Ec141, Spring 2019

Midterm 2

Please read each question carefully. Start each question on a new bluebook page. The use of calculators and other computational aides is not allowed. Good luck!

[1] **[10 Points]** Please write your full name on this exam sheet and turn it in with your bluebook.

[2] **[30 Points]** Let Y equal tons of banana's harvested in a given season for a randomly sampled Honduran banana planation. Output is produced using labor and land according to $Y = AL^{\alpha_0}D^{1-\alpha_0}$, where L is the number of employed workers and D is the size of the plantation in acres and we assume that $0 < \alpha_0 < 1$. The price of a unit of output is P, while that of a unit of labor is W. These prices may vary across plantations (e.g., due to transportation costs, labor market segmentation etc.). We will treat D as a fixed factor; A captures sources of plantation-level differences in farm productivity due to unobserved differences in, for example, soil quality and managerial capacity. Plantation owners choose the level of employed labor to maximize profits. The observed values of L are therefore solutions to the optimization problem:

$$L = \arg\max_{l} P \cdot Al^{\alpha_0}D^{1-\alpha_0} - W \cdot l.$$

[a] **[2 Points]** Show that the amount of employed labor is given by

$$L = \left\{\alpha_0 \frac{P}{W} A\right\}^{\frac{1}{1-\alpha_0}} D. \tag{1}$$

[b] **[3 Points]** Let $a_0 = \frac{1}{1-\alpha_0}\ln\alpha_0 + \frac{1}{1-\alpha_0}\mathbb{E}[\ln A]$, $b_0 = \frac{1}{1-\alpha_0}$, and $V = \frac{1}{1-\alpha_0}\{\ln A - \mathbb{E}[\ln A]\}$. Show that the log of the labor-land ratio is given by

$$\ln\left(\frac{L}{D}\right) = a_0 + b_0 \ln\left(\frac{P}{W}\right) + V \tag{2}$$

and that, letting $c_0 = \mathbb{E}[\ln A]$ and $U = \ln A - \mathbb{E}[\ln A]$, the log of planation yield (output per unit of land) is given by

$$\ln\left(\frac{Y}{D}\right) = c_0 + \alpha_0 \ln\left(\frac{L}{D}\right) + U. \tag{3}$$

[c] **[7 Points]** Briefly discuss **[4-6 sentences]** the content and plausibility of the restriction

$$\mathbb{E}[\ln A|\ln(P/W)] = \mathbb{E}[\ln A]. \tag{4}$$

[d] **[8 Points]** Using (2), (3) and (4) show that the coefficient on $\ln(L/D)$ in $\mathbb{E}^*[\ln(Y/D)|\ln(L/D)]$ equals

$$\alpha_0 + (1-\alpha_0)\frac{\mathbb{V}(\ln A)}{\mathbb{V}(\ln A) + \mathbb{V}(\ln(P/W))}.$$

Provide some economic intuition for this result **[4-6 sentences]**..

[e] **[8 Points]** Using (2), (3) and (4) show that the coefficient on $\ln(L/D)$ in $\mathbb{E}^*[\ln(Y/D)|\ln(L/D), V]$ equals α_0. Provide some economic intuition for this result **[4-6 sentences]**.

[f] **[2 Points]** Assume that all plantations face the same output price (P) and labor cost (W). What value does the coefficient on $\ln(L/D)$ in $\mathbb{E}^*[\ln(Y/D)|\ln(L/D)]$ equal now? Why? **[1-3 sentences]**.

[3] **[40 Points]** This question is inspired by the Moving to Opportunity (MTO) social mobility experiment. Consider a population of public housing residents living in high poverty neighborhoods at baseline. As part of a social experiment, each resident is randomly assigned to receive a restricted housing voucher ($Z = 1$) or not ($Z = 0$). The voucher subsidizes rent for any housing unit located in a low poverty neighborhood; the voucher cannot be used in a high poverty neighborhood. At follow-up respondents either live in a low poverty neighborhood ($W = 1$) or not ($W = 0$). Let Y be an outcome of interest, for example earnings or a measure of academic achievement. We will call respondents in low poverty neighborhoods "treated" and all others "controls".

