

南京大学 ACM-ICPC 集训队代码模版库



Linux-4.15.0-50-generic-x86_64-with-Ubuntu-18.04-bionic
XeTeX 3.14159265-2.6-0.99998 (TeX Live 2017/Debian)
CPython 2.7.15rc1
2019-05-21 15:46:17.803900, build 0058

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1 General

1.1 Code library checksum

```
ab14 #!/usr/bin/python3
c502 import re, sys, hashlib
427e
f7db for line in sys.stdin.read().strip().split("\n") :
ddf5     print(hashlib.md5(re.sub(r'\s|//[.]*', '', line).encode('utf8')).hexdigest()
        [-4:], line)
```

1.2 Makefile

```
dab2 .PHONY : run
427e
207e $(t) : $(t).cpp
2d16     g++ --std=c++14 -Wall -D__LOCAL_DEBUG__ -fsanitize=undefined -fsanitize=
        address -ggdb -pipe -o $@ $<
427e
5f25 run : $(t)
bf3e     ./$$(t) < $(t).in
```

1.3 .vimrc

```
914c set nocompatible
733d syntax on
6bbc colorscheme slate
7db5 set number
b0e3 set cursorline
061b set shiftwidth=2
8011 set softtabstop=2
a66d set tabstop=2
d23a set expandtab
5245 set magic
740c set smartindent
bee8 set backspace=indent,eol,start
815d set cmdheight=1
0a40 set laststatus=2
1c67 set whichwrap=b,s,<,>,[,]
```

1.4 Stack

```
const int STK_SZ = 2000000;
char STK[STK_SZ * sizeof(void)];
void *STK_BAK;

#if defined(__i386__)
#define SP "%esp"
#elif defined(__x86_64__)
#define SP "%rsp"
#endif

int main() {
    asm volatile("movl SP, %0; movl %1, SP: "=g"(STK_BAK):"g"(STK+sizeof(STK)):")
    ;

    // main program

    asm volatile("movl %0, SP: "=g"(STK_BAK));
    return 0;
}
```

1.5 Template

```
#include <bits/stdc++.h>
using namespace std;

#ifdef __LOCAL_DEBUG__
# define _debug(fmt, ...) fprintf(stderr, "[%s] " fmt "\n", \
    __func__, __VA_ARGS__)
#else
# define _debug(...) ((void) 0)
#endif

#define rep(i, n) for (int i=0; i<(n); i++)
#define Rep(i, n) for (int i=1; i<=(n); i++)
#define range(x) begin(x), end(x)
typedef long long LL;
typedef unsigned long long ULL;
```

2 Miscellaneous Algorithms

2.1 2-SAT

Usage:

init(*n*) Initialize the solver with *n* variables.
 add_clause(*x*, *xval*, *y*, *yval*) Add a clause (*x* == *xval*)-> (*y* == *yval*).
 solve() Solve the problem. Return **true** if SAT, or **false** if UN-SAT.
 operator[](*i*) Get the value of *i*-th variable.

```

0f42 const int MAXN = 100005;
03a9 struct twoSAT {
5c83     int n;
8f72     vector<int> G[MAXN*2];
d060     bool mark[MAXN*2];
b42d     int S[MAXN*2], c;
427e
d34f     void init(int n) {
b985         this->n = n;
f9ec         for (int i=0; i < n*2; i++) G[i].clear();
0609         memset(mark, 0, sizeof(mark));
95cf     }
427e
3bd5     bool dfs(int x) {
bd70         if (mark[x^1]) return false;
c96a         if (mark[x]) return true;
fd23         mark[x] = true;
4bea         S[c++] = x;
bd55         for (int u : G[x]) if (!dfs(u)) return false;
3361         return true;
95cf     }
427e
5894     void add_clause(int x, bool xval, int y, bool yval) {
6afe         x = x * 2 + xval;
e680         y = y * 2 + yval;
81cc         G[x^1].push_back(y);
95cf     }
427e
d0cb     bool solve() {
7c39         for (int i=0; i<n*2; i+=2) {
e63f             if (!mark[i] && !mark[i+1]) {
88fb                 c = 0;

```

```

        if (!dfs(i)) {
            while (c > 0) mark[S[--c]] = false;
            if (!dfs(i+1)) return false;
        }
    }
    return true;
}

bool operator[] (int x) { return mark[2*x+1]; }
};

```

f4b9
 3f03
 86c5
 95cf
 95cf
 95cf
 3361
 95cf
 427e
 fb3b
 329b

2.2 Mo's algorithm

All intervals are closed on both sides. When running functions enter() and leave(), the global *l* and *r* has not changed yet. **Assume the data structure is initialized for empty interval.**

Usage:

add_query(*id*, *l*, *r*) Add *id*-th query [*l*, *r*].
 run() Run Mo's algorithm.
 yield(*id*) **TODO.** Yield answer for *id*-th query.
 enter(*o*) **TODO.** Add *o*-th element.
 leave(*o*) **TODO.** Remove *o*-th element.

```

constexpr int BLOCK_SZ = 300;

struct query { int l, r, id; };
vector<query> queries;

void add_query(int id, int l, int r) {
    queries.push_back(query{l, r, id});
}

int l, r;

// ----- functions to implement -----
inline void yield(int id);
inline void enter(int o);
inline void leave(int o);

void run() {
    if (queries.empty()) return;
    sort(range(queries), [](query lhs, query rhs) {

```

5194
 427e
 3ec4
 d26a
 427e
 1e30
 54c9
 95cf
 427e
 9f6b
 427e
 427e
 50e1
 b20d
 13af
 427e
 37f0
 ab0b
 8508

```

c7f8      int lb = lhs.l / BLOCK_SZ, rb = rhs.l / BLOCK_SZ;
03e7      if (lb != rb) return lb < rb;
0780      return lhs.r < rhs.r;
b251    });
6196    l = queries[0].l;
9644    r = queries[0].r;
38e6    for (int i = l; i <= r; i++) enter(i);
5bc9    for (query q : queries) {
f422      while (l > q.l) enter(--l);
39fb      while (r < q.r) enter(++r);
46b3      while (l < q.l) leave(l++);
6234      while (r > q.r) leave(r--);
82f5      yield(q.id);
95cf    }
95cf  }

```

2.3 Matroid Intersection

Find the maximum cardinality common independent set of two matroids. Matroids are given by independence oracle.

Usage:

MatroidOracle	The independence oracle maintaining an independent set. Note that the default constructor must properly initialize inner state to an empty set.
insert(x)	Insert element labeled x to the independent set.
test(x)	Test whether the set is still independent if x is inserted.
MatroidIntersection<MT1, MT2>(n)	Construct the matroid intersection solver with n elements labeled from 0 and matroid oracles MT1 and MT2.
run()	Run the algorithm and return the matroid intersection.

```

0935 struct MatroidOracle {
297b   MatroidOracle() { /* TODO */ }
53e5   void insert(int x) { /* TODO */ }
ff18   bool test(int x) const { /* TODO */ }
329b };
427e
a015 const int MAXN = 8192;
94cc template <typename MT1, typename MT2>
3288 struct MatroidIntersection {
5c83   int n;
5550   bool in[MAXN] = {}, t[MAXN], vis[MAXN];
fe84   int pre[MAXN];

```

```

vector<int> adj[MAXN];
queue<int> q;

MatroidIntersection(int n) : n(n) { }

vector<int> getcur() {
  vector<int> ret;
  rep (i, n) if (in[i]) ret.push_back(i);
  return ret;
}

void enqueue(int x, int p) {
  if (vis[x]) return;
  vis[x] = true; pre[x] = p; q.push(x);
  if (t[x]) throw x;
};

vector<int> run() {
  while (true) {
    vector<int> cur = getcur();
    fill(vis, vis + n, 0);
    rep (i, n) adj[i].clear();
    MT2 mt2;
    for (int i : cur) mt2.insert(i);
    rep (i, n) t[i] = mt2.test(i);
    vector<MT1> mt1s(cur.size());
    vector<MT2> mt2s(cur.size());
    rep (i, cur.size()) rep (j, cur.size()) if (i != j) {
      mt1s[i].insert(cur[j]);
      mt2s[i].insert(cur[j]);
    }
    rep (i, n) if (!in[i]) rep (j, cur.size()) {
      if (mt1s[j].test(i)) adj[cur[j]].push_back(i);
      if (mt2s[j].test(i)) adj[i].push_back(cur[j]);
    }
    q = {};
    try {
      MT1 mt1;
      for (int i : cur) mt1.insert(i);
      rep (i, n) if (mt1.test(i)) enqueue(i, -1);
      while (q.size()) {
        int u = q.front(); q.pop();
        for (int v : adj[u]) enqueue(v, u);
      }

```

```

0b32
93d2
427e
c152
427e
2ed1
995a
a585
ee0f
95cf
427e
ca2b
e5da
f4a6
ff59
329b
427e
9081
1026
c40f
6f47
943b
0e02
3e54
191d
e167
46d2
660b
3cd7
9680
95cf
e8d7
3fe9
645e
95cf
cf76
85eb
2f4f
2f34
4053
1c7d
c048
a697
95cf

```

```

5a9a         } catch (int v) {
a8f3             while (v >= 0) { in[v] ^= 1; v = pre[v]; }
b333             continue;
95cf         }
6173         break;
329b     };
f2de     return getcur();
95cf }
329b };

```

3 String

3.1 Knuth-Morris-Pratt algorithm

```

2836 const int SIZE = 10005;
427e
d02b struct kmp_matcher {
2d81     char p[SIZE];
9847     int fail[SIZE];
57b7     int len;
427e
60cf void construct(const char* needle) {
aaa1     len = strlen(p);
3a87     strcpy(p, needle);
3dd4     fail[0] = fail[1] = 0;
d8a8     for (int i = 1; i < len; i++) {
147f         int j = fail[i];
3c79         while (j && p[i] != p[j]) j = fail[j];
4643         fail[i + 1] = p[i] == p[j] ? j + 1 : 0;
95cf     }
95cf }
427e
c464 inline void found(int pos) {
427e     // ! add codes for having found at pos
95cf }
427e
2daf void match(const char* haystack) { // must be called after construct
700f     const char* t = haystack;
8482     int n = strlen(t);
8fd0     int j = 0;
be8e     rep(i, n) {

```

```

while (j && p[j] != t[i]) j = fail[j];
if (p[j] == t[i]) j++;
if (j == len) found(i - len + 1);
    }
}
};

```

```

4e19
b5d5
f024
95cf
95cf
329b

```

3.2 Manacher algorithm

```

struct Manacher {
    int Len;
    vector<int> lc;
    string s;

    void work() {
        lc[1] = 1;
        int k = 1;

        for (int i = 2; i <= Len; i++) {
            int p = k + lc[k] - 1;
            if (i <= p) {
                lc[i] = min(lc[2 * k - i], p - i + 1);
            } else {
                lc[i] = 1;
            }
            while (s[i + lc[i]] == s[i - lc[i]]) lc[i]++;
            if (i + lc[i] > k + lc[k]) k = i;
        }
    }

    void init(const char *tt) {
        int len = strlen(tt);
        s.resize(len * 2 + 10);
        lc.resize(len * 2 + 10);
        s[0] = '*';
        s[1] = '#';
        for (int i = 0; i < len; i++) {
            s[i * 2 + 2] = tt[i];
            s[i * 2 + 1] = '#';
        }
        s[len * 2 + 1] = '#';
        s[len * 2 + 2] = '\0';
    }
}

```

```

81d4
cd09
9255
b301
427e
ec07
c033
6bef
427e
491f
7957
5e04
24a1
8e2e
e0e5
95cf
74ff
2b9a
95cf
95cf
427e
bfd5
aaaf
f701
7045
8e13
ae54
1321
e995
69fd
95cf
43fd
75d1

