RESEARCH STATEMENT

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I am fascinated by number theory, especially developing mathematical algorithms to solve problems in connections between number theory to harmonic analysis, algebraic geometry, complex dynamics, hyperbolic manifolds, thermodynamic formalism, Kleinian groups, and knot theory. I was fortunate to conduct some preliminary work at the junction of these academic fields during my senior year independent study on prime number theorem with Prof. Paul Zeitz and continuing research with Prof. Chun-Kit Lai for my master thesis. These initial steps, as well as my future plans, are explained here.

My Initial Steps Since I studied quantum field theory and statistical mechanics at National Taiwan University, I have been trying to figure out what is causing the distribution of non-trivial zeros in systems described by the Lee-Yang circle theorem and prime number theorem. Instead of applying Wilsonian renormalization group treatment in statistical field theory, the Lee-Yang circle theorem provides an alternate method for studying first-order phase transition. The exciting part is that for a finite ferromagnetic Ising model, zeros of the partition function of the system cluster around a unit circle. When we apply the thermodynamics limit to this system, the zeros shift to a unit circle on the complex plane. In other words, if we consider the vertical line $z = \frac{1}{2} + it$ to be a circle with infinite radius containing non-trivial zeros of Riemann zeta function, and the zeta function to be a partition function of a thermodynamic system, we can see the two systems share the same question: what are the fundamental rules that determine the distribution?

The development of a technique to precisely compute the critical exponent $\delta(G)$ of a Poincaré series of a geometrically finite non-elementary Fuchsian group G has a long history. Motivated by this curiosity, I began my master thesis on a generalized problem by investigating the line indicated by $z = \delta(G) + it$, where G is a Kleinian group. $\delta(G)$ plays a crucial role in the prime geodesic theorem, similar to the line $z = 1 + it$ in the prime number theorem where the Riemann zeta function has its only singularity at the point $z = 1$ and has no zeros on this line proved by Von Mangoldt in 1895. In Patterson-Sullivan theory, $\delta(G)$ is the Hausdorff dimension of the limit set of G. Furthermore, $\delta(G)$ uniquely determines the first resonance of the corresponding Selberg zeta function of G, i.e. $\lambda_1 = \delta(G)(1 - \delta(G))$, where λ_1 is the lowest eigenvalue of eigenfunctions of the Laplacian operated on a Riemann surface (algebraic curve) $\mathbb{H}^2/G$.

In my MA thesis, a geometric method was developed to compute $\delta(G)$ exactly by computing the lower bound and upper bound of $\delta(G)$, where G are some Schottky groups that satisfy a predefined set of conditions. Then, this might be the first time to derive an exact value of λ_1 of a specific Schottky group. Furthermore, after reading Ahlfors' lecture notes [1] and other related developments about the actions of Kleinian groups in higher dimensions, recently we are conjecturing and proving that this two-dimensional result is also true in some higher dimensions. Since $\delta(G)$ is the convergence exponent of Poincaré series of G, and this series can be written into a Dirichlet L-function, plus eigenfunctions of $\mathbb{H}^2/G$ are automorphic forms, hence an exact connection between number theory and harmonic analysis can be established. It also can be considered as an example that follows the philosophy of Langlands program that Dirichlet characters, algebraic curves, and automorphic forms are interrelated.

Future Plans Beyond the scope of my master's thesis, I would like to discover some generic norms for more general setups in order to better understand the distribution of non-trivial zeros, as well as provide more solid examples to lay the conceptual basis for linkages between mathematics and physics. Mathematics is the language of physics. To unify general relativity and quantum field theory, we might need a dictionary to translate different dialects physicists used in varied areas of mathematics to construct a rigid mathematical foundation. I am excited to see what Langlands Program can tell us about this unification challenge. Mathematics progresses as new objects to explore are discovered, as well as new structures that embody some of the most important relationships - those between geometry, topology, algebra, and analysis. Quantum field theory (QFT) provides both. Almost 300 years ago, Newton attempted to understand Kepler's planetary motion equations and establish a systematic means of thinking about infinitesimal change. This effort resulted in the creation of classical mechanics and calculus, which mathematics assimilated and improved. I aim to do something similar for QFT. This includes defining the essential features of QFT such that future mathematicians do not need to examine the physical context in which the theory evolved.

Not only does the critical exponent of the Poincaré series $\delta(G)$ plays an important role in phase transition of ferromagnetic Ising models, as described by quantum field theory, but it also has a crucial role in topological black holes, as described by the theory of general relativity. Furthermore, because Ising models are widely used in quantum computing, it would be intriguing to examine what consequences research in this area may have.

My long-term goal is to become a professor. Obtaining a PhD will allow me to continue my research while also acquiring more teaching experience. After earning my PhD, I hope to work in academia as a postdoctoral researcher or assistant professor, where I can combine my interests in research and teaching. I aim to be able to give back to Taiwan by inspiring students and developing their interest and enthusiasm for mathematics.

References

1. Lars Valerian Ahlfors, *Mobius transformations in several dimensions*, Lecture Notes at University of Minnesota (1981).