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Example: $p(x) = \text{const.} = k$

$$C_n = \frac{-2}{L}\int_0^L \frac{(B-A)}{L}\, x \sin\left(\frac{n\pi}{L}x\right) dx + \frac{2}{L}\int_0^L (k-A) \sin\left(\frac{n\pi}{L}x\right) dx$$

Note that: $\int_a^b x\sin(\alpha x)\,dx = \frac{1}{\alpha} x\cos(\alpha x)\Big|_b^a + \frac{1}{\alpha^2}\sin(\alpha x)\Big|_a^b$

$$= \frac{-2(B-A)}{L^2}\left\{\frac{L}{n\pi}\left[-L\cos(n\pi)\right] + \left(\frac{L}{n\pi}\right)^2 \cdot \underbrace{\sin(n\pi)}_{=0}\right\}$$

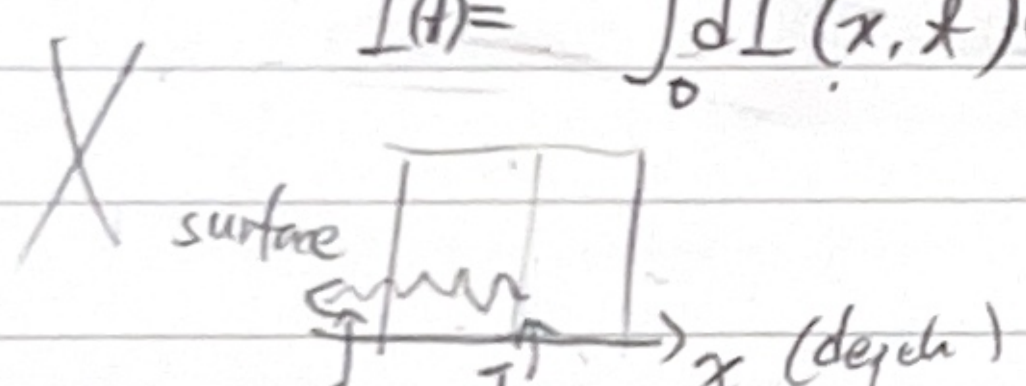
$$+ \frac{2}{L}(k-A)\left(\frac{-L}{n\pi}\right)\left[\cos(n\pi) - 1\right] \quad ; \quad \cos(n\pi) = (-1)^n$$

$$= \frac{2(B-A)}{n\pi}(-1)^n + \frac{2(k-A)}{n\pi}\left[1-(-1)^n\right]$$

$$= \frac{2(k-A)}{n\pi} + \frac{2}{n\pi}(-1)^n\left[B-k\right] \overset{\text{or}}{=} \frac{2}{n\pi}\left[(k-A) - (-1)^n(k-B)\right]$$

* XPS intensity over time

$I(t) = \int_0^L dI(x,t)\,dx$ $dI(x,t)$ = e^- intensity from depth x



Assume $dI(x,t) = u(x,t) \times P$

$\Rightarrow$ Homogeneous Dirishlet boundary condition

General sol.

$$u(x,t) = \left(\frac{B-A}{L}\right)x + A + \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{L}x\right) e^{-\frac{\pi^2 n^2}{L^2}Dt} \; ; \; C_n = \frac{2}{L}\int_0^L \left[p(x) - \left(\frac{B-A}{L}\right)x - A\right]\sin\left(\frac{n\pi}{L}x\right)dx$$

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$$\int_a^b x\sin(\alpha x)\,dx = -\frac{1}{\alpha}x\cos(\alpha x)\Big|_a^b + \frac{1}{\alpha^2}\sin(\alpha x)\Big|_a^b$$

$x \quad + \quad \sin(\alpha x)$

$1 \quad - \quad -\frac{1}{\alpha}\cos(\alpha x)$

$0 \quad \pm \quad -\frac{1}{\alpha^2}\sin(\alpha x)$

$$\int_a^b x e^{-rx}\,dx$$

$$= -\frac{1}{r}e^{-rx}\Big|_a^b - \frac{1}{r^2}e^{-rx}\Big|_a^b$$

$$= \frac{e^{-rx}}{r}\left(1+\frac{1}{r}\right)\Big|_b^a$$

$x \quad + \quad e^{-rx}$

$1 \quad - \quad -\frac{1}{r}e^{-rx}$

$0 \quad + \quad \frac{1}{r^2}e^{-rx}$

$-\frac{1}{\alpha}\cos(\alpha x)\Big|_a^b$

$\sin(x) \quad + \quad e^{-rx}$

$\alpha\cos(x) \quad - \quad \frac{1}{r}e^{-rx}$

$-\alpha^2\sin(x) \quad \pm \quad \frac{1}{r^2}e^{-rx}$

$1 \quad + \quad \sin(\alpha x)$

$0 \quad - \quad -\frac{1}{\alpha}\cos(x)$

$$\int_a^b \sin(\alpha x)e^{-rx}\,dx$$

$$= -\frac{1}{r}\sin(\alpha x)e^{-rx}\Big|_a^b - \frac{\alpha}{r^2}\cos(\alpha x)e^{-rx}\Big|_a^b$$

$$-\frac{\alpha^2}{r^2}\int_a^b \sin(x)e^{-rx}\,dx$$

$x \sin(\alpha x)$

$\alpha = \frac{n\pi}{L}$

$$\int_a^b \sin(\alpha x)e^{-rx}\,dx\left[1+\frac{\alpha^2}{r^2}\right] = \frac{e^{-rx}}{r}\left[\sin(\alpha x)+\frac{\alpha}{r}\cos(\alpha x)\right]\Big|_b^a$$

$x \quad + \quad \sin(\alpha x)$

$1 \quad - \quad -\frac{1}{\alpha}\cos(\alpha x)$

$0 \quad + \quad -\frac{1}{\alpha^2}\sin(\alpha x)$

$$\boxed{\int_a^b e^{-rx}\,dx = \frac{e^{-rx}}{r}\Big|_b^a}$$

B

A

$$\int_0^L x\sin(\alpha x) = -\frac{x}{\alpha}\cos(\alpha x)\Big|_0^L + \frac{1}{\alpha^2}\sin(\alpha x)\Big|_0^L$$

$\frac{(A+B)L}{2}$

$$= \frac{1}{2i}\left[\frac{1}{i\alpha-\beta}\left(e^{(i\alpha-\beta)L}-1\right) + \frac{1}{i\alpha+\beta}\left(e^{(-i\alpha-\beta)L}-1\right)\right]$$

$i\alpha+\beta$

$= \frac{L}{n\pi}$

$\left(\frac{L}{n\pi}\right)^2$

$$\int_0^L \frac{1}{2i}\left(e^{i\alpha x}-e^{-i\alpha x}\right)e^{-\beta x}\,dx$$

$-\frac{L^2}{n\pi}\cos(n\pi)$

$= \frac{L^2}{n\pi}(-1)^{n+1}$

$$\frac{1}{2i}\int e^{(i\alpha-\beta)x}\,dx - \frac{1}{2i}\int e^{(-i\alpha-\beta)x}\,dx$$

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$$= \frac{1}{2i}\frac{1}{i\alpha-\beta}e^{(i\alpha-\beta)x}\Big|_0^L + \frac{1}{2i}\frac{1}{i\alpha+\beta}e^{(-i\alpha-\beta)x}\Big|_0^L$$