```

```

61f7     Len = len * 2 + 2;
3e7a     work();
95cf }
427e
b194 pair<int, int> maxpal(int l, int r) {
901a     int center = l + r + 1;
ffb2     int rad = lc[center] / 2;
ab54     int rmid = (l + r + 1) / 2;
17e4     int rl = rmid - rad, rr = rmid + rad - 1;
3908     if ((r ^ l) & 1) {
69f3     } else rr++;
69dc     return {max(l, rl), min(r, rr)};
95cf }
329b };

```

3.3 Aho-corasick automaton

```

a1ad struct AC : Trie {
9143     int fail[MAXN];
daca     int last[MAXN];
427e
8690 void construct() {
93d2     queue<int> q;
a7a6     fail[0] = 0;
ce3c     rep(c, CHARN) {
b1c6         if (int u = tr[0][c]) {
a506             fail[u] = 0;
3e14             q.push(u);
f689             last[u] = 0;
95cf         }
95cf     }
cc78     while (!q.empty()) {
31f0         int r = q.front();
15dd         q.pop();
ce3c         rep(c, CHARN) {
ab59             int u = tr[r][c];
0ef5             if (!u) {
9d58                 tr[r][c] = tr[fail[r]][c];
b333                 continue;
95cf             }
3e14             q.push(u);
b3ff             int v = fail[r];

```

```

while (v && !tr[v][c]) v = fail[v];
fail[u] = tr[v][c];
last[u] = tag[fail[u]] ? fail[u] : last[fail[u]];
    }
}
}

void found(int pos, int j) {
    if (j) {
        // ! add codes for having found word with tag[j]
        found(pos, last[j]);
    }
}

void find(const char* text) { // must be called after construct()
    int p = 0, c, len = strlen(text);
    rep(i, len) {
        c = id(text[i]);
        p = tr[p][c];
        if (tag[p])
            found(i, p);
        else if (last[p])
            found(i, last[p]);
    }
}
};

```

d2ea
c275
654c
95cf
95cf
95cf
427e
7752
043e
427e
4a96
95cf
95cf
427e
9785
80a4
9c94
b3db
f119
f08e
389b
1e67
299e
95cf
95cf
329b

3.4 Trie

```

const int MAXN = 12000;
const int CHARN = 26;

inline int id(char c) { return c - 'a'; }

struct Trie {
    int n;
    int tr[MAXN][CHARN]; // Trie tree, 0 denotes fail
    int tag[MAXN];

    Trie() {
        memset(tr[0], 0, sizeof(tr[0]));
        tag[0] = 0;
    }
};

```

e6f1
dd87
427e
8ff5
427e
a281
5c83
f4f5
35a5
427e
4fee
3ccc
4d52

```

46bf     n = 1;
95cf     }
427e
427e     // tag should not be 0
30b0     void add(const char* s, int t) {
d50a         int p = 0, c, len = strlen(s);
9c94         rep(i, len) {
3140             c = id(s[i]);
d6c8             if (!tr[p][c]) {
26dd                 memset(tr[n], 0, sizeof(tr[n]));
2e5c                 tag[n] = 0;
73bb                 tr[p][c] = n++;
95cf             }
f119             p = tr[p][c];
95cf         }
35ef         tag[p] = t;
95cf     }
427e
427e     // returns 0 if not found
427e     // AC automaton does not need this function
216c     int search(const char* s) {
d50a         int p = 0, c, len = strlen(s);
9c94         rep(i, len) {
3140             c = id(s[i]);
f339             if (!tr[p][c]) return 0;
f119             p = tr[p][c];
95cf         }
840e         return tag[p];
95cf     }
329b };

```

3.5 Suffix array

The character immediately after the end of the string **MUST** be set to the **UNIQUE SMALLEST** element.

Usage:

s[] the source string
sa[i] the index of starting position of i -th suffix
rk[i] the number of suffixes less than the suffix starting from i
h[i] the longest common prefix between the i -th and $(i-1)$ -th lexicographically smallest suffixes
n size of source string
m size of character set

```

void radix_sort(int x[], int y[], int sa[], int n, int m) {
    static int cnt[1000005]; // size > max(n, m)
    fill(cnt, cnt + m, 0);
    rep(i, n) cnt[x[y[i]]]++;
    partial_sum(cnt, cnt + m, cnt);
    for (int i = n - 1; i >= 0; i--) sa[--cnt[x[y[i]]]] = y[i];
}

void suffix_array(int s[], int sa[], int rk[], int n, int m) {
    static int y[1000005]; // size > n
    copy(s, s + n, rk);
    iota(y, y + n, 0);
    radix_sort(rk, y, sa, n, m);
    for (int j = 1, p = 0; j <= n; j <= 1, m = p, p = 0) {
        for (int i = n - j; i < n; i++) y[p++] = i;
        rep(i, n) if (sa[i] >= j) y[p++] = sa[i] - j;
        radix_sort(rk, y, sa, n, m + 1);
        swap_ranges(rk, rk + n, y);
        rk[sa[0]] = p = 1;
        for (int i = 1; i < n; i++)
            rk[sa[i]] = ((y[sa[i]] == y[sa[i-1]] and y[sa[i]+j] == y[sa[i-1]+j])
                ? p : ++p);
        if (p == n) break;
    }
    rep(i, n) rk[sa[i]] = i;
}

void calc_height(int s[], int sa[], int rk[], int h[], int n) {
    int k = 0;
    h[0] = 0;
    rep(i, n) {
        k = max(k - 1, 0);
        if (rk[i] while (s[i+k] == s[sa[rk[i]-1]+k]) ++k;
        h[rk[i]] = k;
    }
}

```


3.6 Rolling hash

PLEASE call `init_hash()` in `int main()`!

Usage:

`build(str)` Construct the hasher with given string.
`operator()(l, r)` Get hash value of substring $[l, r)$.

```
1e42 const LL mod = 1006658951440146419, g = 967;
9f60 const int MAXN = 200005;
0291 LL pg[MAXN];
427e
dfe7 inline LL mul(LL x, LL y) { return __int128_t(x) * y % mod; }
427e
599a void init_hash() { // must be called in `int main()`
286f     pg[0] = 1;
4af8     for (int i = 1; i < MAXN; i++) pg[i] = mul(pg[i-1], g);
95cf }
427e
7e62 struct hasher {
534a     LL val[MAXN];
427e
4554     void build(const char *str) { // assume lower-case letter only
f937         for (int i = 0; str[i]; i++)
9645             val[i+1] = (mul(val[i], g) + str[i]) % mod;
95cf     }
427e
19f8     LL operator() (int l, int r) { // [l, r)
9986         return (val[r] - mul(val[l], pg[r-l]) + mod) % mod;
95cf     }
329b };
```

4 Math

4.1 Extended Euclidean algorithm and Chinese remainder theorem

```
4fba void exgcd(LL a, LL b, LL &g, LL &x, LL &y) {
7db6     if (!b) g = a, x = 1, y = 0;
037f     else {
ffca         exgcd(b, a % b, g, y, x);
d798         y -= x * (a / b);
95cf     }
95cf }
```

```
LL crt(LL r[], LL p[], int n) {
    LL q = 1, ret = 0;
    rep (i, n) q *= p[i];
    rep (i, n) {
        LL m = q / p[i];
        LL d, x, y;
        exgcd(p[i], m, d, x, y);
        ret = (ret + y * m * r[i]) % q;
    }
    return (q + ret) % q;
}
```

427e
e491
84e6
00d9
be8e
98b4
9f4f
b082
3cd3
95cf
2e47
95cf

4.2 Linear basis

```
const int MAXD = 30;
struct linearbasis {
    ULL b[MAXD] = {};

    bool insert(LL v) {
        for (int j = MAXD - 1; j >= 0; j--) {
            if (!(v & (1ll << j))) continue;
            if (b[j]) v ^= b[j]
            else {
                for (int k = 0; k < j; k++)
                    if (v & (1ll << k)) v ^= b[k];
                for (int k = j + 1; k < MAXD; k++)
                    if (b[k] & (1ll << j)) b[k] ^= v;
                b[j] = v;
                return true;
            }
        }
        return false;
    }
};
```

8b44
03a6
3558
427e
1566
9b2b
de36
ee78
037f
7836
f0b4
b0aa
46c9
8295
3361
95cf
95cf
438e
95cf
329b

4.3 Gauss elimination over finite field

```
const LL p = 1000000007;

LL powmod(LL b, LL e) {
```

b784
427e
2a2c

```

95a2 LL r = 1;
3e90 while (e) {
1783     if (e & 1) r = r * b % p;
5549     b = b * b % p;
16fc     e >>= 1;
95cf }
547e return r;
95cf }

427e
c130 typedef vector<LL> VLL;
42ac typedef vector<VLL> VVLL;
427e
2c62 LL gauss(VVLL &a, VVLL &b) {
561b     const int n = a.size(), m = b[0].size();
a25e     vector<int> irow(n), icol(n), ipiv(n);
2976     LL det = 1;
427e
be8e     rep (i, n) {
d2b5         int pj = -1, pk = -1;
6b4a         rep (j, n) if (!ipiv[j])
e582             rep (k, n) if (!ipiv[k])
6112                 if (pj == -1 || a[j][k] > a[pj][pk]) {
a905                     pj = j;
657b                     pk = k;
95cf                 }
d480         if (a[pj][pk] == 0) return 0;
0305         ipiv[pk]++;
8dad         swap(a[pj], a[pk]);
aad8         swap(b[pj], b[pk]);
be4d         if (pj != pk) det = (p - det) % p;
d080         irow[i] = pj;
f156         icol[i] = pk;
427e
4ecd         LL c = powmod(a[pk][pk], p - 2);
865b         det = det * a[pk][pk] % p;
c36a         a[pk][pk] = 1;
dd36         rep (j, n) a[pk][j] = a[pk][j] * c % p;
1b23         rep (j, m) b[pk][j] = b[pk][j] * c % p;
f8f3         rep (j, n) if (j != pk) {
e97f             c = a[j][pk];
c449             a[j][pk] = 0;
820b             rep (k, n) a[j][k] = (a[j][k] + p - a[pk][k] * c % p) % p;
f039             rep (k, m) b[j][k] = (b[j][k] + p - b[pk][k] * c % p) % p;
95cf         }

```

```

}

for (int j = n - 1; j >= 0; j--) if (irow[j] != icol[j]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[j]], a[k][icol[j]]);
}
return det;
}

```

95cf
427e
37e1
50dc
95cf
f27f
95cf

4.4 Berlekamp-Massey algorithm

Call `berlekamp()` with input sequence $(x_0, x_1, \dots, x_{n-1})$. Return a vector of coefficients $(c_0 = 1, c_1, \dots, c_{m-1})$ with minimum m , such that $\sum_{i=0}^m c_i x_{j-i} = 0$ for all possible j .

```

LL mod = 1000000007;
vector<LL> berlekamp(const vector<LL>& a) {
    vector<LL> p = {1}, r = {1};
    LL dif = 1;
    rep (i, a.size()) {
        LL u = 0;
        rep (j, p.size()) u = (u + p[j] * a[i-j]) % mod;
        if (u == 0) {
            r.insert(r.begin(), 0);
        } else {
            auto op = p;
            p.resize(max(p.size(), r.size() + 1));
            LL idif = powmod(dif, mod - 2);
            rep (j, r.size())
                p[j+1] = (p[j+1] - r[j] * idif % mod * u % mod + mod) % mod;
            dif = u; r = op;
        }
    }
    return p;
}

```

6e50
97db
8904
075b
8bc9
1b35
bd0b
eae9
b14c
8e2e
0c78
02f6
0a2e
9b57
dacc
bcd1
95cf
95cf
e149
95cf

