Training Contest 2 Editorial

May 1, 2021

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Bytedance-Moscow Workshops Training Camp, 2021

A. Another Copy of the Polygon

We are given a convex polygon A given as a sequence of vertices $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ listed in the order of the traversal and a translation vector d. We have to find the total length of the perimeter of the union of A and B, which is A translated by vector d.

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Note this is not necessarily true for two general convex polygons, so we should use the fact that B is A transalted by d. Consider any line l parallel to vector d. Its intersection with A is some segment s_A, while its intersection with B is some segment s_B. Moreover, s_B is s_A translated by d.

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Now we can assume that the set of points of the border of the union belonging to A is not connected and this will result the contradiction with the fact that A is convex.

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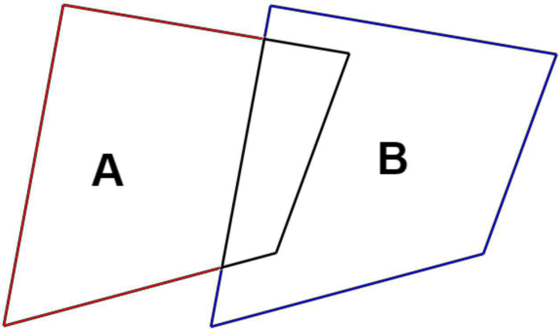

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However, we can use the fact that points of A that lie inside B form a consecutive segment, thus we can first find any point inside and any point outside and then apply binary search algorithm and linear time "point inside polygon" check. To get one point of A inside B and one point of A outside B we can take the leftmost and the rightmost points of A if projected on any line parallel to vector d.

B. Byteland Routes

We are given an unweighted tree. *Importance* of a vertex v is $\min(d(v,1), d(v,2))$, where $d(v,u)$ is the distance between v and u, importance of a path is the minimal importance over all vertices in the path. Count total importance for all paths in the tree.

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The latter is equal to $\sum \binom{s_i}{2}$, where the sum ranges over connected components over active edges, and s_i is the size of a component.

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This solution has complexity $O(n \log n)$ (for sorting the vertices by a_v), but can be optimized to $O(n\alpha(n))$, where $\alpha(n)$ is the inverse Ackermann function.

C. Complete Graph

Divide edges of a complete graph on n vertices (n is odd) into $(n-1)/2$ disjoint Hamiltonian cycles.

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Let us instead divide a complete graph on $n - 1$ vertices into Hamiltonian *paths*. The answer to the original problem is then obtained by adding one more vertex and looping each Hamiltonian path through the new vertex.

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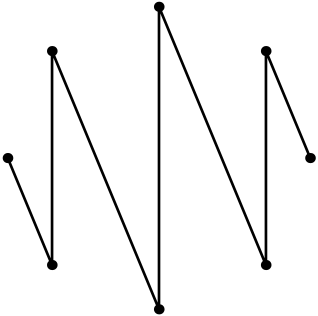
One possible construction of dividing an even-sized complete graph into Hamiltonian paths is as follows. Let us draw the vertices on a circle equally spaced. A single Hamiltonian path can be obtained as follows:



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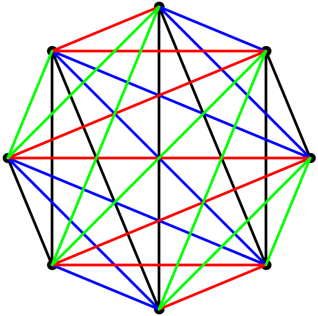
Now all remaining paths can obtained from this path by “rotating” around the center:



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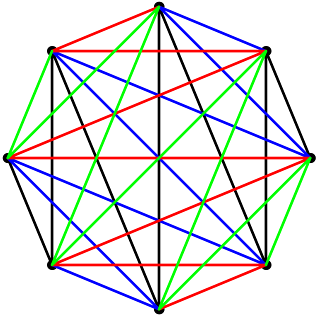

To understand why no two paths have common edges, notice that all parallel edges belong to the same path, and different slopes belong to different paths.

