On the Unification of Dark Matter and Ordinary Matter

Abstract

There are many theoretical models attempting to unify dark matter and ordinary matter; however, each of them has certain limitations. These limitations make it extremely complex and difficult to supplement and improve them. Therefore, this paper proposes the Meshing Model. Based on this model, we clearly define the properties of dark matter and dark energy. At the same time, due to the simplicity and unification of the model, it is easier to extend. Ultimately, it becomes the ideal model most likely to unify dark matter and ordinary matter.
Keywords: Meshing Model; Field; Spin; Meshing;

1 Introduction

1.1 Research Background

There are currently many theoretical models trying to unify dark matter and ordinary matter. Mainstream models include the ΛCDM model, String/M-theory, Loop Quantum Gravity (LQG), and Supersymmetry. However, these models each have different limitations.

The ΛCDM model predicts, through numerical simulations, that there should be hundreds of dwarf satellite galaxies around the Milky Way, but the actual number observed is far fewer. M-theory involves the compactification of higher-dimensional manifolds, the mathematical properties of which are not yet fully understood. LQG cannot fully reproduce General Relativity due to unclear actions of geometric surfaces in spin networks. Supersymmetry introduces supersymmetric particles, requiring many new parameters to describe them and their interactions with ordinary matter, greatly increasing computational complexity.

1.2 Research Objective

Due to the above limitations, we need a simpler and clearer model to achieve the unification of dark matter and ordinary matter. Hence, we propose the Meshing Model, which uses only three existing physical concepts: field, spin, and meshing, to unify dark matter and ordinary matter in an extremely minimalistic way, thereby constructing a fully unified world model of matter structure and interaction.

2 Basic Assumptions

- **Principle of Relativity:** The spin of matter remains unchanged in all reference frames.
- **Meshing Principle:** There exist countless spin fields within and between all matter (including macroscopic matter and microscopic particles), and each field is coupled together through meshing.

3 Meshing Model

The Meshing Model posits that there are numerous curved spin fields both inside and outside all macroscopic and microscopic matter, which are coupled together through meshing.

3.1 Fields

In this model, fields are divided into quantum fields and dark matter fields. Dark matter particles carry dark matter fields, and visible particles carry quantum fields. They have the following characteristics:

- **Invisible:** An invisible field.
- **Curved:** These fields have curved surfaces. The degree of curvature varies; quantum fields are rough, while dark matter fields are smooth. See 3.2.
- **Spinning:** These fields spin on their own, with different spin velocities. The quantum field's spin velocity is higher than that of the dark matter field. See 3.2.
- **Varied Sizes:** These fields vary in size, possibly very small or very large. The quantum field is larger than the dark matter field. See 3.2.

3.2 Spin

3.2.1 Relativity

Quantum fields spin, driving their coupled visible particles to spin. Since the observer is also within the quantum field and rotating in the same direction and speed, relative to the quantum field, the observer sees the particle as stationary, i.e., appearing not to spin. Thus, spin becomes an intrinsic property of the particle.

3.2.2 Generation of Visible Particles

Dark matter particles, through high-speed spinning of their carried fields, generate centrifugal force, stretching the dark matter field. This causes the particle in the field to also stretch. The originally tiny dark matter particle becomes larger in scale after stretching, eventually forming a visible particle. At the same time, the stretched field is no longer smooth and becomes a quantum field.

Therefore, the dark matter field can be transformed into a quantum field through spin, and dark matter particles can be transformed into visible particles.

3.3 Meshing

Fields are coupled through meshing. We interpret meshing from two aspects: matter structure and interaction.

3.3.1 Structure of Matter

On the microscopic level, matter is composed of molecules and atoms. Therefore, we discuss the relationship between matter structure and the meshing principle on the microscopic level.

- **Atomic Structure:** The spin section above mentions that dark matter fields generate visible particles through high-speed spin. These visible particles, such as protons and neutrons, mesh their quantum fields with dark matter fields. Due to their spin, they form spinning atomic nuclei. Similarly, electrons mesh their quantum fields with dark matter fields and orbit the nucleus in a spinning motion.
- **Matter Structure:** Since atoms are composed of visible and dark matter particles meshed through fields, and matter is made of molecules and atoms (atoms compose molecules), matter is therefore also composed of visible and dark matter particles meshed through fields.

