$S = 52$ $\quad \sigma = .25$ $\quad \delta = .02$ $\quad n = 2$

$K = 50$ $\quad r = 0.05$ $\quad T = 1$ $\quad h = T$

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{(.05-.02)(.5) + .25\sqrt{\frac{1}{2}}}$$

$$= 1.2114$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{(.05-.02)(.5) - .25\sqrt{.5}}$$

$$= 0.8506$$

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = 0.4559$$

$S_{uu} = 76.3095$

$C_{uu} = 26.3095$

$S_u = 62.9928$

$C_u = 13.5990$

$S_0 = 52$

$C_0 = 6.8918$

$S_{ud} = 53.5817$

$C_{ud} = 3.5817$

$S_d = 44.2312$

$C_d = 1.5926$

$S_{dd} = 37.6231$

$C_{dd} = 0$

$$C_u = e^{-rh}\left[p^* C_{uu} + (1-p^*) C_{ud}\right]$$

$$= e^{-.05(.5)}\left[.4559(26.3095) + (1-.4559)(3.5817)\right]$$

$$\left[11.9945 + 1.9488\right]$$

$$= 0.9753$$

$= 13.5990$

$C_d = e^{-rh}\left[p^* C_{ud} + (1-p^*) C_{dd}\right]$

$= 0.9753\left[.4559(3.5817) + (1-.4559)(0)\right]$

$= 1.5926$

$C_0 = e^{-rh}\left[p^* C_u + (1-p^*) C_d\right]$

$= .9753\left[.4559(13.5990) + (1-.4559)(1.5926)\right]$

$= 6.8918$

$S_{uu} = 76.3095$
$P_{uu} = 0$

$S_u = 62.9928$
$P_u = 0$

$S_0 = 52$
$P_0 = 3.4854$

$S_{ud} = 53.5817$
$P_{ud} = 0$

$S_d = 44.2312$
$P_d = 6.5680$

$S_{dd} = 37.6231$
$P_{dd} = 12.3769$

$P_d = (e^{-rh})\left[(1-p^*)(P_{dd})\right]$

$= 6.5680$

$P_0 = e^{-rh}\left[(1-p^*)(6.5680)\right]$

$P_0 = 3.4854$

$$C - P = PV(F_{0,T} - K)$$

$$6.8918 - 3.4854 = e^{-.05(1)}\left(52e^{(.05-.02)(1)} - 50\right)$$

$$3.4064 = 3.4089$$

American Binomial Tree

$S = 62$ $\quad \sigma = 0.3$ $\quad \delta = 0.04$ $\quad n = 2$

$K = 60$ $\quad r = 0.03$ $\quad T = 1$ $\quad$ Call

$S_{uu} = 93.8219$

$C_{uu} = 33.8219$

$S_u = 76.2690$

$C_u = 16.2690$

$S_{ud} = 61.3831$

$C_{ud} = 1.3831$

$S_0 = 62$

$C_0 = 7.4984$

$S_d = 49.8991$

$C_d = 0.6093$

$S_{dd} = 40.1600$

$C_{dd} = 0$

$u = 1.2301$ $\quad d = 0.8048$ $\quad p^* = 0.4472$

$$C_u = e^{-rh}\left[(p^*)C_{uu} + (1 - p^*)C_{ud}\right]$$

$$= e^{-.03(.5)}\left[(.4472)(33.8219) + (1 - .4472)(1.3831)\right]$$

$= \cancel{15}.6520$ reject. exercise now.

When does early exercise make sense?

① European Call, no dividends

assume we own $C(K,t)$. At time t, what is the value of extending expiration to time T, $T > t$?

Steps to evaluate:

① State Put-Call Parity

$$C(K, T-t) - P(K, T-t) = PV(F_{t,T} - K)$$

② expand PV of fwd

$$\left[S_t \cancel{e^{(r-\delta)(T-t)}}\right] \cancel{e^{-r(T-t)}} = S_t$$

③ expand PV of strike

$$PV(K) = Ke^{-r(T-t)}$$

④ Restate P-C parity, move put to RHS

$$C(K, T-t) = S_t - Ke^{-r(T-t)} + P(K, T-t)$$

⑤ Add and subtract K from RHS

$$C(K, T-t) = S_t - Ke^{-r(T-t)} + P(K, T-t) + K - K$$

⑥ Rearrange

$$C(K, T-t) = S_t - K + K(1-e^{-r(T-t)}) + P(K, T-t)$$

7. Look at RHS.

   a. $S_t - K$ is the value of immediate exercise

   b. $K(1-e^{-r(T-t)})$ is interest accrued on the strike price between old and new expiration dates.

   c. $P(K, T-t)$ is the implicit insurance against the possibility that $S_T < K$.

