

7T1: Stochastic Model

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Stochastic signals

- Described by the laws of probability; mean, variance, probability distributions
- Autocorrelation

$$Z_{xx}[k] = \sum_{n=0}^{N-1} x[n]x[n+k] \quad k = -N+1, \dots, N-1$$

- Power spectral density

$$Xp[k] = \lim_{N \rightarrow \infty} |X[k]|^2$$

$$\text{where } X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N} \quad k = 0, \dots, N-1$$

Stochastic model

$$yst[n] = \sum_{k=0}^{N-1} u[n] h[n-k]$$

$u[n]$: white noise

$h[n]$: impulse response of filter approximating input signal $x[n]$

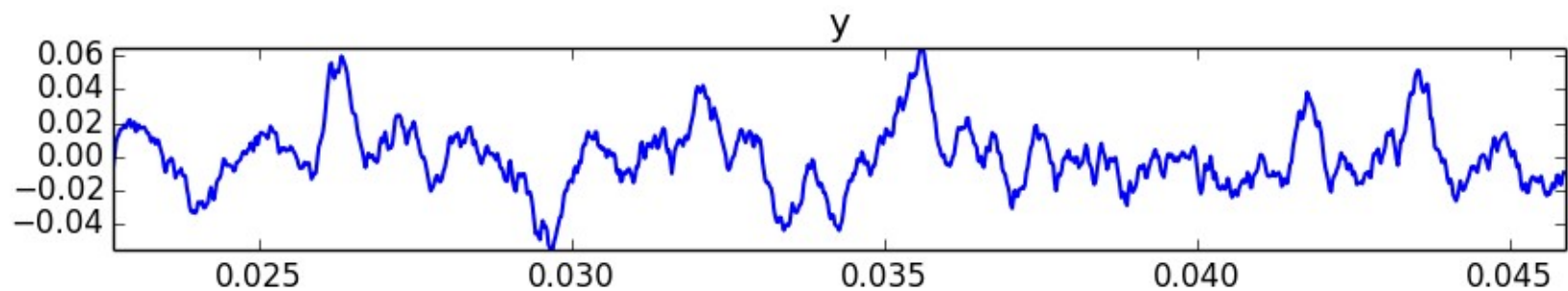
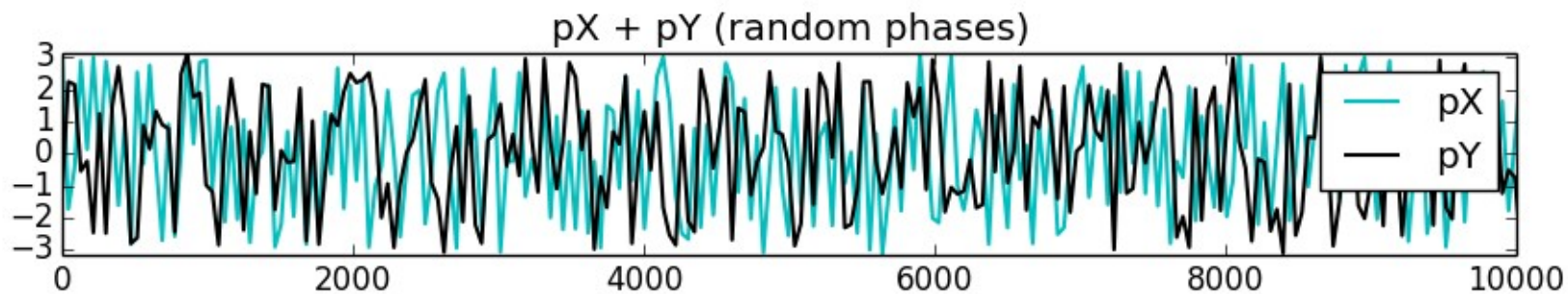
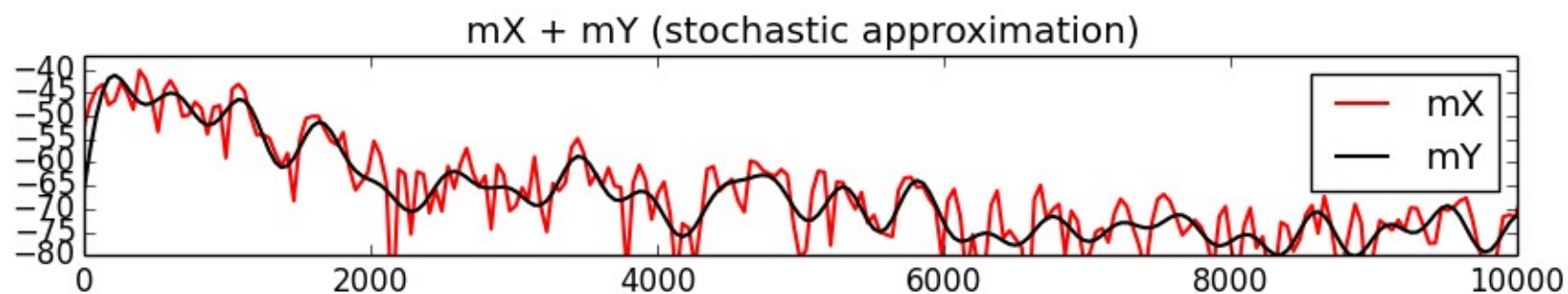
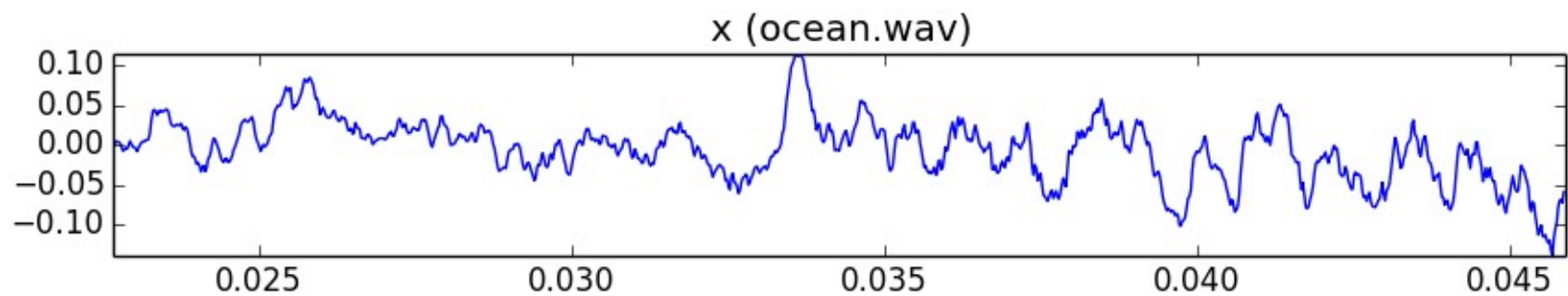
Spectral view:

