

# 5T3: Sinusoidal Model

## (3 of 3)

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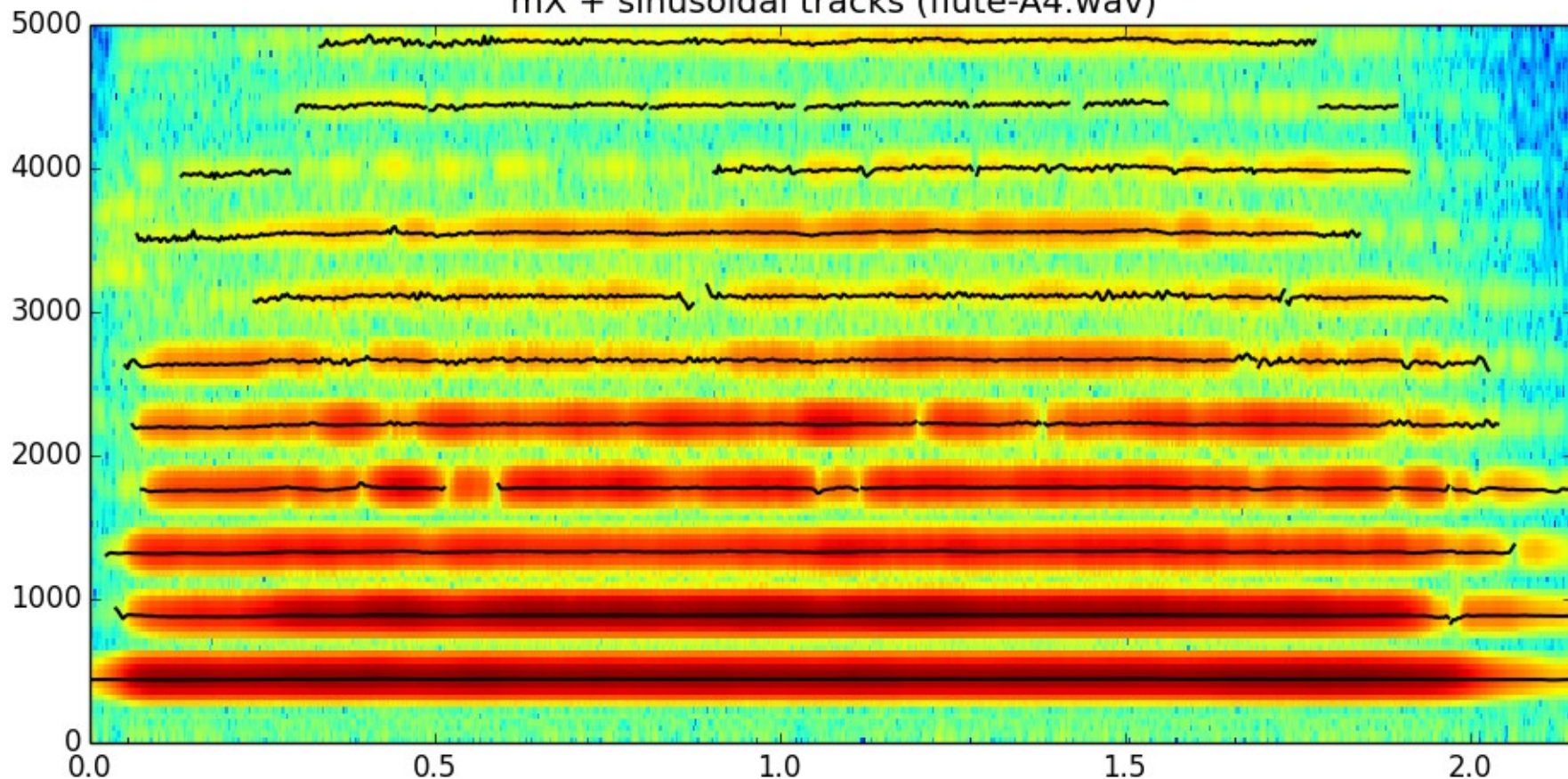
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- Sinusoidal model and spectral peak tracks
- Sinusoidal (additive) synthesis
- Sinusoidal model system

# Sinusoidal model

$$y[n] = \sum_{r=1}^R A_r[n] \cos(2\pi f_r[n]n)$$

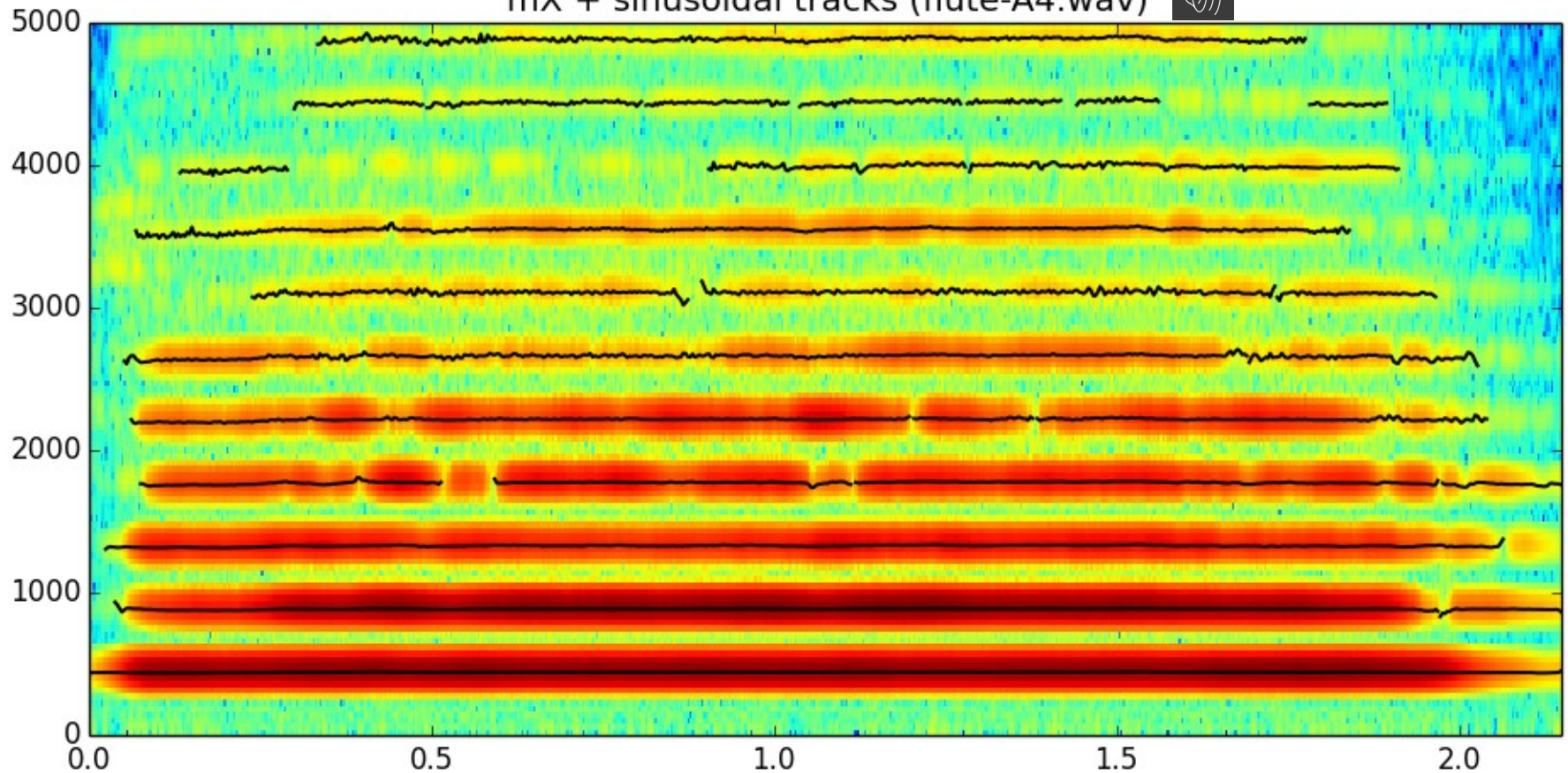
mX + sinusoidal tracks (flute-A4.wav)



