# Assignment



In this assignment, you will analyze the distribution of returns and a number of trading strategies. As part of the preliminary assignment, you have generated a weekly returns stock file for your stock and a weekly returns file for the S&P-500 (using the symbol "spy"). For both files, you have data for five years.

In this assignment, you will investigate some questions concerning weekly and monthly returns:

1. If you buy a stock for just one week, what is the best and worst week (on the average) of the month to do so? Recall that Weekly returns would always mean returns from 'Friday" close price to next Friday's "close" price.

2. What is the best and worst month to be invested in your stock and in S&P-500? Should you take a break from investing in the summer months?

3. comparison of returns if you have an "oracle" that can statistically predict the future behavior of your stock price for next week

We start with introducing some notation. Let $R = \{r_1, \ldots, r_n\}$ denote your weekly returns for your stock for $n$ weeks. The mean of the weekly returns

$$\mu(R) = \frac{r_1 + \cdots + r_n}{n}$$

To compute the standard deviation $\sigma(R)$ we can use

$$\sigma^2(R) = \frac{r_1^2 + \cdots + r_n^2}{n} - \mu^2(R)$$

Let us split the weekly returns $R$ into two sets

1. $R^-$: all negative weekly returns

2. $R^+$: all non-negative weekly returns

Finally, let $|R^-|$ denote the number of weeks with negative returns, and $|R^+|$ denote the number of weeks with non-negative returns.

## Question 1:

1. for each of the five years, compute the mean and standard deviation for the sets $R$, $R^-$ and $R^+$ of weekly returns for your stock for each week of the month

2. summarize your results in the table as shown below (5 tables total).

| Week | $\mu(R)$ | $\sigma(R)$ | $|R^-|$ | $\mu(R^-)$ | $\sigma(R^-)$ | $|R^+|$ | $\mu(R^+)$ | $\sigma(R^+)$ |
|---|---|---|---|---|---|---|---|---|
| Week 1 | | | | | | | | |
| Week 2 | | | | | | | | |
| Week 3 | | | | | | | | |
| Week 4 | | | | | | | | |

3. are there more weeks with negative or non-negative returns?

4. does your stock lose more on a "down" week than it gains on an "up" week.

5. are these results the same weeks of the month?

**Question 2:** Examine your 5 tables.

1. Are there any patterns across weeks in a month?

2. Are there any patterns across different years for the week in a month?

3. What are the best and worst weeks of the month to be invested for each year?

4. do these weeks change from year to year for your stock?

**Question 3:** Compute the aggregate table across all 5 years, one table for both your stock and one table for S&P-500 (using data for "spy").

1. What are the best and worst weeks of the month for each?

2. are these weeks the same for your stock as they are for S&P-500?

3. in 5 years, you have about 250 weeks. If weekly returns were normally distributed with mean $\mu$ and standard deviation $\sigma$, then 5% of these weeks will have returns $r$ with $r < \mu - 2\sigma$ or $r > \mu + 2\sigma$. For how many weeks do you have with weekly returns outside of $\mu \pm 2\sigma$ range (for your stock and for S&P-500)? Is it consistent with the normality of returns?

For the following questions, suppose that you have an **"oracle"**. This oracle knows all "True" labels for any stock for any week in the future. In particular, this oracle could tell you (statistically, of course) whether the price of any stock would increase or decrease the following week (Friday-to-Friday). Let $p$ be the probability that your oracle predicts the "True" label correctly for each day. Assume that you have no transaction costs.

To implement such an oracle, we do the following: given probability $p$, generate a random number $r$ in interval $[0, 1]$. If $r \leq p$ then Oracle gives the "True" label, and if $r > p$ then Oracle gives the opposite of the "True" label.

If $p = 1$, then Oracle gives you the correct "True" label for each week. Therefore, you will never lose if you "week trade" according to these labels. On the other extreme, if $p = 0$, then Oracle would tell you the opposite of "True" labels. In this case, you will lose on every week that you are invested. For the general case $0 < p < 1$, the oracle can only predict "True" labels statistically with probability $p$ of predicting the label correctly.