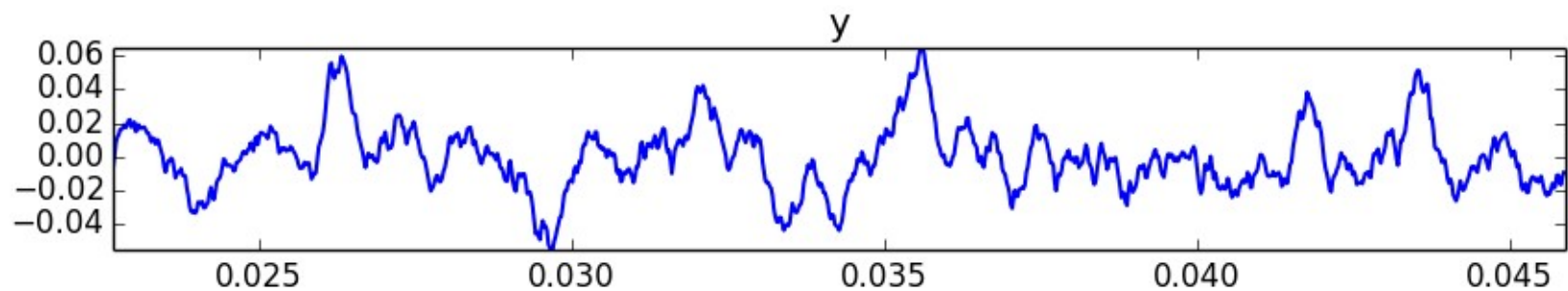
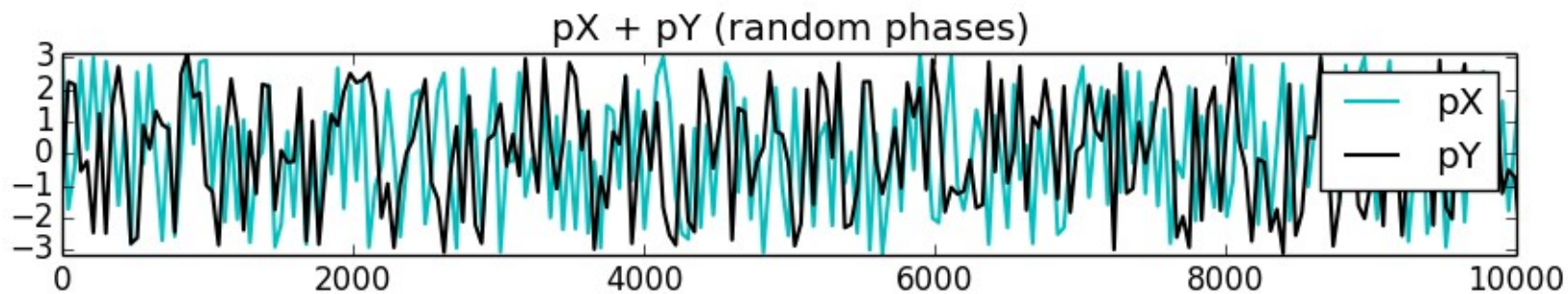
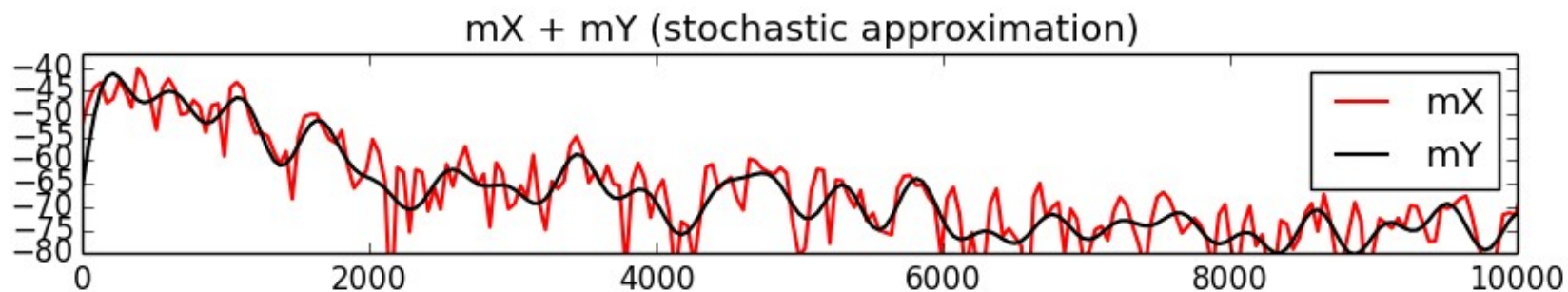
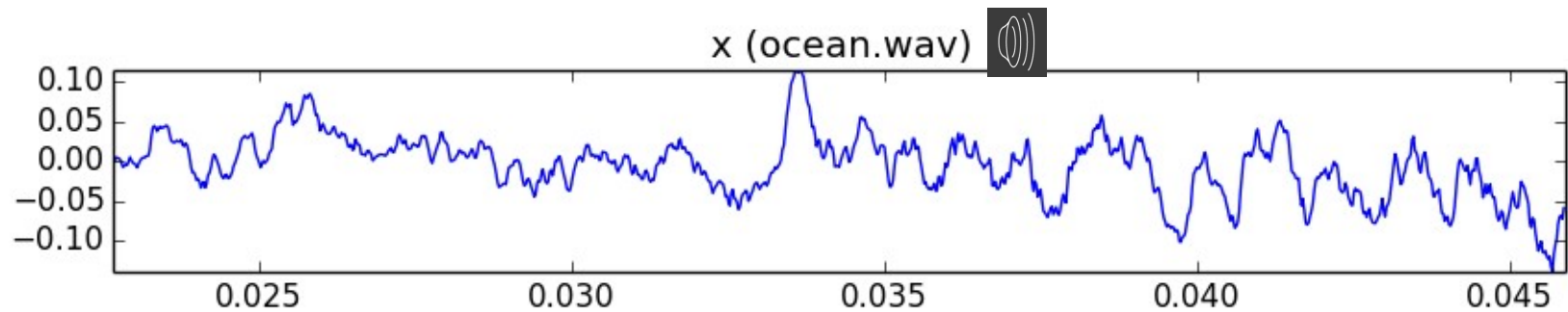
$$Yst_l[k] = |H_l[k]| |U[k]| e^{j(\angle H[k] + \angle U[k])} = |\tilde{X}_l[k]| e^{j\angle U[k]}$$