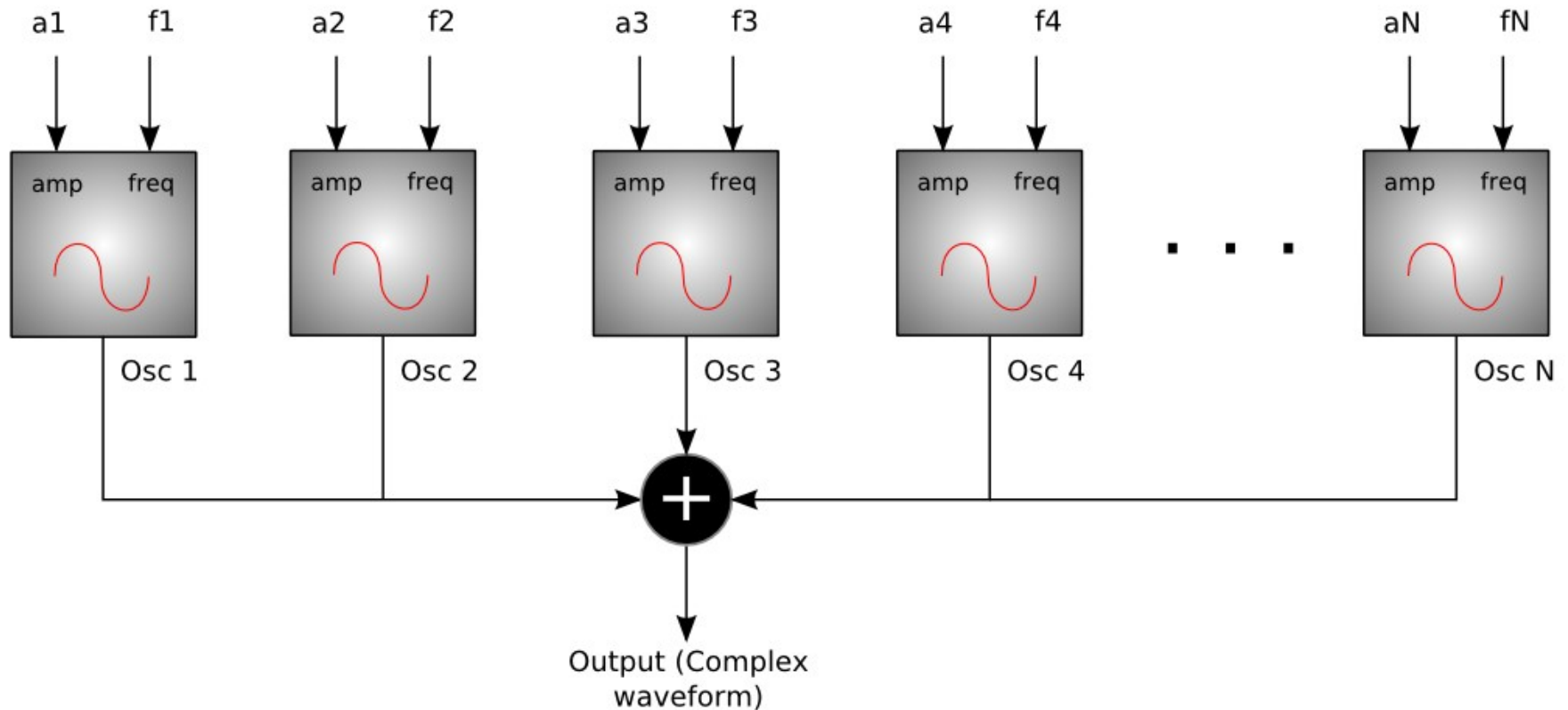
# Sinusoidal model

$$y[n] = \sum_{r=1}^R A_r[n] \cos(2\pi f_r[n]n)$$

mX + sinusoidal tracks (flute-A4.wav)



# Sinusoidal (additive) synthesis



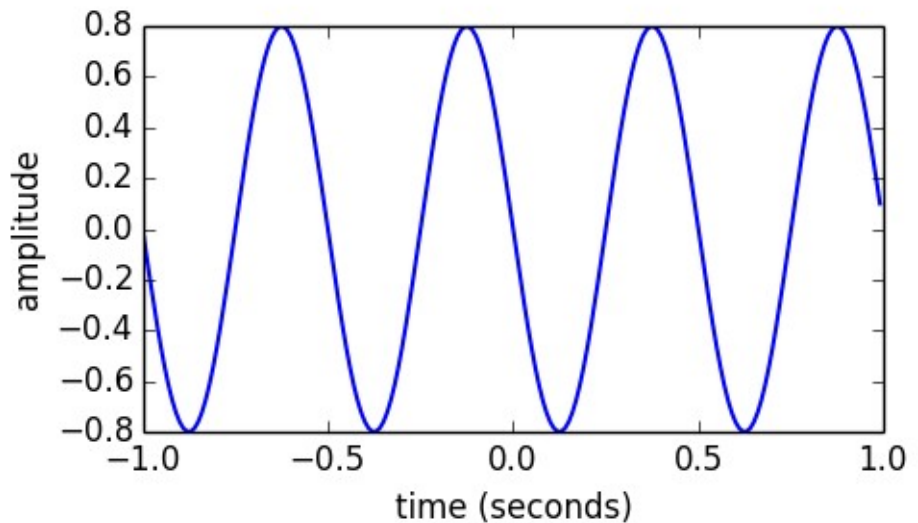


# Sinusoidal synthesis

$$y[n] = A_r[n] \cos(2\pi f_r[n]n + \varphi_r)$$

$A_r[n]$ : instantaneous amplitude ;  $f_r[n]$ : instantaneous frequency  
 $\varphi_r$ : initial phase

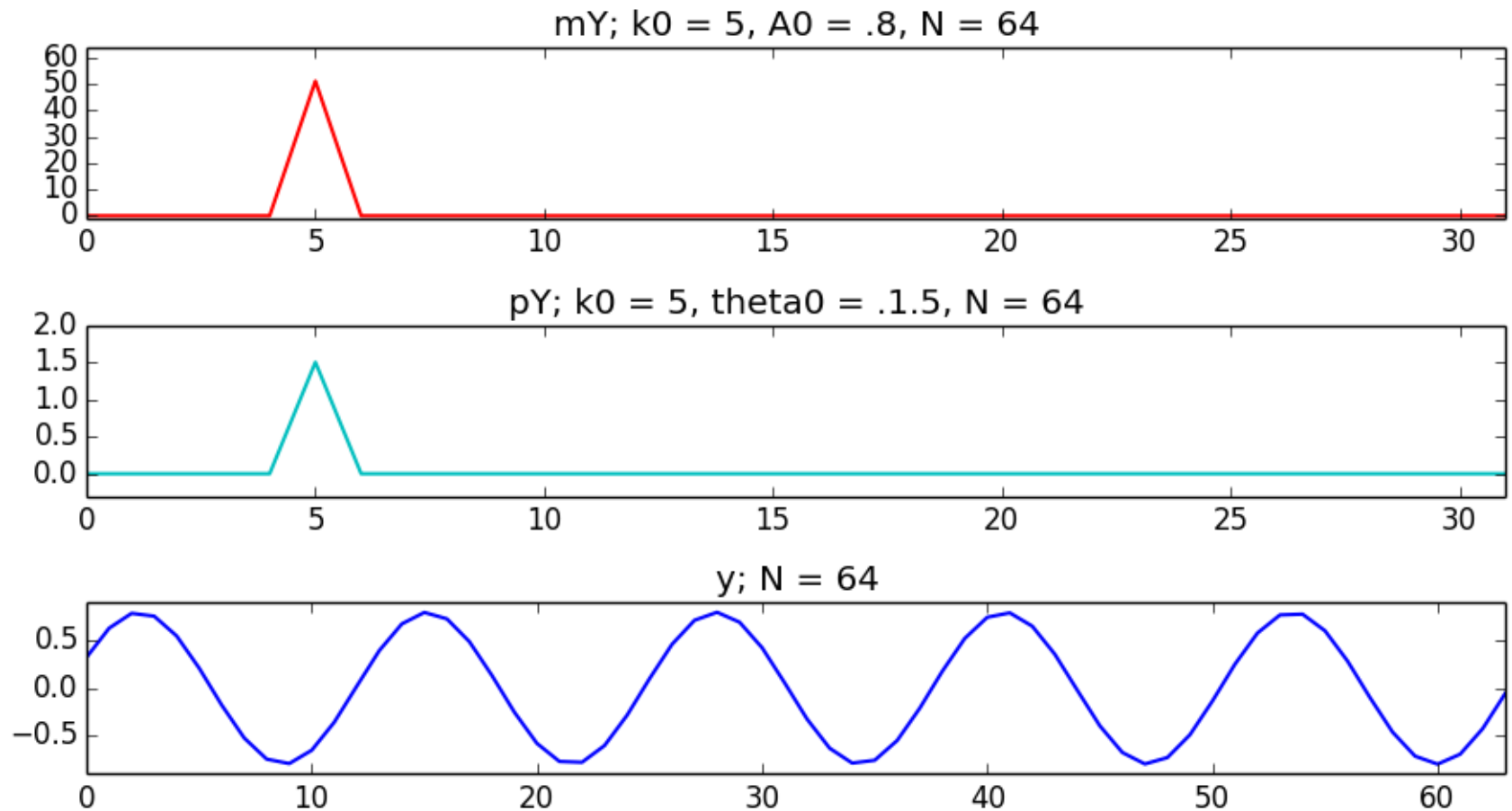
```
Ar = .8  
fr = 2.0  
phi = pi/2  
fs = 100  
t = arange(-1, 1, 1.0/fs)  
x = Ar * cos(2*pi*fr*t+phi)
```