4.5 Fast Walsh-Hadamard transform

```

void fwt(int* a, int n){
    for (int d = 1; d < n; d <= 1)
        for (int i = 0; i < n; i += d < 1)
            rep (j, d){
                int x = a[i+j], y = a[i+j+d];

```

061e
5595
05f2
b833
7796

```

427e          // a[i+j] = x+y, a[i+j+d] = x-y;    // xor
427e          // a[i+j] = x+y;                    // and
427e          // a[i+j+d] = x+y;                    // or
95cf      }
95cf  }
427e
4db1 void ifwt(int* a, int n){
5595     for (int d = 1; d < n; d <= 1)
05f2         for (int i = 0; i < n; i += d << 1)
b833             rep (j, d){
7796                 int x = a[i+j], y = a[i+j+d];
427e                 // a[i+j] = (x+y)/2, a[i+j+d] = (x-y)/2;    // xor
427e                 // a[i+j] = x-y;                            // and
427e                 // a[i+j+d] = y-x;                            // or
95cf             }
95cf }
427e
2ab6 void conv(int* a, int* b, int n){
950a     fwt(a, n);
e427     fwt(b, n);
8a42     rep(i, n) a[i] *= b[i];
430f     ifwt(a, n);
95cf }

```

4.6 Fast fourier transform

```

4e09 const int NMAX = 1<<20;
427e
3fbf typedef complex<double> cplx;
427e
abd1 const double PI = 2*acos(0.0);
12af struct FFT{
c47c     int rev[NMAX];
27d7     cplx omega[NMAX], oinv[NMAX];
9827     int K, N;
427e
1442     FFT(int k){
e209         K = k; N = 1 << k;
b393         rep (i, N){
7ba3             rev[i] = (rev[i>>1]>>1) | ((i&1)<<(K-1));
1908             omega[i] = polar(1.0, 2.0 * PI / N * i);
a166             oinv[i] = conj(omega[i]);

```

```

    }
}

void dft(cplx* a, cplx* w){
    rep (i, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int l = 2; l <= N; l *= 2){
        int m = l/2;
        for (cplx* p = a; p != a + N; p += l)
            rep (k, m){
                cplx t = w[N/l*k] * p[k+m];
                p[k+m] = p[k] - t; p[k] += t;
            }
    }
}

void fft(cplx* a){dft(a, omega);}
void ifft(cplx* a){
    dft(a, oinv);
    rep (i, N) a[i] /= N;
}

void conv(cplx* a, cplx* b){
    fft(a); fft(b);
    rep (i, N) a[i] *= b[i];
    ifft(a);
}
};

```

95cf
95cf
427e
b941
a215
ac6e
2969
b3cf
c24f
fe06
ecbf
95cf
95cf
95cf
427e
617b
a123
3b2f
57fc
95cf
427e
bdc0
6497
12a5
f84e
95cf
329b

4.7 Number theoretic transform

```

const int NMAX = 1<<21;

// 998244353 = 7*17*2^23+1, G = 3
const int P = 1004535809, G = 3; // = 479*2^21+1

struct NTT{
    int rev[NMAX];
    LL omega[NMAX], oinv[NMAX];
    int g, g_inv; // g: g_n = G^((P-1)/n)
    int K, N;

    LL powmod(LL b, LL e){

```

4ab9
427e
427e
fb9a
427e
87ab
c47c
0eda
81af
9827
427e
2a2c

```

95a2     LL r = 1;
3e90     while (e){
6624         if (e&1) r = r * b % P;
489e         b = b * b % P;
16fc         e >>= 1;
95cf     }
547e     return r;
95cf }
427e
f420     NTT(int k){
e209         K = k; N = 1 << k;
7652         g = powmod(G, (P-1)/N);
4b3a         g_inv = powmod(g, N-1);
e04f         omega[0] = oinv[0] = 1;
b393         rep (i, N){
7ba3             rev[i] = (rev[i>>1]>>1) | ((i&1)<<(K-1));
ad4f             if (i){
8d8b                 omega[i] = omega[i-1] * g % P;
9e14                 oinv[i] = oinv[i-1] * g_inv % P;
95cf             }
95cf         }
427e     }
9668     void _ntt(LL* a, LL* w){
a215         rep (i, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
ac6e         for (int l = 2; l <= N; l *= 2){
2969             int m = l/2;
7a1d             for (LL* p = a; p != a + N; p += l)
c24f                 rep (k, m){
0ad3                     LL t = w[N/l*k] * p[k+m] % P;
6209                     p[k+m] = (p[k] - t + P) % P;
fa1b                     p[k] = (p[k] + t) % P;
95cf                 }
95cf             }
95cf         }
427e
92ea     void ntt(LL* a){_ntt(a, omega);}
5daf     void intt(LL* a){
1f2a         LL inv = powmod(N, P-2);
9910         _ntt(a, oinv);
a873         rep (i, N) a[i] = a[i] * inv % P;
95cf     }
427e
3a5b     void conv(LL* a, LL* b){

```

```

        ntt(a); ntt(b);
        rep (i, N) a[i] = a[i] * b[i] % P;
        intt(a);
    }
};

```

```

ad16
e49e
5748
95cf
329b

```

4.8 Sieve of Euler

```

const int MAXX = 1e7+5;
bool p[MAXX];
int prime[MAXX], sz;

void sieve(){
    p[0] = p[1] = 1;
    for (int i = 2; i < MAXX; i++){
        if (!p[i]) prime[sz++] = i;
        for (int j = 0; j < sz && i*prime[j] < MAXX; j++){
            p[i*prime[j]] = 1;
            if (i % prime[j] == 0) break;
        }
    }
}

```

```

cfc3
5861
73ae
427e
9bc6
9628
1ec8
bf28
e82c
b6a9
5f51
95cf
95cf
95cf

```

4.9 Sieve of Euler (General)

```

namespace sieve {
    constexpr int MAXN = 10000007;
    bool p[MAXN]; // true if not prime
    int prime[MAXN], sz;
    int pval[MAXN], pcnt[MAXN];
    int f[MAXN];

    void exec(int N = MAXN) {
        p[0] = p[1] = 1;

        pval[1] = 1;
        pcnt[1] = 0;
        f[1] = 1;

        for (int i = 2; i < N; i++) {
            if (!p[i]) {

```

```

b62e
6589
e982
6ae8
cbf7
6030
427e
76f6
9628
427e
8a8a
bdda
c6b9
427e
a643
01d6

```

```

b2b2     prime[sz++] = i;
37d9     for (LL j = i; j < N; j *= i) {
758c         int b = j / i;
81fd         pval[j] = i * pval[b];
e0f3         pcnt[j] = pcnt[b] + 1;
a96c         f[j] = _____; // f[j] = f(i^pcnt[j])
95cf     }
95cf     }
34c0     for (int j = 0; i * prime[j] < N; j++) {
f87a         int x = i * prime[j]; p[x] = 1;
20cc         if (i % prime[j] == 0) {
9985             pval[x] = pval[i] * prime[j];
3f93             pcnt[x] = pcnt[i] + 1;
8e2e         } else {
cc91             pval[x] = prime[j];
6322             pcnt[x] = 1;
95cf         }
6191         if (x != pval[x]) {
d614             f[x] = f[x / pval[x]] * f[pval[x]]
95cf         }
5f51         if (i % prime[j] == 0) break;
95cf     }
95cf     }
95cf     }
95cf }

```

4.10 Miller-Rabin primality test

The array `a[]` (excluding sentinel, i.e. `LLONG_MAX`) should be

{2}	when $n < 2,047$.
{2, 7, 61}	when $n < 4,759,123,141 (2^{32})$.
{2, 3, 5, 7, 11}	when $n < 2.1 \times 10^{12}$.
{2, 325, 9375, 28178, 450775, 9780504, 1795265022}	when $n < 2^{64}$.

```

f16f bool test(LL n){
59f2     if (n < 3) return n==2;
427e     // ! The array a[] should be modified if the range of x changes.
3f11     const LL a[] = {2LL, 7LL, 61LL, LLONG_MAX};
c320     LL r = 0, d = n-1, x;
f410     while (~d & 1) d >>= 1, r++;
2975     for (int i=0; a[i] < n; i++){
ece1         x = powmod(a[i], d, n); // ! powmod must use for 64bit mulmod

```

```

        if (x == 1 || x == n-1) goto next;
        rep (i, r) {
            x = mulmod(x, x, n);
            if (x == n-1) goto next;
        }
        return false;
next;;
    }
    return true;
}

```

```

7f99
e257
d7ff
8d2e
95cf
438e
d490
95cf
3361
95cf

```

4.11 Integer factorization (Pollard's rho)

```
ULL gcd(ULL a, ULL b) {return b ? gcd(b, a % b) : a;}
```

```

2e6b
427e
54a5
45eb
d3e5
3c69
0964
4753
5952
9e5b
33d5
e1bf
e1bf
a313
95cf
95cf
5d89
95cf

```

```

ULL PollardRho(ULL n){
    ULL c, x, y, d = n;
    if (~n&1) return 2;
    while (d == n){
        x = y = 2;
        d = 1;
        c = rand() % (n - 1) + 1;
        while (d == 1){
            x = (mulmod(x, x, n) + c) % n;
            y = (mulmod(y, y, n) + c) % n;
            y = (mulmod(y, y, n) + c) % n;
            d = gcd(x>y ? x-y : y-x, n);
        }
    }
    return d;
}

```

4.12 Adaptive Simpson's Method

The Simpson's formula has order 3 algebraic precision.

Usage:

<code>integrate(l, r, eps, est, fn)</code>	Integrate the function <code>fn</code> on interval $[l, r]$. <code>eps</code> is the estimated precision, while <code>est</code> is the current estimation, which can be set to arbitrary value initially.
--	---

```
template <typename T>
```

```
b7ec
```

```

9c6c double simpson(double l, double r, T&& f) {
38f4     double mid = (l + r) / 2;
2075     return (f(l) + 4 * f(mid) + f(r)) * (r - l) / 6.0;
95cf }
427e
b7ec template <typename T>
9cbb double integrate(double l, double r, double eps, double est, T&& f) {
38f4     double mid = (l + r) / 2;
5d09     double lv = simpson(l, mid, f), rv = simpson(mid, r, f);
d589     if (fabs(lv + rv - est) <= 15.0 * eps)
036c         return lv + rv + (lv + rv - est) / 15.0;
13c4     return integrate(l, mid, eps, lv, f) + integrate(mid, r, eps, rv, f);
95cf }

```

4.13 Linear Programming (Simplex)

This function solves the following linear program

$$\begin{array}{ll}
 \max & c^T x \\
 \text{s.t.} & Ax \leq b \\
 & x \geq 0
 \end{array}$$

If the program is infeasible, NAN is returned; if the program is unbounded, DBL_MAX is returned; otherwise, the optimal target is returned and the arguments are stored in x.

```

db00 typedef vector<double> VD;
9952 typedef vector<VD> VVD;
89a3 typedef vector<int> VI;
05b7 const double EPS = 1e-9;
427e
5eb7 double LPSolve(VVD A, VD b, VD c, VD& x) {
f1f6     int m = b.size(), n = c.size();
1684     VI B(m), N(n+1);
319d     VVD D(m+2, VD(n+2));
7f8f     rep (i, m) rep (j, n) D[i][j] = A[i][j];
6b6c     rep (i, m) { B[i] = n + i; D[i][n] = -1; D[i][n+1] = b[i]; }
9166     rep (j, n) { N[j] = j; D[m][j] = -c[j]; }
0def     N[n] = -1; D[m+1][n] = 1;
427e
e0f7     auto pivot = [&] (int r, int s) {
3c4b         double inv = 1.0 / D[r][s];
e090         rep (i, m+2) if (i != r) rep (j, n+2) if (j != s)

