

The Short-Time Fourier Transform (1 of 2)

Xavier Serra

Universitat Pompeu Fabra, Barcelona

&

Stanford University

Index

- Short-time Fourier Transform equation
- Analysis window

Short-time Fourier Transform

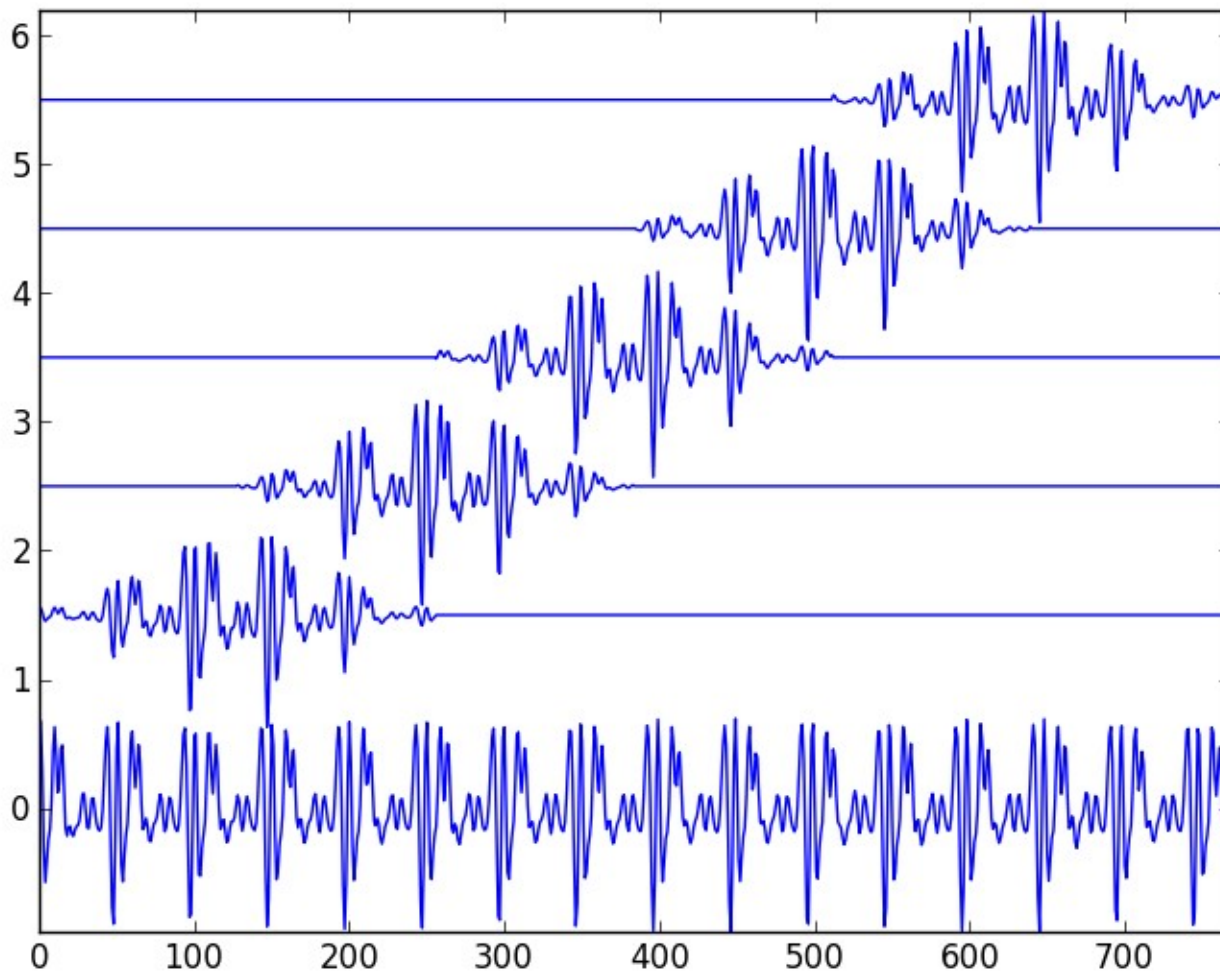
$$X_l[k] = \sum_{n=-N/2}^{N/2-1} w[n] x[n+lH] e^{-j2\pi kn/N} \quad l=0,1,\dots,$$