# Sinusoidal synthesis: discrete frequency

$$y[n] = \text{IDFT} \left( mY[k] * e^{j * pY[k]} \right)$$

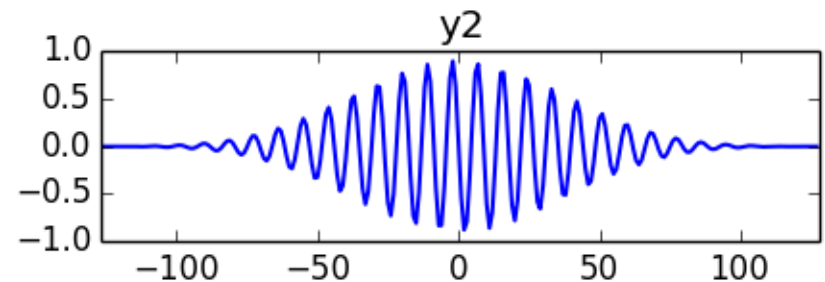
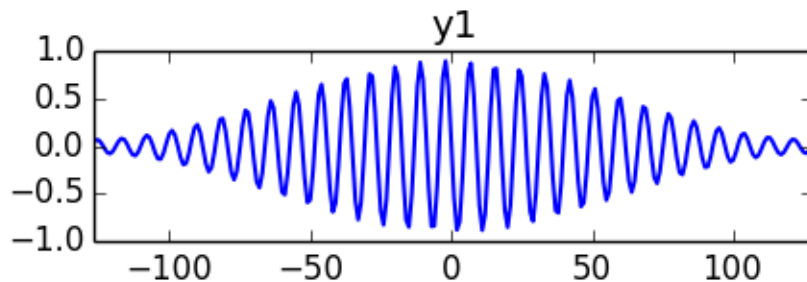
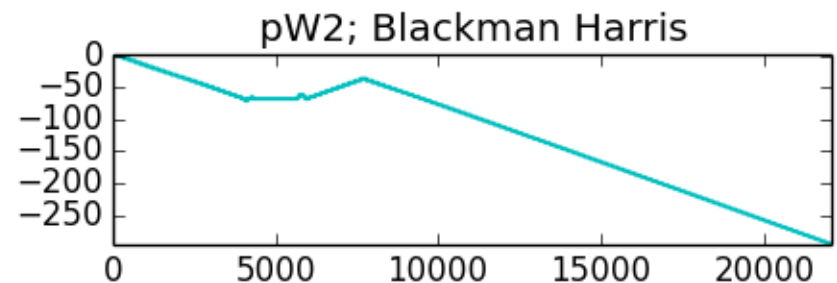
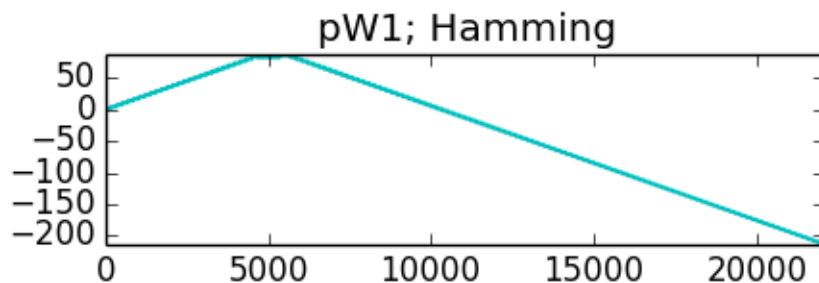
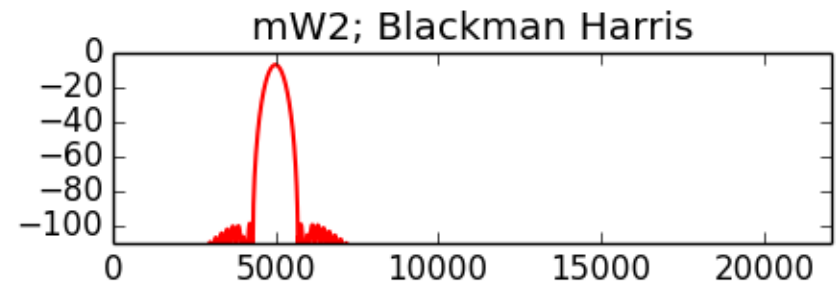
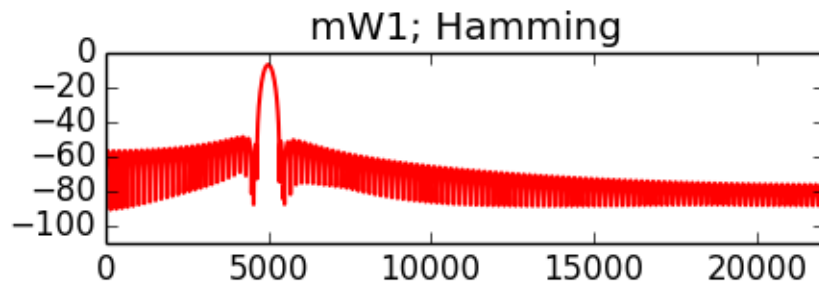
$$mY[k] = A_0 \text{ for } k = k_0 \text{ and } 0 \text{ for } k \neq k_0; \quad pY[k] = \varphi_0 \text{ for } k = k_0 \text{ and } 0 \text{ for } k \neq k_0$$



