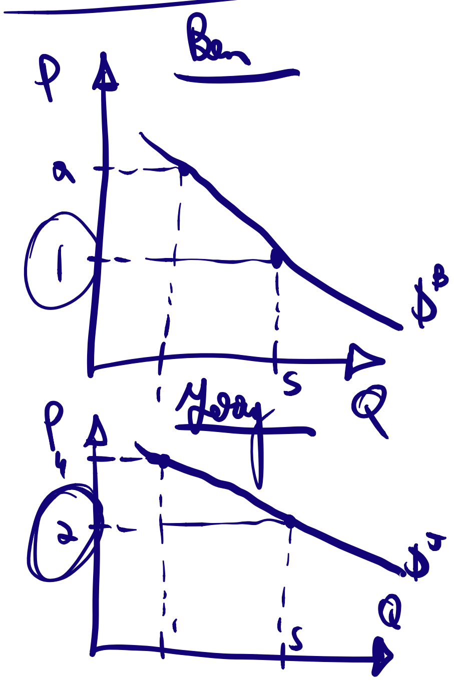
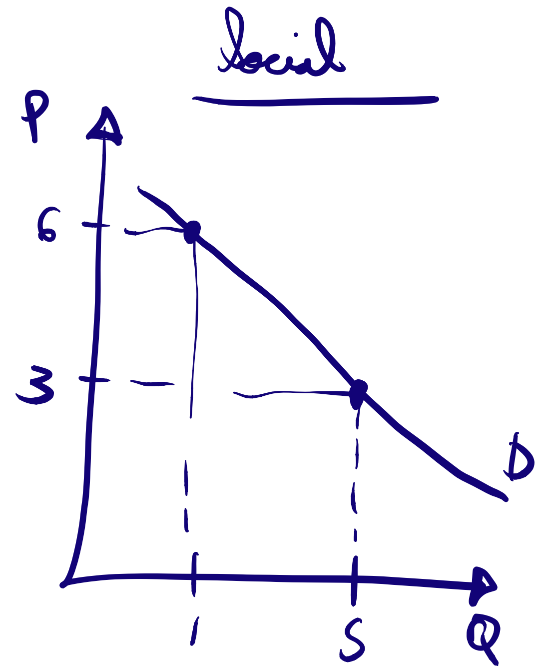
Section 10





2.1) G, Ch.7, Q.13

$$U_i = 4 \log X_i + 2 \log (M)$$

Income = 100

$$M = M_H + M_B$$

Bert:

$$\text{Max } 4 \log (X_B) + 2 \log (M_H + M_B)$$

$$\text{s.t.}: 100 = X_B + M_B \Rightarrow X_B = 100 - M_B$$

$$\max_{n_B} \; 4\log(100 - n_B) + 2\log(n_H + n_B)$$

$$[n_B]: \frac{-4}{100 - n_B} + \frac{2}{n_H + n_B} = 0$$

$$\frac{2}{n_H + n_B} = \frac{2}{100 - n_B}$$ ✓

$$100 - n_B = 2(n_H + n_B)$$

$$3n_B = 100 - 2n_H$$

$$n_B = \frac{100}{3} - \frac{2}{3} n_H \quad (1)$$ ← Response function

$$n_H = \frac{100}{3} - \frac{2}{3} n_B \quad (2)$$

$$M_B = \frac{100}{3} - \frac{2}{3}\underbrace{\left(\frac{100}{3} - \frac{2}{3}M_B\right)}_{M_H}$$

$$M_B = \frac{100}{3} - \frac{200}{9} + \frac{4}{9}M_B$$

$$\frac{5}{9}M_B = \frac{100}{9} \Rightarrow \boxed{M_B = 20}$$

$$\Rightarrow \boxed{M_H = 20}$$

$$\Rightarrow \boxed{M = 40}$$

b) Social Planner:

$$\max_{(x_H, x_B, m_B, m_H)} U_H + U_B$$

s.t.: $100 = x_H + m_H \Rightarrow x_H = 100 - m_H$

$100 = x_B + m_B$

$$\max_{M_H, M_B} \underbrace{4\log(\overbrace{100 - M_H}^{x_i}) + 2\log(M_H + M_B)}_{U_H} + \underbrace{4\log(100 - M_B) + 2\log(M_H + M_B)}_{U_B}$$

$$[M_H]: \frac{-4}{100 - M_H} + \frac{2}{M_H + M_B} + \frac{2}{M_H + M_B} = 0$$

$$\frac{4}{100 - M_H} = \frac{4}{M_H + M_B}$$

$$M_H + M_B = 100 - M_H$$

$$2M_H = 100 - \underbrace{M_B}_{= M_H \text{ (due to symmetry)}}$$

$$3M_H = 100 \Rightarrow \boxed{M_H = 33.3}$$

$$\Rightarrow \boxed{M_B = 33.3}$$

$$\Rightarrow M = 100\left(\frac{2}{3}\right)$$

2.2) G, Ch. 7, Q.15

$B_I = 100, \forall M \geq 1 \Rightarrow MB_I = 0$ (1)

$B_{II} = 200 + 30M - 1.5M^2 \Rightarrow MB_{II} = 30 - 3M$ (2)

$B_{III} = 150 + 90M - 4.5M^2 \Rightarrow MB_{III} = 90 - 9M$ (3)

$MC = 3{,}600$ so $[(1)+(2)+(3)]$

$SMC = SMB$

$3{,}600 = 50(0) + 50(30-3M) + 50(90-9M)$

$3{,}600 = 1{,}500 - 150M + 4{,}500 - 450M$

$600M = 6000 - 3{,}600$

$M = 4$