

3T2: Fourier Transform properties (2 of 2)

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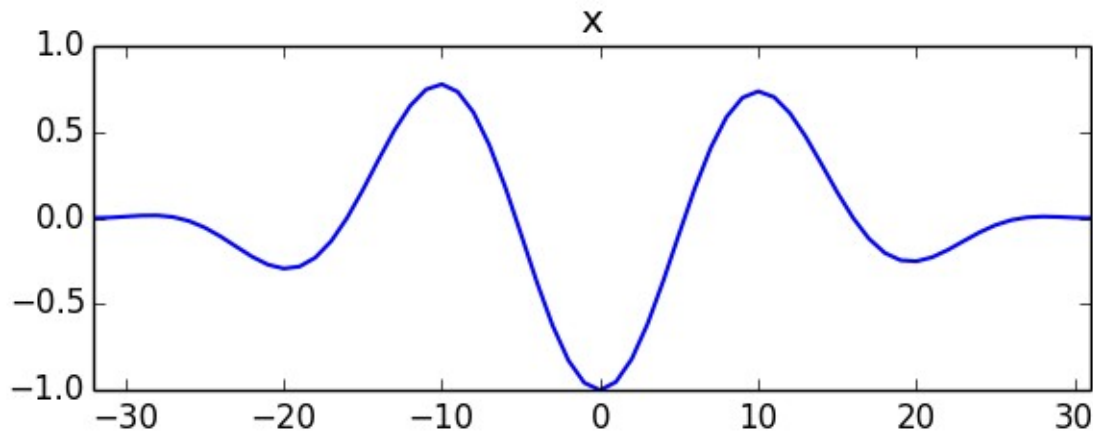
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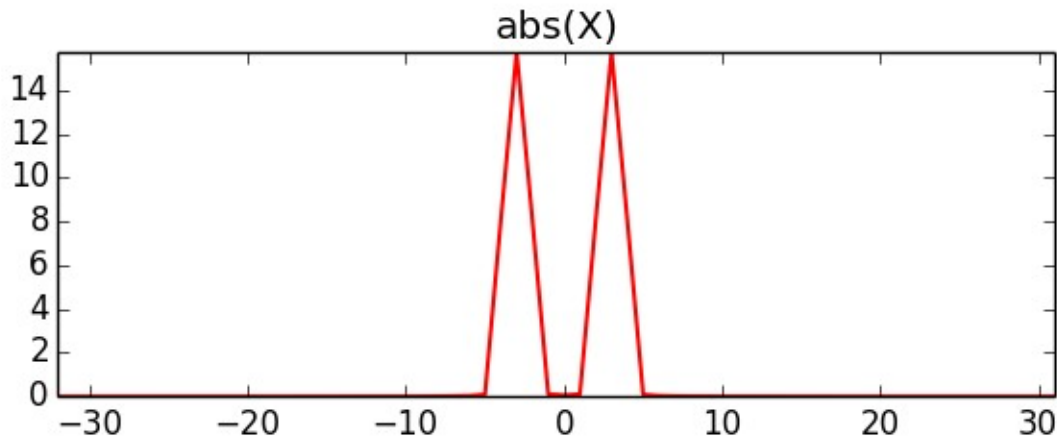
- Energy conservation & decibels
- Phase unwrapping
- Zero padding
- Fast Fourier Transform (FFT)
- FFT and zero-phase windowing
- Analysis/synthesis