```

```

        D[i][j] -= D[r][j] * D[i][s] * inv;
        rep (j, n+2) if (j != s) D[r][j] *= inv;
        rep (i, m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv; swap(B[r], N[s]);
    };

    auto simplex = [&](int phase) {
        int x = m + (phase == 1);
        while (true) {
            int s = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 and N[j] == -1) continue;
                if (s == -1 or D[x][j] < D[x][s] or
                    D[x][j] == D[x][s] and N[j] < N[s]) s = j;
            }
            if (s < 0 or D[x][s] > -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] < EPS) continue;
                if (r == -1 or D[i][n+1] / D[i][s] < D[r][n+1] / D[r][s] or
                    D[i][n+1] / D[i][s] == D[r][n+1] / D[r][s] and
                    B[i] < B[r]) r = i;
            }
            if (r == -1) return false; else pivot(r, s);
        }
    };

    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] <= -EPS) {
        pivot(r, n);
        if (!simplex(1) or D[m+1][n+1] < -EPS) return NAN;
        rep (i, m) if (B[i] == -1) {
            int s = -1;
            for (int j = 0; j <= n; j++) if (s == -1 or D[i][j] < D[i][s]
                or D[i][j] == D[i][s] and N[j] < N[s]) s = j;
            pivot(i, s);
        }
    }
    if (!simplex(2)) return DBL_MAX;
    x = VD(n);
    rep (i, m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return D[m][n+1];
}

```

48ea
79f3
73cf
82f1
329b
427e
3f89
adb8
1026
0676
7e4d
30f5
537c
3262
95cf
083a
bfc5
356f
691d
6855
26b3
412f
95cf
d829
95cf
329b
427e
7c08
468b
8257
d48d
0175
fc91
0676
1e86
a48f
c4cd
95cf
95cf
e566
8720
3232
bbe4
95cf

5 Graph Theory

5.1 Strongly connected component

```

837c const int MAXV = 100005;
427e
2ea0 struct graph{
88e3     vector<int> adj[MAXV];
9cad     stack<int> s;
3d02     int V; // number of vertices
8b6c     int pre[MAXV], lnk[MAXV], scc[MAXV];
27ee     int time, sccn;
427e
bfab     void add_edge(int u, int v){
c71a         adj[u].push_back(v);
95cf     }
427e
d714     void dfs(int u){
7e41         pre[u] = lnk[u] = ++time;
80f6         s.push(u);
18f6         for (int v : adj[u]){
173e             if (!pre[v]){
5f3c                 dfs(v);
002c                 lnk[u] = min(lnk[u], lnk[v]);
6068             } else if (!scc[v]){
d5df                 lnk[u] = min(lnk[u], pre[v]);
95cf             }
95cf         }
8de2         if (lnk[u] == pre[u]){
660f             sccn++;
3c9e             int x;
a69f             do {
3834                 x = s.top(); s.pop();
b0e9                 scc[x] = sccn;
6757             } while (x != u);
95cf         }
95cf     }
427e
4c88     void find_scc(){
f4a2         time = sccn = 0;

```

```

         memset(scc, 0, sizeof scc);
         memset(pre, 0, sizeof pre);
         Rep (i, V){
             if (!pre[i]) dfs(i);
         }
     }

     vector<int> adjc[MAXV];
     void contract(){
         Rep (i, V)
             rep (j, adj[i].size()){
                 if (scc[i] != scc[adj[i][j]])
                     adjc[scc[i]].push_back(scc[adj[i][j]]);
             }
     }
};

```

8de7
8c2f
6901
56d1
95cf
95cf
427e
27ce
364d
1a1e
21a2
b730
b46e
95cf
95cf
329b

5.2 Vertex biconnected component

```

const int MAXN = 100005;
struct graph {
     int pre[MAXN], iscut[MAXN], bccno[MAXN], dfs_clock, bcc_cnt;
     vector<int> adj[MAXN], bcc[MAXN];
     set<pair<int, int>> bcce[MAXN];

     stack<pair<int, int>> s;

     void add_edge(int u, int v) {
         adj[u].push_back(v);
         adj[v].push_back(u);
     }

     int dfs(int u, int fa) {
         int lowu = pre[u] = ++dfs_clock;
         int child = 0;
         for (int v : adj[u]) {
             if (!pre[v]) {
                 s.push({u, v});
                 child++;
                 int lowv = dfs(v, u);
                 lowu = min(lowu, lowv);
                 if (lowv >= pre[u]) {

```

0f42
2ea0
33ae
848f
6b06
427e
76f7
427e
bfab
c71a
a717
95cf
427e
7d3c
9fe6
ec14
18f6
173e
e7f8
fdcf
f851
189c
b687

```

6323         iscut[u] = 1;
57eb         bcc[bcc_cnt].clear();
90b8         bcce[bcc_cnt].clear();
a147         while (1) {
a6a3             int xu, xv;
a0c3             tie(xu, xv) = s.top(); s.pop();
0ef5             bcce[bcc_cnt].insert({min(xu, xv), max(xu, xv)});
3db2             if (bccno[xu] != bcc_cnt) {
e0db                 bcc[bcc_cnt].push_back(xu);
d27f                 bccno[xu] = bcc_cnt;
95cf             }
f357             if (bccno[xv] != bcc_cnt) {
752b                 bcc[bcc_cnt].push_back(xv);
57c9                 bccno[xv] = bcc_cnt;
95cf             }
7096             if (xu == u && xv == v) break;
95cf         }
03f5         bcc_cnt++;
95cf     }
7470     } else if (pre[v] < pre[u] && v != fa) {
e7f8         s.push({u, v});
f115         lowu = min(lowu, pre[v]);
95cf     }
95cf }
e104 if (fa < 0 && child == 1) iscut[u] = 0;
1160 return lowu;
95cf }
427e
17be void find_bcc(int n) {
8c2f     memset(pre, 0, sizeof pre);
e2d2     memset(iscut, 0, sizeof iscut);
40d3     memset(bccno, -1, sizeof bccno);
fae2     dfs_clock = bcc_cnt = 0;
5c63     rep (i, n) if (!pre[i]) dfs(i, -1);
95cf }
329b };

```

5.3 Cut vertices

If the graph is unconnected, the algorithm should be run on each component. One may run
Rep (i, n)if (!dfn[i])tarjan(i, i) for unconnected graph.

Usage:

add_edge(u, v) Add an undirected edge (u, v).
tarjan(u, fa) Run Tarjan's algorithm on tree rooted at fa. Please call
with identical u and fa.
cut[v] Whether v is a cut vertex.

```

const int MAXN = 200005;
vector<int> adj[MAXN];
int dfn[MAXN], low[MAXN], idx;
bool cut[MAXN];

void add_edge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}

void tarjan(int u, int fa) {
    dfn[u] = low[u] = ++idx;
    int child = 0;
    for (int v : adj[u]) {
        if (!dfn[v]) {
            tarjan(v, fa); low[u] = min(low[u], low[v]);
            if (low[v] >= dfn[u] && u != fa) cut[u] = true;
            child += u == fa;
        }
        low[u] = min(low[u], dfn[v]);
    }
    if (u == fa && child > 1) cut[u] = true;
}

```

5.4 Minimum spanning arborescence, faster

All vertices are 1-based. Clear the fields when reuse the struct.

Usage:

add_edge(u, v, w) Add an edge from u to v with weight w .
run(n, rt) Compute the total weight of MSA rooted at rt. If not
exist, return LLONG_MIN.

Time Complexity: $O(|E| \log^2 |V|)$

```

const int MAXN = 300005;
typedef pair<LL, int> pii;
struct MDST {
    priority_queue<pii, vector<pii>, greater<pii>> heap[MAXN];
    LL shift[MAXN];
}

```



```

fc06     int fa[MAXN], vis[MAXN];
427e
38dd     int find(int x) { return fa[x] == x ? x : fa[x] = find(fa[x]); }
427e
29b0     void unite(int x, int y) {
0c14         x = find(x); y = find(y); fa[y] = x; if (x == y) return;
6fa0         if (heap[x].size() < heap[y].size()) {
9c26             swap(heap[x], heap[y]);
2ffc             swap(shift[x], shift[y]);
95cf         }
9959         while (heap[y].size()) {
175b             auto p = heap[y].top(); heap[y].pop();
c0c5             heap[x].emplace(p.first - shift[y] + shift[x], p.second);
95cf         }
95cf     }
427e
0bbd     void add_edge(int u, int v, LL w) { heap[v].emplace(w, u); }
427e
a526     LL run(int n, int rt) {
f7ff         LL ans = 0;
81f2         iota(fa, fa + n + 1, 0);
19b3         Rep (i, n) if (find(i) != find(rt)) {
a7b1             int u = find(i);
010e             stack<int, vector<int>> s;
eff5             while (find(u) != find(rt)) {
0dda                 if (vis[u]) while (s.top() != u) {
c593                     vis[s.top()] = 0; unite(u, s.top()); s.pop();
83c4                 } else { vis[u] = 1; s.push(u); }
c76e                 while (heap[u].size()) {
b385                     ans += heap[u].top().first - shift[u];
dde2                     shift[u] = heap[u].top().first;
da47                     if (find(heap[u].top().second) != u) break;
9fbb                     heap[u].pop();
95cf                 }
6961                 if (heap[u].empty()) return LLONG_MIN;
87e6                 u = find(heap[u].top().second);
95cf             }
2d46             while (s.size()) { vis[s.top()] = 0; unite(rt, s.top()); s.pop(); }
95cf         }
4206         return ans;
95cf     }
329b };

```

5.5 Maximum flow (Dinic)

Usage:

add_edge(u, v, c) Add an edge from u to v with capacity c .

max_flow(s, t) Compute maximum flow from s to t .

Time Complexity: For general graph, $O(V^2E)$; for network with unit capacity, $O(\min\{V^{2/3}, \sqrt{E}\}E)$; for bipartite network, $O(\sqrt{VE})$.

```

struct edge{
    int from, to;
    LL cap, flow;
};

const int MAXN = 1005;
struct Dinic {
    int n, m, s, t;
    vector<edge> edges;
    vector<int> G[MAXN];
    bool vis[MAXN];
    int d[MAXN];
    int cur[MAXN];

    void add_edge(int from, int to, LL cap) {
        edges.push_back(edge{from, to, cap, 0});
        edges.push_back(edge{to, from, 0, 0});
        m = edges.size();
        G[from].push_back(m-2);
        G[to].push_back(m-1);
    }

    bool bfs() {
        memset(vis, 0, sizeof(vis));
        queue<int> q;
        q.push(s);
        vis[s] = 1;
        d[s] = 0;
        while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int i = 0; i < G[x].size(); i++) {
                edge& e = edges[G[x][i]];
                if (!vis[e.to] && e.cap > e.flow) {
                    vis[e.to] = 1;
                    d[e.to] = d[x] + 1;
                    q.push(e.to);

```

```

bcf8
60e2
5e6d
329b
427e
e2cd
9062
4dbf
9f0c
b891
bbb6
b40a
ddcc
427e
5973
7b55
1db7
fe77
dff5
8f2d
95cf
427e
1836
3b73
93d2
5d13
2cd2
721d
cc78
66ba
3b61
b510
bba9
cd72
cf26
ca93

