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A discrete model of market interaction in the presence of social networks and price discrimination[☆]

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ABSTRACT

In this paper, we provide a model to study the equilibrium outcome in a market characterized by the competition between two firms offering horizontally differentiated services, in a context where consumers are the basic unit of decision on the demand side and are related through a social network. In the model, we consider that consumers make optimal choice, participation and consumption decisions, while firms optimally decide their tariffs, eventually using nonlinear pricing schemes. This approach permits us to identify and model two different kinds of network externalities, one associated with tariff-mediated network externalities and the other related to participation network externalities. We apply the model to the telecommunication industry, where we study the impact of alternative regulatory interventions. We provide numerical evidence suggesting that policies designed to reduce horizontal differentiation might be more effective than those designed to limit access charges; this result seems robust to the presence of different forms of price discrimination. We should interpret these findings cautiously due to the existence of potential implementation costs for each policy.

1. Introduction

In recent decades, a growing stream of the literature has focused on social interactions modeled through the use of network structures or graphs, in which agents are represented by nodes, and their relationships are represented by arcs between those nodes. These network structures play an important role in many economic situations and have been widely studied. Jackson (2010) provides an outstanding summary of theory and applications. However, as far as we know, no other authors have focused on modeling customer preferences and participation decisions in an oligopolistic and differentiated industry, in which customers interact through a social network, and firms may use nonlinear pricing schemes.

By treating customers in a social network environment, we allow for the number of interaction events (calls, messages, visits, games, social meetings, money transfers, etc.) between any pair of individuals to depend not only on operator prices and the level of differentiation in services but also on how "close" they are in the social network. This context has been shown to be relevant not only in determining more realistic market outcomes but also in studying how regulation should be implemented when it is needed. Over the last several years, some influential articles in the telecommunications literature have focused on the study of equilibrium interconnection strategies, in a framework with recognized consumer heterogeneity (see, for example, Dessein, 2004; Hahn, 2004). These approaches represent a significant improvement in the effort to obtain more realistic models. However, they lack a consumer social network structure—heterogeneity is instead modeled by varying the consumer propensity to interact.

Several papers are closely related to this article. The seminal ones are in the telecommunication literature: Laffont et al. (1998a,b) and Armstrong (1998, 2002). The equilibrium behavior of competing firms in the presence of heterogeneous consumers has been analyzed by Dessein (2004), Hahn (2004), Hurkens and Jeon (2009), Hoernig et al. (2011), and Jullien et al. (2013), among others. Cambini and Valletti (2008) model information exchange where the closer the parties are in social terms, the higher the intensity of information exchange, but a social network is not explicitly modeled. The use of social networks to model the connections among consumers was introduced by Harrison et al. (2006) but in a context of linear pricing schemes and in the absence of agents' participation decisions.

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In this article, we model a game where consumers are connected through a social network, and they have to make optimal participation and preference decisions in a market characterized by the presence of two competing firms (see Hellmann and Staudigl (2014), for a survey of general coevolutionary models of network formation and play). As our main goal is to develop a tool to study the impact of a social network on the demand side of the market, we adapt the model developed in Laffont et al. (1998a,b), accounting for how consumers are connected through the network. In particular, while firms offer horizontally differentiated products to maximize profits, consumers optimally decide to sign a service contract with only one of the firms based on the connection each firm offers in the network. For example, when two firms offer horizontally differentiated services, consumers will optimally decide to which firm to subscribe (if any). Even more, in this decision, consumers will take into account not only each firm's pricing scheme, but also their own position in the network. Our modeling strategy seeks to be flexible enough to study a variety of economic problems while simplifying exposure by using profit functions similar to those in the telecommunications industry. It is clear, however, that profit functions can be easily changed to represent other industries characterized by the presence of social networks on the demand side. Similar approaches have been used in other regulated industries in the presence of strategic interaction (Safari et al., 2014).[1]

In particular we are interested in analyzing the role of firms' price discrimination because different forms of price discrimination are prevalent in network markets. We look at two prevalent nonlinear pricing schemes: the first is price discrimination that depends on whether the counterpart in the transaction is a client. This is the case for on-net–off-net prices that exist in a variety of industries including telecoms, social clubs (with access to restaurants, cinemas, etc.) or sport clubs (golf, tennis, etc.).[2] The second is when two-part tariffs are feasible, which is in fact even more common than the previous scheme.[3] At first glance, adding price discrimination to the problem may seem unnecessary; however, we will show that this pricing scheme has important effects on the way agents interact in network markets.

Finally, we illustrate the applicability of the model by studying the effectiveness of two alternative regulatory interventions under different pricing schemes in telecommunication markets. First, we provide numerical evidence that suggests the traditional regulatory intervention (reduced access charges) produces a positive impact on competition and welfare by reducing the equilibrium price faced by consumers. Second, we consider an alternative policy intervention in which horizontal differentiation is reduced. Our numerical findings also suggest that policies designed to reduce horizontal differentiation might be more effective than those designed to limit access charges. Welfare also increases when the social network is more dense, but we do not consider this network characteristic to be subject to policy manipulation.

The rest of the paper is organized as follows: in Section 2, we develop the basic economic model, including pricing schemes, agents' optimal decisions, horizontal differentiation and the general firm problem. In Section 3, we develop an example application in the regulation of the telecom industry. Finally, the conclusions are stated in Section 4.

2. Economic model

In the model, we assume the existence of a social network, represented by a graph g. The nodes in the graph represent agents (indexed by $i \in I$), and a link between a pair of agents represents a social connection between them.

There are two firms, A and B, offering horizontally differentiated services, and consumers have to decide whether they participate in the market and, in such a case, to which firm to subscribe. To make their decisions, consumers take into account the pricing schemes offered by each firm and their own preferences for the services provided. In addition, preferences are modeled in a similar way to a standard Hotelling horizontally differentiated model: each agent i in the social network (i.e., each node in g) is endowed with a realization of a taste variable x_i, randomly assigned from a uniform grid with support in [0, 1]. In what follows, we assume that firm A is "located"in 0 and that firm B in 1. None of them provides the "ideal service"to agent i, positioned in x_i, unless x_i is zero or one. In this approach, there is a cost associated with the distance between the existing services and the ideal service. This cost is known as the transportation cost.

2.1. Agent demand

Consider the affiliation decision problem of agent i. If agent i decides to subscribe to network $l = A, B$ then we will say that she belongs to the set $I_l \subseteq I$ of subscribers to l. Agent i's demand is represented by the vector $q_i = (q_{ij})_{j \in I, j \neq i}$, where the generic element q_{ij} is the number of connections that agent i makes to agent j. Then the gross utility of agent i from q_i connections is given by the following[4]:

$$U_i(q_i) = \sum_{j \in I, j \neq i} \delta^{t_{ij}} u(q_{ij}) \qquad \text{with} \qquad u(q_{ij}) = \frac{q_{ij}^{1-1/\eta}}{1 - 1/\eta} \tag{1}$$

where

δ : a discount in utility when agent i connects other agents located one step farther in network g. Accordingly, it satisfies $0 < \delta < 1$.

t_{ij} : the number of steps in the shortest path connecting agents i and j in the social network. We consider $t_{ij} = 0, 1, 2, \ldots$ so that if the agents are direct neighbors, the discount factor is $\delta^0 = 1$. On the other hand, if agents i and j are not socially connected, then $t_{ij} = \infty$.[5]

1 However, our problem presents some particularities (consumers make optimal participation and affiliation decisions under strategic interaction through a social network) that prevent us, as far as we know, from using traditional algorithmic methods to find Nash equilibria (for a general view of algorithmic game theory, see Nisan et al., 2007).

2 Golf club members are usually permitted to invite a nonmember to a game, and the nonmember price is higher (either in the form of a direct entry fee or higher parking fee, for example). Similar concepts arise in the credit card industry. For example, a transaction is called "on us" when it is originated and liquidated in the same bank and "off us" otherwise.

