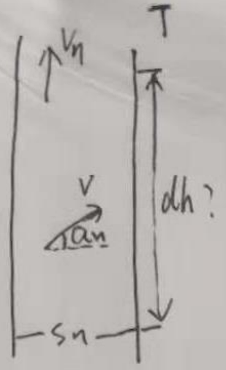
3. Mathematics:



已知：
- 河宽：$S_1, S_2 \cdots S_n$
- 流速：$V_1, V_2 \cdots V_n$
- 总时间为 T
- 游速为 V

未知：游泳角度：$a_1, a_2 \cdots a_n$

求： max distance

解:

$$h(a) = \sum_{i}^{n} \left(V \cdot \sin a_i + V_i \right) \cdot \left(\frac{S_i}{V \cdot \cos a_i} \right)$$

约束条件：$g(a) = \sum_{i}^{n} \left(\frac{S_i}{V \cdot \cos a_i} \right) - T = 0$

根据拉格朗日乘数法可得：

$$\begin{aligned} H(a, \lambda) &= h(a) + \lambda(g(a)) \\ &= \sum_{i}^{n} \left(V \cdot \sin a_i + V_i \right) \cdot \left(\frac{S_i}{V \cdot \cos a_i} \right) + \\ &\quad \lambda \left(\sum_{i}^{n} \left(\frac{S_i}{V \cdot \cos a_i} \right) - T \right) \end{aligned}$$

$$\begin{aligned} \frac{\partial H(a, \lambda)}{\partial a_i} &= \frac{S_i}{\cos^2 a_i} + \frac{S_i V_i}{V} \cdot \frac{\sin a_i}{\cos^2 a_i} + \frac{\lambda S_i \sin a_i}{V \cos^2 a_i} \\ &= S_i \cdot \frac{V + V_i \sin a_i + \lambda \sin a_i}{V \cos^2 a_i} = 0 \end{aligned}$$

$$\frac{\partial H(a, \lambda)}{\partial \lambda} = \frac{S_i}{V \cos a_i} - T = 0$$