CRITICAL ORIENTATION OF SEISMIC FORCE FOR FLOOR-MOUNTED NONSTRUCTURAL COMPONENT ANCHORAGE

Robert Wang, PE
Degenkolb Engineers
Oakland, CA

Dan Miller, PhD, PE
Degenkolb Engineers
Sacramento, CA

Edward Gil, PE
Degenkolb Engineers
Oakland, CA

Abstract

The design of nonstructural component anchorage depends on both magnitude and direction of the seismic force ($\mathbf{F_p}$), the latter of which is the subject of this paper. In recent years, research efforts led by ATC (2017) have greatly improved the estimation of seismic demand, resulting in a revamped $\mathbf{F_p}$ equation in the 2022 version of ASCE/SEI-7. As for direction, the code offers limited guidance and states that $\mathbf{F_p}$ shall be applied in the direction that produces the most critical load effects. Alternatively, the code permits the use of the empirical "100%-30%" directional combination like the one used in the seismic analysis of building structures. In this paper, we explore the surprisingly nuanced topic of critical load orientation for design of floor-mounted component anchorage. The study began with a rigorous definition of how the load effects – namely anchor shear and tension demand – are calculated, addressing variabilities in assumptions and methods in industry practice. The formulations were then incorporated into a standalone python package to streamline calculations. Using this program, a series of parametric studies were conducted to tackle the key question: "how does one determine the critical force direction for floor-mounted component anchorage?". An example problem is provided at the end to illustrate the concepts discussed herein.

For a specific archetype of floor-mounted component, namely those with rectangular base restrained by a rectangular array of anchors, the following conclusions were made:

- If the component is directly bearing on the floor, the critical direction that maximizes anchor tension is typically along one of the flat edges. The 100%-30% combination rule is always conservative in this context since the critical direction is in one of the principal directions. It is often readily apparent which one is worse.
- If the component is supported on legs (i.e. isolators, snubbers, columns, etc.), the critical direction that maximizes anchor tension is always along a diagonal. Furthermore, it can be explicitly calculated using equation [33]. There is very little reason to use the 100%-30% combination rule in this context since the simple closed-form solution serves as a better alternative.
- The critical direction that maximizes anchor shear does not exist if shear demand due to in-plane torsion is ignored. Otherwise, it can be determined using the procedures outlined in this paper. Most notably, the critical direction that maximizes anchor shear or tension are often not the same.

For situations that do not fit the archetype described above, more deliberation about loading direction is recommended. Two such scenarios are presented in this paper along with other helpful tips for practicing engineers.

Notations

$\boldsymbol{C}$	=	resultant compression force from bearing surface
$\boldsymbol{c_x}$	=	distance from centroid to an anchor in the x direction
$\boldsymbol{c_y}$	=	distance from centroid to an anchor in the y direction
$\boldsymbol{d_w}$	=	distance from neutral axis to the component center of mass
$\boldsymbol{d_i}$	=	distance from neutral axis to an anchor
$\boldsymbol{d_N}$	=	distance from neutral axis to the furthest anchor
$\boldsymbol{e_x}$	=	eccentricity between component center of mass (COM) and anchor group center of rigidity (COR) along the x-direction
$\boldsymbol{e_y}$	=	eccentricity between component center of mass (COM) and anchor group center of rigidity (COR) along the y-direction
$\boldsymbol{F_h}$	=	design horizontal force on component with all applicable ASCE 7-22 factors.
$\boldsymbol{F_v}$	=	design vertical force on component with all applicable ASCE 7-22 factors.
$\boldsymbol{I_x}$	=	anchor group moment of inertia about the x-axis
$\boldsymbol{I_y}$	=	anchor group moment of inertia about the y-axis
$\boldsymbol{I_{xy}}$	=	anchor group product moment of inertia
$\boldsymbol{J}$	=	anchor group polar moment of inertia
$\boldsymbol{M_{OT}}$	=	overturning moment induced by horizontal force (rigid base method and elastic method)
$\boldsymbol{M_R}$	=	resisting moment induced by vertical force (rigid base method)
$\boldsymbol{M_{net}}$	=	net overturning moment (rigid base method)
$\boldsymbol{M_{weight}}$	=	gravity-induced overturning moment (elastic method)
$\boldsymbol{M_{total}}$	=	total overturning moment (elastic method)
$\boldsymbol{M_{torsion}}$	=	in-plane torsion
$\boldsymbol{N_{anchors}}$	=	total number of anchors in anchor group
$\boldsymbol{P_i}$	=	axial demand for an anchor (+P for tension, -P for compression)
$\boldsymbol{P_{weight}}$	=	axial demand for an anchor due to vertical force
$\boldsymbol{P_M}$	=	axial demand for an anchor due to overturning moment
$\boldsymbol{T_i}$	=	tension demand for an anchor
$\boldsymbol{T_N}$	=	tension demand for the anchor furthest away from the neutral axis
$\boldsymbol{T_{max}}$	=	maximum anchor tension demand
$\boldsymbol{V_i}$	=	shear demand for an anchor
$\boldsymbol{V_{direct}}$	=	shear demand due to a concentrically applied horizontal force
$\boldsymbol{V_{torsion}}$	=	shear demand due to in-plane torsion
$\boldsymbol{x_i}$	=	x coordinate of an anchor
$\boldsymbol{\bar{x}_i}$	=	x coordinate of an anchor relative to the anchor group center of rigidity
$\boldsymbol{x_{COM}}$	=	x coordinate of the component center of mass
$\boldsymbol{x_{COR}}$	=	x coordinate of the anchor group center of rigidity
$\boldsymbol{y_i}$	=	y coordinate of an anchor
$\boldsymbol{\bar{y}_i}$	=	y coordinate of an anchor relative to the anchor group center of rigidity
$\boldsymbol{y_{COM}}$	=	y coordinate of the component center of mass
$\boldsymbol{y_{COR}}$	=	y coordinate of the anchor group center of rigidity
$\boldsymbol{z_{COM}}$	=	z distance from floor to component center of mass
$\boldsymbol{\alpha}$	=	angle of the equipment orientation measured counterclockwise from the x-axis
$\boldsymbol{\theta}$	=	angle of the applied force measured counterclockwise from the x-axis
$\boldsymbol{\theta_p}$	=	offset angle to the anchor group principal orientation

Introduction

In recent years, growing interest in resilience and post-earthquake functional recovery has highlighted the importance of seismic bracing of nonstructural components. Nonstructural components and contents can make up as much as 80% of the total building construction cost (Taghavi & Miranda, 2003). Given their potential impact on decision variables like repair cost, and repair time, the design of nonstructural anchorage is a critical part of the building design process that warrants more attention from practicing engineers. The design of nonstructural component anchorage relies on estimation of both magnitude and <u>direction</u> of seismic force (***F_p***), the latter of which is the subject of this paper. Design equation for ***F_p*** is provided in chapter 13 of ASCE/SEI-7-22 *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*. As for direction, the code offers limited guidance and states that ***F_p*** shall be applied in the direction that produces the most critical load effects. Alternatively, the code permits the use of the empirical "100%-30%" directional combination like the one used in seismic analysis of building structures.

In this paper, we examine the surprisingly nuanced topic of load orientation for floor-mounted component anchorage and attempt to answer the question: "how does one determine the critical force direction for floor-mounted component anchorage?". To that end, parametric studies were conducted along with an example problem based on real equipment data obtained from the OSHPD Preapproval of Manufacturer's Certification (OPM) catalog. Based on the results of our studies, we offer practical design guidance and highlight common calculation pitfalls in industry practice.

Overview of Floor-Mounted Component Anchorage

As the name suggests, floor-mounted nonstructural components are seismically restrained to the building's floor system. The intent of this restraint is to prevent sliding and overturning failure. In other words, the component must not slide out of position, and it must not tip over. In practice, this means applying a horizontal seismic force (***F_p***), typically as a concentrated force at the component's center of mass, then translating the applied force and the associated overturning effects to local tension and shear demands at the individual anchors. If the mounting substrate is concrete, then the concrete anchors are subject to the limit states outlined in Chapter 17 of ACI 318-19 *Building Code Requirements for Structural Concrete*. Before we can determine the direction that would cause the most critical load effects, we must first establish the procedures for calculating these load effects in the first place; namely anchor tension and shear demands.

Tension Demand: Rigid Base Method

There are two common methods for computing anchor tension demand. In the first method, the equipment is idealized as a rigid box with a fully rigid base. As the component tips over, a linear tension profile is assumed along the base. The free body diagram of this condition is shown in figure 1.

