form expectations optimally
- Does not mean that your forecasts are always right, it means your forecasts are right on average
- Formally, for a random variable $X_{t+1}$, we have:

$$\mathbb{E}(X_{t+1}) = X_{t+1} + \varepsilon_{t+1}$$

- $\varepsilon_{t+1}$ is a forecast error and is (i) zero on average and (ii) unpredictable

# Efficient Markets

- Suppose that the expected return consistent with the SDF on an asset is $R_t^*$. This could differ across assets due to risk, liquidity, etc. $R_t^*$ is the required return on the asset
- Suppose that $R_t^e > R_t^*$ for this asset. What should a smart investor do? Buy more of that asset until $R_t^e = R_t^*$. Doing so will drive the price of that asset, $P_t$, up, and hence the yield down
- Vice-versa if $R_t^e < R_t^*$
- Smart investors ought to eliminate arbitrage opportunities
- Price of asset should be set such that $R_t^e = R_t^*$
- Implication: there is no such thing as an under- or over-valued stock according to efficient markets!

# Random Walk Hypothesis

- An implication of efficient markets is something known as the random walk hypothesis
- The basic idea of the random walk hypothesis is that changes in stock prices ought to be unpredictable
- Suppose a stock pays no dividend, so the price satisfies:

$$P_t = \mathbb{E}\left[\frac{P_{t+1}}{1+\kappa_e}\right]$$

- This implies that, approximately:

$$\mathbb{E}\left[\frac{P_{t+1} - P_t}{P_t}\right] = \kappa_e$$

- The stock ought to be priced where the expected growth rate (more generally, return if stock pays dividend) equals the discount rate
- If you thought you had information that $P_{t+1}$ was going to increase, increasing the expected return, market participants ought to buy the stock, raising $P_t$ and eliminating the high return

# Bubbles

- Recall from earlier that successively substituting in gave us the expression for a stock price:

$$P_t = \mathbb{E}\left[\sum_{j=1}^{\infty} \frac{D_{t+j}}{(1+\kappa_e)^j}\right] + \mathbb{E}\left[\lim_{T\to\infty} \frac{P_{t+T}}{(1+\kappa_e)^T}\right]$$

- Define the bubble term as the last part:

$$B_t = \mathbb{E}\left[\lim_{T\to\infty} \frac{P_{t+T}}{(1+\kappa_e)^T}\right]$$

- In what we had done earlier, we ruled this out by setting $B_t$ to 0. Can we always do that?
- Define the fundamental price, $P_t^F$, as the PDV of dividends:

$$P_t^F = \mathbb{E}\left[\sum_{j=1}^{\infty} \frac{D_{t+j}}{(1+\kappa_e)^j}\right]$$

- Actual price is sum of fundamental price and bubble:

$$P_t = P_t^F + B_t$$

# Bubbles Continued

- Recall that we can write:

$$P_t = \mathbb{E}\left[\frac{D_{t+1} + P_{t+1}}{1 + \kappa_e}\right]$$

- Using facts that $P_t = P_t^F + B_t$, as well as fact that $P_t^F = \mathbb{E}\left[\frac{D_{t+1} + P_{t+1}^F}{1+\kappa_e}\right]$, we can conclude:

$$\mathbb{E}\left[B_{t+1}\right] = (1 + \kappa_e) B_t$$

- If bubble exists ($B_t \neq 0$), then it must be expected to grow at the discount rate for equity
- This is why we call it a "bubble" – in expectation, it must get bigger
- Intuition: you would "overpay" for an asset (pay more than fundamental value) if and only if you think you can sell it to someone else in the future who will overpay by more

# Bubbles Bursting

- Note that, if one exists, a bubble must grow in expectation, but this doesn't mean that the bubble will in actuality last forever
- To see this, suppose that $B_t = 1$ and $\kappa_e = 0.05$
- Suppose that the bubble "bursts" with probability $p$ (meaning $B_{t+1} = 0$) and continues with probability $1 - p$
- We can solve for what the realized value of $B_{t+1}$ must be in the event it does not burst by noting:

$$\mathbb{E}(B_{t+1}) = p \times 0 + (1 - p) \times B_{t+1} = (1 + \kappa_e) B_t$$

