

Equivalence between Pareto Optimality (Social Planner's Allocation)
and Competitive Equilibria
Applications to Risk Sharing in China
Econ 43750

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This material is covered in

- Curtis and Mark, “Business Cycles, Consumption, and Risk Sharing: How Different is China?” (mainly the risk sharing part).

Topic Overview

- **Social Optimum vs. Competitive Equilibrium**
- **Centralized vs. Decentralized** solution to resource allocation problem.
- **Socialism vs. Capitalism**
 - Socialism is public ownership of capital. Not dictatorship.
 - Marx: From each according to his ability, to each according to his need. (today's language: insurance).
 - In 1950s and 1960s, England was socialist-nationalized many industries/firms (British Airways, British Leyland, British Rail, British Shipbuilders, etc. etc. etc.).
- In the 1930s, 1940s, 1950s, it was unclear which system was better.
 - Dissatisfaction with capitalism, and the market, in aftermath of the Great Depression (which was world-wide).
- Economic theory shows, under certain conditions (assumptions), the two systems are equivalent.
 - Two giants of economics, **Kenneth Arrow** and **Gerald Debreu**.
- How much **risk sharing** is there in China? **Curtis and Mark** paper.
- How much **misallocation** is there in China? **Hsieh and Klenow** paper.

A couple of definitions

- **A Pareto Optimum:** an allocation of resources where it is impossible to reallocate to make someone better off without making someone else worse off.
 - Who does the allocating? A benevolent Social Planner maximizes social welfare by commanding allocations to achieve a Pareto Optimum
- **Competitive Equilibrium:** The allocation that results when all markets (goods, factors, asset) are perfectly competitive, when people have complete and perfect information, no market failures, and factors are perfectly mobile
 - A Competitive Equilibrium can attain a Pareto Optimum.
- China has operated under two systems-social planner system and decentralized market system. Let's compare!

The Two Fundamental Theorems of Welfare Economics

- ① Any **competitive equilibrium** leads to a **Pareto optimal** allocation of resources
- ② Any **Pareto optimal** allocation can be sustained by a **competitive equilibrium**

Paraphrasing from **the Wealth of Nations**: each individual, pursuing his own self interest is led, as if by an **invisible hand**, to achieve the best for all.

We explore these ideas to interpret central planning and the reallocations from Deng's reforms.

Figure: Who's this guy Pareto?



Figure: Pareto

Pareto Optimal Risk Sharing, Complete Markets

- **Perfect risk sharing** in environment of **complete markets** attains the **social optimum** in the sense of Pareto.
- The concept forms the basis of all of **modern finance theory**.
- We want to
 - **compare** risk-sharing under **central planning** and **post reform** economies of China.
 - **Central tenet of communism**: From each according to one's ability, to each according to one's needs—slogan popularized by Marx.
 - We want to apply this idea to risk sharing.
 - Did Mao's central plan or Deng's decentralized economy do a better job of sharing risk?
 - How much risk-sharing was there in the two regimes?

Optimal Risk Sharing

- A single-good endowment economy. Good is nonstorable.
- Two states of nature, $\{1, 2\}$ each with equal probability $p(1) = p(2) = 1/2$.

Louie and Kai's endowments in the two states

state	probability	Louie	Kai
1	0.5	2	4
2	0.5	4	2

- Louie and Kai have log utility. Planner weights them equally
 - They are risk averse.

$$\underbrace{\frac{1}{2} \ln(2) + \frac{1}{2} \ln(4)}_{2.08} < \underbrace{\frac{1}{2} \ln(3) + \frac{1}{2} \ln(3)}_{2.19} \quad (1)$$

Social Planner Problem

- The problem is

$$\max \ln(c_1) + \ln(c_2) \quad (2)$$

subject to

$$y = c_1 + c_2 \quad (3)$$

- Lagrangian,

$$L = \ln(c_1) + \ln(c_2) + \lambda(y - c_1 - c_2) \quad (4)$$

- First-order conditions

$$c_1 : \frac{1}{c_1} - \lambda = 0 \quad (5)$$

$$pc_2 : \frac{1}{c_2} - \lambda = 0 \quad (6)$$

$$\lambda : y - c_1 - c_2 = 0 \quad (7)$$

- Hence, efficient or optimal risk-sharing says,

$$c_1 = c_2 \quad (8)$$

This is what Mao should have done. Maybe he did.

