



Row-sparse blind compressed sensing for reconstructing multi-channel EEG signals

Ankita Shukla, Angshul Majumdar*

Indraprastha Institute of Information Technology, Delhi, India

ARTICLE INFO

Article history:

Received 24 April 2014

Received in revised form 29 June 2014

Accepted 4 September 2014

Available online 24 January 2015

Keywords:

Compressed sensing
EEG

ABSTRACT

This communication concentrates on application of blind compressed sensing (BCS) framework for reconstruction of multichannel electroencephalograph (EEG) signal for wireless body area networks (WBANs). Compressed sensing (CS) based techniques employ a known sparsifying basis (wavelet/DCT/Gabor). BCS learns the sparsifying dictionary while recovering the signal. The BCS framework was proposed for recovering sparse signals. A recent work showed that, EEG signals can be better recovered by exploiting inter-channel correlation. This led to a row-sparse recovery problem. In this work, we modify the basic BCS framework for recovering row-sparse signal ensembles – this leads to better EEG reconstruction accuracy compared to prior CS recovery methods. The success of this technique enables reducing the energy expenditure of the sensor nodes of the WBAN.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

EEG signals are important indicators for several pathological conditions [1,2]. Recently there has been an interest in tele-monitoring of EEG signals over wireless body area networks (WBAN). In this scenario, the EEG sensors are wireless portable nodes which sample and transmit the data to a nearby device (say a smart phone) which reconstructs the signals and finally transmits them to a remote healthcare facility for monitoring and analysis. In such a system, the EEG sensors are small, portable and highly power efficient.

Compressed sensing based techniques [3–5] are usually employed for computationally cheap, energy efficient transmission of these signals. The EEG signal is sampled, and the sampled signal is projected onto a lower dimension by a random matrix – this effectively compresses the signal with minimal computational requirement (matrix–vector products can be efficiently implement at the circuit level without the requirement of specialized DSP's or FPGA's). Finally the compressed sensing is transmitted to the base station (healthcare facility).

EEG signal compression can be modeled as follows:

$$y_{m \times 1} = A_{m \times n} x_{n \times 1} + \eta_{m \times 1}, \quad m < n \quad (1)$$

* Corresponding author. Tel.: +91 9126907451.

E-mail addresses: ankita1292@iiitd.ac.in (A. Shukla), angshul@iiitd.ac.in (A. Majumdar).

Here x is the EEG signal, A is the random projection matrix, y is the compressed signal (to be transmitted) and η is the noise.

At the receiver, the problem is to reconstruct the signal x after receiving y and given A .

Compressed sensing (CS) based techniques exploit the sparsity of the signal in a transform domain in order to recover it, i.e. CS assumes that the transform coefficients $\alpha = \Psi x$ is sparse. Here it is assumed that the sparsifying transform (Ψ) is either orthogonal or tight-frame.¹ Incorporating the transform into (1), we get:

$$y = A \Psi^T \alpha + \eta \quad (2)$$

The sparse transform coefficients can be recovered by solving l_1 -minimization problem:

$$\hat{\alpha} = \min_{\alpha} \|y - A \Psi^T \alpha\|_2^2 + \lambda \|\alpha\|_1 \quad (3)$$

The l_1 -norm is defined as the sum of absolute values of the vector.

One of the first works in this area [3] proposed this technique and employed Gabor basis for sparsifying the signal and a Gaussian basis for compressing the signal. Once the sparse transform coefficients are recovered at the receiver, the EEG signal can be reconstructed by: $\hat{x} = \Psi^T \alpha$.

EEG signals are always acquired via multiple channels. Thus the acquired signals are correlated across the channels. Therefore

¹ Orthogonal : $\Psi^T \Psi = I = \Psi \Psi^T$
Tight-frame : $\Psi^T \Psi = I \neq \Psi \Psi^T$

instead of recovering the signals piecemeal, it was proposed in [4] to recover the full ensemble instead. The data acquisition model for each channel is the same as (2):

$$y_c = A\Psi^T\alpha_c + \eta \quad (4)$$

The difference between [4] and [3] is that the compression matrix A , in [4] is a sparse random matrix which is more efficient to store and operated with. The compression can be succinctly represented as:

$$aY = A\Psi^TZ + N \quad (5)$$

where Y and Z are formed by stacking the y_c 's and α_c 's as columns respectively.

