Training Contest 1 Editorial, April 30, 2021

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A. Algorithmia's Sky

Given n points $P = \{p_1, p_2, ..., p_n\}$ on the plane; each point p_i moves with speed v_i. The problem is to find a day between 0 and given D such that diameter of set of points at this day is minimal.

A. Algorithmia's Sky

Let $dst2_{ij}(t)$ be a square of distance between p_i and p_j at day t, and $diam2(t)$ be a square of diameter at that day. Obviously, $dst2_{ij}(t)$ is convex downward function; $diam2(t)$ is convex downward too as maximum of $n(n-1)/2$ convex downward functions. Then, we can use ternary search to find a minimal possible diameter.

For any fixed day t, the square of diameter $diam2(t)$ can be found in $O(nlogn)$ time (full description of the algorithm can be found in `http://euro.ecom.cmu.edu/people/faculty/mshamos/1978ShamosThesis.pdf`, pp.76-82). Then, the complexity of our solution is $O(n\ log\ n\ log\ D)$.

B. Build The Trees

Given undirected graph G with n enumerated vertices and m edges, find number of different trees on n vertices containing G as subgraph.

B. Build The Trees

If there are some cycles in G, the answer is 0.

Otherwise, suppose that G contains k components of connection, and sizes of these components are $s_1, s_2, \cdots, s_k$. Then, the answer is

$answer = s_1 s_2 \ldots s_k n^{k-2}$.

This formula is a simple generalization of Cayley's formula about a number of trees and can be proved using Prufer's code. Full proof can be found, for example, here: https://acmcairoscience.wordpress.com/2015/04/07/prufer-code-cayley-formula-the-number-of-ways-to-make-a-graph-connected/

C. Contest on TV

There is a two-stage competition, with total rankings determined by sum of scores. We know a — the results of the first stage, and c — shuffled results of the second stage. How many assignments between second stage results and participants are there so that top-k stays the same as in the ranking only by the first stage?

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- $a_i + c_{\pi(i)} \geqslant x$ for $i \leqslant k$;
- $a_i + c_{\pi(i)} \leqslant x$ for $i > k$.

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Graphically, we mark cells (i, j) with green if $\pi(j) = i$ is possible according to criterion above. Our task is to count the number of assignments in which all corresponding cells $(\pi(j), j)$ are green; one such assignment is denoted by circled cells.

$a \backslash c$	1	2	4	6	8	9
1	2	3	5	7	9	10
3	4	5	7	9	11	12
4	5	6	8	10	12	13
5	6	7	9	11	13	14
6	7	8	10	12	14	15
7	8	9	11	13	15	16

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This is the same as the *permanent* of the matrix with 1's in green

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From the constraints on the green cells we notice that it is possible to draw a vertical line so that the top-left and bottom-right parts of the table are all green (let m denote the number of columns to the left of the line).

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The strategy is now as follows: for all x count the number of ways to circle x elements in top-right and bottom-left parts independently, then try to place the rest the all-green parts (they are easier).

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We do the similar DP to count the same quantities in the top-right part.

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The top-left green part has $n - k - y$ free rows and $m - x$ free columns. To fill these rows and columns completely we must have $n - k - y = m - x = t$, and the number of ways to do it in this case is $t!$.

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Altogether, we are able to obtain the answer in $O(n^2)$ time for a single x.

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To do this, we perform the previous computation, and subtract the number of assignments that satisfy:

- $a_i + c_{\pi(i)} > x$ for $i \leqslant k$;
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Finally, sum the answers over all x. The resulting complexity is $O(an^2)$, where a is the maximal score.

D. Damage

Given n skills, each of them helps hero to do d_i damage to the monster with probability p_i. Each round, hero chooses skills equiprobably one-by-one and try to use it; round is over when skills are over or some skills does damage successfully; each skill can't be chosen more than once. The question is: what it the mathematical expectation E of damage after one round?

