2T1: The Discrete Fourier Transform (1 of 2)

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Discrete Fourier Transform

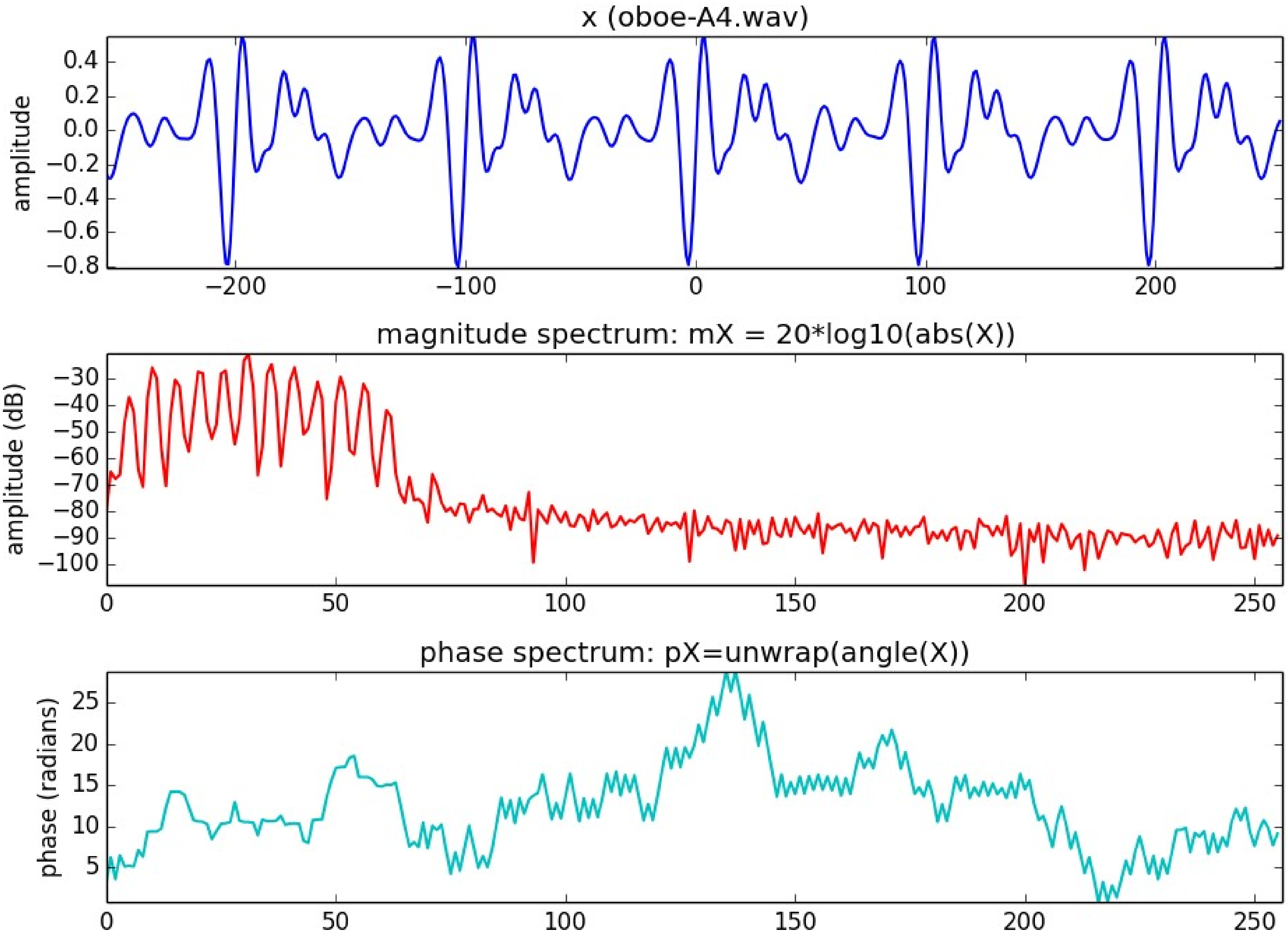
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \qquad k = 0, \ldots, N-1$$