3 Other kinds of price discrimination have been studied in the literature. For example, Yang and Ng (2010) have considered bundling strategies, but their analysis was performed in the absence of strategic effects on the supply side and social networks structures on the demand side.

4 Note that we are assuming in this formulation that all consumers in the network can receive connections even if they are not affiliated with A or B. This assumption is made for tractability, but it is not so demanding. For example, in the telecom market, a person with a prepaid phone can always receive calls without payments in a calling party pays (CPP) regime. In the golf club, a person can play with a club member even if he is not a member of any club. In what follows, however, we will discuss what happens if we depart from this assumption.
On the other hand, we are not considering the utility arising from receiving connection requests. The introduction of these kinds of externalities is direct, however, through the addition of a term $v(q_{ji})$.

5 Note that it would be equivalent to take an exogenous set of parameters MB_{ij} that scale the marginal benefit of calls between i and j to replace each $\delta^{t_{ij}}$ terms. Despite this equivalence, we see the social network as a key ingredient in this context.

η : a constant parameter representing the elasticity of demand, which is assumed to be constant, greater than 1 and independent of j.[6]

A typical and general pricing scheme applied for firm A (analogous for B) is given by $T(q_A, \widehat{q}_A) = F_A + p_A q_A + \widehat{p}_A \widehat{q}_A$, where F_A is a fixed charge and p_A is the price per connection for a subscriber in network A when she connects another subscriber in network A (on-net call in telecom services), while $\widehat{p}_A$ is the price per connection for a subscriber in network A when she connects a subscriber of B (off-net call in telecom). The notation q_A and $\widehat{q}_A$ refer to the corresponding levels of connections.

The model is flexible enough to consider such a general pricing scheme, but in this article, we focus on the impact of some particular but meaningful schemes. First, we study a simple linear nondiscriminatory pricing scheme (case 1), where $F_A = 0$ and $p_A = \widehat{p}_A$ (analogous for B). Then, we consider two types of nonlinear pricing schemes. In case 2, we study the role of price discrimination when we permit $p_A \neq \widehat{p}_A$, but we set F_A and F_B as equal to zero, and finally, in case 3, we study the role of two-part tariffs, but we impose $p_A = \widehat{p}_A$ and $p_B = \widehat{p}_B$.

For practical reasons, we are assuming that an individual who is not affiliated with any firm can still be connected to perform a transaction, and in such a case, the requesting party pays the corresponding on-network price (this is a natural assumption in many network markets, but our model is trivially extended when this assumption does not hold).

Suppose that after observing the price schemes offered by the firms, agent i has to decide to affiliate herself with a firm or to remain out of the market. She makes this decision based on a comparison of net consumer surplus under each firm. If she decides to affiliate with firm A, the vector of calls $q_i = (q_{ij})_{j\in I, j\neq i}$ to all her contacts in network g is defined by the following:

$$W_i(p_A, \widehat{p}_A) = \max_{q_i} \left\{ U_i(q_i) - p_A \sum_{\substack{j\in I\setminus I_B \\ j\neq i}} q_{ij} - \widehat{p}_A \sum_{j\in I_B} q_{ij} \right\} \tag{2}$$

Solving this maximization problem, we obtain the components of her demand:

$$q_{ij}(p) = \left(\frac{p}{\delta^{t_{ij}}}\right)^{-\eta} \quad \text{with} \quad p = p_A \text{ o } \widehat{p}_A \tag{3}$$

Intuitively, for the same price p, agent i makes more (fewer) connection requests with contacts located closer to (farther from) her in social network g. Moreover, the number of connections also depends on the possibility for discrimination based on where agent j is affiliated. Therefore, we plug it into Eq. (2) and obtain the indirect utility function:

$$W_i(p_A, \widehat{p}_A) = \sum_{\substack{j\neq i \\ j\in I\setminus I_B}} \delta^{\eta t_{ij}} \frac{p_A^{1-\eta}}{\eta - 1} + \sum_{j\in I_B} \delta^{\eta t_{ij}} \frac{\widehat{p}_A^{1-\eta}}{\eta - 1} \tag{4}$$

and an analogous result arises for firm B.

If agent i, located in x_i, decides to subscribe to network A located in 0 or network B located in 1, she has to pay a transportation cost, given by τ per unit of "distance". The total cost of selecting network A is assumed to be $x_i \tau \sum_{\substack{j\in I \\ j\neq i}} \delta^{t_{ij}}$, and the total cost of selecting B is $(1 - x_i)\tau \sum_{\substack{j\in I \\ j\neq i}} \delta^{t_{ij}}$. It is important to note that, unlike the standard Hotelling model, we assume that agent i incurs a discounted disutility for connections due to imperfect matching between her preferences and the service provided. The discount is due to the fact that the imperfection becomes more annoying as agents j and i become closer within the network. The total cost of imperfect matching is the sum of all the pairwise-discounted costs. In addition, note that the cost to agent i of an imperfect service in the connection with agent j is assumed to be independent of the number of connections.[7]

Let us define the net surplus for consumer i when affiliates to firm A and B as follows:

$$w_i(p_A, \widehat{p}_A, F_A, x_i) \equiv W_i(p_A, \widehat{p}_A) - F_A - \tau x_i \sum_{\substack{j\in I \\ j\neq i}} \delta^{t_{ij}}$$

$$w_i(p_B, \widehat{p}_B, F_B, 1 - x_i) \equiv W_i(p_B, \widehat{p}_B) - F_B - \tau(1 - x_i) \sum_{\substack{j\in I \\ j\neq i}} \delta^{t_{ij}}$$

The preference for A or B depends on whether x_i is to the right or to the left of a critical value x_i^* given by the following:

$$w_i(p_A, \widehat{p}_A, F_A, x_i^*) = w_i(p_B, \widehat{p}_B, F_B, 1 - x_i^*)$$

If $x_i < x_i^*$ (resp. $x_i > x_i^*$), agent i prefers network A (resp. B) even though network A does not provide the ideal service (and has to pay $\tau x_i \sum_{\substack{j\in I \\ j\neq i}} \delta^{t_{ij}}$ for imperfect matching). Solving for x_i^* yields the following:

$$x_i^* = \frac{1}{2} + \sigma_i \left[W_i(p_A, \widehat{p}_A) - F_A - (W_i(p_B, \widehat{p}_B) - F_B)\right]$$
$$\left(\text{with } \sigma_i = \frac{1}{2\tau \sum_{\substack{j\in I \\ j\neq i}} \delta^{t_{ij}}}\right) \tag{5}$$

Let us define $\alpha_i = 0$ if agent i prefers network A and $\alpha_i = 1$ if agent i prefers network B.

Accordingly, the preference decision can be written as follows:

$$\alpha_i = \begin{cases} 0 & \text{if } x_i < x_i^* \\ 0 \text{ or } 1 & \text{if } x_i = x_i^* \\ 1 & \text{if } x_i > x_i^* \end{cases} \tag{6}$$

However, we also have to consider agent i's option to keep out of the market, i.e., the participation decision. Agent i participates in the market if and only if

$$Max\left\{w_i(p_A, \widehat{p}_A, F_A, x_i), w_i(p_B, \widehat{p}_B, F_B, 1 - x_i)\right\} \geq 0 \tag{7}$$

Equivalently, we can define the following:

$$\Omega_i(p_A, \widehat{p}_A, p_B, \widehat{p}_B, F_A, F_B, x_i, \alpha_i) = (1 - \alpha_i) w_i(p_A, \widehat{p}_A, F_A, x_i) + \alpha_i w_i(p_B, \widehat{p}_B, F_B, 1 - x_i)$$

and then, the participation decision for agent i is modeled by β_i such that the following holds:

$$\beta_i = \begin{cases} 0 & \text{if } \Omega_i < 0 \\ 1 & \text{if } \Omega_i \geq 0 \end{cases} \tag{8}$$

Accordingly, for agent i to contract with firm A, it is necessary that she prefers A to B ($\alpha_i = 0$) and that agent i participates in the market ($\beta_i = 1$).