w : analysis window

l : frame number

H : hop-size

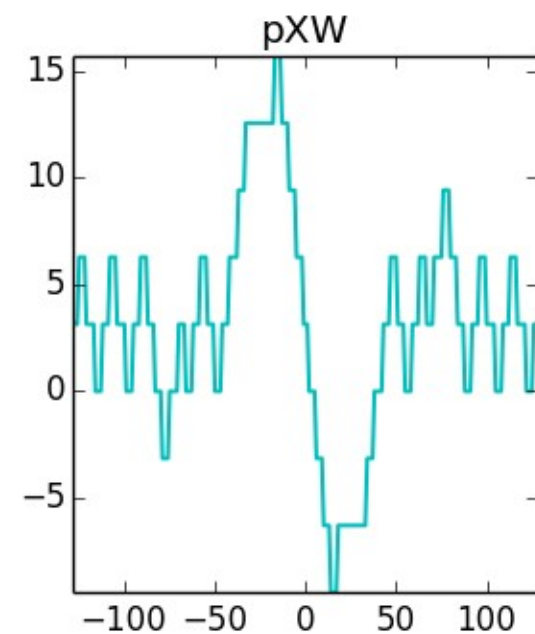
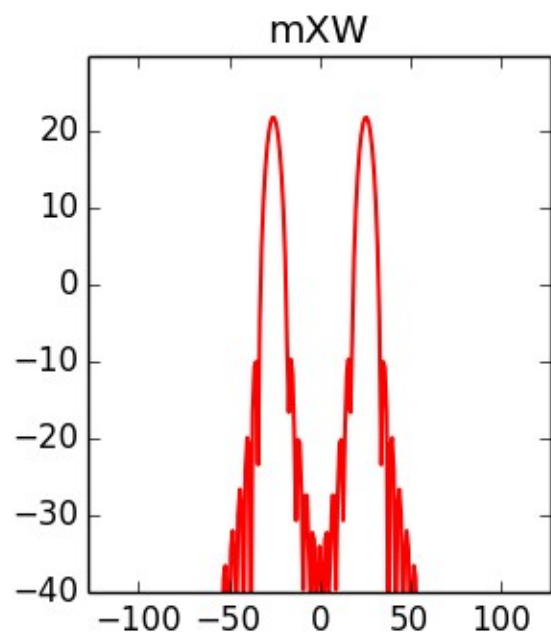
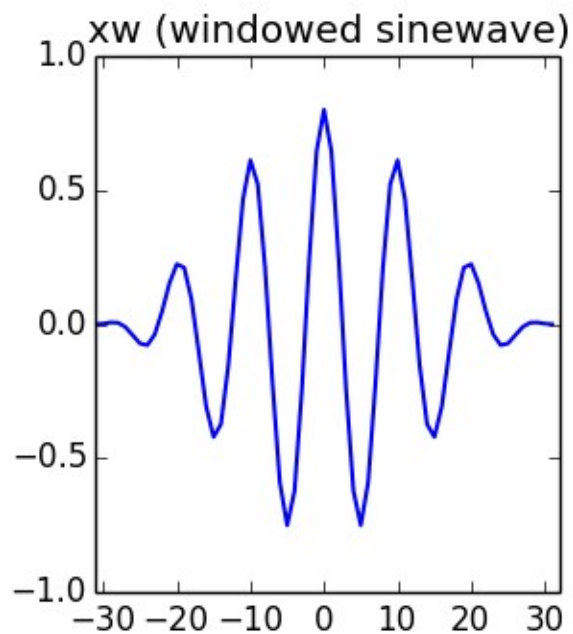
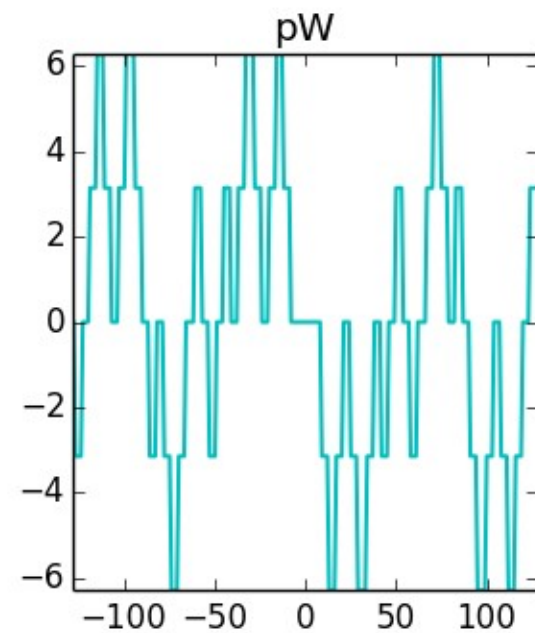
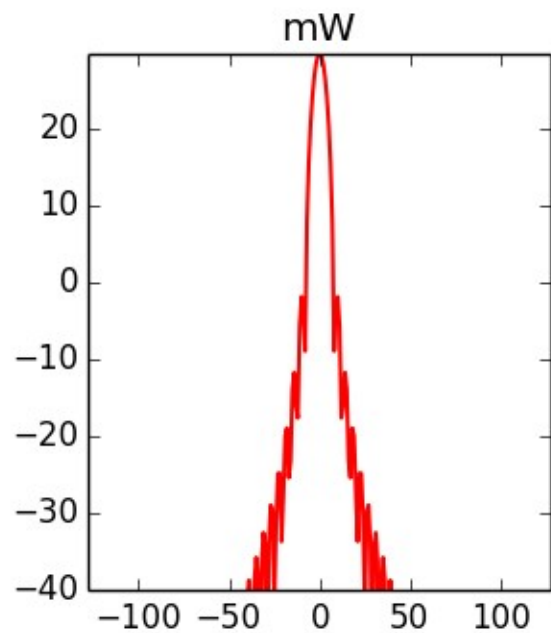
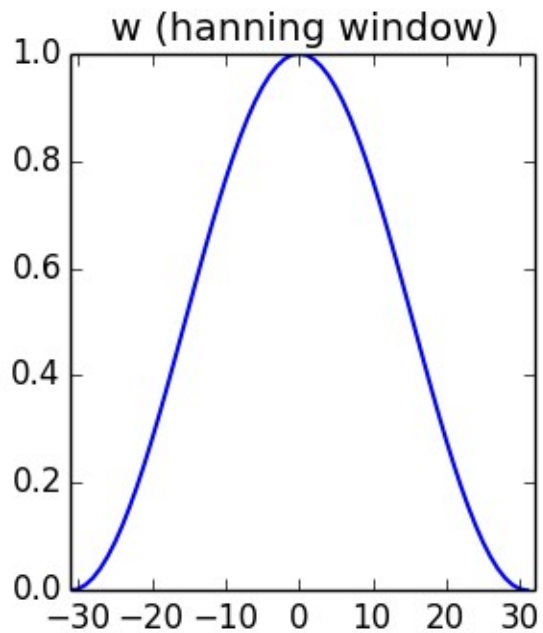
$$xw_l[n] = w[n]x[n + lH] \quad l = 0, 1, \dots,$$