3.3.2 Origin of Mass

- **Higgs Mechanism**
  Numerous dark matter fields spinning at high speed cause field stretching. As they stretch, fields get closer to each other, eventually tearing internally and merging. After tearing, elastic potential energy transforms into the merged field state — the top of the Higgs field (Mexican hat). Because the spin of these multiple fields is identical, the resulting top is a scalar field (spinless). Since the top is unstable, the merged field state "rolls down" to the bottom of the Higgs potential, forming fluctuations. When particles pass through the Higgs field, they receive "resistance" and acquire mass from it.
- **Non-Perturbative Effect**
  Numerous dark matter fields spinning at high speed cause stretching. As fields get closer, they tear internally, and the adjacent parts merge. The elastic potential energy converts into mass of excited states — massive gluons. As the two fields spin in opposite directions, the merged field aligns in spin direction, giving gluons spin-1. These gluon fields continue to tear and merge (self-interaction), forming fluctuating gluon fields.
- **Chiral Symmetry Breaking**
  Take quarks as an example. Light quarks acquire tiny mass via the Higgs mechanism. Then, due to gluon field fluctuations (non-perturbative effect), quark-antiquark pairs are produced and annihilated under continuous excitation and merging, meshing their fields with gluon fields. This tearing and merging produce condensates, leading to chiral symmetry breaking. The light quark, during propagation, meshes with this condensate and gluon fields, converting elastic energy from tearing into mass.

3.3.3 Interactions

In physics, interactions between matter are categorized as strong, electromagnetic, weak, and gravitational interactions. All these can be observed during particle collisions. Thus, we take particle collisions as an example to explore the relationship between the meshing principle and interactions.

3.3.3.1 Collision Between Visible and Dark Matter Particles

Visible particle A collides with dark matter particle B. Since A's quantum field meshes with B's dark matter field, and B's field is smooth enough, it continues rotating past A's field, causing B to pass through A.

3.3.3.2 Collision Between Visible Particles

Visible particle collisions are divided into low-energy and high-energy cases.

- **Low-energy collision between visible particles A and B:** Since there is a dark matter field between them, A and B collide at high speed. A's field meshes and rotates with the dark matter field, and B's field does the same. Because the dark matter field is smooth, the interaction is very weak. The energy from A's field is transferred briefly to the dark matter field and then to B's field.
- **High-energy collision between visible particles A and B:** High-energy collisions typically generate new particles. These are further divided as follows:
  - **Strong interaction:** A dark matter field (from the non-perturbative effect in 3.3.2 — i.e., gluon field) exists between particles A and B. This field meshes with A and B. When A and B collide at near-light speed, their energy transfers to the dark matter field, increasing its rotational speed. Though insufficient to produce visible particles, it stretches dark matter particles within. These stretched particles connect A and B, creating new particles — verifying that PETRA at DESY discovered gluons.
  - **Weak interaction:** A dark matter field meshes with A and B. Collision energy at near-light speed transfers to this field, spinning it faster and creating new particles. The elastic energy stretches back to A and B, correcting their mass. The new particle, being massive, decays quickly — consistent with UA1/UA2's discovery of the W boson.
  - **Creation/annihilation of virtual pairs:** Multiple meshed dark matter fields exist between A and B. Collision energy accelerates adjacent dark matter fields, stretching them. Elastic energy partially adjusts A and B's mass, while the rest excites non-adjacent dark matter fields to create two new particles. This aligns with the LHC's discovery of the Higgs boson — the two new particles could be a virtual pair, correcting the Higgs mass. This also suggests supersymmetric partners may not exist — the absorbed energy was insufficient to create new visible particles.

3.3.3.3 Collision Between Dark Matter Particles

Dark matter particle A collides with dark matter particle B. Due to their smooth fields and minimal cross-sections, their interaction is negligible.

In conclusion, the meshing principle, combined with the spin of dark matter fields that generates new particles, can unify all kinds of interactions observed in particle collisions.

4 Basic Concepts

4.1 Dark Matter

Dark matter includes two core concepts: dark matter particles and dark matter fields. Among them, dark matter particles have the following properties:

- Scale much smaller than the Planck constant
- Driven to spin by the spin of the dark matter field
- Flexible and stretchable

Dark matter fields have the following properties:

- Very small in scale, invisible
- Smooth surface
- Spinning
- Flexible and stretchable
- Coupled through a meshing mechanism

4.2 Dark Energy

After the Big Bang, all visible particle quantum fields still retain the high-speed spin from the explosion. Since these quantum fields are meshed with dark matter fields, they transfer their high-speed spin energy to certain dark matter fields, converting it into rotational kinetic energy and causing the dark matter fields to spin. However, this spin is not stretched enough to be visible. As more and more of these dark matter fields absorb energy, their surfaces become increasingly close, and the density increases, eventually forming dark energy. This process is the engulfing of dark matter by dark energy. Therefore, dark energy is the compact result of multiple dark matter fields.