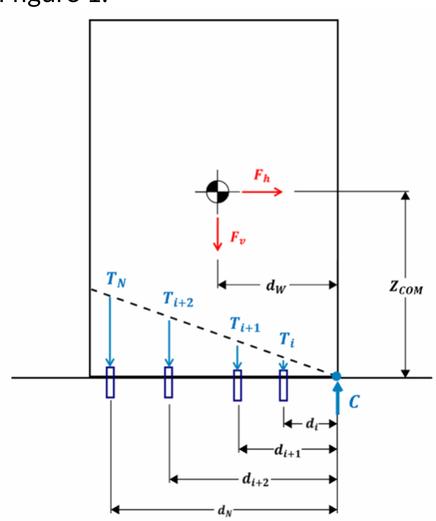

Figure 1: Free Body Diagram for the Rigid Base Method

While this method is conceptually simple – requiring only rudimentary statics – embedded in this straightforward method is an assumption of neutral axis depth, whose determination is a topic of considerable variability in industry practice. Some engineers opt for the simple approach of resolving the net overturning moment with a tension/compression couple and an assumed internal lever arm, while others invoke the use of iterative solvers with rectangular stress blocks, triangular stress blocks, or refined concrete material models to more accurately locate the neutral axis.

The simplest approach, and the one favored by the authors of this paper, is to assume axis of rotation at the very edge of the component as seen in (Lobos & Tokas, 2023). In figure 1, notice how the concrete bearing reaction (**C**) is represented as a concentrated point load, rather than a distributed load whose resultant lies some distance away from the edge. This simplification recognizes that, in most cases, the depth of compression stress block is relatively small compared to the component footprint due to ample bearing area and low tensile capacity of concrete anchors (relative to fully developed rebar). Thus, the calculated tension demands are not particularly sensitive to a few inches of lever arm reduction. With the edge assumed as the axis of rotation, we greatly simplify the problem without significant loss in accuracy. Nevertheless, it may be prudent to check if the component footprint is indeed large enough relative to the neutral axis depth. A more conservative approach would be to reduce the anchor lever arm distances ($\mathbf{d_i}$) by an assumed fraction of the total depth (analogous to the parameter "jd" in concrete beam design). Derivation of the rigid base method is presented below.

In the free-body diagram presented in figure 1, the applied horizontal seismic force ($\boldsymbol{F_h}$) is equal to $\boldsymbol{F_p}$ with applicable overstrength and ASD/LRFD load combination factors per ASCE 7-22. This horizontal force induces an overturning moment ($\mathbf{M_{OT}}$). $\mathbf{Z_{COM}}$ is the distance between the component's center of mass and the floor.

$$M_{OT} = F_h \times Z_{COM} \tag{1}$$

The vertical force ($\boldsymbol{F_v}$) is equal to the component weight ($\mathbf{W_p}$) plus vertical seismic effects ($\mathbf{E_v}$), with applicable LRFD/ASD load combination factors per ASCE 7-22. This vertical force induces a restoring moment which counteracts the overturning moment. $\mathbf{d_w}$ is the distance between the component's center of mass to the axis of rotation which is assumed to be at the edge of the component.

$$M_R = F_v \times d_W \tag{2}$$

The resulting net overturning moment ($\mathbf{M_{net}}$) is calculated as the difference between $\mathbf{M_R}$ and $\mathbf{M_{OT}}$.

$$M_{net} = M_{OT} - M_R \tag{3}$$

If $\mathbf{M_{net}}$ is negative, then the component is not at risk of tipping over (i.e. T = 0). Otherwise, tension demands are calculated as follows. The anchor furthest away from the neutral axis has the highest tension demand and is denoted with subscript "N" ($\mathbf{d_N}$, $\mathbf{T_N}$). Tension demands for the other anchors vary linearly from 0 at the neutral axis, to $\mathbf{T_N}$ at $\mathbf{d_N}$.

$$T_i = \frac{d_i}{d_N} \times T_N \tag{4}$$

With this linear relationship established, substitute equation [4] into the moment equilibrium equation and solve for $\mathbf{T_N}$.

$$\sum M = 0 = -F_h H_{cg} + F_v d_w + \sum T_i d_i \tag{5}$$

$$F_h H_{cg} - F_v d_w = \sum \left(\frac{d_i}{d_N} \times T_N \right) d_i \tag{6}$$

$$M_{net} = \frac{T_N}{d_N} \sum d_i^2 \tag{7}$$

$$T_N = T_{max} = \frac{M_{net} d_N}{\sum d_i^2} \tag{8}$$

Once we have the max anchor tension, use Eq [4] to obtain demands at the intermediate anchors (Eq [8] can also be used with $\mathbf{d_i}$ substituted for $\mathbf{d_N}$). It's important to emphasize that $\mathbf{T_i}$ or $\mathbf{T_N}$ should not be divided by anything. In the formulation above, overlapping anchors with equal distance to neutral axis are accounted for in the denominator with the $\boldsymbol{\sum d_i^2}$ term.

Lastly, the compression resultant (**C**) can be calculated using the force equilibrium equation. The engineer may wish to verify how much area is required to sustain this compressive force and adjust the neutral axis depth accordingly.

$$C = F_v + \sum T_i \tag{9}$$

At diagonal orientations, it becomes somewhat tedious to calculate distance terms $\mathbf{d_i}$ by hand, but this is easy to do programmatically. Figure 2 shows an overturning equipment that is rotated $\boldsymbol{\alpha}$ degrees from the x-axis.

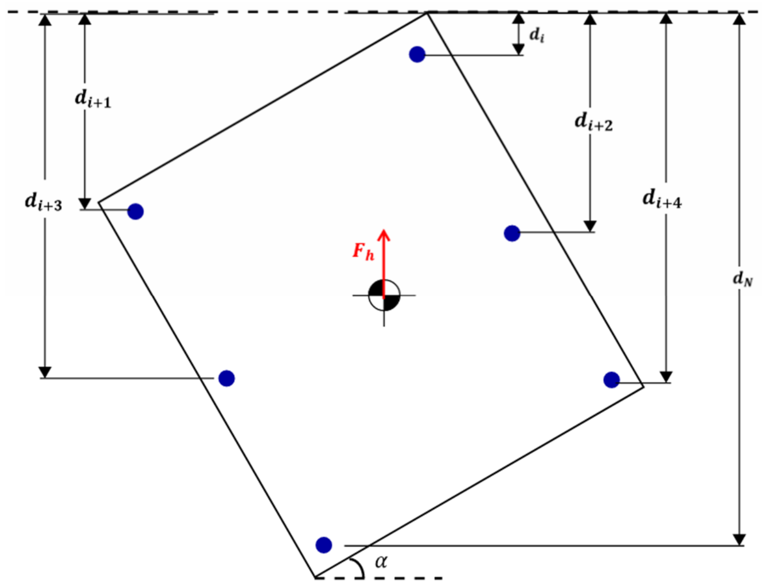

Figure 2: Example of Non-Principal Direction Overturning

There are two nuances worth highlighting about the rigid base method. Firstly, the orientation of the applied moment vector is not necessarily aligned with the orientation of the neutral axis. Physically speaking, this means a 25-degrees oriented load may not necessarily tip the component at 25-degrees. Any asymmetry in the base geometry or anchor layout could produce minor-axis moment in the orthogonal direction. This minor-axis moment maintains equilibrium and slightly offsets the direction of tipping (i.e. neutral axis orientation). The solution strategy presented above uses various simplifications to derive anchor tension without reliance on any material properties information. In addition, it only considers a two-dimensional slice of a three-dimensional problem, which means moment equilibrium in the orthogonal direction is not really addressed. The most accurate solution would require the use of material models as well as iterative solvers that can locate both the depth and orientation of the neutral axis. An inexact, but simpler approach would be to rotate the component geometry to encompass all possible neutral axis orientations. At each orientation, the full $\mathbf{F_p}$ force is applied in the direction perpendicular to the neutral axis. Minor-axis force/moment is ignored since it will only increase the magnitude of the applied force resultant.

Secondly, recall that the principle of superposition is only valid when the underlying physical relationship is linear. Unfortunately, linearity is not preserved when we vary load orientations for a component on a compression-only bearing surface. In practice, many engineers prefer to rotate the applied force vector rather than the neutral axis. Given that superposition is not allowed, it is not appropriate to resolve the total applied moment into components along two orthogonal axes (e.g. along the flat edges), analyze them separately, then superimpose the anchor demands afterwards. Consider the analogous problem of a rectangular concrete column subjected to bi-axial bending. The correct solution requires the determination of a diagonally oriented neutral axis. Superposition of results about a major and minor direction is not applicable to concrete sections, nor to the overturning rigid box above.