Transform of a windowed sinewave

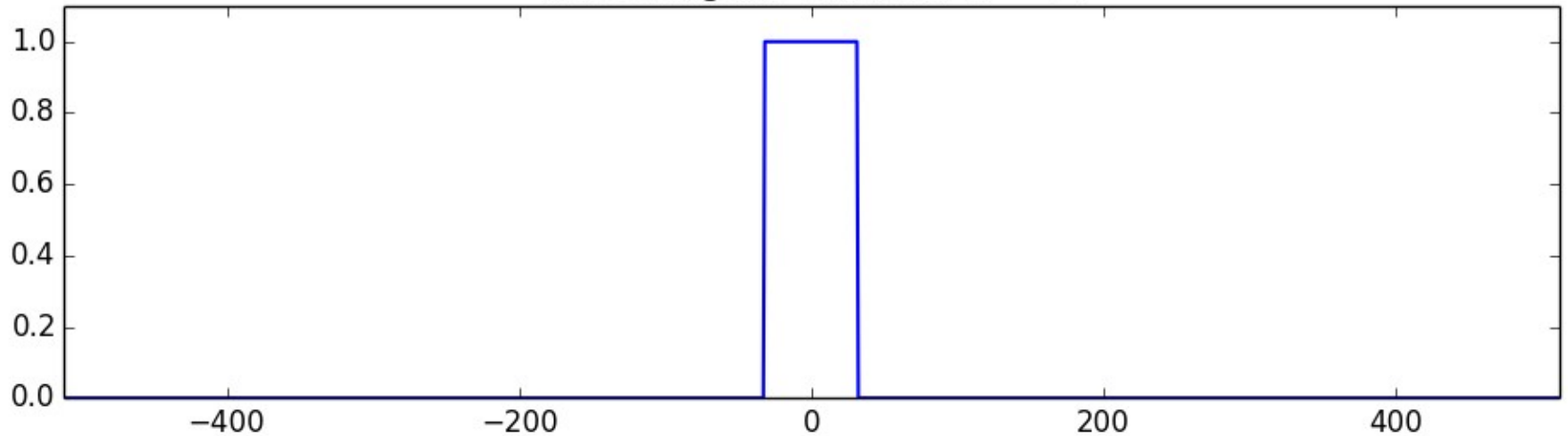
$$x[n] = A_0 \cos(2\pi k_0 n/N) = \frac{A_0}{2} e^{j2\pi k_0 n/N} + \frac{A_0}{2} e^{-j2\pi k_0 n/N}$$

$$\begin{aligned} X[k] &= \sum_{n=-N/2}^{N/2-1} w[n] x[n] e^{-j2\pi kn/N} \\ &= \sum_{n=-N/2}^{N/2-1} w[n] \left(\frac{A_0}{2} e^{j2\pi k_0 n/N} + \frac{A_0}{2} e^{-j2\pi k_0 n/N} \right) e^{-j2\pi kn/N} \\ &= \sum_{n=-N/2}^{N/2-1} w[n] \frac{A_0}{2} e^{j2\pi k_0 n/N} e^{-j2\pi kn/N} + \sum_{n=-N/2}^{N/2-1} w[n] \frac{A_0}{2} e^{-j2\pi k_0 n/N} e^{-j2\pi kn/N} \\ &= \frac{A_0}{2} \sum_{n=-N/2}^{N/2-1} w[n] e^{-j2\pi(k-k_0)n/N} + \frac{A_0}{2} \sum_{n=-N/2}^{N/2-1} w[n] e^{-j2\pi(k+k_0)n/N} \\ &= \frac{A_0}{2} W[k-k_0] + \frac{A_0}{2} W[k+k_0] \end{aligned}$$