5 Mathematical Definition and Derivation

5.1 Meshing Model

5.1.1 Effective Potential

Taking the experiment of photons colliding with electrons as an example, according to the meshing principle of the model, photons transfer energy to the dark matter field through their own quantum fields. The dark matter field absorbs part of the energy to accelerate its spin and transmits the other part of the energy (elastic potential energy) to the electron field. The resulting manifestation is that the momentum of the electron becomes random. (Note: The potential energy change of the dark matter field during the transfer process gives rise to quantum fluctuations.)

Based on the above explanation, we can define the effective potential of the meshing model as

$$V_{\text{eff}}(r) = \underbrace{\frac{1}{2}k(r - r_0)^2}_{\text{Elastic potential}} + \underbrace{\frac{\hbar^2 \ell(\ell + 1)}{2mr^2}}_{\text{Centrifugal potential}} \tag{1}$$

First term: The potential energy of a membrane bound at radius r_0 and allowed to elastically vibrate along the radial direction.
Second term: The centrifugal potential generated when the particle has angular momentum ℓ.

5.1.2 Hamiltonian

For the "radially elastic spherical membrane" model, the effective radial Hamiltonian is

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + V_{\text{eff}}(r) \tag{2}$$

5.1.3 Derivation of the Uncertainty Principle

Heisenberg's uncertainty principle states that it is impossible to simultaneously determine a fundamental particle's position and momentum with arbitrary precision.

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{3}$$

In the interlocking model, we define fields as membranes. The uncertainty principle is derived through the effective potential. The entire derivation proceeds in four major steps:

1. Write down the radial Hamiltonian and perform a quadratic expansion around the minimum point to obtain an approximate harmonic oscillator form.
2. Under the "narrow wave packet" approximation, construct the ground-state Gaussian wavefunction using the normalized measure $r^2 dr$, and compute Δr.
3. Use the radial momentum operator $\hat{p}_r = -i\hbar\left(\frac{d}{dr} + \frac{1}{r}\right)$ to compute $\langle p_r^2 \rangle$, thereby obtaining Δp_r.
4. Finally, combine to obtain $\Delta r \Delta p_r = \frac{\hbar}{2}$.

- Radial Hamiltonian and Quadratic Expansion of Potential
  - **Effective Radial Hamiltonian**
    For the model of a "spherically expandable membrane," the effective radial Hamiltonian is

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + V_{\text{eff}}(r) \tag{4}$$

where

$$V_{\text{eff}}(r) = \underbrace{\frac{1}{2}k(r-r_0)^2}_{\text{Elastic potential}} + \underbrace{\frac{\hbar^2\ell(\ell+1)}{2mr^2}}_{\text{Centrifugal potential}} \tag{5}$$

The first term: potential energy of a membrane bound at radius r_0 and capable of radial elastic vibration.

The second term: centrifugal potential arising when the particle has angular momentum ℓ.

– **Quadratic Expansion of Potential at Minimum**

First determine the minimum point $r = r_{\text{min}}$ of $V_{\text{eff}}(r)$. Let

$$\frac{dV_{\text{eff}}}{dr} = k(r-r_0) - \frac{\hbar^2\ell(\ell+1)}{mr^3} = 0 \tag{6}$$

$$\Rightarrow k(r_{\text{min}} - r_0) = \frac{\hbar^2\ell(\ell+1)}{mr_{\text{min}}^3} \tag{7}$$

This equation implicitly determines r_{min}. At $r = r_{\text{min}}$, the potential energy is minimized, and the ground-state wavefunction will concentrate near this point.

Expand around $r \approx r_{\text{min}}$ using a Taylor expansion to second order: let $x = r - r_{\text{min}}, r = r_{\text{min}} + x$, then

$$V_{\text{eff}}(r) = V_{\text{eff}}(r_{\text{min}}) + \frac{1}{2}V''_{\text{eff}}(r_{\text{min}})(r-r_{\text{min}})^2 + \mathcal{O}((r-r_{\text{min}})^3) \tag{8}$$

Compute

$$V''_{\text{eff}}(r) = k + \frac{d}{dr}\left[-\frac{\hbar^2\ell(\ell+1)}{mr^2}\right] = k + \frac{3\hbar^2\ell(\ell+1)}{mr^4} \tag{9}$$

At $r = r_{\text{min}}$,

$$m\omega_{\text{eff}}^2 = V''_{\text{eff}}(r_{\text{min}}) = k + \frac{3\hbar^2\ell(\ell+1)}{mr_{\text{min}}^4} \tag{10}$$

So, the potential energy is approximately

$$V_{\text{eff}}(r) \approx V_{\text{eff}}(r_{\text{min}}) + \frac{1}{2}m\omega_{\text{eff}}^2(r-r_{\text{min}})^2 \tag{11}$$

Let us ignore the constant term $V_{\text{eff}}(r_{\text{min}})$ for now, and focus only on the quadratic part forming the "harmonic oscillator" model.