In [4] it is argued that since the individual α_c 's are sparse, Z will be sparse as well. Thus, the inverse problem (5) is solved by solving l_1 -minimization:

$$\hat{Z} = \min_Z \|Z - A\Psi^TZ\|_F^2 + \lambda \|vec(Z)\|_1 \quad (6)$$

It should be noted, that there is no basic difference between [3] and [4]; the latter only recovers the multi-channel EEG signal ensemble – it does not exploit the fact that the signals are correlated.

In a recent work [6], it was argued that since the EEG signals are correlated, they will have a common sparsity pattern in the transform domain, i.e. all the transform coefficient vectors will have high values at the same positions. This assumption leads to a row-sparse recovery problem, i.e. Z is row-sparse, since all the α_c 's have high values at the same positions. This was formulated as an $l_{2,1}$ -minimization problem:

$$\hat{Z} = \min_Z \|Z - A\Psi^TZ\|_F^2 + \lambda \|Z\|_{2,1} \quad (7)$$

Here the $l_{2,1}$ -norm is defined as the sum of the l_2 -norms of the rows. Here the l_2 -norm promotes a dense solution in the selected row, the sum-of- l_2 -norm promotes selection of a very few rows. It was shown in [6] that better results are obtained by exploiting row-sparsity compared to standard CS techniques.

CS assumes sparsity of the signal in a known basis. Blind compressed sensing (BCS) [7] framework combines elements from both compressed sensing (CS) and dictionary learning. It estimates the sparsifying dictionary as well as the sparse signal from the data.

It assumes that the data is sparse in a learned dictionary, i.e. $X = DZ$, where D is the unknown dictionary (to be estimated) and Z is the set of sparse coefficients. Using this formulation, an inverse problem of the form $Y = AX + N$, can be expressed as: $Y = ADZ + N$. In BCS, both D and Z need to be estimated. This is achieved by solving:

$$\min_{D,Z} \|Y - ADZ\|_F^2 + \lambda_1 \|D\|_F^2 + \lambda_2 \|vec(Z)\|_1 \quad (8)$$

In [8] BCS was successfully used for recovering dynamic MRI sequences. One must note the subtle difference between BCS and other dictionary learning methods. Usually in dictionary learning, the learning phase is offline, i.e. the dictionary is first estimated and then the learned dictionary is employed for signal reconstruction. In BCS, both dictionary learning and signal estimation proceed simultaneously.

In this work, we want to exploit inter-channel correlation within the BCS framework for improving recovery. The details of the proposed approach is outlined in the next section. In short, it would lead to a row-sparse BCS problem. Since there is no algorithm to solve it, we propose a split Bregman approach to solve the said problem. Finally we will show that as a consequence of this learning the basis, this technique can be applied for reducing the energy consumption of the sensor network.

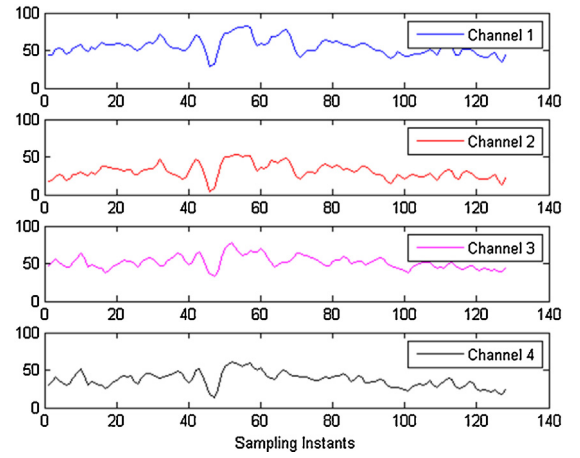


Fig. 1. Similarity of EEG signals from multiple channels.

