A Review of EEG Signal Simulation Methods

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Abstract. This paper describes EEG signal simulation methods. Three main methods have been included in this study: Markov Process Amplitude (MPA), Artificial Neural Network (ANN), and Autoregressive (AR) models. Each method is described procedurally, along with mathematical expressions. By the end of the description of each method, the limitations and benefits are described in comparison with other methods. MPA comprises of three variations; first-order MPA, nonlinear MPA, and adaptive MPA. ANN consists of two variations; feed forward back-propagation NN and multilayer feed forward with error back-propagation NN with embedded driving signal. AR model based filtering has been considered with its variation, genetic algorithm based on autoregressive moving average (ARMA) filtering.

Keywords: ANN · AR · ARMA · EEG simulation · MPA

1 Introduction

The behavior of neurophysical activities could be viewed through a comprehensive signal simulation. Usually, it is represented as a function of time, space, or any other independent variables mathematically [1]. The simulation of electrical activity on the scalp [2] or electroencephalography (EEG) has been proved by the researchers [3–5] in getting the broad and comprehensive sense of the nature of the cerebral activities in term of signal before applying analysis in particular signal processing for future research development. It can also be used for forecasting the future neurological outcome and for data compression [6].

EEG consists of nonstationary [4,7], nonlinearity [8] and stochastic properties. Early researchers conceptualize EEG signal and some of them exhibit simple EEG signal [4]. As time progressed, the computational power become advance and thus, realistically achieving some complex problem which was not technologically possible in the past. It also creates new field of knowledge i.e. parallel computing and high performance computing.

Studies has shown that EEG has its own rhythms classified by certain activities which could be simulated with certain technique. To generate a signal according to certain EEG rhythm, that will resemble the real, measured, and recorded

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signal in real time is very challenging. Nevertheless, there exists method to simulate EEG signals with the mentioned constraints.

The first method is Markov Process Amplitude (MPA). Markov process is a type of stochastic process whereby the conditional probability distribution of future states $x(n+1)$ of a particular system is only depend on the present state $x(n)$, without considering the past state of the system [9,10]. Any past history is omitted in the computation. In particular, the significant of EEG simulation using Markov process could be viewed as interpreting the serial dependencies of the EEG signal between the adjacent periods. By analysing the probability of interdependence between the adjacent periods, the next state could thus be predicted (future state) while exploiting the event that precedes it.

The adaptation of human neurological system in the field of artificial intelligence has brought neural network methods. The nonlinearity, complexity, and parallel computing capabilities of brain inspires scientists and researchers forming a groundwork for Artificial Neural Network based approach in Artificial Intelligence. According to Haykin [11], neural network has the capability to manifest the actual brain learning activity, nonlinearity, input-output mapping, adaptivity, and contextual information [11]. The modeling of EEG signal based on ANN has been extensively researched by Adeli et al. [12,13].

The last method considered in this work is based on Autoregressive (AR) model. This model is one where the current value of a variable depends only upon the values that the variable took in previous periods plus an error term [14]). AR model is a part of linear prediction models with the main objective of finding a set of model parameters that best describe the signal generation system [4]. The signal in AR modelling is described to be linearly related with respect to a number of its previous samples.

The structure of this paper is started with the introduction, followed by each methods with its variations. And lastly, the conclusion.

2 Markov Process Amplitude

In this section, we will describe MPA with its variations; First-Order MPA [7], followed by Nonlinear MPA model 7, and lastly the Adaptive MPA [6].

2.1 First-Order MPA

Nishida et al. pioneered this method in 1986 by exploiting the sinusoidal waves with the Markov process amplitude is utilized to simulate the EEG signal [7]. The estimated MPA EEG output $x(n\Delta t)$ is composed by K different oscillations $(k = 1, 2, \ldots, K)$ as

$$x(n\Delta t) = \sum_{k=1}^{K} a_k(n\Delta t) sin(2\pi m_k n\Delta t + \phi_k) \tag{1}$$

where $a_k(n\Delta t)$ is the model amplitude of the first-order Markov process is, m_k is the kth average frequency, ϕ_k is the initial phase which assumed to be zero. The definition of the following model amplitude estimation

$$\begin{gathered} a_k((n+1)\Delta t) = \gamma_k a_k(n\Delta t) + \xi_k(n\Delta t) \\ 0 < \gamma_k < 1, k = 1, 2, \ldots, K \end{gathered} \tag{2}$$

where $\xi_k(n\Delta t)$ is the independent increments of Gaussian distribution with zero mean and unity variance. γ_k is the coefficient of the first order Markov process. For stability, it is proved that γ_k satisfies the condition $0 < \gamma_k < 1$.

