Inequality Snowballing

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April 2, 2023

ABSTRACT: It has long been argued that efficient policies tend to provide larger legal entitlements to the rich than to the poor. This article shows how efficient legal rules can become even more skewed against the poor *over time* by sowing the seeds of their own vicious cycles. Repeated application over time of these rules can lead to increasingly adverse outcomes for the poor, which the article calls "policy snowballing."

Consider a set of polluters choosing between locating in places with rich versus poor people and facing a strict liability rule for harm to earnings. Polluters will disproportionately locate in the poor area, where they face lower damages. That disproportionate share of polluters in the poor area can make it cheaper to harm the poor in the next period, making subsequent polluters locate yet more disproportionately in poor neighborhoods, driving down the poor's earnings further.

We identify the conditions for snowballing and explore its dynamics. When compensation for the harm is incomplete, policy snowballing can lead to spiraling income inequality. As a result, government transfers to the poor to compensate for the change in legal regime would be inadequate if calculated in a way that ignores the snowballing. The article raises the intriguing prospect that legal rules could generate state dependence in the legal costs of harm, and that efficient policymaking could be a contributing factor to increasing inequality over time.

Keywords: taxes and transfers; economic inequality; torts; dynamics; efficiency

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1 Introduction

The United States has long-term clusters of poverty and pollution. For example, Richmond, California, has had multiple rounds of polluters enter over a nearly 150-year-long history (Walker, 2001; Moore, 2000). Throughout, it has been one of the poorest places in the Bay Area (Allen and Li, 2016). Mining and chemical plants arrived in the late 19th century. Then Standard Oil and other petrochemical industries arrived around the turn of the 20th century. Shipyards arrived with World War II. After the war, the existing industrial footprint continued to expand, especially a Chevron oil refinery. The last thirty years have seen a substantial number of high-profile explosions and mishaps at the industrial sites. These torts have settled, generally at a low per-person dollar amount.

Of course, such poverty-pollution clusters have many causes. This article shows how a distinctive aspect of the law may contribute to such clusters—and other concentrations of poverty—and explores the dynamics of the phenomenon. In particular, this paper develops a model in which inequality "snowballs" due to repeated application of Kaldor-Hicks efficient (that is, economic-surplus-increasing) legal rules, as these rules induce state dependence in the price of firm location and sow the seeds of their own vicious cycles.

To begin, suppose that there is a neighborhood of poor (i.e., low-earning) individuals and a neighborhood of rich (i.e., high-earning) ones. A continuum of toxic-waste dumps that emit a fixed amount of pollution is deciding where to locate. Pollution reduces productivity and thus yearly earnings through, say, lost days of work; earnings are thus reduced in proportion to the pollution to which the workers are exposed.[1] Harm to earnings lasts for two periods.

Firms decide where to locate based on two factors. First, the polluters pay damages resulting from a strict liability rule.[2] In our model, this legal rule is a Kaldor-Hicks improvement to a regime that equalizes pollution across neighborhoods. Damages equal the

[1]This modeling assumption is consistent with evidence showing that pollution can lead to lost days of work (Moretti and Neidell, 2011), causing proportional earnings declines.

[2]Of course, in practice, there may be a negotiated settlement, but—assuming that settlements are negotiated in the shadow of the strict liability rule—the implications should be similar regardless of whether or not parties go to trial.

foregone earnings resulting from the pollution. Second, the firms have a symmetric continuum of preferences (e.g., based on closeness to customers or quality of the land) between the two locations. As a result, a dump indifferent between the two locations for reasons other than the damages it will pay will locate in the poor neighborhood, knowing that the damages will be lower under tort law's economic damages rule, since pollution causes less monetary damage to lower-earning individuals than higher-earning ones.[3] So, all else equal, dumps will tend to locate in the poor neighborhood.

