Econ 204 – Problem Set 4

Due Tuesday, August 11

1. Similarly as it's defined in class, let $C([0,1])$ be the set of all continuous functions whose domain is the unit interval $[0,1]$ and range is $\mathbb{R}$. Let Φ be the subset consisting of all real polynomials (whose domain is restricted to the unit interval) of degree at most two:

$$\Phi \equiv \{ a + bx + cx^2 \mid a, b, c \in \mathbb{R} \}$$

   Note that the set $C([0,1])$ is a vector space over the field of real numbers and the subset Φ is a proper subspace.

   (a) Are the vectors $\{ x, (x^2-1), (x^2+2x+1) \}$ linearly independent over $\mathbb{R}$?

   (b) Find a Hamel basis for the subspace Φ.

   (c) What is the dimension of Φ ? Show that $C([0,1])$ is not finite dimensional!

2. Let V have finite dimension greater than 1. Prove whether or not the set of non-invertible operators is a subspace of $L(V,V)$.

3. Suppose that V is finite dimensional and $T, S \in L(V,V)$. Prove that TS is invertible if and only if both T and S are invertible.

4. $T : M_{2\times2} \to M_{2\times3}$ is defined by:

$$T\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} a_{11} + a_{21} & a_{11} + 3a_{22} & 0 \\ a_{11} - a_{12} & a_{12} + a_{21} & 0 \end{pmatrix}$$

   Determine $\ker T, \dim(\ker T)$, and rank T. Is T one-to-one, onto, both or neither?

5. (a) Prove that the eigenvalues of any upper or lower triangular matrix A are the diagonal entries of A;

   (b) Show that the eigenspace of any matrix A belonging to an eigenvalue λ_i (see de la Fuente, p. 147 for a definition) is a vector space;

   (c) Show that if λ is an eigenvalue of A then λ^k is an eigenvalue of A^k for $k \in \mathbb{N}$;

   (d) Show that if λ is an eigenvalue of the invertible matrix A then $1/\lambda$ is an eigenvalue of A^{-1}.