2. Proposed work

2.1. Proposed method

We can directly apply the BCS technique (8) for recovering EEG signals. But BCS only assumes that the signal is sparse; it does not account for row-sparsity. Therefore BCS is the blind equivalent of (6). Such an approach would not exploit the inter-channel correlations between the channels.

In Fig. 1, we show that EEG signals from 4 different channels. The signals all look very similar to each other; i.e. they are correlated. In [6] it is argued that since all the signals are correlated, they will have a common sparse support in the transform domain. Therefore if we stack the sparse transform coefficients of the multi-channel signals as columns of a matrix, the resulting matrix will have sparse rows. This leads to a row-sparse recovery problem of the following form:

$$\min_{D,Z} \|Y - ADZ\|_F^2 + \lambda_1 \|D\|_F^2 + \lambda_2 \|Z\|_{2,1} \quad (9)$$

There is no algorithm to solve the said problem. We derived a split Bregman type algorithm for solving it.

2.2. Algorithm derivation

The objective is to solve (9). We introduce two proxy variables – P and Q for the two penalty functions respectively. This leads to the Augmented Lagrangian formulation:

$$\begin{aligned} \min_{D,Z,P,Q} & \|Y - ADZ\|_F^2 + \lambda_1 \|P\|_F^2 + \lambda_1 \|Q\|_{2,1} + \mu_1 \|P - D\|_F^2 \\ & + \mu_2 \|Q - Z\|_F^2 \end{aligned} \quad (10)$$

The Augmented Lagrangian enforces equality between the variables and the proxies in every iteration. But we do not want this; initially we want to assign more importance to the Frobenius norm and $l_{2,1}$ -norm penalties (9). When the solution converges, we want the variables and its proxies to be equal. There are two ways to achieve this effect. The first one is to employ fixed point continuation techniques, i.e. to solve for one set of values μ_1 and μ_2 , and progressively increase their values to enforce equality at convergence. The other possibility is to initially relax the equality constraint and enforce it progressively. The second possibility is favored in split Bregman techniques as it automatically adjusts the constraint [10,11].

To achieve the said objective, we add terms relaxing the equality constraints of each quantity and its proxy, and in order to enforce equality at convergence, we introduce Bregman relaxation variables B_1 and B_2 . The new objective function is:

$$\min_{D,Z,P,Q} \|Y - ADZ\|_F^2 + \lambda_1 \|P\|_F^2 + \lambda_1 \|Q\|_{2,1} + \mu_1 \|P - D - B_1\|_F^2 + \mu_2 \|Q - Z - B_2\|_F^2 \quad (11)$$

This substitution allows (11) to be solved via alternating minimization of the following (simpler) sub-problems:

$$\min_D \|Y - ADZ\|_F^2 + \mu_1 \|P - D - B_1\|_F^2 \quad (12a)$$

$$\min_Z \|Y - ADZ\|_F^2 + \mu_2 \|Q - Z - B_2\|_F^2 \quad (12b)$$

$$\min_P \lambda_1 \|P\|_F^2 + \mu_1 \|P - D - B_1\|_F^2 \quad (12c)$$

$$\min_Q \lambda_1 \|Q\|_{2,1} + \mu_2 \|Q - Z - B_2\|_F^2 \quad (12d)$$

The sub-problems (12a)–(12c) are just least squares minimization and can be solved via LSQR. The sub-problem (12d) is a row-sparse recovery problem. The algorithm for solving it is derived in [9]. It is solved via modified iterative soft thresholding, as follows:

$$Q \leftarrow \text{signum}(Z + B_2) \max \left(0, |Z + B_2| - \frac{\lambda_2}{\mu_2} \Lambda \right)$$

where $\Lambda = \text{diag}(\|(Z + B_2)^{j \rightarrow} \|_2^{-1}) \text{abs}(Z + B_2)$; $(X)^{j \rightarrow}$ denotes the j th row of X .

