Amherst College
Department of Economics
Economics 58
Fall 2009

Name_______________

First Hour Test

SOLUTIONS

There are three questions on this 80 minute examination. Each is of equal weight in grading.

1. Suppose that a person consumes two goods, x and y, in fixed proportions. He or she always consumes 1 unit of x together with 3 units of y no matter what the relative prices are.

a. What is the mathematical form for this person's utility function?
Want $y = 3x$ So math form is $U(x, y) = Min[3x, y]$

b. Calculate the Marshallian demand function for both goods for this person.
Substitute $y=3x$ into budget constraint : $I = p_x x + 3p_y x$.

So, Demand functions are $x = \dfrac{I}{p_x + 3p_y}$ $\quad y = 3x = \dfrac{3I}{p_x + 3p_y}$

c. Use the results from part b to calculate the indirect utility function and the expenditure function in this case.

$$V(p_x, p_y, I) = y = \frac{3I}{p_x + 3p_y} \qquad E(p_x, p_y, V) = \frac{V(p_x + 3p_y)}{3}$$

d. In class we discussed why expenditure functions are concave in prices. Is the expenditure function you calculated in part c concave in p_x? Explain your result intuitively.

Here $\dfrac{\partial E}{\partial p_x} = V/3$ $\quad \dfrac{\partial^2 E}{\partial p_x^2} = 0$. Concavity requires the ability to substitute away from a good whose price has risen. In this case there is no substitution. Holding V constant requires holding x constant, so expenditures increase linearly with the price of x.

e. One way to compute the Marshallian price elasticity of demand in this problem is to use the Slutsky equation in elasticity form. Write down that equation and then use it in conjunction with the demand function calculated in part b to derive e_{x,p_x} in this case (note: this will be a function of the prices, not a specific number). What is unusual about the price elasticity in this case? (Note – the price elasticity can also be computed from the Marshall demand function in part b. – you may use this as a check if you wish, but not as your primary proof)
In this example, substitution effects are zero. So the Slutsky equation in elasticity form is:

$e_{x,p_x} = 0 - s_x e_{x,I}$. Using part b, $e_{x,I} = 1$, $s_x = \frac{p_x x}{I} = \frac{p_x}{p_x + 3p_y}$ So the Marshallian price elasticity is: $e_{x,p_x} = -\frac{p_x}{p_x + 3p_y}$. Can also get this directly from part b by applying the definition of elasticity.

2. John Hicks' reformulation of demand theory focused on compensated demand functions rather than on the more widely used Marshallian demand functions.

a. Explain the difference between these two concepts:
Marshallian demand allows the analyst to hold income constant.
Hicks demand allows the analyst to hold utility constant.

b. One advantage of the Hicks approach to demand is that cross price effects are symmetric. Explain what "symmetric" means in this context and prove that this symmetry holds for any pair of goods.

Symmetry means that the effect of the price of x on the quantity of y demanded is the same is the effect of the price of y on the quantity of x demanded. This follows from the envelope theorem and Young's Theorem:

$$\frac{\partial x_i^c}{\partial p_j} = \frac{\partial^2 E}{\partial p_j \partial p_i} = \frac{\partial^2 E}{\partial p_i \partial p_j} = \frac{\partial x_j^c}{\partial p_i}.$$

c. Explain in intuitive terms why Marshallian cross price effects generally are not symmetric.
Marshallian cross price effects contain income effects and these are generally not symmetric. One good may have a larger income effect than another because one is a luxury and the other a necessity. This difference in income elasticities will result in differing gross cross-price effects.

d. Explain why cross-price effects are indeed symmetric for the Marshallian demand functions derived from a Cobb-Douglas utility function. Then prove that this is a special case of the more general result that Marshallian cross-price effects are symmetric if a person always spends a constant share of his or her income on each good regardless of prices. (Hint: You will need to use the cross-price Slutsky equation to show this)

The substitution terms in the Slutsky equation are symmetric by part b. If the individual spends a constant share on each good, $x_i = \frac{s_i I}{p_i}$ $x_j = \frac{s_j I}{p_j}$ The income effects are represented by $-x_j \frac{\partial x_i}{\partial I} = -\frac{s_i s_j I}{p_i p_j} = -x_i \frac{\partial x_j}{\partial I}$. Hence, in this case the income effects are also symmetric. So the overall Marshallian cross-price effects are symmetric.

e. Hicks' "third law" states that "most goods are substitutes". This is proved using Euler's theorem for homogeneous equations which states that if a function $f(x_1,...,x_n)$ is homogeneous of degree k then $x_1 f_1 + ...x_n f_n = kf(x_1,...,x_n)$. Show how this mathematical theorem can be used to prove Hicks result and then show how to state the Hicks result in elasticity terms.

Apply the theorem to the Hicks demand function $x_1^c(p_1...p_n,V)$ which is homogeneous of degree zero in the prices: $p_1 \frac{\partial x_1^c}{\partial p_1} + ... + p_n \frac{\partial x_1^c}{\partial p_n} = 0$. But, since $\frac{\partial x_1^c}{\partial p_1} < 0$, the other terms must be predominately positive – the sign of substitutes.

Dividing the equation in this proof by x_1 yields $e_{x_1^c, p_1} + ... + e_{x_1^c, p_n} = 0$. In words, the sum of all of the compensated price elasticities for a good must be zero. Since the own price elasticity is negative, the cross price elasticities must be predominately positive.