[a] **[2 Points]** Let $W(z)$ for $z \in \{0, 1\}$ denote each a respondent's treatment assignment given "encouragement" $Z = z$. Consider the following table:

	$W(0)$	$W(1)$
Complier	0	1
Defier	1	0
Always-taker	1	1
Never-taker	0	0

Explain how this table divides the population into four subpopulations. Describe these subpopulations in words **[4 sentences]**.

[b] **[3 Points]** Let $Y(w, z)$ denote a unit's potential outcome given treatment $W = w$ and encouragement $Z = z$. Explain, in words, the restriction that $Y(w, z) = Y(w)$ for $w \in \{0, 1\}$ and $z \in \{0, 1\}$. Is this restriction reasonable in the present context? **[2-4 sentences]**.

[c] **[2 Points]** Assume that $W(1) \geq W(0)$. What type of behavior does this restriction rule out? It is reasonable in the present context? **[1-3 sentences]**.

[d] **[3 Points]** Explain how random assignment ensures that

$$(Y(0), Y(1), W(0), W(1)) \perp Z = z \,\text{for}\, z \in \{0, 1\}.$$

What if, instead of random assignment, vouchers were allocated to households by a case-worker? How might the above restriction be violated? **[2-4 sentences]**.

[e] **[5 Points]** Show that, under the restrictions outlined above that

$$\mathbb{E}[W|\, Z = 1] - \mathbb{E}[W|\, Z = 0] = \Pr(W(0) = 0, W(1) = 1).$$

[f] **[7 Points]** Show that, under the restrictions outlined above that

$$\begin{aligned}\beta_{\text{WALD}} &= \frac{\mathbb{E}[Y|\, Z = 1] - \mathbb{E}[Y|\, Z = 0]}{\mathbb{E}[W|\, Z = 1] - \mathbb{E}[W|\, Z = 0]} \\ &= \mathbb{E}[Y(1) - Y(0)|\, W(0) = 0, W(1) = 1].\end{aligned}$$

Comment on your result? How does your interpretation depend on the magnitude of $\Pr(W(0) = 0, W(1) = 1)$? **[2-6 sentences]**.

[g] **[8 Points]** Show that if $\Pr(W(0) = 1, W(1) = 1) = 0$ that

$$\beta_{\text{WALD}} = \mathbb{E}[Y(1) - Y(0)|\, W = 1].$$

Is this likely to be true in the present context? Is a public housing recipient likely to be able to move to a low poverty neighborhood without voucher support? [**2-4 sentences**].

[4] [**20 Points**] Consider the following model of supply and demand:

$$\begin{aligned}
\ln Q_i^D(p) &= \alpha_1 + \alpha_2 \ln(p) + U_i^D \\
\ln Q_i^S(p) &= \beta_1 + \beta_2 \ln(p) + U_i^S,
\end{aligned}$$

with i indexing a generic random draw from a population of 'markets' ; U_i^D and U_i^S are market-specific demand and supply shocks. We assume that $\left(U_i^S, U_i^D\right) \overset{i.i.d}{\sim} F$ for $i = 1, 2, \ldots, N$. In each market the observed price and quantity pair (P_i, Q_i) coincides with the solution to market clearing condition

$$Q_i^D(P_i) = Q_i^S(P_i) = Q_i.$$

[a] [**3 Points**] Provide an economic interpretation of the parameters α_2 and β_2. What signs do you expect them to take? Why? [**1-3 sentences**].

[b] [**10 Points**] Depict the market equilibrium graphically. Solve for the equilibrium values of $\ln Q_i$ and $\ln P_i$ algebraically. How is the market price and quantity related to the demand and supply shocks, U_i^D and U_i^S? Provide some economic content for your answer. Can you use a figure to illustrate it? [**4-5 sentences**].

[c] [**7 Points**] Calculate $\mathbb{E}^*\left[\ln Q_i \middle| \ln P_i\right]$. You may assume that $\mathbb{C}\left(U_i^D, U_i^S\right) = 0$. Evaluate the coefficient on $\ln(P_i)$, does it coincide with an economically interpretable parameter? Assume that $\mathbb{V}\left(U_i^S\right) / \left(\mathbb{V}\left(U_i^S\right) + \mathbb{V}\left(U_i^D\right)\right) \approx 1$, does your answer change? Why? [**4-5 sentences**].