Homework 3

(Due on: Mon, April 30, 8:00PM, by e-mail)

Problem 1. A dynamical system is governed by the stochastic difference equation

$$\underline{x}(k+1) = \begin{bmatrix} 1.5 & 1 \\ -0.7 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1.0 \\ 0.5 \end{bmatrix} w(k) \qquad (1)$$

where $w(k)$ is a sequence of independent Normal (mean=0,std=1) (Gaussian) random variables.

a) Derive the expression for the state covariance and determine the state covariance in the steady state P_∞.

b) Write a MATLAB code that generates $\underline{x}(k)$ for $k = 1..500$ and $\underline{x}(0) = [0\ 0]^T$. Generate five sequences of $\underline{x}(k)$ and plot them on two diagrams, one for $x_1(k)$ and one for $x_2(k)$.

c) Generate 50 trajectories for $\underline{x}(k)$ and compute from them the standard deviation for $x_1(k)$ and $x_2(k)$.

d) Use the analytical results to compute the state $\underline{x}(k)$ covariance matrix $P(k)$ and corresponding standard deviations for $x_1(k)$ and $x_2(k)$. Compare them with the computed values in point c.

Problem 2. A 1D stochastic process $x(t)$ (t is the time) is described by the following stochastic differential equation

$$dx = c \cdot dw \qquad (2)$$

where dw is a Wiener process increment, c is a scaling factor and the variable t in $x(t)$ denotes the time.

(a) If we know that $x(1) = 1$, $x(2) = 2$ and $x(t) = -1$, generate values of that process between $t = 1$ and $t = 3$ using the sample time of $\Delta t = 0.05$ and $c = 1$.

(b) Repeat the process 30 times from (a) for $c = 1$ and plot all trajectories on top of each other on the same diagram. Then plot another 30 trajectories, but using $c = 2$.