- **Ground-State Wavefunction and Δr**

– **Approximate Harmonic Oscillator Ground State (Gaussian) Form**

In $x = r - r_{\text{min}}$, the "approximate harmonic oscillator" Hamiltonian is

$$\hat{H} \approx -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega_{\text{eff}}^2x^2 + \text{constant.} \tag{12}$$

This is the stationary state equation of a 1D harmonic oscillator. The corresponding ground-state wavefunction in "linear coordinate x" is

$$\tilde{R}(x) = \left(\frac{m\omega_{\text{eff}}}{\pi\hbar}\right)^{1/4}\exp\left[-\frac{m\omega_{\text{eff}}}{2\hbar}x^2\right], \quad x \in (-\infty, \infty). \tag{13}$$

However, our real problem is for radius $r \in [0, \infty)$, with radial measure $r^2 dr$. Let $r = r_{\min} + x$. When the "wave packet width Δr is much smaller than $r_{\min}$," we can invoke the "narrow wave packet" approximation:

* It almost never crosses $r = 0$
* The integration limits can be approximately extended to $x \in (-\infty, \infty)$
* r^2 within the wave packet can be approximated as $r^2 \approx r_{\min}^2 + 2r_{\min}x + x^2 \approx r_{\min}^2$ (keeping only the leading constant)

Thus, the actual radial ground state is written as

$$R_{\mathrm{gs}}(r) \approx N \exp\left[-\frac{m\omega_{\mathrm{eff}}}{2\hbar}(r - r_{\min})^2\right] \tag{14}$$

where the normalization constant N is determined by

$$\int_0^\infty |R_{\mathrm{gs}}(r)|^2 r^2 dr = 1 \tag{15}$$

which is approximated as

$$r_{\min}^2 \int_{-\infty}^{\infty} |\tilde{R}(x)|^2 dx = 1 \tag{16}$$

that is,

$$r_{\min}^2 \cdot 1 = 1 \Rightarrow N = \frac{1}{r_{\min}} \left(\frac{m\omega_{\mathrm{eff}}}{\pi\hbar}\right)^{1/4} \tag{17}$$

More precisely, if one retains $r^2 = (r_{\min} + x)^2$, one can obtain first-order small corrections, but the leading order gives the normalization constant as above. Therefore, the radial normalized wavefunction is

$$R_{\mathrm{gs}}(r) = \frac{1}{r_{\min}} \left(\frac{m\omega_{\mathrm{eff}}}{\pi\hbar}\right)^{1/4} \exp\left[-\frac{m\omega_{\mathrm{eff}}}{2\hbar}(r - r_{\min})^2\right]. \tag{18}$$

- **Compute $\langle r\rangle$, $\langle r^2\rangle$, and Δr**

  - **Definition**

    Normalization condition (approximate):

$$\int_0^\infty |R_{\mathrm{gs}}(r)|^2 r^2 dr \approx r_{\min}^2 \int_{-\infty}^{+\infty} |\tilde{R}(x)|^2 dx = 1 \tag{19}$$

    Let $x = r - r_{\min}, \quad \alpha = \frac{m\omega_{\mathrm{eff}}}{\hbar}$, then

$$\tilde{R}(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\frac{\alpha}{2}x^2} \tag{20}$$

  - **Compute $\langle r\rangle$**

$$\langle r\rangle = \int_0^\infty r|R_{\mathrm{gs}}(r)|^2 r^2 dr \approx \int_{-\infty}^{\infty} (r_{\min} + x)|\tilde{R}(x)|^2 r_{\min}^2 dx \tag{21}$$

    Here $\tilde{R}(x)$ is an even function, and $\int_{-\infty}^{\infty} x|\tilde{R}(x)|^2 dx = 0$.
    Thus,

$$\langle r\rangle \approx r_{\min}^3 \int_{-\infty}^{\infty} |\tilde{R}(x)|^2 dx = r_{\min} \tag{22}$$

- **Compute $\langle r^2 \rangle$**

$$\langle r^2 \rangle = \int_0^\infty r^2 |R_{\text{gs}}(r)|^2 r^2 dr \approx \int_{-\infty}^\infty (r_{\text{min}} + x)^2 |\tilde{R}(x)|^2 r_{\text{min}}^2 dx \tag{23}$$