The final step to update the Bregman variables:

$$B_1 \leftarrow P - D - B_1$$

$$B_2 \leftarrow Q - Z - B_2$$

The full algorithm for solving (9) is given below.

<p>Initialize: D, B_1, B_2 In every iteration: Solve: $\hat{Z} = \min_Z \ Y - ADZ\ _F^2 + \mu_2 \ Q - Z - B_2\ _F^2$ using LSQR</p> <p>Solve: $\hat{D} = \min_D \ Y - ADZ\ _F^2 + \mu_1 \ P - D - B_1\ _F^2$ using LSQR</p> <p>Solve: $\hat{P} = \min_P \lambda_1 \ P\ _F^2 + \mu_1 \ P - D - B_1\ _F^2$ using LSQR</p> <p>Update: $Q \leftarrow \text{signum}(Z + B_2) \max \left(0, Z + B_2 - \frac{\lambda_2}{\mu_2} \Lambda \right)$</p> <p>Update: $B_1 \leftarrow P - D - B_1$; $B_2 \leftarrow Q - Z - B_2$</p>
--

The parameters μ_1 and μ_2 need to be specified internally. In this case, we have kept $\mu_1 = 0.1$ and $\mu_2 = 0.001$ since these values yields the best results. B_1, B_2 were initialized to all one's. Fixing D is tricky; the first task is to specify the number of columns in D . We found that the results did not vary as long as the number of columns were larger than 30. In this case we have kept it to be 40. Similar observations were also deduced in [8] for the problem of dynamic MRI reconstruction. D is initialized to random values. The number of iterations for LSQR was fixed at 20.

BCS is not a convex problem owing to its bilinearity. Our algorithm has two stopping criteria. The first one is the maximum number of iterations. This has been fixed at 100. The second criteria are the tolerance of the objective function between successive iterations; when the change in objective function is very small (less than 0.0001), we stop the algorithm.

2.3. A new power efficient sampling paradigm

Let us briefly revisit the problem. The EEG signal is being fully sampled; it is compressed by projecting it onto a sparse binary random matrix. Finally the compressed EEG signal is being transmitted. These operations reduce transmission energy only; but cannot reduce sensing and processing energies. Can we not reduce the said acquisition and processing energy by randomly under-sampling the EEG signal? Using traditional CS this would not be possible.

There are two requirements for CS to work – (i) sparsity of the signal, and (ii) incoherence of the sparsity basis with the measurement basis [12]. EEG signals are sparse in wavelet, DCT or Gabor transforms. These transforms are incoherent with the sparse binary random basis used for compressing the EEG signals. However, if we randomly under-sample the EEG signals, the measurement basis will be the Dirac basis. None of the aforesaid sparsity promoting transforms are incoherent with the Dirac basis. For example, for the given problem the (normalized) coherence between the Wavelet and Dirac basis is 1.98 and between DCT and Dirac is 1.38. Higher coherence implies poorer recovery for a given number of measurements. This is the main drawback in trying to employ CS for recovering under-sampled EEG signals.

The BCS framework is likely to succeed where traditional CS fails. This is because, BCS learns the sparsifying basis. Prior studies in dictionary learning have shown that the empirical learned dictionaries improve upon fixed basis. But this is not the only reason; there is another intuitive reason for BCS to succeed for our problem.

The only penalty on the dictionary D is the Frobenius norm (9). Thus, the dictionary D is a dense matrix. Such a dense matrix will be highly incoherent with the sparse Dirac measurement basis, indicating that BCS would be able to successfully recover EEG signals from randomly under-sampled measurements. We do not require changing the recovery algorithm in any fashion. Only change is in the definition of the measurement basis A (1); for random under-sampling it would correspond to a sampling operator.

