The Kinetic Stress Index: A Distilled Mathematical Framework

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1 I. Model Setup

1.1 Parameters and Asset Universe

The model uses $n = 4$ assets, a correlation window of $w = 60$ days, and a Mahalanobis lookback of $l = 252$ days. While the framework is general, Section IV details an optimal asset selection.

2 II. State Vector Construction ($\mathbf{s}_t$)

2.1 Pairwise Kinetics

This involves the six unique asset pairs (i, j), where $(i, j) \in \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$.

1. **Log-returns:** $r_{i,t} = \ln(P_{i,t}/P_{i,t-1})$ (or simple differences for yields).
2. **Spearman Correlation ($\rho_{ij,t}$):** The rank correlation is computed over the window w.

$$\rho_{ij,t} = \text{rank-corr}(r_{i,t-w+1:t}, r_{j,t-w+1:t})$$

3. **Position ($\boldsymbol{\theta}_t \in \mathbb{R}^6$):** Fisher z-transform is applied to each $\rho_{ij,t}$, where $\theta_{ij,t} \in \mathbb{R}$.

$$\theta_{ij,t} = \tanh^{-1}(\rho_{ij,t})$$

The vector is ordered explicitly:

$$\boldsymbol{\theta}_t = [\theta_{12,t}, \theta_{13,t}, \theta_{14,t}, \theta_{23,t}, \theta_{24,t}, \theta_{34,t}]^\top$$

4. **Velocity ($\mathbf{v}_t \in \mathbb{R}^6$) and Acceleration ($\mathbf{a}_t \in \mathbb{R}^6$):**

$$\mathbf{v}_t = \boldsymbol{\theta}_t - \boldsymbol{\theta}_{t-1} \quad ; \quad \mathbf{a}_t = \mathbf{v}_t - \mathbf{v}_{t-1}$$

2.2 Global Dynamics

1. **Maximum Eigenvalue ($\lambda_{\max,t}$):** This is extracted from the full 4×4 asset correlation matrix, $\mathbf{R}_t$.

$$\mathbf{R}_t = \begin{pmatrix} 1 & \rho_{12,t} & \rho_{13,t} & \rho_{14,t} \\ \rho_{12,t} & 1 & \rho_{23,t} & \rho_{24,t} \\ \rho_{13,t} & \rho_{23,t} & 1 & \rho_{34,t} \\ \rho_{14,t} & \rho_{24,t} & \rho_{34,t} & 1 \end{pmatrix}$$

The largest eigenvalue measures the strength of the dominant, collective mode of correlation in the system.

$$\lambda_{\max,t} = \max(\text{eig}(\mathbf{R}_t))$$

2. **Eigenvalue Velocity ($\Delta\lambda_{\max,t}$):** This is the day-over-day change in the maximum eigenvalue.

$$\Delta\lambda_{\max,t} = \lambda_{\max,t} - \lambda_{\max,t-1}$$

2.3 State Vector Synthesis

The components are concatenated into the final state vector $\mathbf{s}_t \in \mathbb{R}^{20}$.

$$\mathbf{s}_t = \begin{pmatrix} \boldsymbol{\theta}_t \\ \mathbf{v}_t \\ \mathbf{a}_t \\ \lambda_{\max,t} \\ \Delta\lambda_{\max,t} \end{pmatrix}$$

3 III. KSI Calculation and Decomposition

3.1 Adaptive Baseline Estimation

The baseline mean and covariance are estimated over the lookback window l.

$$\boldsymbol{\mu}_{t-1} = \frac{1}{l}\sum_{k=1}^{l} \mathbf{s}_{t-k}$$

The covariance matrix $\boldsymbol{\Sigma}_{t-1}$ is a robust, shrunk estimator (e.g., using Geometric-Inverse Shrinkage or Ledoit-Wolf). A simplified representation is:

$$\boldsymbol{\Sigma}_{t-1} = (1-\alpha)\mathbf{S}_{t-1} + \alpha\mathbf{T}$$

where $\mathbf{S}_{t-1}$ is the sample covariance, $\mathbf{T}$ is a shrinkage target, and $\alpha \in [0,1]$ is the shrinkage intensity.

$$\mathbf{S}_{t-1} = \frac{1}{l-1}\sum_{k=1}^{l}(\mathbf{s}_{t-k} - \boldsymbol{\mu}_{t-1})(\mathbf{s}_{t-k} - \boldsymbol{\mu}_{t-1})^\top \quad ; \quad \mathbf{T} = \mathrm{diag}(\mathrm{diag}(\mathbf{S}_{t-1}))$$

3.2 Kinetic Stress Index (KSI)

The KSI is the Mahalanobis distance between the current state vector $\mathbf{s}_t$ and its historical mean $\boldsymbol{\mu}_{t-1}$.