2.2. Firm problem

To work with specific profit functions, from this section on, we assume that we are considering a telecommunications market where connections mean calls (or messages), and each firm pursues profit maximization. It is clear, however, that we can work with other profit functions in other industries such as sport clubs and social clubs.

[6] Note that this assumption is consistent with the empirical literature. See, for example, Hazlett and Muñoz (2009).

[7] Alternatively, one could make transportation cost dependent on the utility obtained from the connections or from the number of connections. Our selection here is consistent with that of Laffont et al. (1998a).

The telecom industry has the peculiarity that firms are also interconnected. In fact, to complete a call from firm B's subscriber to firm A's subscriber, part of the signal is processed by each firm (the origin of the access charge concept). Firm A's access charge a_A is the per-minute fee that firm A applies to firm B to terminate a call from a subscriber of B to a subscriber of A (a_B is defined analogously). As we will see, access charges can affect final outcomes and are subject to regulatory scrutiny.

When access charges are exogenously given by a_A and a_B, firm A (resp. B) will select its prices $p_A, \widehat{p}_A, F_A$ (resp. $p_B, \widehat{p}_B, F_B$) such that the following holds:

$$\max_{p_A,\widehat{p}_A,F_A\geq 0} \pi_A(p_A, \widehat{p}_A, p_B, \widehat{p}_B, F_A, F_B, a_A, a_B) = \sum_{i\in I_A}\left\{\sum_{\substack{j\in I\setminus I_B\\ j\neq i}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) + \sum_{j\in I_B} q_{ij}(\widehat{p}_A)(\widehat{p}_A - c_A^o - a_B) + F_A - f\right\} + \sum_{i\in I_B}\sum_{j\in I_A} q_{ij}(\widehat{p}_B)(a_A - c_A^f) \tag{9}$$

where[8]:

f : the fixed cost incurred by a firm when it affiliates a new subscriber.

c_A^o : the cost per unit of originating a call for firm A (c_B^o is defined analogously).

c_A^f : the cost per unit of terminating or finishing a call for firm A (c_B^f is defined analogously).

From the previous discussion, it is clear that firms and consumers interact strategically, so the relevant solution concept is a Nash equilibrium of the game.[9] Note that in the firm problem described in Eq. (9), the sets I_A, I_B and $I\setminus I_B$ also depend on the selected prices, through consumer affiliation decisions, so they are endogenously determined in equilibrium. This fact complicates the calculation of the Nash equilibria. Therefore, we need to make these endogenous relations explicit. To do so, our goal will be to express both constraints in linear form, so as to write firm A's problem as follows:

$$\underset{p_A,\widehat{p}_A,F_A\geq 0}{Max}\ \pi_A(p_A, \widehat{p}_A, p_B, \widehat{p}_B, F_A, F_B, a_A, a_B; \alpha, \beta) \tag{10}$$

$$\begin{aligned} s.t.\quad & H\alpha \leq z, \quad \alpha \in \{0,1\}^I \qquad \text{(preference constraint)}\\ & K\beta \leq y, \quad \beta \in \{0,1\}^I \qquad \text{(participation constraint)} \end{aligned}$$

With this goal in mind, we separate the problem in two parts. First, we write preference and participation constraints as a system of inequality constraints, and then, we write the objective function as in (10). Even so, the dimension of the problem makes it unsolvable in the general case, so we focus on three particular schemes: a simple linear nondiscriminatory pricing scheme as a benchmark (case 1) and then two types of nonlinear pricing schemes (cases 2 and 3).

Case 1 - Linear Nondiscriminatory Prices: In this case $p_A = \widehat{p}_A$, $p_B = \widehat{p}_B$ and $F_A = F_B = 0$. It is easy to check that all the agents satisfy the participation constraint, i.e., $\beta_i = 1\,\forall i \in I$, and then, preferences become affiliation decisions. This result is a direct consequence of the absence of a fixed charge and the structure of the demand functions given in (3). Then, firm A's problem can be written as follows[10]:

$$\underset{p_A\geq 0}{Max}\ \pi_A^1(p_A, p_B, a_A, a_B; \alpha) \tag{11}$$

$$s.t.\quad H_1\alpha \leq z_1, \quad \alpha \in \{0,1\}^I \qquad \text{(affiliation constraint)}$$

Case 2 — On–Off Price Discrimination: In this case, $F_A = F_B = 0$. As in the previous case, we have $\beta_i = 1\,\forall i \in I$, and preferences become affiliation decisions. In other words, all agents satisfy the participation constraint. Then, the problem for firm A becomes the following[11]:

$$\underset{p_A,\widehat{p}_A\geq 0}{Max}\ \pi_A^2(p_A, \widehat{p}_A, p_B, \widehat{p}_B, a_A, a_B; \alpha) \tag{12}$$

$$s.t.\quad H_2\alpha \leq z_2, \quad \alpha \in \{0,1\}^I$$

Case 3 — Two-Part Tariffs: In this case, $p_A = \widehat{p}_A$ and $p_B = \widehat{p}_B$. The main difference from the previous case is the participation decision—the presence of a fixed part in the tariff can prevent the participation of some consumers. In other words, β could now be different from a vector of ones, so we need to model the participation decision. Accordingly, the problem for firm A becomes the following[12]:

$$\underset{p_A,F_A\geq 0}{Max}\ \pi_A^3(p_A, p_B, F_A, F_B, a_A, a_B; \alpha, \beta) \tag{13}$$

$$\begin{aligned} s.t.\quad & H_3\alpha \leq z_3, \quad \alpha \in \{0,1\}^I\\ & K_3\beta \leq y_3, \quad \beta \in \{0,1\}^I \end{aligned}$$

3. Example of application: Regulatory interventions in telecommunications markets

In the following analysis, we consider a regulatory application of our model. In particular, we consider the standard regulatory approach, where access charges are defined by the authority, and only the final prices are the result of market interactions. We define a regulatory benchmark case in which the authority selects access charges equal to marginal termination costs (i.e., $a_A = c_A^f$ and $a_B = c_B^f$), leaving final prices defined by competition between firms A and B. For simplicity, we also assume symmetric firms, so $c_A^f = c_B^f$. Using the model developed in the previous sections, we study two alternative regulatory interventions that depart from the benchmark case:

1. The authority can set access charges below marginal termination costs to enhance competition. Under this policy, the firms have an additional incentive to reduce prices because a net outflow of calls is more profitable than a balanced pattern.

2. The authority can implement policies to reduce horizontal differentiation, which intensify rivalry for affiliate consumers. We consider a reduction of transportation costs as a proxy for such a policy.

Absent the implementation costs of these policies, we are allowed to conduct welfare analysis. Furthermore, this analysis could be reduced to a simple comparison between the results of both regulatory interventions, but it is also illustrative to compare the results with those obtained in meaningful benchmarks. In what follows, we put implementation costs aside (or we consider them to be significantly low) to compare these two policies with other benchmarks. A first benchmark is given by the standard

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8 In what follows, when we solve an optimization problem, we always assume that the social network, as well f, c_A^o, c_B^o, c_A^f, c_B^f, $\{x_i\}_{i=1}^I$ and τ are all given exogenously.

9 The best response functions of the firms can eventually generate multiple Nash equilibria. We reduce this possibility when assuming a uniform grid for x_i, instead of a random realization of them. However, we cannot guarantee, at least theoretically, uniqueness even in the symmetric case.

10 See a detailed discussion in Appendix B.1.

11 See a detailed discussion in Appendix B.2.

12 See a detailed discussion in Appendix B.3.

access charge regulation described above, where final prices are defined by competition in the final market. A second benchmark considers again the standard access charge regulation, but final prices are defined so as to maximize social welfare. A third benchmark considers again the standard access charge regulation, but final prices are set monopolically. Finally, a fourth benchmark is the Ramsey approach, where consumer surplus is maximized subject to an industry breakeven constraint. In this section, we provide the fundamental tools to solve the benchmark cases.

