

MIDTERM EXAM

The exam consists of two parts. There are 80 points total. Part I has 12 points and Part II has 68 points.

Some parts of the exam are harder than others. If you get stuck on one part, do the best you can without spending too much time, and then work on other parts of the exam.

On the multiple choice (Part I): If you wish, you may add a BRIEF explanation of your answer to AT MOST ONE question. In that case, your grade on that question will be based on your answer and explanation together. This means that an explanation can either raise or lower a grade.

**PART I. Multiple choice (12 points)** In your blue book, give the best answer to 3 of the following 4 questions. (If you answer all 4 questions, your overall score will be based on your average, not on your 3 best scores.)

1. Among the things that Charles Jones and Jihee Kim discuss in “A Schumpeterian Model of Top Income Inequality” are:

A. The “Laibson racecar,” a comparison of income inequality in Spain and Portugal, and “exogenous government deficits.”

B. The “Ramey drone,” a comparison of income inequality in Sweden and Germany, and “endogenous self-fulfilling prophecies.”

C. The “Pistaferri chariot,” a comparison of income inequality in China and India, and “exogenous cartels.”

D. The “Luttmer rocket,” a comparison of income inequality in the United States and France, and “endogenous creative destruction.”

2. Consider an economy described by the Solow model where initially capital per unit of effective labor,  $k$ , is less than its balanced-growth-path value. A permanent increase in the depreciation rate,  $\delta$ , will cause output per worker to be:

A. Lower than it would have been otherwise during the transition to the balanced growth path.

B. Lower than it would have been otherwise when the economy reaches its balanced growth path.

C. (A) and (B).

D. None of the above.

3. A first-order Taylor expansion of the equation for  $\dot{k}(t)$  in the Solow model around the balanced-growth-path value of  $k$ ,  $k^*$ , can be written as:

A.  $\dot{k}(t) \cong [sf'(k^*) - (n + g + \delta)] \left[ \frac{dk(t)}{dt} \right].$

B.  $\dot{k}(t) \cong [1 - (n + g + \delta)] \left[ \frac{dk(t)}{dt} \right].$

C.  $\dot{k}(t) \cong [sf'(k(t)) - (n + g + \delta)k(t)] + [sf'(k^*) - (n + g + \delta)][k(t) - k^*].$

D.  $\dot{k}(t) \cong \left[ \frac{sf'(k^*)k^*}{f(k^*)} - \frac{(n+g+\delta)k^*}{f(k^*)} \right] \frac{f(k^*)}{k^*} [k(t) - k^*].$

4. Consider the Diamond overlapping-generations model with the depreciation rate,  $\delta$ , strictly less than 1. Let  $K$  denote capital,  $W$  the wage per worker (not per unit of effective labor),  $C_{1t}$  the consumption of a representative individual born in period  $t$  in the first period of their life,  $L_t$  the number of people born in period  $t$ ,  $r_{t+1}$  the real interest rate from period  $t$  to period  $t + 1$ , and  $n$  the rate of population growth. Then the capital stock in period  $t + 1$  is given by:

A.  $K_{t+1} = (W_t - C_{1t})L_t.$

B.  $K_{t+1} = (1 - \delta)K_t + (W_t - C_{1t})L_t.$

C.  $K_{t+1} = (1 + r_{t+1})[(1 - \delta)K_t + (W_t - C_{1t})L_t].$

D.  $K_{t+1} = (1 + r_{t+1})[(1 - \delta)\left[\frac{K_t}{1+n}\right] + (W_t - C_{1t})L_t].$

**PART II. Problems (68 points)**

**DO ALL 3 PROBLEMS.**

**(13 points) 5. Growth with forced saving.** Consider a Ramsey-Cass-Koopmans economy that is on its balanced growth path.

Suppose that (perhaps because of some type of national emergency) at some time,  $t_0$ , the government imposes an unexpected forced saving plan of fixed, known duration. Specifically, for  $t_0 < t < t_1$  (where  $t_1 > t_0$ ), the government requires households to set  $c(t) = \bar{c}$  (where, as usual,  $c$  is consumption per unit of effective labor, and where  $\bar{c}$  is strictly positive, and strictly less than the value of  $c$  on the balanced growth path).

Describe the behavior of  $k$  and  $c$  both from time  $t_0$  to time  $t_1$  and after time  $t_1$ . (Note: Phase diagrams, sketches of  $k$  and  $c$  as functions of time, and qualitative answers are all that is expected or desired.)

