5T1: Sinusoidal model (1 of 3)

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Sinusoidal model

$$y[n]=\sum_{r=1}^{R} A_r[n]\cos\left(2\pi f_r[n]n\right)$$

R: number of sinewaves
$A_r[n]$: instantaneous amplitude
$f_r[n]$: instantaneous frequency (Hz)

Spectrum of sinewave

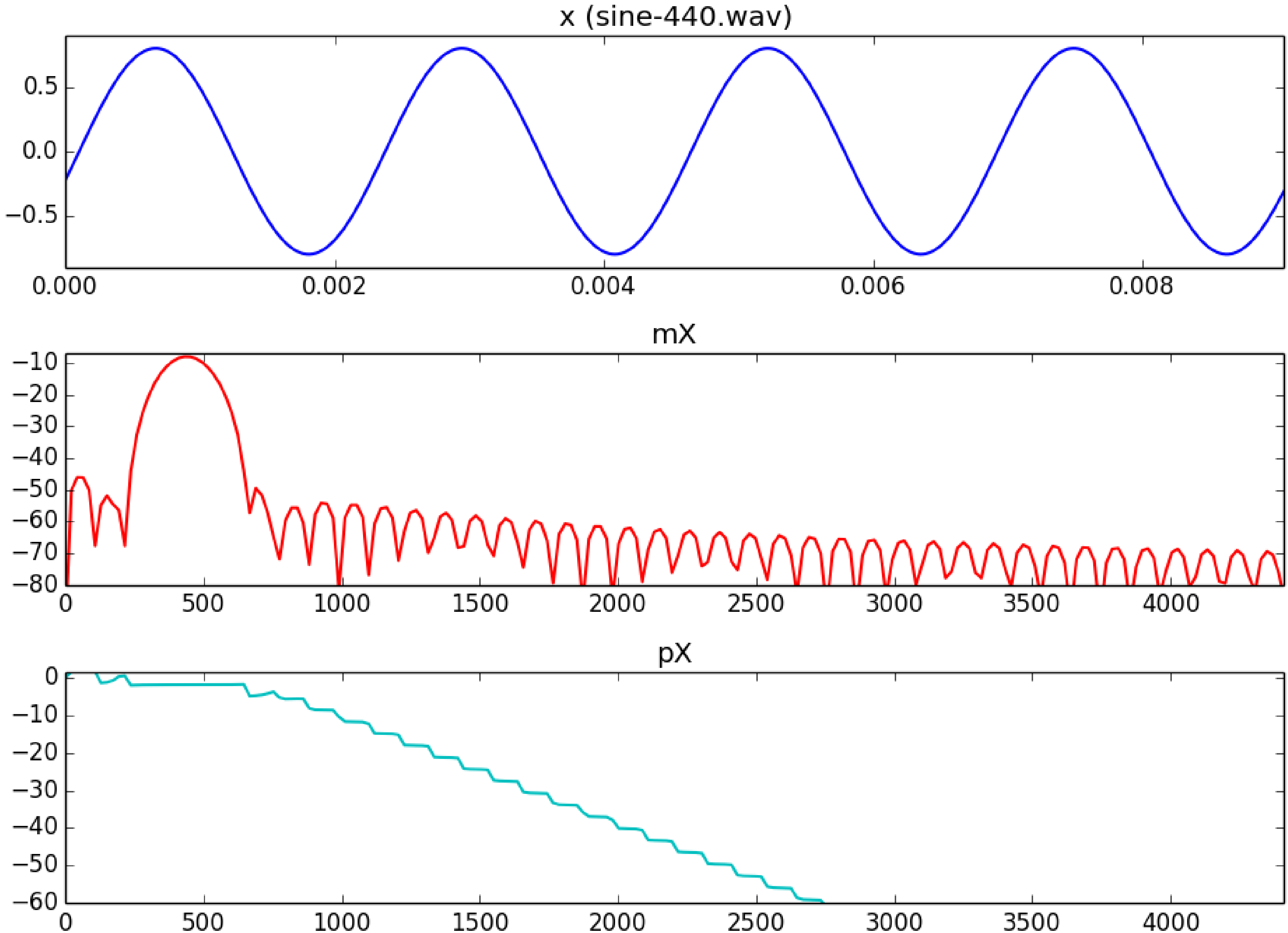
$$x[n] = A\cos(2\pi k_0 n/N + \varphi)$$

$$X[k] = A\sum_{n=0}^{N-1} w[n]\frac{1}{2}\left(e^{j2\pi k_0 n/N} + e^{-j2\pi k_0 n/N}\right)e^{-j2\pi kn/N}$$