For any pair of prices (p_A, p_B), we can evaluate consumer surplus as follows:

$$CS(p_A, \widehat{p}_A, p_B, \widehat{p}_B, F_A, F_B) = \sum_{i \in I_A} w_i(p_A, \widehat{p}_A, F_A, x_i) + \sum_{i \in I_B} w_i(p_B, \widehat{p}_B, F_B, 1 - x_i) \tag{14}$$

Accordingly, total welfare is given by the following:

$$TW(p_A, p_B) = CS(p_A, p_A, p_B, p_B, F_A, F_B) + \pi_A(p_A, p_A, p_B, p_B, F_A, F_B, c_A^f, c_B^f) + \pi_B(p_A, p_A, p_B, p_B, F_A, F_B, c_A^f, c_B^f)$$

It is noteworthy that, to obtain total welfare, we are not considering on-net–off-net price discrimination and that F_A and F_B are just transfers from consumers to firms, so they should not affect welfare as long as all consumers are served. Then, we obtain the maximum achievable welfare by solving the following:

$$\underset{p_A,\ p_B}{Max}\ TW(p_A, p_B)$$

The previous approach could result in subsidies to the firms, an outcome that in some cases is unfeasible. An alternative approach that permits us to avoid this problem is the Ramsey approach[13]:

$$\underset{p_A,\ p_B \geq 0}{Max}\ CS(p_A, p_A, p_B, p_B, 0, 0) \tag{15}$$

$s.t.$

$$\pi_A(p_A, p_A, p_B, p_B, 0, 0, c_A^f, c_B^f) + \pi_B(p_A, p_A, p_B, p_B, 0, 0, c_A^f, c_B^f) = 0$$

Access charges have been set as equal to marginal termination costs, and tariffs are linear.

Finally, it is also illustrative to use the monopoly case under linear tariffs as another benchmark. In this case, the affiliation decision is irrelevant, and the firm simply solves the following:

$$\max_{p \geq 0} \pi_M(p) = \left\{ (p - c_M^o - c_M^f) p^{-\eta} \sum_{i \in I} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}} - \sum_{i \in I} f \right\}$$

where the subindex M denotes the monopoly levels.

3.1. Numerical results

In this section, we report the main simulation results for alternative regulatory interventions.[14] It should be noted that problems (10), (22) and (24) are nonlinear not only in the objective function but also in the constraints because they depend on prices. However, for any given vector of prices, the constraint can be solved in α and/or β as a linear problem. Once α and/or β have/has been selected, we can evaluate the goal function for the corresponding vector of prices, access charges, α and β. In what follows, we look for a symmetric Nash equilibria in all the settings.

Table 1
Default parameter values.

Elasticity of demand	$-\eta = -1.2$
Discount factor	$\delta = 0.9$
Origination cost	$c_A^o = c_B^o = 0.75$
Termination cost	$c_A^f = c_B^f = 0.75$
Fixed cost	$f = 50$
Access charges	$a_A = a_B = 0.75$
Number of individuals	$I = \begin{cases} 1000 \text{ in case 1} \\ 100 \text{ in cases 2 and 3} \end{cases}$
Transportation cost	$\tau = \begin{cases} 0.5 \text{ in case 2} \\ 0.25 \text{ in cases 1 and 3} \end{cases}$

In the simulation, the social network is generated using random regular graphs (see Bollobas, 2001), where the connectivity degree d of graph g represents the average number of social connections between agents.[15]

The default values for the parameters are given in Table 1. In the subsequent analysis below, we depart from this setting in some key variables associated with different regulatory interventions.

All parameters were selected based on the relevant literature or "reasonableness."For example, Ingraham and Sidak (2004) estimated that the elasticity of demand in the US for wireless services is between −1.12 and −1.29. The fixed cost (f) was selected to represent 10% of ARPU (average revenue per user). On the other hand, the origination, termination and transportation costs are of the same order of magnitude as those reported by De Bijl and Peitz (2002) in their simulations.

Table 2 summarizes the parameters used for the analysis of regulatory interventions. The first column corresponds to the standard case where regulatory authorities set access prices equal to marginal termination costs. The second column contains the parameters for the Ramsey approach, while the last two columns contain the settings in which the intervention occurs in access charges (scheme 1) and transportation costs (scheme 2).

3.2. Results for Case 1: Linear nondiscriminatory prices

In the simulation, we consider 15 random networks generated for each level of connectivity degree (d). Our results show that (a) equilibrium prices and Ramsey prices are both decreasing in d; (b) consumer surplus is increasing in connectivity degree; and (c) the gaps between, on the one hand, the Ramsey consumer surplus and equilibrium consumer surplus and, on the other hand, the last and monopoly consumer surplus are both increasing in d. These results show that the importance of procompetitive regulation increases when individuals become more socially connected.

In relation to regulatory recommendations, Fig. 1 provides support for setting access charges below marginal costs because equilibrium prices approach Ramsey levels when access charges are reduced. It is clear, according to our simulations, that lowering access charges below marginal termination costs permits us to increase social welfare.[16] It is also clear that as the degree of connectivity increases, equilibrium prices drop,[17] and the

[13] The same approach for the Ramsey option is followed by Laffont et al. (1998a), where the problem does not contain a social structure among customers.

[14] A full report of the results is available on our webpages or upon request.

[15] Observe that as d rises, the marginal benefit of each call remains unchanged, but there are more connections between consumers, so they are "socially closer".

[16] The welfare increase comes from a reduction in oligopoly rents rather than calling externalities, which are not modeled here.

[17] The average rate of reduction over connectivity degrees was 1.95%.

Table 2
Basic parameters under regulatory interventions.

Parameters	Standard	Ramsey	Scheme 1	Scheme 2
Access charges ($a_A = a_B$)	0.75	0.75	*variable*	0.75
Transportation cost (τ)	as in Table 1	as in Table 1	as in Table 1	*variable*
Connectivity degree (d)	*variable*	*variable*	*variable*	*variable*

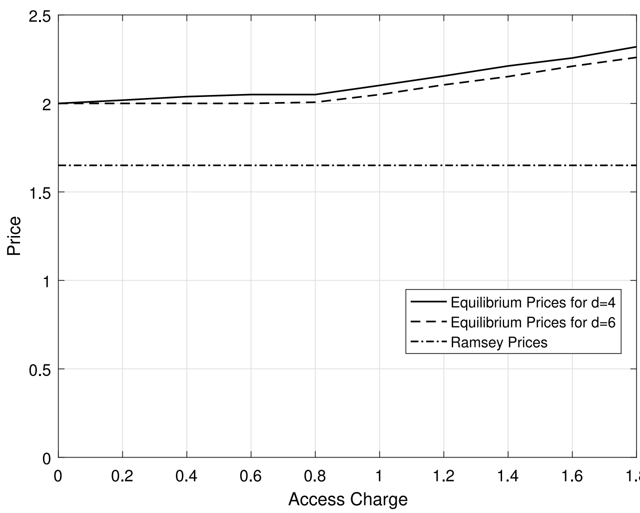

Fig. 1. Equilibrium prices and access charges.

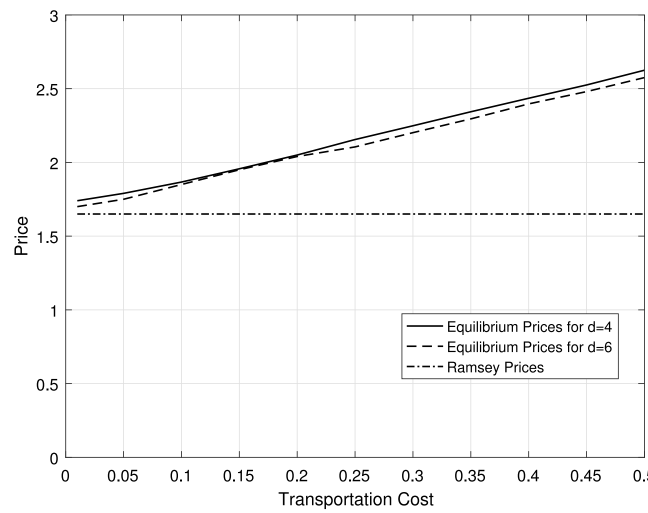

Fig. 2. Equilibrium prices and transportation costs.

policy becomes more effective. However, this is not the only policy intervention that can be evaluated. Fig. 2 shows the effect of a policy where the authority focuses on reducing horizontal differentiation, intensifying the competition for customers. Our simulations show that these kinds of policies are even more effective than access charge regulation in generating equilibrium prices closer to the Ramsey benchmark case. As before, the effectiveness of the policy increases with connectivity; however, the rate of price reduction is lower than in the access charge policy approach,[18] probably because the benchmark situation (with $d = 4$) is already close to Ramsey prices.