$|\tilde{X}[k]|$: approximation of magnitude spectrum of input signal $x[n]$

$\angle U[k]$: spectral phases of noise signal

l : frame number

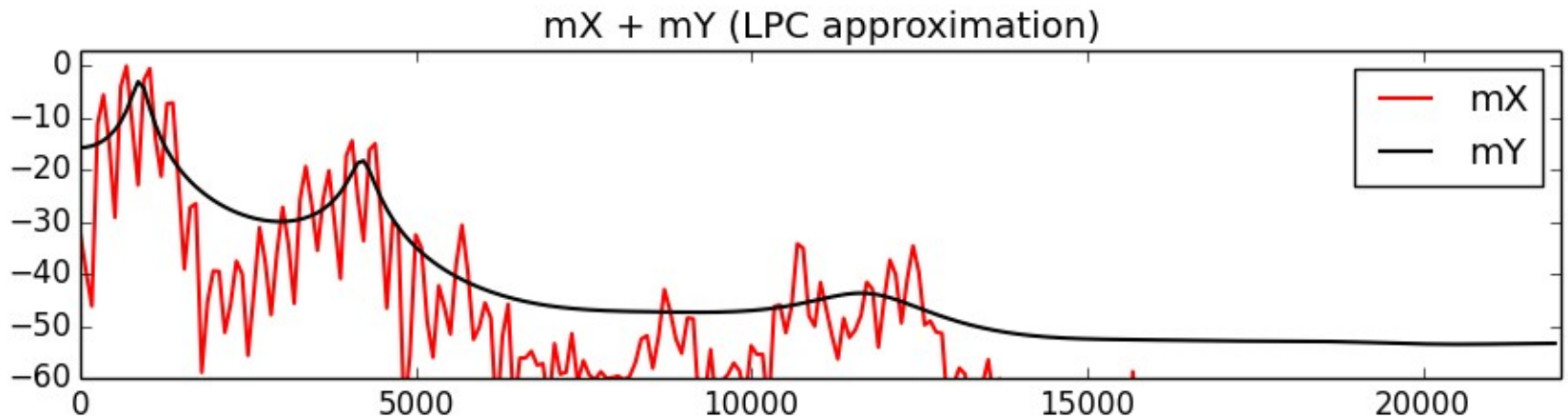
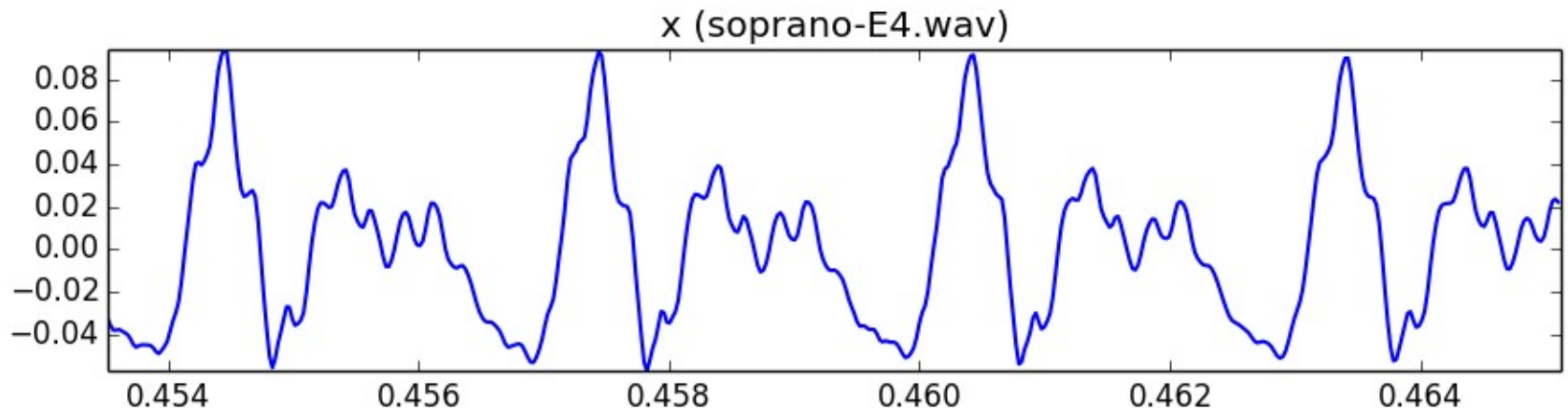