```

```

95cf    }
95cf    }
95cf    }
b23b    return vis[t];
95cf    }
427e
9252    LL dfs(int x, LL a) {
6904        if (x == t || a == 0) return a;
8bf9        LL flow = 0, f;
f515        for (int& i = cur[x]; i < G[x].size(); i++) {
b510            edge& e = edges[G[x][i]];
2374            if(d[x] + 1 == d[e.to] && (f = dfs(e.to, min(a, e.cap-e.flow))) > 0)
                {
1cce                e.flow += f;
e16d                edges[G[x][i]^1].flow -= f;
a74d                flow += f;
23e5                a -= f;
97ed                if(a == 0) break;
95cf            }
95cf        }
84fb        return flow;
95cf    }
427e
5bf2    LL max_flow(int s, int t) {
590d        this->s = s; this->t = t;
62e2        LL flow = 0;
ed58        while (bfs()) {
f326            memset(cur, 0, sizeof(cur));
fb3a            flow += dfs(s, LLONG_MAX);
95cf        }
84fb        return flow;
95cf    }
427e
c72e    vector<int> min_cut() { // call this after maxflow
1df9        vector<int> ans;
df9a        for (int i = 0; i < edges.size(); i++) {
56d8            edge& e = edges[i];
46a2            if(vis[e.from] && !vis[e.to] && e.cap > 0) ans.push_back(i);
95cf        }
4206        return ans;
95cf    }
329b    };

```

5.6 Maximum cardinality bipartite matching (Hungarian)

```

#include <bits/stdc++.h>
using namespace std;

#define rep(i, n) for (int i = 0; i < (n); i++)
#define Rep(i, n) for (int i = 1; i <= (n); i++)
#define range(x) (x).begin(), (x).end()
typedef long long LL;

struct Hungarian{
    int nx, ny;
    vector<int> mx, my;
    vector<vector<int>> > e;
    vector<bool> mark;

    void init(int nx, int ny){
        this->nx = nx;
        this->ny = ny;
        mx.resize(nx); my.resize(ny);
        e.clear(); e.resize(nx);
        mark.resize(nx);
    }

    inline void add(int a, int b){
        e[a].push_back(b);
    }

    bool augment(int i){
        if (!mark[i]) {
            mark[i] = true;
            for (int j : e[i]){
                if (my[j] == -1 || augment(my[j])){
                    mx[i] = j; my[j] = i;
                    return true;
                }
            }
        }
        return false;
    }

    int match(){
        int ret = 0;

```

```

302f
421c
427e
0d6c
cfe3
8843
5cad
427e
84ee
fbf6
9ec6
9d4c
edec
427e
8324
c1d1
f9c1
ac92
3f11
1023
95cf
427e
4589
486c
95cf
427e
0c2b
207c
dae4
6a1e
0892
9ca3
3361
95cf
95cf
95cf
438e
95cf
427e
3fac
5b57

```

```

b0f1     fill(range(mx), -1);
b957     fill(range(my), -1);
4ed1     rep (i, nx){
13a5         fill(range(mark), false);
cc89         if (augment(i)) ret++;
95cf     }
ee0f     return ret;
95cf     }
329b };
    
```

5.7 Maximum matching of general graph (Edmond's blossom)

Usage:

`init(n)` Initialize the template with n vertices, numbered from 1.

`add_edge(u, v)` Add an undirected edge uv .

`solve()` Find the maximum matching. Return the number of matched edges.

`mate[]` The mate of a matched vertex. If it is not matched, then the value is 0.

Time Complexity: $O(|V|^3)$, but extremely fast in practice.

```

c041 const int MAXN = 1024;
6ab1 struct Blossom {
0b32     vector<int> adj[MAXN];
93d2     queue<int> q;
5c83     int n;
0de2     int label[MAXN], mate[MAXN], save[MAXN], used[MAXN];
427e
2186 void init(int nv) {
3728     n = nv; for (auto& v : adj) v.clear();
477d     fill(range(label), 0); fill(range(mate), 0);
bb35     fill(range(save), 0); fill(range(used), 0);
95cf }
427e
c2dd void add_edge(int u, int v) { adj[u].push_back(v); adj[v].push_back(u); }
427e
2a48 void rematch(int x, int y) {
8af8     int m = mate[x]; mate[x] = y;
1aa4     if (mate[m] == x) {
f4ba         if (label[x] <= n) {
740a             mate[m] = label[x]; rematch(label[x], m);
8e2e         } else {
    
```

```

        int a = 1 + (label[x] - n - 1) / n;
        int b = 1 + (label[x] - n - 1) % n;
        rematch(a, b); rematch(b, a);
    }
}

void traverse(int x) {
    Rep (i, n) save[i] = mate[i];
    rematch(x, x);
    Rep (i, n) {
        if (mate[i] != save[i]) used[i] ++;
        mate[i] = save[i];
    }
}

void relabel(int x, int y) {
    Rep (i, n) used[i] = 0;
    traverse(x); traverse(y);
    Rep (i, n) {
        if (used[i] == 1 and label[i] < 0) {
            label[i] = n + x + (y - 1) * n;
            q.push(i);
        }
    }
}

int solve() {
    Rep (i, n) {
        if (mate[i]) continue;
        Rep (j, n) label[j] = -1;
        label[i] = 0; q = queue<int>(); q.push(i);
        while (q.size()) {
            int x = q.front(); q.pop();
            for (int y : adj[x]) {
                if (mate[y] == 0 and i != y) {
                    mate[y] = x; rematch(x, y); q = queue<int>(); break;
                }
                if (label[y] >= 0) { relabel(x, y); continue; }
                if (label[mate[y]] < 0) {
                    label[mate[y]] = x; q.push(mate[y]);
                }
            }
        }
    }
}
    
```

```

95cf    }
8abb    int cnt = 0;
b52f    Rep (i, n) cnt += (mate[i] > i);
6808    return cnt;
95cf    }
329b    };

```

5.8 Minimum cost maximum flow

```

bcf8    struct edge{
60e2        int from, to;
d698        int cap, flow;
32cc        LL cost;
329b    };
427e
cc3e    const LL INF = LLONG_MAX / 2;
2aa8    const int MAXN = 5005;
c6cb    struct MCMF {
9ceb        int s, t, n, m;
9f0c        vector<edge> edges;
b891        vector<int> G[MAXN];
f74f        bool inq[MAXN]; // queue
8f67        LL d[MAXN];    // distance
9524        int p[MAXN];    // previous
b330        int a[MAXN];    // improvement
427e
f7f2        void add_edge(int from, int to, int cap, LL cost) {
24f0            edges.push_back(edge{from, to, cap, 0, cost});
95f0            edges.push_back(edge{to, from, 0, 0, -cost});
fe77            m = edges.size();
dff5            G[from].push_back(m-2);
8f2d            G[to].push_back(m-1);
95cf        }
427e
3c52        bool spfa(){
93d2            queue<int> q;
8494            fill(d, d + MAXN, INF); d[s] = 0;
fd48            memset(inq, 0, sizeof(inq));
5e7c            q.push(s); inq[s] = true;
2dae            p[s] = 0; a[s] = INT_MAX;
cc78            while (!q.empty()){
b0aa                int u = q.front(); q.pop(); inq[u] = false;

```

```

        for (int i : G[u]) {
            edge& e = edges[i];
            if (e.cap > e.flow && d[e.to] > d[u] + e.cost){
                d[e.to] = d[u] + e.cost;
                p[e.to] = G[u][i];
                a[e.to] = min(a[u], e.cap - e.flow);
                if (!inq[e.to]) q.push(e.to), inq[e.to] = true;
            }
        }
    }
    return d[t] != INF;
}

void augment(){
    int u = t;
    while (u != s){
        edges[p[u]].flow += a[t];
        edges[p[u]^1].flow -= a[t];
        u = edges[p[u]].from;
    }
}

#ifdef GIVEN_FLOW
bool min_cost(int s, int t, int f, LL& cost) {
    this->s = s; this->t = t;
    int flow = 0;
    cost = 0;
    while (spfa()) {
        augment();
        if (flow + a[t] >= f){
            cost += (f - flow) * d[t]; flow = f;
            return true;
        } else {
            flow += a[t]; cost += a[t] * d[t];
        }
    }
    return false;
}
#else
int min_cost(int s, int t, LL& cost) {
    this->s = s; this->t = t;
    int flow = 0;
    cost = 0;
    while (spfa()) {

```

```

3bba
56d8
3601
55bc
0bea
8249
e5d3
95cf
95cf
95cf
6d7c
95cf
427e
71a4
06f1
b19d
db09
25a9
e6c9
95cf
95cf
427e
6e20
5972
590d
21d4
23cb
22dc
bcdb
a671
b14d
3361
8e2e
2a83
95cf
95cf
438e
95cf
a8cb
f9a9
590d
21d4
23cb
22dc

```

```

bcd b      augment();
2a83      flow += a[t]; cost += a[t] * d[t];
95c f      }
84f b      return flow;
95c f      }
1937 #endif
329 b };

```

5.9 Fast LCA

All indices of the tree are 1-based.

Usage:

preprocess(root) Initialize with tree rooted at root.
lca(u, v) Query the lowest common ancestor of u and v .

```

0e34 const int MAXN = 500005;
0b32 vector<int> adj[MAXN];
fcc b int id[MAXN], nid;
1356 pair<int, int> st[MAXN << 1][33 - __builtin_clz(MAXN)];
427e
e16d void dfs(int u, int p, int d) {
0df2     st[id[u] = nid++][0] = {d, u};
18f6     for (int v : adj[u]) {
bd87         if (v == p) continue;
f58c         dfs(v, u, d + 1);
08ad         st[nid++][0] = {d, u};
95c f     }
95c f }
427e
3d1b void preprocess(int root) {
3269     nid = 0;
91e1     dfs(root, 0, 1);
5e98     int l = 31 - __builtin_clz(nid);
213b     rep (j, l) rep (i, 1+nid-(1<<j))
1131         st[i][j+1] = min(st[i][j], st[i+(1<<j)][j]);
95c f }
427e
0f0b int lca(int u, int v) {
cfc4     tie(u, v) = minmax(id[u], id[v]);
be9b     int k = 31 - __builtin_clz(v-u+1);
8ebc     return min(st[u][k], st[v-(1<<k)+1][k]).second;
95c f }

```

5.10 Heavy-light decomposition

Time Complexity: The decomposition itself takes linear time. Each query takes $O(\log n)$ operations.

```

const int MAXN = 100005;
vector<int> adj[MAXN];
int sz[MAXN], top[MAXN], fa[MAXN], son[MAXN], depth[MAXN], id[MAXN];

void dfs1(int x, int dep, int par){
    depth[x] = dep;
    sz[x] = 1;
    fa[x] = par;
    int maxn = 0, s = 0;
    for (int c: adj[x]){
        if (c == par) continue;
        dfs1(c, dep + 1, x);
        sz[x] += sz[c];
        if (sz[c] > maxn){
            maxn = sz[c];
            s = c;
        }
    }
    son[x] = s;
}

int cid = 0;
void dfs2(int x, int t){
    top[x] = t;
    id[x] = ++cid;
    if (son[x]) dfs2(son[x], t);
    for (int c: adj[x]){
        if (c == fa[x]) continue;
        if (c == son[x]) continue;
        else dfs2(c, c);
    }
}

void decomp(int root){
    dfs1(root, 1, 0);
    dfs2(root, root);
}

void query(int u, int v){

```

0f42
0b32
42f2
427e
be5c
7489
2ee7
adb4
b79d
c861
fe45
fd2f
b790
f0f1
c749
fe19
95cf
95cf
0e08
95cf
427e
ba54
3644
8d96
d314
c4a1
c861
9881
5518
13f9
95cf
95cf
427e
0f04
9fa4
1c88
95cf
427e
2c98

```
03a1 while (top[u] != top[v]){
45ec     if (depth[top[u]] < depth[top[v]]) swap(u, v);
427e     // id[top[u]] to id[u]
005b     u = fa[top[u]];
95cf }
6083 if (depth[u] > depth[v]) swap(u, v);
427e // id[u] to id[v]
95cf }
```

5.11 Centroid decomposition

Note that the centroid here is not the exact centroid of the graph. It only guarantees that the size of each subtree does not exceed half of that of the original tree. This is enough to guarantee the correct time complexity. All vertices are numbered from 1. Call `decomp(root)` to use.