Of these, b and c must be non-negative.

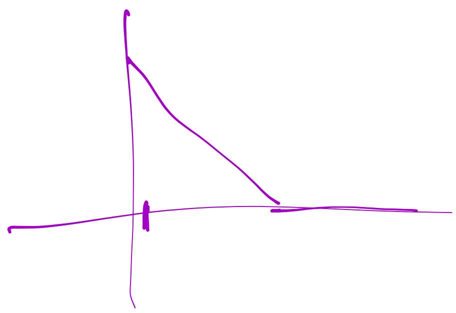
So $C(K, T-t) \geq S_t - K$, i.e. a longer call is always worth at least as much as a shorter one, on an asset w/ no dividends.

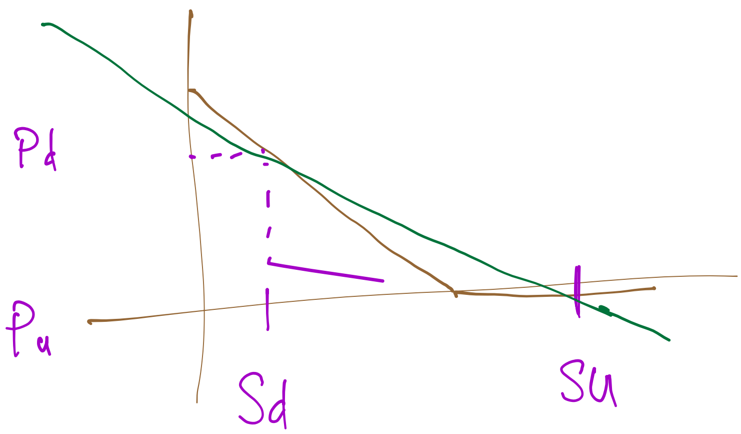
2. Call on an asset with dividends

It is possible that exercise just before the dividend is paid may be more valuable than a call expiring after the dividend.

3. European puts with longer terms can be less valuable than otherwise equal puts with shorter terms

if the asset value is so low that exercise is inevitable.





Early Exercise on American Option

① Call on non-dividend-~~paying asset~~ would never be exercised early.

② Puts can be exercised early if the asset value is ~~low enough to~~ ensure eventual exercise.

3) Calls on a dividend ~~-paying asset~~

$$C(K,T-t) = P(K,T-t) + S_t - K - \left[K\left(1-e^{-r(T-t)}\right) - S_t\left(1-e^{-\delta(T-t)}\right)\right]$$

How other option properties affect value.

1) Different Strike Prices

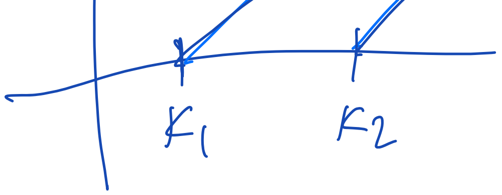
Assume we have strike prices $K_1 < K_2 < K_3$ with corresponding Calls and Puts $C(K_i), P(K_i)$

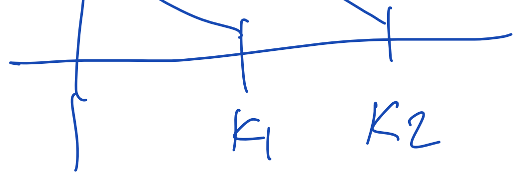
These conditions must hold:

Rule #1:

a) $C(K_1) \geq C(K_2)$

b) $P(K_2) \geq P(K_1)$





Rule #2:

a) $C(K_1) - C(K_2) \leq K_2 - K_1$

b) $P(K_2) - P(K_1) \leq K_2 - K_1$

Arbitrage: Suppose $C(45) - C(46) = 3.5 - 2$

$T = t$

Action	$t=0$	$S_T < 45$	$45 \leq S_T < 46$	$S_T \geq 46$
Sell $C(45)$	3.5	0	$45 - S_T$	$45 - S_T$
Buy $C(46)$	-2	0	0	$S_T - 46$
	1.5	0	$45 - S_T$	-1

must be between 0 and 1

must be less than FV(1.5)