The elastic method presented in the next section, however, does allow for superposition of anchor demands, and it is the preferred method for many engineers due to its similarity with familiar concepts in theory of elasticity.

Tension Demand: Elastic Method

The elastic method is commonly used in the design of bolted steel connections. In essence, a bolt group – or an anchor group in the context of component anchorage – is treated like an elastic section with geometric properties like centroid and moment of inertias. With this assumption in mind, finding anchor tension demands is entirely analogous to finding normal stresses using the combined elastic stress formulas presented in introductory mechanics of material courses (i.e. $\sigma = P/A + M_x/S_x + M_y/S_y$). Unlike a cross-section, which has continuous area, anchors are assumed to have equal unitary area at discrete locations. Equations for anchor group geometric properties are summarized in table 1.

Table 1: Equations for Anchor Group Geometric Properties

Geometric Property	Equations
Centroid	$x_{COR} = \sum x_i/N_{anchors}$ $y_{COR} = \sum y_i/N_{anchors}$
Moment of Inertias	$I_x = \sum(y_i - y_c)^2$ $I_y = \sum(x_i - x_c)^2$
Polar Moment of Inertia	$J = I_x + I_y$
Product Moment of Inertia	$I_{xy} = \sum(x_i - x_c)(y_i - y_c)$
Angle to Principal Orientation	$\theta_p = 0.5 \times \text{atan}\,(I_{xy}/(I_x - I_y)/2)$

There are two major assumptions that underlies the elastic method. First, the neutral axis is assumed to coincide with the anchor group centroidal axis. Second, overturning moment is resolved entirely through the anchors alone without consideration of any bearing surface. In other words, anchors are assumed to take compression, and must do so to maintain equilibrium. In fact, the component footprint is completely irrelevant to the calculation and can be ignored.

Theoretically speaking, the elastic method is only applicable to components on legs (isolators, snubbers, columns, etc.), such as vibration-isolated equipment or elevated tanks. However, it is common to see it used for all base bearing conditions in industry practice. When the elastic method is applied to non-legged components, the predicted anchor tension tends to be conservative. Despite the conservatism and ease of application, it is the author's opinion that elastic method should not be used for non-legged, base-bearing components because the assumption that anchors take compression is erroneous from the outset. Refer to figure 3 for a free body diagram of an anchor group.

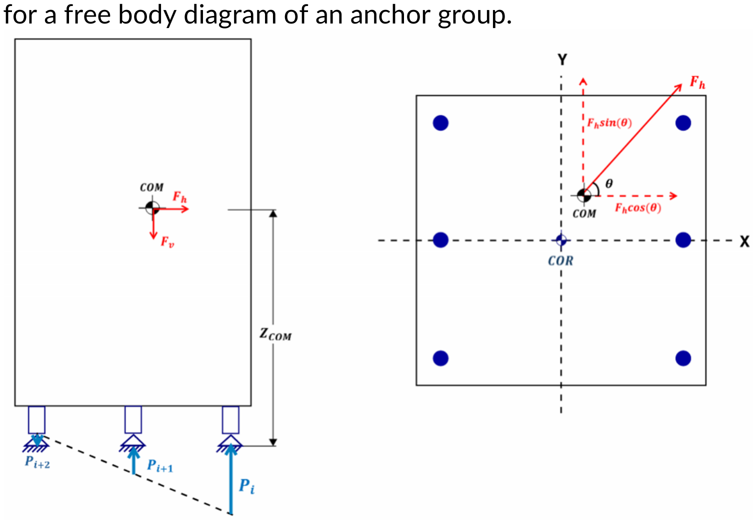

Figure 3: Elastic Anchor Group Free Body Diagram Elevation and Plan View

The word “anchors” is used loosely in this context. The blue dots in figure 3 can also be individual legs, columns, isolators, snubbers, etc. The underlying assumption of the elastic method is that tension demand varies linearly, emanating from zero at the centroid outwards to the extreme fibers/anchors/legs. With legged components, two analyses may be required: one for the connection between component and legs, and another for the connection between leg and floor. Alternatively, the component and legs are treated as a combined assembly. The second approach assumes that the linear tension profile described above extends to the floor, which is only accurate when the component-to-legs connection is sufficiently rigid to fully couple the assembly for overturning. $\mathbf{Z_{COM}}$ either includes or excludes the legs depending on the approach taken.

Notice how the anchor demands in figure 3 are denoted by the letter $\mathbf{P_i}$ instead of $\mathbf{T_i}$. This is a conscious decision meant to emphasize how the anchors can now take both compressions (**-P**) and tension (**+P**). Derivation of axial demand using the elastic method is presented below. Unlike the rigid base method, signs become very important to track in the elastic method. $\mathbf{F_v}$ and $\mathbf{F_h}$ are always positive. Negative signs are inserted where applicable to maintain right-hand convention.

Axial demand due to self-weight is simply the vertical force divided by the number of anchors ($\mathbf{N_{anchors}}$). The vertical force ($\boldsymbol{F_v}$) is equal to the weight of the component ($\mathbf{W_p}$) plus vertical seismic effects ($\mathbf{E_v}$), with applicable LRFD/ASD load combination factors per ASCE 7-22.

$$P_{weight} = -F_v/N_{anchors} \quad [10]$$

The applied horizontal seismic force ($\boldsymbol{F_h}$) is equal to $\boldsymbol{F_p}$ with applicable overstrength and LRFD/ASD load combination factors per ASCE 7-22. At any orientation, $\boldsymbol{F_h}$ is resolved into its component along the principal X and Y axis, causing overturning in both directions ($\mathbf{M_{OT,x}}$ and $\mathbf{M_{OT,y}}$).

$$M_{OT,x} = -F_h \sin(\theta) \times Z_{COM} \quad [11]$$

$$M_{OT,y} = F_h \cos(\theta) \times Z_{COM} \quad [12]$$

If the anchor group centroid (**COR**) is offset from the component center of mass (**COM**) in plan, then there is an additional self-weight-induced overturning demand. Where applicable, this moment shifts the weight distribution such that $\mathbf{F_v}$ is no longer evenly distributed amongst the anchors.

$$M_{weight,x} = -F_v(y_{COM} - y_{COR}) \quad [13]$$

$$M_{weight,y} = F_v(x_{COM} - x_{COR}) \quad [14]$$

The total overturning moment demand is:

$$M_{total,x} = M_{OT,x} + M_{weight,x} \quad [15]$$

$$M_{total,y} = M_{OT,y} + M_{weight,y} \quad [16]$$

The axial demand due to overturning for an anchor located at ($\mathbf{x_i}$, $\mathbf{y_i}$) is:

$$P_{Mx} = \frac{M_{total,x}(y_i - y_{COR})}{I_x} \quad [17]$$

$$P_{My} = \frac{-M_{total,y}(x_i - x_{COR})}{I_y} \quad [18]$$

Lastly, apply the principle of superposition to calculate the total anchor axial demand.

$$P_i = P_{weight} + P_{Mx} + P_{My} \quad [19]$$

It’s important to note that the equations presented above are only valid when the anchor group is in its principal orientation. In other words, the selection of X and Y axis cannot not arbitrary. An anchor group is in its principal orientation if the product moment of inertia ($\mathbf{I_{xy}}$) is equal to 0. If not, the geometry must be rotated by $\boldsymbol{\theta_p}$ before the above equations can be used. In addition, the applied force must also be resolved into components about the principal axes.