The prospective power savings we get by changing to this new sampling paradigm is significant. We analyze the overall energy saving by following a power model proposed in [13]. The total power of a CS EEG unit comprises of three major factors:

$$P_{\text{tot}} = P_{\text{sense}} + P_{\text{proc}} + P_{\text{comm}} \quad (13)$$

The sensing is comprised of two portions – amplification (P_{amp}) and analog-to-digital conversion (P_{ADC}); therefore $P_{\text{sense}} = C(P_{\text{amp}} + P_{\text{ADC}})$, where C is the total number of channels. The processing consists of two operations – random number generation (P_{RNG}) and matrix–vector multiplication (P_{mult}); thus, $P_{\text{proc}} = P_{\text{RNG}} + P_{\text{mult}}$. It is difficult to uniquely model the power requirement for communication (transmission) because it is dependent on the communication protocol. In general the communication power is expressed as $P_{\text{comm}} = C J f_s R$ where J is the transmission power per bit, f_s is the sampling frequency (bits per second) of the ADC and R is the number of bits per sample (resolution).

The sensing power and the communication power increases linearly with the number of channels, but the processing power does not scale with the number of channels. CS based techniques can only reduce the transmission power by compressing an n dimensional signal to m samples via matrix–vector multiplications. Thus, the total power consumption by a CS based EEG WBAN will consume:

$$P_{\text{tot}} = C(P_{\text{amp}} + P_{\text{ADC}}) + P_{\text{RNG}} + P_{\text{mult}} + C(m/n) J f_s R \quad (14)$$

Following the same arguments, our proposed unit will only consume:

$$P_{\text{tot}} = C(P_{\text{amp}} + (m/n)P_{\text{ADC}}) + C(m/n)f_s R \quad (15)$$

The amplifier being an analog device cannot be switched on and off at a fast rate (power gating may be possible but we have not considered it here). We do not need the matrix–vector product for compression so the corresponding power terms vanish; we compress right when we sample. The transmission power remains the same as that of a CS based unit.

Establishing hard values for power consumption gives some idea regarding the actual power cost and how much we save. For the ADC, we get the values from [14] – for a 12 bits per sample (R), 0.5 ksample/s sampling rate (f_s) and $P_{\text{ADC}} = 0.2 \mu\text{W}$. The specifications of the power amplifier (for EEG signals) is obtained from [15] – 30 dB gain and 30 Hz bandwidth require $P_{\text{amp}} = 0.9 \mu\text{W}$. The power consumption for the random number generator is $3 \mu\text{W}$ [16]. In [13] the power cost of matrix–vector multiplication is pegged at $352 \mu\text{W}$ assuming that the DSP chip is of TI MSP430 family. The transmission energy required per bit is estimated to be 5 nJ per bit [11].

With these figures, we compute the energy requirement for a CS based system using (14). $64 \times (0.2 + 0.9) + 3 + 352 + 64 \times 0.2 \times 0.5 \times 5 \times 12 = 809 \mu\text{W}$. Here it is assumed that the compression ratio is 5:1 (0.2). Now, considering our proposed random under-sampling, the power consumption (for same compression ratio) is computed using (15): $64 \times (0.2 \times 0.2 + 0.9) + 64 \times 0.2 \times 0.5 \times 5 \times 12 = 444 \mu\text{W}$; this is about half the power consumption required by CS techniques. In other words, with our proposed methodology, the WBAN is expected to last almost twice longer. If the sampling ratio is increased to 5:2 (0.4), the power requirement for the CS based method is 1.19 mW and that of our proposed technique is 0.83 mW.

3. Results

The experiments are carried out on the BCI III dataset 1 [17]. During the BCI experiment, the subject had to perform imagined movements of either the left small finger or the tongue. The time series of the electrical brain activity was picked up during these trials using a 8×8 ECoG platinum electrode grid which was placed on the contralateral (right) motor cortex. The grid was assumed to cover the right motor cortex completely. All recordings were performed with a sampling rate of 1000 Hz. After amplification the recorded potentials were stored as microvolt values. Every trial consisted of either an imagined tongue or an imagined finger movement and was recorded for 3 s duration. To avoid visually evoked potentials being reflected by the data, the recording intervals started 0.5 s after the visual cue had ended.