D. Day For Picnic

There are n distinct special points in the plane. Build a quadrilateral with distinct vertices in special points with area at most l, out of these choose one with maximal area.

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Similarly, another set of numbers is available to the other side.

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Let us sort A and B. Now the problem can be solved in $O(|A| + |B|)$ time by the "two pointers" approach: for each position i in the first sequence maintain the largest position j such that $a_i + b_j \leqslant L$. As i increases, j can only decrease.

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The resulting solution has complexity $O(n^3 \log n)$.

E. Easy Guessing Game

This was interactive problem. Jury's program thinks of one number from 1 to n. Your program can ask whether jury's number is less than x given by you. You are playing this game k times, you have to guess all k numbers.

The number of questions shouldn't exceed $k(\log(n+1) + \frac{1}{10})$

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  - Then build full binary tree from left elements

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- Say n is equal to number of leaves, $m = 2^k \leqslant n < 2^{k+1}$ and $n = m + t$

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- Say n is equal to number of leaves, $m = 2^k \leqslant n < 2^{k+1}$ and $n = m + t$
- We are to prove: $k(m - t) + (k + 1) \cdot 2t \leqslant (m + t)(\log(m + t) + \frac{1}{10})$

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So $f(x) < 0.1$

F. Fred's Parquet

A rectangular border is drawn in the plane with a regular shifted square tiling. Find the number of squares that have a common point with the border.

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With a reckless approach, the number of corner cases in a solution of this problem can be huge. Let's try to minimize the number of these cases (in fact, we pretty much won't have any).

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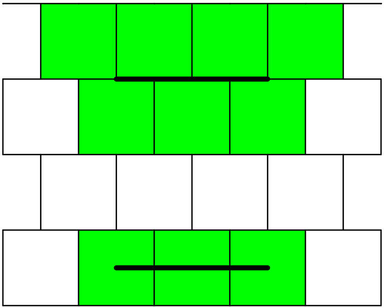
Let us consider only the horizontal sides first. For each horizontal segment, the set of concerned squares is one or two horizontal ranges of squares.



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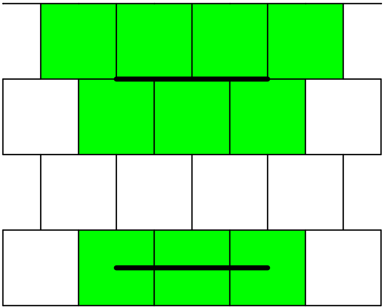

Notice that when the segments are too close a range may correspond to both of them, so we have to eliminate repetitions.

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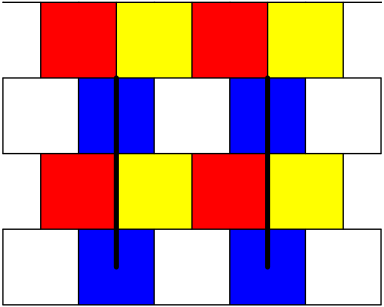
We have a similar situation for the vertical sides, except that it is now convenient to define a vertical range as a set of squares with vertical shift 2 from each other (marked with different colors on the picture below).



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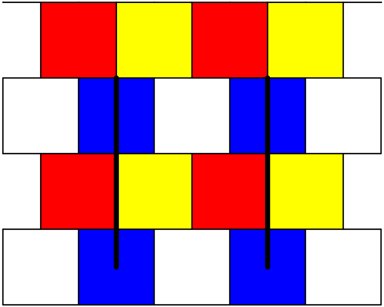

Similarly, we have to eliminate repetitions among vertical ranges.

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Finally, the answer is almost the sum of lengths of all ranges (horizontal and vertical), except for horizontal-vertical intersections.

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Since each square belongs to at most one horizontal and at most one vertical range, we have to consider all horizontal-vertical pairs of ranges and subtract 1 for each intersection (remember to account for parity of vertical ranges).

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Time complexity is $O(1)$.