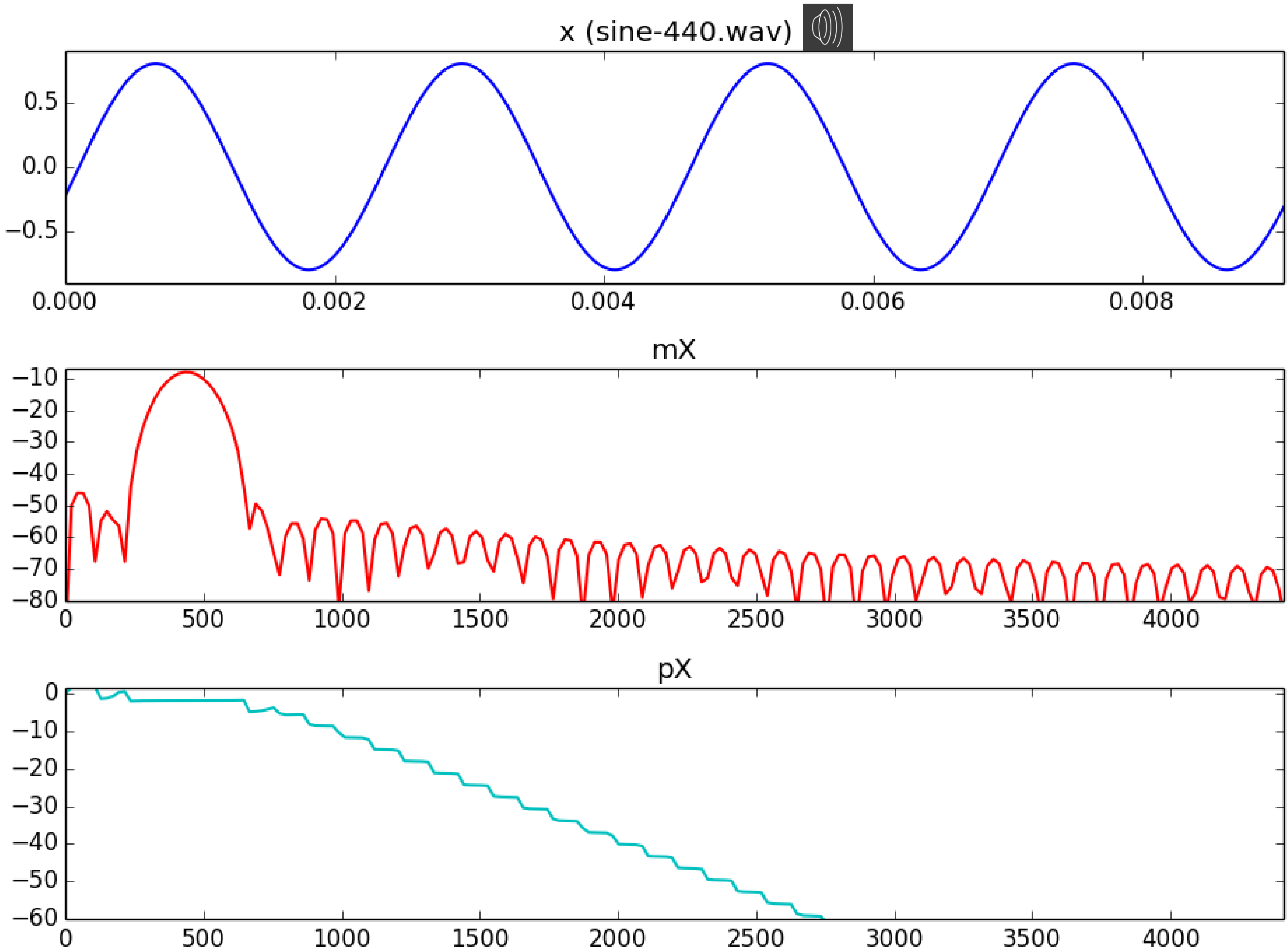
$$= \frac{A}{2}\sum_{n=0}^{N-1} w[n]e^{j2\pi k_0 n/N}e^{-j2\pi kn/N} + \frac{A}{2}\sum_{n=0}^{N-1} w[n]e^{-j2\pi k_0 n/N}e^{-j2\pi kn/N}$$

$$= \frac{A}{2}\sum_{n=0}^{N-1} w[n]e^{-j2\pi(-k_0+k)n/N} + \frac{A}{2}\sum_{n=0}^{N-1} w[n]e^{-j2\pi(k_0+k)n/N}$$