LPC approximation

$$\hat{x}[n] = \sum_{k=1}^K a_k x[n-k]$$

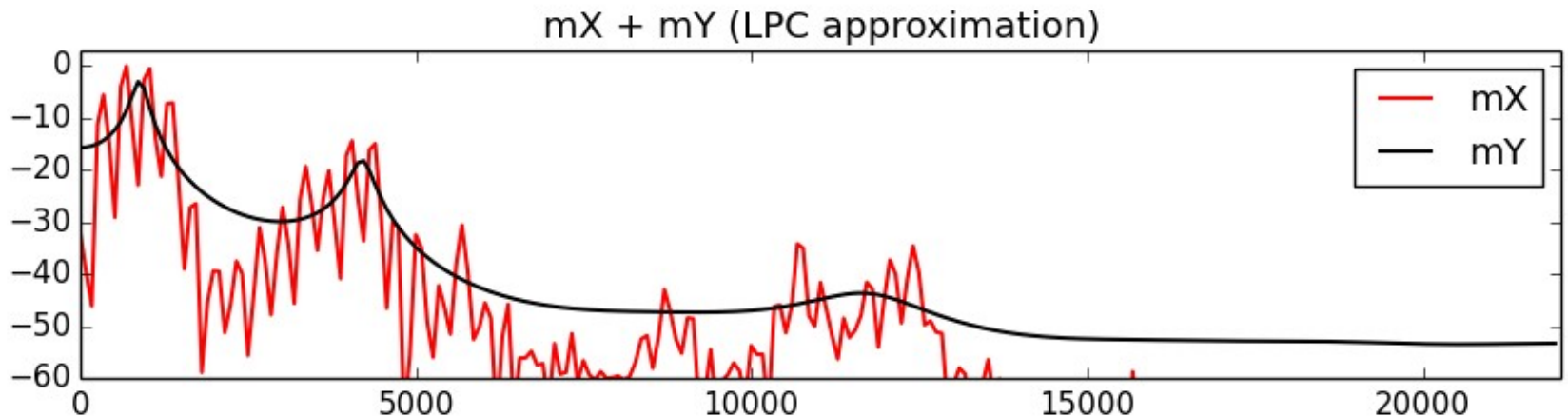
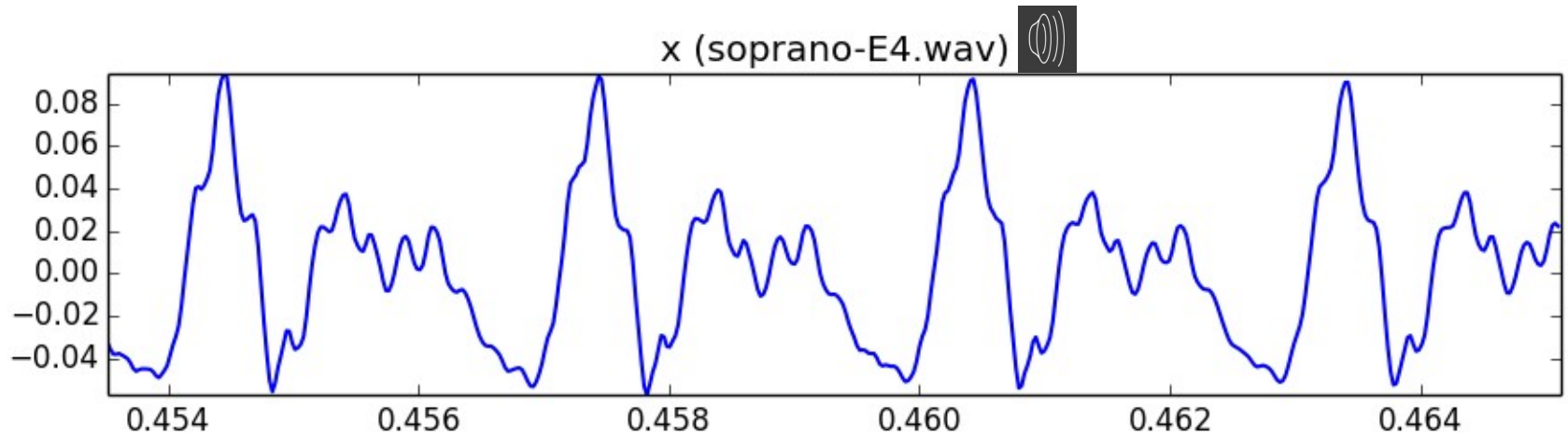
$$Error = \sum_{n=-\infty}^{\infty} \left(x[n] + \sum_{k=1}^K a_k x[n-k] \right)^2$$