D. Damage

Consider a scenario in which damage was done by skill i, and before it, k different skills $j_1, j_2, ..., j_k$ was chosen in some order and was unsuccessful. The probability of this scenario is $p_i * k! * \frac{(1-p_{j_1})*(1-p_{j_2})*\cdots*(1-p_{j_k})}{n*(n-1)*...*(n-k+1)}$. So, the answer is

$$E = \sum_{i=1}^{n} d_i p_i \sum_{k=0}^{n-1} (k! * f(i,k)), \text{ where}$$
$$f(i,k) = \sum\{(1-p_{j_1})...(1-p_{j_k}) | 1 \leqslant j_1 < j_2 < ... < j_k \leqslant n,\ each\ j_l - s\ is\ not\ equal\ to\ i\}.$$

So, if we know $f(i,k)$ for all i and k, then we can solve our problem in $O(n^2)$. But how to find $f(i,k)$?

D. Damage

Let $d[i][k]$ be $\sum\{(1-p_{j_1})\ldots(1-p_{j_k})|1 \leqslant j_1 < j_2 < \ldots < j_k \leqslant i$.

Then, $d[i][k] = d[i-1][k] + (1-p_i) * d[i-1][k-1]$ for $k, i \geqslant 1$; also, $d[i][0] = 1$ for any i. It means that:

1. all $d[i][k]$ can be found in $O(n^2)$;
2. $f(i,k) = d[n][k] - f(i,k-1) * (1-p_i)$ ("sum of products without i-th is sum of products minus sum of products with i-th"); so, all $f(i,k)$ can be calculated in $O(n^2)$ too.

So, complexity of calculating $f(i,k)$-s and the whole solution is $O(n^2)$.

E. Engines and Pumps

There are n red points and m blue points on the real line ($n \leqslant m$). We have to assign a blue point to each read point so that:

- no blue point is assigned to more than one red point;
- sum of the distances between assigned pairs is smallest possible.

E. Engines and Pumps

Consider a particular assignment f of a blue point $f(x)$ to each red point x. Let us say that a red point x is of *type* L if $f(x) \leqslant x$, and otherwise x is of *type* R.

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If f is an optimal assignment, then:

- If x is a red point of type R, y is a red point of type L, and $x < y$, then $f(x) < f(y)$ (otherwise we can swap $f(x)$ and $f(y)$ to obtain a better assignment);

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- Therefore, if x is, say, an L-type red point, and another red point y satisfies $f(x) \leqslant y < x$, then y is an L-type point as well.
- If $f(x) < x$ for a red point x, then all blue points in the range $(f(x), x]$ are assigned to type-L red points.

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It follows that the balance of each prefix of such subsegment (where red point counts as $+1$, and a blue point as -1) is either always non-negative or non-positive (depending of L/R type of red points). If the balance reaches zero at a position other than endpoints, we can decompose the segment into smaller ones.

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To make a transition, we consider all sensible segments of points (see above). Suppose that a segment $[prev(p), p]$ contains red points with numbers in $[l, r]$. We then have to try to improve dp_r with $dp_{l-1}+$the cost of assigning points inside $[prev(p), p]$.

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F. Funny Lottery

There are R red, G green and B blue marble in the lottery wheel; on each step, some random marble is spat from the wheel; if it's red marble, then we do nothing more on this step; otherwise, we return this marble to the wheel. The problem is to find a mathematical expectation of number of step in which blue marble will be spat in K-th time by modulo $MOD = 10^9 + 7$.

F. Funny Lottery

Let $d[r][k]$ be an answer to the problem for $R = r, K = k$, G and B are the same as in input data. Then, it follows from the statement that for all natural r, k:

$$d[0][0] = d[r][0] = 0,\ d[0][k] = k,$$

$d[r][k] =$
$1 + \frac{r}{r+G+B} * d[r-1][k] + \frac{B}{r+G+B} * d[r][k-1] + \frac{G}{r+G+B} * d[r][k]$,
i.e.

$$d[r][k] = \frac{r+G+B+R*d[r-1][k]+B*d[r][k-1]}{r+G+B}.$$

Then, it can be proven by induction and calculated in $O(logk + logMOD)$ that

$$answer = d[R][K] = R + K + \frac{KG}{B} - R(\frac{B}{B+1})^K.$$

The details of the proof are left to the reader.