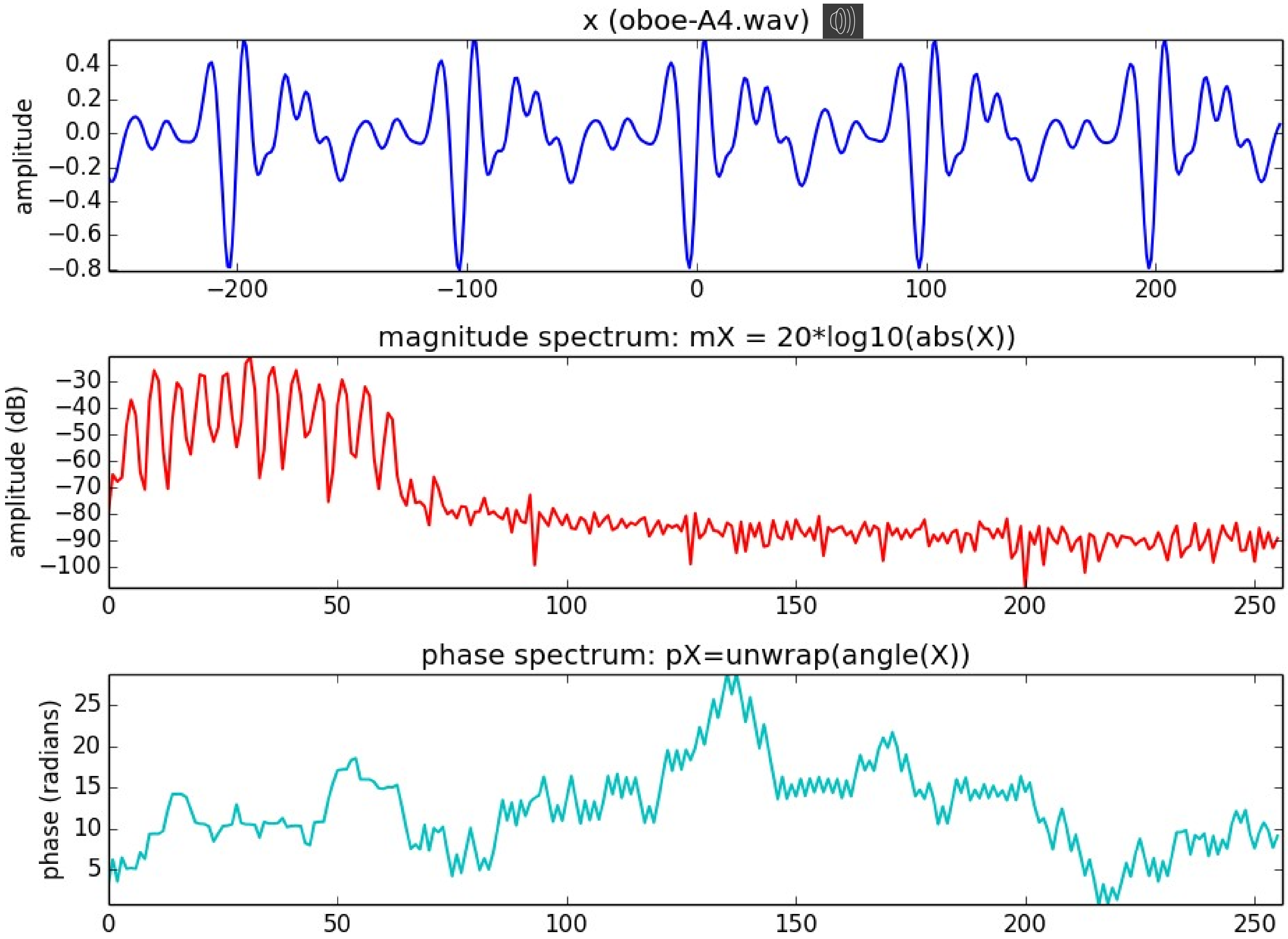
n : discrete time index (normalized time, $T = 1$)

k : discrete frequency index

$\omega_k = 2\pi k/N$: frequency in radians

$f_k = f_s k/N$: frequency in Hz (f_s : sampling rate)


x (oboe-A4.wav)
amplitude
magnitude spectrum: mX = 20*log10(abs(X))
amplitude (dB)
phase spectrum: pX=unwrap(angle(X))
phase (radians)