Decentralized, Complete Markets

- (Completing the market) If there is a demand (for a product, commodity, security), a market will emerge to provide it.
- Intertemporal setting. Securities trading occurs **before** state of nature is revealed—before uncertainty is resolved.
- Assets are **state-contingent** securities (bonds) that pay one unit of the consumption good if that state of nature occurs, and expires worthless if that state does not occur.
 - Think of insurance contract. Purchase security with premium, and payoffs if state occurs (accident) or not.
- Prime denotes next period.
- $s = 1, 2$ is state of nature. Louie and Kai's endowments $y_1(s), y_2(s)$

	State 1	State 2
Louie	$y_1(1) = 2$	$y_1(2) = 4$
Kai	$y_2(1) = 4$	$y_2(2) = 2$

Decentralized, Complete Markets

- What if Louie buys one State 1 security and sells one State 2 security? He will have perfectly insured himself.
- What if Kai sells one State 1 security and buys one State 2 security? He too, will have perfectly insured himself.
- Through the securities market, Louie and Kai have insured each other.

Here's the Formal Treatment: Decentralized, Complete Markets

- State-contingent securities. Each security pays one unit of consumption good if the state occurs. For $s = \{1, 2\}$,
 - $b_1(s)$: No. of state- s securities bought (+) or sold (-) by Louie.
 - $b_2(s)$: No. of state- s securities bought (+) or sold(-) by Kai.
 - $q(s)$: Price of state-securities. $q(1)$ for $s = 1$ security. $q(2)$ for $s = 2$ security.
- Louie's budget constraints
Today: Assume current state $s = 2$

$$y_1(2) + \underbrace{b_1(2)}_{\text{payoff}} = c_1(2) + \underbrace{(q(1)b'_1(1) + q(2)b'_1(2))}_{\text{Investment Portfolio}} \quad (9)$$

$$y'_1(s) + b'_1(s) = c'_1(s) + (q'(1)b''_1(1) + q'(2)b''_1(2)) \quad (10)$$

- Kai's budget constraints: replace 1 subscript with 2.

$$y_2(2) + \underbrace{b_2(2)}_{\text{payoff}} = c_2(2) + \underbrace{(q(1)b'_2(1) + q(2)b'_2(2))}_{\text{Investment Portfolio}} \quad (11)$$

$$y'_2(s) + b'_2(s) = c'_2(s) + (q'(1)b''_2(1) + q'(2)b''_2(2)) \quad (12)$$

- Louie's problem. Maximize expected utility subject to his budget constraints.
- Assume today in state $s = 2$. Lagrangian,

$$\begin{aligned}
 L = & \ln(c_1(2)) + \left[\frac{1}{2} \ln(c'_1(1)) + \frac{1}{2} \ln(c'_1(2)) \right] \\
 & + \lambda_1 (y_1(2) + b_1(2) - c_1(2) - [q(1)b'_1(1) + q(2)b'_2(2)]) \\
 & + \frac{1}{2} \lambda'_1 (y'_1(1) + b'_1(1) - c'_1(1) - [q'(1)b''_1(1) + q'_2(2)b''_1(2)]) \\
 & + \frac{1}{2} \lambda'_1 (y'_1(2) + b'_1(2) - c'_1(2) - [q'(1)b''_1(1) + q'_2(2)b''_1(2)])
 \end{aligned}$$