Energy conservation

$$\sum_{n=-N/2}^{N/2-1} |x[n]|^2 = \frac{1}{N} \sum_{k=-N/2}^{N/2-1} |X[k]|^2$$

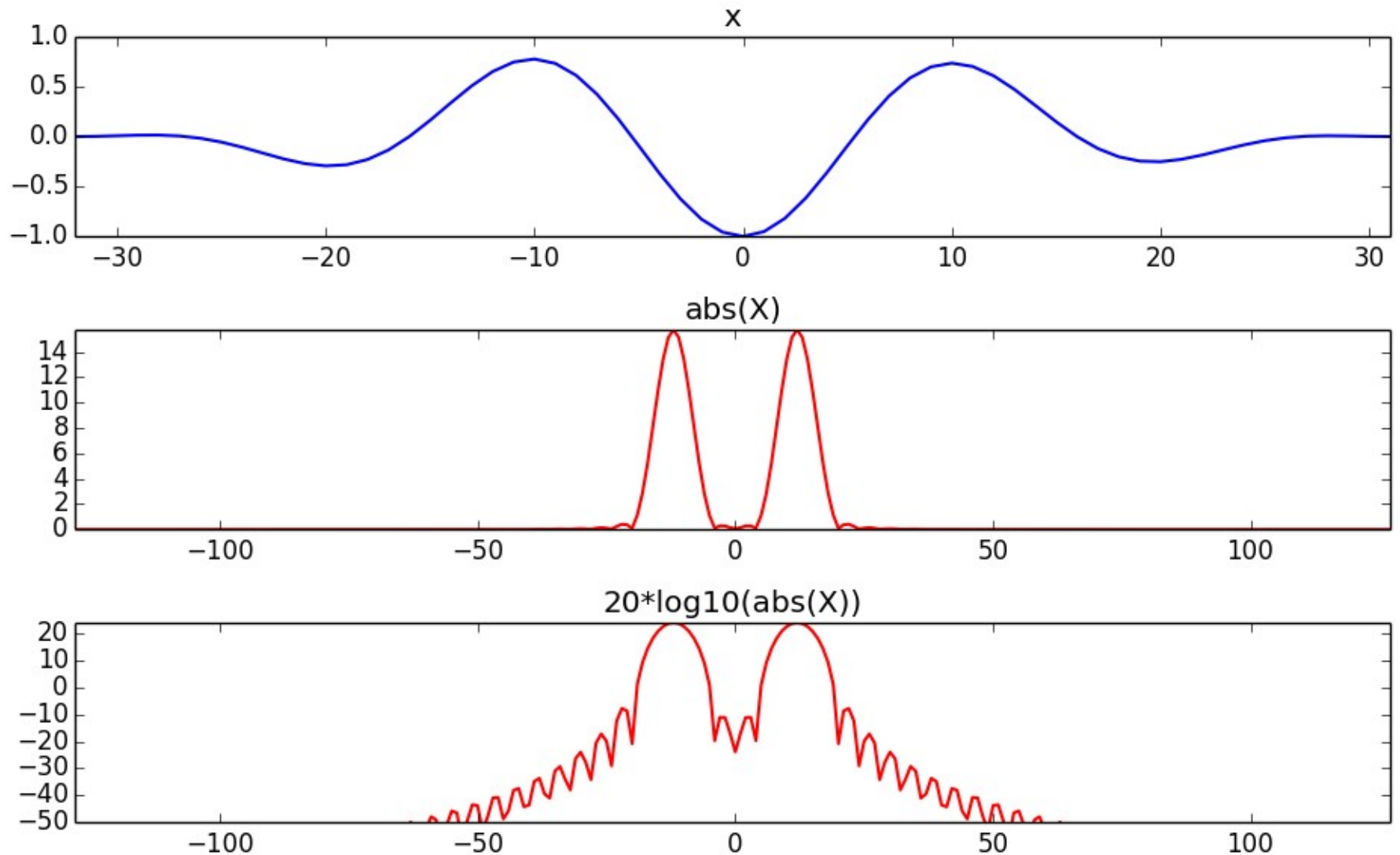