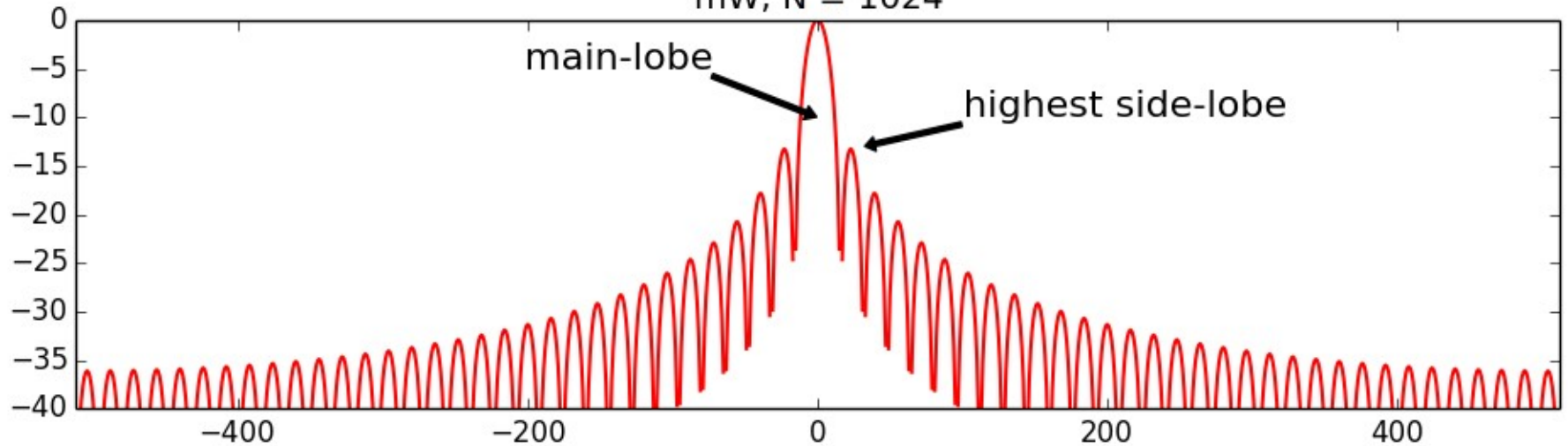


Analysis window

w (rectangular window), $M = 64$



mW, $N = 1024$



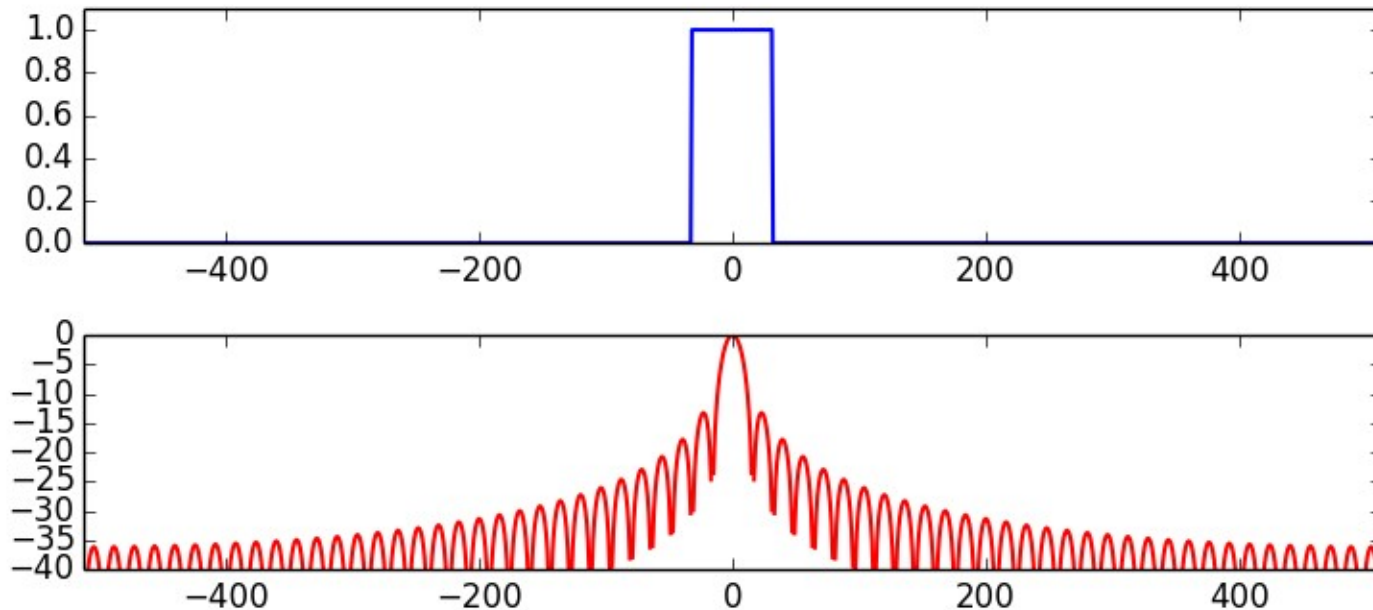
Window functions in Scipy

<code>barthann (M[, sym])</code>	Return a modified Bartlett-Hann window.
<code>bartlett (M[, sym])</code>	Return a Bartlett window.
<code>blackman (M[, sym])</code>	Return a Blackman window.
<code>blackmanharris (M[, sym])</code>	Return a minimum 4-term Blackman-Harris window.
<code>bohman (M[, sym])</code>	Return a Bohman window.
<code>boxcar (M[, sym])</code>	Return a boxcar or rectangular window.
<code>chebwin (M, at[, sym])</code>	Return a Dolph-Chebyshev window.
<code>flattop (M[, sym])</code>	Return a flat top window.
<code>gaussian (M, std[, sym])</code>	Return a Gaussian window.
<code>general-gaussian (M, p, sig[, sym])</code>	Return a window with a generalized Gaussian shape.
<code>hamming (M[, sym])</code>	Return a Hamming window.
<code>hann (M[, sym])</code>	Return a Hann window.
<code>kaiser (M, beta[, sym])</code>	Return a Kaiser window.
<code>nuttall (M[, sym])</code>	Return a minimum 4-term Blackman-Harris window according to Nuttall.
<code>parzen (M[, sym])</code>	Return a Parzen window.
<code>slepian (M, width[, sym])</code>	Return a digital Slepian window.
<code>triang (M[, sym])</code>	Return a triangular window.