LPC approximation

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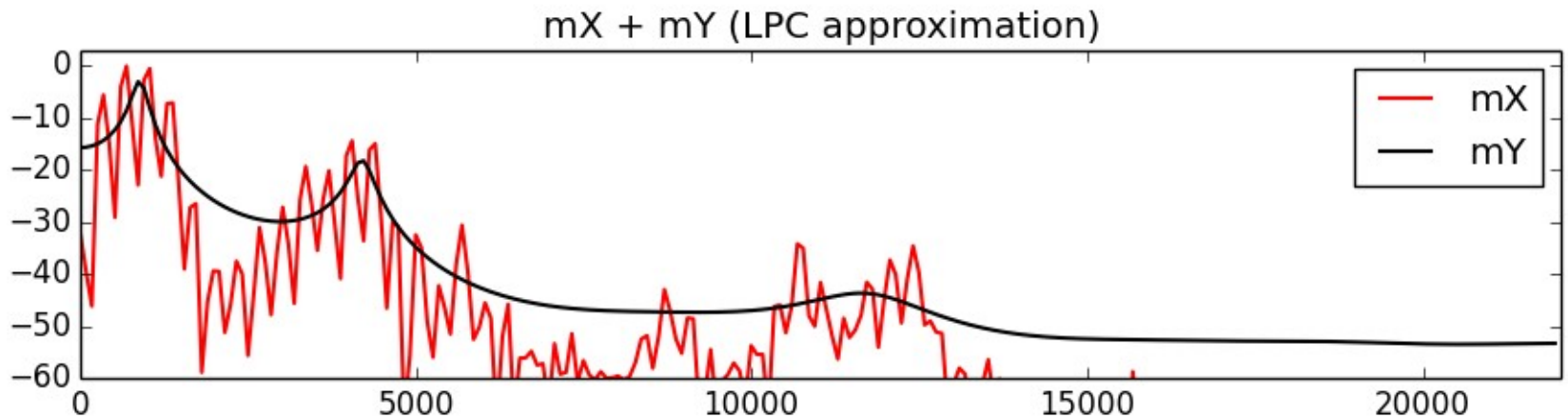
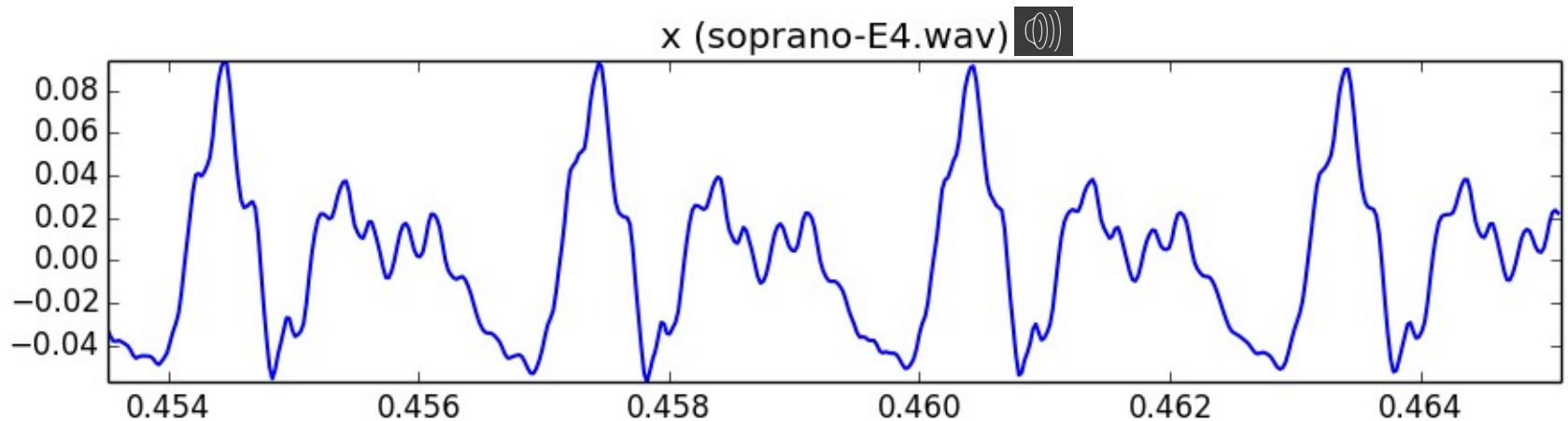
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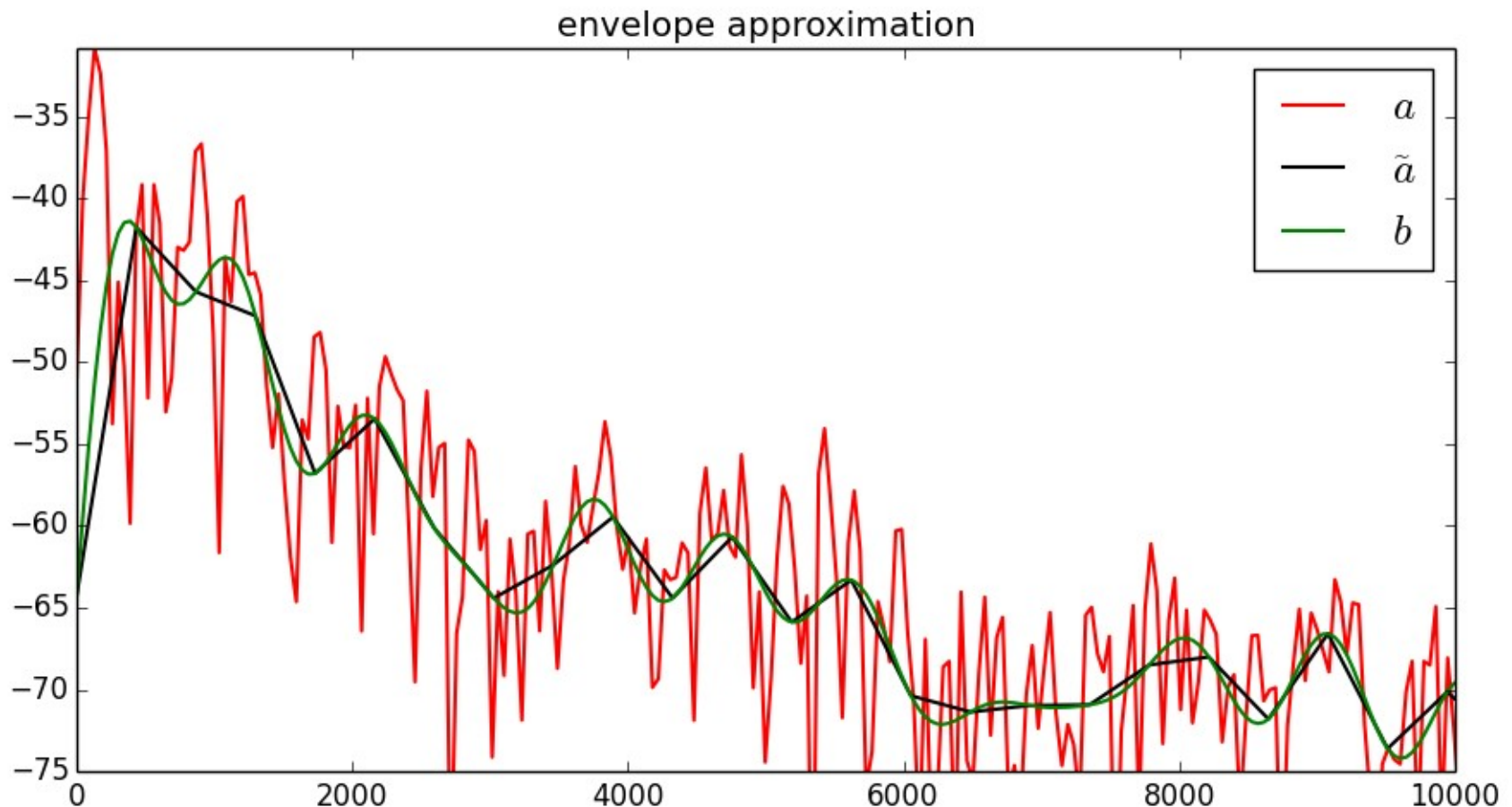
$$Error = \sum_{n=-\infty}^{\infty} \left(x[n] + \sum_{k=1}^K a_k x[n-k] \right)^2$$



Envelope approximation

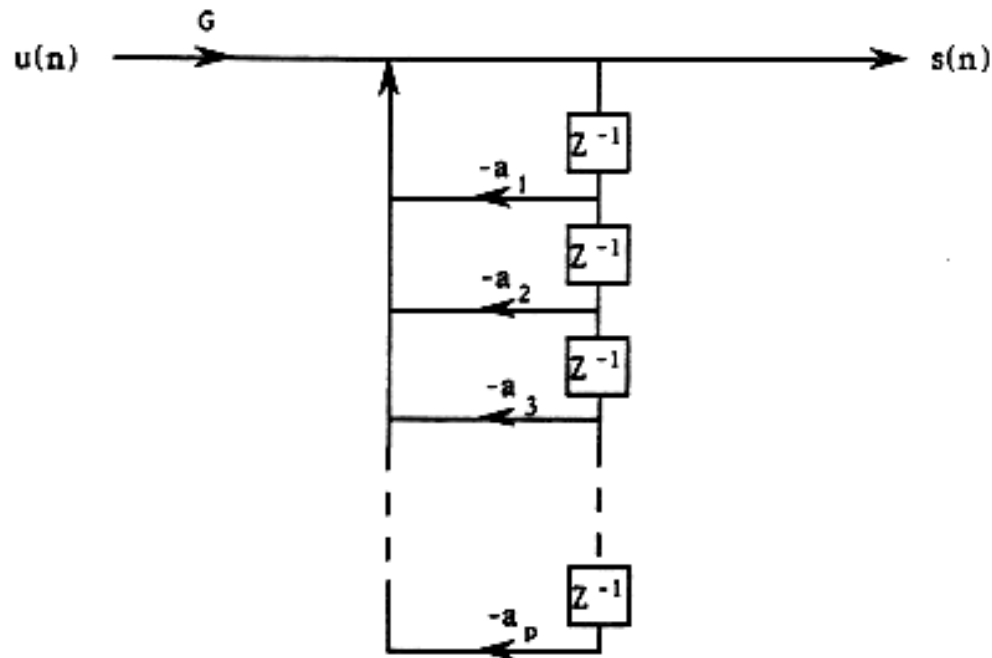
$$\tilde{a}[k] = IDFT(LP(DFT(a[k]))) \quad b[k] = IDFT(ZP(DFT(\tilde{a}[k])))$$