In the BCI competition, the task was that of a binary classification, i.e. to predict what the subject was imagining – tongue movement or finger movement. However, for our problem, the interest is in reconstruction accuracy. The metric for measuring reconstruction accuracy is normalized mean squared error defined as: $\text{NMSE} = \frac{\| \text{original} - \text{reconstructed} \|}{\| \text{original} \|}$.

We carried out two kinds of experiment. The first one is the standard method used in all previous studies – the full signal is acquired and then compressed by projecting onto a random binary sparse matrix. The second experiment, randomly under-samples the signal itself. This constitutes the case where we are reducing sampling and processing energy. For both kinds the experiments, the compression/sampling ratio is fixed at 50%. The experimental results are shown in Table 1.

We find that our proposed method is significantly better than previous CS based techniques. It has lower error rates. What is

Table 1
Reconstruction error.

Methods	Compression		Under-sampling	
	Mean	St. dev.	Mean	St. dev.
Piecemeal signal recovery [3]	0.1263	0.0576	0.1667	0.1008
Sparse CS recovery [4]	0.1667	0.0745	0.2239	0.1107
Row-sparse CS recovery [6]	0.1142	0.0721	0.0967	0.0458
Row-sparse BCS recovery	0.0122	0.0327	0.0379	0.0225

Table 2
Classification accuracy.

Methods	Compression	Under-sampling
No compression (ground-truth)	81%	
Piecemeal signal recovery [3]	73%	68%
Sparse CS recovery [4]	70%	66%
Row-sparse CS recovery [6]	76%	74%
Row-sparse BCS recovery	79%	78%

interesting to note is that all prior CS based techniques falter when the EEG signal is under-sampled instead of compressed – the recovery results deteriorate considerably. However, for our proposed row-sparse BCS technique there is hardly any difference between under-sampling and compression.

Signal recovery is not the final outcome. In practice, the recovered EEG signals are analyzed further. The dataset we are using was used for automatic classification of two activities – tongue movement and left small finger movement. This is basically a binary classification problem. The training set consists of 278 labeled trials (samples) and the test set consists of 100 trials (samples). We carried out the classification using algorithm [18] on the BCI III dataset. The classification was carried out on the groundtruth as well as on the reconstructed EEG data. The classification results are shown in Table 2. This algorithm was available from the BCI competition website.

The results corroborate our previous observations. Both for compression and under-sampling, our proposed method yields the best results. For the other techniques the recovery deteriorates as we move from compression to under-sampling.

4. Conclusion

There are two major contributions of this work. This is the first time, we employ the BCS framework to reconstruct EEG signals. But instead of applying the sparse BCS framework, we modify it for row-sparse problems. This modification is inspired by a recent study which showed that instead of reconstructing the EEG signals individually, better recovery is achieved when inter-channel correlations are accounted for [6]; in this study it is assumed that the sparsity basis for the signal is known. The results from our proposed method show that there is significant improvement compared to prior techniques.

The second contribution of this work is far reaching. Previous CS techniques could only reduce the data transmission energy; they could not reduce acquisition and processing energies. We propose to further reduce the energy requirement by directly under-sampling the signal – this reduces sensing energy and processing energy in one step. Unfortunately previous CS based methods cannot be used in such a sensing paradigm, because the fixed sparsifying basis (wavelet/Gabor/DCT) are not incoherent with under-sampling basis and therefore violates the basic requirement of CS [10]. The BCS framework learns the dictionary from the data itself, this learned dictionary is highly incoherent with the under-sampling basis, making it a good solution to our problem. The results corroborate our assumptions. The reconstruction

accuracy from prior methods deteriorate drastically as we move from the compression paradigm to the under-sampling paradigm; but the results from the proposed BCS technique remain almost the same. In short, our method can be used to bring down the power consumption at the sensor nodes without compromising on the quality of the reconstructed signal.