The electrical amplitude of $(n+1)\Delta t$ this model depends solely on the previous value $n\Delta t$ with the coefficient γ_k and independent for any time before $n\Delta t$. Thus, first-order Markov process is the process with the independent increments. This method is best in simulating stationary EEG signals since the first-order exhibit the linear properties of the signal, and the parameters are determined based on the power spectrum for both spontaneous and mutually coupled components of the EEG signals [7].

2.2 Nonlinear MPA

The existing MPA is further improvised to overcome the limitation of distinguishing within the same frequency band of nonlinearly coupled frequencies from spontaneously excited signals by utilizing the delta and alpha rhythm of recorded EEG signals to determine nonlinear coupling phenomena [7]. The linear spontaneous oscillations of linear MPA EEG model is viewed in [7] while according to Wiener [15] the most general nonlinearity apart from many other complicated nonlinear features; is inspired by quadratic coupling features [16] of the spontaneously activated oscillations in EEG [7]. The nonlinear coupling is composed of two oscillatory waves that pass through a nonlinear square system and thus generates self-coupling frequencies and cross-coupling frequencies [7]

$$\begin{aligned} x(n\Delta t) = & \sum_{k=1}^{K} a_k(n\Delta t) sin(2\pi m_k n\Delta t + \phi_k) \\ & + \sum_{k=1}^{K} \epsilon_k^s a_k(n\Delta t) sin(2\pi 2 m_k n\Delta t + 2\phi_k') \\ & + \sum_{i,j\in K, i\neq j}^{K} \{\epsilon_{ij}^{c1} a_i(n\Delta t) a_j(n\Delta t) cos[2\pi(m_i - m_j)n\Delta t + (\phi_i - \phi_j)] \\ & \epsilon_{ij}^{c2} a_i(n\Delta t) a_j(n\Delta t) cos[2\pi(m_i + m_j)n\Delta t + (\phi_i + \phi_j)]\} \end{aligned} \tag{3}$$

The power spectrum is obtained by transforming the digital EEG into Fourier components with Kaiser-Bessel [17,18] by Fast Fourier Transformation with the same length of the window and the FFT as the identically divided EEG data. The Welch method with Kaiser-Bessel window is used to obtain low variance of the EEG power spectrum. Thus, by knowing frequency resolution of the power spectrum, it is then used to determine the parameters and coefficients of the model. Next, the minimization of the square sum of the difference between the nonlinear EEG model and continuous EEG, and power spectrum is done by algorithm in [19], which exhibit rapid convergence for minimization property. The model is evaluated by a criterion of percentage error [7].

This method is used under the consideration of the nonlinearly coupled frequency components of EEG [7]. It is proved that for self-coupling part, this experiment exhibit the same properties of quadratic nonlinear as mentioned by Nunez [20] interactions of delta and alpha rhythm of EEG signal; as the delta and alpha oscillations are separated by the nonlinear MPA EEG model. The regarded nonlinear cross-coupling components could be view as a good match to the power spectrum of continuous EEG signal [7].

2.3 Adaptive MPA

The adaptive MPA model utilizes the least mean square algorithm to determine the parameters adaptively dated in 2004. This model should free from the stationary limitations and the need to repeatedly and manually compute the MPA model parameters. In this method, some parameter in the equation of first-order MPA is simplified [7]. if $s(n)$ is the EEG signal to be modeled, then the instantaneous error of the adaptive system is viewed as

$$e(n) = s(n) - y(n) \tag{4}$$

The core of this method, least-mean-square algorithm uses the mean square error (MSE). It could be viewed as follow

$$\begin{aligned} J = & \frac{1}{2}E(e(n)^2) = \frac{1}{2}E((s(n) - y(n))^2) \\ = & \frac{1}{2}Rs - \sum_{j=1}^{K} a_j(n)Rsx_j + \frac{1}{2}\sum_{i=1}^{K}\sum_{j=1}^{K} a_i(n)a_j(n)Rxx_{i,j} \\ = & \frac{1}{2} - \sum_{j}^{K} \big(\gamma_j(n-1)a_j(n-1) + \mu_j(n-1)\xi_j(n-1)\big)Rsx_j \\ & + \frac{1}{2}\sum_{i=1}^{K}\sum_{j=1}^{K} \big(\gamma_i(n-1)a_j(n-1) + \mu_i(n-1)\xi_i(n-1)\big) \\ & \times \big(\gamma_j(n-1)a_j(n-1) + \mu_j(n-1)\xi_j(n-1)\big)Rxx_{i,j} \end{aligned} \tag{5}$$

