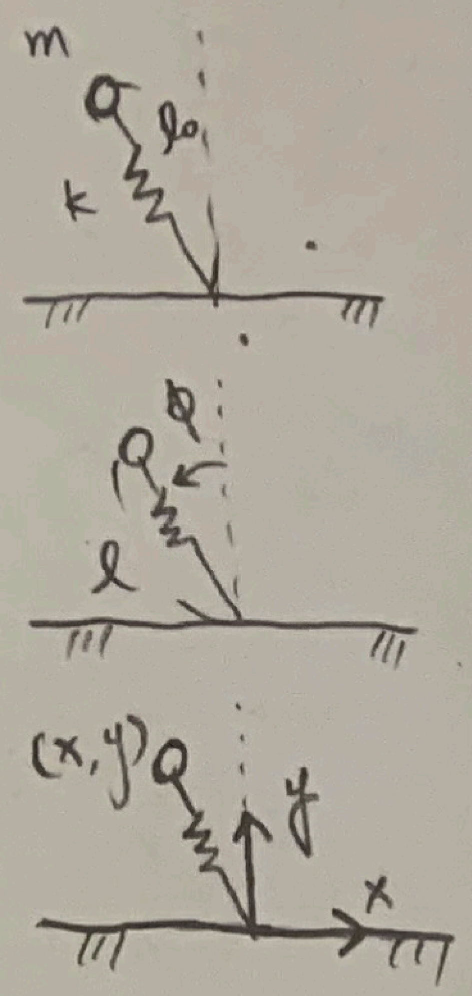
§ Step 1 : Generate data

1. Construct a simulated SLIP model



- parameter : m, k, l_0
  (I use $l_0 = 1$, $m = 60$, $k = 5600$ here)

- generalized coordinates : $q = \begin{bmatrix} \phi \\ l \end{bmatrix}$
  ∵ For the EOM in stance phase, it is easier to use ϕ and l.

- states : $\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$

Flight phase EOM :

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -g \end{bmatrix}$$

Stance phase EOM :

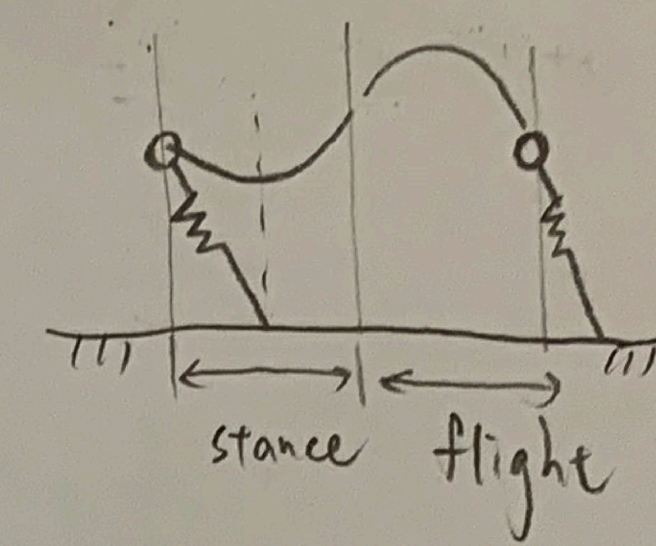
$$\begin{cases} x = -l \sin\phi \\ y = l \cos\phi \end{cases}$$

$$L = T - V = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + mg(l\cos\phi) - \frac{1}{2} k (l - l_0)^2$$

$$\text{EOM: } \frac{\partial}{\partial q}\left(\frac{\partial L}{\partial \dot{q}}\right) \ddot{q} + \frac{\partial}{\partial q}\left(\frac{\partial L}{\partial \dot{q}}\right) \dot{q} - \frac{\partial L}{\partial q} = 0.$$

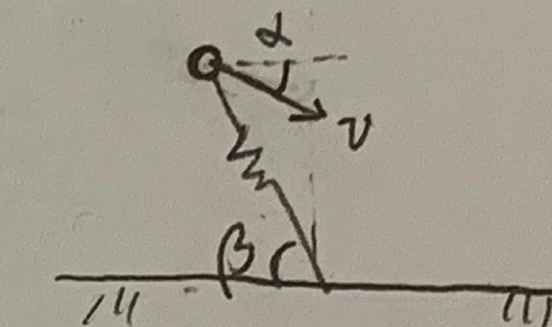
⇒ Using ode45 and EOMs, we can simulate the dynamics of SLIP model in stance and flight phase.

2. Collect data.



- "sucessful" one step = stance + flight.

touch down.

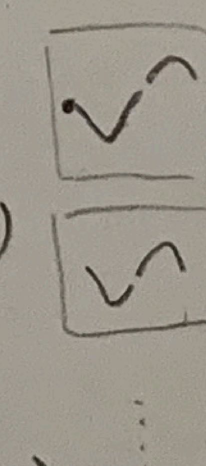


- I have tried $\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$ grid to describe touchdown condition (initail condition) but it is hard to find a range of "sucessful" one step. Thus, I use (v, α, β) to describe touch down condition so I can find a "sucessful range".

- Grid : $v = 1.5, 1.75, 2$
  $\alpha = 10°, 15°, 20°$
  β = proper range for each case

ex : As $v = 1.5$, $\alpha = 10°$
set β = linspace (63°, 70°, 5)
As $v = 1.5$, $\alpha = 15°$
set. β = linspace (64°, 71°, 5)

(v_1, α_1, β_1)
(v_2, α_2, β_2)
⋮
(v_n, α_n, β_n)
⋮



⇒ Dataset.