G. Giant Aquarium

An aquarium is filled with water. The bottom of the aquarium is an axes-aligned polyline with non-decreasing x coordinate. We can punch k holes in the bottom, the water will then flow out where it can. Find the maximal amount of water we can get to flow out.

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When we build the whole decomposition, the problem is reduced to the following:

There is a weighted rooted tree. Choose k disjoint vertical paths so that the total weight of covered vertices is largest possible.

(Vertex weights correspond to the top water body volume in the corresponding subproblem.)

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- Construct an RMQ structure on y's of segments. Find the highest segment, descend recursively. $O(n \log n)$.
- Process segments from left to right with stack. $O(n)$.

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The answer to the problem is simply taking k best paths.

H. Hospital

We are given a weighted tree, with two special vertices containing hospitals. There are a_i people living at i-th vertex. Each person takes the shortest path to one of the hospitals when needed. We are allowed to shorten some of the edges by 1 at most B times, but can not make any edge shorter than L this way. Spend the money so that:

- sum of the distances to the closest hospital for all people is minimal;
- the largest distance is minimal.

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Minimizing total distance: when we shorten an edge, the total distance decreases by s_i — total number of people in the subtree. Thus each edge allows us to perform $\max(0, w_i - L)$ operations that bring profit s_i. Now we sort the operations by their profit and perform them while we can.

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Minimizing maximal distance: binary search on the answer. For each vertex of the tree store the longest path down. Naturally, to get rid of all paths greater than x it makes sense to shorten higher edges first. The total cost of satisfying the restriction can be found with a single DFS.

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The total complexity is $O(n \log A)$, where A is the total length of all edges.

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The complexity is $O(n^2 \log A)$.

I. ICPC and Bulbs

There is a rotateable tile in each of the puzzle board cells. Each tile contains a lightbulb, a wire connecting two sides of the tile, or nothing. Rotate all tiles so that the two lightbulbs are connected, and all wires are used.

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If we have a q cell, this information determines the rotation uniquely.

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First, try all possible cases of rotating the lightbulbs. Process all the remaining cells in row-major order, that is, from top row to bottom, and from left to right in each row.

We claim that there is at most one possible rotation of each tile that agrees with everything we already determined.

Indeed, when we reach a cell, we must know (by induction hypothesis) if a wire enters the cell from the left side and from the top side.

If we have a `q` cell, this information determines the rotation uniquely.

If we have a `l` cell, either the rotation is unique, or we have a conflict in any case.

I. ICPC and Bulbs

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Complexity is $O(nm)$.

J. Jokes of Loki

We are trying to guess a number x by asking queries "is $x \leqslant q$?". We are allowed to make at most a_x queries. Can we win, and what are the possible options for the first question?

J. Jokes of Loki

Suppose that $x \in [l, r]$. If $l = r$, no more questions are needed. Otherwise, we choose m and ask if $x \leqslant m$; the possible scenarios are to proceed to segments $[l, m]$ and $[m + 1, r]$.

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For a certain strategy of choosing m we can build a binary rooted tree of all scenarios. The number of questions to guess the number x is exactly the *depth* of the leaf $[x, x]$, that is, the distance to the root.

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Thus, we can win the game iff we can construct a tree with n leaves with constraints on their respective depths.

J. Jokes of Loki

Proposition

If d_x is the depth of a leaf x in a binary rooted tree, then $\sum_x 2^{-d_x} = 1$.

Proof: induction.

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Sketch of a proof: just place each leaf as deep as possible, observe that everything is fine.