LP : low-pass filter ZP : zero-padding

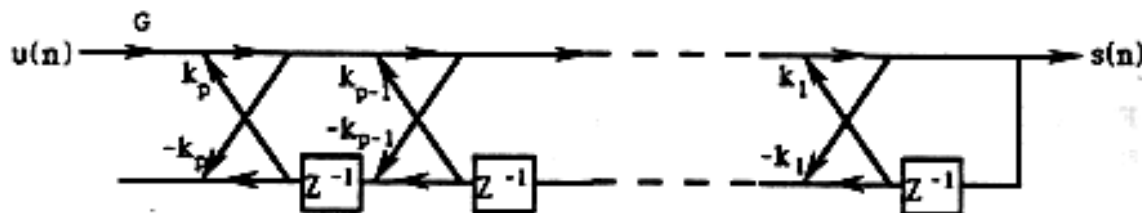


Stochastic synthesis using LPC

$$yst[n] = \sum_{k=1}^K a_k u[n-k], \quad a_k: \text{filter coefficients}; u[n]: \text{white noise}$$



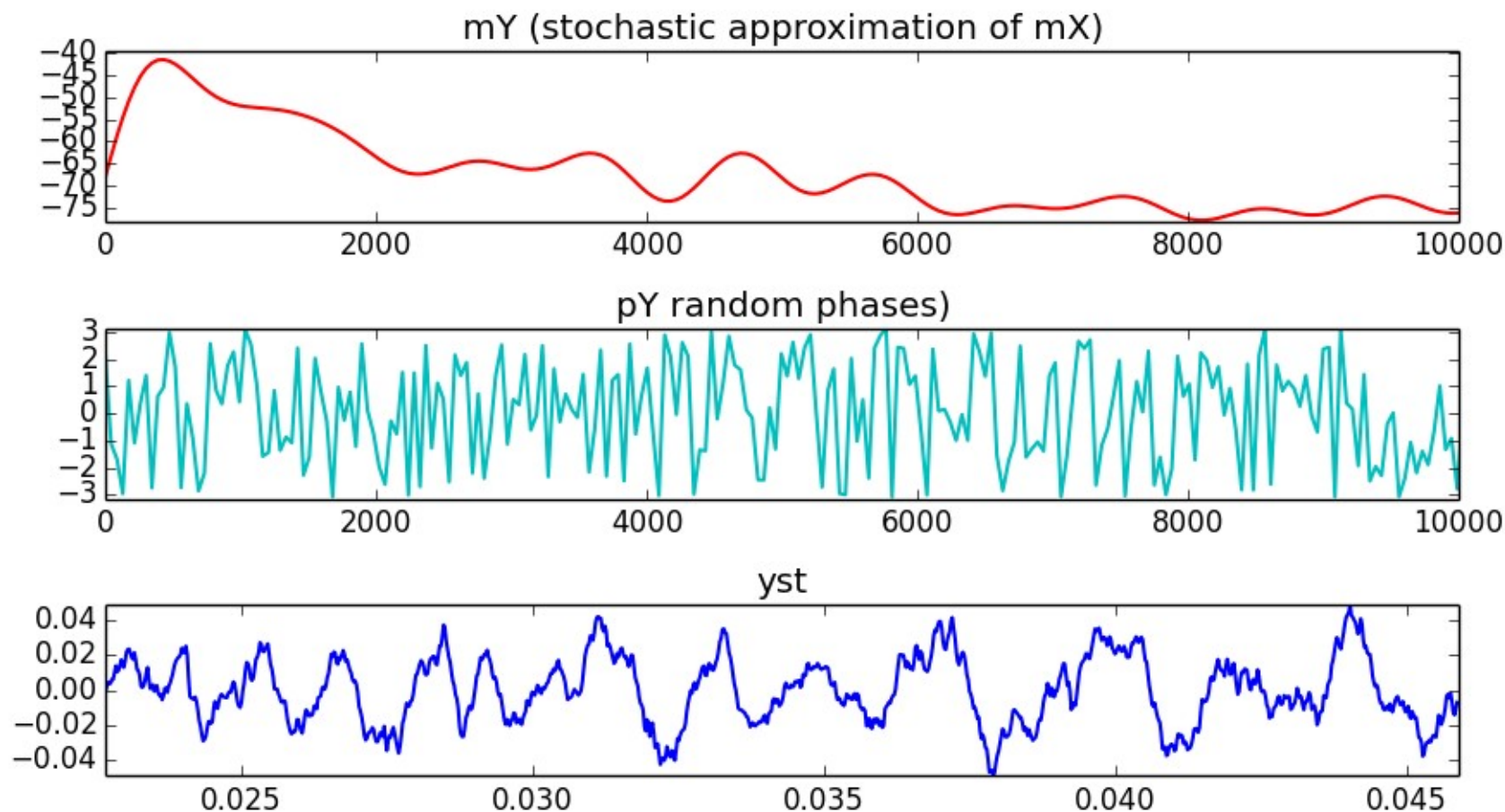
direct form