# Sinusoidal synthesis: any frequency

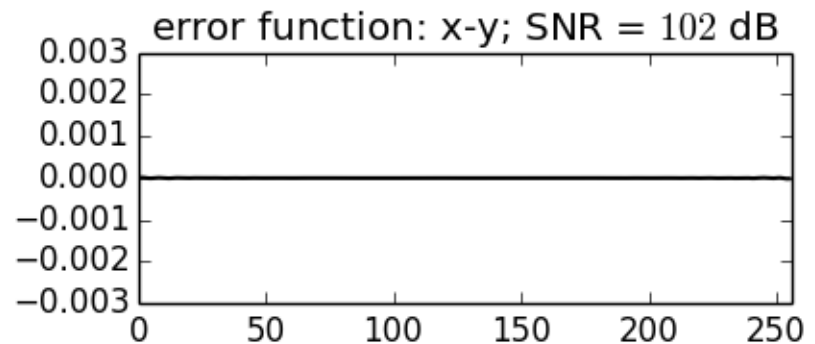
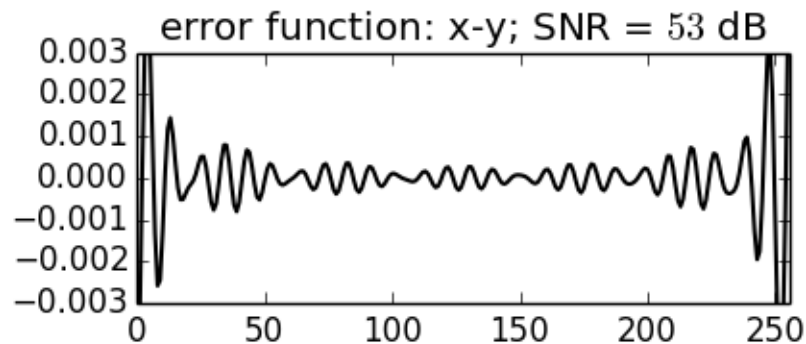
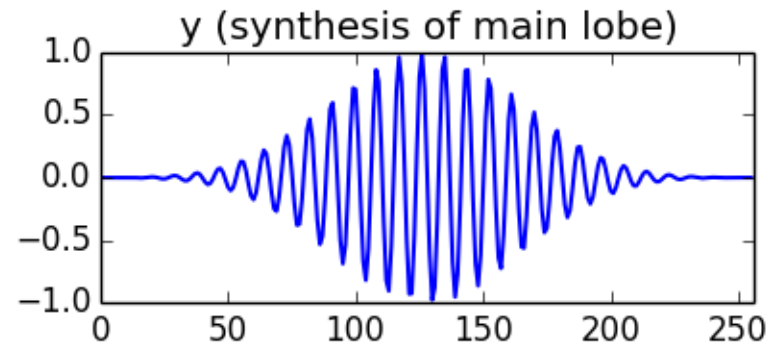
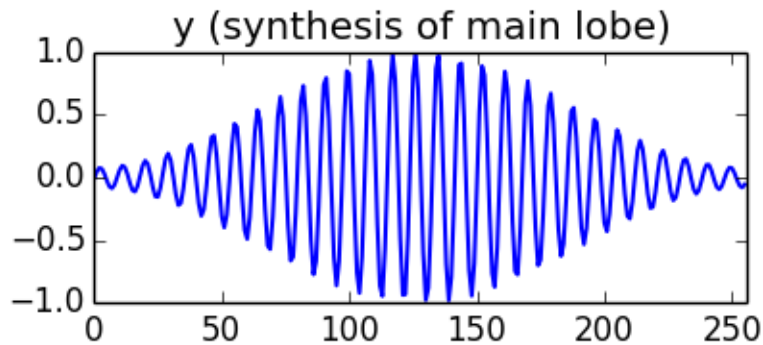
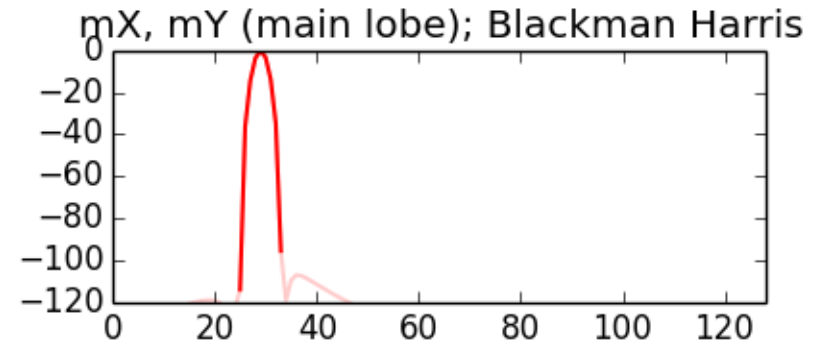
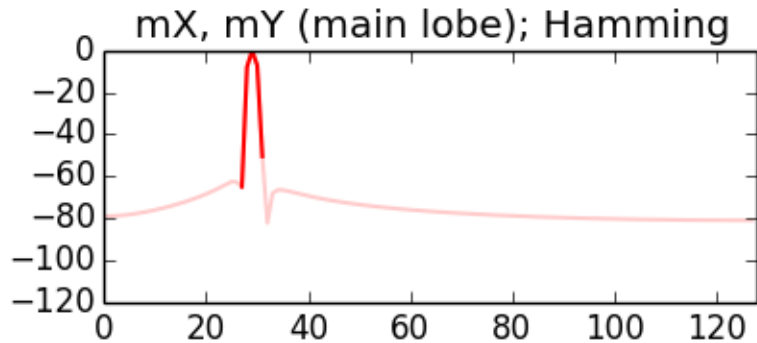
$$y[n] = IDFT \left( A_0 * mW[k - k_0] * e^{j * (pW[k - k_0] + \varphi_0)} \right)$$