In the next period, after the existing toxic-waste dumps are full, polluters again decide where to locate. The pivotal question for further increases in the share of firms in the poor neighborhood is how past pollution affects the harm from current pollution. Here we introduce the critical "feedback" parameter. It could be the case that pollution in the previous period has no bearing on the harm from pollution in the current period. In that scenario, there is no state dependence in the price of firm location; we would see no further changes in the share of firms located in the poor neighborhood. However, we show that if pollution from the last period reduces the marginal harm to earnings from pollution this period, then the legal rule begets state dependence in the legal costs from exerting harm. In the subsequent period, yet more polluting firms will locate in the poor neighborhood. And the same snowballing occurs for the third, fourth, etc. periods: more pollution leads to lower earnings and lower marginal harm from pollution, leading to more pollution. We call this feedback process in which the harm governed by the policy becomes increasingly adverse to the poor by operation of an efficient legal rule "policy snowballing."

Importantly, policy snowballing is invariant to the compensation received by those harmed. The driver of policy snowballing is the diverging marginal harm of pollution across neighborhoods because of the increasing pollution on the poor. Given the application of an efficient

[3] One response might be that the parties can have insurance. However, first, it is unlikely that such insurance exists in the real world (at least to a sufficient extent) due to adverse selection and moral hazard. Second and more importantly, after risk adjustment, the poor would have to pay a larger amount in premia to gain actuarially fair insurance (due to the greater risk of having the dump placed nearby). So while insurance does remove the risk component, it does not solve the distributive issue.

legal rule, compensation—in the form of either damages from polluters or transfers from the government—does not change the increase in past pollution, which drives the policy snowballing, and thus does not slow the snowballing.

In the model, there may be incomplete compensation because a share of damages goes to legal fees, as is the case with the "American Rule" under which each party pays its own legal fees. (Note that, to be efficient, legal rules need not involve any compensation at all.) And furthermore, in the baseline model, there is no compensation through taxes and transfers, for example due to political economy failures (Fennell and McAdams, 2016; Liscow, 2018a). We show that incomplete compensation will lead policy snowballing to produce increasing income inequality which we call "income snowballing."[4]

The article's main, novel contribution is identifying the possibility that not only are efficient legal rules not neutral, but also that a given rule may become increasingly harmful to the poor with time by inducing state dependence in the price of harm, exacerbating income inequality. The article also has four additional contributions.

First, we identify the key feedback parameter that is a necessary and sufficient condition for policy snowballing and a further necessary and sufficient condition—incomplete compensation—for income snowballing.

Second, we characterize the equilibrium level of pollution in the rich and poor communities and explore its comparative statics. For example, we show that, the higher the level of initial earnings inequality, the more unequal the equilibrium pollution will be. We also identify the parameters that determine the speed at which the equilibrium is reached.

Third, we identify a simple formula for the extent to which ignoring dynamics—that is, the additional harm from pollution that arises over time from snowballing—causes a failure

[4]One response might be that the pollution will lower rents, benefitting the poor renters. That is probably true to some extent, but there is little reason to believe that there would be full offset of the income loss by lower rents. For example, with an infinitely elastic housing supply, there should be no price response at all. Furthermore, recent work by Kline and Moretti has emphasized the importance of "infra-marginal" individuals who are not on the margin between moving between one place and another (Kline and Moretti, 2014a; Kline and Moretti, 2014b). These individuals are harmed when the quality of their current residence declines in value, since they are staying there and paying the rent regardless.

to capture harm to the poor. It is proportional to the same key feedback parameter. We show in an example that these losses can be quite large. So, part of this article's policy stakes is that, when a more efficient legal rule is adopted, policymakers trying to compensate for losses may calculate the wrong distributional consequences if they only account for the one-period distributional effects rather than the dynamic distributional effects that flow from the new legal regime.

Finally, we develop a formula for welfare analysis. Depending on the welfare weights and the level of existing inequality, the social planner may prefer an *inefficient* policy that treats everyone as if they have the same earnings, as proposed by Sharkey (2021), because of the dynamics that can immiserate the poor over time.

As the article explains, similar state dependence dynamics in which inequality affects the outcome of the legal rule, which in turn exacerbates inequality, may exist in many areas of law, including transportation spending, healthcare spending, eminent domain law, the provision of amenities like parks, and a host of others. Take the example of efficient cost-benefit analysis of transportation spending across rich and poor areas, which has similar features to the torts example (Liscow, 2022). In particular, it has the three key features for snowballing: First, all else equal, it is efficient to spend more in rich places than poor ones (because the rich are willing to pay more for faster transportation) and, second, application of the legal rule may induce state dependence in the value of transportation funding, because the rich receive more transportation funding and thus access to higher incomes in the next period. These two features set up policy snowballing. If there is additionally a third factor that there is insufficient compensation for the greater spending on the rich, there may be income snowballing. So cost-benefit analysis for transportation spending is promising for snowballing as well. As the article explains, other areas are not promising, including those where efficient legal rules do not benefit the rich more than the poor and those where there is no state dependence induced.

