MATH 540 2021 PROJECT 1
DUE IN CLASS OCTOBER 21

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In this project you will look at solving the Allen-Cahn equation

$$u_t = \epsilon u_{xx} + u - u^3, \quad a < x < b. \tag{1}$$

$\epsilon > 0$ is a user specified parameter.

(1) Write a Julia module for solving (1) which supports:
   (a) Sparse finite difference discretizations;
   (b) Periodic and Neumann boundary conditions;
   (c) Backwards Euler and Crank-Nicolson time stepping.

   It should also support user defined values for
   (a) The domain (a, b);
   (b) ϵ, the parameter;
   (c) n_x, determining the mesh;
   (d) Δt, the time step, and n_t, the number of time steps;

   Within a Jupyter notebook, demonstrate that calling these functions for construction, assembly, and running the problem work correctly.

(2) The uniform state, $u = 1$, is an exact solution to this with both periodic boundary conditions and homogeneous boundary conditions. Build a test unit for your module for all combinations of spatial discretizations and time stepping and boundary conditions. For the test unit use $\epsilon = 0.1$, $(a, b) = (0, 1)$, $\Delta x = 0.1$, $\Delta t = 0.1$ and $n_t = 100$. The discretization at the end should be, within floating point, $u(x, t_{\max}) = 1$ with $t_{\max} = 1$ for convenience.

   You should report your test results in a Jupyter notebook with the code:

```
using Pkg
Pkg.test("AllenCahn")
```

   in a Jupyter cell.

(3) Focusing on the case of periodic boundary conditions, with $\epsilon = 0.1$ and $(a, b) = (0, 1)$, assess the convergence in space and time for the problem, checking both spatial discretizations and both time stepping schemes. Choose Δx and Δt (independently) to compare against a high resolution solution for the initial condition $u_0 = \sin(2\pi x)$. Take $t_{\max} = 1$ here, for convenience.

Date: October 7, 2021.

What convergence rates do you see in the different cases? Are these what you would anticipate?

Visualize the results in your Jupyter notebook.

(4) Picking reasonable values of Δx and Δt, with $\epsilon = 0.1$, $(a, b) = (0, 1)$, and $t_{\max} = 10$, integrate $u_0 = \sin(4\pi x)$. Visualize the result on a space-time plot using `contourf`; it should look vaguely like the figure on the cover of Lord, Powell, & Shardlow.

Your results should be submitted with a link to the GitHub repository in which you developed the code. Enclosed in the repository, in a folder separate from `src` and `test`, you should have a Jupyter notebook showing the above results. A portion of your grade will be based on seeing that you effectively made use of git during development.