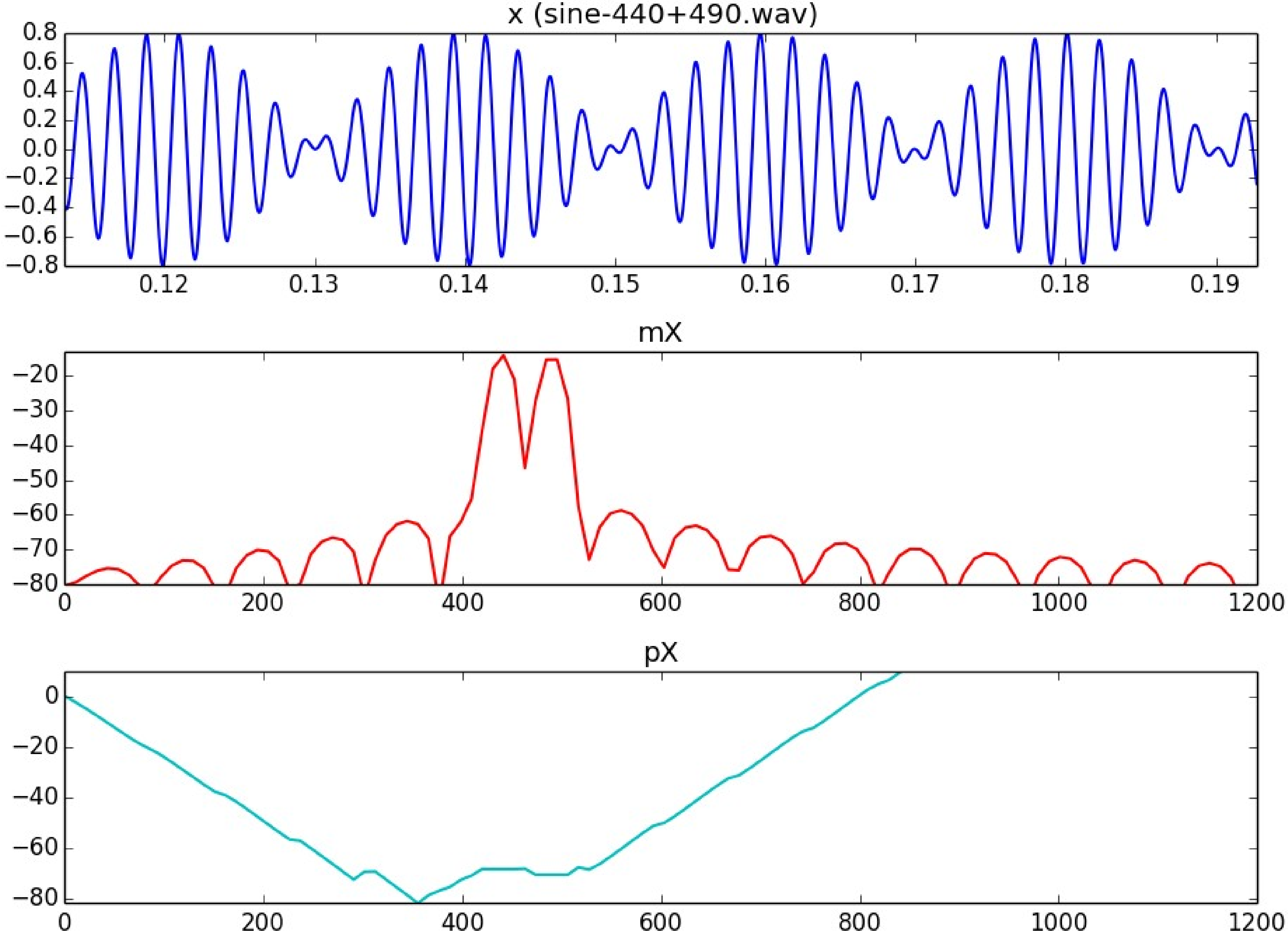
$$= \frac{A}{2}W[k-k_0] + \frac{A}{2}W[k+k_0]$$



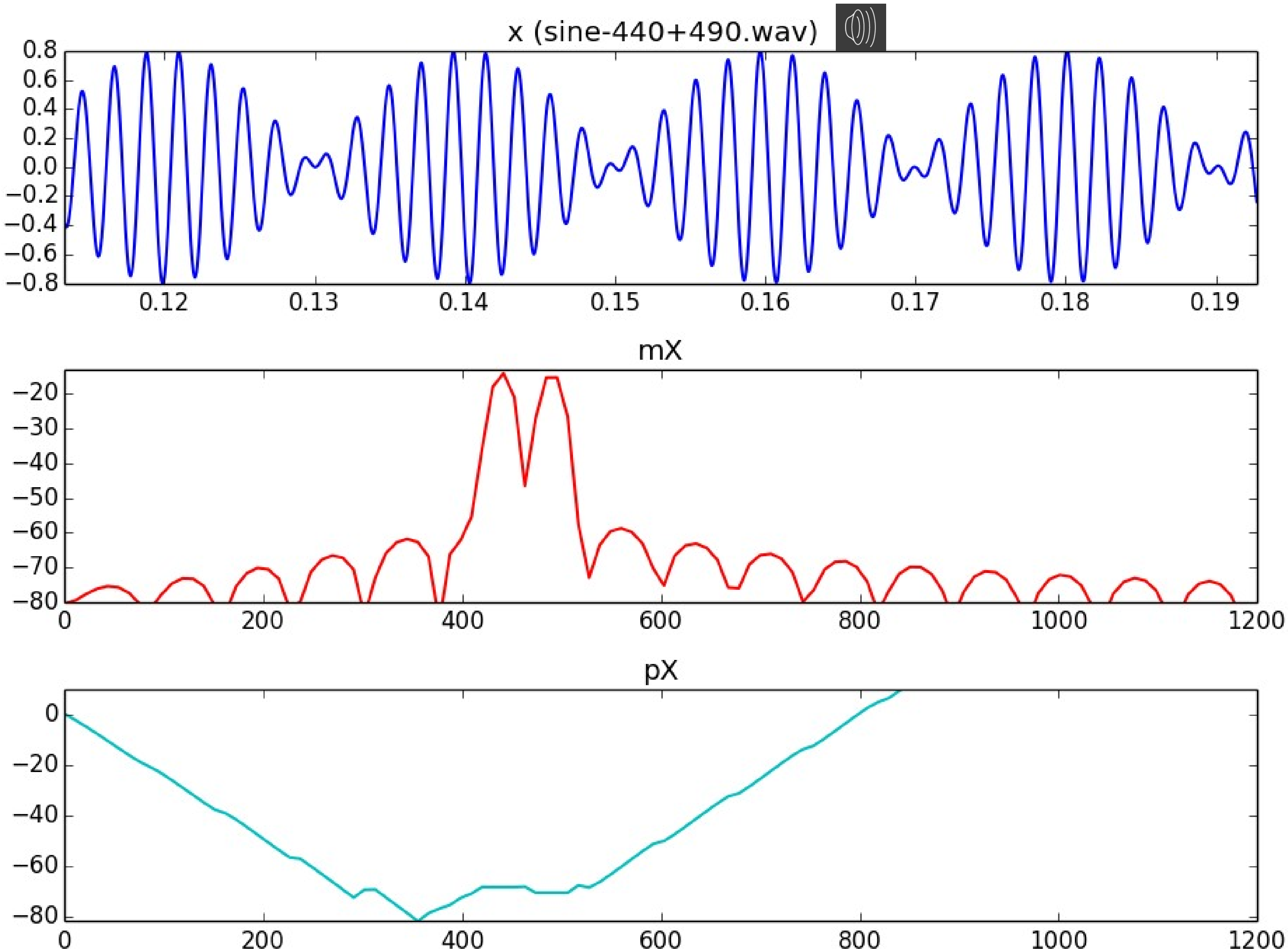
x (sine-440.wav)
0.5
0.0
−0.5
0.000
0.002
0.004
0.006
0.008
mX
−10
−20
−30
−40
−50
−60
−70
−80
0
500
1000
1500
2000
2500
3000
3500
4000
pX
0
−10
−20
−30
−40
−50
−60
0
500
1000
1500
2000
2500
3000
3500
4000