Inequality rose in developed countries over the past few decades (Piketty, 2014; Piketty and Saez, 2003).[5] There are many proposed explanations for the increase in inequality, like the institutional forces described by Piketty (2014), skill-biased technological change (Autor, Katz, and Kearney, 2008), globalization of trade (Harrison, McLaren, and McMillan, 2011), and others. In this article, we raise the intriguing possibility that the adoption of more efficient legal rules could lead to repeated multiplication of inequality over time, suggesting that the dynamics of efficient laws may be an additional possible explanation. We believe that this is the first economics article to lay out a model providing a proof of possibility for how such multiplication would happen—without making any claim as to whether such a mechanism has in fact been at play in increasing income inequality.

As a proof of possibility, this model of course does not account for all relevant factors. One important factor is the absence of potential benefits from having a toxic waste dump located nearby, like employment. The extensions section shows that labor demand can easily be incorporated into the model, producing similar results so long as the benefits to earnings from the positive labor demand shock do not outweigh the harm to earnings from pollution. Other potentially relevant aspects of reality, some of which could mitigate the severity or eliminate the presence of snowballing, are discussed below.

The paper touches on several literatures. The distributional impacts of Kaldor-Hicks preferred legal rules have been much discussed—quite sensibly, since the Kaldor-Hicks efficient prescription is the standard one in law and economics (Posner, 2014; Shavell, 2004; Cooter and Ulen, 2012). Perhaps most famously, Ronald Dworkin (1980) described how it could be efficient to take a book from a poor person and give it to a rich person who values it more. Essentially, because the rich are typically willing to pay more for things by virtue of their greater wealth, there is a tendency for efficient legal rules to allocate more to the rich, since efficient legal rules are based on willingness to pay (Liscow, 2018a). While valuable, this analysis is static, in a one-period model, and thus does not consider the dynamics of how repeated application of efficient legal rules could amplify or counteract these distributional

[5] Note that others argue that claims about increasing income inequality are overstated (Burkhauser et al, 2012).

consequences.

There has, of course, been considerable work on dynamics in economics (e.g., Stokey, Lucas, and Prescott, 1989). And there has long been speculation about the long-run effects of efficient rulemaking (e.g., Hicks, 1941).[6] But little has been done formally on the dynamics of efficient legal rules, which is the focus of this paper. We also contribute to the literature on state dependence in economic outcomes (i.e., for those who experience an event, influential economic variables change going forward), of which the classic example is employment status (Heckman, 1981). State dependence has also been explored in a variety of economic contexts such as product choice (Dubé et al., 2010) and innovation (Acemoglu, 2002). We add to this literature by showing how legal rules can induce state dependence in the price that those causing harm pay in one place versus another.

Additionally, the "environmental justice" literature has long discussed how governments may tend to pollute more on the poor than the rich, perhaps due to racism (e.g., Bullard, 1994). This article introduces a new legal mechanism explaining disproportionate environmental harm to the poor, formally models it, and explores its dynamics.

Finally, while it has been noted that tort law's assessment of economic damages based on income tends to lead to higher damages for the rich than the poor (e.g., Chamallas, 2005; Sharkey, 2021), the consequent dynamics over time developed here are new. Likewise, the American Rule's requirement that parties pay their own fees has been criticized for under-compensating those who are harmed, but again the dynamics over time described here, including the interaction with economic damages, are new (Ehrenzweig, 1966).