$$\sum_{n=-N/2}^{N/2-1} |x[n]|^2 = 11.81182$$

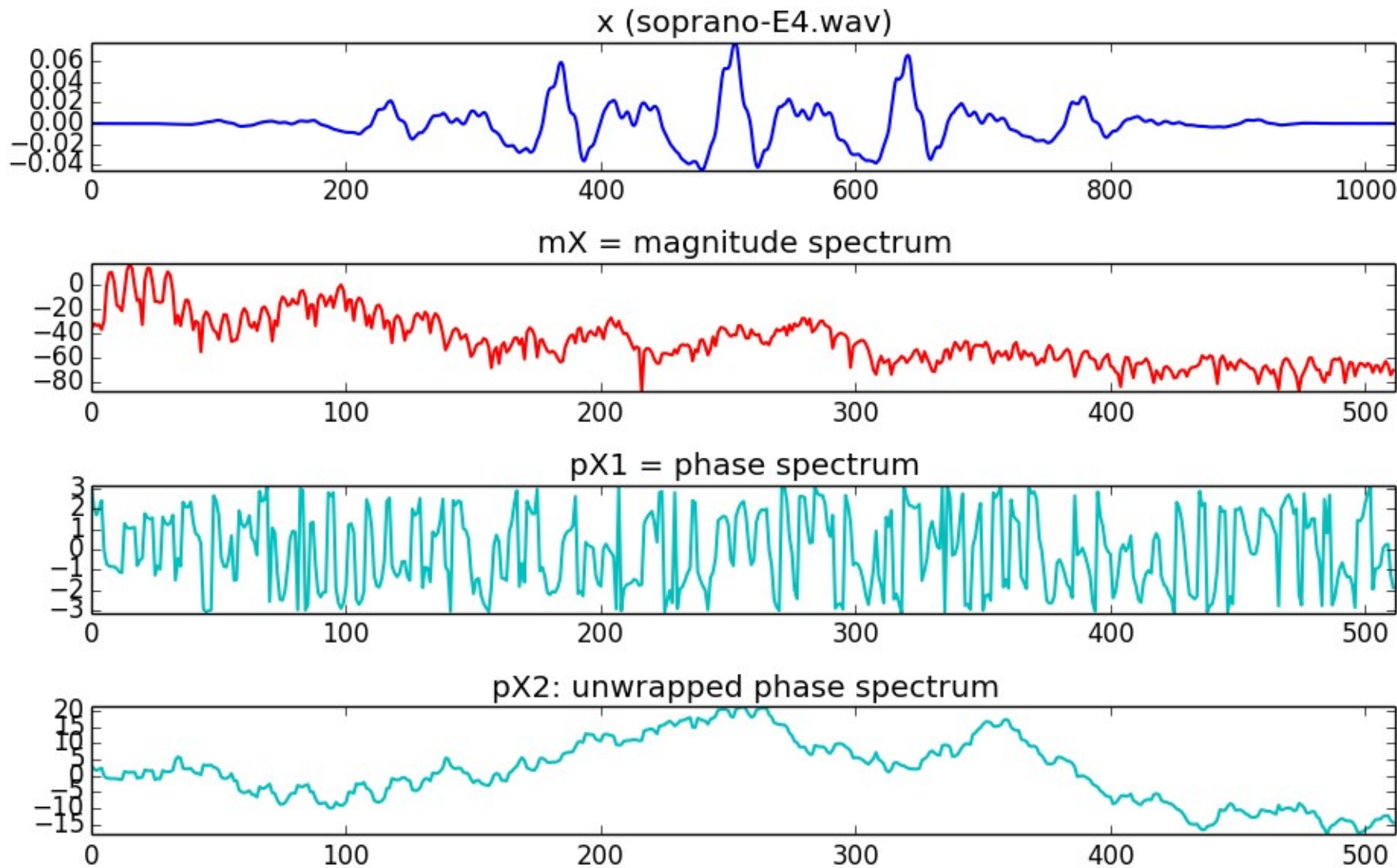


$$\frac{1}{N} \sum_{k=-N/2}^{N/2-1} |X[k]|^2 = 11.81182$$

Amplitude in decibels (dB)