Shear Demand

So far in this paper, we have only discussed tension demand. As for shear demand, it is common to assume perfect alignment between anchor group COR and the component COM. In such cases, shear demand is simply the total horizontal force divided by the number of anchors. Reliance on frictional resistance is not permitted per ASCE 7-22 section 13.4.

$$V_i = F_h / N_{anchors} \quad [20]$$

In reality, COR and COM are often misaligned which produces additional shear demand due to in-plane torsion. The additional "torsional" shear demand can be calculated using the elastic method. This process is analogous to rigid diaphragm analysis where the total shear is equal to direct shear plus an additional torsional shear.

$$V_i = V_{direct} + V_{torsion} \quad [21]$$

Direct shear is calculated with the equations below.

$$V_{direct,x} = -F_h \cos(\theta) / N_{anchors} \quad [22]$$

$$V_{direct,y} = -F_h \sin(\theta) / N_{anchors} \quad [23]$$

In-plane torsion (**$M_{torsion}$**) due to in-plane eccentricity (**e**) is calculated as follows.

$$e_x = x_{COM} - x_{COR} \quad [24]$$

$$e_y = y_{COM} - y_{COR} \quad [25]$$

$$M_{torsion} = -F_h \cos(\theta)\, e_y + F_h \sin(\theta)\, e_x \quad [26]$$

Torsional shear is calculated as follows for an anchor located at (**x_i, y_i**). The anchor group polar moment of inertia (**J**) can be calculated using the equation provided in table 1.

$$V_{torsion,x} = \frac{M_{torsion}(y_i - y_{COR})}{J} \quad [27]$$

$$V_{torsion,y} = \frac{-M_{torsion}(x_i - x_{COR})}{J} \quad [28]$$

The resultant anchor shear demand is the sum of the terms above added together vectorially.

$$V_i = \sqrt{\left(V_{direct,x} + V_{torsion,x}\right)^2 + \left(V_{direct,y} + V_{torsion,y}\right)^2} \quad [29]$$

Python Package for Floor-Mounted Component Anchorage Calculations

The calculation procedures presented above were incorporated into a standalone python package called *ezanchor*. All results presented hereafter were obtained using ezanchor. The program is open-source and publicly available on GitHub: https://github.com/wcfrobert/ezanchor. Animated gifs are also available at the provided link, which some readers may be more elucidating than the static images presented in this paper.

The program can generate anchor demand envelope curves for floor-mounted components with arbitrary footprint and anchor layout. In the generated plots, the y-axis shows the anchor demands, whereas the x-axis shows equipment orientation ($\boldsymbol{\alpha}$) or applied force orientation ($\boldsymbol{\theta}$) depending on the methods used. For the rigid base method, the entire equipment geometry is rotated while the applied force is kept constant (in the +Y direction). For the elastic method, the applied force vector is rotated the equipment orientation is kept constant. One point of discrepancy in *ezanchor* is that $\boldsymbol{\theta}$ is measured counterclockwise from the +Y axis which is offset 90 degrees from the conventions presented in this paper (measured from +X axis). There is no rationale for this inconsistency. The convention presented in this paper is mathematically better and easier. Demand curve for the governing anchor is shown in dark blue. Demand curves for other anchors are shown in light blue. The envelope demand curve is shown in dark red. For clarity, the program also draws the component/anchor geometry on the lefthand side of the plot.

Critical Orientation: Tension Demand Using Rigid Base Method

Back to the original question: "what direction produces the most critical load effects?". The answer depends on whether the "load effect" under consideration is shear or tension. Furthermore, it depends on the method with which tension demand is calculated. Let's look at critical direction for tension demand first (as calculated using the rigid base method). As a starting point, consider this somewhat contrived example of a circular base component. Hypothetically speaking, all orientation would yield the same maximum tension. This is indeed what we observe in figure 4. As we increase the number of anchors along the perimeter to form a continuous ring, the envelope curve approaches a straight line.

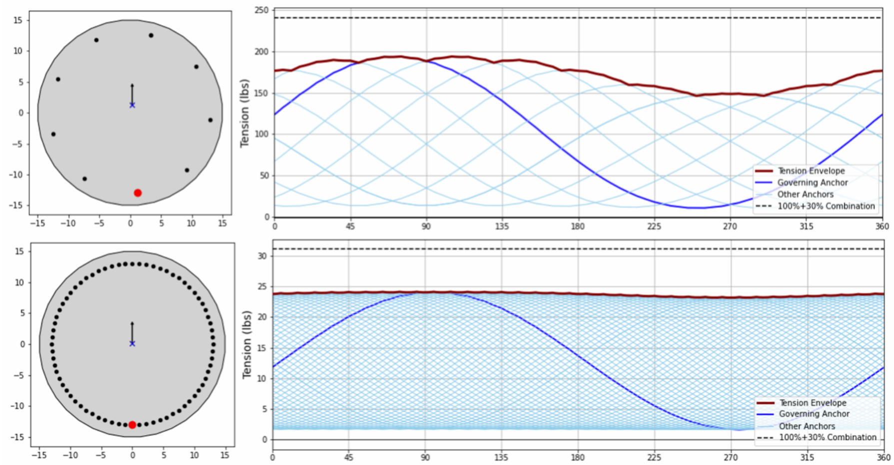

Figure 4: Anchor Tension Demand for Circular Base Component with Varying Number of Anchors

Notice how the individual anchor demand curves in figure 4 can still be represented by non-piecewise sinusoidal equations. As we morph the base from circular to square in figure 5, the demand curves that were previously differentiable get "squished" with clear cusps forming at the principal orientations (0, 90, 180, and 270 degrees). Demands at non-principal orientations form the "valleys" between the peaks.

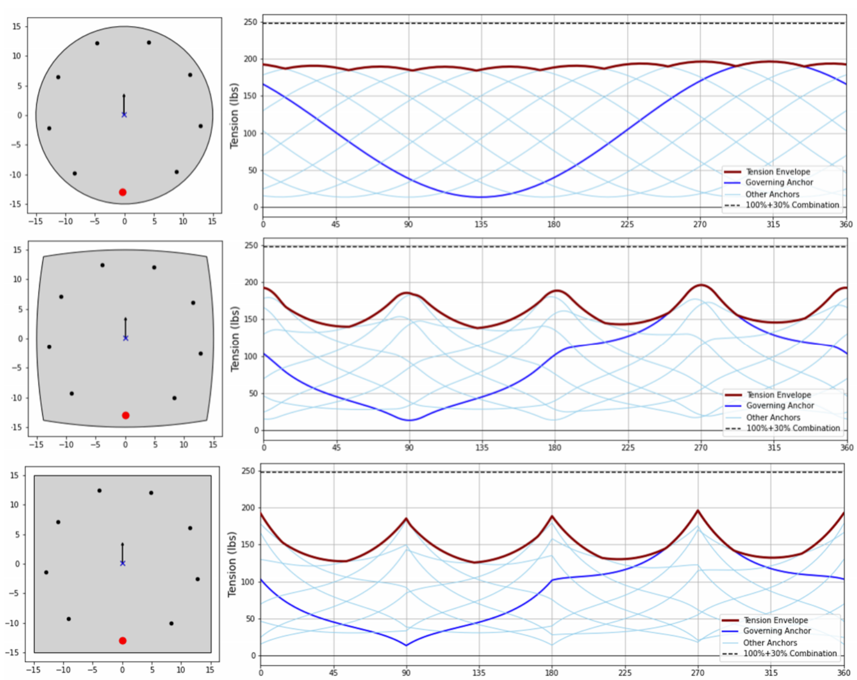

Figure 5: Anchor Tension Demand for Circular vs. Rectangular-Base Component

In figure 6, as we increase the aspect ratio for a rectangular-based component, overturning in the minor principal axis (short direction) clearly governed. Most notably, overturning in non-principal directions (i.e. along a diagonal) always yields lower tension demand. The *shape* of the envelope curve is always preserved for a given anchor/base geometry. Parameters like component weight (**W_p**) or level of seismicity (**S_{DS}**) do not affect the underlying pattern and only scales the y-axis.

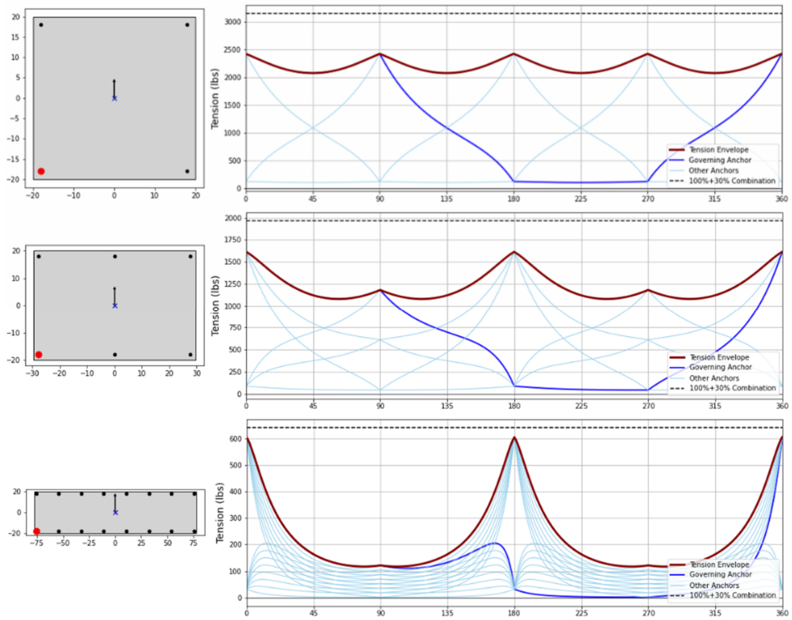

Figure 6: Anchor Tension Demand for Rectangular Component with Varying Aspect Ratio

Based on the studies above, we can conclude the following about the critical direction for anchor tension:

- For rectangular base components not on legs, restrained by a rectangular array of anchors, with tension demand calculated using the rigid base method, the critical load orientation is along one of the flat edges (i.e. principal directions). This archetype covers most practical cases. Furthermore, 100%-30% orthogonal combination is unnecessary in this context since we know the critical direction is either 0, 90, 180, or 270 degrees. It is often readily apparent which one is the worst, in which case analysis in one direction is enough.