Usage:

`decomp(u, p)` Decompose the tree rooted at u with parent p .

Time Complexity: The decomposition itself takes $O(n \log n)$ time.

```
1fb6 vector<int> adj[100005];
88e0 int sz[100005], sum;
427e
f93d void getsz(int u, int p) {
5b36     sz[u] = 1; sum++;
18f6     for (int v : adj[u]) {
bd87         if (v == p) continue;
e3cb         getsz(v, u);
8449         sz[u] += sz[v];
95cf     }
95cf }
427e
67f9 int getcent(int u, int p) {
d51f     for (int v : adj[u])
76e4         if (v != p and sz[v] > sum / 2)
18e3             return getcent(v, u);
81b0     return u;
95cf }
427e
4662 void decompose(int u) {
618e     sum = 0; getsz(u, 0);
303c     u = getcent(u, 0); // update u to the centroid
427e
18f6     for (int v : adj[u]) {
```

```
// get answer for subtree v
}
// get answer for the whole tree
// don't forget to count the centroid itself

for (int v : adj[u]) { // divide and conquer
    adj[v].erase(find(range(adj[v]), u));
    decompose(v);
    adj[v].push_back(u); // restore deleted edge
}
}
```

427e
95cf
427e
427e
427e
18f6
c375
fa6b
a717
95cf
95cf

5.12 DSU on tree

This implementation avoids parallel existence of multiple data structures but requires that the data structure is invertible. To use this template, implement `merge`, `enter`, `leave` as needed; first call `decomp(root, 0)`, then call `work(root, 0, false)`. Labels of vertices start from 1.

Usage:

`decomp(u, p)` Decompose the tree u .
`work(u, p, keep)` Work for subtree u . When `keep` is set, information is not cleared.

Time Complexity: $O(n \log n)$ times the complexity for `merge`, `enter`, `leave`.

```
vector<int> adj[100005];
int sz[100005], son[100005];

void decomp(int u, int p) {
    sz[u] = 1;
    for (int v : adj[u]) {
        if (v == p) continue;
        decomp(v, u);
        sz[u] += sz[v];
        if (sz[v] > sz[son[u]]) son[u] = v;
    }
}

template <typename T>
void trav(T fn, int u, int p) {
    fn(u);
    for (int v : adj[u]) if (v != p) trav(fn, v, u);
}
```

1fb6
901d
427e
5559
50c0
18f6
bd87
a851
8449
d28c
95cf
95cf
427e
b7ec
62f5
4412
30b3
95cf

```

427e
7467 #define for_light(v) for (int v : adj[u]) if (v != p and v != son[u])
33ff void work(int u, int p, bool keep) {
72a2     for_light(v) work(v, u, 0); // process light children
427e
427e     // process heavy child
427e     // current data structure contains info of heavy child
9866     if (son[u]) work(son[u], u, 1);
427e
18a9     auto merge = [u] (int c) { /* count contribution of c */ };
1ab0     auto enter = [] (int c) { /* add vertex c */ };
f241     auto leave = [] (int c) { /* remove vertex c */ };
427e
3d3b     for_light(v) {
74c6         trav(merge, v, u);
c13d         trav(enter, v, u);
95cf     }
427e
427e     // count answer for root and add it
427e     // Warning: special check may apply to root!
c54f     merge(u);
9dec     enter(u);
427e
427e     // Leave current tree
4e3e     if (!keep) trav(leave, u, p);
95cf }

```

6 Data Structures

6.1 Fenwick tree (point update range query)

```

9976 struct bit_purq { // point update, range query
d7af     int N;
99ff     vector<LL> tr;
427e
456d     void init(int n) { tr.resize(N = n + 5); }
427e
63d0     LL sum(int n) {
f7ff         LL ans = 0;
6770         while (n) { ans += tr[n]; n &= n - 1; }
4206         return ans;

```

```

    }
    void add(int n, LL x){
        while (n < N) { tr[n] += x; n += n & -n; }
    }
};

```

6.2 Fenwick tree (range update point query)

```

struct bit_rupq{ // range update, point query
    int N;
    vector<LL> tr;

    void init(int n) { tr.resize(N = n + 5);}

    LL query(int n) {
        LL ans = 0;
        while (n < N) { ans += tr[n]; n += n & -n; }
        return ans;
    }

    void add(int n, LL x) {
        while (n) { tr[n] += x; n &= n - 1; }
    }
};

```

6.3 Segment tree

```

LL p;
const int MAXN = 4 * 100006;
struct segtree {
    int l[MAXN], m[MAXN], r[MAXN];
    LL val[MAXN], tadd[MAXN], tmul[MAXN];

#define lson (o<<1)
#define rson (o<<1|1)

    void pull(int o) {
        val[o] = (val[lson] + val[rson]) % p;
    }

```

```

e4bc void push_add(int o, LL x) {
5dd6     val[o] = (val[o] + x * (r[o] - l[o])) % p;
6eff     tadd[o] = (tadd[o] + x) % p;
95cf }
427e
d658 void push_mul(int o, LL x) {
b82c     val[o] = val[o] * x % p;
aa86     tadd[o] = tadd[o] * x % p;
649f     tmul[o] = tmul[o] * x % p;
95cf }
427e
b149 void push(int o) {
3159     if (l[o] == m[o]) return;
0a90     if (tmul[o] != 1) {
0f4a         push_mul(lson, tmul[o]);
045e         push_mul(rson, tmul[o]);
ac0a         tmul[o] = 1;
95cf     }
1b82     if (tadd[o]) {
9547         push_add(lson, tadd[o]);
0e73         push_add(rson, tadd[o]);
6234         tadd[o] = 0;
95cf     }
95cf }
427e
471c void build(int o, int ll, int rr) {
0e87     int mm = (ll + rr) / 2;
9d27     l[o] = ll; r[o] = rr; m[o] = mm;
ac0a     tmul[o] = 1;
5c92     if (ll == mm) {
001f         scanf("%lld", val + o);
e5b6         val[o] %= p;
8e2e     } else {
7293         build(lson, ll, mm);
5e67         build(rson, mm, rr);
ba26         pull(o);
95cf     }
95cf }
427e
4406 void add(int o, int ll, int rr, LL x) {
3c16     if (ll <= l[o] && r[o] <= rr) {
db32         push_add(o, x);
8e2e     } else {
c4b0         push(o);

```

```

        if (m[o] > ll) add(lson, ll, rr, x);
        if (m[o] < rr) add(rson, ll, rr, x);
        pull(o);
    }
}

void mul(int o, int ll, int rr, LL x) {
    if (ll <= l[o] && r[o] <= rr) {
        push_mul(o, x);
    } else {
        push(o);
        if (ll < m[o]) mul(lson, ll, rr, x);
        if (m[o] < rr) mul(rson, ll, rr, x);
        pull(o);
    }
}

LL query(int o, int ll, int rr) {
    if (ll <= l[o] && r[o] <= rr) {
        return val[o];
    } else {
        push(o);
        if (rr <= m[o]) return query(lson, ll, rr);
        if (ll >= m[o]) return query(rson, ll, rr);
        return query(lson, ll, rr) + query(rson, ll, rr);
    }
}
} seg;

```

```

4305
d5a6
ba26
95cf
95cf
427e
48cd
3c16
e7d0
8e2e
c4b0
d1ba
67f3
ba26
95cf
95cf
427e
0f62
3c16
6dfe
8e2e
c4b0
462a
5cca
bbf9
95cf
95cf
4d99

```

6.4 Treap

Self-balanced binary search tree which supports split and merge.

Usage:

<code>push(x)</code>	Push lazy tags to children.
<code>pull(x)</code>	Update statistics of node x .
<code>Init(x, v)</code>	Initialize node x with value v .
<code>Add(x, v)</code>	Apply addition to subtree x .
<code>Reverse(x)</code>	Apply reversion to subtree x .
<code>Merge(x, y)</code>	Merge trees rooted at x and y . Return the root of new tree.
<code>Split(t, k, x, y)</code>	Split out the left k elements of tree t . The roots of left part and right part are stored in x and y , respectively.
<code>init(n)</code>	Initialize the treap with array of size n .
<code>work(op, l, r)</code>	Range operation over $[l, r)$.
Time Complexity: Expected $O(\log n)$ per operation.	

```

const int MAXN = 200005;
mt19937 gen(time(NULL));

struct Treap {
    int ch[MAXN][2];
    int sz[MAXN], key[MAXN], val[MAXN];
    int add[MAXN], rev[MAXN];
    LL sum[MAXN] = {0};
    int maxv[MAXN] = {INT_MIN}, minv[MAXN] = {INT_MAX};

    void Init(int x, int v) {
        ch[x][0] = ch[x][1] = 0;
        key[x] = gen(); val[x] = v; pull(x);
    }

    void pull(int x) {
        sz[x] = 1 + sz[ch[x][0]] + sz[ch[x][1]];
        sum[x] = val[x] + sum[ch[x][0]] + sum[ch[x][1]];
        maxv[x] = max({val[x], maxv[ch[x][0]], maxv[ch[x][1]]});
        minv[x] = min({val[x], minv[ch[x][0]], minv[ch[x][1]]});
    }

    void Add(int x, int a) {
        val[x] += a; add[x] += a;
        sum[x] += LL(sz[x]) * a; maxv[x] += a; minv[x] += a;
    }

    void Reverse(int x) {
        rev[x] ^= 1;
        swap(ch[x][0], ch[x][1]);
    }
}

```

```

void push(int x) {
    for (int c : ch[x]) if (c) {
        Add(c, add[x]);
        if (rev[x]) Reverse(c);
    }
    add[x] = 0; rev[x] = 0;
}

int Merge(int x, int y) {
    if (!x || !y) return x | y;
    push(x); push(y);
    if (key[x] > key[y]) {
        ch[x][1] = Merge(ch[x][1], y); pull(x); return x;
    } else {
        ch[y][0] = Merge(x, ch[y][0]); pull(y); return y;
    }
}

void Split(int t, int k, int &x, int &y) {
    if (t == 0) { x = y = 0; return; }
    push(t);
    if (sz[ch[t][0]] < k) {
        x = t; Split(ch[t][1], k - sz[ch[t][0]] - 1, ch[t][1], y);
    } else {
        y = t; Split(ch[t][0], k, x, ch[t][0]);
    }
    if (x) pull(x); if (y) pull(y);
}

} treap;

int root;

void init(int n) {
    Rep (i, n) {
        int x; scanf("%d", &x);
        treap.Init(i, x);
        root = (i == 1) ? 1 : treap.Merge(root, i);
    }
}

void work(int op, int l, int r) {
    int tl, tm, tr;
    treap.Split(root, l, tl, tm);
    treap.Split(tm, r - l, tm, tr);
}

```

1a53
5fe5
fd76
7a53
95cf
49ee
95cf
427e
9d2c
1b09
cd7e
bffa
a3df
8e2e
bf9e
95cf
95cf
427e
dc7e
6303
f26b
3465
ffdf
8e2e
8a23
95cf
89e3
95cf
b1f4
427e
24b6
427e
d34f
34d7
7681
0ed8
bcc8
95cf
95cf
427e
d030
6639
b6c4
8de3

```

3658     if (op == 1) {
c039         int x; scanf("%d", &x); treap.Add(tm, x);
1dcb     } else if (op == 2) {
ae78         treap.Reverse(tm);
581d     } else if (op == 3) {
e092         printf("%lld_%d_%d\n",
867f             treap.sum[tm], treap.minv[tm], treap.maxv[tm]);
95cf     }
6188     root = treap.Merge(treap.Merge(tl, tm), tr);
95cf }

```

6.5 Link/cut tree

Dynamic connectivity of undirected acyclic graph. Support single-vertex update, path aggregation and relative LCA query. Vertices are numbered from 1. Zero initialization is enough except for the statistic information.