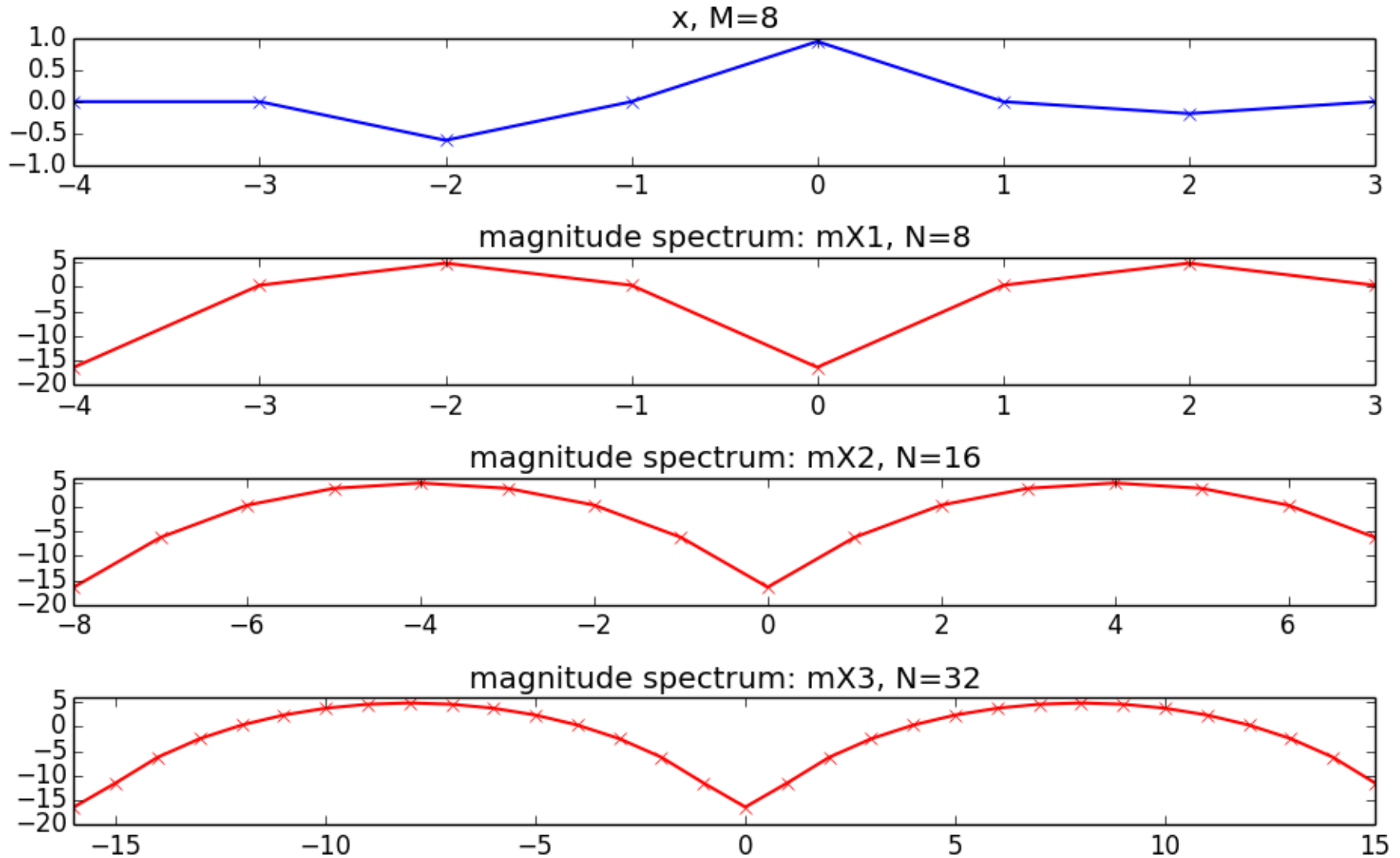


Phase unwrapping



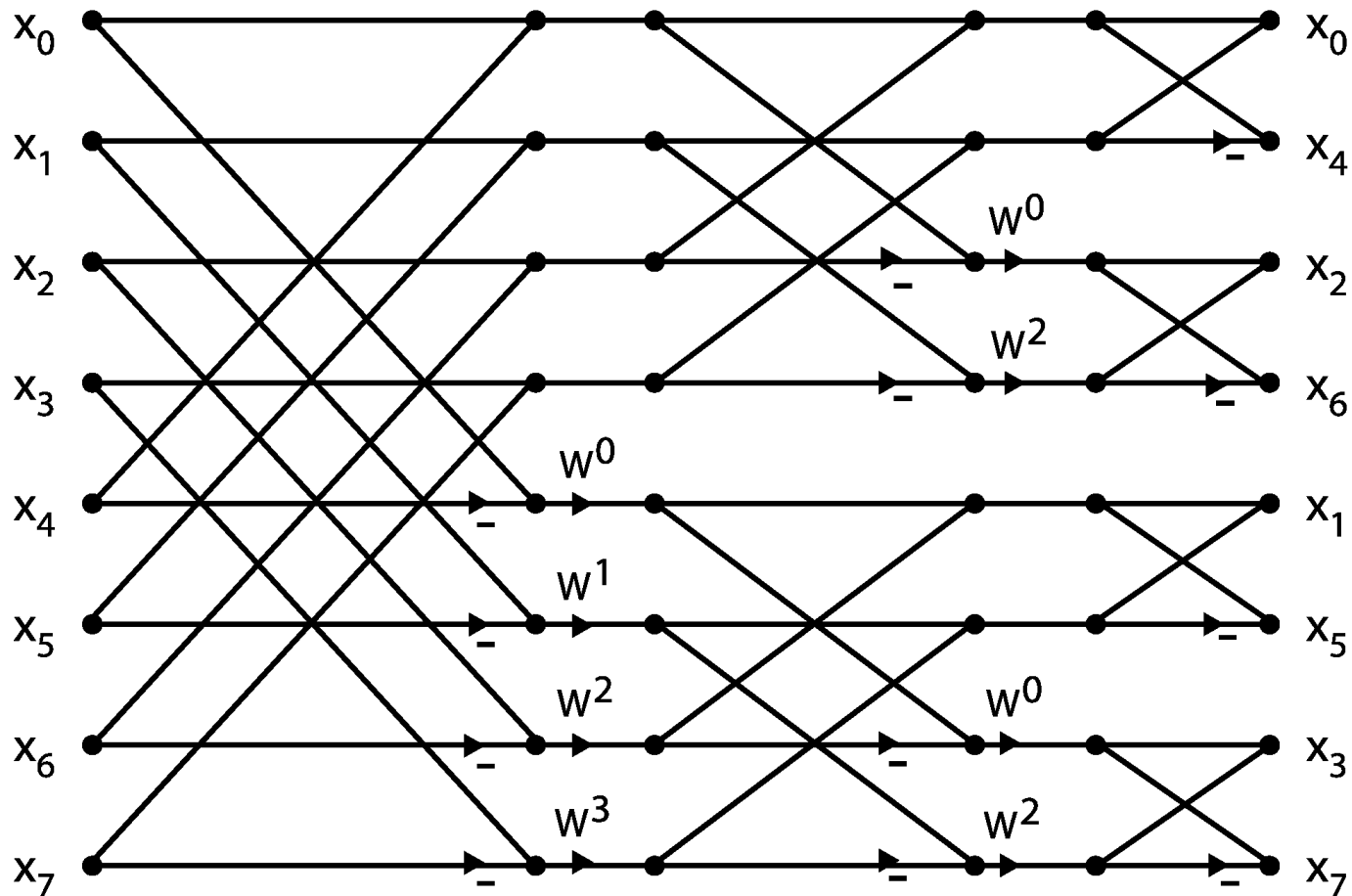
Zero-padding

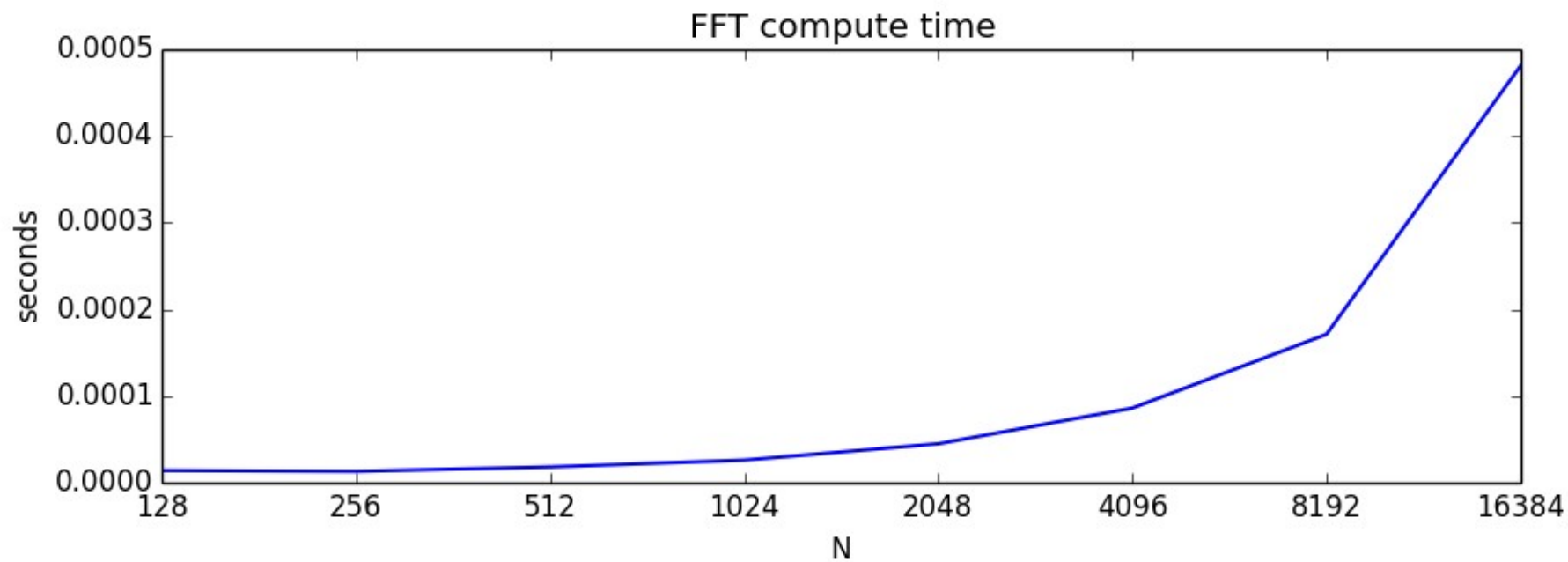