- Choices to make today: $c_1(2)$, $b'_1(1)$, $b'_1(2)$. First-order conditions

$$c_1(2) : \frac{1}{c_1(2)} - \lambda_1 = 0$$

$$b_1(1) : q(1)\lambda_1 - \frac{1}{2}\lambda'_1(1) = 0$$

$$b_1(2) : q(2)\lambda_1 - \frac{1}{2}\lambda'_1(2) = 0$$

- Eliminate the multipliers

$$q(1) = \frac{1}{2} \frac{c_1(2)}{c'_1(1)} \quad (13)$$

$$q(2) = \frac{1}{2} \frac{c_1(2)}{c'_1(2)} \quad (14)$$

- Do the same for Kai. We get,

$$q(1) = \frac{1}{2} \frac{c_2(2)}{c'_2(1)} \quad (15)$$

$$q(2) = \frac{1}{2} \frac{c_2(2)}{c'_2(2)} \quad (16)$$

- The law-of-one price

$$\frac{c_1(2)}{c'_1(s)} = \frac{c_2(2)}{c'_2(s)} \quad (17)$$

for any state $s = 1, 2$ in the future. If today was state 1, replace the 2 with a 1 in (17). So regardless of what state of nature we are in today, the growth rate in consumption will be equalized. A solution to (17) is to set consumption equal across individuals in every state of nature,

$$c_1(s) = c_2(s) \quad (18)$$

- The complete markets equilibrium achieves the social optimum

Overview of China's Financial System

- Mao: No financial markets
- Since Mao: Increasingly evolving financial system.
- Banks (indirect finance) and capital markets—stocks and bonds (direct finance).
- 1978-2009: Finance dominated by SOBs. Increasing and saving rates, channeled into SOBs that paid low interest, then made loans to SOEs. (Financial repression).
- High household saving rate, fueled investment. Demographic transition, life-cycle and precautionary saving motives.
- Stock and bond markets weren't important until after 2009.

The Stock Market

- Shenzhen and Shanghai exchanges (early 1990s).
- Early trading plagued by transparency issues, market manipulation.
- Majority of large firms are state controlled. Shareholders not in control.
- Circulating shares: A-shares. Denominated in RMB, available only to citizens.
B-Shares. Denominated in foreign currency, originally available only to foreigners, but now citizens can hold.
- H-shares are listings on markets outside of China (e.g., Hong Kong).
- Segmentation motivated by inconvertibility of the RMB and desire to keep degree of international financial integration low.
- A-shares denominated in RMB. B-shares in foreign currency (US dollars in Shanghai and Hong Kong dollars in Shenzhen). For a long time, A-shares closed to foreign investors, B-shares open only to foreigners.
2001, B-share market opened to individual Chinese investors.
2003, select foreign institutions allowed to buy A-shares.
2014, Shanghai and Hong Kong markets integrated

Bond Market

- Bond issues (primarily) by the government or SOBs. Not an important source of corporate financing.
- These bonds can be foisted upon commercial banks who hold until maturity.

Shadow Banking

- 2010-2012: Emergence of non-bank financial institutions and off-balance sheet products.
- Wealth management products: Available to household. Bank uses to buy other risky financial assets. A method for households to participate more broadly in financial markets for higher return.

Risk-Sharing under Mao and Deng

This material is from Curtis and Mark, "Business Cycles, Consumption and Risk Sharing: How Different is China?"

How does per capita consumption growth rate vary across subperiods

- Per capita consumption growth

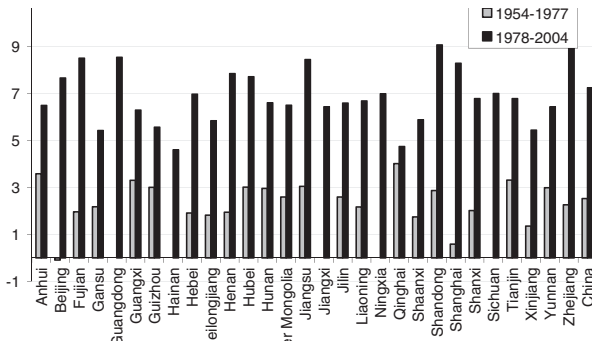


Figure: Pre- and Post-Reform Consumption Growth

- Growth is much higher in post-reform period
- Implies that saving is higher in post-reform period. Why? (Ans. Precautionary motive and buffer-stock saving.)