Usage:

<code>pull(x)</code>	Update statistics of node x .
<code>Root(u)</code>	Get the root of tree where vertex u is in.
<code>Link(u, v)</code>	Link two unconnected trees.
<code>Cut(u, v)</code>	Cut an existent edge.
<code>Query(u, v)</code>	Path aggregation.
<code>Update(u, x)</code>	Single point modification.
<code>LCA(u, v, root)</code>	Get the lowest common ancestor of u and v in tree rooted at root.

Time Complexity: $O(\log n)$ per operation

```

2e73 const int MAXN = 1000005;
ca06 struct LCT {
6a6d     int fa[MAXN], ch[MAXN][2], val[MAXN], sum[MAXN];
c6e1     bool rev[MAXN];
427e
eba3     bool isroot(int x) { return ch[fa[x]][0] == x || ch[fa[x]][1] == x; }
f19f     void pull(int x) { sum[x] = val[x] ^ sum[ch[x][0]] ^ sum[ch[x][1]]; }
1c4d     void reverse(int x) { swap(ch[x][0], ch[x][1]); rev[x] ^= 1; }
1a53     void push(int x) {
89a0         if (rev[x]) rep (i, 2) if (ch[x][i]) reverse(ch[x][i]); rev[x] = 0;
95cf     }
425f     void rotate(int x) {
51af         int y = fa[x], z = fa[y], k = ch[y][1] == x, w = ch[x][!k];
e1fe         if (isroot(y)) ch[z][ch[z][1] == y] = x;
1e6f         ch[x][!k] = y; ch[y][k] = w; if (w) fa[w] = y;

```

```

        fa[y] = x; fa[x] = z; pull(y);
    }
    void pushall(int x) { if (isroot(x)) pushall(fa[x]); push(x); }
    void splay(int x) {
        int y = x, z = 0;
        for (pushall(y); isroot(x); rotate(x)) {
            y = fa[x]; z = fa[y];
            if (isroot(y)) rotate((ch[y][0] == x) ^ (ch[z][0] == y) ? x : y);
        }
        pull(x);
    }
    void access(int x) {
        int z = x;
        for (int y = 0; x; x = fa[y = x]) { splay(x); ch[x][1] = y; pull(x); }
        splay(z);
    }
    void chroot(int x) { access(x); reverse(x); }
    void split(int x, int y) { chroot(x); access(y); }

    int Root(int x) {
        for (access(x); ch[x][0]; x = ch[x][0]) push(x);
        splay(x); return x;
    }
    void Link(int u, int v) { chroot(u); fa[u] = v; }
    void Cut(int u, int v) { split(u, v); fa[u] = ch[v][0] = 0; pull(v); }
    int Query(int u, int v) { split(u, v); return sum[v]; }
    void Update(int u, int x) { splay(u); val[u] = x; }
    int LCA(int x, int y, int root) {
        chroot(root); access(x); splay(y);
        while (fa[y]) splay(y = fa[y]);
        return y;
    }
};

```

6d09
95cf
52c6
f69c
d095
c494
ceef
4449
95cf
78a0
95cf
6229
1548
8854
7afd
95cf
a067
126d
427e
d87a
f4f1
0d77
95cf
9e46
7c10
0691
a999
1f42
6cb2
02e5
c218
95cf
329b

6.6 Balanced binary search tree from pb_ds

```

#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;

tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update>
    rkt;
// null_tree_node_update

```

0475
332d
427e
43a7
427e

```

427e // SAMPLE USAGE
190e rkt.insert(x);      // insert element
05d4 rkt.erase(x);      // erase element
add5 rkt.order_of_key(x); // obtain the number of elements less than x
b064 rkt.find_by_order(i); // iterator to i-th (numbered from 0) smallest element
c103 rkt.lower_bound(x);
4ff4 rkt.upper_bound(x);
b19b rkt.join(rkt2);    // merge tree (only if their ranges do not intersect)
cb47 rkt.split(x, rkt2); // split all elements greater than x to rkt2

```

6.7 Persistent segment tree, range k-th query

```

f1a7 struct node {
2ff6     static int n, pos;
427e
7cec     int value;
70e2     node *left, *right;
427e
20b0     void* operator new(size_t size);
427e
3dc0     static node* Build(int l, int r) {
b6c5         node* a = new node;
ce96         if (r > l + 1) {
181e             int mid = (l + r) / 2;
3ba2             a->left = Build(l, mid);
8aaf             a->right = Build(mid, r);
8e2e         } else {
bfc4             a->value = 0;
95cf         }
5ffd         return a;
95cf     }
427e
5a45     static node* init(int size) {
2c46         n = size;
7ee3         pos = 0;
be52         return Build(0, n);
95cf     }
427e
93c0     static int Query(node* lt, node *rt, int l, int r, int k) {
d30c         if (r == l + 1) return l;
181e         int mid = (l + r) / 2;

```

```

if (rt->left->value - lt->left->value < k) {
    k -= rt->left->value - lt->left->value;
    return Query(lt->right, rt->right, mid, r, k);
} else {
    return Query(lt->left, rt->left, l, mid, k);
}
}

```

```

static int query(node* lt, node *rt, int k) {
    return Query(lt, rt, 0, n, k);
}

```

```

node *Inc(int l, int r, int pos) const {
    node* a = new node(*this);
    if (r > l + 1) {
        int mid = (l + r) / 2;
        if (pos < mid)
            a->left = left->Inc(l, mid, pos);
        else
            a->right = right->Inc(mid, r, pos);
    }
    a->value++;
    return a;
}

```

```

node *inc(int index) {
    return Inc(0, n, index);
}
nodes[8000000];

```

```

int node::n, node::pos;
inline void* node::operator new(size_t size) {
    return nodes + (pos++);
}

```

6.8 Block list

All indices are 0-based. All ranges are left-closed right-open.

Usage:

cb5a
8edb
2412
8e2e
0119
95cf
95cf
427e
c9ad
9e27
95cf
427e
b19c
5794
ce96
181e
203d
f44a
649a
1024
95cf
2b3e
5ffd
95cf
427e
e80f
c246
95cf
865a
427e
99ce
1987
bb3c
95cf

block::fix() Apply tags to the current block.
 Init(l, r) Range initializer.
 Reverse(l, r) Reverse the range.
 Add(l, r, x) Add x to the range.
 Query(l, r) Range aggregation.

```

fd9e  const int BLOCK = 800;
76b3  typedef vector<int> vi;
427e
a771  struct block {
8fbc    vi data;
e3b5    LL sum; int minv, maxv;
41db    int add; bool rev;
427e
d7eb    block(vi&& vec) : data(move(vec)),
1f0c        sum(accumulate(range(data), 0ll)),
8216        minv(*min_element(range(data))),
527d        maxv(*max_element(range(data))),
6437        add(0), rev(0) { }
427e
b919    void fix() {
0694        if (rev) reverse(range(data));      rev = 0;
0527        if (add) for (int& x : data) x += add; add = 0;
95cf    }
427e
8bc4    void merge(block& another) {
b895        fix(); another.fix();
f516        vi temp(move(data));
d02c        temp.insert(temp.end(), range(another.data));
88ea        *this = block(move(temp));
95cf    }
427e
42e8    block split(int pos) {
3e79        fix();
ccab        block result(vi(data.begin() + pos, data.end()));
861a        data.resize(pos); *this = block(move(data));
56b0        return result;
95cf    }
329b };
427e
2a18  typedef list<block>::iterator lit;
427e
ce14  struct blocklist {
5540    list<block> blk;

```

```

void maintain() {
    lit it = blk.begin();
    while (it != blk.end() && next(it) != blk.end()) {
        lit it2 = it;
        while (next(it2) != blk.end() &&
            it2->data.size() + next(it2)->data.size() <= BLOCK) {
            it2->merge(*next(it2));
            blk.erase(next(it2));
        }
        ++it;
    }
}

lit split(int pos) {
    for (lit it = blk.begin(); ; it++) {
        if (pos == 0) return it;
        while (it->data.size() > pos)
            blk.insert(next(it), it->split(pos));
        pos -= it->data.size();
    }
}

void Init(int *l, int *r) {
    for (int *cur = l; cur < r; cur += BLOCK)
        blk.emplace_back(vi(cur, min(cur + BLOCK, r)));
}

void Reverse(int l, int r) {
    lit it = split(l), it2 = split(r);
    reverse(it, it2);
    while (it != it2) {
        it->rev ^= 1;
        it++;
    }
    maintain();
}

void Add(int l, int r, int x) {
    lit it = split(l), it2 = split(r);
    while (it != it2) {
        it->sum += LL(x) * it->data.size();
        it->minv += x; it->maxv += x;
    }
}

```

427e
 7b8e
 3131
 4628
 852d
 188c
 3600
 93e1
 e1fa
 95cf
 5771
 95cf
 95cf
 427e
 b7b3
 2273
 5502
 8e85
 2099
 a5a1
 427e
 95cf
 95cf
 427e
 1c7b
 9919
 8950
 95cf
 427e
 a22f
 997b
 dfd0
 8f89
 6a06
 5283
 95cf
 b204
 95cf
 427e
 3cce
 997b
 8f89
 e927
 03d3

```

4511         it->add += x; it++;
95cf     }
b204     maintain();
95cf }
427e
3ad3 void Query(int l, int r) {
997b     lit it = split(l), it2 = split(r);
c33d     LL sum = 0; int minv = INT_MAX, maxv = INT_MIN;
8f89     while (it != it2) {
e472         sum += it->sum;
72c4         minv = min(minv, it->minv);
e1c4         maxv = max(maxv, it->maxv);
5283         it++;
95cf     }
b204     maintain();
8792     printf("%lld_%d_%d\n", sum, minv, maxv);
95cf }
958e } lst;

```

6.9 Persistent block list

Block list that supports persistence. All indices are 0-based. All ranges are left-closed right-open. `std::shared_ptr` is used to ease memory management. One should modify the constructor of `block` to maintain extra information. Here we use this policy that the size of each block does not exceed `BLOCK`, while the sum of sizes of two adjacent blocks does not less than `BLOCK`.

When some operation that breaks block list property, please call `maintain` in time to restore the property.

Usage:

<code>maintain()</code>	Maintain the block list property.
<code>split(pos)</code>	Split the block list at position <code>pos</code> . Returns an iterator to a block starting at <code>pos</code> .
<code>sum(l, r)</code>	An example function of list traversal between $[l, r)$.