$$\mathrm{KSI}_t = \sqrt{(\mathbf{s}_t - \boldsymbol{\mu}_{t-1})^\top \boldsymbol{\Sigma}_{t-1}^{-1} (\mathbf{s}_t - \boldsymbol{\mu}_{t-1})}$$

Simple 2D Mahalanobis Example: Consider data with mean $\boldsymbol{\mu} = \left(\begin{smallmatrix}0\\0\end{smallmatrix}\right)$ and covariance $\boldsymbol{\Sigma} = \left(\begin{smallmatrix}4&0\\0&1\end{smallmatrix}\right)$. Two points $\mathbf{A} = \left(\begin{smallmatrix}2\\0\end{smallmatrix}\right)$ and $\mathbf{B} = \left(\begin{smallmatrix}0\\2\end{smallmatrix}\right)$ have the same Euclidean distance (2) from the centre, but different Mahalanobis distances:

$$D_M(\mathbf{A}) = \sqrt{\left(\begin{smallmatrix}2&0\end{smallmatrix}\right)\begin{pmatrix}1/4&0\\0&1\end{pmatrix}\left(\begin{smallmatrix}2\\0\end{smallmatrix}\right)} = \sqrt{1} = 1 \tag{1}$$

$$D_M(\mathbf{B}) = \sqrt{\left(\begin{smallmatrix}0&2\end{smallmatrix}\right)\begin{pmatrix}1/4&0\\0&1\end{pmatrix}\left(\begin{smallmatrix}0\\2\end{smallmatrix}\right)} = \sqrt{4} = 2 \tag{2}$$

Point $\mathbf{B}$ is "more surprising" because it deviates along the direction with smaller historical variance.

Decomposition via Eigendecomposition

The squared KSI can be perfectly decomposed by re-expressing the problem in the natural coordinate system of the covariance matrix.

1. **The Eigenvector Equation:** The eigenvectors $\mathbf{v}_j$ and eigenvalues ℓ_j of $\mathbf{\Sigma}_{t-1}$ define the principal axes of the data's variance.

$$\mathbf{\Sigma}_{t-1}\mathbf{v}_j = \ell_j \mathbf{v}_j, \quad \text{for } j = 1, \ldots, 20 \quad (\text{where } \ell_1 \geq \ell_2 \geq \cdots \geq \ell_{20} > 0)$$

2. **Matrix Decomposition:** The covariance matrix can be expressed using its eigenvectors ($\mathbf{V}$) and eigenvalues ($\mathbf{L}$).

$$\mathbf{\Sigma}_{t-1} = \mathbf{V}\mathbf{L}\mathbf{V}^\top, \quad \text{where } \mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_{20}], \quad \mathbf{L} = \operatorname{diag}(\ell_1, \ldots, \ell_{20})$$

3. **Projection:** Project the raw deviation vector onto each eigenvector direction $\mathbf{v}_j$ to find the magnitude of deviation along that mode.

$$y_j = \mathbf{v}_j^\top(\mathbf{s}_t - \boldsymbol{\mu}_{t-1}) \quad \Longrightarrow \quad \mathbf{y} = \mathbf{V}^\top(\mathbf{s}_t - \boldsymbol{\mu}_{t-1})$$

4. **Substitution and Simplification:** Substitute the decomposed inverse, $\mathbf{\Sigma}_{t-1}^{-1} = \mathbf{V}\mathbf{L}^{-1}\mathbf{V}^\top$, into the KSI^2 formula.

$$\begin{aligned}
\mathrm{KSI}_t^2 &= (\mathbf{s}_t - \boldsymbol{\mu}_{t-1})^\top(\mathbf{V}\mathbf{L}^{-1}\mathbf{V}^\top)(\mathbf{s}_t - \boldsymbol{\mu}_{t-1}) && (3)\\
&= ((\mathbf{s}_t - \boldsymbol{\mu}_{t-1})^\top\mathbf{V})\,\mathbf{L}^{-1}\,(\mathbf{V}^\top(\mathbf{s}_t - \boldsymbol{\mu}_{t-1})) && (4)\\
&= (\mathbf{V}^\top(\mathbf{s}_t - \boldsymbol{\mu}_{t-1}))^\top\,\mathbf{L}^{-1}\,(\mathbf{V}^\top(\mathbf{s}_t - \boldsymbol{\mu}_{t-1})) && (5)\\
&= \mathbf{y}^\top\mathbf{L}^{-1}\mathbf{y} = \sum_{j=1}^{20}\frac{y_j^2}{\ell_j} && (6)
\end{aligned}$$

Interpretation: The contribution of each independent mode, $\mathrm{Contrib}_j$, to the total squared stress is the ratio:

$$\mathrm{Contrib}_j = \frac{y_j^2}{\ell_j} = \frac{(\text{Deviation along mode } j)^2}{\text{Typical variance of mode } j}$$

4 IV. Application: The Orthogonal Tension Matrix and a Concrete Trading Signal

To demonstrate the maximum power of the KSI model, we move from a generic asset universe to a specific, mathematically optimized quartet: the **Orthogonal Tension Matrix**. This selection is designed to maximize the signal clarity derived from the kinetic decomposition.