This article proceeds as follows. Section 2 sets up the model. Section 3 defines the two forms of inequality snowballing—policy snowballing and income snowballing—and describes the broad circumstances under which the model exhibits either one. Section 4 discusses model

[6]The question of how efficient legal rules affect distribution in aggregate over time is one that has divided scholars. For example, Richard Zerbe and Tyler Scott (2014) argue that cost-benefit analysis yields results that are close to Pareto superior. See also Polinsky (1972). Indeed, Hicks (1941, pg. 111) called this notion the "classical creed," meaning that if society made "all alterations" that met the Kaldor-Hicks criterion, then "there would be a strong probability that almost all (individuals) . . . would be better off after the lapse of a sufficient length of time." See also Persky (2001) for further exploration of the "classical creed."

dynamics, comparative statics, the importance of dynamics for calculating needed taxes and transfers after a change in legal rules, and the implications for welfare. Section 5 offers an extension to the model, including localized labor demand impacts from firms' locational choices. Section 6 discusses factors that were not considered in the model and how they might impact the presence of snowballing. Section 7 considers other policy scenarios that could similarly lead to inequality snowballing.

2 Model Setup

The Effect of Pollution on Earnings

We conceptualize pollution as affecting the health stock of a worker, like in Isen, Rossin-Slater, and Walker (2017). The health stock, in turn, affects earnings by reducing a worker's productivity or working capacity. For example, a worker could lose days of work through illness, work fewer hours due to diminished capacity, or simply have a lower return to work. This is consistent with a variety of evidence that pollution reduces workers' productivity (Heyes, Neidell, and Saberian, 2016; Moretti and Neidell, 2011; Sanders, 2012). In particular, we allow for a dynamic effect of pollution on health stock and, therefore, earnings. Drawing again on Isen, Rossin-Slater, and Walker (2017), in the model, past pollution can affect current health by causing a persistent illness that is expensive or difficult to ameliorate, such as respiratory illness.

Evidence supports this dynamic conception of pollution's effect on earnings. Using the Clean Air Act of 1970, Isen, Rossin-Slater, and Walker (2017) found that early childhood exposure to air pollution lowered earnings and labor force participation at age 30. Though it does not deal with earnings, Sanders (2012) also found evidence for the persistent effects of early childhood air pollution, which he found worsened later-life educational outcomes. In the model, we assume that only pollution in the previous period and the current period affect earnings, implicitly assuming that the health stock fully recovers after two periods from pollution. We make this choice for the sake of simplicity.

Neighborhoods and Earnings

There are two equally populated neighborhoods, one high-earning, denoted by H, and one low-earning, L. People are immobile and infinitely lived. Workers supply one unit of labor inelastically each time period, so, in a world without pollution, the earnings of a worker in neighborhood $N \in \{H, L\}$ are $\overline{e}_N$. By assumption, an individual in the high-earnings neighborhood begins with higher earnings than one in the poor neighborhood: $\overline{e}_H > \overline{e}_L$. Each time period, there is a unit measure of polluting firms that decide where to locate. Think of the firms as toxic waste dumps that do not employ any labor and that fill up each period.

All firms produce the same, fixed amount of pollution that reduces the earnings of workers in the neighborhood in which they locate by reducing their productivity. The total harm to the earnings of individuals in the neighborhoods depends on the share of firms that locate (and have located) in the neighborhood. Specifically, let s_t be the share of polluting firms that locate in the low-earnings neighborhood at time t, making $1 - s_t$ the share of polluting firms that locate in the high-earnings neighborhood. The realized earnings of individuals in the low and high income neighborhoods at time t are

$$e_{Lt} = [1 - \theta_1 s_t - \theta_2 s_{t-1} + \theta_3 s_t s_{t-1}] \, \overline{e}_L. \tag{1}$$

$$e_{Ht} = [1 - \theta_1(1 - s_t) - \theta_2(1 - s_{t-1}) + \theta_3(1 - s_t)(1 - s_{t-1})] \, \overline{e}_H \tag{2}$$

Harm from pollution thus may last two periods, with $\theta_1 s_t$ the harm from pollution this period, $\theta_2 s_{t-1}$ the harm from pollution last period, and $\theta_3 s_t s_{t-1}$ the harm from the interaction of this period and the last one.[7] For example, when base earnings $\overline{e}_L$ are multiplied by $\theta_1 s_t$,

[7] For space considerations, we proceed by only showing equations for the low-earnings neighborhood, but the analogous equations for the high-earnings neighborhood can easily be obtained by replacing "$[s]$" with "$[1 - s]$."

this multiplies pollution, s_t, by the harm from that pollution, θ_1, reducing realized earnings. We call θ_1 and θ_2 "main effects." Again, it is natural to think about this as pollution reducing the share of days that someone can work. Note that, since it is added rather than subtracted, a higher θ_3 *decreases* the marginal harm of pollution today for a given level of pollution yesterday. We call this the "feedback" effect. This effect might seem like a relief to the harmed parties, since it reduces harm; we will show that it can, in fact, have the opposite effect.

