Natural-Deduction Examples
in Propositional Logic

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1 Example: $P \vdash Q \rightarrow (P \wedge Q)$

1	P	premise
2	Q	assume
3	$P \wedge Q$	$\wedge$i $1, 2$
4	$Q \rightarrow (P \wedge Q)$	$\rightarrow$i

2 Example: $P \rightarrow (Q \rightarrow R) \vdash P \wedge Q \rightarrow R$

1	$P \rightarrow (Q \rightarrow R)$	premise
2	$P \wedge Q$	assume
3	P	$\wedge$e$_1$ 2
4	$Q \rightarrow R$	$\rightarrow$e $1, 3$
5	Q	$\wedge$e$_2$ 2
6	R	$\rightarrow$e $4, 5$
7	$P \wedge Q \rightarrow R$	$\rightarrow$i

3 Example: $P \wedge Q \to R \vdash P \to (Q \to R)$

1	$P \wedge Q \to R$	premise
2	P	assume
3	Q	assume
4	$P \wedge Q$	$\wedge$i 2, 3
5	R	$\to$e 1, 4
6	$Q \to R$	$\to$i
7	$P \to (Q \to R)$	$\to$i

4 Example: $P \to (Q \to R) \vdash (P \to Q) \to (P \to R)$

1	$P \to (Q \to R)$	premise
2	$P \to Q$	assume
3	P	assume
4	Q	$\to$e 2, 3
5	$Q \to R$	$\to$e1, 3
6	R	$\to$e 5, 4
7	$P \to R$	$\to$i
8	$(P \to Q) \to (P \to R)$	$\to$i

5   Example: $P \wedge \neg Q \rightarrow R, \ \neg R, \ P \ \vdash \ Q$

1	$P \wedge \neg Q \rightarrow R$	premise
2	$\neg R$	premise
3	P	premise
4	$\neg Q$	assume
5	$P \wedge \neg Q$	$\wedge$i $3, 4$
6	R	$\rightarrow$e $1, 5$
7	$\bot$	$\neg$e $6, 2$
8	$\neg\neg Q$	$\neg$i
9	Q	$\neg\neg$e 8