G. Game With Mirroring

Find the number of solutions to $x + rev(x) = z$, where $rev(x)$ is the number obtained from x by reversing its decimal representation.

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Let us decide the value of l — the length of x (notice that is either length(z) or length$(z) - 1$).

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Deciding rest of the digits is essentially solving the same problem for $z' = (z - (d + d') \times (1 + 10^{l-1}))/10$ under $\text{length}(x) = l - 2$. The answer to this subproblem has to be multiplied by the number of ways to partition $d + d'$ into digits d and d'.

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A bit of care is needed to forbid leading zeros in x.

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Many other solutions are possible, including digit-wise DP or optimized testing of all possible x.

H. How Long Till Connection?

Given a bipartite graph: the first part has n vertices and the second one has m vertices.

Initially there are k edges in it.

Then you add edge one by one, on i-th step: vertex ($i \bmod n$) from the first part is connected to the vertex ($i \bmod m$) from the second part.

Find out, after how many added edges the graph becomes connected.

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Reducing the problem

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- So after connecting $i \bmod (n + m)$ and $(i + n) \bmod (n + m)$, we get i and $(i \bmod m) + n$ connected

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Solving the new problem

- So we have a vertices and we connect i with $(i+b) \bmod a$
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- If it's not connected, then reduce the number of vertices and solve the same problem for b and $a \bmod b$
- Otherwise use binary search to find the answer
  - Consider you added t edges, then $a-b-t$ vertices left to connect with initial k edges
  - So each check is always done in $O(k)$

I. Imoaix

In a $(n+1) \times (n+1)$ grid each cell contains a number of coins. We have to make a circular tour of $4n$ steps that visits each of the corners so that each step is a chess king move, while collecting as much coins as possible.

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We can assume that the circular tour visits corner in clockwise order. Indeed, we can reverse the tour if its counter-clockwise, and if the tour includes going from a corner straight to its opposite, then the tour must have a self-intersection in the center of the grid, which allows us to swap parts of the tour (possible with reverse) to get rid of traversing the diagonal.

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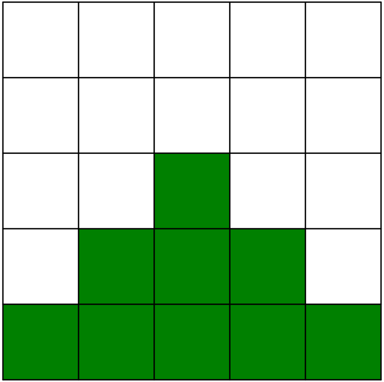
For a pair of adjacent corners each shortest path connecting them must lie inside a “quarter-square”. We call finding a path for each of these a *subproblem*.



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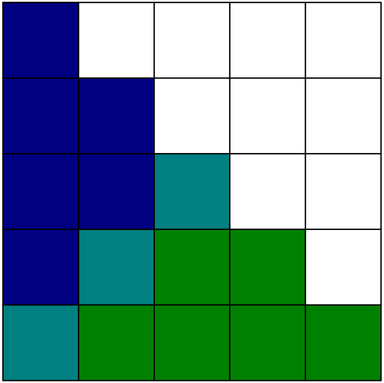
Cells reachable from two subproblems corresponding to adjacent sides of the square are half the diagonal (including the center when $n+1$ is odd).



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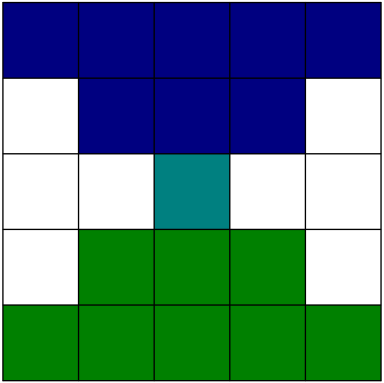
And for opposite sides only the center is possibly reachable in both subproblems.



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Similarly, only one of the four subproblems make use of the center.

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To allow for all possible solutions, let us try all options of allowing diagonal parts to corresponding subproblems, and allowing the center to one of them. In each of these options, the subproblems become independent, and thus can be solved in $O(n^2)$ time.