Time Complexity: When `BLOCK` is properly selected, the time complexity is $O(\sqrt{n})$ per operation.

```

a19e constexpr int BLOCK = 800;
76b3 typedef vector<int> vi;
0563 typedef shared_ptr<vi> pvi;
013b typedef shared_ptr<const vi> pcvi;
427e
a771 struct block {

```

```

pcvi data;
LL sum;

// add information to maintain
block(pcvi ptr) :
    data(ptr),
    sum(accumulate(ptr->begin(), ptr->end(), 0ll))
{ }

void merge(const block& another) {
    pvi temp = make_shared<vi>(data->begin(), data->end());
    temp->insert(temp->end(), another.data->begin(), another.data->end());
    *this = block(temp);
}

block split(int pos) {
    block result(make_shared<vi>(data->begin() + pos, data->end()));
    *this = block(make_shared<vi>(data->begin(), data->begin() + pos));
    return result;
}
};

typedef list<block>::iterator lit;

struct blocklist {
    list<block> blk;

    void maintain() {
        lit it = blk.begin();
        while (it != blk.end() and next(it) != blk.end()) {
            lit it2 = it;
            while (next(it2) != blk.end() and
                    it2->data->size() + next(it2)->data->size() <= BLOCK) {
                it2->merge(*next(it2));
                blk.erase(next(it2));
            }
            ++it;
        }
    }

    lit split(int pos) {
        for (lit it = blk.begin(); ; it++) {
            if (pos == 0) return it;
            while (it->data->size() > pos) {

```

2989
8fd0
427e
427e
a613
24b5
0cf0
e93b
427e
5c0f
0b18
ac21
6467
95cf
427e
42e8
dac1
01db
56b0
95cf
329b
427e
2a18
427e
ce14
5540
427e
7b8e
3131
5e44
852d
0b03
029f
93e1
e1fa
95cf
5771
95cf
95cf
427e
b7b3
2273
5502
d480

```

2099         blk.insert(next(it), it->split(pos));
95cf     }
a1c8     pos -= it->data->size();
95cf     }
95cf     }
427e
fd38     LL sum(int l, int r) { // traverse
48b4         lit it1 = split(l), it2 = split(r);
ac09         LL res = 0;
9f1d         while (it1 != it2) {
8284             res += it1->sum;
61fd             it1++;
95cf         }
b204         maintain();
244d         return res;
95cf     }
329b };

```

6.10 Sparse table, range minimum query

The array is 0-based and the range is left-closed right-open.

```

db63 const int MAXN = 100007;
cefd int a[MAXN], st[MAXN][30];
427e
d34f void init(int n){
c73d     int l = log2(n);
cf75     rep (i, n) st[i][0] = a[i];
426b     rep (j, l) rep (i, 1+n-(1<<j))
1131         st[i][j+1] = min(st[i][j], st[i+(1<<j)][j]);
95cf }
427e
c863 int rmq(int l, int r){
f089     int k = log2(r - l);
6117     return min(st[l][k], st[r-(1<<k)][k]);
95cf }

```

7 Geometrics

7.1 2D geometric template

```

#include <bits/stdc++.h>
using namespace std;

typedef int T;
typedef struct pt {
    T x, y;
    T operator , (pt a) { return x*a.x + y*a.y; } // inner product
    T operator * (pt a) { return x*a.y - y*a.x; } // outer product
    pt operator + (pt a) { return {x+a.x, y+a.y}; }
    pt operator - (pt a) { return {x-a.x, y-a.y}; }

    pt operator * (T k) { return {x*k, y*k}; }
    pt operator - () { return {-x, -y}; }
} vec;

typedef pair<pt, pt> seg;

bool ptOnSeg(pt& p, seg& s){
    vec v1 = s.first - p, v2 = s.second - p;
    return (v1, v2) <= 0 && v1 * v2 == 0;
}

// 0 not on segment
// 1 on segment except vertices
// 2 on vertices
int ptOnSeg2(pt& p, seg& s){
    vec v1 = s.first - p, v2 = s.second - p;
    T ip = (v1, v2);
    if (v1 * v2 != 0 || ip > 0) return 0;
    return (v1, v2) ? 1 : 2;
}

// if two orthogonal rectangles do not touch, return true
inline bool nIntRectRect(seg a, seg b){
    return min(a.first.x, a.second.x) > max(b.first.x, b.second.x) ||
           min(a.first.y, a.second.y) > max(b.first.y, b.second.y) ||
           min(b.first.x, b.second.x) > max(a.first.x, a.second.x) ||
           min(b.first.y, b.second.y) > max(a.first.y, a.second.y);
}

// >0 in order
// <0 out of order
// =0 not standard

```

```

7538 inline double rotOrder(vec a, vec b, vec c){return double(a*b)*(b*c);}
427e
31ed inline bool intersect(seg a, seg b){
427e // ! if (nIntRectRect(a, b)) return false; // if commented, assume that a
      and b are non-collinear
cb52 return rotOrder(b.first-a.first, a.second-a.first, b.second-a.first) >= 0 &&
059e rotOrder(a.first-b.first, b.second-b.first, a.second-b.first) >= 0;
95cf }
427e
427e // 0 not intersect
427e // 1 standard intersection
427e // 2 vertex-line intersection
427e // 3 vertex-vertex intersection
427e // 4 collinear and have common point(s)
4d19 int intersect2(seg& a, seg& b){
5dc4 if (nIntRectRect(a, b)) return 0;
42c0 vec va = a.second - a.first, vb = b.second - b.first;
2096 double j1 = rotOrder(b.first-a.first, va, b.second-a.first),
72fe j2 = rotOrder(a.first-b.first, vb, a.second-b.first);
5ac6 if (j1 < 0 || j2 < 0) return 0;
9400 if (j1 != 0 && j2 != 0) return 1;
83db if (j1 == 0 && j2 == 0){
6b0c if (va * vb == 0) return 4; else return 3;
fb17 } else return 2;
95cf }
427e
2c68 template <typename Tp = T>
5894 inline pt getIntersection(pt P, vec v, pt Q, vec w){
6850 static_assert(is_same<Tp, double>::value, "must_be_double!");
7c9a return P + v * (w*(P-Q)/(v*w));
95cf }
427e
427e // -1 outside the polygon
427e // 0 on the border of the polygon
427e // 1 inside the polygon
cbdd int ptOnPoly(pt p, pt* poly, int n){
5fb4 int wn = 0;
1294 for (int i = 0; i < n; i++) {
427e
3cae T k, d1 = poly[i].y - p.y, d2 = poly[(i+1)%n].y - p.y;
b957 if (k = (poly[(i+1)%n] - poly[i])*(p - poly[i])){
8c40 if (k > 0 && d1 <= 0 && d2 > 0) wn++;
3c4d if (k < 0 && d2 <= 0 && d1 > 0) wn--;
aad3 } else return 0;

```

```

    }
    return wn ? 1 : -1;
}

istream& operator >> (istream& lhs, pt& rhs){
    lhs >> rhs.x >> rhs.y;
    return lhs;
}

istream& operator >> (istream& lhs, seg& rhs){
    lhs >> rhs.first >> rhs.second;
    return lhs;
}

```

```

95cf
0a5f
95cf
427e
d4a3
fa86
331a
95cf
427e
07ae
5cab
331a
95cf

```

8 Appendices

8.1 Primes

8.1.1 First primes

p	$g(p)$	p	$g(p)$	p	$g(p)$	p	$g(p)$	p	$g(p)$
2	1	3	2	5	2	7	3	11	2
13	2	17	3	19	2	23	5	29	2
31	3	37	2	41	6	43	3	47	5
53	2	59	2	61	2	67	2	71	7
73	5	79	3	83	2	89	3	97	5
101	2	103	5	107	2	109	6	113	3
127	3	131	2	137	3	139	2	149	2
151	6	157	5	163	2	167	5	173	2
179	2	181	2	191	19	193	5	197	2
199	3	211	2	223	3	227	2	229	6

8.1.2 Arbitrary length primes

$\lg p$	p	$g(p)$	p	$g(p)$
3	967	5	1031	14
4	9859	2	10273	10
5	96331	10	102931	3
6	958543	6	1031137	5
7	9594539	2	10169651	2
8	96243449	3	103211039	7
9	980483981	2	1042484357	2
10	9858935453	2	10261276009	7
11	95748666809	3	101759940101	2
12	950781833849	3	1012797784423	5
13	9739822952371	7	10037217092377	7
14	96181051140397	5	104974966380359	11
15	981030138360889	13	1029038416465403	2
16	9655206098080843	3	10116299875820773	2
17	97687777921994419	3	101506415998163437	2

8.1.3 $\sim 1 \times 10^9$

p	$g(p)$	p	$g(p)$	p	$g(p)$
954854573	3	967607731	2	973215833	3
975831713	3	978949117	2	980766497	3
983879921	3	985918807	3	986608921	29
991136977	5	991752599	13	997137961	11
1003911991	3	1009775293	2	1012423549	6
1021000537	5	1023976897	7	1024153643	2
1037027287	3	1038812881	11	1044754639	3
1045125617	3	1047411427	3	1047753349	6

8.1.4 $\sim 1 \times 10^{18}$

p	$g(p)$	p	$g(p)$
951970612352230049	3	963284339889659609	3
967495386904694119	3	969751761517096213	2
983238274281901499	2	984647442475101409	23
989286107138674069	11	1002507954383424641	3
1006658951440146419	2	1020152326159075903	3
1034876265966119449	7	1042753851435034019	2
1043609016597371563	2	1045571042176595707	2
1048364250160580293	2	1049495624119026949	2

8.2 Pell's equation

$x^2 - ny^2 = 1$, where n is a positive nonsquare integer.

Let (x_0, y_0) be the smallest positive solution of the equation, then the k -th solution is:

$$\begin{pmatrix} x_k \\ y_k \end{pmatrix} = \begin{pmatrix} x_0 & ny_0 \\ y_0 & x_0 \end{pmatrix}^k \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

Some smallest solutions to Pell's equation:

n	2	3	5	6	7	8	10	11	12	13	14	15	17	18	19	20
x	3	2	9	5	8	3	19	10	7	649	15	4	33	17	170	9
y	2	1	4	2	3	1	6	3	2	180	4	1	8	4	39	2

8.3 Burnside's lemma and Polya's enumeration theorem

The Burnside's lemma says that

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

where G is a group acting on X , X^g is the set of elements in X that are fixed by g , i.e. $X^g = \{x \in X : gx = x\}$.

The unweighted version of Pólya enumeration theorem says that

$$|Y^X/G| = \frac{1}{|G|} \sum_{g \in G} m^{c_g}$$

where $m = |X|$ is the number of colors, c_g is the number of the cycles of permutation g .

8.4 Supnick TSP

Given f and $x_1 \leq x_2 \leq \dots \leq x_n$, if f is Supnick, then

$$\sum_{i=1}^n f(x_{\pi(i)}, x_{\pi(i+1)})$$

1. is minimized when $\pi = (1, 3, 5, 7, \dots, 8, 6, 4, 2)$.
2. is maximized when $\pi = (n, 2, n-2, 4, \dots, 5, n-3, 3, n-1, 1)$.

8.5 Lagrange's interpolation

For sample points $(x_0, y_0), \dots, (x_k, y_k)$, define

$$l_j(x) = \prod_{0 \leq m \leq k, m \neq j} \frac{x - x_m}{x_j - x_m}$$

then the Lagrange polynomial is

$$L(x) = \sum_{j=0}^k y_j l_j(x).$$

To use the script below, type two lines

```
x0 x1 x2 ... xn
y0 y1 y2 ... yn
```

the script will print the fractional coefficient of the polynomial in ascending exponent order.

```
#!/usr/bin/python2
from fractions import *

def polymul(a, b) :
    p = [0] * (len(a)+len(b)-1)
    for e1, c1 in enumerate(a) :
        for e2, c2 in enumerate(b) :
            p[e1+e2] += c1*c2
    return p

x, y = [map(Fraction, raw_input().split()) for _ in 0,0]
n = len(x)
lj = [reduce(polymul, [[-x[m]/(x[j]-x[m]), 1/(x[j]-x[m])]
    for m in range(n) if m != j]]) for j in range(n)]
print '_'.join(map(str, map(sum, zip(*map(
    lambda a, b : [x*a for x in b], y, lj)))))
```

```
6dc9
4b2b
427e
796b
83e4
f697
156c
dfce
5849
427e
f06d
e80a
a649
9dfa
3cae
7c0d
```