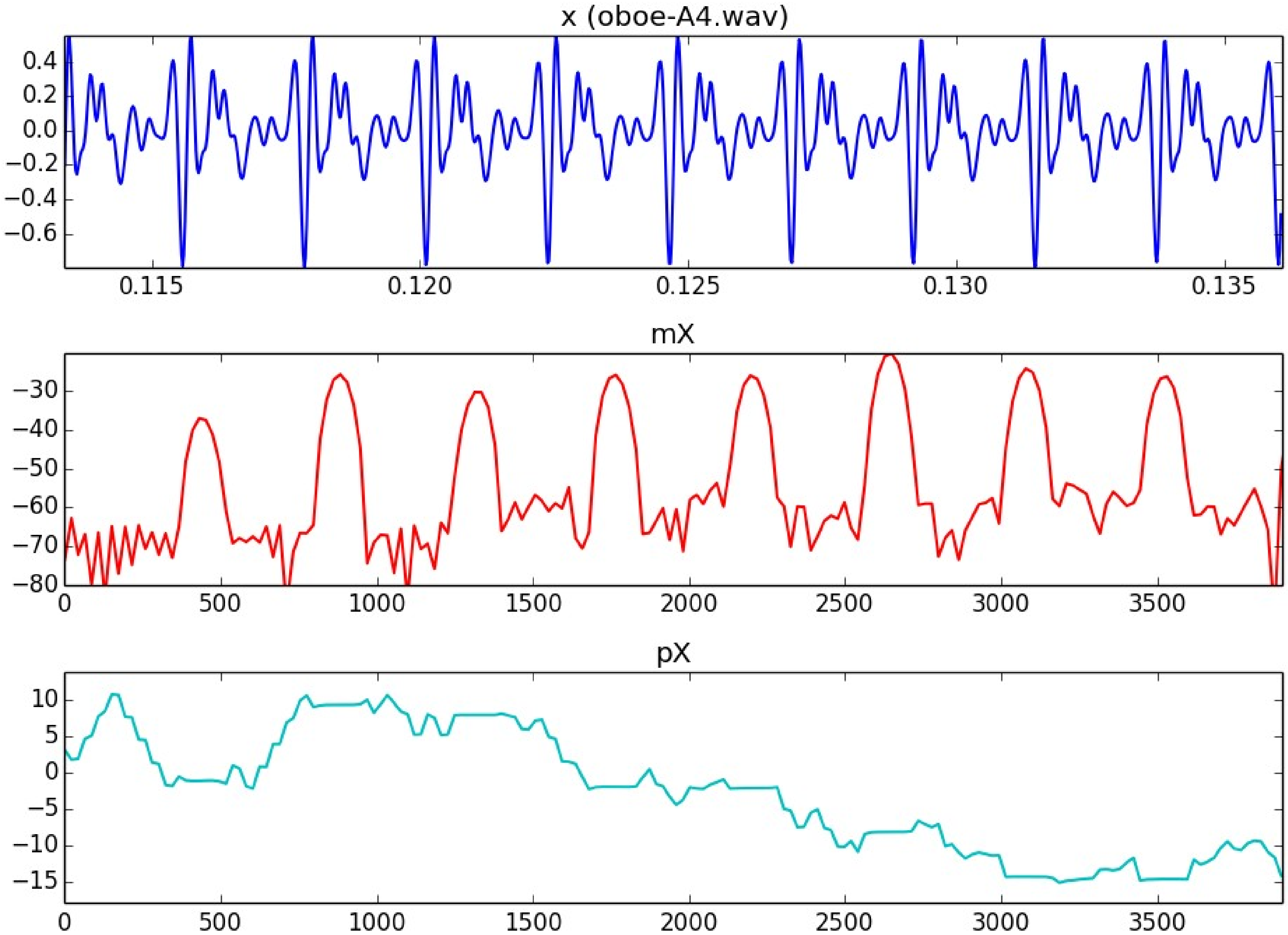
x (sine-440.wav)
0.5
0.0
−0.5
0.000
0.002
0.004
0.006
0.008
mX
−10
−20
−30
−40
−50
−60
−70
−80
0
500
1000
1500
2000
2500
3000
3500
4000
pX
0
−10
−20
−30
−40
−50
−60
0
500
1000
1500
2000
2500
3000
3500
4000