$mW[k]$ ,  $pW[k]$  magnitude and phase spectrum of window





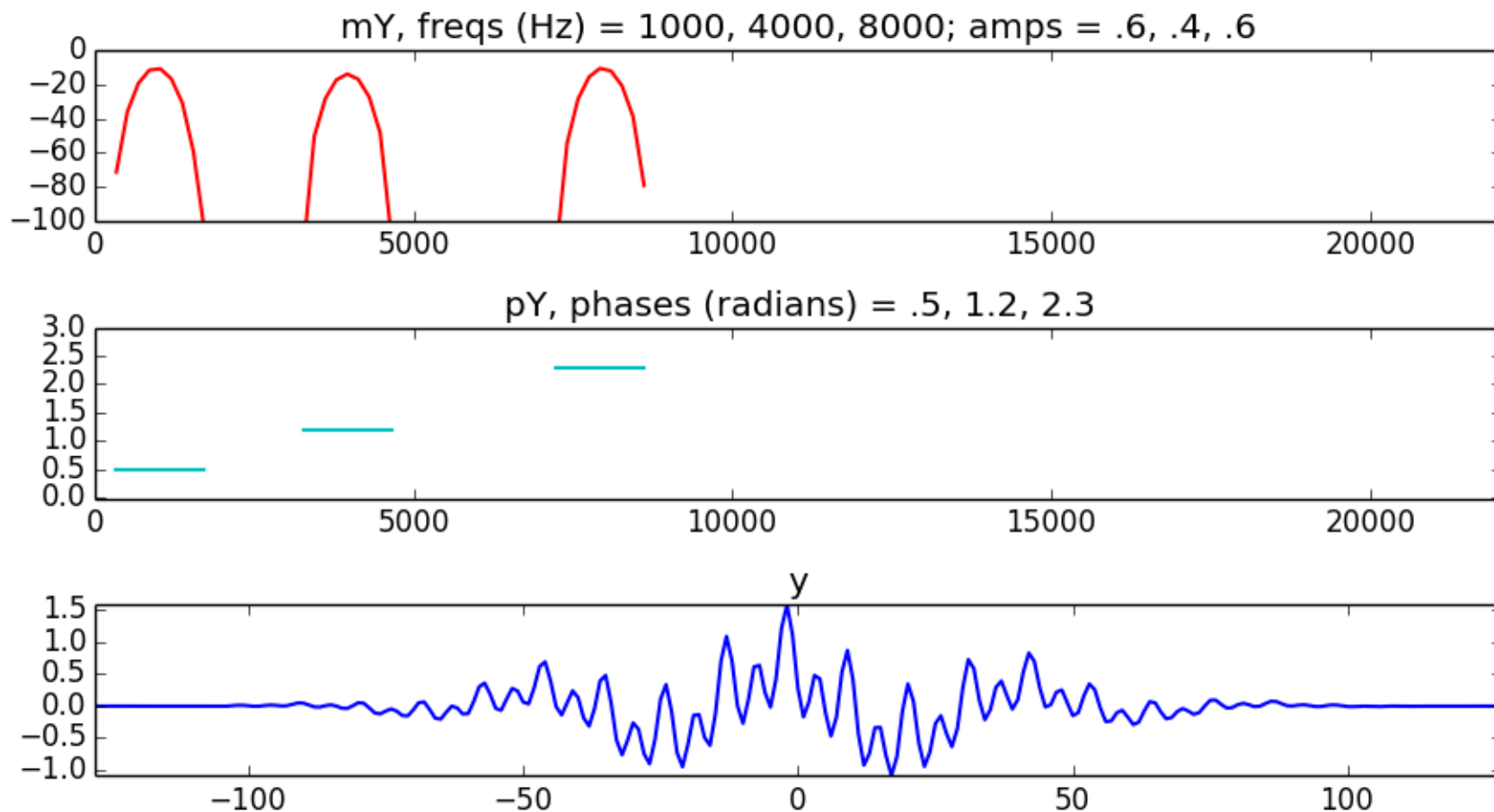
# Sinusoidal synthesis: only main lobe



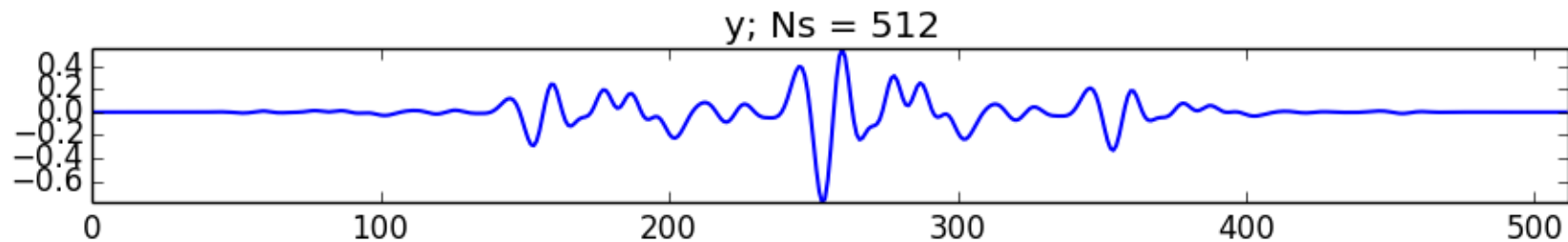
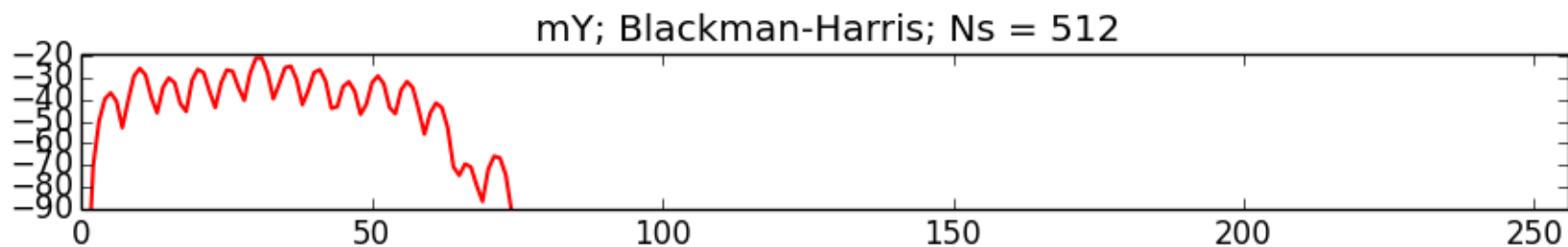
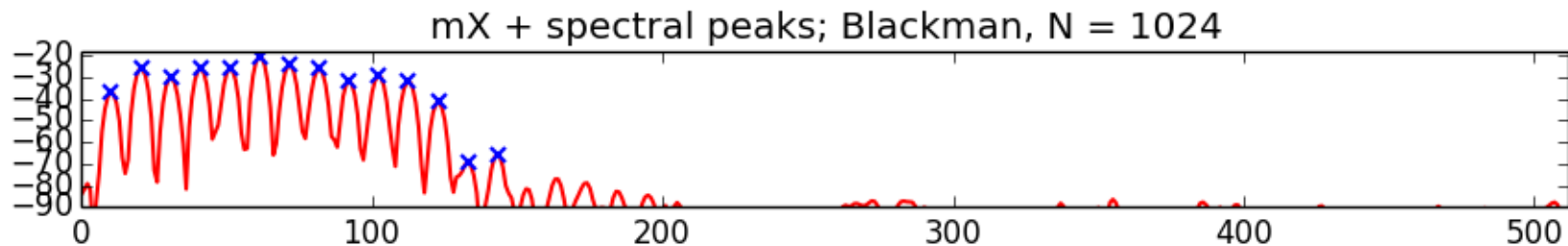
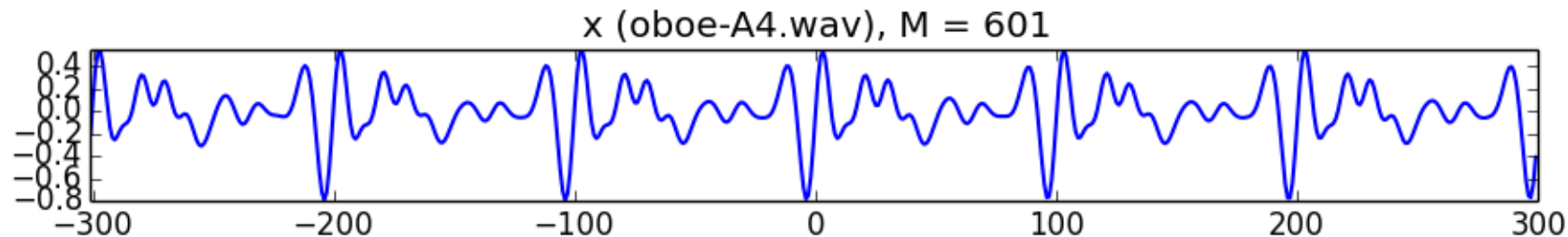
# Additive synthesis

$$y[n] = IDFT \left( \sum_{r=0}^R A_r * mWl[k - k_r] * e^{j * (pWl[k - k_r] + \varphi_r)} \right)$$

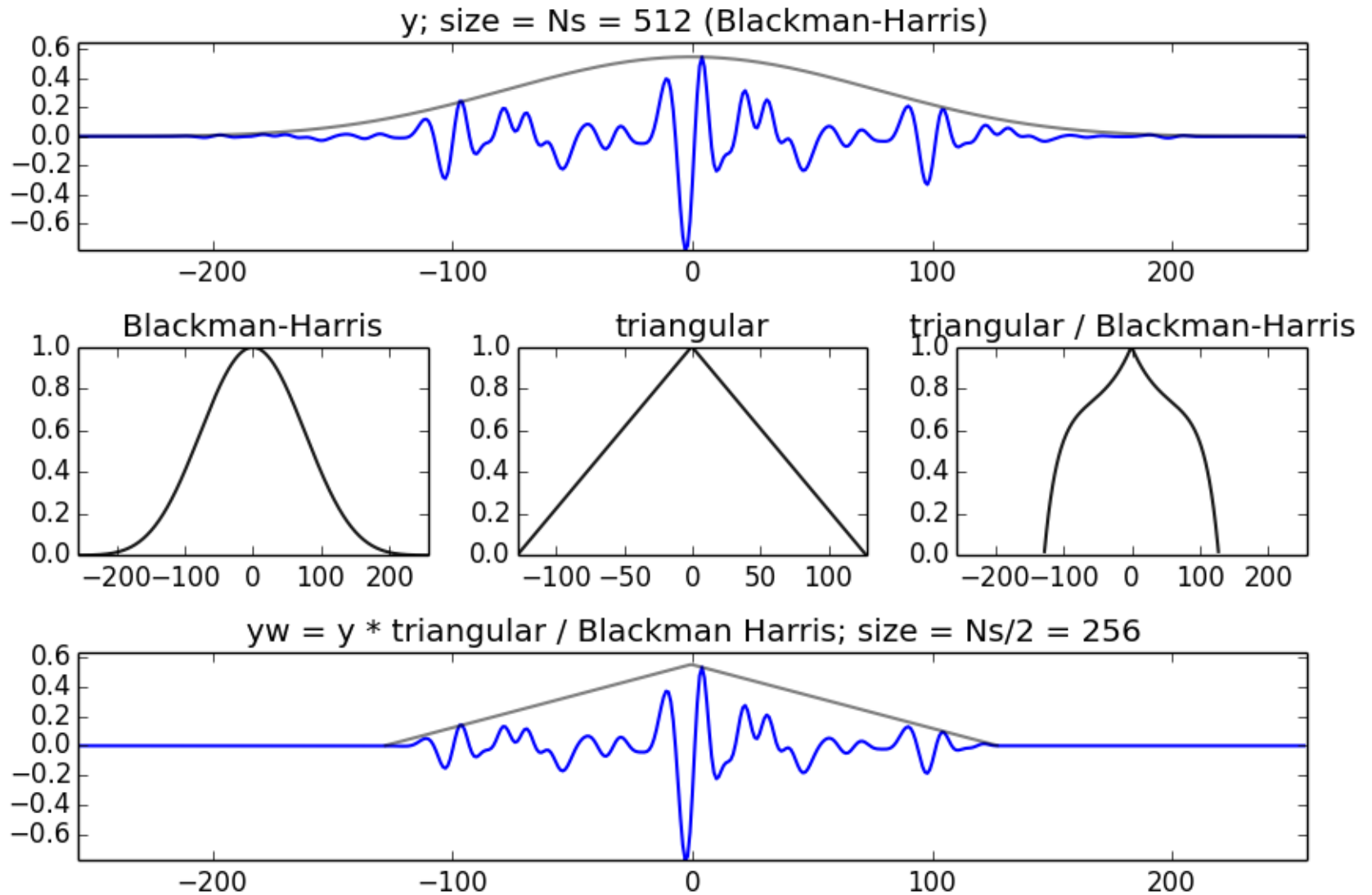
$mWl[k]$ ,  $pWl[k]$  magnitude and phase spectrum of window main lobe