Rectangular window

$$w[n] = \begin{cases} 1, & n = -M/2, \dots, 0, \dots, M/2 \\ 0, & n = \text{elsewhere} \end{cases}$$

$$W[k] = \frac{\sin(\pi k)}{\sin(\pi k/M)}$$

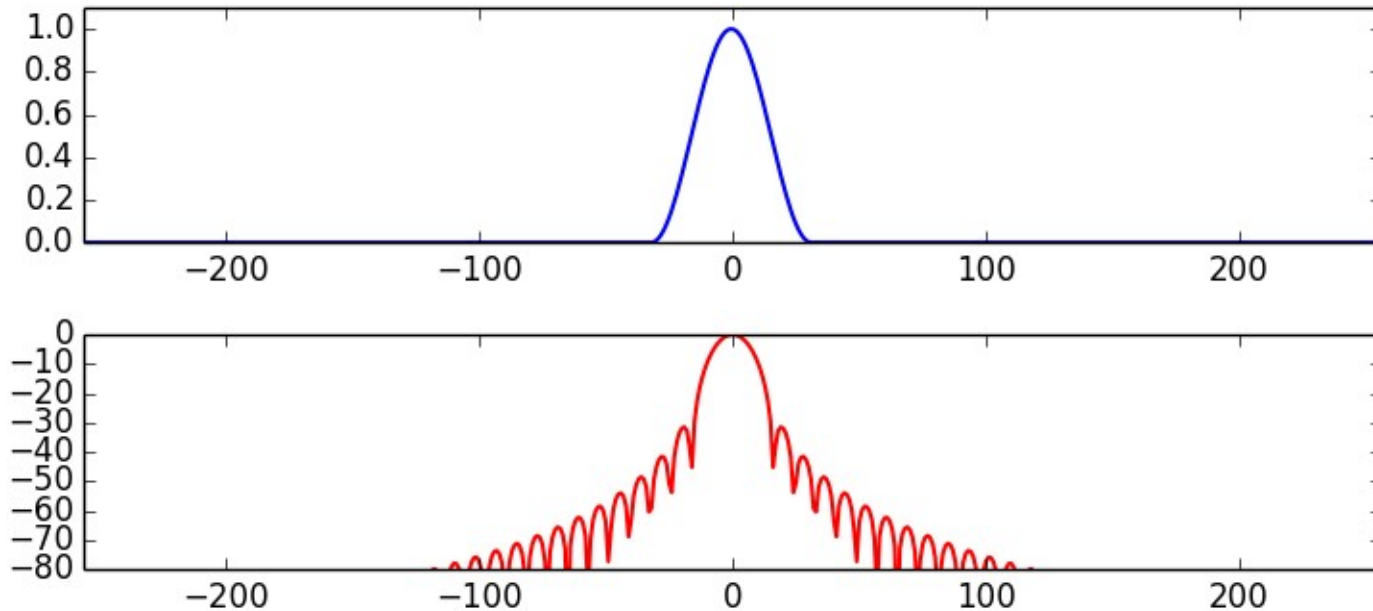


main-lobe width: 2 bins
side-lobe level: -13.3 dB

Hanning window

$$w[n] = .5 + .5 \cos(2\pi n/M), \quad n = -M/2, \dots, 0, \dots, M/2$$

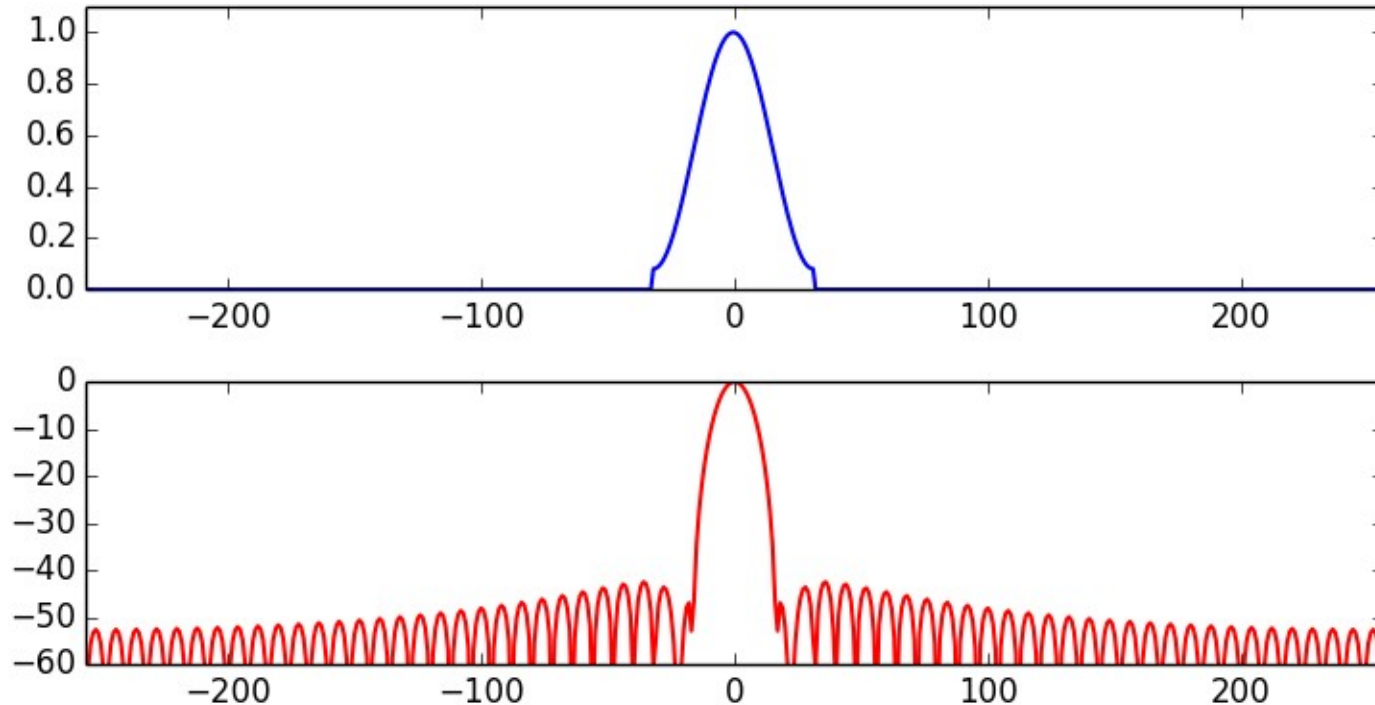
$$W[k] = .5 D[k] + .25 (D[k-1] + D[k+1]) \quad \text{where } D[k] = \frac{\sin(\pi k)}{\sin(\pi k/M)}$$