3.3. Results for Case 2: Discrimination by call destination

In this case, the structure of matrix H_2 is complex enough for the constraint to allow for multiple equilibria. Here, we select α so as to minimize $\sum_{i=1}^{I}\alpha_i$, in other words, firm A's most favorable selection in terms of market share. As in the previous case, social networks matter, but we focus here on the impact of the alternative regulatory interventions on equilibrium prices p and $\widehat{p}$.

Fig. 3 reports the equilibrium p and $\widehat{p}$ when access charges are permitted to change. Both of them are above the Ramsey prices and by far below monopoly prices, but the most interesting finding is that for sufficiently low access charges, it is cheaper to call off-net than on-net. The reason is simple: receiving calls from the rival firm is expensive because the termination cost is higher than the access charge. As a result, firms try to attract high-demand customers to avoid a high incoming flow from the rival network.

Fig. 4 reports the equilibrium p and $\widehat{p}$ when transportation costs are permitted to change, as a proxy for horizontal differentiation. Both of them are above the Ramsey prices, but they are

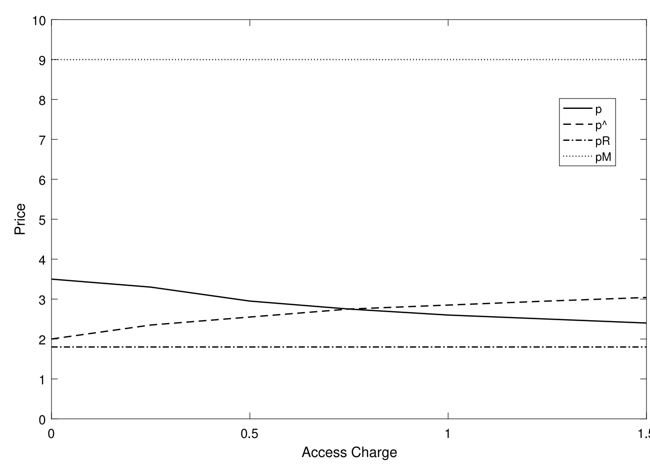

Fig. 3. Equilibrium prices (with on-net–off-net discrimination) and access charges.

closer than in Fig. 3. It is interesting that for low transportation costs it is cheaper to call off-net than on-net. The reason for this behavior, however, is quite different from the case of access charge regulation described in Fig. 3. If, in equilibrium, $\widehat{p}$ were higher than p, all consumers would jump to one network, and in the static framework under analysis, competition would intensify because the firm that loses the battle is out of the market. Moreover, given the rule to select among multiple equilibria, A is the surviving firm.

3.4. Results for Case 3: Two-part tariffs

The case of two-part tariffs is different from the previous ones in several aspects. First, the participation decision is nontrivial ($\beta_i = 1$ versus 0). Second, the consumer affiliation decision is guaranteed to be unique. This is because both matrix H_3 and K_3

[18] In this case the rate of reduction was 1.63%.

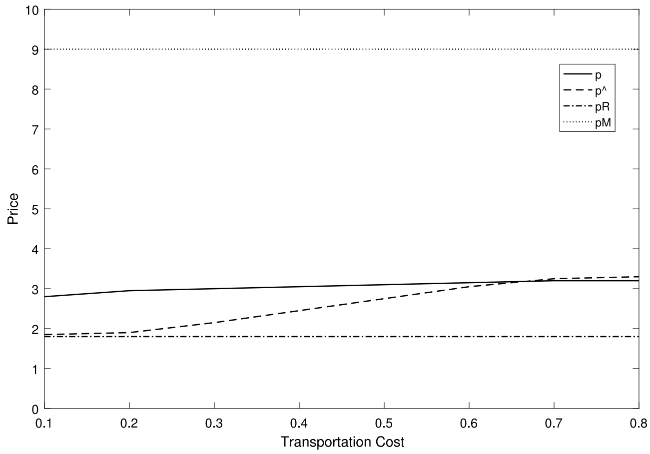

Fig. 4. Equilibrium prices (with on-net–off-net discrimination) and transportation costs.

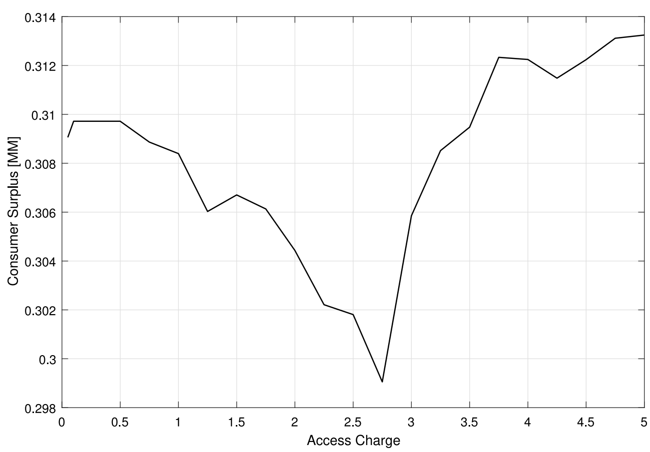

Fig. 5. Consumer surplus and access charges.

are "bidiagonal". The net effect is that the algorithms are simpler than those developed for case 2.

It is easy to verify that, as in the previous cases, social structure matters, but again, here, we are interested in studying the relative efficiency of the two kinds of regulatory interventions. To do that, and because of the presence of a fixed part of the tariff, it is convenient to focus the analysis on the effect on consumer surplus. Fig. 5 shows that access charge regulation is not longer useful as a tool to increase consumer surplus, while Fig. 6 shows that reductions in transportation costs have a positive and predictable effect on consumer surplus. The intuition is that a reduction in access charges reduces the variable part of the tariff but increases the fixed part, and in equilibrium, some customers can be excluded from the market.[19]

Observe that our numerical exercise – exposed in Fig. 5 – suggests that policies aiming to reduce access charge might fail to be beneficial to consumers. That is – unlike in 6 – consumer welfare is not monotone decreasing in access charge.

4. Conclusion

In this paper, we develop a model to study the competition between two firms offering horizontally differentiated services in

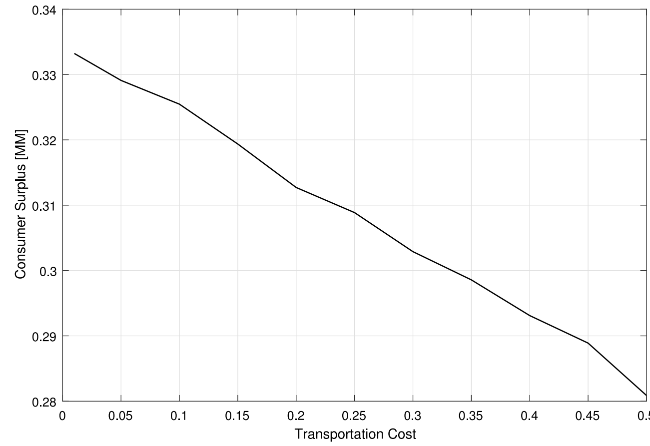

Fig. 6. Consumer surplus and transportation costs.

a context where consumers are related through a social network. In the model, we consider that consumers make optimal preference, participation and consumption decisions, while firms offer horizontally differentiated services and optimally decide their tariffs, eventually using nonlinear pricing schemes. The model is flexible enough to study different network industries, differing in terms of profit function, regulatory environment and/or the presence of social networking between consumers, among others. The main difference with the existing literature is on the demand side—we treat rational consumers as the basic decision unit, and they are related through a social network.