Rule #3 "Convexity"

a) $$\frac{C(K_1) - C(K_2)}{K_2 - K_1} \geq \frac{C(K_2) - C(K_3)}{K_3 - K_2}$$

b) $$\frac{P(K_2) - P(K_1)}{K_2 - K_1} \leq \frac{P(K_3) - P(K_2)}{K_3 - K_2}$$

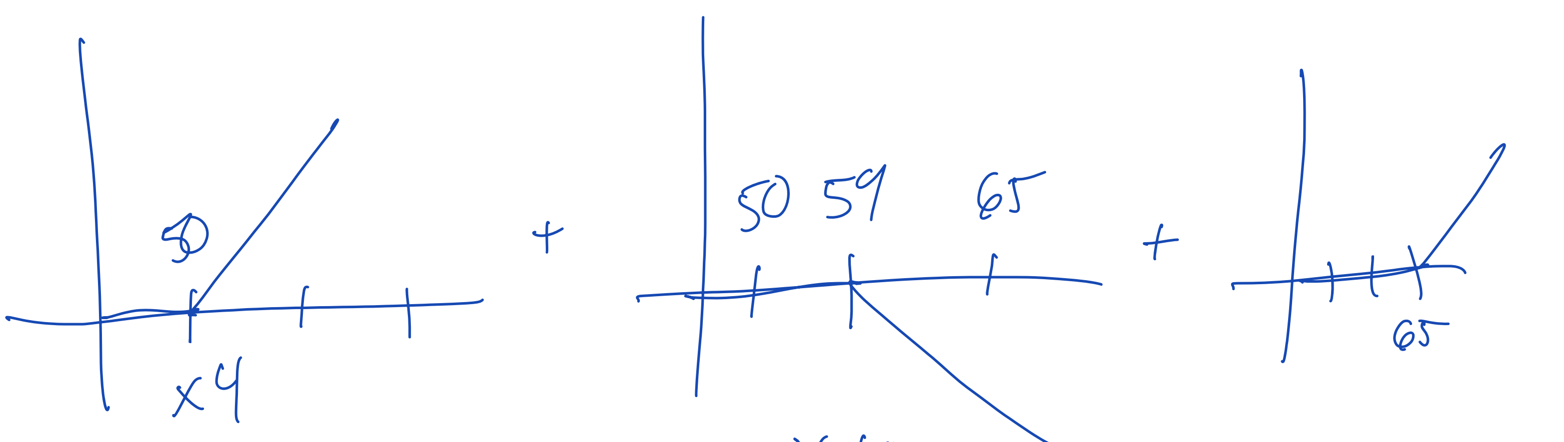
Arbitrage: Suppose:

$K_1 = 50$ $\qquad C(50) = 14$

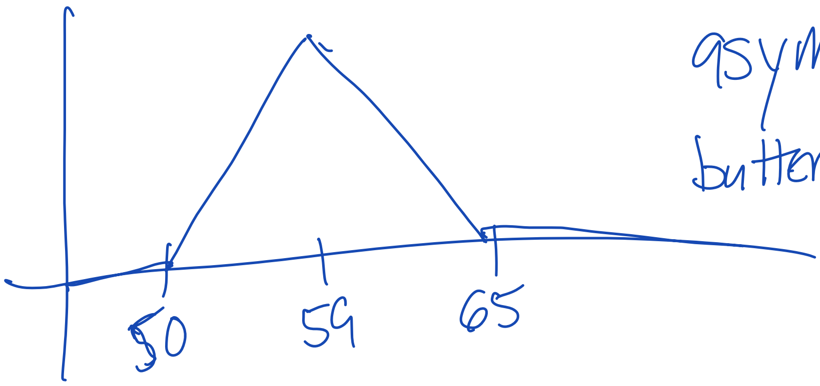
$K_2 = 59$ $\qquad C(59) = 8.9$

$K_3 = 65$ $\qquad C(65) = 5$

Action	$t=0$	$S_T < 50$	$50 \leq S_T < 59$	$59 \leq S_T < 65$	$65 \leq S_T$
Buy 4 $C(50)$	-4×14	0	$4(S_T - 50)$	$4(S_T - 50)$	$4(S_T - 50)$
Sell 10 $C(59)$	$+10 \times 8.9$	0	0	$10(59 - S_T)$	$10(59 - S_T)$
Buy 6 $C(65)$	-6×5	0	0	0	$6(S_T - 65)$
	3	0	$4(S_T - 50)$	$6(65 - S_T)$	0



