# One-Curve Prism & a Clean Hedge: Borrow JPY $\to$ Short JGB $\to$ Long USD Bond

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## Notation (fixed throughout)

We value everything in **USD**. Let $t$ be today and $T > t$ a future date.

- $P_{\mathrm{USD}}(t,T)$, $P_{\mathrm{JPY}}(t,T)$: USD and JPY discount factors to $T$ (OIS-style).
- $S_t^{\mathrm{USD/JPY}}$: *USD per 1 JPY* at time $t$ (note: this is the inverse of the standard market quote "JPY per USD").
- $F_t^{\mathrm{USD/JPY}}(T)$: *USD per 1 JPY* outright forward that settles at $T$.

**One-curve object (the "prism edge").**

$$\boxed{C(t,T) \;:=\; \frac{F_t^{\mathrm{USD/JPY}}(T)}{S_t^{\mathrm{USD/JPY}}}} \qquad \text{(dimensionless).} \tag{1}$$

**Covered interest parity (CIP) and basis.**

$$\boxed{C(t,T) = \frac{P_{\mathrm{JPY}}(t,T)}{P_{\mathrm{USD}}(t,T)} \;\times\; B(t,T)}, \qquad B \equiv 1 \text{ iff CIP holds exactly.} \tag{2}$$

**USD value of JPY cash flows (the only pricing rule you need).** For a JPY cash flow $c^{\mathrm{JPY}}(T)$ paid at $T$,

$$\boxed{{}_{\mathrm{USD}}\big[c^{\mathrm{JPY}}(T)\big] \;=\; c^{\mathrm{JPY}}(T)\, P_{\mathrm{USD}}(t,T)\, S_t^{\mathrm{USD/JPY}}\, C(t,T)}. \tag{3}$$

Equivalently, the USD price of a JPY zero-coupon paying $1_{\mathrm{JPY}}$ at $T$ is $Z^{\mathrm{USD}\leftarrow\mathrm{JPY}}(t,T) = P_{\mathrm{USD}}(t,T)\, S_t^{\mathrm{USD/JPY}}\, C(t,T)$.

**FX forwards are not extra inputs.**

$$F_t^{\mathrm{USD/JPY}}(T) = S_t^{\mathrm{USD/JPY}}\, C(t,T). \tag{4}$$

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## Hedge the trade: Borrow JPY, Short JGB, Long USD bond

### Goal

Eliminate **FX** and **JPY interest-rate** risk so that P&L reduces (to first order) to a clean carry between the USD bond and JPY funding converted through $C$.

### Setup

- **Liability (funding in JPY):** repay a stream $\{L_i^{\mathrm{JPY}}\}$ at dates $\{T_i\}$ (principal/coupons).
- **USD asset:** a USD bond with coupons $\{A_j^{\mathrm{USD}}\}$ at dates $\{U_j\}$.
- **Hedge instruments:** (i) the forward strip $F_t^{\mathrm{USD/JPY}}(\cdot)$, (ii) a short position in a JGB (or a JPY fixed-income basket) with cash flows $\{B_k^{\mathrm{JPY}}\}$ at $\{V_k\}$.

### Step 1 — Lock the FX: convert the USD bond to a JPY cash-flow stream

For each USD bond cash flow $A_j^{\mathrm{USD}}$ at $U_j$, enter a forward that *sells USD / buys JPY* at $U_j$ for notional $A_j^{\mathrm{USD}}$. The resulting **synthetic JPY** cash flow at $U_j$ is

$$\boxed{A_j^{\mathrm{JPY,syn}} \;=\; \frac{A_j^{\mathrm{USD}}}{F_t^{\mathrm{USD/JPY}}(U_j)} \;=\; \frac{A_j^{\mathrm{USD}}}{S_t^{\mathrm{USD/JPY}}\, C(t,U_j)}}, \tag{5}$$

which is fixed at inception. *Conclusion:* after the forward strip, the USD bond has no FX or JPY-rate uncertainty; it is a deterministic JPY cash-flow ladder $\{A_j^{\mathrm{JPY,syn}}\}$.

### Step 2 — Neutralise JPY curve risk with a JGB short

Your JPY funding leg $\{L_i^{\mathrm{JPY}}\}$ still has JPY interest-rate (curve) risk. Hedge it by shorting JGBs to match DV01 in *JPY* terms:

$$\boxed{N_{\mathrm{JGB}} \;=\; \frac{\mathrm{DV01}_{\mathrm{JPY}}\big(\{L_i^{\mathrm{JPY}}\}\big)}{\mathrm{DV01}_{\mathrm{JPY}}\big(\{B_k^{\mathrm{JPY}}\}\big)}}, \tag{6}$$

with sign chosen so the JGB is *short*. If dates do not align, use a small basket of JGBs (or a JPY IRS) to match the key-rate DV01s; the idea is the same.