Our modeling choice for earnings is a flexible generalization of harm to earnings. For example, consider an earnings process in which pollution affects earnings multiplicatively. Such a process could be expressed as

$$e_{Lt} = (1 - \theta s_t)(1 - \theta s_{t-1})\,\overline{e}_L.$$

This is the case of earnings function (1) in which $\theta_1 = \theta_2 = \theta$ and $\theta_3 = \theta^2$. Here, pollution reduces earnings this period a certain percent, and next period the same percent. But, because the pollution effects are applied multiplicatively (by reducing days worked), the past pollution effectively reduces the earnings base on which the current pollution acts. Pollution in the previous period makes the base on which pollution acts on $(1 - \theta s_{t-1})\,\overline{e}_L$ instead of $\overline{e}_L$. A lower base means a smaller absolute reduction in earnings from a given level of pollution. The θ^2 term in this special case captures the feedback effect.

Our setup allows for pollution harm to have different effects depending on time since pollution. Such a process could be expressed as

$$e_{Lt} = (1 - \theta_1 s_t)(1 - \theta_2 s_{t-1})\,\overline{e}_L.$$

Now the impact of past and present pollution is allowed to vary but it is still applied multiplicatively. For example, here $\theta_3 = \theta_1 \cdot \theta_2$.

Equation 1 also nests the possibility that there is no feedback effect at all. In such a case, the harm from pollution today, and the harm from pollution yesterday are always applied to the same base: $\overline{e}_L$. The process could be expressed as:

$$e_{Lt} = (1 - \theta_1 s_t - \theta_2 s_{t-1})\, \overline{e}_L.$$

This is the case of earnings function (1) in which $\theta_3 = 0$.

We set the following conditions on the earnings parameters:

$$0 \leq \theta_1 \leq 1, \qquad 0 \leq \theta_2 \leq 1, \qquad \theta_3 < \theta_1, \qquad \theta_3 < \theta_2, \qquad \theta_1 + \theta_2 - \theta_3 \leq 1 \tag{3}$$

We restrict θ_3 to be less than both θ_1 and θ_2. This ensures that an increase in pollution, either of s_t or s_{t-1}, cannot increase the realized wage of the individual. The constraint $\theta_1 + \theta_2 - \theta_3 \leq 1$ ensures that, for any possible combination of past and present pollution, current wages can never be negative.

Legal Regime and Damages

Suppose that, for many periods in the past, the law set a limit on the amount of pollution that each neighborhood could receive, and firms had strict liability for harm caused by their pollution. This limit was at a level such that half of the firms located in the low-earnings neighborhood and the other half in the high-earnings neighborhood. Thus, the original law produced equality in the distribution of pollution across neighborhoods. This equal share of firms in the neighborhood at time $t = 0$ sets the initial condition for the model, namely $s_0 = \frac{1}{2}$.

This initial regime is a simple setting for a less efficient, but more egalitarian legal rule. It also resembles existing pollution policy. One of the EPA's main forms of regulating air quality is to set ceilings on detected pollutants in counties. In cases in which a county exceeds limits for regulated pollutants, the law "imposes strict regulations on polluters" in that county, often leading to dramatic falls in pollution (Chay and Greenstone, 2005). Our initial setting is one in which a legal ceiling on pollutants leads to equal shares of polluters across the two neighborhoods.

The government, starting in time period $t = 1$, decides to switch to a more efficient (in the Kaldor-Hicks sense) legal regime. Starting in that period, it uncaps the pollution limit while still enforcing strict liability from harm. This is more Kaldor-Hicks efficient as firms are made to pay for the harm they cause, and so if firms move it must be that total surplus increases from that move.