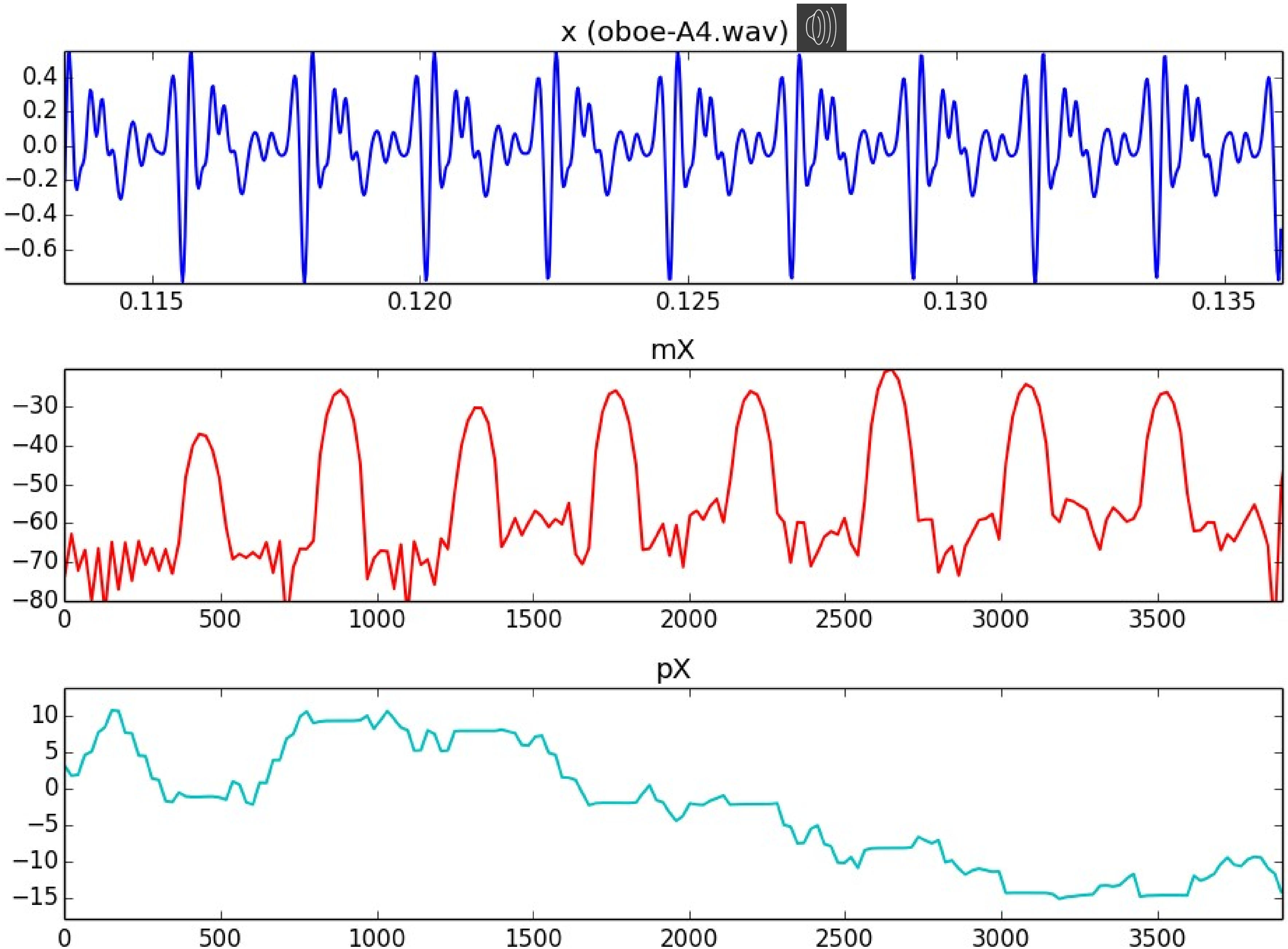
x (sine-440+490.wav)
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
mX
−20
−30
−40
−50
−60
−70
−80
0
200
400
600
800
1000
1200
pX
0
−20
−40
−60
−80
0
200
400
600
800
1000
1200



x (sine-440+490.wav)
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
mX
−20
−30
−40
−50
−60
−70
−80
0
200
400
600
800
1000
1200
pX
0
−20
−40
−60
−80
0
200
400
600
800
1000
1200



x (oboe-A4.wav)
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.115
0.120
0.125
0.130
0.135
mX
−30
−40
−50
−60
−70
−80
0
500
1000
1500
2000
2500
3000
3500
pX
10
5
0
−5
−10
−15
0
500
1000
1500
2000
2500
3000
3500



x (oboe-A4.wav)
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.115
0.120
0.125
0.130
0.135
mX
−30
−40
−50
−60
−70
−80
0
500
1000
1500
2000
2500
3000
3500
pX
10
5
0
−5
−10
−15
0
500
1000
1500
2000
2500
3000
3500

Sinewaves as spectral peaks

- Sinusoid → peak in magnitude spectrum
- Frequency resolution: 1/2 bin
- Improvement of frequency resolution by:
  - zero-padding
  - spectral interpolation