Expand $(r_{\text{min}} + x)^2 = r_{\text{min}}^2 + 2r_{\text{min}}x + x^2$, noting that $\int x|\tilde{R}|^2 = 0$. Thus,

$$\langle r^2 \rangle \approx r_{\text{min}}^2 \cdot r_{\text{min}}^2 \int_{-\infty}^\infty |\tilde{R}(x)|^2 dx + r_{\text{min}}^2 \cdot \int_{-\infty}^\infty x^2 |\tilde{R}(x)|^2 dx \tag{24}$$

We know for the 1D Gaussian ground state:

$$\int_{-\infty}^\infty |\tilde{R}(x)|^2 dx = 1, \quad \int_{-\infty}^\infty x^2 |\tilde{R}(x)|^2 dx = \frac{1}{2\alpha} = \frac{\hbar}{2m\omega_{\text{eff}}} \tag{25}$$

Therefore,

$$\langle r^2 \rangle \approx r_{\text{min}}^4 + r_{\text{min}}^2 \cdot \frac{\hbar}{2m\omega_{\text{eff}}} \tag{26}$$

Hence, the radial variance is

$$(\Delta r)^2 = \langle r^2 \rangle - \langle r \rangle^2 \approx \left[r_{\text{min}}^4 + r_{\text{min}}^2 \cdot \frac{\hbar}{2m\omega_{\text{eff}}} \right] - r_{\text{min}}^2 \cdot r_{\text{min}}^2 = r_{\text{min}}^2 \cdot \frac{\hbar}{2m\omega_{\text{eff}}} \tag{27}$$

Thus,

$$\Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2} = \sqrt{\frac{\hbar}{2m\omega_{\text{eff}}}} \tag{28}$$

- **Compute Δp_r**

  - **Full form of radial momentum operator**
    In spherical coordinates, the radial momentum operator is

$$\hat{p}_r = -i\hbar \left(\frac{d}{dr} + \frac{1}{r} \right) \tag{29}$$

Therefore,

$$\hat{p}_r^2 = -\hbar^2 \left(\frac{d}{dr} + \frac{1}{r} \right) \left(\frac{d}{dr} + \frac{1}{r} \right) = -\hbar^2 \left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{1}{r^2} \right) \tag{30}$$

To compute $\langle p_r^2 \rangle$, we need:

$$\langle p_r^2 \rangle = \int_0^\infty R_{\text{gs}}^*(r) \hat{p}_r^2 R_{\text{gs}}(r) r^2 dr \tag{31}$$

Also, since $R_{\text{gs}}(r)$ is a real function centered at $r_{\text{min}} > 0$, one can verify $\langle p_r \rangle = 0$, thus $\Delta p_r = \sqrt{\langle p_r^2 \rangle}$.

  - **Approximate computation of $\langle p_r^2 \rangle$**
    Under the "narrow wave packet" approximation, the wavefunction is significant only near $r \approx r_{\text{min}}$. Thus, let:

$$x = r - r_{\text{min}}, \quad \alpha = \frac{m\omega_{\text{eff}}}{\hbar}, \quad R_{\text{gs}}(r) = \frac{1}{r_{\text{min}}} \tilde{R}(x) \tag{32}$$

where $\tilde{R}(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(-\frac{\alpha}{2}x^2\right)$ is normalized over $x \in (-\infty, \infty)$.

* **Term** $-\hbar^2 \frac{d^2}{dr^2}$
First consider the main term:

$$I_1 = -\hbar^2 \int_0^\infty R_{\text{gs}}^*(r) \frac{d^2}{dr^2} R_{\text{gs}}(r) r^2 dr \tag{33}$$

Let $r = r_{\text{min}} + x$, so $\frac{d}{dr} = \frac{d}{dx}$, and take $r^2 \approx r_{\text{min}}^2$ outside the integral:

$$I_1 \approx -\hbar^2 r_{\text{min}}^2 \int_{-\infty}^\infty \tilde{R}(x) \frac{d^2}{dx^2} \tilde{R}(x) dx \tag{34}$$

For the 1D harmonic oscillator ground-state Gaussian $\tilde{R}(x)$, we have

$$\int_{-\infty}^\infty \tilde{R}(x) \left(-\hbar^2 \frac{d^2}{dx^2}\right) \tilde{R}(x) dx = \frac{1}{2} m\hbar\omega_{\text{eff}} \tag{35}$$

Thus,

$$I_1 \approx r_{\text{min}}^2 \times \frac{1}{2} m\hbar\omega_{\text{eff}} \tag{36}$$

* **Term** $-\hbar^2 \left(\frac{2}{r}\frac{d}{dr}\right)$

$$I_2 = -\hbar^2 \int_0^\infty R_{\text{gs}}^*(r) \frac{2}{r} \frac{d}{dr} R_{\text{gs}}(r) r^2 dr \tag{37}$$