The use of the model is illustrated in an application to the telecommunications industry, where firms are required to be interconnected in order to provide services to their customers. In this application, we showed that a social network structure in demand significantly affects industry performance and appropriate regulation.

Our numerical findings suggest that if rates are linear, an access charge lower than marginal cost reduces the equilibrium price to consumers, improving competition. However, an alternative policy intervention that reduced horizontal differentiation was much more effective at any connectivity degree, in the sense that it brought final prices closer to a second-best solution given by the Ramsey approach. Under nonlinear pricing schemes, additional effects arise. For example, when on–off price discrimination is permitted, setting access charges below marginal costs implies that in equilibrium off-net calls would become cheaper than on-net calls. On the other hand, when two-part tariffs are feasible, consumer surplus is effectively increased by any policy that reduces horizontal differentiation, but a policy focused on reducing access charges is not longer useful. Still, we should interpret these findings cautiously due to the existence of potential implementation costs for each policy.

Finally, in relation to future work, the most immediate application would be in other industries where social networks play a role. Still, it is also possible to extend the analysis to two-sided markets (or multisided markets) in which different types of agents have to make optimal demand-side decisions in the presence of cross-network effects. This is the case, for example, in credit card markets, where customers and merchants have to decide whether to participate in the market and, if so, choose a credit card platform, in the presence of platform price discrimination and cross-network externalities (Armstrong, 2006). In the same line, it is possible to generalize the sources of utility by incorporating membership value along with traditional interaction

19 This negative effect increases if the excluded customer is not reachable by the others in our model.

value (Weyl, 2010). This should improve representation of particular network markets. The case in which a consumer derives utility from receiving a contact request can be incorporated into the current model. We could also treat the case in which more than two firms compete on the supply side.

Appendix A. Definitions of M and N

The goal of this section is to define valid values for the bounds M and N introduced in Eqs. (18) and (23), respectively.

In the case of M, we consider the following:

$$\left|b_i - L_i^t \alpha_{-i}\right| \le |b_i| + \left|L_i^t \alpha_{-i}\right| \le \frac{1}{2} + \frac{\sigma_i}{\eta - 1}\left(\underline{p}^{1-\eta} - \overline{p}^{1-\eta}\right)\sum_{\substack{j\in I\\ j\neq i}} \delta^{\eta t_{ij}} + 2\frac{\sigma_i}{\eta - 1}\left(\underline{p}^{1-\eta} - \overline{p}^{1-\eta}\right)\sum_{\substack{j\in I\\ j\neq i}} \delta^{\eta t_{ij}}$$

using $0 < \delta < 1$ and $\eta > 1$, we have the following:

$$\le \frac{1}{2} + \frac{3\sigma_i}{\eta - 1}(I-1)\left[\underline{p}^{1-\eta} - \overline{p}^{1-\eta}\right]$$

where the underbar and upperbar represent the minimum and maximum possible values for the corresponding variable, respectively.

Assuming that individual i is connected to the network $\left(\sum_{j\neq i} \delta^{t_{ij}} \ge 1\right)$ we have[20]: $\sigma_i \le \sigma \equiv \frac{1}{2\tau}$ and then, M can be chosen as follows:

$$M \equiv \frac{1}{2} + \frac{3\sigma}{\eta - 1}(I-1)\left[\underline{p}^{1-\eta} - \overline{p}^{1-\eta}\right]$$

On the other hand, from Eq. (23) and the definition of Ω_i, we can write the following:

$$\begin{aligned}
&|\Omega_i(p_A, p_A, p_B, p_B, F_A, F_B, x_i, \alpha_i)| \\
&\quad = |(1-\alpha_i) w_i(p_A, p_A, F_A, x_i) + \alpha_i w_i(p_B, p_B, F_B, 1 - x_i)| \\
&\quad < |w_i(p_A, p_A, F_A, x_i)| + |w_i(p_B, p_B, F_B, 1 - x_i)| \\
&\quad \le W_i(p_A, p_A) + F_A + \tau x_i \sum_{\substack{j\in I\\ j\neq i}} \delta^{t_{ij}} + W_i(p_B, p_B) \\
&\quad + F_B + \tau(1-x_i)\sum_{\substack{j\in I\\ j\neq i}} \delta^{t_{ij}} \\
&\quad \le I\frac{\left[\underline{p}_A^{1-\eta} + \underline{p}_B^{1-\eta}\right]}{\eta - 1} + 2\left[\overline{F} + \tau I\right] \\
&\quad \le 2\left[I\frac{\underline{p}^{1-\eta}}{\eta - 1} + \overline{F} + tI\right] \equiv N
\end{aligned}$$

Appendix B. The model under relevant pricing schemes

B.1. Case 1: Linear nondiscriminatory prices

In this case, it is easy to verify that all agents satisfy the participation constraint, i.e., $\beta_i = 1\, \forall i \in I$, and then, preferences become affiliation decisions. This result is a direct consequence of the absence of a fixed charge and the structure of the demand functions given in (3). Then, firm A's problem can be written as follows:

$$\begin{aligned}
&\underset{p_A \ge 0}{Max}\ \pi_A^1(p_A, p_B, a_A, a_B; \alpha) && (16)\\
&s.t.\quad H_1\alpha \le z_1, \quad \alpha \in \{0, 1\}^I \qquad \text{(affiliation constraint)}
\end{aligned}$$

Note that the previous structure is not warranted in general because we are requiring that affiliation constraints (now equivalent to preferences) allow for a linear representation. The gains from obtaining such a neat representation of the problem are very important. First, despite the introduction of social networks and the requirement that all agents makes optimal decisions, the problem is kept simple; second, we will be able to expand the set of situations where the model applies without changing the structure; and, third, it helps us to find a solution algorithm. With this goal in mind, we separate the problem into two parts. First, we write the vector of optimal affiliation decisions as the solution of the linear inequality constraint in (16), and then, we write the corresponding objective function, making the dependence of the objective function on the vector of affiliation decisions (alpha) explicit. The following sections are devoted to these tasks.

B.1.1. The constraint

It is easy to verify that (5) can be reduced to the following:

$$x_i^* = \frac{1}{2} + \sigma_i \frac{(p_A^{1-\eta} - p_B^{1-\eta})}{\eta - 1}\sum_{\substack{j\neq i\\ j\in I}} \delta^{\eta t_{ij}}$$

Noting that the values of x_i^* do not depend on the affiliation decisions of agents other than i,[21] and using the definition of α_i in (6), we have the following:

$$\alpha_i = \begin{cases} 0 & \text{if} \quad b_i < 0 \\ 0 \text{ or } 1 & \text{if} \quad b_i = 0 \\ 1 & \text{if} \quad b_i > 0 \end{cases} \tag{17}$$

where $b_i = x_i - x_i^*$, which for our purposes is convenient to write as follows:

$$b_i = x_i - \frac{1}{2} - \mathbf{1}^t e_{-i}(p_A, p_B)$$

with

$$e_{-i}(p,q) = \begin{pmatrix} e_{i,1}(p,q) \\ \vdots \\ e_{i,i-1}(p,q) \\ e_{i,i+1}(p,q) \\ \vdots \\ e_{i,I}(p,q) \end{pmatrix}_{I-1} \qquad \mathbf{1} = \begin{pmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}_{I-1}$$

$$e_{i,j}(p,q) = \frac{\sigma_i \delta^{\eta t_{ij}}}{\eta - 1}\left(p^{1-\eta} - q^{1-\eta}\right)$$

The optimal affiliation decisions are then formally characterized, but they are still nonlinear. To linearize them, consider $M \in \mathbb{R}_+$ sufficiently high such that, for a given i, the constraint (17) is equivalent to the following pair of inequalities[22]:

$$\begin{aligned}
&0 \ge b_i - M\alpha_i && (18)\\
&0 \le b_i + M(1-\alpha_i)
\end{aligned}$$

In effect, when $b_i < 0$ holds, agent i is forced to choose $\alpha_i = 0$, otherwise (i.e., by selecting $\alpha_i = 1$) the second inequality in (18) is violated. An analogous argument applies when $b_i > 0$. In the case when $b_i = 0$, the inequalities in (18) hold with $\alpha_i = 0$ or

[20] If individual i is disconnected from the social network, then, without loss of generality, he can be removed from the set of consumers.