§ Step 2. Find Koopman operator

1. Overview.

From hybrid system paper, the switched system

$$x_{k+1} = f_{\lambda_k}(x_k, u_k)$$

$$\lambda_{k+1} = g(x_k, u_k, \lambda_k).$$

The Koopman operator representation is

$$\psi_x(x_{k+1}) = K_x^{\lambda_k}\,\psi_x(x_k) + K_{xu}^{\lambda_k}\,\psi_{xu}(x_k, u_k)$$

$$\lambda_{k+1} = g(x_k, u_k, \lambda_k).$$

Here assume passive SLIP so we don't have u_k. In the paper, they didn't talk about how they get function g so I just assume, in our case,

$$\lambda_{k+1} = g(x_k, \lambda_k) = \begin{cases} 0 & \text{if } \lambda_k = 1 \cap \sqrt{x_k(1)^2 + x_k(2)^2} > l_0 \\ 1 & \text{if } \lambda_k = 0 \cap x_k(2) < l_0 \cos\phi_0 \\ \lambda_k & \text{else.} \end{cases}$$

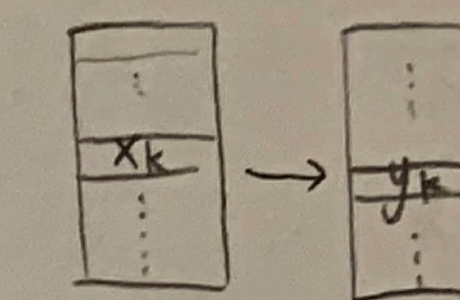
($\lambda_k = 1$ for stance, $\lambda_k = 0$ for flight).

Therefore the remain goal is to find $K_x^{\lambda_k=1}$ and $K_x^{\lambda_k=0}$.

2. Calculate Koopman operator. (Dan's paper).

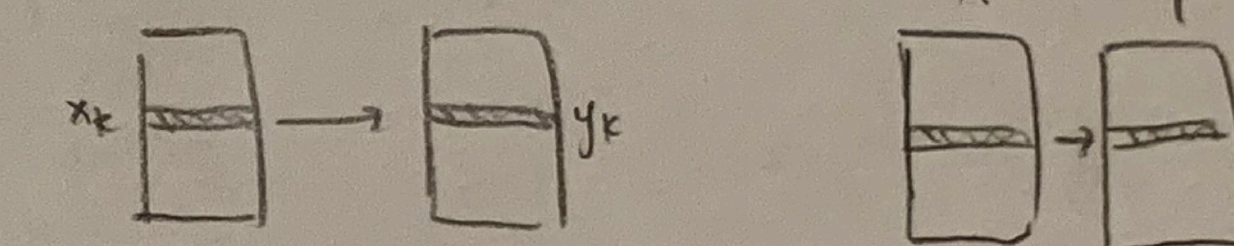
(1) arrange data into "snapshot pairs" $\{(x_k), (y_k)\}$

flow map: $x \xrightarrow{\Delta t} y$.



and seperate data by λ_k. So, we have

$X^{\lambda_k=1}$ $Y^{\lambda_k=1}$ $X^{\lambda_k=0}$ $Y^{\lambda_k=0}$



(2) lift data

$$\Psi_x^{\lambda_k=1} = \begin{bmatrix} \psi(x_1)^T \\ \psi(x_2)^T \\ \vdots \end{bmatrix}, \quad \Psi_y^{\lambda_k=1} = \begin{bmatrix} \psi(y_1)^T \\ \psi(y_2) \\ \vdots \end{bmatrix}$$

$$\Psi_x^{\lambda_k=0} = \begin{bmatrix} \psi(x_1)^T \\ \vdots \end{bmatrix}, \quad \Psi_y^{\lambda_k=0} = \begin{bmatrix} \psi(y_1)^T \\ \vdots \end{bmatrix}$$

(3) Koopman operator.

$$K_x^{\lambda_k=1} = \left(\Psi_x^{\lambda_k=1}\right)^{\dagger} \Psi_y^{\lambda_k=1}$$

$$K_x^{\lambda_k=0} = \left(\Psi_x^{\lambda_k=0}\right)^{\dagger} \Psi_y^{\lambda_k=0}$$

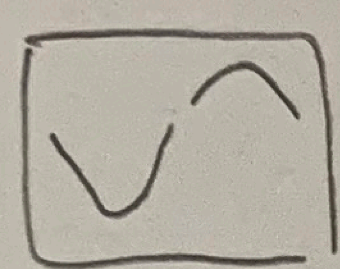
$$A^{\lambda_k=1} = \frac{1}{T_s} \log K_x^{\lambda_k=1}$$

$$A^{\lambda_k=0} = \frac{1}{T_s} \log K_x^{\lambda_k=0}$$

§ Step 3 : Estimate

1. Test data.

select an arbitrary touchdown condition that can sucessful for one step. Collect one-step dynamic data of simulated SLIP model as reference.

$(v, \alpha, \beta) \rightarrow$  $\rightarrow$ test reference

2. Predict by Koopman operator.

Given the states $\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$ at touchdown moment as initial states x_1. Let $x_{k+1} = F(x_k) = \left(\frac{\partial \psi(x)}{\partial x}\right)^{\dagger} (A^{\lambda_k})^T \psi(x_k)$.

Iteratively predict the one-step dynamics starting from x_1.

initial states x_1, $\lambda_1 = 1$

$\downarrow$

(x_k, λ_k)

$\boxed{x_{k+1} = F(x_k)}$ $\boxed{\lambda_{k+1} = g(x_k, \lambda_k)}$

$\downarrow$ $\downarrow$

(x_{k+1}, λ_{k+1})