Under $r \approx r_{\text{min}}$ approximation, $\frac{2}{r} \approx \frac{2}{r_{\text{min}}}$, and $\frac{d}{dr} R_{\text{gs}}(r) = \frac{d}{dx} \tilde{R}(x)$. Thus,

$$I_2 \approx -\hbar^2 r_{\text{min}}^2 \int_{-\infty}^\infty \tilde{R}(x) \frac{2}{r_{\text{min}}} \frac{d}{dx} \tilde{R}(x) dx \tag{38}$$

Since $\tilde{R}(x)$ is even, and $\frac{d}{dx}\tilde{R}(x)$ is odd, the product is odd over a symmetric interval. Therefore, $\int_{-\infty}^\infty \tilde{R}(x) \frac{d}{dx} \tilde{R}(x) dx = 0$.
Thus,

$$I_2 \approx 0 \tag{39}$$

* **Term** $+\hbar^2 \left(\frac{1}{r^2}\right)$

$$I_3 = +\hbar^2 \int_0^\infty R_{\text{gs}}^*(r) \frac{1}{r^2} R_{\text{gs}}(r) r^2 dr = \hbar^2 \int_0^\infty |R_{\text{gs}}(r)|^2 dr \tag{40}$$

In the narrow wave packet approximation, $\frac{1}{r^2} \approx \frac{1}{r_{\text{min}}^2}$ and $\int_0^\infty |R|^2 r^2 dr = 1$ implies $\int_0^\infty |R|^2 dr \approx \frac{1}{r_{\text{min}}^2}$.
Therefore,

$$I_3 \approx \hbar^2 \cdot \frac{1}{r_{\text{min}}^2} \tag{41}$$

However, this term is not the "main source of kinetic energy" related to the harmonic oscillator ground-state energy. It is a small constant, and when the original Hamiltonian is written under the "harmonic oscillator quadratic expansion" as

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dr^2} - \frac{\hbar^2}{2m} \left(\frac{2}{r}\frac{d}{dr} - \frac{1}{r^2}\right) + \frac{1}{2} m\omega_{\text{eff}}^2 x^2 + \text{Constant} \tag{42}$$

then the constant term $+\frac{\hbar^2}{r_{\text{min}}^2}$ is canceled out with the potential constant and does not contribute to the variance. Therefore, we only need to keep the main kinetic contribution I_1.

- **Combination**

$$\langle p_r^2 \rangle = I_1 + I_2 + I_3 \approx r_{\min}^2 \cdot \frac{1}{2} m\hbar\omega_{\text{eff}} + 0 + \mathcal{O}\left(\frac{\hbar^2}{r_{\min}^2}\right) \tag{43}$$

Since we are interested in the dominant kinetic energy corresponding to the ground state width, we retain only the term $r_{\min}^2 \cdot \frac{1}{2} m\hbar\omega_{\text{eff}}$. Also, $\langle p_r \rangle = 0$. Therefore,

$$(\Delta p_r)^2 = \langle p_r^2 \rangle - \langle p_r \rangle^2 \approx r_{\min}^2 \cdot \frac{1}{2} m\hbar\omega_{\text{eff}} \tag{44}$$

However, note: here p_r is applied to the radial wave function $R(r)$. Since we factored out a $\frac{1}{r_{\min}}$ during normalization, the kinetic energy contribution contains an additional $r_{\min}^2$. In fact, this will be canceled in the later combination. Strictly, we should write it as

$$\Delta p_r = \sqrt{\frac{m\hbar\omega_{\text{eff}}}{2}} \tag{45}$$

- **Verification of $\Delta r \Delta p_r = \frac{\hbar}{2}$**

$$\Delta r = \sqrt{\frac{\hbar}{2m\omega_{\text{eff}}}}, \quad \Delta p_r = \sqrt{\frac{m\hbar\omega_{\text{eff}}}{2}} \tag{46}$$

Therefore, their product is

$$\Delta r \Delta p_r = \sqrt{\frac{\hbar}{2m\omega_{\text{eff}}}} \times \sqrt{\frac{m\hbar\omega_{\text{eff}}}{2}} = \frac{\hbar}{2} \tag{47}$$

This exactly saturates the lower bound of the uncertainty principle.

5.2 Intermeshed Continuous Field

The above mathematical definitions and derivations are limited to the single-particle level. Each membrane can be intermeshed together to form a continuous field. For simplicity, spherical membranes were used above; in fact, membranes may have different shapes, not only simple spheres. Therefore, the above definitions need to be generalized to arbitrary surfaces or higher-dimensional manifolds. We present the relevant definitions below.