[21] In the following section, we study the discriminatory case, where x_i^* actually depends on the affiliation decisions of all the agents, and the problem becomes much more complicated.

[22] A feasible definition of M is given in Appendix A.

$\alpha_i = 1$. As a result, the vector of affiliation decisions must satisfy the following system of linear inequalities:

$$H_1\alpha \le z_1$$

where

$$H_1 = \begin{bmatrix} -M & & \\ M & & \\ & -M & \\ & M & \\ \vdots & \vdots & \vdots \\ & & -M \\ & & M \end{bmatrix}_{2I\times I} \qquad z_1 = \begin{bmatrix} -b_1 \\ b_1+M \\ -b_2 \\ b_2+M \\ \vdots \\ -b_I \\ b_I+M \end{bmatrix}_{2I\times 1}$$

$$\alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_I \end{pmatrix}_{I\times 1}$$

It is convenient to emphasize that H_1 is independent of a particular vector of prices (p_A, p_B). On the other hand, z_1 depends on the vector of prices because b_i does so for each i. Accordingly, we should write the constraint as follows: $H_1\alpha \le z_1(p_A, p_B)$.

B.1.2. The objective function

Consider the problem for firm A,[23] established in Eq. (16), under the affiliation constraint. By replacing the optimal values for q_{ij} defined in Eq. (3), it becomes the following:

$$\max_{p_A\ge 0}\pi_A^1(p_A, p_B, a_A, a_B) = (p_A - c_A^o - c_A^f)p_A^{-\eta}\sum_{i\in I_A}\sum_{\substack{j\in I_A\\ j\ne i}}\delta^{\eta t_{ij}} + (p_A - c_A^o - a_B)p_A^{-\eta}\sum_{i\in I_A}\sum_{j\in I_B}\delta^{\eta t_{ij}} - \sum_{i\in I_A} f + (a_A - c_A^f)p_B^{-\eta}\sum_{i\in I_B}\sum_{j\in I_A}\delta^{\eta t_{ij}}$$

It is important to remember that the previous structure of the objective function is inadequate because the sets I_A and I_B represent the group of consumers affiliated with the corresponding firms, which are endogenous to the vector of prices (p_A, p_B). The objective function can be simplified by incorporating the variables α_i identifying the affiliation decisions. If we include the fact that affiliation decisions are also optimal for consumers, firm A's problem is given by the following:

$$\begin{aligned} \max_{p_A\ge 0}&\pi_A^1(p_A, p_B, a_A, a_B; \alpha) \\ &= (p_A - c_A^o - c_A^f)p_A^{-\eta}\sum_{i\in I}\sum_{\substack{j\in I\\ j\ne i}}\delta^{\eta t_{ij}}(1-\alpha_i)(1-\alpha_j) \qquad (19)\\ &+ (p_A - c_A^o - a_B)p_A^{-\eta}\sum_{i\in I}\sum_{j\in I}\delta^{\eta t_{ij}}(1-\alpha_i)\alpha_j \\ &- \sum_{i\in I}(1-\alpha_i)f + (a_A - c_A^f)p_B^{-\eta}\sum_{i\in I}\sum_{j\in I}\delta^{\eta t_{ij}}\alpha_i(1-\alpha_j) \\ s.t.\quad & H_1\alpha \le z_1(p_A, p_B), \qquad \alpha \in \{0,1\}^I \end{aligned}$$

where H_1, z_1 and α were defined in the previous subsection. It is clear that problem (19) has the structure required in (10).

The analogous problem for firm B is easily obtained. Note that the constraint for firm B is the same that in Eq. (19), even when the objective function changes according to the definition of α_i.

B.2. Case 2: On–off price discrimination

In this case, and following the same arguments described in the previous section, we have $\beta_i = 1\,\forall i \in I$, and preferences become affiliation decisions. In this case, we can write the following:

$$\alpha_i = \begin{cases} 0 & \text{if} \quad b_i < L_i^t\alpha_{-i} \\ 0 \text{ or } 1 & \text{if} \quad b_i = L_i^t\alpha_{-i} \\ 1 & \text{if} \quad b_i > L_i^t\alpha_{-i} \end{cases} \qquad (20)$$

where α_{-i} is an $I-1$ column vector containing the affiliation decisions of agents other than i; L_i is an $I-1$ column vector; and $b_i \in \mathbb{R}$ with[24]:

$$b_i = x_i - \frac{1}{2} - \mathbf{1}^t e_{-i}(p_A, \widehat{p}_B)$$
$$L_i = e_{-i}(\widehat{p}_A, p_B) - e_{-i}(p_A, \widehat{p}_B)$$

Note that different from Eq. (17), the condition in Eq. (20) depends on where consumer ís contacts are affiliated. In particular, if $x_i > x_j$ and consumer i affiliates with firm A, it does not imply that individual j would prefer firm A. The reason for this is that consumer j́s social contacts could be affiliated with B, so j could prefer B to take advantage of more convenient on-net prices. This result shows that a consumption inefficiency arises as a consequence of on-net–off-net price discrimination.

The constraint (20) is still difficult to incorporate into an optimization program. We would like to have a linearized version of this constraint, which should be imposed $\forall i \in I$.

Consider a sufficiently high M[25] such that, for a given i, the expression (20) is equivalent to the following pair of inequalities:

$$\begin{aligned} L_i^t\alpha_{-i} &\ge b_i - M\alpha_i \qquad (21)\\ L_i^t\alpha_{-i} &\le b_i + M(1-\alpha_i) \end{aligned}$$

In effect, when $b_i < L_i^t\alpha_{-i}$ holds, agent i is forced to choose $\alpha_i = 0$, and otherwise (i.e., by selecting $\alpha_i = 1$), the second inequality in (21) is violated. An analogous argument applies when $b_i > L_i^t\alpha_{-i}$. If $b_i = L_i^t\alpha_{-i}$, the inequalities in (21) are satisfied with either $\alpha_i = 0$ or $\alpha_i = 1$.

As a result, the vector of affiliation decisions must satisfy the following system of linear equations:

$$H_2\alpha \le z_2$$

where

$$H_2 = \begin{bmatrix} -M & -(L_1^D)^t & \\ M & (L_1^D)^t & \\ -(L_2^U)^t & -M & -(L_2^D)^t \\ (L_2^U)^t & M & (L_2^D)^t \\ \vdots & \vdots & \vdots \\ -(L_i^U)^t & -M & -(L_i^D)^t \\ (L_i^U)^t & M & (L_i^D)^t \\ \vdots & \vdots & \vdots \\ & -(L_I^U)^t & -M \\ & (L_I^U)^t & M \end{bmatrix}_{2I\times I}$$

$$z_2 = \begin{bmatrix} -b_1 \\ b_1+M \\ -b_2 \\ b_2+M \\ \vdots \\ \vdots \\ -b_I \\ b_I+M \end{bmatrix}_{2I\times 1} \qquad \alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_I \end{pmatrix}_{I\times 1}$$

---

23 Firm B's problem is totally analogous.

24 Note that in this case, b_i does not necessarily coincide with $x_i - x_i^*$. However, in the absence of on–off price discrimination, $L_i = 0\ \forall i \in I$, so we recover case 1, particularly $b_i = x_i - x_i^*$.