main-lobe width: 4 bins
side-lobe level: -31.5 dB

Hamming window

$$w[n] = .54 + .46 \cos(2\pi n/M), \quad n = -M/2, \dots, 0, \dots, M/2$$

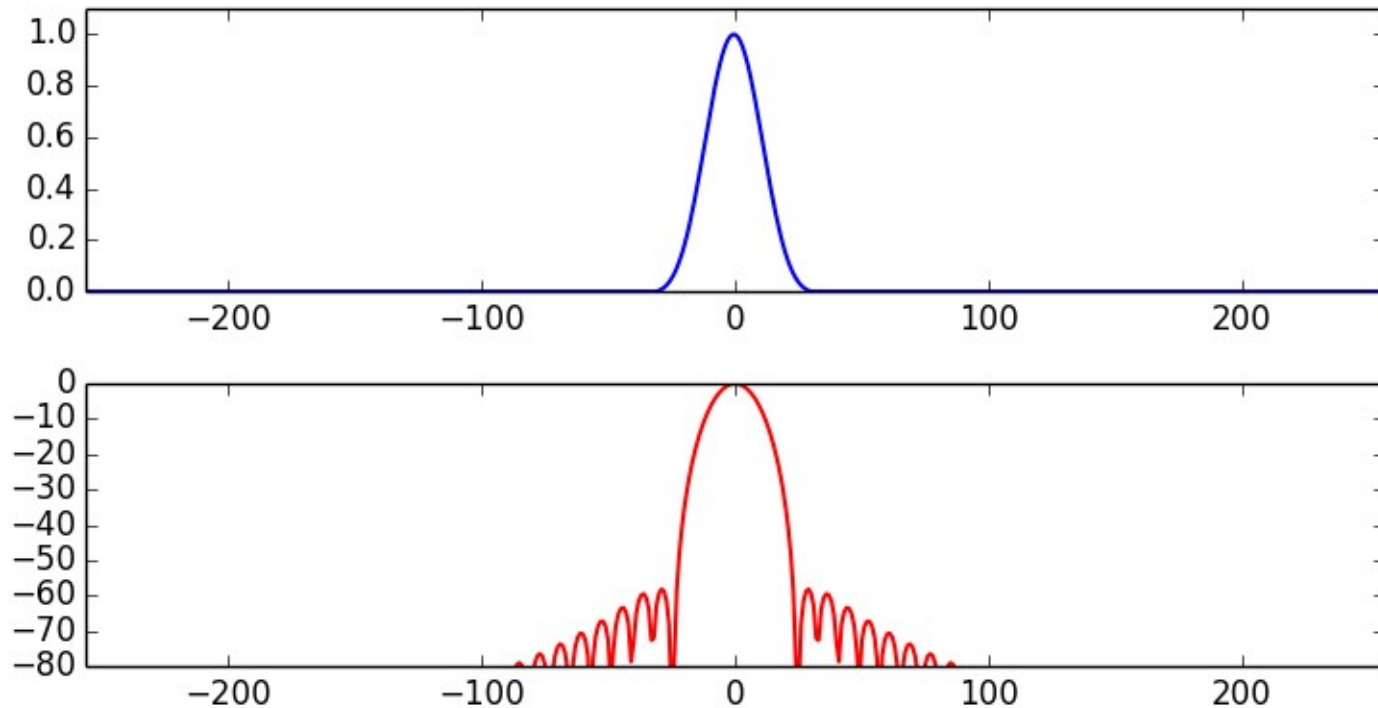


main-lobe width: 4 bins

side-lobe level: -42.7 dB

Blackman window

$$w[n] = 0.42 - 0.5 \cos(2\pi n/M) + 0.08 \cos(4\pi n/M)$$

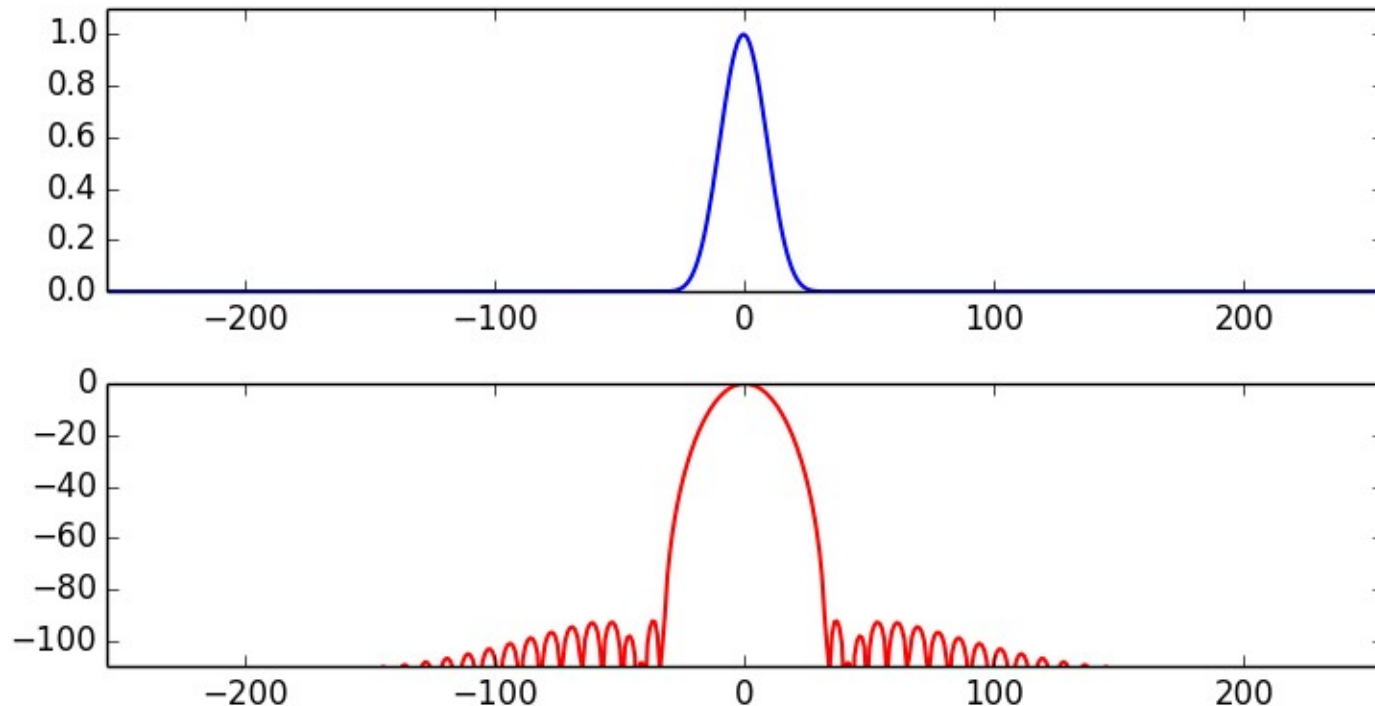


main-lobe width: 6 bins
side-lobe level: -58 dB

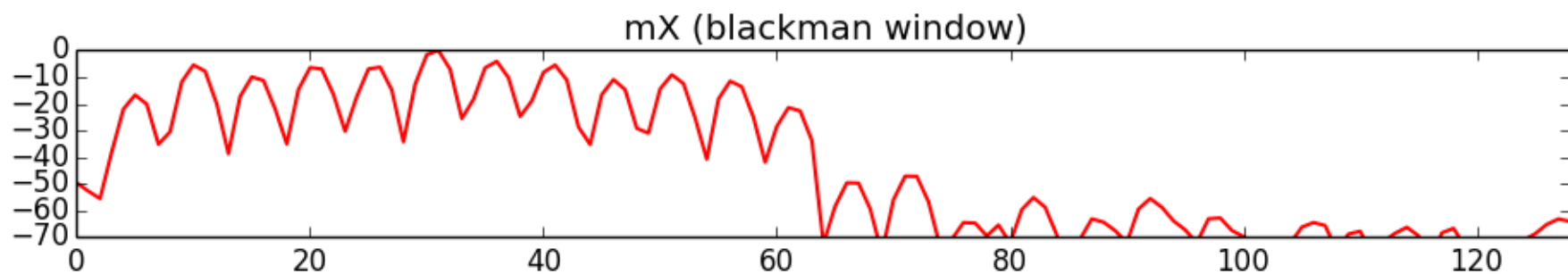
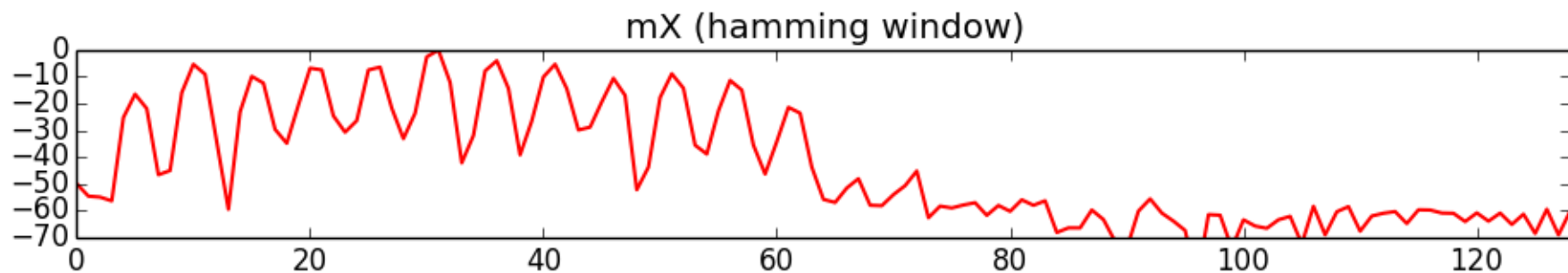