Let D_{Lt} and D_{Ht} be the damages a given firm has to pay if it locates in the low-earnings and high-earnings neighborhoods, respectively, at time t. Then

$$D_{Lt} = (\theta_1 + \theta_2 - \theta_3 s_{t-1})\, \overline{e}_L \tag{4}$$

Note that the firms have to pay for damage that will be realized next period but is caused by pollution this period (the θ_2 term in the damages equation). There are no punitive damages.

There are a couple of points to note about the damages charged to a firm locating in neighborhood N. The first is that damages (and harm) are a percentage of the base earnings $\overline{e}_N$, since with "economic damages" in tort law, higher wages mean higher damages for the same lost number of hours. This will tend to push firms toward the low-earnings neighborhood. For example, suppose that $\theta_3 = 0$. Then, a firm will cause the same harm as a percentage of earnings in either neighborhood. Because $\overline{e}_H > \overline{e}_L$, this means that absolute damages are higher in the high-earnings neighborhood than in the low-earnings neighborhood. Since the choice of neighborhood depends on the absolute harm and not relative harm (as we will describe below), this will tend to push firms toward the low-earnings neighborhood.

The second aspect to note about the damages from current-period pollution shown in (4) is that an increase in s_{t-1} (pollution in the neighborhood in the *previous* period) leads to a decrease in damages as long as $\theta_3 > 0$. As polluting firms move into the neighborhood, the marginal harm from a polluter locating in that neighborhood in the next period decreases. In a strict liability legal regime, this, in turn, will tend to push more firms into the neighborhood. This basic dynamic drives policy and inequality snowballing.

Third, the uncapping of the pollution limit is a Kaldor-Hicks efficiency improvement (see Result 7), but it is not the Kaldor-Hicks optimal policy. This is because pollution in the current period changes the marginal cost of pollution in the next period through θ_3. Firms, however, are not made to take into account how their pollution this period changes the number of firms that will move into the neighborhood in the next period. We do not elaborate on the Kaldor-Hicks optimal path for a couple of reasons. First, it is difficult to see how the courts would plausibly calculate such an amount without knowing the path that firm location decisions will take. To make such a calculation, the courts would require as much information as the hypothetical social planner, and would have to foresee perfectly the path of firm decisions. The second reason is for analytical simplicity—the path produced by our definition of damages produces a first-order linear recurrence, which is much simpler to analyze than the efficient path, which is a second-order linear recurrence.

We focus on the Kaldor-Hicks efficiency criterion because this is the dominant paradigm for efficiency in law and economics. And part of the "classical creed" of Kaldor-Hicks efficiency is that such improvements should be pursued (Persky, 2001).

There is precedent in the U.S. for switching to more efficient policies that redistribute from the poor to the rich. We discuss two here: changes in trade policy in the 20th century and changes in law in the 19th century.

One example of a shift to a more Kaldor-Hicks-efficient policy that arguably had adverse distributional consequences is trade liberalization. Trade liberalization has long been one of economists' go-to examples of a welfare-improving policy that should be pursued, despite the known potentially negative distributional impacts (Krugman, 1997; Autor, Dorn, and Hanson, 2016). In the second half of the 20th century, the United States government liberalized its trade policy, greatly reducing its tariffs on goods from other countries and eliminating many barriers to trade. While studies have argued that these trade liberalizations have been total welfare improving (see, for example, Caliendo and Parro (2015)), these trade liberalizations, in particular the rising role of China in international trade, have also been shown to have increased unemployment and decreased wages in areas of the United States with manufacturing jobs most exposed to international trade (Autor, Dorn, and Hanson, 2016). In an attempt to smooth over the distributional impacts of trade liberalization, the US introduced Trade Adjustment Assistance (TAA) beginning in 1962, and has expanded the program with successive waves of trade liberalization (Decker and Corson, 1995). Evaluations of TAA have argued that its ability to reduce unemployment and increase earnings is low (Decker and Carson, 1995; Reynolds and Patalucci, 2012). In this way, the change in American trade policy in the late 20th century is an example of a shift to a more efficient regime that had negative distributional consequences and incomplete compensation of harms.