How has volatility of consumption growth changed?

- An informal measure of riskiness of economic life

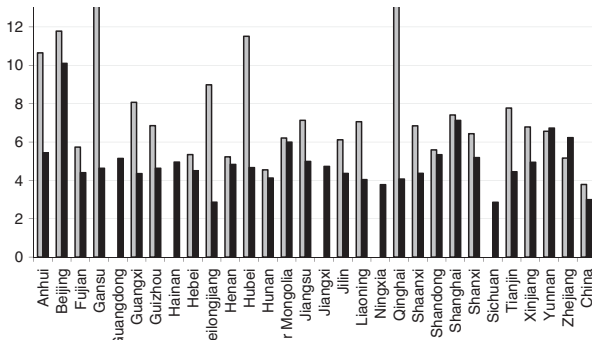


Figure: Consumption Growth Volatility

- Volatility declines in every province except for the southern province of Yunnan and Zhejiang bordering Shanghai to the south on the eastern coast. Gansu, an economy based heavily on mining in the interior western region, shows a huge decline in consumption growth volatility.

How has GDP growth changed?

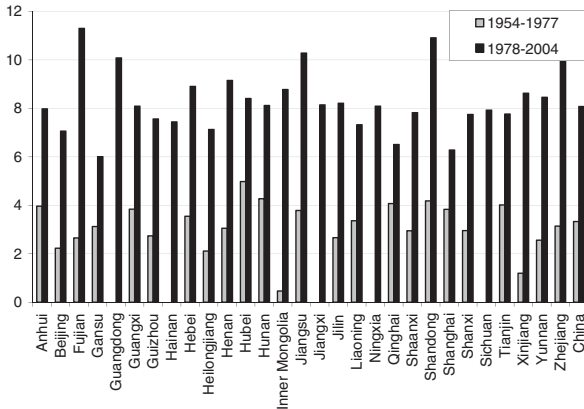


Fig. 3. Growth rate of output.

Figure: GDP Growth

GDP volatility

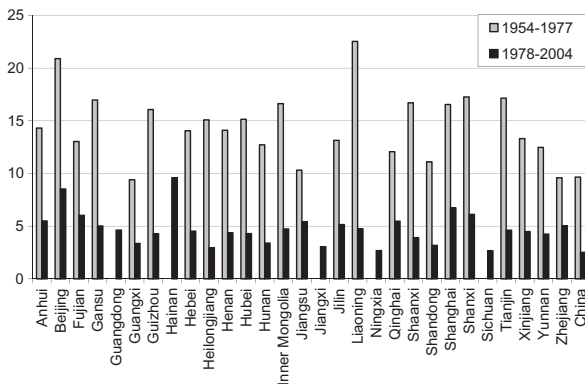


Fig. 4. Volatility of output growth.

Figure: GDP Volatility

In post-reform period, is about the same volatility as in the U.S. before the Great Moderation.

Correlation between provincial and aggregate GDP growth

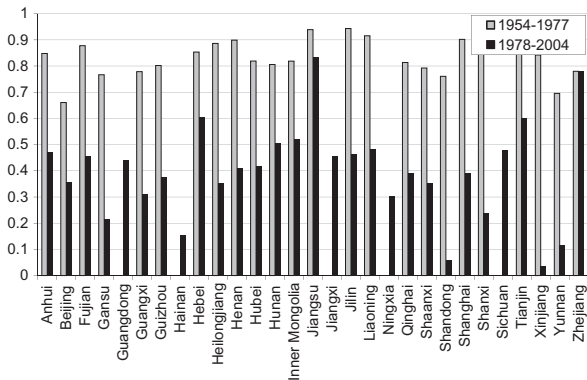


Fig. 5. Correlation between provincial aggregate output growth.