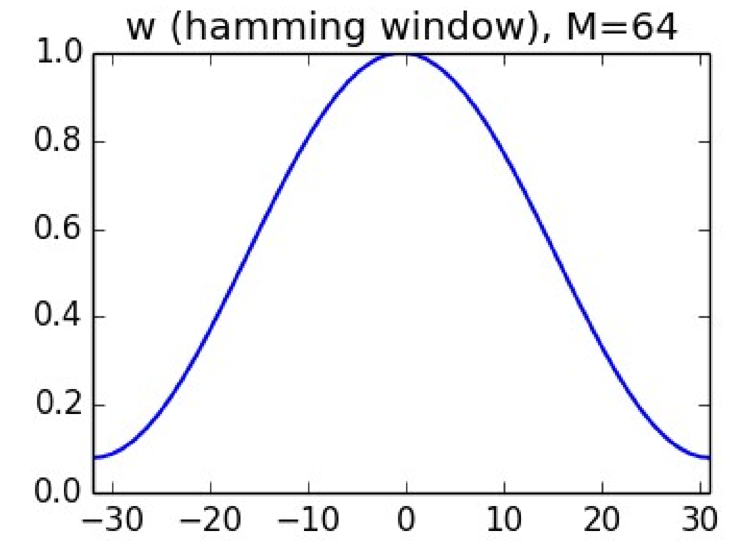
Spectral peaks and window-size

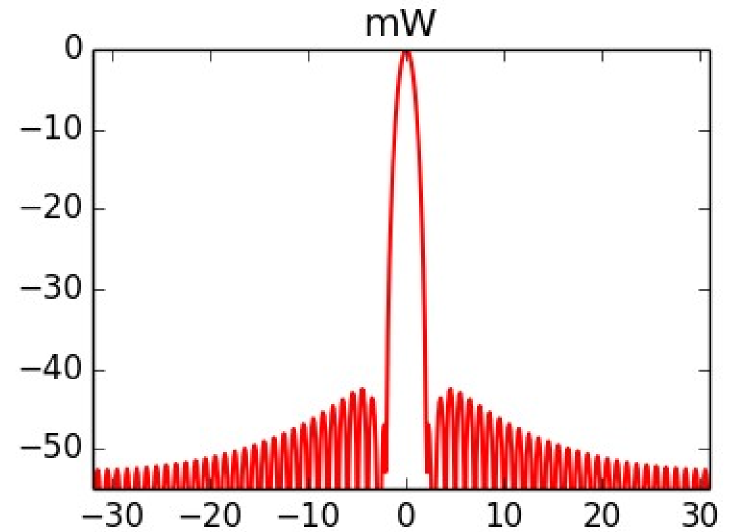
If $B_f = B_s f_s / M$ and $\Delta = |f_{k+1} - f_k|$

$B_s =$ main-lobe bandwidth of window
$f_s =$ sampling rate in Hz
$M =$ window size
f_k and $f_{k+1} =$ frequency of sinusoids in Hz

$$M \geqslant B_s \frac{f_s}{\Delta} = B_s \frac{f_s}{|f_{k+1} - f_k|}$$

If $f_0 = \Delta$, then $B_f \leqslant f_0$
and $M \geqslant B_s f_s / f_0$, or $M \geqslant B_s P$,
where $P =$ period in samples