×10 ×6



asymmetric butterfly spread

To construct: ① Sell one $C(K_2)$

② Buy $\left(\frac{K_2-K_1}{K_3-K_1}\right) C(K_3)$

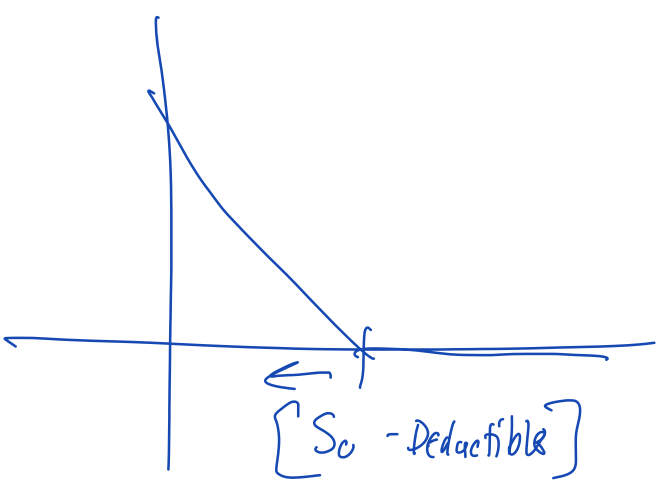
③ Buy $\left(\frac{K_3-K_2}{K_3-K_1}\right) C(K_1)$

$K_1 = 50 \quad K_2 = 59 \quad K_3 = 65$

① Sell $C(59)$ $\times 10 = 10$

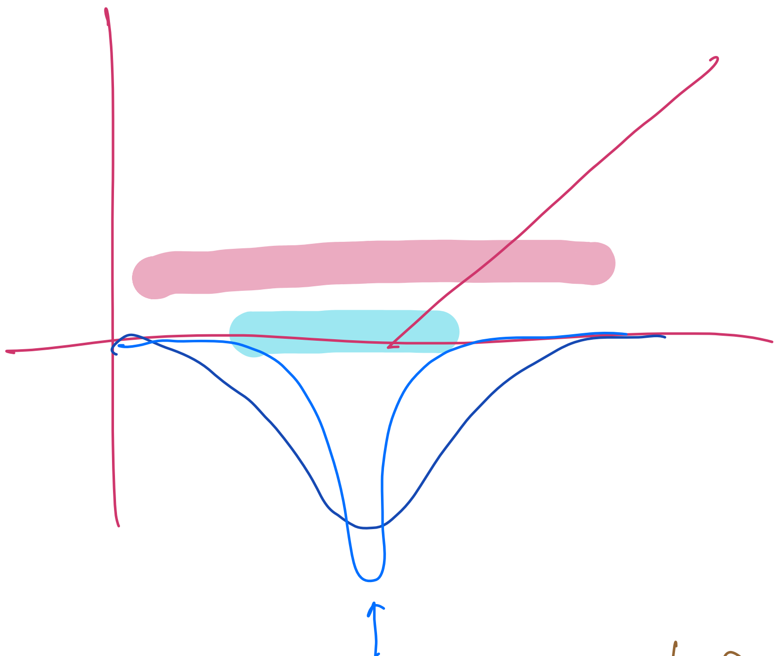
② Buy $\left(\frac{59-50}{65-50}\right) = \frac{3}{5}\, C(65)$ $\times 10 = 6$

③ Buy $\left(\frac{65-59}{65-50}\right) = \frac{2}{5}\, C(50)$ $\times 10 = 4$



Volatility

Std dev. of price changes



Higher volatility means higher payoff but not lower losses.

Interest Rates / Dividend Yield

These are essentially the same force, just acting on different assets.

An increase in the interest rate makes the strike price relatively more valuable as it stays invested.
A call allows us to delay purchase,

so is more valuable as interest rises.

Conversely, a larger dividend yield makes holding the asset more desirable, so the put, which allows deferred sale, becomes more valuable.