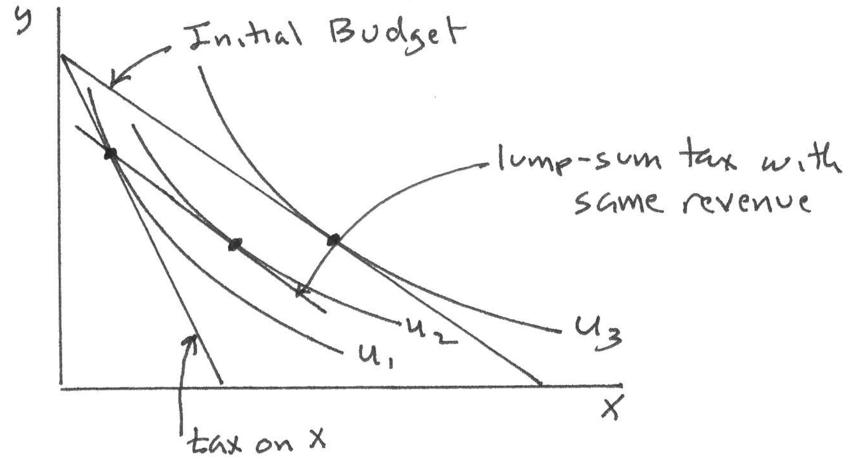
3. Much of the theory of optimal taxation is based on principles that can be derived from simple demand theory. In this question you are asked to take a primarily graphical approach to this topic.

a. Define the terms "lump sum principle" and "excess burden of a tax".

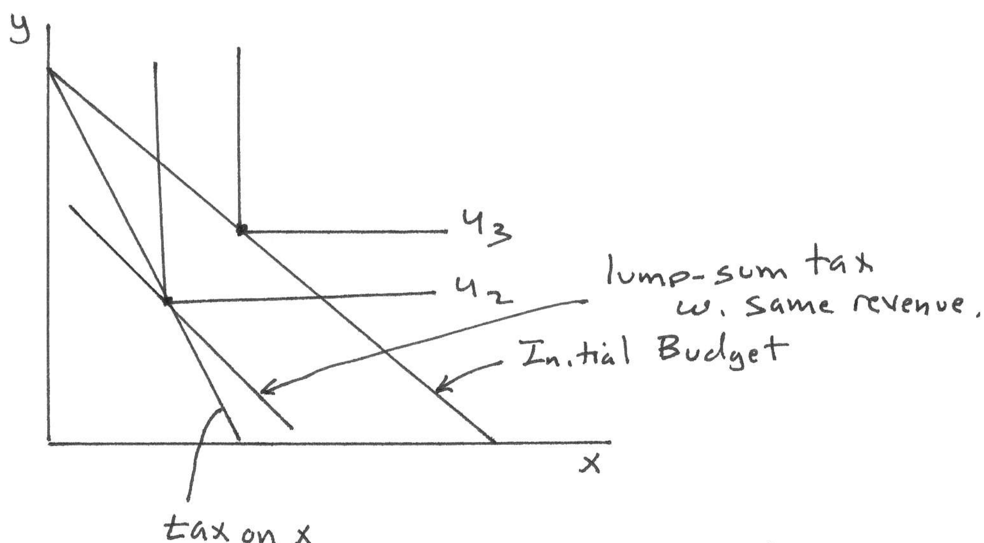
The "lump-sum principle states that non-distortionary taxes on general purchasing power involve smaller losses in utility than do taxes on individual goods. The "excess burden of a tax is the extra utility lost from a given tax relative to a lump-sum tax that raises the same revenue.

b. Show graphically how a simple two good model of utility maximization can be used to illustrate these two concepts. That is show the excess burden of a tax on a single item graphically. (Note: This question asks you to use the utility maximization graph – not a demand curve). Be sure to explain your answer fully.

$u_2 - u_1 =$ excess burden.



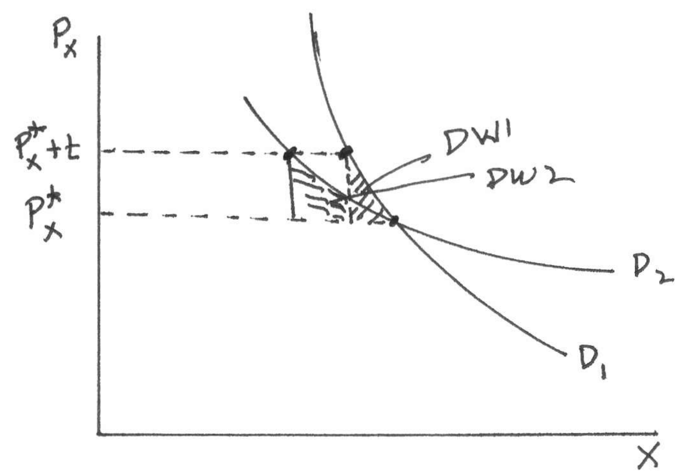
c. If x and y are consumed in fixed proportions, the lump sum principle does not hold and there is no excess burden from a commodity tax. Show this result with a carefully drawn graph.



No excess burden.

d. What conclusion do you draw from comparing your answers to parts b and c? That is, what determines the size of the excess burden from a tax? Illustrate your conclusion with a demand curve graph. Be sure you indicate whether you are drawing a Marshallian or Hicksian demand curve for your illustration.

Excess burden arises from the substitution effects induced by a tax. If there are no substitution effects there is no excess burden. The graph below shows two Hicks demand curves. One for a situation with large substitution effects, the other for a situation with small substitution effects. Notice that the excess burden is much smaller in the second case.



e. Another application of this line of theory concerns optimal pricing of electricity. If users exhibit different responses to electricity prices, how should prices be arranged so as to yield the minimal deadweight loss from an electricity monopoly?
The utility could charge higher prices to consumers with less elastic demands. This would minimize the deadweight loss from such monopoly pricing. The graph would look much like that in part d. This sort of pricing is sometimes called "Ramsey pricing" after the economist frank Ramsey who discovered the idea in the 1920's.