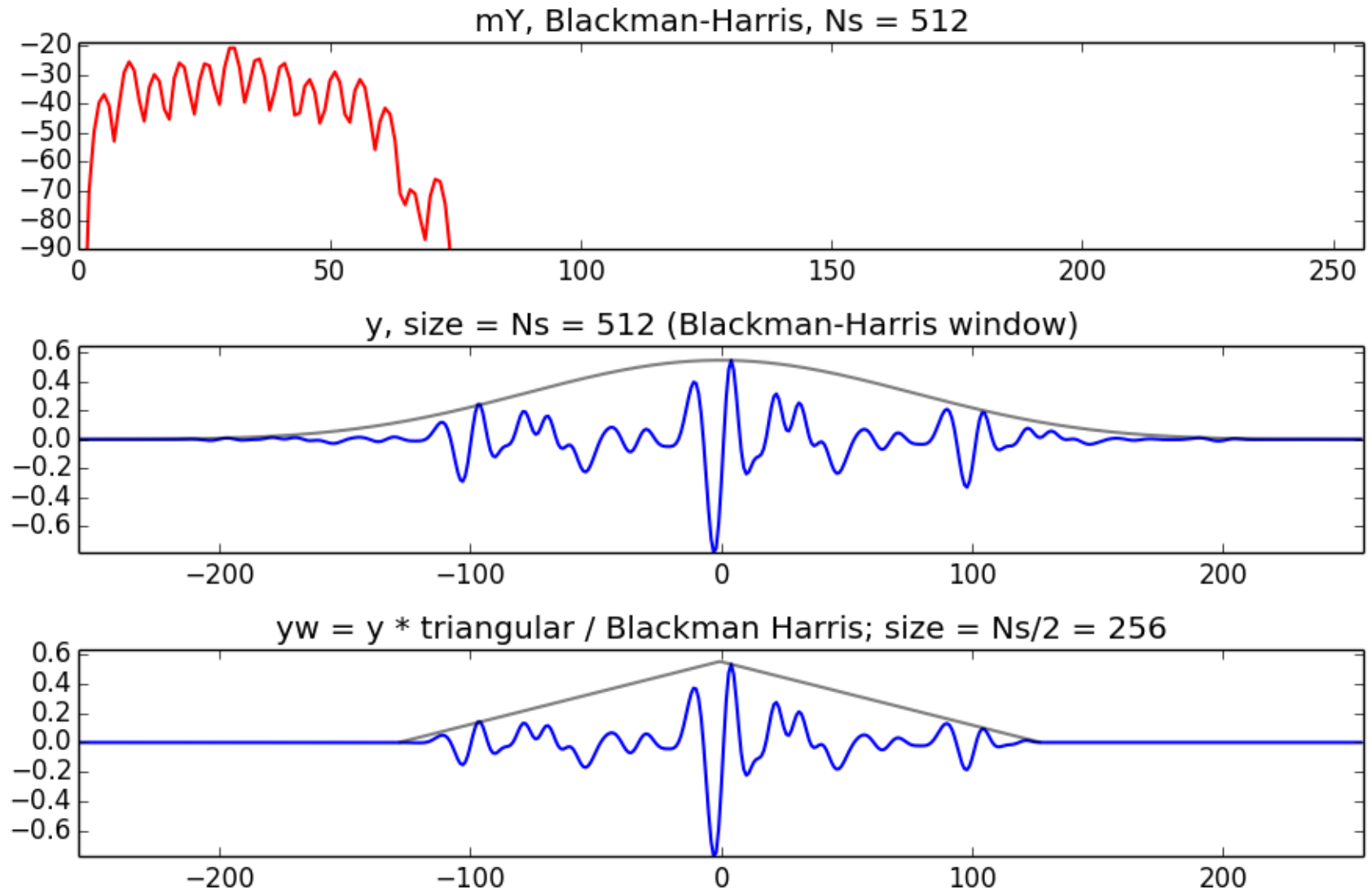
# Analysis / Synthesis



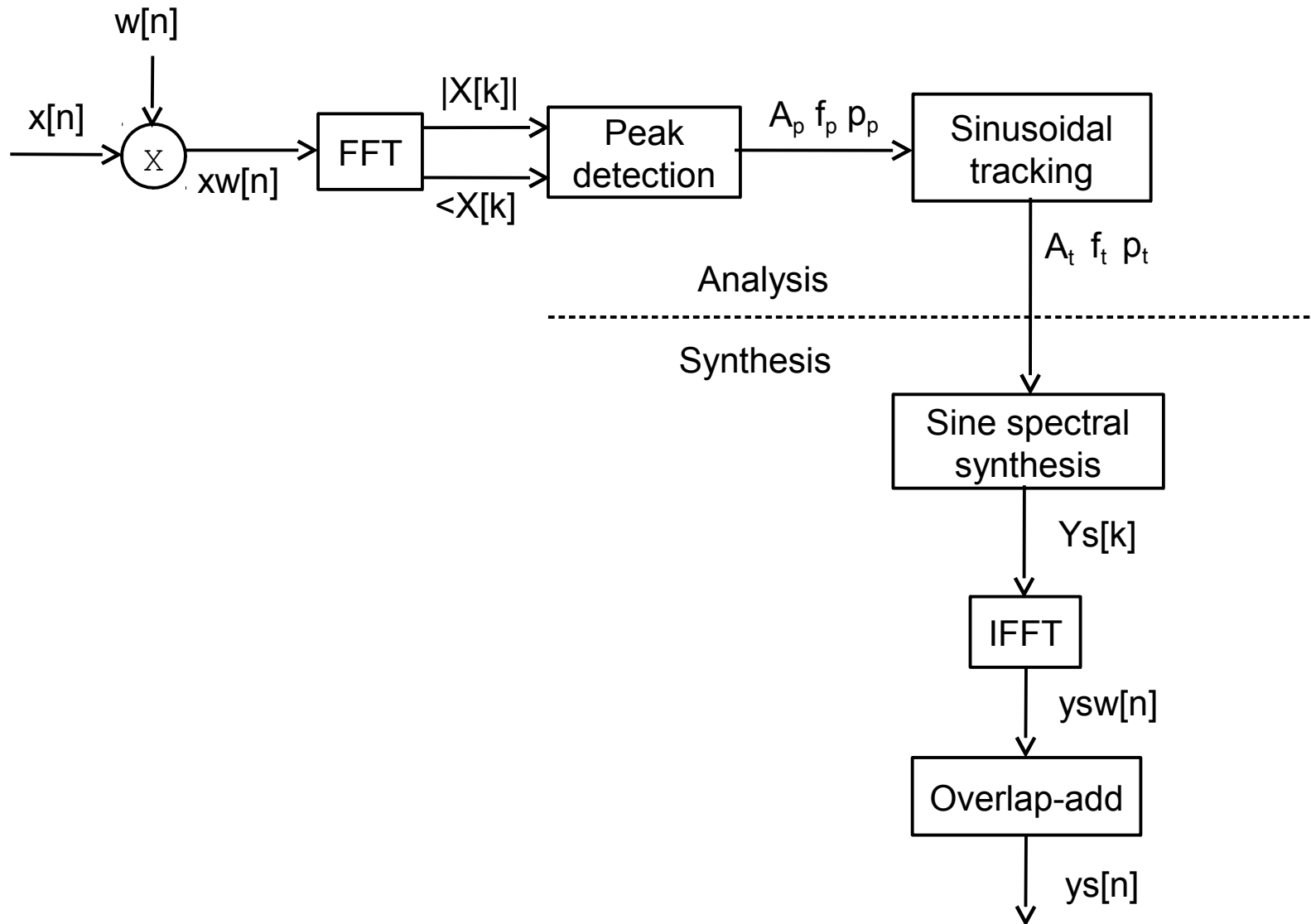
# Synthesis window



# Synthesis for overlap of 25%

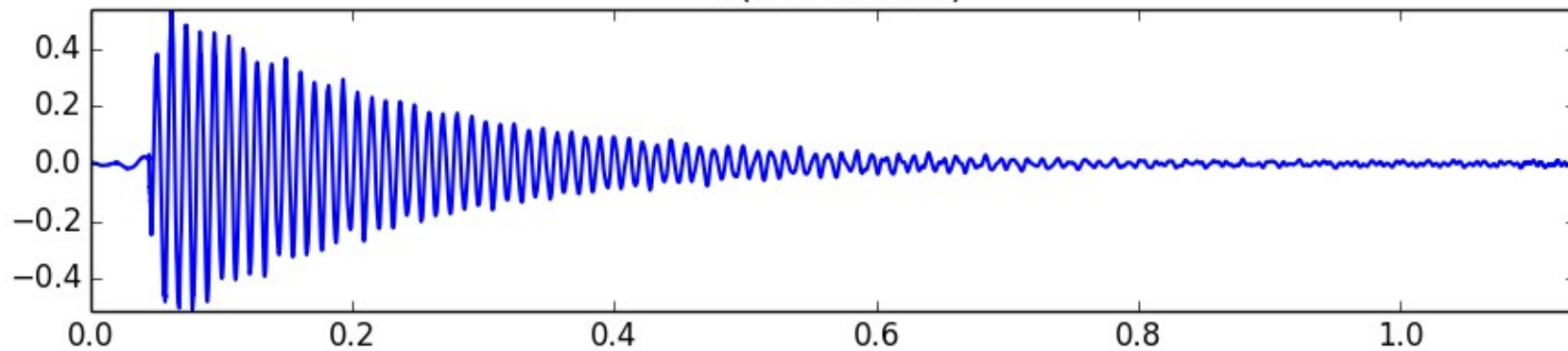


