6 Weitzman's Dismal Theorem: REStats and AER PP

1. Let S_1 be climate sensitivity.It measures the climate response to sustained radiative forcing. IT is the global average surface warming following a doubling of CO2 concentration (ranging between 2 to 4.5 degrees C).

$$\Delta T = \left(\frac{S_1}{\ln(2)}\right) \Delta \ln CO_2$$

2. Much uncertainty surrounding estimates of S_1 from climate studies. Upper 5% probability level averaged over 22 climate sensitivity studies, cited by Weitzman is 7 degrees C. The distribution of S_1 estimates is fat-tailed. Warming over 4 degrees C can have disasterous effects.

3. Two-period model.

   (a) C is consumption, adjusted for welfare by subtracting out damages from climate change. Adaptation and mitigation included in C.

   Today's consumption $C_0 = 1$.

$$U(C) = \frac{C^{1-\eta}}{1-\eta}$$
$$U'(C) = C^{-\eta}$$
$$Y = \ln(C) = \Delta \ln(C)$$

   (b) Let G and γ be parameters. Assume effect of temperature change on consumption growth is,

$$Y = G - \gamma \Delta T$$

   (c) Stochastic discount factor

$$M = \beta \frac{U'(C)}{U'(1)} = \beta C^{-\eta} = \beta e^{-\eta \ln(C)} = \beta e^{-\eta Y}$$

   (d) $E(M)$ is current consumption the agent is willing to give up to get one extra unit of future consumption

$$E(M) = \beta E\left(e^{-\eta Y}\right)$$

   View as the shadow price for discounting future costs and benefits.

   An overall indicator of the present cost of future uncertainty. Gives the same answer as welfare-equivalent deterministic consumption or willingness to pay to avoid uncertainty. . This is the metaphor for understanding what drives the results of all utility-based welfare calculations of potentially unlimited exposure to catastrophic impacts.

   (e) Upper case denotes a random variable. Lower case is the realization. Formally, the SDF is

$$E(M) = \beta \int_{-\infty}^{\infty} e^{-\eta y} f(y)\, dy$$

   Look! It's the moment generating function of $f(y)$, the pdf. Let $\beta = 1/(1+\delta)$ .If C is log normal, $Y \sim N\left(\mu, s^2\right)$, then

$$E(M) = e^{\left(-\delta - \eta\mu - \frac{1}{2}\eta^2 s^2\right)}$$

(f) In asset pricing, $E(M) = 1/\left(1+r^f\right)$, where r^f is the risk-free interest rate,

$$r^f = \delta + \eta\mu - \frac{1}{2}\eta^2 s^2$$

(g) This is the social interest rate used for intergenerational cost-benefit discounting of policies to mitigate GHG emissions. Debate about what is the ethical rate of pure time preference, δ. In Weitzman's paper for any $\eta > 0$, the value of δ won't matter.

(h) The uncertainty surrounding s gives rise to fat-tails in teh distribution of future consumption. Turn now to a more intuitive and heuristic treatment of fat-tails.

4. Switch to AEA papers and proceedings paper

   (a) Life-time utility
   $$W = \frac{C_o^{1-\eta}}{1-\eta} + \beta E\left(\frac{C^{1-\eta}}{1-\eta}\right)$$
   where $C_0 = 1$ normaliztion is used. C^* is the catastropically low effective consumption which occurs with probability p. One extra unit of carbon abatement shifts future consumption (including C^*) up by $\theta > 0$. In the disaster state, consumption is $(1+\theta)\,C^*$. The SCC (social cost of carbon) is how much C_0 you give up today for extra expected future consumption that leaves welfare unchanged.
   $$\text{SCC} = \beta\theta p\,(C^*)^{1-\eta}$$
   This is the expected benefit in the disaster state. Analyze what happens when $p \to 0$ and $C^* \to 0$.

   (b) Let x measure how deep into the bad tail we are.
   $$\begin{aligned} x &= -\ln(C^*) \\ C^*(x) &= e^{-x} \\ p(x) &= \text{Prob}(x) \\ \text{SCC}(x) &= \beta\theta p(x)\left(e^{-x}\right)^{1-\eta} = \beta\theta p(x)\,e^{(\eta-1)x} \end{aligned}$$
   As $x \to \infty$, $C^* \to 0$. What happens as $x \to \infty$? If $\eta > 1, e^{(\eta-1)x} \to \infty$. What happens to SCC depends on how fast $p(x) \to 0$.

   (c) If $p(x)$ goes to zero faster than the exponential part, (like in the normal distribution), then $p(x)$ is thin-tailed. SCC(x) $\to$finite number.

   (d) If $p(x)$ is fat-tailed (like Student-t or Cauchy), then SCC(x) $\to \infty$. Hence,
   $$\lim_{x\to\infty} \text{SCC}(x) = \infty$$

5. This is called the 'dismal theorem'.

   (a) The dismal theorem is an absurd result. Society would not pay an infinite amount to abate one unit of carbon.

   (b) What could be wrong? Maybe the limiting probabilities are not fat-tailed. Maybe utility is not CRRA. Maybe you cant use expected presnet discounted utility to study extreme problems such as this.

6. What useful implications can be drawn?
   (a) An investment in abatement has the potential to be highly valuable. It could potentially dominate SCC calculations, if there are fat tails.
   (b) Beware of IAMs with no uncertainty, or with thin-tailed distributions.
   (c) IAMs need to seriously probe the fat-tailed probabilities of super-catastrophic impacts.