Q1 (a) $d(0.5) = \frac{101}{104}$ $PV(1) = 1 \cdot d(0.5) = 0.971$

(b) $104 = 5 \cdot d(0.5) + 105 \cdot d(1)$

$PV = d(1) = \frac{104 - 5 \cdot 0.971}{105} = 0.944$

(c) $PV = 3 \cdot d(0.5) + 103 \cdot d(1) = 3 \cdot 0.971 + 103 \cdot 0.944$
$= 100.145$

the bond is underpriced, borrow at term structure rate and buy the bond.

(d) $100 = 100 \cdot \frac{c}{2} \cdot d(0.5) + 100 \cdot (1 + \frac{c}{2}) \cdot d(1)$

$100 = 48.55c + 94.4 + 47.2c$

$c = 5.85\%$

(e) $i_2(0.5) = 2 \cdot \left(\frac{1}{d(0.5)} - 1\right) = 5.97\%$

(f) $i_2(1) = 2 \cdot \left(\frac{1}{d(1)^{\frac{1}{2}}} - 1\right) = 5.85\%$

Q2: We don't know which one has the higher coupon since we don't know about their prices.

Q3. $P_{0.5} = \frac{30}{1.04} + \cdots + \frac{1030}{1.04^{59}} = 774.716$

$\frac{payoff}{1000} = \frac{4.716}{1000} = 0.004716 = 0.4716\%$

$i_2(1) = 0.9432\%$

Q4. (a) $f_2^3 = 2 \cdot \left(\frac{1}{0.92^{\frac{1}{2}}} - 1\right) = 8.51\%$

(b) New $d(2) = 0.93$ $d(1) = 0.96$

$F = \frac{d(2)}{d(1)} \cdot 10^5 = 96,875$

X can sell the contract at t_1 for a profit of $4,875 \cdot 0.96 = 4,680$

Q5. (a) $d(0.5) = \frac{1}{1.05}$ $d(1) = \frac{1}{1.06^2}$

$= 0.952$ $= 0.890$

= price of 0.5 yr zero = price of 1 yr zero

price of 100 par coupon bond $= 6 \cdot d(0.5) + 106 \cdot d(1)$

$= 5.712 + 94.34 = 100.052$

(b) \$1 0.5 yr: $DD = \frac{0.5}{1.05^2} = 0.454$

\$1 1 yr: $DD = \frac{1}{1.08^2} = 0.840$

\$100 coupon bond:

$100.052 = \frac{6}{(1+\frac{y}{2})} + \frac{106}{(1+\frac{y}{2})^2}$ $y = 11.94$

$DD = 100\left(\frac{0.12}{0.1194^2}\left(1 - \frac{1}{1.0597^2}\right) + \left(1 - \frac{0.12}{0.1194}\right)\left(\frac{1}{1.0597^2}\right)\right)$

$= 91.75$

(c) $MD = \frac{DD}{P} = \frac{0.454}{0.952} = 0.477$ \$1, 0.5 yr

$= \frac{0.89}{0.89} = 0.944$ \$1, 1 yr

$= \frac{91.75}{100.052} = 0.917$ \$100 coupon

Q6. (a) $d(2) = 0.9$ $i_2(2) = 2 \cdot \left(\frac{1}{0.9^{\frac{1}{4}}} - 1\right) = 5.39\%$

(b) $DD = \frac{2}{1.02675} = 1.753$

(c) $d(3) = 0.84$ $i_2(3) = 2 \cdot \left(\frac{1}{0.84^{\frac{1}{6}}} - 1\right) = 5.90\%$

(d) $DD = \frac{3}{1.02957} = 2.448$

(e) $DC = \frac{10.5}{1.02958} = 8.321$

(f) $\Delta P \simeq 10^6 \cdot (-2.448) \cdot (-0.005) = 12,240$

(g) $\Delta P \simeq 12,240 + \frac{1}{2} \cdot 8.321 \cdot (-0.005)^2 \cdot 10^6 = 12,344$

Q7. (a) $\Delta p \simeq -6 \cdot 10^6 \cdot (-0.004) = 24,000$

(b) $\Delta P \simeq 24,000 + \frac{1}{2} \cdot 50 \cdot 10^6 \cdot (-0.004)^2 = 24,400$

(c). $100a + 200b + 300c = 10^6$

$2 \cdot \frac{100a}{10^6} + 2 \cdot \frac{200b}{10^6} + 10 \cdot \frac{300c}{10^6} = 6$

$4 \cdot \frac{100a}{10^6} + 5 \cdot \frac{200b}{10^6} + 200 \cdot \frac{300c}{10^6} = 50$

$$\begin{pmatrix} 100 & 200 & 300 \\ 2 \cdot 10^{-4} & 4 \cdot 10^{-4} & 3 \cdot 10^{-3} \\ 4 \cdot 10^{-4} & 10^{-3} & 6 \cdot 10^{-2} \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 10^6 \\ 6 \\ 50 \end{pmatrix}$$

$a = 525,000$

$b = -260,000$

$c = 1666.7$