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The number of options is roughty bounded by $2^4 \cdot 4$, hence the solution is fast enough.

J. Journey Of Cat

An $2^n \times 2^{n+1}$ rectangle is divided recursively into 4^n tiles of size 1×2 each. We follow a walk on this rectangle. For each step determine if we've changed a tile on this step or not.

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If the subrectangle is divided further, we find the next subrectangle similarly, and so on until we arrive at the 1×2 tile. When descending, two possible orientations of a rectangle should be treated separately.

Hence, the problem is easily solved in $O(nl)$ time, where l is the length of the walk.

K. King Size

We are given an $r \times c$ table containing decimal digits and q queries "sum in rectangle". As the table is large, each row is given by its period of length no more than $s \leqslant 100$.

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One rectangle sum query can be split in four prefix-rectangle queries, i.e.

$$sum(l, u, r, d) = psum(r, d) - psum(l-1, d) - psum(r, u-1) + psum(l-1, u$$

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For each $psum(a, b)$ and fixed x we know can compute the number of times the whole period will fit into the sum plus the length of the remaining prefix. For each y from 0 to x and i from 0 to m_x compute the sum of prefixes of length y for first i rows with period equal to x. This can be done in $O(m_x \cdot x)$ time, thus in $O(rs)$ in

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The overall running time of the whole solution will be $O(s(r + q))$.

L. Laura's Function

For a square $n \times n$ matrix A, the value $f(A)$ is equal to the sum of absolute values of differences of all n^4 ordered pairs of elements of A. Find the sum of $f(A)$ over all contiguous $k \times k$ submatrices of the given $n \times m$ matrix B.

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Since a_i are ordered, we can expand the brackets in each absolute value with a certain sign. After summing similar terms, we have that the sum is equal to $-\sum_{i=1}^{n^2} 2(n^2 - 2i - 1)a_i$.

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For the current element x, iterate over all submatrices it belongs to. For a particular submatrix, we increase its counter $cnt_{i,j}$ by one. At this point, we know that x contributes $-2x(k^2 - 2cnt_{i,j} - 1)$ to the total.

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Alternatively, we can keep the value of $w_{i,j} = -(k^2 - 2cnt_{i,j} - 1)$ for each submatrix, then each subsequent element increases $w_{i,j}$ by 2

L. Laura's Function

Let's optimize this solution. We can see that the problem is reduced to 2D range queries of two types:

- take the sum of $w_{i,j}$ over a subrectangle;
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A data structure that allows to perform each of these operations in $O(\log n \log m)$ is 2D binary indexed tree (aka BIT, Fenwick tree) with range addition. The rest of the discussion is a description of this structure.

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A range query "add x to $a_l, \ldots, a_r$" can be interchanged with two prefix queries "add x to $a_1, \ldots, a_r$" and "add $-x$ to $a_1, \ldots, a_{l-1}$". We only consider prefix additions from now on.

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Let's see how adding x to first k elements changes sums on every prefix. If we denote $s_i = a_1 + \ldots + a_i$, we have that s_i increases by $x \cdot i$ when $i \leqslant k$, and by $x \cdot k$ when $i > k$.

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To allow for quick changes, we introduce two auxiliary arrays p_i and q_i. At any point we want to maintain the condition

$$s_i = i \cdot (p_1 + \ldots + p_i) + (q_1 + \ldots + q_i).$$

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One can check that after performing operations:

- p_1 += x, p_{k+1} −= x,
- q_{k+1} += $x \cdot k$,

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Since we have reduced to $O(1)$ single-element updates and/or prefix sums queries for p_i and q_i per each query of the original problem, storing p_i and q_i in BITs clearly does the trick.

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The same idea can be carried over to 2D (and, in fact, any dimension) range queries. Let $s_{i,j}$ be the two-dimensional "prefix sums":

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2D BIT allows to perform each single-element update and sum query in $O(\log n \log m)$ time, which finishes the story.