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How can we check the condition $\sum_x 2^{-a_x} \leqslant 1$ precisely? Store the number of repetitions for each x, then perform the carries in a binary representation of the sum.

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When then answer is inf? In this case at least one our winning option is to waste a question, so $\sum_x 2^{-a_x} \leqslant 1/2$.

K. Kitchen-based Economy

Given cactus - undirected connected graph with N vertices and M edges with no edge lie in more than one simple cycle. For each vertex v, there exist numbers a_v and b_v of people and restaurants in the vertex, respectively. Each man choose a restaurant with equal probability and go there using one of the shortests paths (with equal probability). The problem is to find all edges with maximum expected value of people going through it.

K. Kitchen-based Economy

Run DFS on our graph and find all cycles; it will be simple because any cycle will consist on some path in DFS-tree from ancestor to descendant and one back edge. Write these cycles in vector *cycles* (in fact, vector of vectors), for each vertex write all numbers cycles vertex lies in and a position of it's number, and for each edge write a number of cycle it lies in. Also, we will consider each bridge as cycle of length 2.

Consider some cycle $v_0, v_1, \ldots, v_{k-1}$, $k \geqslant 2$. If we delete all edges of this cycle, graph will decay to components of connection $C_0, C_1, \ldots, C_{k-1}$ containing vertices $v_0, v_1, \ldots, v_{k-1}$ respectively. Let $csuma_i$ and $csumb_i$ be summary numbers of people and restaurants in component C_i.

K. Kitchen-based Economy

ByteDance Moscow Workshops

Find an expected number $E_{v_{k-1}v_0}$ of people going through edge $e = (v_{k-1}, v_0)$ strictly in this direction. For each concrete man m, probability p_{em} of visiting e depends only on number of C_i of start and finish of his journey; let it be C_s and C_f, respectively. Any path of our man goes through vertices v_s and v_f and choose shortest of two paths. if $s \leqslant f$, then $p_{em} = 0$; otherwise $p_{em} = 1$ if $2 * (f + k - 1 - s) < k$, $p_{em} = 0.5$ if $2 * (f + k - 1 - s) = k$ and $p_{em} = 0$ if $2 * (f + k - 1 - s) > k$.

It means that $E_{v_{k-1}v_0} B * F_{v_{k-1}v_0}$,
$F_{v_{k-1}v_0} = (2 * \sum\limits_{0 \leqslant s,f < k; f+k-1-s<k} (suma_s * sumb_f) + \sum\limits_{0 \leqslant s,f < k; f+k-1-s=k}) * (csuma_s * csumb_f))$; here B is a positive constant (equal to $0.5/(b_1 + b_2 + \cdots + b_n)$) which doesn't depend on concrete edge).

K. Kitchen-based Economy

Then, the $O(n^3)$ solution is:

1) find all cycles in the graph;

2) run second $DFS(v, parentCycleNumber)$ on vertices and cycles; for each vertex v, find $suma_v$ and $sumb_v$ - sum of a-s and b-s for all vertices connected with v by path which doesn't contain edges of cycle $parentCycleNumber$. To launch this DFS, just call $DFS(0, -1)$; then $DFS(v, parentCycleNumber)$ will go through all cycles contains v, except $parentCycleNumber$. For each such cycle C and vertices u_i of it, run $DFS(u_i, C)$ and sum up calculated $suma_{u_i}$ and $sumb_{u_i}$ to $suma_v$ and $sumb_v$. At the end, add a_v to $suma_v$ and b_v to $sumb_v$ and finish DFS;

K. Kitchen-based Economy

3) Having *suma*-s and *sumb*-s, try to find an answer for each edge. Let $C = (v_0, v_1, v_2, \ldots, v_{k-1})$ be a cycle, and v_0 is a vertex which was first visited by *DFS* on second step. Then, $csuma_i = suma_{v_i}, csumb_i = sumb_{v_i}$ for $i = 1, 2, \ldots, k-1$; then, $suma_0 = suma - \sum_{i=1}^{k-1} csuma_i$, $sumb_0 = sumb - \sum_{i=1}^{k-1} csumb_i$; here, *suma* and *sumb* are full number of people and restaurants in the whole graph.