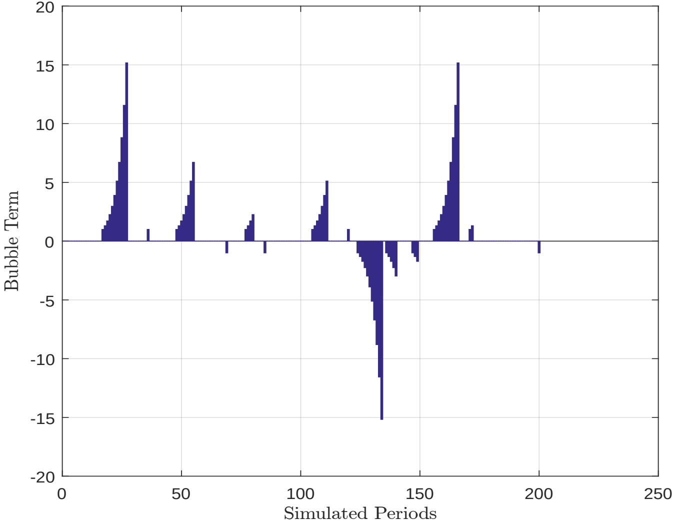
- So, if $p = 0.2$ for example:

$$B_{t+1} = \frac{(1 + \kappa_e) B_t}{1 - p} = \frac{1.05}{0.8} = 1.3125$$

# Simulating Bubbles

- Suppose I have a bubble process
- If $B_t = 0$, there is a $p$ probability of entering a positive bubble ($B_{t+1} = 1$), and a $q$ probability of entering a negative bubble ($B_{t+1} = -1$). Hence a $1 - p - q$ probability you stay out of a bubble
- If you're in a bubble, $B_t \neq 0$, in expectation the bubble must grow at $(1 + \kappa_e)$, but you exit the bubble (go back to 0) with probability $r$
- Assume $\kappa_e = 0.05$, $p = q = 0.05$, and $r = 0.2$

# Simulating Bubbles



# Bubbles Continued

- Recall that the bubble term is:

$$B_t = \mathbb{E}\left[\lim_{T\to\infty} \frac{P_{t+T}}{(1+\kappa_e)^T}\right]$$

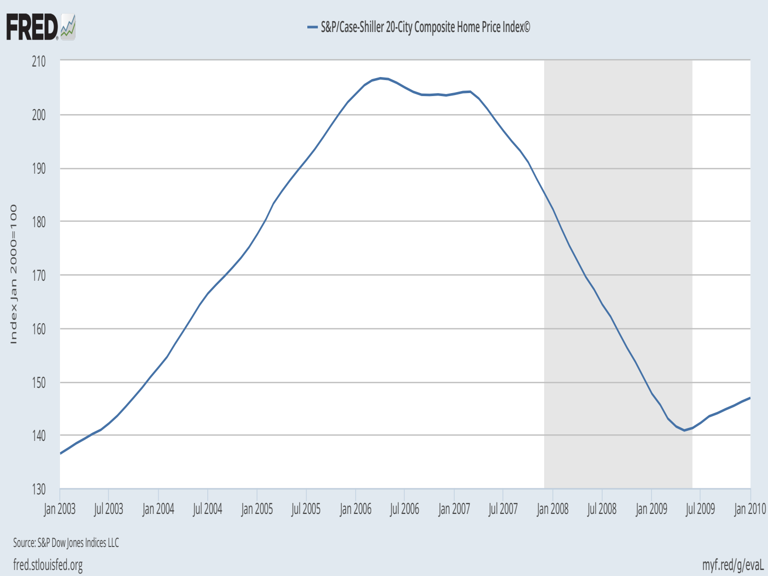
- If the asset in question has a finite "life span" (e.g. a bond with a known maturity), there cannot be bubbles
- Why? The value of the asset at maturity is zero, so $P_{t+T} = 0$ at maturity, and therefore $B_t = 0$
- We should not observe bubbles for assets with known maturities (e.g. bonds, cars), but may see them in assets without maturities (e.g. stocks, land/housing)