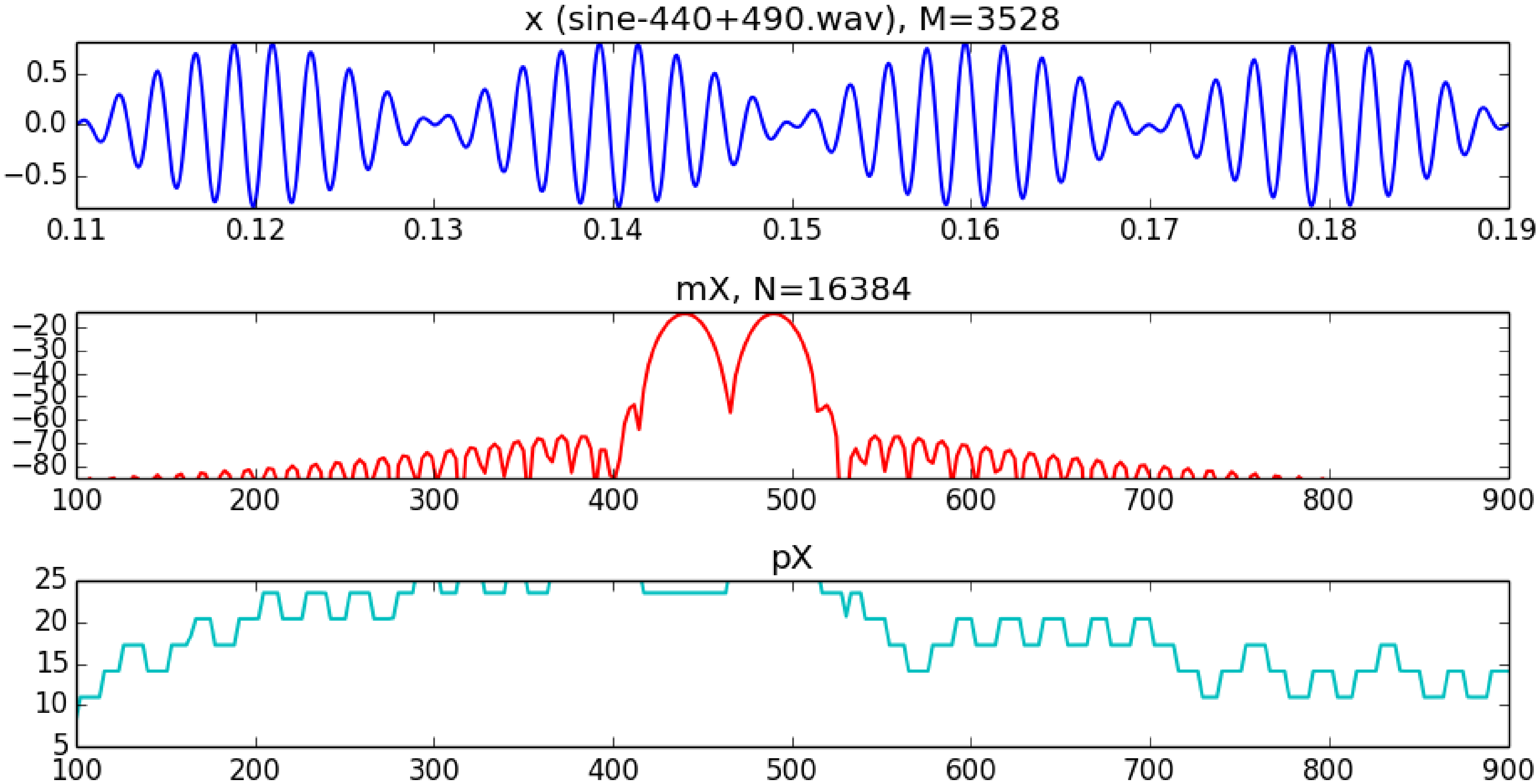


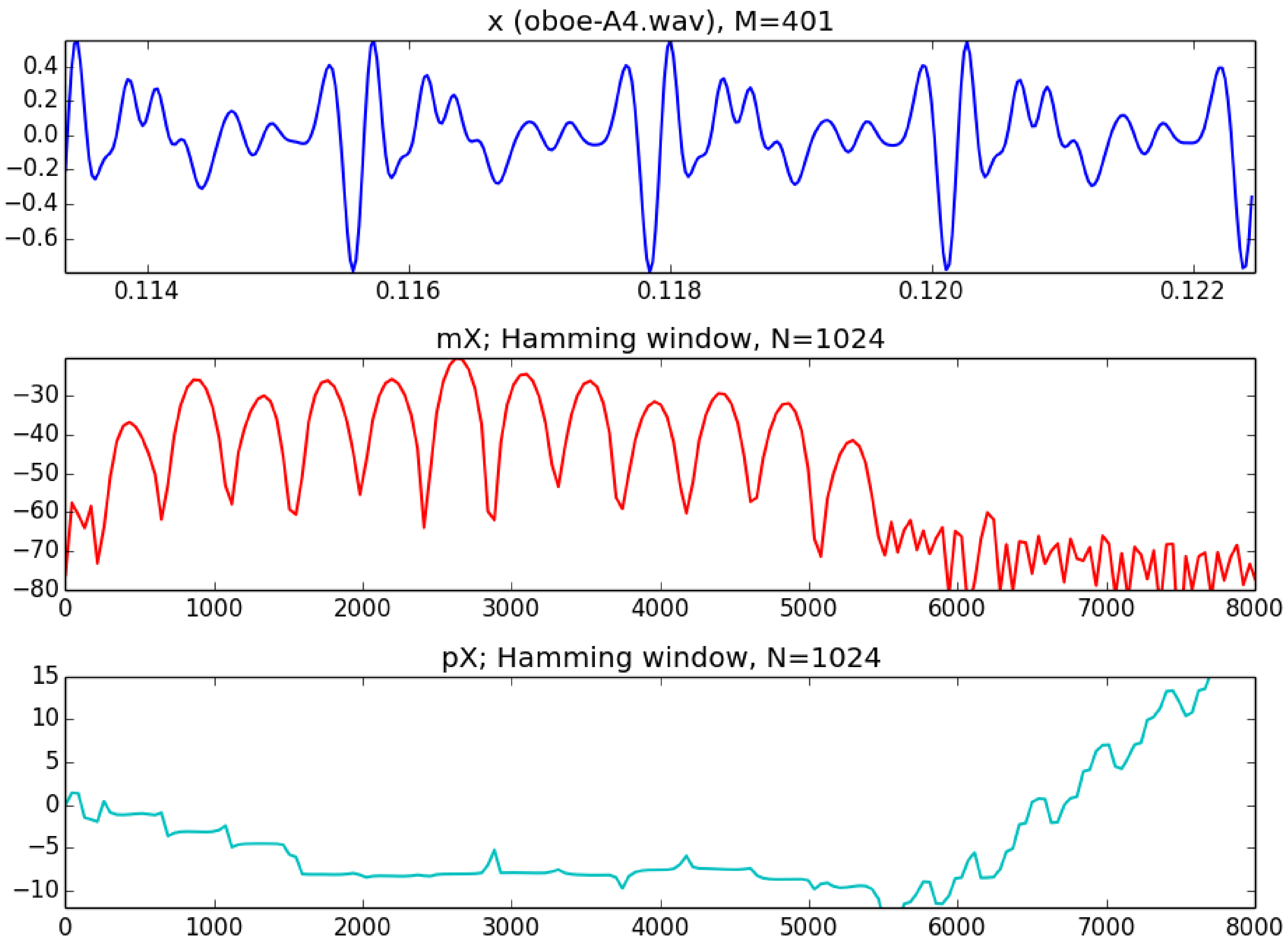
Hamming window : $B_s = 4$

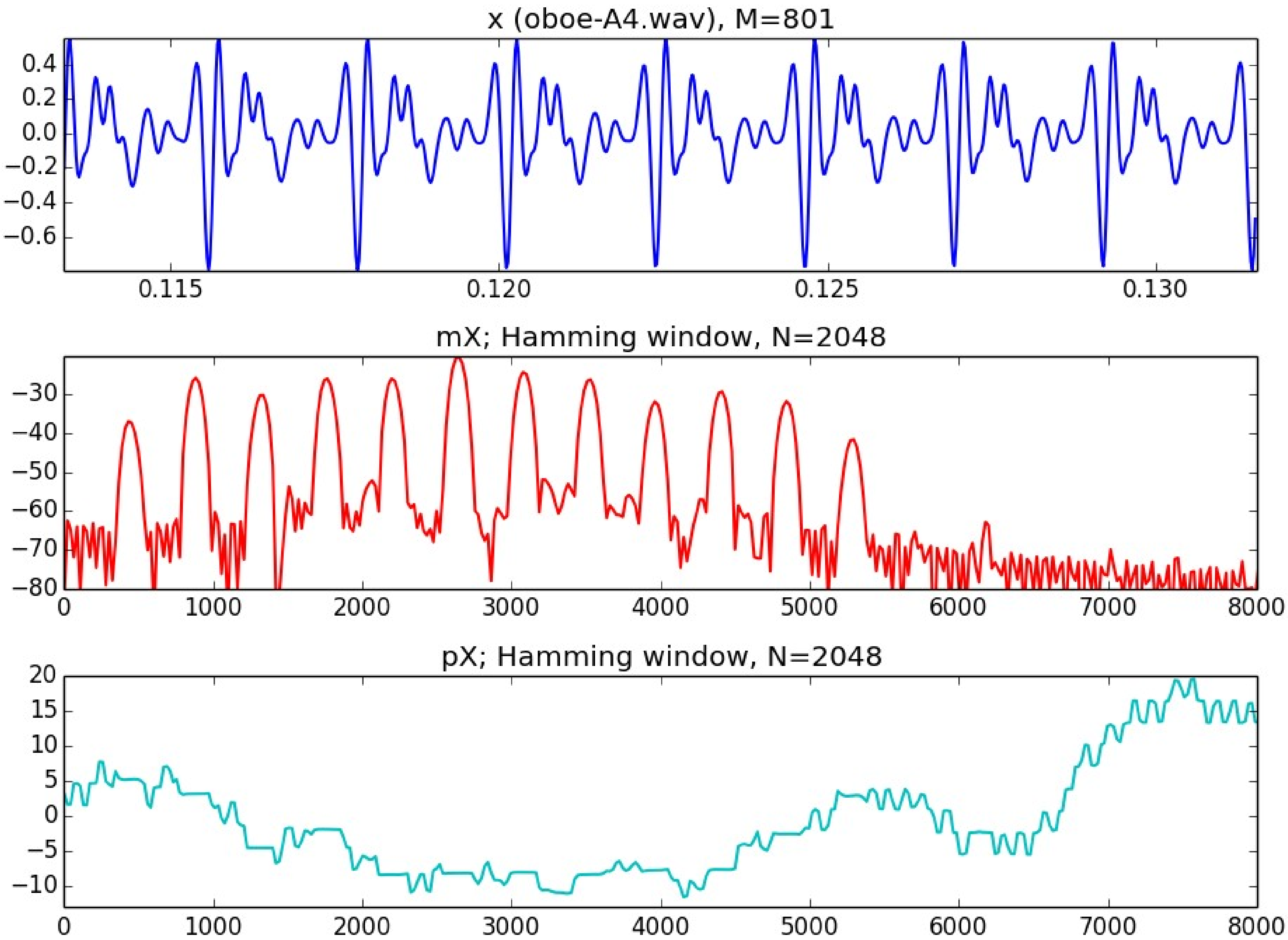
$f_s = 44100\,\text{Hz}$

$f_k = 440\,\text{Hz}\,;\ f_{k+1} = 490\,\text{Hz}$

$$M \geqslant B_s \frac{f_s}{|f_{k+1} - f_k|} = 4\frac{44100}{|490-440|} = 3528$$




x (oboe-A4.wav), M=401
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.114
0.116
0.118
0.120
0.122
mX; Hamming window, N=1024
−30
−40
−50
−60
−70
−80
0
1000
2000
3000
4000
5000
6000
7000
8000
pX; Hamming window, N=1024
15
10
5
0
−5
−10
0
1000
2000
3000
4000
5000
6000
7000
8000



x (oboe-A4.wav), M=801
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.115
0.120
0.125
0.130
mX; Hamming window, N=2048
−30
−40
−50
−60
−70
−80
0
1000
2000
3000
4000
5000
6000
7000
8000
pX; Hamming window, N=2048
20
15
10
5
0
−5
−10
0
1000
2000
3000
4000
5000
6000
7000
8000

References and credits

- More information in:
  - http://en.wikipedia.org/wiki/Sinusoidal_model
- Reference on sinusoidal modeling by Julius O. Smith: https://ccrma.stanford.edu/~jos/sasp/Spectrum_Analysis_Sinusoids.html
- Sounds from: http://www.freesound.org/people/xserra/packs/13038/
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