4) Having *csuma*-s and *csumb*-s, find each F in $O(k^2)$. So, our algorithm works in $O(m^3)$ time.

But $O(m^3)$ is too slow for us ($m \leqslant 200000$). How to improve it?

K. Kitchen-based Economy

The only thing we should optimize is calculating F-s on cycle C - other parts of algorithm works in linear time.

First of all, calculate prefix sums on arrays *suma* and *sumb*; then, for any path on cycle (containing edge (v_{k-1}, v_0) or not) we can find sum of *suma*-s and *sumb*-s of it's vertices in linear time.

Then, suppose that $k = 2 * l$ (case $k = 2 * l + 1$ is very similar and simpler). Then,

$F_{v_{k-1},v_0} = 2 * (suma_{k-l+1} * sumb_0 + suma_{k-l+2} * (sumb_0 + sumb_1) + \cdots + suma_{k-1} * (sumb_0 + sumb_1 + \cdots + sumb_{k-2})) + (suma_{k-l} * sumb_0 + suma_{k-l+1} * sumb_1 + \cdots + suma_{k-1} * sumb_{l-1})$.

K. Kitchen-based Economy

Hence, F_{v_{k-1},v_0} can be calculated in $O(k)$ using partial sums, and now we can solve the problem in $O(m^2)$. The idea which gives us a linear solution is that difference between neighbouring F-s can be calculated in $O(1)$, because, for example, $F_{v_0,v_1} - F_{v_{k-1},v_0} = 2 * suma_0 * (sumb_1 + sumb_2 + \cdots + sumb_{l-1}) + suma_0 * sumb_l - 2 * sumb_0 * (suma_{k-1} + suma_{k-2} + \cdots + suma_{k-l+1}) - sumb_0 * suma_{k-l}$ (to move to F_{v_0,v_1} from F_{v_{k-1},v_0}, we just added short paths starting in v_0 and subtract short paths finishing in v_0). So, we can calculate F_{v_{k-1},v_0} in linear time and then find each new $F_{v_i,v_{i+1}}$ from previous one in $O(1)$ with partial sums. The complexity of our last version of algorithm is $O(m)$.

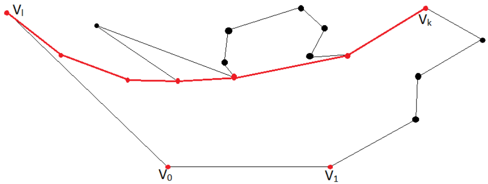
L. Light Sources

Given a polygon with N vertices $v_0, v_1, \ldots, v_{n-1}$; whole polygon is illuminated by at least one of light sources in vertices v_0 and v_1, and also two number a and b. The problem is to find lexicographically minimal path from v_a to v_b.

L. Light Sources

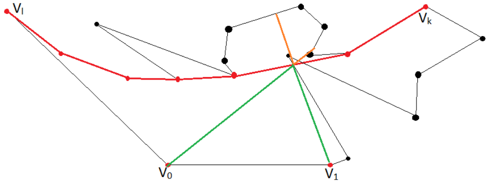
Replace number of v_0 from 0 to n; so, we have vertices $v_1, v_2, \cdots, v_n$.

Then, suppose that $a < b$. Then, go through vertices from v_a to v_b using stack and build something similar to building lower part of convex hull in Andrew's algorithm:



L. Light Sources

It seems intuitively obvious and can be proven that polyline we've built is an optimal path among paths lying non-higher than points $v_a, v_{a+1}, \ldots, v_b$; this polyline must lie in the polygon, because otherwise, we go to contradiction with constraint of visibility:



L. Light Sources

So, our algorithm works in a linear time. A last small detail is that three points on the same line is bad for optimal way, and this case should be considered in implementation.