# Sinusoidal model system

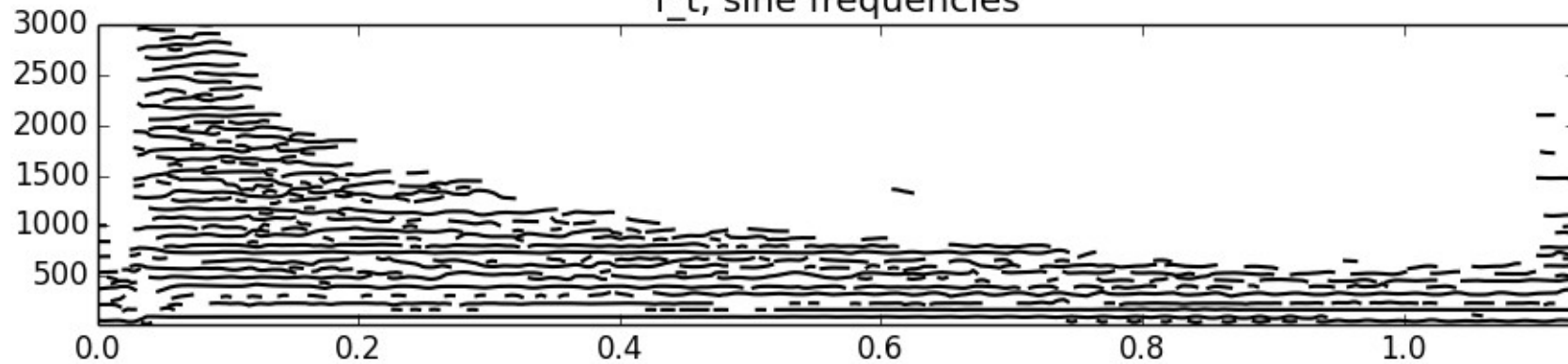




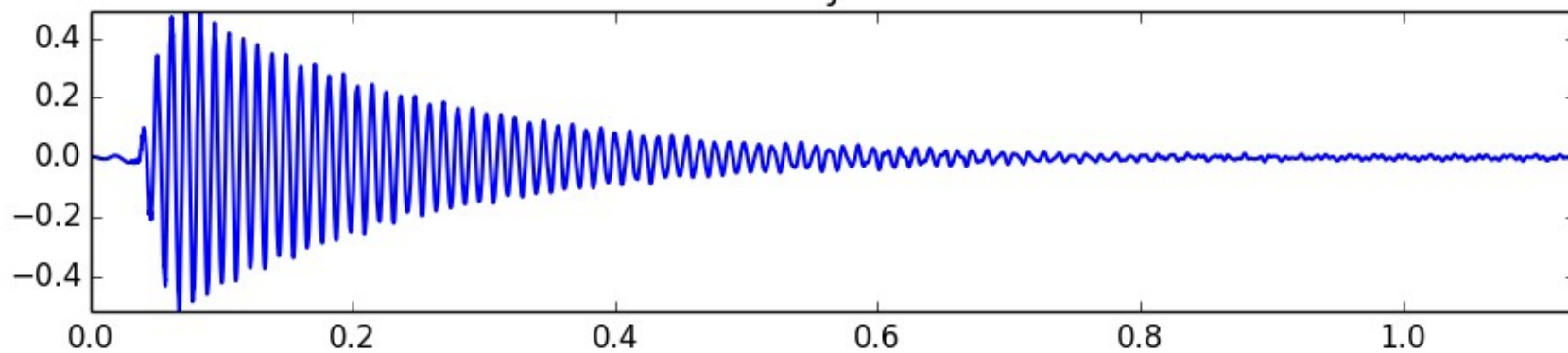
x (bendir.wav)

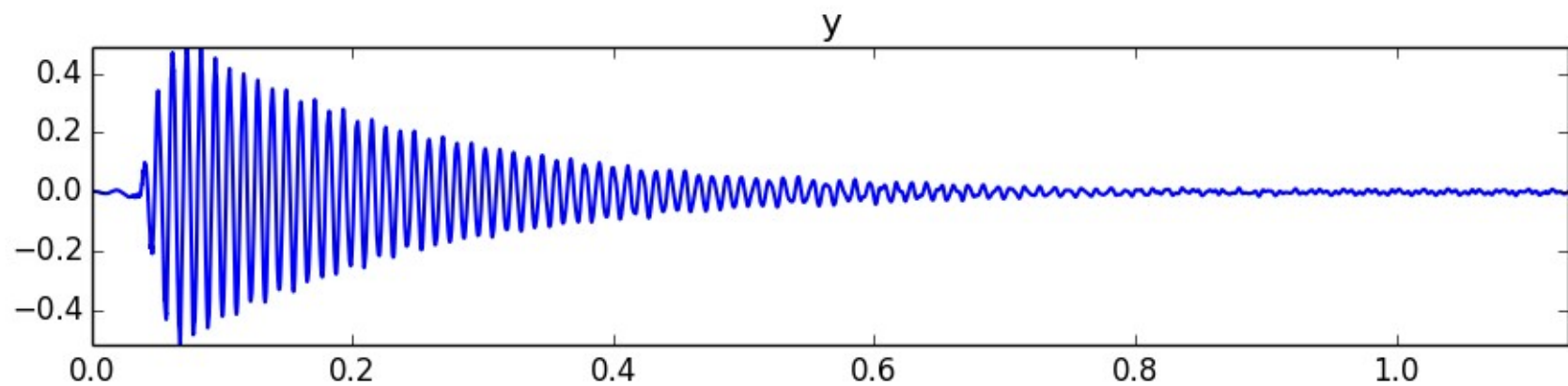
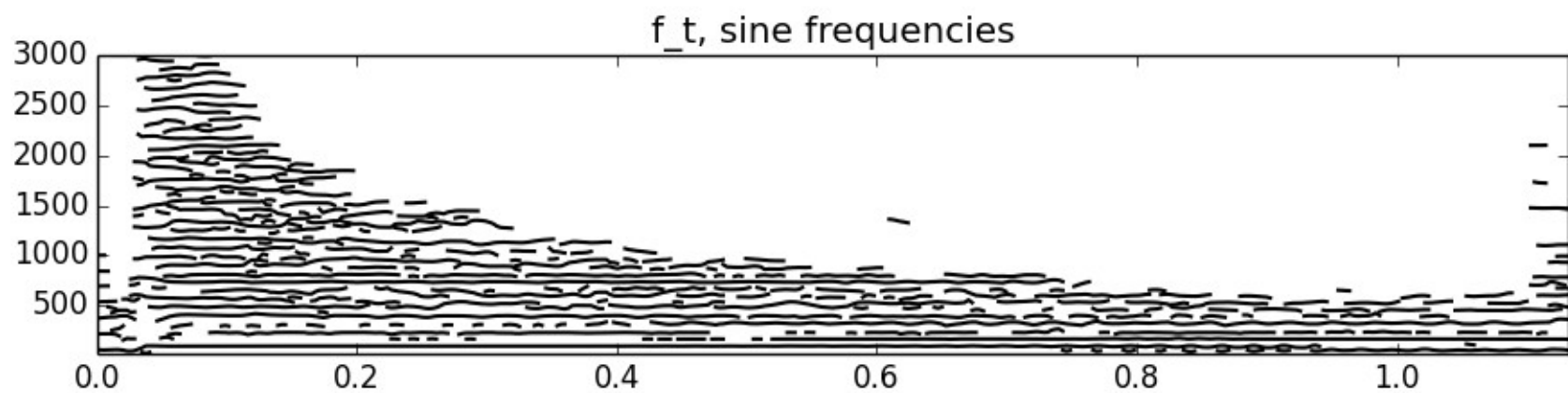
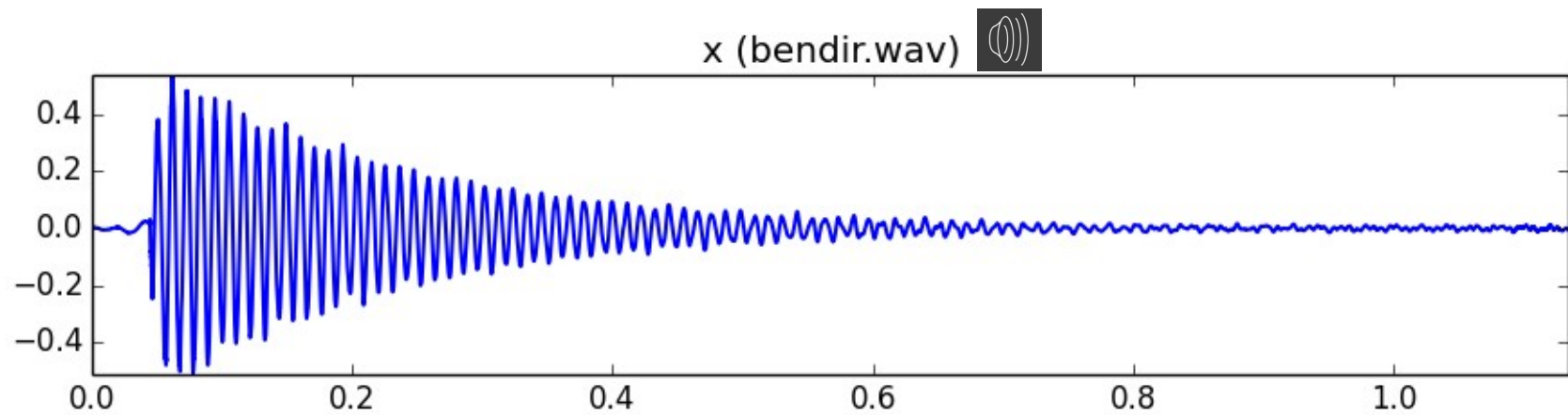