Figure: GDP Correlations

Correlation is higher in pre-reform period.

Correlation b/t provincial and aggregate cons. growth

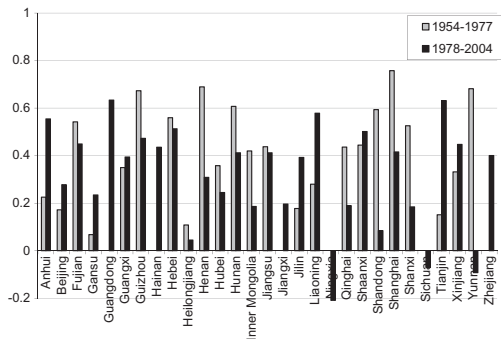


Fig. 7. Correlation between provincial and aggregate consumption growth.

Figure: Provincial and Aggregate Consumption Correlation

Correlations are generally pretty low in the first place. Correlations decline in 15 of 24 provinces.

Implement test of perfect risk-sharing

- **Null** hypothesis: Provincial level and aggregate level consumption growth are perfectly correlated.
- **Alternative** hypothesis: There is no risk sharing, and each province is a permanent income consumer.
- Let $\Delta c_{j,t}$ consumption growth in province j . (The change in log per capita consumption). ΔC_t is aggregate consumption growth. $y_{j,t}^p$ is a measure of permanent income. (Skipping the details of construction). Nest the two hypotheses in a regression.

$$\Delta c_{j,t} = \alpha_j + \lambda_j \Delta C_t + (1 - \lambda_j) \Delta y_{j,t}^p + \epsilon_{j,t}$$

- Null hypothesis: $\lambda_j = 1$ (perfect risksharing)
- Alternative hypothesis $0 \leq \lambda_j \leq 1$ (imperfect risk sharing).
- Alternative hypothesis: $0 = \lambda_j$ (no risk sharing).
- Compute 95 percent confidence interval for λ_j . Ask if it includes 1 or 0.

Results

Results in Table 6. I,II, and III refer to different ways of measuring permanent income. Let's just look at I:

Table 6. Tests of perfect risk sharing on 24 Chinese provinces

Income model	$\bar{\lambda}$	Average S.E. ($\bar{\lambda}$)	Values R^2	No. of provinces for which a 95% confidence interval contains the value of λ such that			
				$\lambda = 1$	$0 < \lambda < 1$	$\lambda = 0$	$\lambda = 0$ or 1
<i>1954–1977</i>							
I	0.45	0.43	0.38	6	2	8	8
II	0.39	0.40	0.42	6	1	9	8
III	0.33	0.38	0.43	5	2	11	6
<i>1978–2004</i>							
I	0.50	0.32	0.26	9	2	5	7
II	0.49	0.32	0.27	8	1	7	8
III	0.43	0.31	0.27	6	1	10	7

Figure: Empirical Results

Table 7. Tests of perfect risk sharing on US states and G-7 countries

Income model	$\bar{\lambda}$	Average S.E. ($\bar{\lambda}$)	Values R^2	No. of provinces for which a 95% confidence interval contains the value of λ such that			
				$\lambda = 1$	$0 < \lambda < 1$	$\lambda = 0$	$\lambda = 0$ or 1
<i>US states, 1972–1990</i>							
I	0.94	0.31	0.51	31	3	2	9
II	0.84	0.34	0.50	29	2	3	13
III	0.88	0.32	0.50	33	3	2	10
<i>G-7 countries, 1972–1990</i>							
I	0.60	0.26	0.45	2	2	2	1
II	0.44	0.44	0.57	1	2	4	0
III	0.37	0.37	0.57	1	2	4	0

Figure: Empirical Results

- Cannot reject perfect risk sharing for more provinces in post reform period than pre-reform period.
- Cannot reject no risk sharing for more provinces in pre-reform period than in post-reform period.
- Conclude that risk-sharing (while far from perfect) improved in the post-Mao period.