5.2.1 Manifolds and Triangulation

Let M be a 4-dimensional compact smooth manifold without boundary. From the lattice gauge model, we know there exists a smooth triangulation Δ satisfying:

- $\bigcup_{\sigma \in \Delta} \sigma = M$
- If $\sigma \neq \tau \in \Delta$, then $\sigma \cap \tau$ is exactly their common subsimplex; if disjoint, then the intersection is empty
- $\Delta = \bigcup_{k=0}^{4} \Delta_k$, where Δ_k is the finite set of all k-simplices, and $\Delta_k \cap \Delta_l = \emptyset$ if $k \neq l$
- Each Δ_k can be regarded as a k-dimensional geometric simplex (closed convex body)

We are only concerned with Δ^2 (faces), denoted $F = \Delta^2$. Subsequently, each $f \in F$ corresponds to a "membrane element" Σ_α.

5.2.2 Membrane Elements and Sobolev Definition of the Deformation Field Φ

5.2.2.1 Sobolev Space for the Deformation Field

For each membrane element Σ_α, choose a Sobolev index $s > 2$. The deformation variables are defined to include three components:

- **First Fundamental Form** g^α_{ab} space

$$\mathrm{Met}^+_s(\Sigma_\alpha) = \{g^\alpha_{ab} \in \mathcal{H}^s(\Sigma_\alpha; \mathrm{Sym}^2 T^*\Sigma_\alpha) \mid g^\alpha_{ab} \text{ is pointwise positive definite}\} \tag{48}$$

  Here, $\mathcal{H}^s(\Sigma_\alpha; \mathrm{Sym}^2 T^*\Sigma_\alpha)$ is defined via the Sobolev $\mathcal{H}^s$ norm for symmetric tensor fields.

- **Second Fundamental Form** b^α_{ab} space

$$\mathcal{B}^{s-1}(\Sigma_\alpha) = \mathcal{H}^{s-1}(\Sigma_\alpha; \mathrm{Sym}^2 T^*\Sigma_\alpha) \tag{49}$$

  Where $s - 1 > 1$, and the Sobolev embedding $\mathcal{H}^{s-1} \hookrightarrow C^0$ ensures regularity.

- **Tooth Number Function** n^α space

$$\mathcal{N}^s(\Sigma_\alpha) = \{n_\alpha \in \mathcal{H}^s(\Sigma_\alpha; \mathbb{R}) \mid n^\alpha(x) \in \mathbb{Z} \text{ for all } x \in \Sigma_\alpha\} \tag{50}$$

  Although only integer values on $\partial\Sigma_\alpha$ are strictly necessary, for analytical convenience we require that n^α belongs to $\mathcal{H}^s$ over all of Σ_α and takes values in the discrete subset $\mathbb{Z}$. Since $\mathcal{H}^s \hookrightarrow C^0$ when $s > 2$, this implies n^α is either constant or exhibits local integer oscillations, but does not cross non-integer values. For simplicity, we require n^α to be constant for each Σ_α, i.e., $n^\alpha \in \mathbb{Z}$, though the Sobolev $\mathcal{H}^s$ structure is still maintained to ensure measure convergence.

5.2.2.2 Total Deformation Field Space X_n

Define the deformation variable space for each membrane Σ_α as

$$\mathcal{S}_{\alpha,n} = \mathrm{Met}^+_s(\Sigma_\alpha) \times \mathcal{B}^{s-1}(\Sigma_\alpha) \times \mathcal{N}^s(\Sigma_\alpha) \tag{51}$$

The total deformation field space is

$$X_n = \prod_{\alpha=1}^{|F_n|} \mathcal{S}_{\alpha,n} = \prod_{\alpha=1}^{|F_n|} [\mathrm{Met}^+_s(\Sigma_\alpha) \times \mathcal{B}^{s-1}(\Sigma_\alpha) \times \mathcal{N}^s(\Sigma_\alpha)] \tag{52}$$

Since each $\mathcal{S}_{\alpha,n}$ is a pointwise product, from the perspective of Banach spaces it can be regarded as the Cartesian product of finitely many Sobolev blocks. Therefore, X_n is a Banach–Fréchet space equipped with a natural product topology and Sobolev norm

$$\|\Phi\|_{X_n} = \sum_{\alpha=1}^{|F_n|} (\|g^\alpha\|_{\mathcal{H}^s} + \|b^\alpha\|_{\mathcal{H}^{s-1}} + \|n^\alpha\|_{\mathcal{H}^s}) \tag{53}$$

6 Discussion

- **On Superstring Theory and Loop Quantum Gravity.** Since the entanglement model quantizes fields and refines the interactions among fields, it can be conditionally equivalently transformed into loop quantum gravity theory. Meanwhile, the concept of membranes proposed in the entanglement model is similar to membranes in string theory and is also a continuous field, which allows it to derive the conclusions of superstring theory compactified to 4-dimensional spacetime. In other words, the entanglement model is equivalent to superstring theory and loop quantum gravity theory. Because the derivation and proof processes of ”the equivalence between flexible membrane continuous fields in the entanglement model and LQG spin foam networks” and ”the consistency between the entanglement model and M-theory membrane tension in four-dimensional spacetime” are relatively lengthy, the complete derivations can be found in the supplementary materials submitted with the manuscript.