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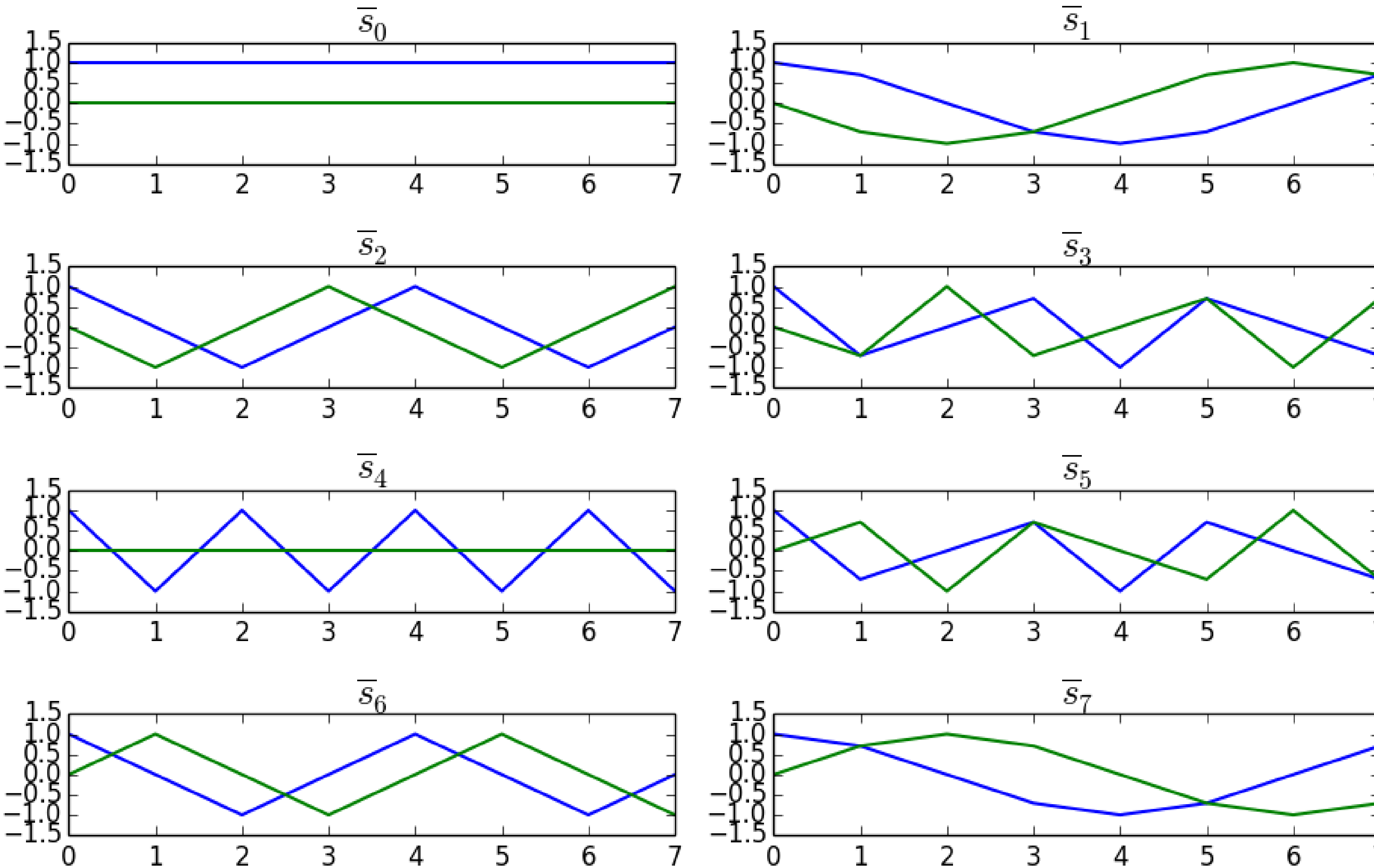
DFT: complex exponentials

$$\bar{s}_k = e^{-j2\pi kn/N} = \cos(2\pi kn/N) - j\sin(2\pi kn/N)$$

for $N=4$, thus for $n=0,1,2,3;\quad k=0,1,2,3$

$$\bar{s}_0 = \cos(2\pi*0*n/4) - j\sin(2\pi*0*n/4) = [1,1,1,1]$$
$$\bar{s}_1 = \cos(2\pi*1*n/4) - j\sin(2\pi*1*n/4) = [1,-j,-1,j]$$
$$\bar{s}_2 = \cos(2\pi*2*n/4) - j\sin(2\pi*2*n/4) = [1,-1,1,-1]$$
$$\bar{s}_3 = \cos(2\pi*3*n/4) - j\sin(2\pi*3*n/4) = [1,j,-1,-j]$$

DFT: complex exponentials



DFT: scalar product

$$\langle x, s_k \rangle = \sum_{n=0}^{N-1} x[n]\bar{s}_k[n] = \sum_{n=0}^{N-1} x[n] e^{-j 2\pi kn/N}$$