# Bubbles in the Press

- Economists have a precise definition of a bubble – deviation of price from fundamental value, where fundamental value is PDV of cash flows
- In the press and in the media, a "bubble" is more loosely defined as a situation in which an asset (e.g. stocks, housing) experiences very rapid price growth, followed by a subsequent decline (i.e. a bursting of the bubble)
- Real life examples:
    1. Tech boom and bust of late 1990s
    2. Housing boom and bust of mid-2000s

# Tech Boom and Bust

# Housing Boom and Bust



# Were These Episodes Actual Bubbles?

- Very hard to say, especially in "real time," but even after the fact
- Evidence of prices rising and then subsequently falling is not necessarily evidence of a bubble which subsequently burst
- People could have expected dividends (rents, in the case of housing) to grow in future, and this didn't materialize
- Alternatively, people could have had temporarily low discount rates which subsequently increased

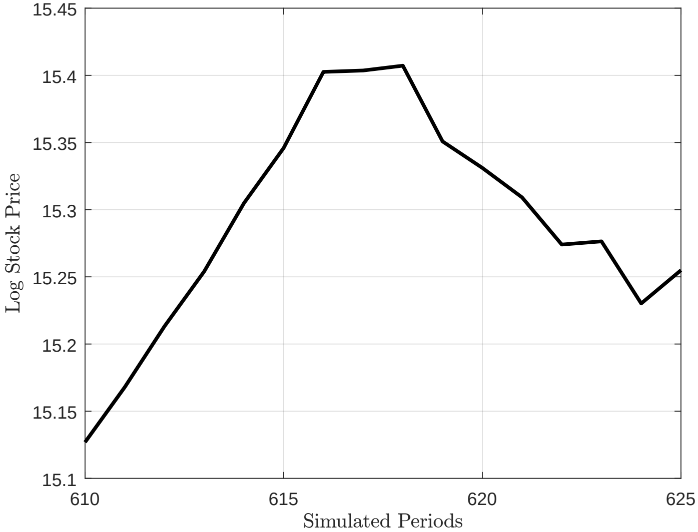
# Analysis on Simulated Data

- I created a computer program to simulate stock prices
- I assume a constant discount rate, $\kappa^e = 0.07$
- I assume dividends are given by $D_{t+1} = (1 + g_t)D_t$, where $g_t$ is the growth rate, which follows a stochastic process:

$$g_t = (1 - \rho)g^* + \rho g_{t-1} + \varepsilon_t$$

- $g^*$ is the average growth rate, $\rho$ is a measure of persistence, and $\varepsilon_t$ is an iid shock drawn from a normal distribution with standard deviation $s_g$
- I set $\rho = 0.8$, $g^* = 0.02$, and $s_g = 0.01$
- I simulate a process for dividends, then at each date, given current dividends and the known process for the growth rate, I forecast future dividends and discount those to compute the price at each date, assuming no bubble