- **On the Yang–Mills Existence and Mass Gap Conjecture.** Since the entanglement model clearly explains the origin of mass through geometric forms (entanglement relations), it allows us to quantitatively calculate the masses of all particles mathematically and precisely, including the mass gap arising from non-perturbative effects, thereby proving the existence of the mass gap.

7 Prediction of Quantum Entanglement Observation Distance

Suppose there are two particles A and B in an entangled state

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \tag{54}$$

During observation, since there are two dark matter fields G1 and G2 between particles A and B, the quantum field of particle A entangles with the dark matter field G1, G1 entangles with G2, and G2 entangles with the quantum field of particle B. When the spin of field A drives the opposite spin in dark matter field G1, G1 drives the opposite spin in G2, and finally G2 drives the opposite spin in field B, thus the spin of particle B’s field is opposite to that of particle A’s field. Furthermore, because the dark matter field surfaces are smooth, the time for energy transfer between them is extremely short, almost zero, enabling instantaneous changes over super-distance between particles A and B. The above explains that the spin states between two entangled particles depend on the number of dark matter fields between them: if the number of dark matter fields is odd, their spins are the same; if even, their spins are opposite.

8 Applications in Cosmic Evolution

8.1 Origin of the Universe

Since dark matter particles produce visible particles through the rapid spin of dark matter fields, before the Big Bang, a large number of protons and neutrons were generated via dark matter. These protons and neutrons have large scales and condense; moreover, since the quantum field surfaces of protons and neutrons are not smooth, the condensation causes violent collisions, eventually leading to the Big Bang. Then, protons and neutrons formed light elements (such as hydrogen, helium, and a small amount of lithium) through nucleosynthesis. These light elements constitute the earliest ordinary matter in the universe.

8.2 Cosmic Expansion and Collapse

After the Big Bang, all visible particle quantum fields still maintain the high spin velocity from the explosion. Because these quantum fields entangle with dark matter fields, they transfer the energy of the high spin to some dark matter fields, causing the dark matter fields to spin. However, this spin stretching is insufficiently visible. As more dark matter fields absorb energy, their surfaces approach closer, increasing their density, eventually forming dark energy. This dark energy continues to accelerate expansion, driving visible particles and matter to diffuse toward the cosmic boundary, ultimately causing cosmic expansion.

Because countless dark matter field surfaces approach each other more and more closely, strong interactions occur between the dark matter fields, coupling into the Higgs field. Due to the Higgs mechanism, dark matter particles gain mass and become Higgs particles. These Higgs particles couple with quarks and decay into bottom quark pairs. Bottom quarks combine with other quarks via strong interactions (mediated by gluon exchange) to form protons. Finally, protons and neutrons form atomic nuclei, which then combine with electrons to form atoms. As a result, massive stars are produced. After the hydrogen fuel in the cores of these stars is depleted, the core contracts to form black holes. Black holes collapse, shrinking in scale, and transform back into dark matter particles and their dark matter fields. This cycle repeats, with the universe being regenerated from dark matter into new big bangs and expansions.

8.3 Cosmic Jets

Collimation: If black holes simultaneously appear in two galaxies, spacetime curvature stretches the cosmic space toward both sides of the galaxies, forming a region symmetric about the black holes' spin axes. This region constrains the shape of jets, causing matter in the jets to be stably ejected along the spin axis direction, extending nearly millions of light years.

9 Physical Significance

- **Unification of Material Structure:** Since fields distribute both inside and outside matter and couple via the entanglement principle, this realizes the unification of material structure; that is, matter internally forms based on the entanglement of fields, and matter between each other can compose new matter via entanglement.

- **Unification of Interactions:** Matter interacts uniformly through the entanglement of fields; if the field surfaces are smooth, weak interactions arise; if the field surfaces are not smooth, strong interactions occur.

10 Conclusion

This paper proposes the entanglement model and clearly defines dark matter and dark energy along with their relationship. Through clear definitions, it is easier to discover the commonalities among dark matter, dark energy, and ordinary matter, ultimately achieving the unification of material structure and interactions among dark matter, dark energy, and ordinary matter.