structure



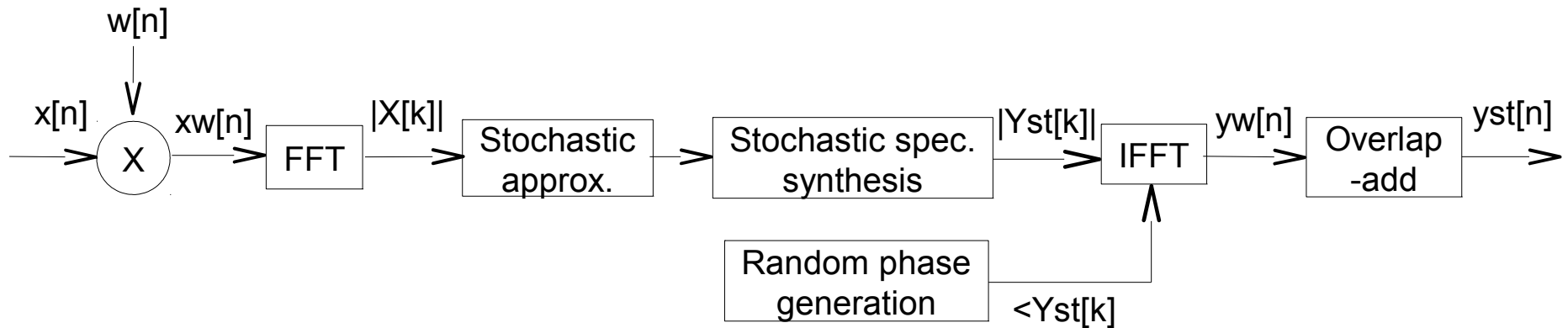
lattice
structure

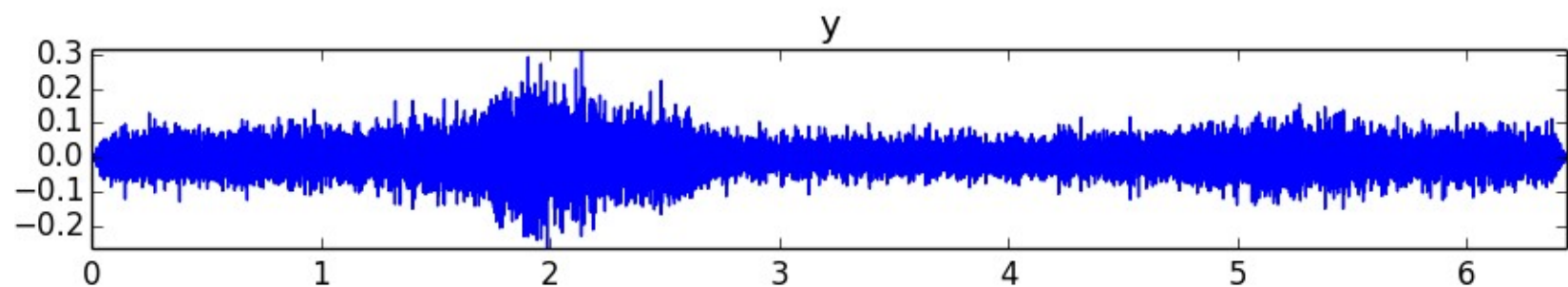
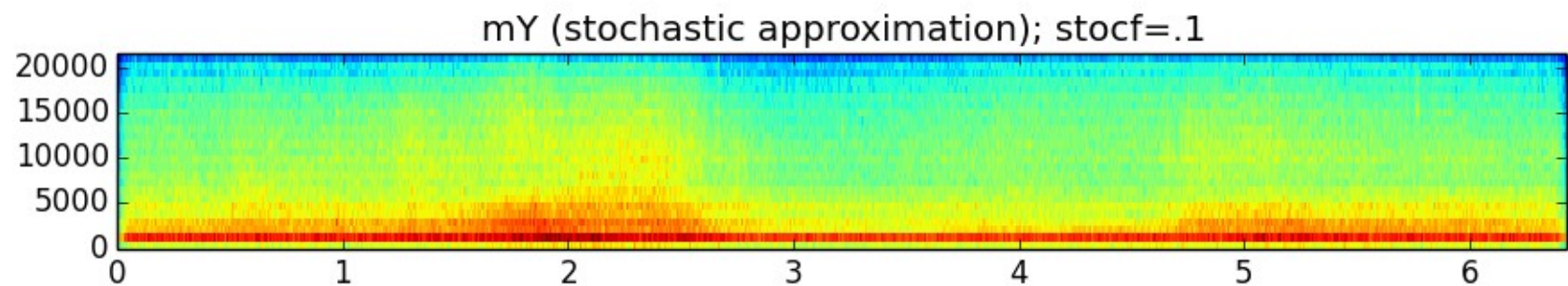
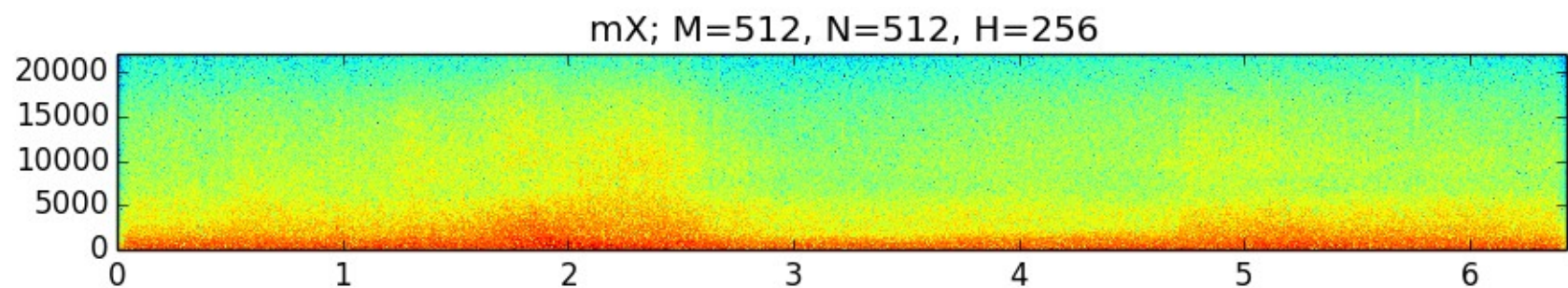
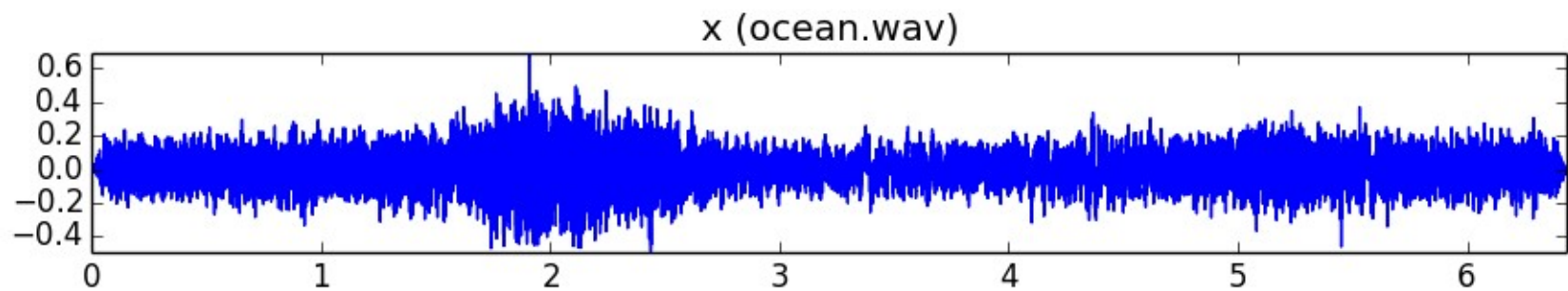
Stochastic synthesis using envelopes