LMS [21] is used to adjust γ and μ adaptively. Due to high degree of nonstationarity in each EEG segment, this model could not track the distinction of the domain. However, it could track transient EEG activities. Also, the calculated NMSE for AMPA is proved to be better than MPA model. Another variation of MPA method is proposed in [22] synergizing the neural network knowledge to determine nonlinearities of the EEG signal variations.

3 Artificial Neural Network

This method comprises of two variations started with multi-layered back-propagation Neural Network [22], and followed by Neural Network based approach with embedded Driving Signal (DS) concept [23] in the existing time-delayed neural network estimation model [24].

3.1 Multi-layered Back Propagation Neural Network

In this method, the same procedure for recording EEG signals as in [6]. It described the approximated EEG output $y(n)$ is a function of K different oscillations. NN architecture is designed with the standard back-propagation learning algorithm with a hidden layer of 80 nodes. The result is analyzed by comparing the EEG segments' power spectral density (PSD) and normalized mean squared error (NMSE) value. NMSE is used to determine the deviation between the predicted values and the actual values [25]. It is reported that this method could liberate the generated signal from stationary constraints and to determine the model nonlinearity parameter [22].

3.2 Multi-layered Feedforward with Back Propagation Neural Network + Driving Signal (DS)

The multilayer NN is chosen due its capabilities to performed tasks in time series modelling, prediction and estimation [24,26]. The recorded EEG signal, $x(t)$ is averaged to set its baseline variations, $u(t)$ [23]. EEG background activity without variations, $r(t)$ is used throughout this research, can be obtained from deducting the baseline variation signal from the recorded signal [23]. Then, the autocorrelation function (ACF) [27] is used to determine the sample length between EEG values $r(t)$. The correlated number of EEG samples, L will be used as the length of ANN input vector is obtained by counting the correlated EEG samples until the ACF dropped for the first time below the correlation threshold.

Next, the commencement of training and validation procedure is done to adjust the weight and bias of the network. The measured EEG data is set as training data set. In the training procedure [24], EEG values are normalized into the range of [-1,$+1$]. For the weight coefficients and biases, the algorithm in [28] is used. The stopping criteria, MSE is calculated between generated and target output EEG values. Then, the incremental training procedure is performed as the weight coefficients is adjusted after every randomly selected input/output pair. Next, the input vector is initialized by randomly segmenting EEG value $r(t)$ with L length. It is then propagated into the EEG simulator with generated DS value as the last element of the input vector. Only one value is generated at a time. The output value is then reiterated into the input vector and is propagated again along with the newly generated DS value as the last element into the EEG simulator to produce a new value. This is called time-delayed neural network [24] or shifting process, in particular the new output will replace the last element while omitting the first value of the input vector.

DS is used to implement the randomness into the algorithm due to ANN output entered stationary state after several iterations in the closed loop prediction. Since DS is also used to navigate the prediction signal, the value generated from DS is used in determining the direction of the predicted signal. DS element consists of two values; -1 and $+1$ with negative and positive value represent the descending and ascending side of slope respectively [23]. DS is generated from DS extractor and DS generator. Extractor composed of a couple of units; difference and transfer. In [23], the function of the first unit is to determine the difference between the two successive EEG values while the second unit transfers the output value from the first unit into one of the conditional values using sign function[1] [29].

Next is DS generator. This part essentially to generate output in the value of either -1 or $+1$ based on preceding values. The sequence of values with determined length is used to predict the output. The conditional probabilities is exploited for this reason [30]. For every combination of the sequence, the probabilities of the next values were computed and stored in the lookup table [23]. The algorithm worked when the input sequence with N length is compared with values in the lookup table to produce certain output. To avoid biasness, the classical roulette wheel method [31] is used with random values in the range of [0, 1]. The next output were calculated based on the first output in the closed loop system. The first output will be included into the sequence of input as the last element, and the first element is then omitted thus the shifting process occurred. Then, the process of determining output based on the stored probability in the lookup table is occurred and determining output in the selection test based on classical roulette wheel method.