**Why DV01 in JPY, not USD?** Because after Step 1 the converted USD-bond leg has *no* JPY curve sensitivity (the forwards fix all FX rates at inception). The only remaining JPY-rate sensitivity is on the funding leg, so hedge it directly in JPY.

### Step 3 — Size the USD bond

Two equivalent sizing rules:

1. **Cash-flow match (preferred if dates align):** choose the USD notional $N_{\mathrm{USD}}$ so that the synthetic JPY ladder equals the liability ladder date-by-date,

$$\sum_{j:U_j=T_i} \frac{N_{\mathrm{USD}}\, A_j^{\mathrm{USD}}}{S_t^{\mathrm{USD/JPY}}\, C(t,T_i)} \;=\; L_i^{\mathrm{JPY}} \quad \forall i. \tag{7}$$

   If the calendars differ, use short JPY MM/IRS to re-time small differences.

2. **PV match (works for any schedules):** choose $N_{\mathrm{USD}}$ so that USD PVs balance at $t$:

$$\boxed{N_{\mathrm{USD}} \sum_j A_j^{\mathrm{USD}}\, P_{\mathrm{USD}}(t,U_j) \;=\; \sum_i L_i^{\mathrm{JPY}} \underbrace{P_{\mathrm{USD}}(t,T_i)\, S_t^{\mathrm{USD/JPY}}\, C(t,T_i)}_{\text{USD price of } 1_{\mathrm{JPY}} \text{ at } T_i}}. \tag{8}$$

### Resulting economics (first-order)

After Steps 1–3:

- **FX risk:** none (every USD cash flow is pre-sold for JPY via forwards).
- **JPY curve risk:** neutral (by the JGB DV01 hedge).
- **Carry & spread:** your deterministic carry is the difference between the USD bond yield (in USD PV terms) and the JPY funding cost converted into USD via $P_{\mathrm{USD}}\, S_t^{\mathrm{USD/JPY}}\, C$, net of the JGB short's carry (coupons minus financing). Symbolically, with PV matching (8):

$$\text{Carry} \approx N_{\mathrm{USD}} \sum_j A_j^{\mathrm{USD}}\, P_{\mathrm{USD}}(t,U_j)\ \underbrace{y_{\mathrm{USD}}}_{\text{USD bond yield}} - \sum_i L_i^{\mathrm{JPY}}\, P_{\mathrm{USD}}(t,T_i)\ \underbrace{y_{\mathrm{JPY}}^{\mathrm{fund}}}_{\text{JPY funding}} - N_{\mathrm{JGB}} \times \mathrm{carry}_{\mathrm{JGB}}.$$

**Note on CIP-basis.** The forward strip already embeds $B$ via (2) and (4). Once locked, basis moves do not create *pathwise* FX P&L on the hedge; they only affect mark-to-market if you revalue at new forwards.

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## Minimal cheat sheet

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| One curve | $C(t,T) = \dfrac{F_t^{\mathrm{USD/JPY}}(T)}{S_t^{\mathrm{USD/JPY}}}.$ |
| CIP (+ basis) | $C = \dfrac{P_{\mathrm{JPY}}}{P_{\mathrm{USD}}}\, B.$ |
| USD price of $1_{\mathrm{JPY}}$ at $T$ | $Z^{\mathrm{USD}\leftarrow\mathrm{JPY}} = P_{\mathrm{USD}}\, S_t^{\mathrm{USD/JPY}}\, C.$ |
| FX lock for USD coupon $A^{\mathrm{USD}}$ at $U$ | $A^{\mathrm{JPY,syn}} = \dfrac{A^{\mathrm{USD}}}{F_t^{\mathrm{USD/JPY}}(U)} = \dfrac{A^{\mathrm{USD}}}{S_t^{\mathrm{USD/JPY}} C(t,U)}.$ |
| JPY DV01 hedge size | $N_{\mathrm{JGB}} = \dfrac{\mathrm{DV01}_{\mathrm{JPY}}(\text{funding leg})}{\mathrm{DV01}_{\mathrm{JPY}}(\mathrm{JGB})}.$ |
| USD notional (PV match) | $N_{\mathrm{USD}} = \dfrac{\sum_i L_i^{\mathrm{JPY}}\, P_{\mathrm{USD}}(t,T_i)\, S_t^{\mathrm{USD/JPY}}\, C(t,T_i)}{\sum_j A_j^{\mathrm{USD}}\, P_{\mathrm{USD}}(t,U_j)}.$ |