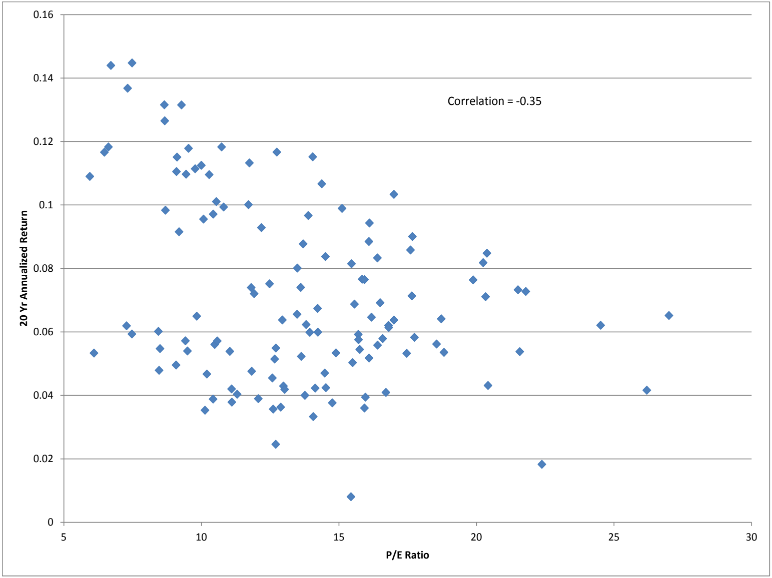
# Looks Like a Bubble, But Not



# Detecting Bubbles in the Data

- Robert Shiller (Nobel Prize Winner) is an advocate of the existence of bubbles
- One empirical test he proposes is to look at correlation between P/E ratios and subsequent realized returns
- Basic idea: if there is a bubble, P/E ratio will be high (or low) but the bubble will eventually burst, so realized returns will be low (or high)
- In other words, bubbles would manifest as negative correlations between P/E ratios and subsequent realized returns
- Other ways to try to do this (such as try to calculate fundamental price and compare deviation from observed prices)
- Look at S&P 500 stock market, evidence roughly consistent with this

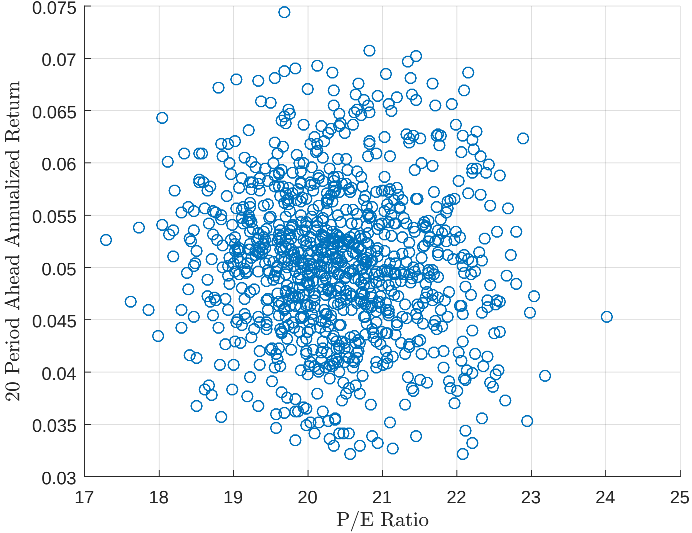
# P/E Ratios and Subsequent 20 Year Annualized Returns: Data



# Is This a Good Test?

- To see whether this test makes sense, I return to the model simulation
- I use the same parameter values, and assume no bubbles
- I calculate P/E ratios and subsequent 20 period returns and produce a scatter plot
- It's really a "blob" – no obvious correlation between PE ratios and subsequent returns
- Also, not an enormous amount of variation in P/E ratios

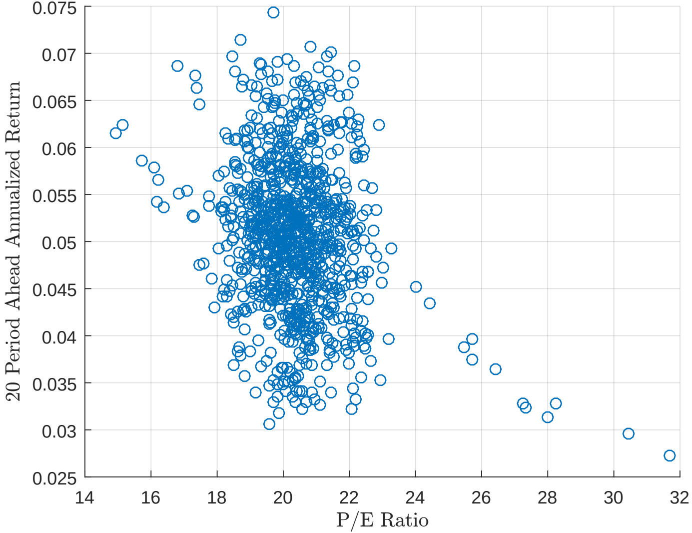
# P/E Ratios and Subsequent 20 Year Annualized Returns: Model