For situations that do not fit the archetype described above, more deliberation about load direction is recommended. Two such cases are presented in figure 7, though this is by no means exhaustive. In the first example, the component base is L-shaped. In the second example, the component center of mass is extremely off-centered (perhaps even falling outside of the component footprint). Where this occurs, the restoring moment (**M_R**) per equation [2] is minimized or could even contribute to the net overturning moment.

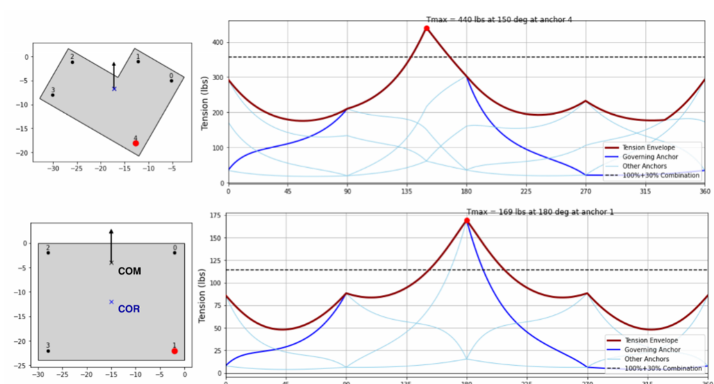

Figure 7: Examples Where Critical Orientation Requires More Deliberation

Critical Orientation: Tension Demand Using Elastic Method

As stated earlier, the elastic method is technically only applicable to floor-mounted components on legs. Given the difference in formulation compared to the rigid base method, the critical direction is also quite different. The inherent linearity of the elastic method means a closed-form solution is possible.

Start with equation [19] and write the anchor axial demand as a function of applied load orientation ($\boldsymbol{\theta}$). Distances between anchors and anchor group COR are calculated as $\boldsymbol{\bar{x}_i = x_i - x_{COR}}$ and $\boldsymbol{\bar{y}_i = y_i - y_{COR}}$

$$P_i = -\frac{F_v}{N_{anchors}} + \frac{(-F_h \sin(\theta)\, Z_{COM} + M_{weight,x}) \times \bar{y}_i}{I_x} + \frac{-(F_h \cos(\theta)\, Z_{COM} + M_{weight,y}) \times \bar{x}_i}{I_y} \qquad [30]$$

The critical point can be found by setting the derivative of equation [30] to zero.

$$\frac{dP_i}{d\theta} = 0 = \frac{-F_h Z_{COM} \bar{y}_i}{I_x}\cos(\theta) + \frac{F_h Z_{COM} \bar{x}_i}{I_y}\sin(\theta) \qquad [31]$$

$$\frac{\bar{x}_i}{I_y}\sin(\theta) = \frac{\bar{y}_i}{I_x}\cos(\theta) \qquad [32]$$

$$\theta = \operatorname{atan}\left(\frac{\bar{y}_i I_y}{\bar{x}_i I_x}\right) \qquad [33]$$

Therefore, in the context of the elastic method, the critical angle of load application ($\boldsymbol{\theta}$) can be calculated explicitly with a simple arctan function. However, notice that the arctan function is also dependent on the coordinates (**x_i, y_i**) of a selected anchor. In other words, in addition to finding the critical direction, the engineer also must select the critical *anchor*. Fortunately, this is easy for a symmetrical rectangular anchor group (which covers most practical cases) where the most critical anchors are at the corners.

The same three rectangular components from figure 6 are reproduced in figure 8 below with tension demands calculated using the elastic method. Unlike the rigid base method, the critical orientation is no longer at 0 or 90 degrees and is instead <u>always along a diagonal</u> and predicted exactly by equation [33].

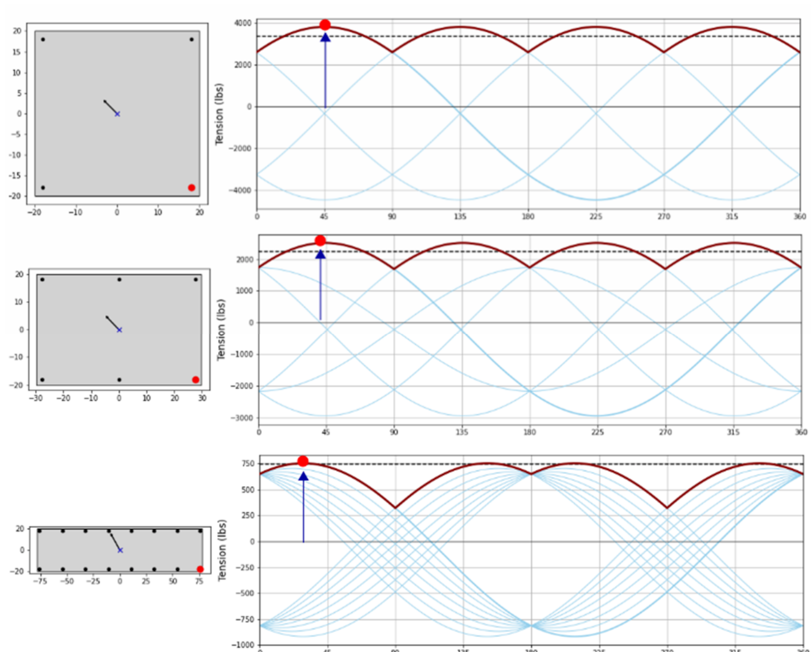

Figure 8: Anchor Tension Demand for Rectangular Component with Varying Aspect Ratio (Elastic Method)

Another notable observation is how the 100%-30% orthogonal combination rule proposed by the ASCE 7-22 – shown as dotted black lines in figure 8 – is unconservative for anchor groups with aspect ratio close to 1 (i.e. square). However, there is scarcely any reason to use 100%-30% combination for the elastic method when a better alternative in the form of equation [33] is available. We can reformulate equation [33] into the "$\boldsymbol{\beta_x}\% + \boldsymbol{\beta_y}\%$" combination rule, where $\boldsymbol{\beta_x}$ is equal to $100\cos(\theta)$, and $\boldsymbol{\beta_y}$ is equal to $100\sin(\theta)$.

Critical Orientation: Shear Demand

For cases where anchor group COR and the component COM are perfectly aligned, as is commonly assumed in industry practice, a critical orientation for shear does not exist because anchor shear demands are the same at all load orientations. This condition is shown in figure 9.

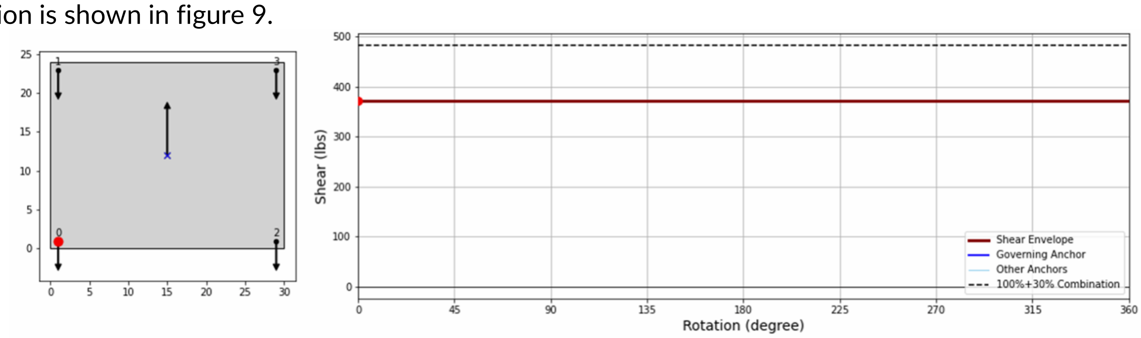

Figure 9: Anchor Shear Demand for Component Without In-Plane Eccentricities

Where COM and COR are misaligned, a critical direction does exist and can be calculated exactly by setting the derivative of equation [29] ($\boldsymbol{dV_i/d\theta}$) to zero. However, the resulting formula is lengthy and not very practical. Instead, it is much easier to implement equations [20] to [29] programmatically (i.e. in a spreadsheet) and compute results for all possible orientations. Alternatively, a reasonable but inexact approximation of the critical direction for shear is the direction that maximizes in-plane torsion ($\mathbf{M_{torsion}}$); which occurs when the applied force is perpendicular to the line connecting COR to COM as shown in figure 10.