25 A feasible definition of M is given in Appendix A.

and

$$L_i = \begin{pmatrix} L_{i,1} \\ \vdots \\ L_{i,i-1} \\ L_{i,i+1} \\ \vdots \\ L_{i,I} \end{pmatrix} \equiv \begin{pmatrix} L_i^U \\ L_i^D \end{pmatrix} \qquad \alpha_{-i} \equiv \begin{pmatrix} \alpha_{-i}^U \\ \alpha_{-i}^D \end{pmatrix}$$

$$\alpha = \begin{pmatrix} \alpha_{-i}^U \\ \alpha_i \\ \alpha_{-i}^D \end{pmatrix}$$

It is convenient to emphasize that H_2 depends on the vector of prices $(p_A, p_B, \widehat{p}_A, \widehat{p}_B)$ because L_i does for each i. Analogously, z_2 also depends on the vector of prices because b_i also depends on the price vector for each i.

Consider now the objective function for firm A (the case for firm B is analogous), established in Eq. (9), but in the absence of fixed charges:

$$\begin{aligned}\pi_A^2(p_A, \widehat{p}_A, p_B, \widehat{p}_B, a_A, a_B) = {} & (p_A - c_A^o - c_A^f)p_A^{-\eta} \sum_{i\in I_A} \sum_{\substack{j\in I_A \\ j\neq i}} \delta^{\eta t_{ij}} \\ & + (\widehat{p}_A - c_A^o - a_B)\widehat{p}_A^{-\eta} \sum_{i\in I_A} \sum_{j\in I_B} \delta^{\eta t_{ij}} \\ & - \sum_{i\in I_A} f + (a_A - c_A^f)\widehat{p}_B^{-\eta} \sum_{i\in I_B} \sum_{j\in I_A} \delta^{\eta t_{ij}}\end{aligned}$$

Moreover, using the definition of α_i in (6), we can write the objective function as follows:

$$\begin{aligned}&\pi_A^2(p_A, \widehat{p}_A, p_B, \widehat{p}_B, a_A, a_B; \alpha) \\ &\quad = (p_A - c_A^o - c_A^f)p_A^{-\eta} \sum_{i\in I} \sum_{\substack{j\in I \\ j\neq i}} \delta^{\eta t_{ij}}(1-\alpha_i)(1-\alpha_j) \\ &\quad + (\widehat{p}_A - c_A^o - a_B)\widehat{p}_A^{-\eta} \sum_{i\in I} \sum_{j\in I} \delta^{\eta t_{ij}}(1-\alpha_i)\alpha_j \\ &\quad - \sum_{i\in I}(1-\alpha_i)f + (a_A - c_A^f)\widehat{p}_B^{-\eta} \sum_{i\in I} \sum_{j\in I} \delta^{\eta t_{ij}}\alpha_i(1-\alpha_j)\end{aligned}$$

Accordingly, the problem for firm A has been transformed into the following:

$$\begin{aligned}&\underset{p_A,\widehat{p}_A\geq 0}{Max}\ \pi_A^2(p_A, \widehat{p}_A, p_B, \widehat{p}_B, a_A, a_B; \alpha) \qquad (22) \\ &s.t. \quad H_2\alpha \leq z_2, \quad \alpha \in \{0,1\}^I\end{aligned}$$

It is important to note that the linear nondiscriminatory case (the benchmark) can be obtained as a special case of this problem when $p_A = \widehat{p}_A$ and $p_B = \widehat{p}_B$.

B.3. Case 3: Two-part tariffs

The preference constraint now arises from an analysis similar to the previous subsection, but with $p_A = \widehat{p}_A$ and $p_B = \widehat{p}_B$. In this case, $L_i = 0\ \forall i \in I$ so matrix H_3 is considerably simpler. Additionally, the expression for b_i naturally becomes $b_i = x_i - \frac{1}{2} - \mathbf{1}^t e_{-i}(p_A, p_B) - \sigma_i(F_B - F_A)$. The main difference from the previous case is the participation decision—the presence of a fixed part in the tariff can prevent the participation of some consumers. In other words, β could now be different from a vector of ones, so we need to model the participation decision.

In a procedure analogous to the one used previously, let us establish an N sufficiently high such that Eq. (8) is equivalent to the following:

$$\begin{aligned}&0 \geq \Omega_i - N\beta_i \qquad (23) \\ &0 \leq \Omega_i - N(1-\beta_i)\end{aligned}$$

As a result, the vector of market participation decisions must satisfy the following system of linear inequalities:

$$K_3\beta \leq y_3$$

where

$$K_3 = \begin{bmatrix} -N & & & \\ N & & & \\ & -N & & \\ & N & & \\ \vdots & \vdots & \vdots & \\ & & -N & \\ & & N & \end{bmatrix}_{2I\times I} \qquad y_3 = \begin{bmatrix} -\Omega_1 \\ \Omega_1 + N \\ -\Omega_2 \\ \Omega_2 + N \\ \vdots \\ -\Omega_I \\ \Omega_I + N \end{bmatrix}_{2I\times 1}$$

$$\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_I \end{pmatrix}_{I\times 1}$$

In this case, it is convenient to emphasize that K_3 is independent of a particular vector of prices (p_A, p_B). However, y_3 now depends on the vector of prices, fixed charges and also on α_i because Ω_i does for each i.

Consider now the objective function for firm A (the case for firm B is analogous), established in Eq. (9), but in the absence of on–off price discrimination:

$$\begin{aligned}&\pi_A^3(p_A, p_B, F_A, F_B, a_A, a_B) = \\ &\sum_{i\in I_A}\left\{\sum_{\substack{j\in I\setminus I_B \\ j\neq i}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) \right. \\ &\qquad \left. + \sum_{j\in I_B} q_{ij}(p_A)(p_A - c_A^o - a_B) + F_A - f\right\} + \\ &\sum_{i\in I_B}\sum_{j\in I_A} q_{ij}(p_B)(a_A - c_A^f)\end{aligned}$$

Moreover, using the definitions of α_i, β_i and the demand functions, we have the following:

$$\begin{aligned}&\pi_A^3(p_A, p_B, F_A, F_B, a_A, a_B) = \\ &\sum_{i\in I}\beta_i(1-\alpha_i)\left\{p_A^{-\eta}(p_A - c_A^o - c_A^f)\sum_{\substack{j\in I \\ j\neq i}}\left[1-\alpha_j\beta_j\right]\delta^{\eta t_{ij}} \right. \\ &\qquad \left. + p_A^{-\eta}(p_A - c_A^o - a_B)\sum_{j\in I}\alpha_j\beta_j\delta^{\eta t_{ij}} + F_A - f\right\} \\ &+ p_B^{-\eta}(a_A - c_A^f)\sum_{i\in I}\sum_{j\in I}\beta_i\beta_j\alpha_i(1-\alpha_j)\delta^{\eta t_{ij}}\end{aligned}$$

Accordingly, the problem for firm A has been transformed into the following:

$$\begin{aligned}&\underset{p_A,F_A\geq 0}{Max}\ \pi_A^3(p_A, p_B, F_A, F_B, a_A, a_B; \alpha, \beta) \qquad (24) \\ &s.t. \quad H_3\alpha \leq z_3, \quad \alpha \in \{0,1\}^I \\ &\qquad\ K_3\beta \leq y_3, \quad \beta \in \{0,1\}^I\end{aligned}$$

It is interesting to note here that if we remove the assumption that unaffiliated individuals can receive calls, then some terms can appear out of the bidiagonal in matrix K_3. The analogy with the difference between H_1 and H_2 is evident. In that case, the difference is given by the presence of *tariff-mediated network externalities*, which make the selection of a provider an interdependent decision. In the current case, when unaffiliated individuals are not able to receive calls (or more generally, to obtain service through a connection request by a client), then market participation decisions become interdependent because if some individual decides to remain out of the market, the utility for other individuals would diminish, especially those socially closer. As a result, a *participation network externality* arises.

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