**(28 points) 6.** Consider the optimization problem of a representative household in the Ramsey-Cass-Koopmans model. For simplicity, population growth and technological progress are both absent (so  $n = g = 0$ ). As usual, the consumer supplies 1 unit of labor inelastically at each point in time, and so labor income at time  $t$  is  $W(t)$ . Thus, in the absence of any complications, the consumer's objective function would be

$$(1) \quad U = \int_{t=0}^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta}}{1-\theta} dt, \quad \rho > 0, \quad \theta > 0,$$

and the consumer's flow budget constraint would be

$$(2) \quad \dot{D}(t) = r(t)D(t) + W(t) - C(t).$$

In addition,  $D(0)$  (the consumer's initial assets) would be given, and the consumer would have to satisfy its lifetime budget constraint, which would imply

$$(3) \quad \lim_{t \rightarrow \infty} e^{-R(t)} D(t) \geq 0,$$

where, as usual,

$$(4) \quad R(t) \equiv \int_{\tau=0}^t r(\tau) d\tau.$$

This problem asks you to consider a variation on this set-up: the price that the consumer pays for consumption goods at time  $t$ ,  $P(t)$ , rather than always being equal to 1, is an increasing function of the consumer's consumption:  $P(t) = P(C(t))$ ,  $P(\cdot) > 0$ ,  $P'(\cdot) \geq 0$ . Thus the total amount the consumer pays at time  $t$  is  $P(C(t))C(t)$  rather than just  $C(t)$ . (Intuitively, imagine that when the consumer is consuming more, they have less time to shop, and so end up paying more on average for what they buy.)

a. How, if at all, does this change in assumptions affect equations (1)–(4)?

b. What is the Hamiltonian? (You are welcome to use either the current-value or the present-value Hamiltonian, but please state which you are using.)

c. Find the conditions for optimality.

d. Use your results to find an expression for  $\dot{C}(t)/C(t)$  analogous to our usual expression, which would be (since  $n = g = 0$ )  $\dot{C}(t)/C(t) = [r(t) - \rho]/\theta$ . (Hint: Your answer should simplify to  $\dot{C}(t)/C(t) = [r(t) - \rho]/\theta$  in the special case  $P(C(t)) = 1$  for all  $C(t)$ .)

e. Explain intuitively why the expression you found in part (d) does or does not differ from the usual expression, and the nature of any differences.

**(27 points) 7.** Consider the P. Romer model of endogenous technological change as presented in the reading and in lecture. Some of the key equilibrium conditions of that model, expressed in words, are:

1. Workers in the R&D sector earn their marginal revenue products.
2. Workers in the intermediate-input producing sector earn their marginal revenue products.
3. Wages in the two sectors are equal.
4. The sum of employment in the two sectors equals population.
5. The marginal revenue product of a worker in the R&D sector is equal to research productivity (new ideas per worker employed in R&D) times the present value of the profits from a new idea.
6. Total consumption equals output.
7. The growth rate of consumption equals the real interest rate minus households' discount rate.
8. The growth rate of output equals  $(1 - \phi)/\phi$  times the growth rate of ideas plus the growth rate of the number of workers producing intermediate inputs.
9. The price that each intermediate-input producer charges for their input is proportional to the wage (with constant of proportionality  $1/\phi$ ).

a. Explain what equilibrium condition (or conditions) listed above, if any, would be directly affected by each of the following changes to the model. (For example, the condition, "The growth rate of consumption equals the real interest rate minus households' discount rate" is not directly affected if that condition still holds but the growth rate of consumption, the real interest rate, and/or households' discount rate change; it is only directly affected if it is no longer the case that the growth rate of consumption equals the real interest rate minus households' discount rate.)

i. There is an economy-wide union that for some reason (such as trying to raise wages) prohibits some fraction of workers from taking jobs. As a result, at any time an exogenous fraction,  $\bar{u}$ , of the members of each household are unemployed (where  $0 < \bar{u} < 1$ ).

ii. Patents only last for a fixed length of time  $T$ . That is, if a new idea is discovered at time  $t$ , beginning at time  $t + T$  anyone can hire labor to produce the intermediate input that embodies that idea without paying the discoverer of the idea.

iii. The government taxes labor income (regardless of whether it is earned in the R&D sector or in the intermediate-input producing sector) at rate  $\tau$  ( $0 < \tau < 1$ ). The tax revenues are paid to households through lump-sum transfers (that is, the amount of each household's transfer does not depend on any decision that it makes).

b. Suppose you had to choose one of the three changes in part (a) to analyze. Which one do you think would be most interesting to consider? (That is, which one do you think would be most likely to yield interesting, unexpected, and/or non-obvious insights?) Defend your answer.