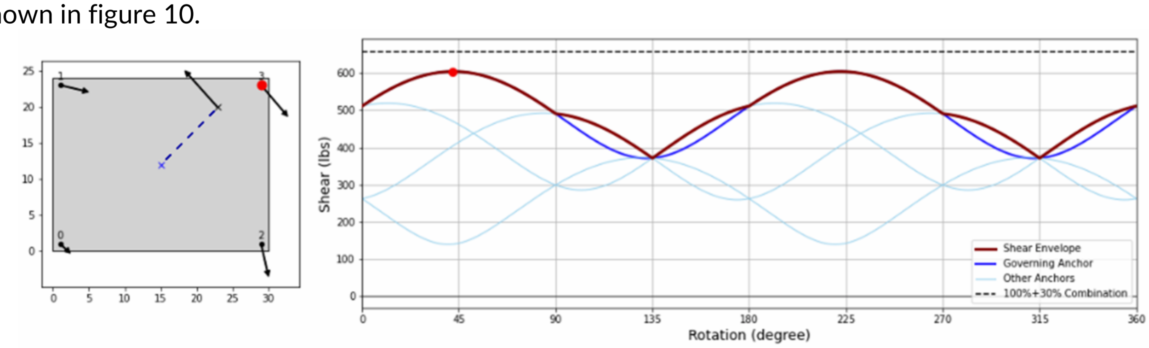

Figure 10: Anchor Shear Demand for Component with In-Plane Eccentricities

The existence of a separate critical direction for shear – often different from the critical direction for tension – adds additional layers of complexity. Which direction governs the final anchorage design? What if the governing tension and shear demand occurs at different anchors? And what if those anchors have different capacities? (Such as when one anchor is closer to the slab edge and thus has smaller shear breakout capacity, or when the shear force vector points towards or away from the edge). Should all anchors be evaluated? These are all sources of ambiguity that necessitates additional engineering judgement from practitioners. Every loading scenario produces a tension-shear demand pair that should be evaluated using anchor tension-shear interaction equations such as the one provided in section 17.8.3 of ACI 318-19. The simplest, and most conservative approach is to assume that the maximum tension/shear demand occurs simultaneously, in the same loading direction, to the same anchor with the lowest capacities. These layers of conservatism can then be trimmed away as needed.

Calculation Example

Using the methodologies outlined in this paper, determine the critical directions and the associated load effects for the floor-mounted component shown in figure 11.

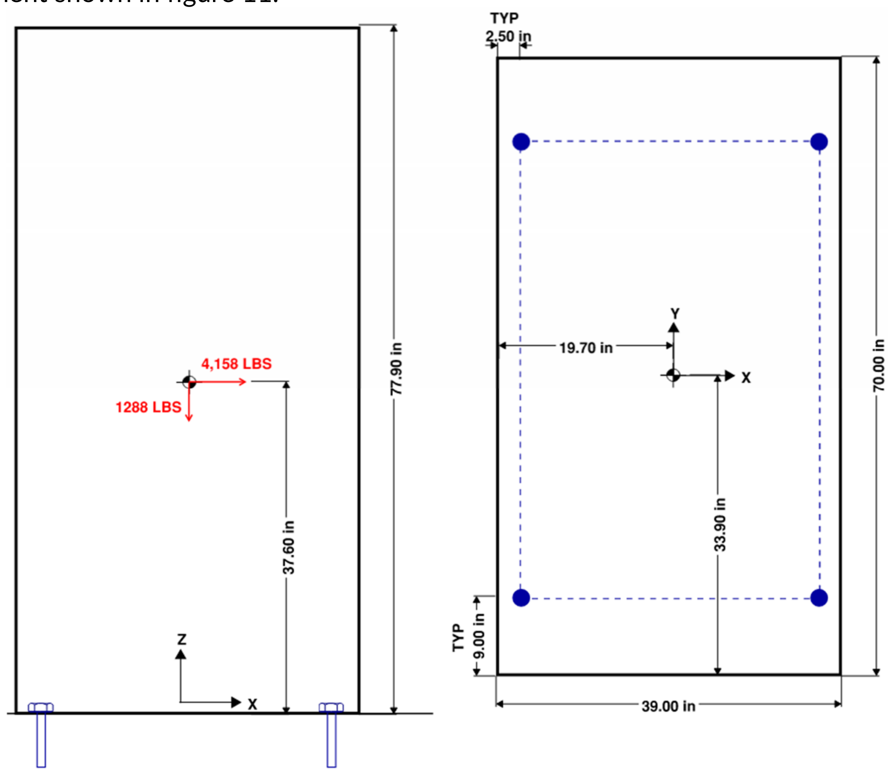

Figure 11: Floor-Mounted Component Geometry and Load Information

Critical Orientation for Tension Demand

Based on the results presented in this paper, we know that the critical load orientation is along one of the flat edges for a base-bearing component. In this case, overturning in the short direction obviously governs. **Therefore, the critical load direction for tension is at 0 degrees (i.e. F_h applied in the +X direction)**. Let's calculate the anchor tension demands using the rigid base method.

Net overturning moment can be determined using equation [3]:

$$M_{net} = M_{OT} - M_R = (4{,}158)(37.6) - (1{,}288)(19.3) = 131{,}483\ lbs$$

The maximum anchor tension demand can be calculated using equation [8]

$$T_N = \frac{M_{net} d_N}{\sum d_i^2} = \frac{(131{,}483)(36.5)}{2 \times 36.5^2 + 2 \times 2.5^2} = \mathbf{1{,}793\ lbs}$$

The other anchor tension demands can be calculated using equation [5]

$$T_1 = \frac{d_1}{d_N} \times T_N = \frac{2.5}{36.5} \times 1{,}793 = \ 123\ lbs$$

The resultant compression force (**C**) can be calculated using equation [9]. Anchor lever arm distances (**d_i**) may be reduced accordingly if the concrete is not able to provide enough bearing area. We will assume the floor is able to support this force and not iterative further.

$$C = F_v + \sum T_i = 1{,}288 + (2)(1{,}793) + (2)(123) = 5{,}120\ lbs$$

For comparison purposes, let's calculate the anchor demands using the elastic method. Note the elastic method is technically only applicable to components on legs (isolators, snubbers, columns, etc.) which is not the case here. However, we will go through the calculation procedure for illustrative purposes. First calculate the geometric properties of the anchor group using equations provided in table 1. Let the bottom left corner be the origin of the global coordinate system.

$$x_{COR} = \frac{\sum x_i}{N_{anchors}} = \frac{(2.5 \times 2) + (36.5 \times 2)}{4} = 19.5\ in$$

$$y_{COR} = \frac{\sum y_i}{N_{anchors}} = \frac{(9 \times 2) + (61 \times 2)}{4} = 35\ in$$

$$I_x = \sum (y_i - y_c)^2 = 2(9-35)^2 + 2(61-35)^2 = 2{,}704\ in^2$$

$$I_y = \sum (x_i - x_c)^2 = 2(2.5-19.5)^2 + 2(36.5-19.5)^2 = 1{,}156\ in^2$$

$$J = I_x + I_y = 2{,}704 + 1{,}156 = 3{,}860\ in^2$$

$$\begin{aligned} I_{xy} &= \sum (x_i - x_c)(y_i - y_c) \\ &= (2.5-19.5)(9-35) + (2.5-19.5)(61-35) \\ &\qquad + (36.5-19.5)(9-35) + (36.5-19.5)(61-35) \\ &= 442 - 442 - 442 + 442 \\ &= 0 \end{aligned}$$

$$\theta_p = 0.5 \times \operatorname{atan}\left(I_{xy}/(I_x - I_y)/2\right) = 0.5 \times \operatorname{atan}(0/(2{,}704 - 1{,}156)/2) = 0$$

Because the product moment of inertia (**I_{xy}**) is equal to zero, the anchor group is already at its principal orientations and no rotation is necessary. This is true for all symmetrical rectangular anchor groups. Using equation [33], the critical orientation for tension can be calculated. The upper left corner anchor was selected as the most critical because it maximizes tension demand from self-weight-induced overturning.

$$\theta = \operatorname{atan}\left(\frac{\bar{y}_i I_y}{\bar{x}_i I_x}\right) = \operatorname{atan}\frac{(61-35)(1{,}156)}{(2.5-19.5)(2{,}704)} = -33.2\ deg$$

Therefore, if the component was supported on legs (snubbers, isolators, columns, etc.), the critical load direction for tension is 33.2 degrees clockwise from the x-axis. Now let's calculate anchor tension demand using the elastic method.