You start with \$100 on the first trading day of year 1 to trade your stock and another \$100 to trade "spy". We ignore transaction costs and any taxes.

**Question 4:** Let $p = 1$, and you listen to the oracle and follow its advice. How much money will you have on the last trading week of year 5:

1. your stock?

2. S&P-500 stock?

3. take the last two digits of your BU id and add this number to \$100. Assume that this is the desired amount. For

example, if your BU id ends on 47, your desired amount is \$100 + \$47 = \$147. If $p = 1$, then how many weeks would it take for your stock and for S&P-500 to get to that amount?

**Question 5:** Consider "buy-and-hold" strategy: you buy on the first trading day and sell on the last day. So you do not listen to your oracle at all. As before, assume that you start with \$100 for both your stock and "spy".

1. How much money will you have on the last trading day of year 5?

2. How do these results compare with results obtained in question 4?

3. Summer Vacation for your stock? You implement "buy-and-hold" except that you sell on the last trading day in May and buy on the last trading day in August. You do this every year. In other words, your stock takes a long 3-month summer vacation in each of 5 years. What are the final amounts for your stock and for S&P-500? Is this a good strategy? Summarize your findings in the table:

4. Monthly Vacation for your stock? For each month, you implement "buy-and-hold" except that you sell on the first trading day of that month and buy on the last trading day

| Strategy | Your Stock | S&P-500 |
|---|---|---|
| Buy-and-hold | | |
| Buy-and-hold with Summer Vacation | | |

of that month. You do this for that month every year. In other words, your stock takes a montly vacation. What are the final amounts for your stock and for S&P-500 for such a strategy? Summarize your findings in the table:

| Strategy | Your Stock | S&P-500 |
|---|---|---|
| Buy-and-hold (B&H) | | |
| B&H without January | | |
| B&H without February | | |
| ...... | ...... | ...... |
| ...... | ...... | ...... |
| B&H without December | | |

What is the best and worst month to vacation for your stock and for S&P-500?

**Question 6:** Compute and compare the final value of your portfolio for buy-and-hold strategy with Oracle for different values of $p = \{0, 0.1, 0.2, \ldots, 1\}$. Summarize the results in the table:

As before, assume that you start with $100 for both your stock and "spy".

| Strategy | Your Stock | S&P-500 |
|---|---|---|
| Buy-and-hold | | |
| p = 0 | | |
| p = 0.1 | | |
| p = 0.2 | | |
| ...... | ...... | ...... |
| p = 1.0 | | |

1. for what value of $p$, the results are comparable to buy-and-hold strategy. Is this value the same for your stock as it is for SP=500?

2. examine your table. Any patterns?

**Question 7:** You are given a choice of an oracle. You choose the perfect oracle with $p = 1$. On the way home, you got so excited that you dropped your oracle (clumsy you!!!). The oracle is now very angry with you. It decides to take revenge by giving you wrong advice from time to time. Unlike the general statistical oracle, this "angry" oracle decided to lie to you on the best and worst trading weeks. Specifically, let us consider the following three scenarios:

(a) the angry oracle gave you wrong results for the best ten weeks. In other words, you missed the best ten weeks, and your overall profit will be lower.

(b) the angry oracle gave you wrong results for the worst ten weeks days. In other words, you traded on the worst ten weeks, and your overall profit will be lower.

(c) The angry oracle gave you wrong results for the best five weeks and for the worst five weeks. In other words, you missed the best five trading weeks, and you were invested during the worst five weeks.

Please answer the following questions:

1. for each of the scenarios above (a,b and c), compute the final amount that you will have for both your stock and "spy". Summarize your results in the following table:

| Strategy | Your Stock | S&P-500 |
|---|---|---|
| Buy-and-hold (B&H) | | |
| Your "True" labels | | |
| p = 1 (never lose) | | |
| missed 10 best weeks | | |
| invested during 10 worst weeks | | |
| missed 5 best weeks and invested during 5 worst weeks | | |

2. do you lose more by missing the best weeks or by being invested on the worst weeks?

3. for each part (a),(b), and (c), what are your estimates for the probability $p$ if you used a statistical oracle instead of an angry one?