Neural Network method approach is better than ARMA filtering method in this research with seven determined criteria. Those are the probability density function (PDF) and cumulative distribution function (CDF) [32], level-crossing rates (LCS) average duration of fades (ADF) [33], correlation properties [34], power margin quality measures [35], weighted mean-square autocorrelation error (WMSAE) [36], representation in time domain, and representation in frequency domain [37].

4 Autoregressive (AR) Model

This section comprises white Gaussian noise filtering based on AR model [38] with its variation, Genetic approach with ARMA filtering [39]. The signal in

[1] *signum* function or *sgn* (x)

$$sgn(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}.$$

AR modeling is described to be linearly related with respect to a number of its previous samples

$$y(n) = -\sum_{k=1}^{p} a_k y(n-k) + x(n) \tag{6}$$

where a_k,$k = 1, 2, \ldots, p$, are the linear parameters, n is the discrete sample time normalized to unity, and $x(n)$ is the noise input.

4.1 Autoregressive (AR) Model

Doležal et al. [40] could not manage to model continuous non-movement related EEG based on Markov model. So, it is done via AR analysis [38]. It started with EEG pre-processing; the recorded EEG data were set to be clean from any artefact, analysis of AR and resting EEG modelling. Next is followed by assessing the quality of the EEG model. For modelling EEG signal, the frequency limitation is done with by filtering for getting new frequency and subtracting the DC component. These is done upon recorded EEG signal. It is done to avoid new poles to the processed spectrum. Then, the autocorrelation function as well as the power of the signal is computed and averaged. The modelling filter coefficients is computed by using Yule-Walker equation [41]. Thus, the model is gained by filtering white noise.

4.2 Genetic Approach to Autoregressive Moving Average (ARMA) Filter Synthesis

This model is inspired by Zetterbergs method for simulating EEG signals. The method is simulated as the result of filtering a white noise source of specific characteristics (statistical characteristics and flat spectrum) with an ARMA filter. The linear differential equation [42] represents the filter at order p (where $p \geq q$). ARMA filter is composed by Autoregressive and the Moving Average filters, could be described as the following transfer function

$$H_{ARMA} = \frac{B(z-1, q)}{A(z-1, p)} \tag{7}$$

where the numerator is related to the MA while the denominator to the AR models. AR filter is suitable for the simulation of signals which spectrum displays sharp peaks whereby MA filter is more suitable to signals with deep valleys in the spectrum [39]. For estimating the order of ARMA filter, the approach proposed by Vaz et al. [43] is used.

This method started with generating Gaussian distribution for the white noise source, then the real EEG signal is recorded from a patient by the expert in order to compare with the simulated signal. The following steps is to transformed fitness function from time domain to frequency domain utilizing Discrete Fourier Transform (DFT). Since GA is used in this method, the variables represented in this method were represented as genes or filter coefficients. The fitness function is used to compare between the spectrum of real EEG and simulated EEG signals. Next is selecting the genetic algorithm parameters via tournament selection for slower convergence. This selection is used using 10 % of the population. The process is continued with the simulation and analysis. The order selection criterion is utilized to analyze and validate for real and simulated EEG signals. In the result section, the real signal has smaller average amplitude than simulated signal represented in time domain. It is also shown that ARMA filters are best in both biological signal processing and control systems [39].

5 Conclusion

The original MPA model which can only model stationary and linear EEG model has evolved to suit the nonlinear element with self-coupling and cross coupling into the EEG artificial model. MPA based approach were using instantaneous error of simulated and actual EEG signal. The improvement lead to more comprehensive model of the actual EEG signal. The closest method that resembles the real EEG signal is Neural Network method due to the nature of NN algorithm which utilizes the actual data set for training purpose. The issue with NN training data set is the data availability and data size. One might need a very large data set. Also, since the training data is big, the NN architecture also need to be optimized to avoid undertraining and overtraining issue. On the other hand, AR model which is based on white Gaussian noise filtering has its own uniqueness. Different approaches could be implemented but this method could only model certain type of EEG signals. This method prioritizes the selection of the model order and filter coefficient.

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