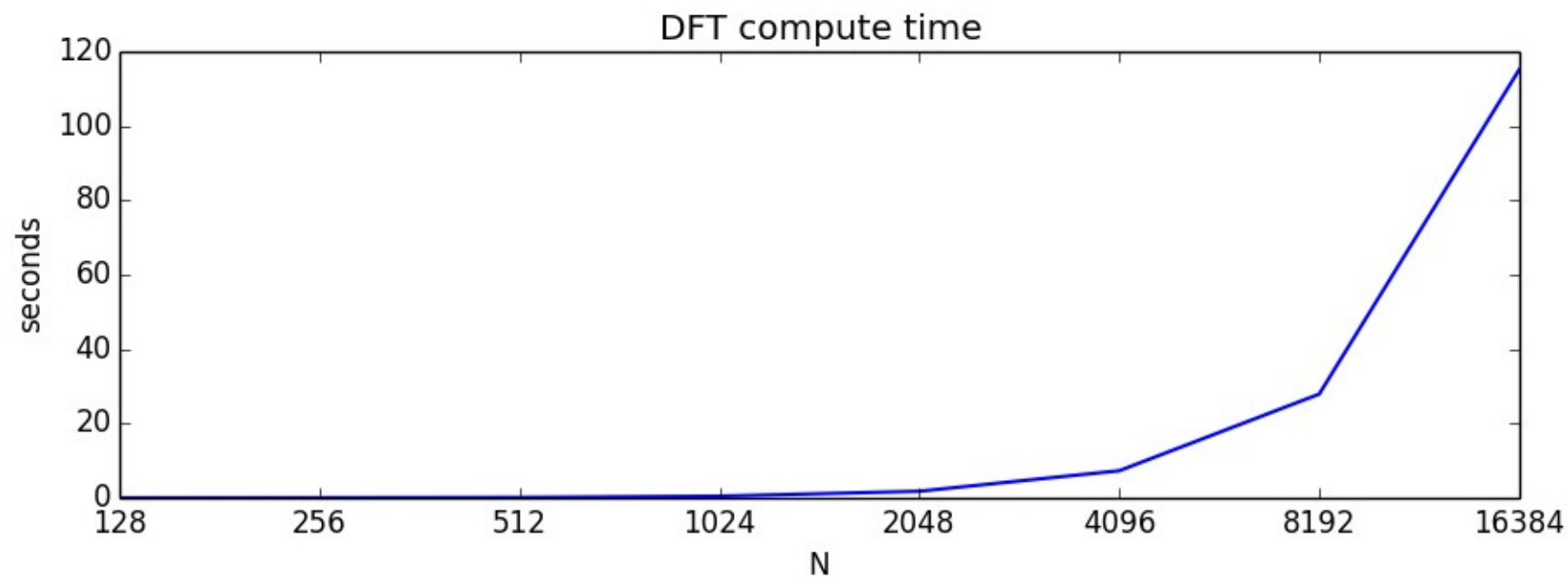
zero padding \leftrightarrow interpolation



Fast Fourier Transform

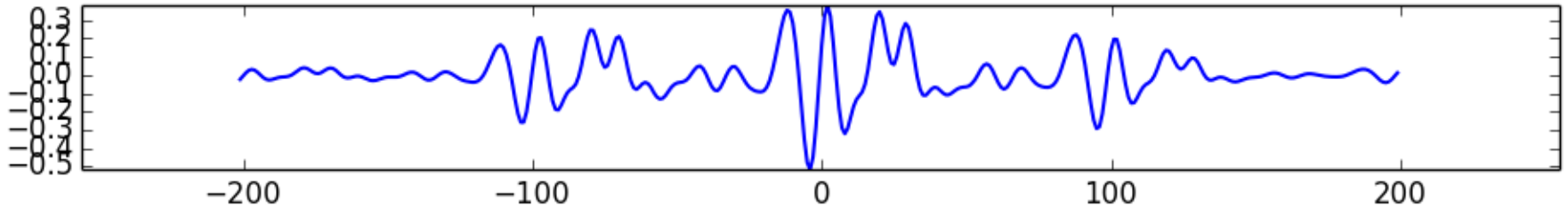
Coley-Tukey algorithm: breaks down recursively the DFT of a power of 2 size into two pieces of size $N/2$.