Anchor axial demand due to self-weight per equation [10]:

$$P_{weight} = -\frac{F_v}{N_{anchors}} = -\frac{1{,}288}{4} = -322\ lbs$$

Seismic force induced overturning moment per equation [11] and [12]:

$$M_{OT,x} = -F_h \sin(\theta) \times Z_{COM} = -4{,}158 \sin(-33.2) \times 37.6 = 85{,}606\ lbs.in$$

$$M_{OT,y} = F_h \cos(\theta) \times Z_{COM} = 4{,}158 \cos(-33.2) \times 37.6 = 130{,}820\ lbs.in$$

Self-weight induced overturning moment per equation [13] and [14]:

$$M_{weight,x} = -F_v(y_{COM} - y_{COR}) = -1{,}288(33.9 - 35) = 1{,}417\ lbs.in$$

$$M_{weight,y} = F_v(x_{COM} - x_{COR}) = 1{,}288(19.7 - 19.5) = 258\ lbs.in$$

The total overturning moment about X and Y axes per equation [15] and [16]:

$$M_{total,x} = M_{OT,x} + M_{weight,x} = 1{,}417 + 85{,}606 = 87{,}023\ lbs.in$$

$$M_{total,y} = M_{OT,y} + M_{weight,y} = 130{,}820 + 258 = 131{,}078\ lbs.in$$

Anchor axial demand due to overturning for the top left corner anchor per equation [17] and [18]:

$$P_{Mx} = \frac{M_{total,x}(y_i - y_{COR})}{I_x} = \frac{(87{,}023)(61 - 35)}{(2{,}704)} = 837\ lbs$$

$$P_{My} = \frac{-M_{total,y}(x_i - x_{COR})}{I_y} = \frac{(131{,}078)(2.5 - 19.5)}{(1{,}156)} = 1928\ lbs$$

Finally, the total tension demand in the upper corner anchor is calculated per equation [19]:

$$P_i = -322 + 837 + 1{,}928 = \mathbf{2{,}443}\ \boldsymbol{lbs}$$

Calculations for the other anchors are left as exercise for the readers. A result summary is provided in figure 12.

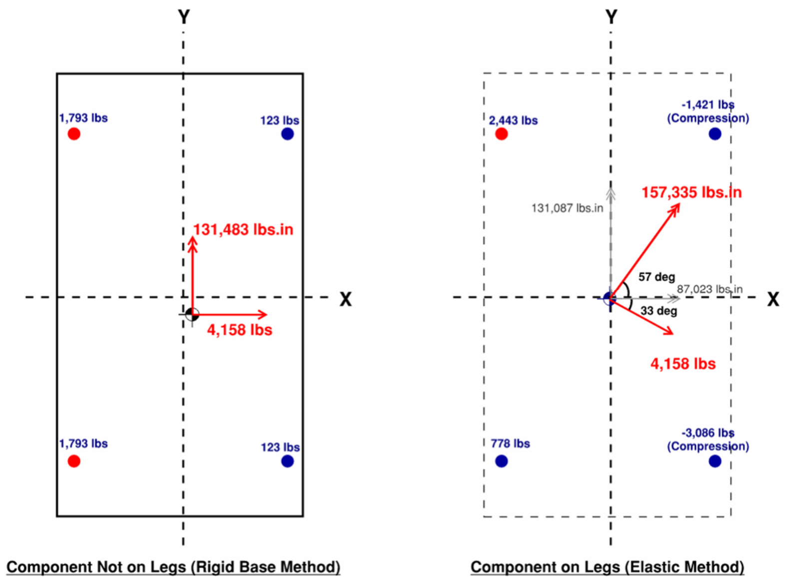

Figure 12: Summary of Critical Load Directions and the Associated Tension Demand

Figure 13 shows the tension demand curves for both methods. Note angles in *ezanchor* are measured counterclockwise from the +Y axis which is offset 90 degrees from the formulations presented in this paper.

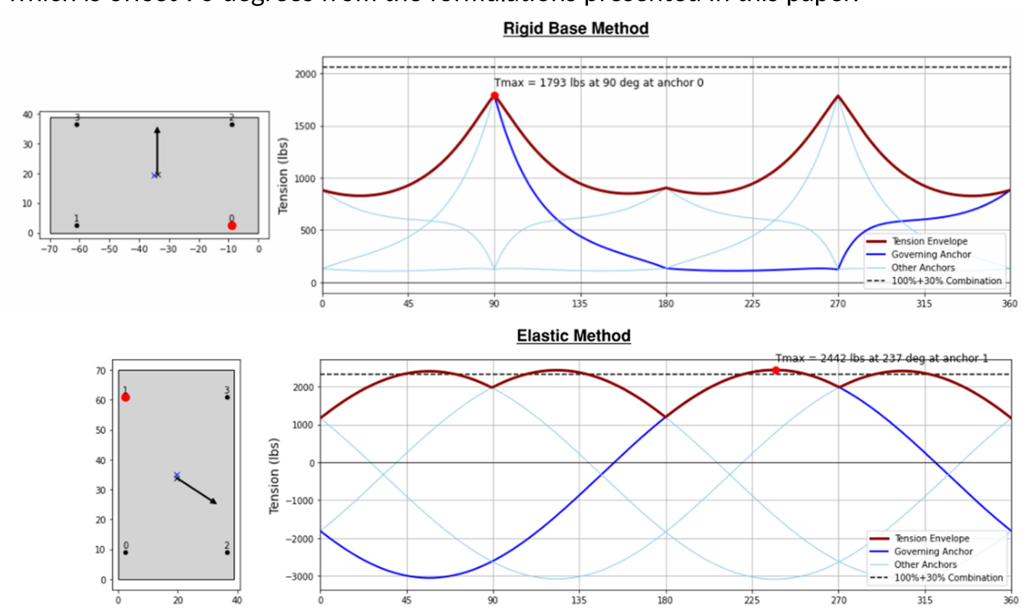

Figure 13: Anchor Tension Demand Curves for All Load Directions

Critical Orientation for Shear Demand

Next, let's look at the critical direction for anchor shear demand. Given the small COM/COR offset, torsional shear demands are likely minimal.

$$e_x = x_{COM} - x_{COR} = 19.7 - 19.5 = 0.2\ in$$

$$e_y = y_{COM} - y_{COR} = 33.9 - 35 = -1.1\ in$$

The exact critical direction for shear can be calculated by setting the derivative of equation [29] ($\boldsymbol{dV_i/d\theta}$) to zero. However, the resulting formula is lengthy and not very practical. Let's instead try to approximate it as the direction perpendicular to the line connecting COR/COM.

$$\theta = \tan^{-1}\left(\frac{0.2}{1.1}\right) = \mathbf{10.3\ deg}$$

Direct shear per equation [22] and [23]:

$$V_{direct,x} = -\frac{F_h \cos(\theta)}{N_{anchors}} = -\frac{4{,}158 \cos 10.3}{4} = -1023\ lbs$$

$$V_{direct,y} = -\frac{F_h \sin(\theta)}{N_{anchors}} = -\frac{4{,}158 \sin 10.3}{4} = -186\ lbs$$

Total in-plane torsion per equation [26]:

$$\begin{aligned} M_{torsion} &= -F_h \cos(\theta)\, e_y + F_h \sin(\theta)\, e_x \\ &= -4{,}158 \cos(10.3)\,(-1.1) + 4{,}158 \sin(10.3)\,(0.2) \\ &= 4{,}649\ lbs.in \end{aligned}$$

Torsional shear is calculated for the bottom right anchor, per equation [27] and [28]:

$$V_{torsion,x} = \frac{M_{torsion}(y_i - y_{COR})}{J} = \frac{4{,}649(9 - 35)}{3{,}860} = -31\ lbs$$

$$V_{torsion,y} = \frac{M_{torsion}(x_i - x_{COR})}{J} = \frac{-4{,}649(36.5 - 19.5)}{3860} = -20\ lbs$$

The total shear demand for the bottom right anchor is calculated per equation [29]:

$$\begin{aligned} V_i &= \sqrt{\left(V_{direct,x} + V_{torsion,x}\right)^2 + \left(V_{direct,y} + V_{torsion,y}\right)^2} \\ &= \sqrt{(-1023 - 31)^2 + (-186 - 20)^2} \\ &= \mathbf{1074.1\ lbs} \end{aligned}$$

The shear demand curve for all possible orientations is plotted in figure 14 below. Note angles in *ezanchor* are measured counterclockwise from the +Y axis which is offset 90 degrees from the formulations presented in this paper.