# Now Add in Bubbles

- Similar setup as above, but slightly different parameters
- Probability of entering a positive or negative bubble is $p = q = 0.005$, so bubbles are pretty rare
- Conditional on being in a bubble, stay in the bubble with probability $1 - r = 0.90$ (10 percent chance of exit)
- If you enter a bubble, its size is proportional to current level of dividends (this ensures bubble term isn't irrelevant later in the sample since dividends and hence price are growing)
- You generate a downward-sloping scatter plot, just as in data

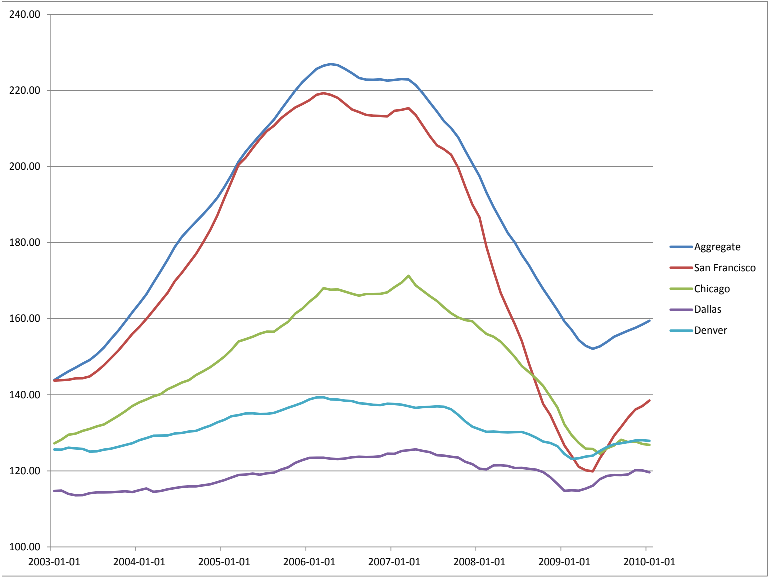
# P/E Ratios and Subsequent 20 Year Annualized Returns: Model with Bubble



# Should Monetary Policy Try to Prevent Bubbles?

- Some argue that (i) the Fed helped fuel the housing bubble by keeping interest rates too low for too long after early 2000s recession and (ii) the Fed should seek to identify bubbles and use monetary policy to burst them before they get too big and before their bursting becomes as painful
- Empirical evidence on (i) is not great – see Dokko et al (2007, "Monetary Policy and the Global Housing Bubble," *Economic Policy*
- What about (ii)? Interest rate is a rather crude tool – it applies to all markets equally, and bubble may not be same in all markets (see next slide)

# Comparing the Housing "Bubble" In Different Markets



# Macroprudential Regulation

- If “bubble” not the same in all markets (it wasn’t), interest rate is a pretty blunt tool
- Macroprudential regulation: macro (as opposed to micro) financial market rules and regulations which try to prevent the kind of financial market upheaval recently witnessed
  1. Loan-to-value ratios
  2. Lending standards
  3. Capital requirements
- In a nutshell, trying to make it difficult to get debt-fueled asset price increases in the first place (through lending standards and loan to value ratios), and trying to make consequences of prices falling less disastrous for other markets (capital requirements) if prices do decline