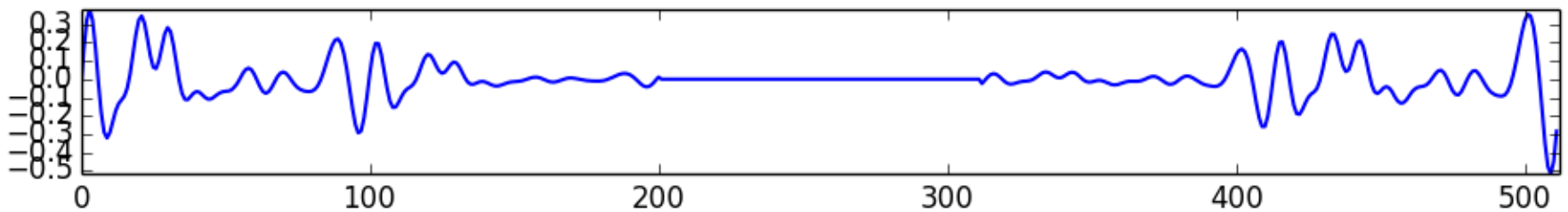


FFT and zero-phase windowing

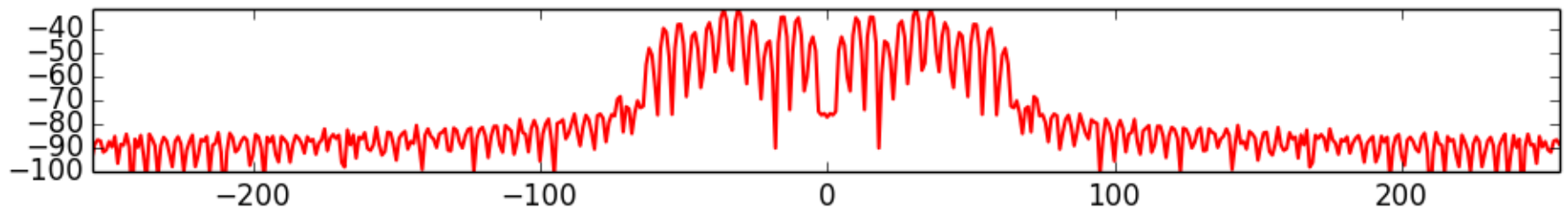
x (oboe-A4.wav), M = 401



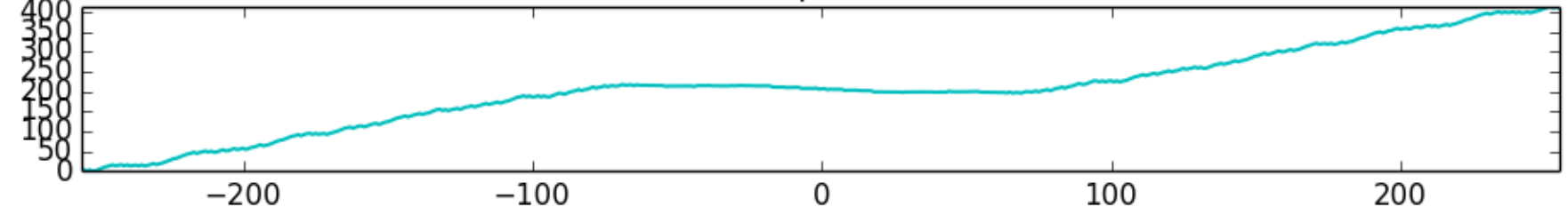
fftbuffer: N = 512



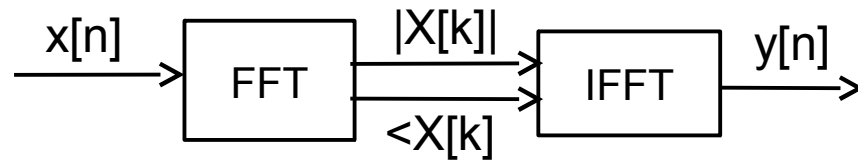
mX



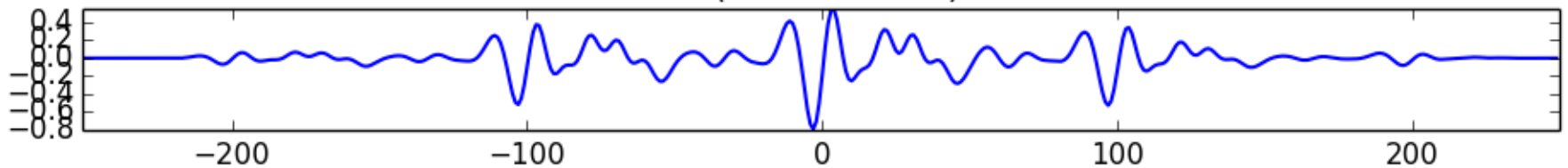
pX



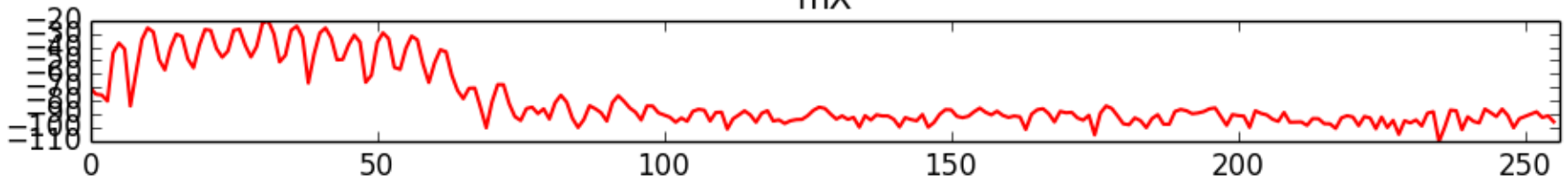
Analysis/synthesis



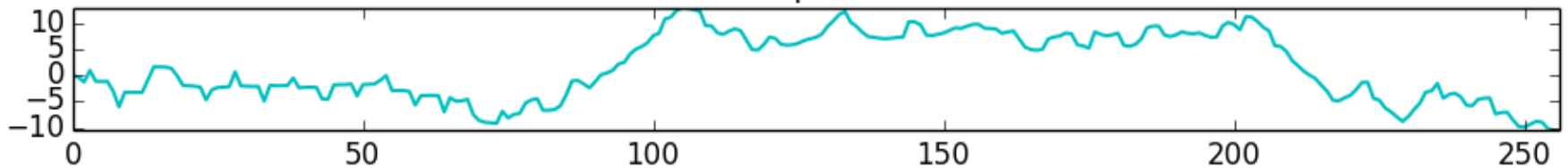
x (oboe-A4.wav)



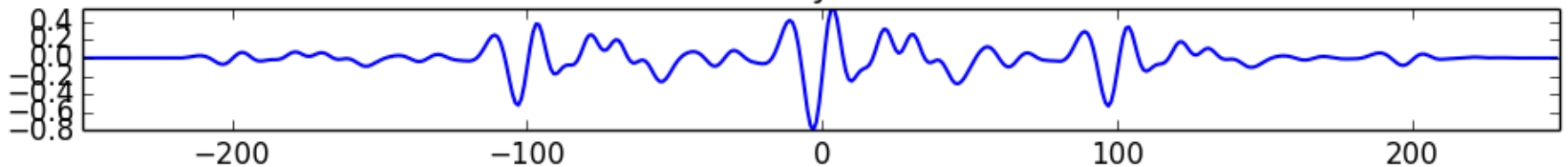
mX



pX



y



References and credits

- More information on:
 - https://en.wikipedia.org/wiki/Discrete_Fourier_transform
 - https://en.wikipedia.org/wiki/Fast_Fourier_transform
- Sounds from:
<http://www.freesound.org/people/xserra/packs/13038/>
- Reference for the DFT by Julius O. Smith:
<https://ccrma.stanford.edu/~jos/mdft/>
- Slides and code released using the CC Attribution-Non Commercial-Share Alike license or the Affero GPL license and available from <https://github.com/MTG/sms-tools>

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