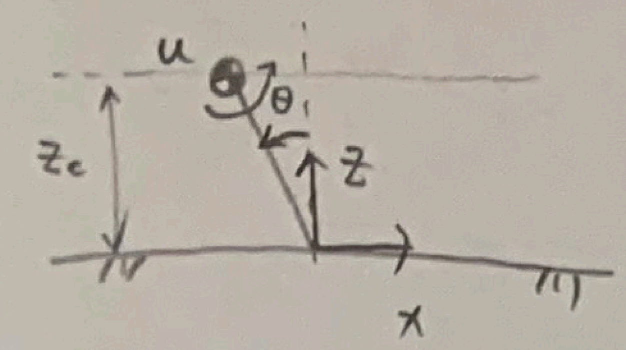
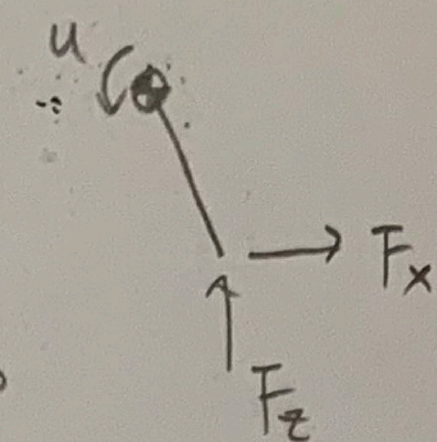
§ Model : LIP

• single stance



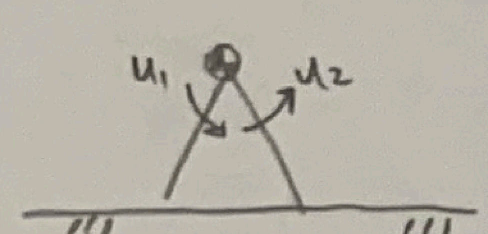
$$\begin{cases} \ddot{x} = \frac{1}{m} F_x \\ \ddot{z} = \frac{1}{m} F_z - g \\ u + F_x l \cos\theta + F_z l \sin\theta = 0 \end{cases}$$



$$\Rightarrow \ddot{x} = \frac{g}{z_c} x + \frac{1}{m z_c} u$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{g}{z_c} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \frac{1}{m z_c} u$$

• double stance



$$\ddot{x} = \frac{g}{z_c} x + \frac{1}{m z_c} (u_1 + u_2).$$

§ Trajectory.

Assume the trajectory in single stance is a cubic.

$$x(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

§ Koopman eq

common

$$\psi_y = (U^t)^T \psi_x$$

$$U^t = (\psi_x^T)^\dagger \psi_y^T$$

switch system with input

$$\psi_y = (U_x^t)^T \psi_x + (U_{xu}^t)^T \psi_{xu}$$

$$\psi_y^T = \psi_x^T U_x^t + \psi_{xu}^T U_{xu}^t$$

$$\begin{bmatrix} U_x^t \\ \hline U_{xu}^t \end{bmatrix} = [\psi_x^T \ \psi_{xu}^T]^\dagger \psi_y^T$$

Recall: $\hat{x} = \arg\min_x \|Ax - b\|_2$

$\hat{x} = A^\dagger b.$

Goal:

$$\arg\min_{U_x^t, U_{xu}^t} \| \psi_x^T U_x^t + \psi_{xu}^T U_{xu}^t - \psi_y^T \|_2$$

$$= \arg\min_{U_x^t, U_{xu}^t} \left\| \begin{bmatrix} \psi_x^T & \psi_{xu}^T \end{bmatrix} \begin{bmatrix} U_x^t \\ U_{xu}^t \end{bmatrix} - \psi_y^T \right\|_2$$

$$\therefore \begin{bmatrix} U_x^t \\ U_{xu}^t \end{bmatrix} = [\psi_x^T \ \psi_{xu}^T]^\dagger \psi_y^T$$