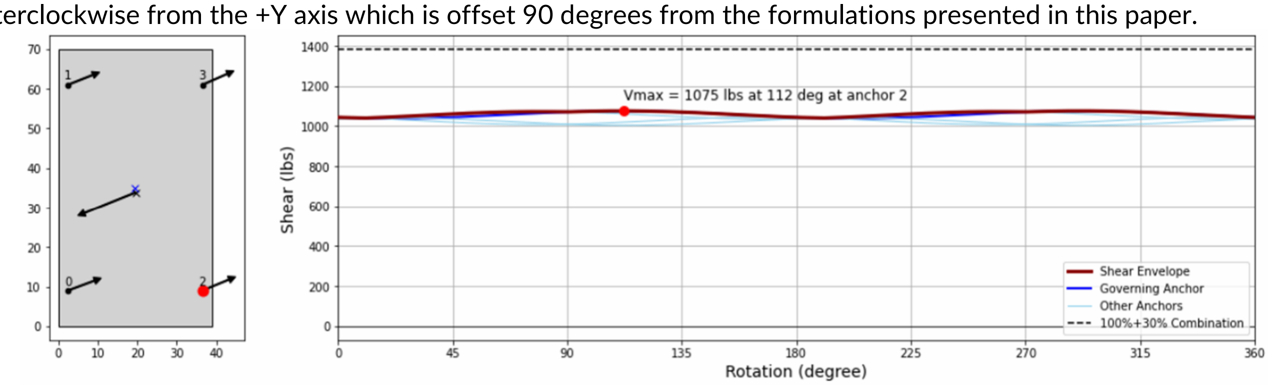

Figure 14: Anchor Shear Demand Curves for All Load Directions

Figure 14 suggests the critical angle for shear is actually 22 degrees measured counterclockwise from the +x axis. Let's check shear demand at this orientation.

Direct shear per equation [22] and [23]:

$$V_{direct,x} = -\frac{F_h \cos(\theta)}{N_{anchors}} = -\frac{4{,}158 \cos 22}{4} = -964\ lbs$$

$$V_{direct,y} = -\frac{F_h \sin(\theta)}{N_{anchors}} = -\frac{4{,}158 \sin 22}{4} = -389\ lbs$$

Total in-plane torsion per equation [26]:

$$\begin{aligned} M_{torsion} &= -F_h \cos(\theta)\, e_y + F_h \sin(\theta)\, e_x \\ &= -4{,}158 \cos(22)\,(-1.1) + 4{,}158 \sin(22)\,(0.2) \\ &= 4{,}552\ lbs.in \end{aligned}$$

Torsional shear is calculated for the bottom right anchor, per equation [27] and [28]:

$$V_{torsion,x} = \frac{M_{torsion}(y_i - y_{COR})}{J} = \frac{4{,}552(9 - 35)}{3{,}860} = -31\ lbs$$

$$V_{torsion,y} = \frac{M_{torsion}(x_i - x_{COR})}{J} = \frac{-4{,}552(36.5 - 19.5)}{3860} = -20\ lbs$$

The total shear demand for the bottom right anchor is calculated per equation [29]:

$$\begin{aligned} V_i &= \sqrt{\left(V_{direct,x} + V_{torsion,x}\right)^2 + \left(V_{direct,y} + V_{torsion,y}\right)^2} \\ &= \sqrt{(-964 - 31)^2 + (-389 - 20)^2} \\ &= \mathbf{1075.5\ lbs} \end{aligned}$$

Indeed, despite the smaller in-plane torsion (4,552 lbs.in vs. 4,649 lbs.in), the total anchor demand is 1.4 lbs higher when the load is applied at 22 degrees.

In summary, the critical direction for shear was estimated to be 10.3 degrees with peak shear demand of 1,074 lbs. The actual critical direction – determined by comprehensively checking all possible force orientation – is 22 degrees with a peak shear demand of 1,076 lbs. The direction that maximizes in-plane torsion appears to be reasonable estimate of maximum anchor shear demands, but not critical direction in this example.

Conclusion and Recommendations

In this paper, we examined the topic of critical force orientation for floor-mounted component anchorage through a series of parametric studies. We began by rigorously defining the formulations for calculating anchorage demands, unifying and addressing some variabilities in industry practice. The calculation procedures were then incorporated into a standalone python package. Using this program, we conducted various parametric studies to address the key question: "which direction produces the most critical load effects?". Based on the results of the studies, we provided guidance for how to determine critical direction for three specific scenarios:

1. Critical direction for anchor tension for components bearing on floor
2. Critical direction for anchor tension for components supported on legs (i.e. isolators, snubbers, columns, etc.)
3. Critical direction for anchor shear

Here are the main takeaways of this paper:

- There are two common methods for calculating anchor tension.
    - The rigid base method is applicable to base-bearing components and resembles methods used in the design of base plates and concrete sections. A neutral axis is placed near the edges of the component with a compression resultant contributing to the overall equilibrium of the overturning component.
    - The elastic method is applicable to components on legs and uses familiar concepts in theory of elasticity. Bearing surfaces are effectively ignored. Anchors are assumed to take compression and must do so to maintain equilibrium.
    - The two methods result in completely different critical load orientations and anchor force distribution. Therefore, while both methods satisfy equilibrium, they should not be used interchangeably.
- For calculating anchor shear, it is common to assume perfect alignment between component center of mass (COM) and anchor center of rigidity (COR), in which case anchor shear is simply the applied seismic force divided by the number of anchors. More often, COR and COM are misaligned which produces additional shear demand due to in-plane torsion.
- The critical force direction for anchorage design depends on if the load effect in question is shear or tension. Furthermore, it depends on the method with which tension demand is calculated. Adding to the complexity is the fact that the critical direction for shear and tension are often different, and the peak shear/tension demand may not occur at the same anchors. These are all sources of ambiguity that necessitates additional engineering judgement from practitioners. For a specific archetype of floor-mounted component, namely those with rectangular base restrained by a rectangular array of anchors, the following conclusions were made:
    - If the component is directly bearing on the floor, the critical direction that maximizes anchor tension is always along one of the flat edges. The 100%-30% combination rule is always conservative in this context since the critical direction is either 0, 90, 180, or 270 degrees. It is often readily apparent which one is the worst.
    - If the component is supported on legs (i.e. isolators, snubbers, columns, etc.), the critical direction that maximizes anchor tension is always along a diagonal and can be explicitly calculated using equation [33]. There is very little reason to use the 100%-30% combination rule in this context since the simple closed-form solution serves as a better alternative.
    - The critical direction that maximizes anchor shear does not exist if shear demand due to in-plane torsion is ignored. Otherwise, it can be determined using the procedures outlined in this paper.
- For situations that do not fit the archetype described above, more deliberation about load direction is recommended. Two such examples are presented in figure 7. The first example is a component with L-shaped base, and the second example is a component with extreme COM/COR offset.

The results presented in this paper are relevant to floor-mounted nonstructural components anchorage only. They should not be applied to ceiling-mounted or wall-mounted components. Furthermore, the analyses relied on several assumptions, and the conclusions drawn are contingent on the accuracy of those assumptions. Additional design consideration such as prying effect, fixity of bracket angles attachment, structural integrity of the component's interior, uneven anchor stiffnesses, presence of shear-only brackets or tension-only straps, and flexibility of equipment base were not within the scope of this paper.

References

ACI, 2019. *Building Code Requirements for Structural Concrete.* Farmington Hills, Michigan.

ASCE/SEI, 2022. *Minimum Design Loads and Associated Criteria for Buildings and Other Structures (ASCE/SEI 7-16).* Reston Virginia.

ATC, 2017. *NIST GCR 17-917-44: Seismic Analysis, Design, and Installation of Nonstructural Components and Systems – Background and Recommendations for Future Work*. Applied Technology Council, Redwood City, California.

Lobo, R., Tokas, C. 2023. *Seismic Anchorage and Bracing of Nonstructural Components – Common Calculation Flaws in Current Practice*. 2023 SEAOC Convention Paper. Structural Engineer s Association of California. Sacramento, California.

Taghavi, S., & Miranda, E. 2003. *Response Assessment of nonstructural building elements.* Pacific Earthquake Engineering Research Center. University of California Berkeley. Berkeley, California.