Example:

$$x[n] = [1, -1, 1, -1]; N = 4$$

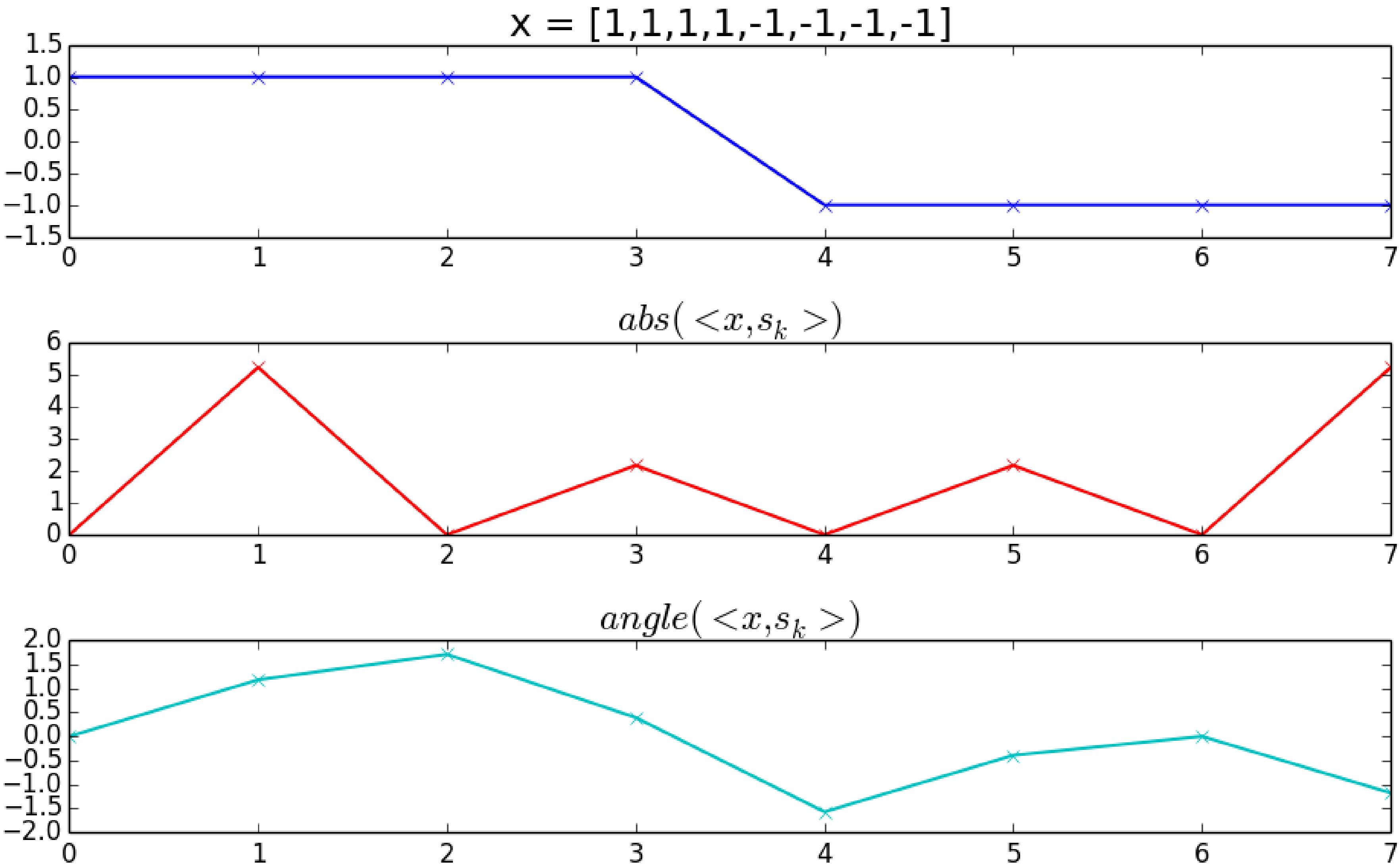
$$\langle x, s_0 \rangle = 1*1 + (-1)*1 + 1*1 + (-1)*1 = 0$$

$$\langle x, s_1 \rangle = 1*1 + (-1)*(-j) + 1*(-1) + (-1)*j = 0$$

$$\langle x, s_2 \rangle = 1*1 + (-1)*(-1) + 1*1 + (-1)*(-1) = 4$$

$$\langle x, s_3 \rangle = 1*1 + (-1)*(-j) + 1*(-1) + (-1)*j = 0$$

DFT: scalar product



References and credits

- More information in:
  - https://en.wikipedia.org/wiki/Discrete_Fourier_transform
- Reference on the mathematics of the DFT from Julius O. Smith: https://ccrma.stanford.edu/~jos/mdft/
- Sounds from: http://www.freesound.org/people/xserra/packs/13038
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