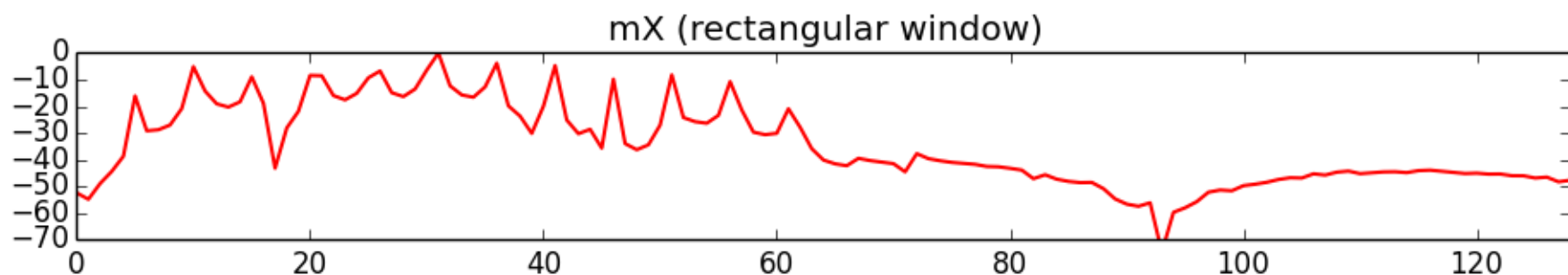
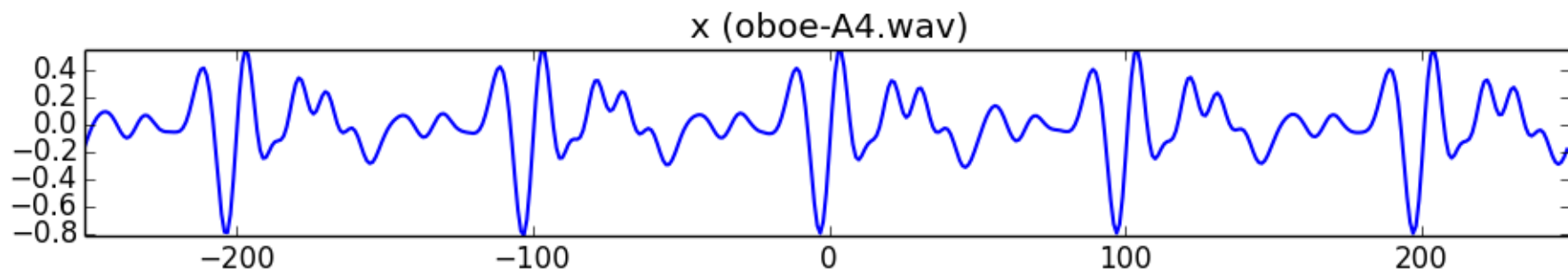
Blackman-Harris window

$$w(n) = \frac{1}{M} \sum_{l=0}^3 \alpha_l \cos(2nl\pi/M), \quad n = -M/2, \dots, 0, \dots, M/2$$

where $\alpha_0 = 0.35875, \alpha_1 = 0.48829, \alpha_2 = 0.14128, \alpha_3 = 0.01168$



main lobe width : 8 bins
side-lobe level : -92dB



References and credits

- More information in:
 - <https://en.wikipedia.org/wiki/STFT>
 - https://en.wikipedia.org/wiki/Window_function
- Reference on the STFT by Julius O. Smith:
<https://ccrma.stanford.edu/~jos/sasp/>
- Sounds from:
<http://www.freesound.org/people/xserra/packs/13038/>
- Slides and code released using the CC Attribution-Noncommercial-Share Alike license or the Affero GPL license and available from <https://github.com/MTG/sms-tools>

The Short-Time Fourier Transform (1 of 2)

Xavier Serra

Universitat Pompeu Fabra, Barcelona

&

Stanford University