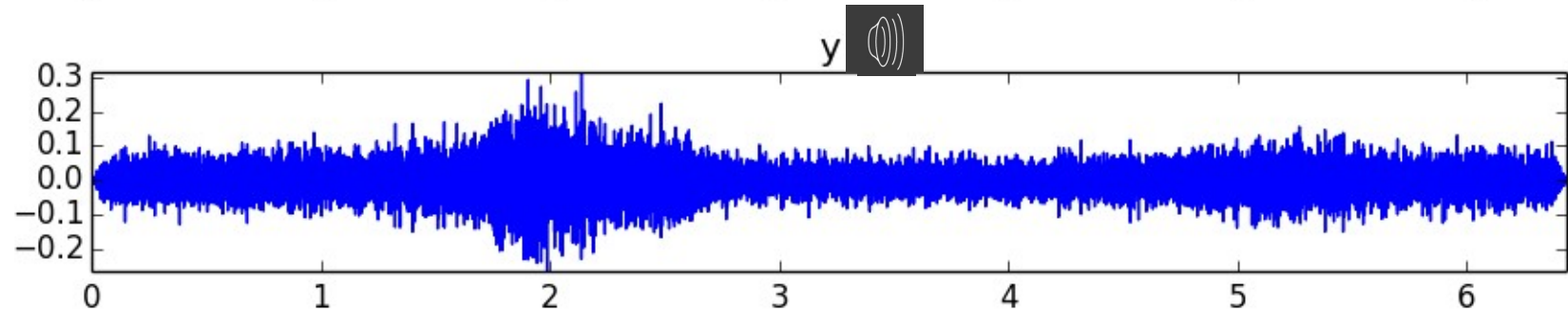
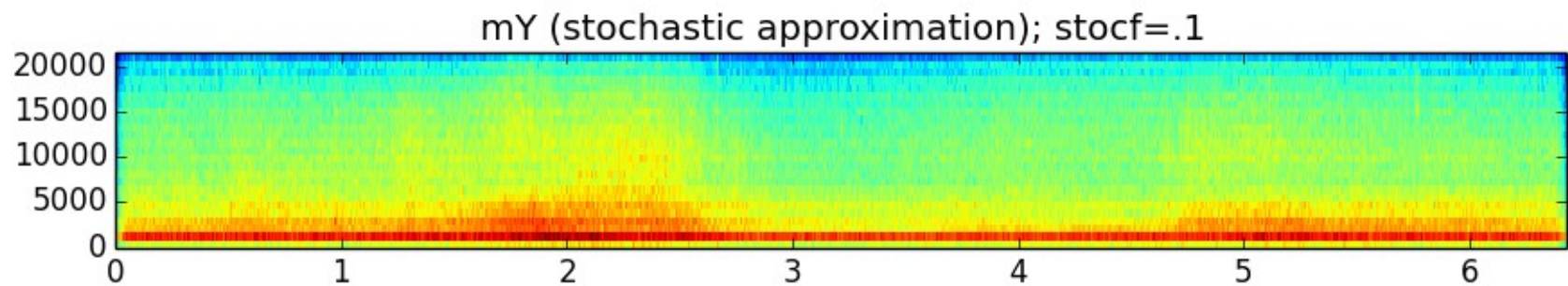
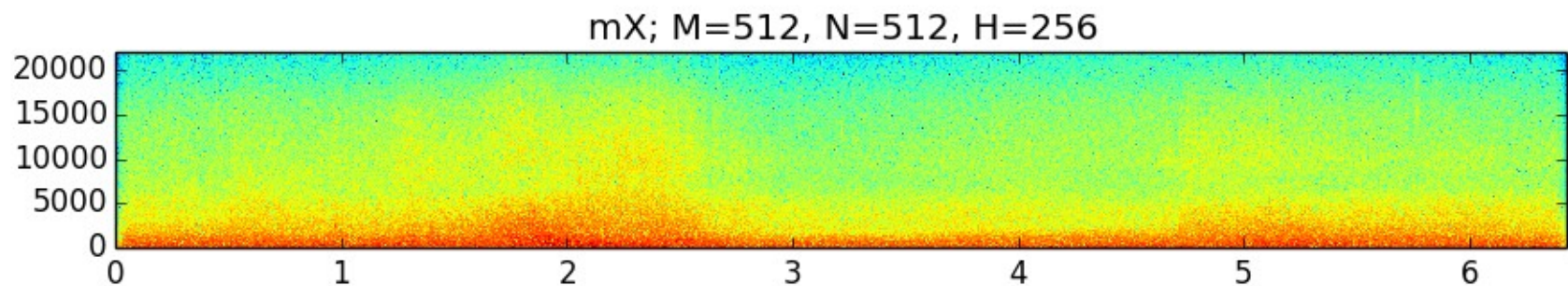
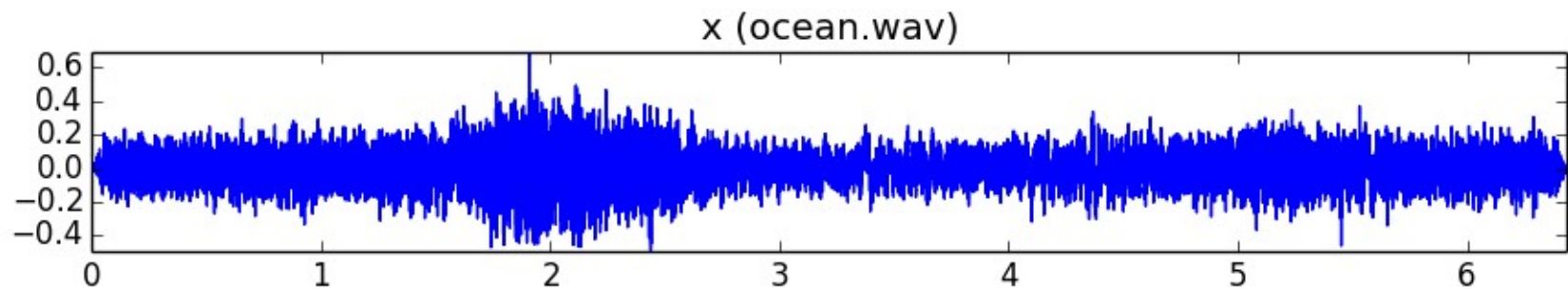
$$yst[n] = IDFT(|\tilde{X}[k]| e^{j\sum U[k]})$$



Stochastic model system







References and credits

- More information in:
 - http://en.wikipedia.org/wiki/Statistical_signal_processing
 - http://en.wikipedia.org/wiki/Stochastic_process
 - http://en.wikipedia.org/wiki/Linear_predictive_coding
- Sounds: <http://www.freesound.org/people/xserra/packs/13038/>
- Slides and code released using the CC Attribution-Noncommercial-Share Alike license or the Affero GPL license and available from <https://github.com/MTG/sms-tools>

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