4.1 The Optimal Asset Universe

The universe is chosen to create two, internally-correlated pairs that are orthogonal (uncorrelated) to each other during normal market regimes.

- **Pair A: The "Risk-On" Axis** ($i = 1, 2$)
  - Asset 1: High-Yield Corporate Bonds ('HYG' ETF)
  - Asset 2: AUD/JPY Exchange Rate ('AUDJPY' Curncy)
- **Pair B: The "Safe-Haven" Axis** ($i = 3, 4$)
  - Asset 3: Long-Duration US Treasuries ('TLT' ETF)
  - Asset 4: CHF/JPY Exchange Rate ('CHFJPY' Curncy)

4.2 Regime-Dependent Correlation Structure

The power of this setup lies in the stark contrast between its correlation matrix, $\mathbf{R}_t$, in two distinct states.

4.2.1 Normal Regime: An Orthogonal, Multi-Factor World

In stable markets, the correlation matrix exhibits a **block-diagonal structure**:

$$\mathbf{R}_{\text{normal}} \approx \begin{pmatrix} 1 & \rho_{12} & \epsilon_{13} & \epsilon_{14} \\ \rho_{12} & 1 & \epsilon_{23} & \epsilon_{24} \\ \epsilon_{13} & \epsilon_{23} & 1 & \rho_{34} \\ \epsilon_{14} & \epsilon_{24} & \rho_{34} & 1 \end{pmatrix} \quad \text{where} \quad \begin{cases} \rho_{12} \approx +0.7 & \text{(High positive correlation)} \\ \rho_{34} \approx +0.4 & \text{(Mild positive correlation)} \\ \epsilon_{ij} \approx 0 & \text{(Near-zero cross-correlation)} \end{cases}$$

The system's variance is explained by at least two significant, independent factors (eigenvectors). The leading eigenvalue, $\lambda_{\max}$, is relatively low, and the kinetic components of the state vector $\mathbf{s}_t$ (velocity $\mathbf{v}_t$, acceleration $\mathbf{a}_t$) are minimal.

4.2.2 Crisis Regime: A Single-Factor World

During a systemic crisis (e.g., a liquidity shock), the orthogonal structure collapses. The "cross-pair" correlations, previously near-zero, become dominant. For example, the correlation between high-yield bonds and treasuries (ρ_{13}) can spike from ~ 0 to a large negative value as panic ensues.

$$\mathbf{R}_{\text{crisis}} = \text{A dense matrix where previously zero correlations become significant.}$$

This structural collapse causes the leading eigenvalue $\lambda_{\max}$ to surge as the system's behavior becomes dominated by a single "panic" factor.

4.3 The Precise Trading Signal via Kinetic Decomposition

A high KSI_t value alone is insufficient. The actionable trading signal—which we term a **"Kinetic Cross-Contamination Event"**—is identified when a peak in KSI_t is primarily driven by the *acceleration* of the previously dormant cross-pair correlations.

Using the decomposition $\text{KSI}_t^2 = \sum_{j=1}^{20} y_j^2/\ell_j$, we look for peaks where the dominant contributions come from modes ($\mathbf{v}_j$) that are heavily loaded on specific kinetic components of the state vector.

The Signal Condition: A trade is triggered if a KSI peak (> 95th percentile) meets these criteria:

1. **Primary Driver ($>50\%$ of KSI^2 contribution):** The stress must be dominated by modes whose largest eigenvector loadings correspond to the **acceleration of the cross-pair correlations**. These are the state vector components tracking the second derivative of correlations like $\rho_{13}, \rho_{14}, \rho_{23}, \rho_{24}$.

$$\text{High Contribution from modes } j \text{ where } \mathbf{v}_j \text{ is aligned with } (\ldots, \mathbf{a}_{\text{cross-pairs}}, \ldots)^\top$$

   This is the mathematical signature of the firewall between the "risk" and "haven" universes shattering at an accelerating rate.

2. **Secondary Confirmation:** A significant contribution must also come from the **velocity of the maximum eigenvalue**, $\Delta\lambda_{\max,t}$. This confirms the system is rapidly consolidating into a single-factor regime.

This signal is a direct, quantitative measure of a market phase transition. It is designed to front-run the explosive release of kinetic energy (i.e., price volatility) that follows such a structural collapse. The resulting trade is a non-directional bet on this impending volatility, typically executed by purchasing straddles or strangles on all four assets.