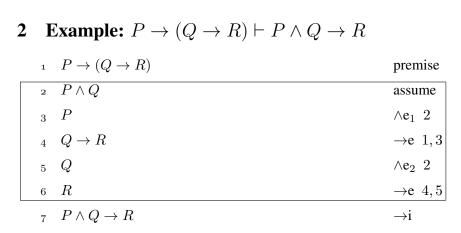
Natural-Deduction Examples in Propositional Logic

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$\textbf{1} \quad \textbf{Example:} \ P \vdash Q \rightarrow (P \land Q)$

1	P	premise	e
2	Q	assume	;
3	$P \wedge Q$	\wedge i 1, 2	
4	$Q \to (P \land Q)$	$ ightarrow \mathrm{i}$	

2 Example: $P \rightarrow (Q \rightarrow R) \vdash P \land Q \rightarrow R$



3 Example: $P \wedge Q \rightarrow R \vdash P \rightarrow (Q \rightarrow R)$

 $\begin{array}{c|cccc} & P & & \text{premise} \\ \hline 2 & P & & \text{assume} \\ \hline 3 & Q & & \text{assume} \\ 4 & P \wedge Q & & \wedge \text{i } 2, 3 \\ 5 & R & & \rightarrow \text{e } 1, 4 \\ \hline 6 & Q \rightarrow R & & \rightarrow \text{i} \\ \hline 7 & P \rightarrow (Q \rightarrow R) & & \rightarrow \text{i} \\ \hline \end{array}$

4 Example: $P \rightarrow (Q \rightarrow R) \vdash (P \rightarrow Q) \rightarrow (P \rightarrow R)$

 $_{\scriptscriptstyle 1}$ $P \to (Q \to R)$ premise

$_{2}$ $P \rightarrow Q$	assume
3 P	assume
$_4$ Q	ightarrowe 2, 3
$_{5}$ $Q \rightarrow R$	\rightarrow e1, 3
6 R	\rightarrow e 5, 4
$_{7}$ $P \rightarrow R$	\rightarrow i

$$8 \quad (P \to Q) \to (P \to R) \qquad \qquad \to i$$

5 Example: $P \wedge \neg Q \rightarrow R, \ \neg R, \ P \vdash Q$

$_{1}$ $P \land \neg Q \rightarrow R$	premise
$_{2}$ $\neg R$	premise
3 P	premise
$_4$ $\neg Q$	assume
$_{5}$ $P \wedge \neg Q$	$\wedge i \ 3,4$
6 R	\rightarrow e 1,5
₇	¬e 6, 2
8 770	⊐i

 $\neg \neg e \ 8$

 $_{9}$ Q