f\_t, sine frequencies

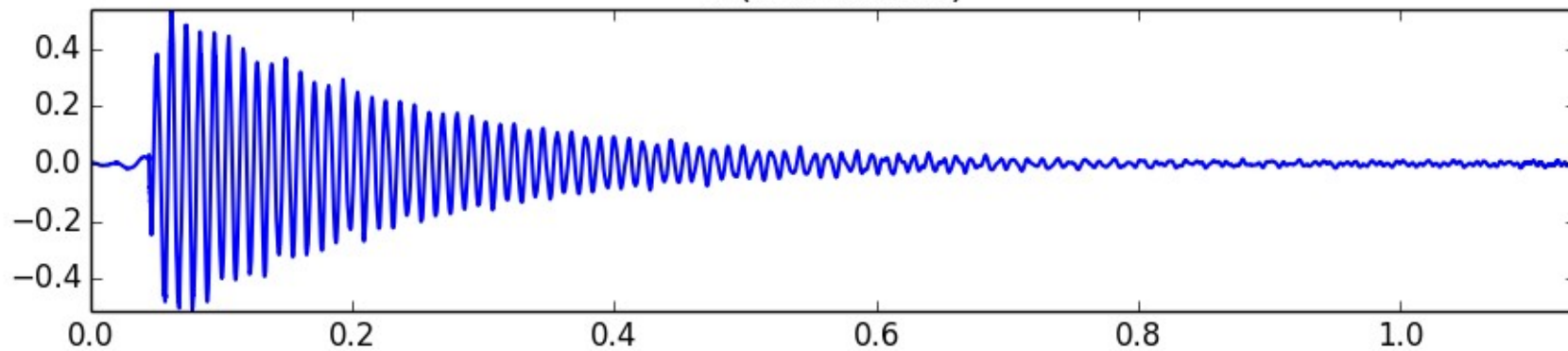


y

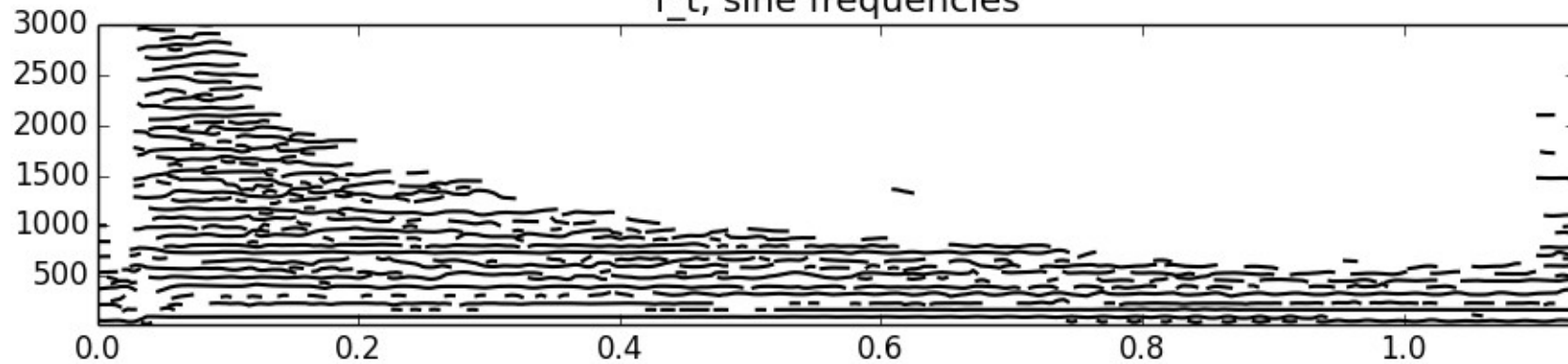




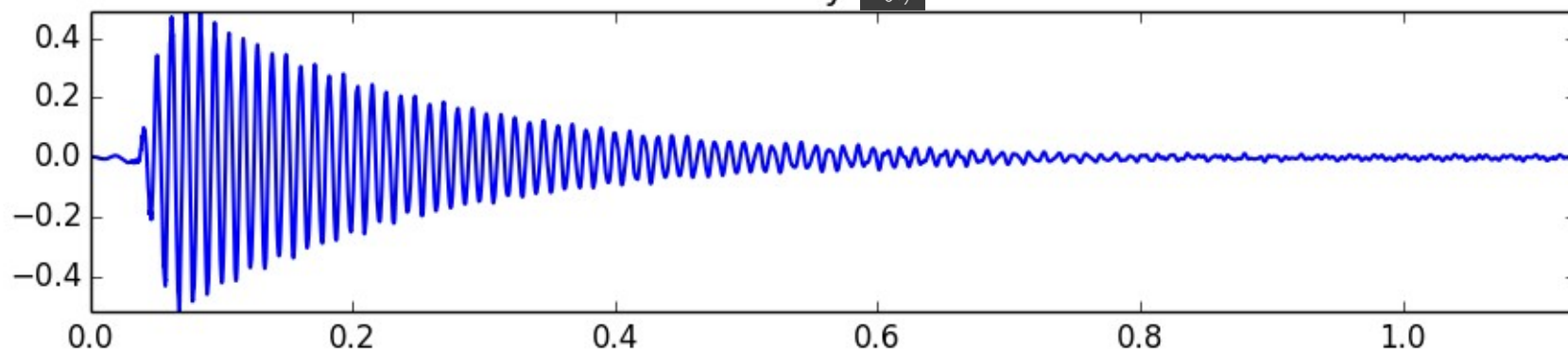
x (bendir.wav)



f\_t, sine frequencies



y 



# References and credits

- More information in:
  - [http://en.wikipedia.org/wiki/Additive\\_synthesis](http://en.wikipedia.org/wiki/Additive_synthesis)
  - [http://en.wikipedia.org/wiki/Sinusoidal\\_model](http://en.wikipedia.org/wiki/Sinusoidal_model)
- Reference on sine modeling by Julius O. Smith:  
[https://ccrma.stanford.edu/~jos/sasp/Spectrum\\_Analysis\\_Sinusoids.html](https://ccrma.stanford.edu/~jos/sasp/Spectrum_Analysis_Sinusoids.html)
- Sounds from:  
<http://www.freesound.org/people/xserra/packs/13038/>
- Slides and code released using the CC Attribution-Noncommercial-Share Alike license or the Affero GPL license and available from <https://github.com/MTG/sms-tools>

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