Our work proposes a scheme for general purpose sensing and transmission of EEG signals. The transmitted signal can be used for any application – seizure detection, brain–computer interface, etc. However if the goal is known a priori, there are tailored techniques to reduce the energy consumption even further. One recent work on this topic [19], showed that it is possible to compute low-energy consuming features for seizure detection and only send these features for further analysis. Such techniques require more computational resources but less transmission power (energy) compared to ours.

References

- [1] A. Temko, C. Nadeu, W. Marnane, G.B. Boylan, G. Lightbody, EEG signal description with spectral-envelope-based speech recognition features for detection of neonatal seizures, *IEEE Trans. Inf. Technol. Biomed.* 15 (6) (2011) 839–847.
- [2] V. Bajaj, R.B. Pachori, Classification of seizure and nonseizure EEG signals using empirical mode decomposition, *IEEE Trans. Inf. Technol. Biomed.* 16 (6) (2012) 1135–1142.
- [3] S. Aviyente, Compressed sensing framework for EEG compression, in: *IEEE/SP SSP*, August, Madison, 2007.
- [4] M.H. Kamal, M. Shooran, Y. Leblebeci, A. Schmid, P. Vandergheynst, Compressive multichannel cortical signal recording, in: *ICASSP*, 2013.
- [5] M. Mohsina, A. Majumdar, Gabor based analysis prior formulation for EEG signal reconstruction, *Biomed. Signal Process. Control* 8 (6) (2013) 951–955.
- [6] A. Majumdar, R.K. Ward, Non-convex row-sparse MMV analysis prior formulation for EEG signal reconstruction, *Biomed. Signal Process. Control* 13 (2014) 142–147.
- [7] S. Gleichman, Y.C. Eldar, Sensor Array and Multichannel Signal Processing Workshop (SAM), *IEEE* (2010) 129–132.
- [8] S.G. Lingala, M. Jacob, Blind compressive sensing dynamic MRI? *IEEE Trans. Med. Imaging* 32 (June (6)) (2013) 1132–1145.
- [9] A. Majumdar, R.K. Ward, Synthesis and analysis prior algorithms for joint-sparse recovery? in: *IEEE International Conference on Acoustics, Speech, and Signal Processing*, 2012, pp. 3421–3424.
- [10] R. Chartrand, B. Wohlberg, A nonconvex ADMM algorithm for group sparsity with sparse groups, in: *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, May, Vancouver, Canada, 2013, pp. 6009–6013.
- [11] R. Chartrand, Nonconvex splitting for regularized low-rank + sparse decomposition *IEEE Trans. Signal Process.* 60 (2012) 5810–5819.
- [12] E. Candès, J. Romberg, Sparsity and incoherence in compressive sampling, *Inverse Probl.* 23 (3) (2007) 969.
- [13] A.M. Abdulghani, A.J. Casson, E. Rodriguez-Villegas, Foundations of Augmented Cognition. *Neuroergonomics and Operational Neuroscience Lecture Notes in Computer Science*, Springer 5638 (2009) 319–328.
- [14] N. Verma, A. Chandrakasan, An ultra low energy 12-bit rate-resolution scalable SAR ADC for wireless sensor nodes, *IEEE J. Solid State Circuits* 42 (6) (2007) 1196–1205.
- [15] R. Harrison, C. Charles, A low-power low-noise CMOS amplifier for neural recording applications, *IEEE J. Solid State Circuits* 38 (6) (2003) 958–965.
- [16] J. Holleman, B. Otis, S. Bridges, A. Mitros, C. Diorio, A 2.92 μ W hardware random number generator, in: *ESSCIRC Solid-State Circuits Conference*, 2006, pp. 134–137.
- [17] <http://www.bbci.de/competition/iii/>
- [18] K. Yang, H. Yoon, C. Shahabi, A supervised feature subset selection technique for multivariate time series, in: *International Workshop on Feature Selection for Data Mining: Interfacing Machine Learning with Statistics*, 2005.
- [19] J. Chiang, R.K. Ward, Energy-efficient data reduction techniques for wireless seizure detection systems? *Sensors* 14 (2) (2014) 2036–2051.