

$$c_{t+p} - c_t = \alpha + \beta s_t + \epsilon_{t+p}$$

Extra credit

$$y_{t+2} = ay_{t+1} + bx_{t+1} + \epsilon_{yt+2}$$

$$x_{t+2} = cy_{t+1} + dx_{t+1} + \epsilon_{xt+2}$$

$$\begin{aligned} E_t(y_{t+2}) &= E_t(ay_{t+1} + bx_{t+1} + \epsilon_{yt+2}) \\ &= aE_t(y_{t+1}) + bE_t(x_{t+1}) + 0 \\ &= a(ay_t + bx_t) + b(cy_t + dx_t) \end{aligned}$$

$$E_t(y_{t+1}) = E_t(ay_t + bx_t + \epsilon_{yt+1}) = ay_t + bx_t$$

$$E_t(x_{t+1}) = E_t(cy_t + dx_t + \epsilon_{xt+1}) = cy_t + dx_t$$

Announcement

Groups find a paper topic. Let me know what it is by 21 October.

Conditional and unconditional distributions

$0 < \rho < 1$ , Stationary AR(1),  $\epsilon_t$  iid  $(0, \sigma^2)$

$$y_t = \rho y_{t-1} + \epsilon_t$$

Unconditional distribution of  $y_t$  is the same for all  $t$ . The unconditional mean

$$y_t = \epsilon_t + \rho\epsilon_{t-1} + \rho^2\epsilon_{t-2} + \dots$$

$$E(y_t) = E(\epsilon_t + \rho\epsilon_{t-1} + \rho^2\epsilon_{t-2} + \dots) = 0$$

But the conditional mean,

$$E_{t-1}(y_t) = E_{t-1}(\rho y_{t-1} + \epsilon_t) = \rho y_{t-1}$$

The conditional distribution is different for every different value of  $y_{t-1}$ .

Unconditional variance

$$\text{Var}(u_t) = E[u_t - E(u_t)]^2$$

Conditional variance definition

$$E_{t-1}(u_t) = E(u_t | u_{t-1}, u_{t-2}, \dots)$$

$$\sigma_t^2 = E_t[u_t - E_{t-1}(u_t)]^2 = E\left([u_t - E(u_t | u_{t-1}, u_{t-2}, \dots)]^2 | u_{t-1}, u_{t-2}, \dots\right)$$

ARCH(1) model

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$$

ARCH(2) model

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2$$

ARCH(p) model includes  $p$  lags of  $u_t^2$