As another example, Horwitz (1977) argued that, around the turn of the 19th century, American law pivoted away from relatively more protections for the disadvantaged in society and towards promoting economic growth and private owners' rights to develop. For instance, this time period saw a shift away from the general compensation principle towards the negligence standard, giving property owners more leeway to develop without having to compensate those harmed. Additionally, the law increasingly considered the relative productivity of a particular land use when adjudicating nuisance, giving particular immunity to railroad companies as they built more track. In both of these examples, a more efficient, growth-friendly doctrine was adopted at the expense of disadvantaged groups in society.

Firm Choices

A firm's choice of neighborhood depends on the value of damages paid as well as firm-specific preferences for either the high-earnings neighborhood or the low-earnings neighborhood. These firm-specific preferences can be thought of as coming out of features of each neighborhood that affect profitability, such as the quality of the geography for building a dump; proximity to natural resources, customers or suppliers; cheap supplies of energy; transportation infrastructure; or low construction costs. We assume that pollution affects only the earnings of the neighborhoods and does not change the characteristics of the neighborhoods that determine the firm's preferences for either neighborhood. We also assume that firm rents are high enough so that even if the firm that has the highest preference for the high-earnings neighborhood locates in the low-earnings neighborhood and pays the associated damages, the firm would rather operate than exit the market.

Specifically, let $\alpha_k \sim F$ be firms k's idiosyncratic (non-damages) relative value from locating in the high-earnings neighborhood compared to the low-earnings neighborhood. F is a smooth cdf whose pdf has compact support, and the firm keeps this preference for the rest of time. We impose an additional condition on F: that it has median 0. This is merely so that, before damages calculations, firm preferences do not tilt the balance of firms to one neighborhood or the other.

Under the strict liability legal regime, firm k will locate in the low-earnings neighborhood if

$$\alpha_k < D_{Ht} - D_{Lt} \tag{5}$$

That is, the firm will locate in the low-earnings neighborhood if the value of its relative preference for the high-earnings neighborhood is smaller than the damage payments loss from locating in the high-earnings neighborhood. The difference in damage payments across the neighborhoods and its comparison to a firm's individual preferences governs a firm's location decision. Because of this, we briefly take a closer look at the damages difference:

$$D_{Ht} - D_{Lt} = \left(\bar{e}_H - \bar{e}_L\right)\left(\theta_1 + \theta_2\right) - \theta_3 \bar{e}_H + \theta_3 s_{t-1}\left(\bar{e}_H + \bar{e}_L\right) \tag{6}$$

First, note that it is only a function of the share of firms that previously located in the low-earnings neighborhood and parameters of the model. Second, absolute increases in θ_3

increase the influence of past location decisions on this period's damage differences. Moreover, a higher share of firms locating in the low-earnings neighborhood in the previous period increases the damages difference.

By the location decision condition (5), the share of firms that will locate in the low-earnings neighborhood at time t is

$$l_t = F\left(D_{Ht} - D_{Lt}\right) \tag{7}$$

This equation is the law of motion for the share of firms in the low-earnings neighborhood.

Note that, in the baseline model, there are no taxes and transfers that compensate for the losses that the poor suffer from the switch in legal regime. Theoretically, there are many reasons for legislative inertia that would lead to incomplete distributional offsetting (Fennell and McAdams, 2016; Liscow, 2018a). It is an open empirical question how much compensation happens after a change in legal rule impacts distribution. And there is evidence both ways (Autor, Dorn, and Hanson, 2013; Boylan and Mocan, 2014; Liscow, 2018b). For our purposes, the point is that there is a plausible case that compensation for the change in legal regime is incomplete.

3 Policy and Income Snowballing

In this section, we describe the conditions in which the switch to the efficient legal regime in our model leads not only to an increase in the share of firms in the low-earnings neighborhood, but also an increasing share of firms in the low-earnings neighborhood over time. We call a situation in which the share of firms in the low-earnings neighborhood is increasing over time because of these dynamics *policy snowballing.* We also provide conditions in which the policy snowballing translates to increasing income inequality over time, which we call *inequality snowballing.* The importance of both of these phenomena is not just that the efficient regime leads to more pollution and income inequality for the low-earnings neighborhood over time.

The dynamic, compounding increase in pollution will also make it difficult for a legislature to redistribute to compensate for the increased inequality, a point we elaborate on in Section 4.

3.1 Policy Snowballing

Formally, we define *policy snowballing* as occurring when $s_t > s_{t-1}$ for all $t \geq 1$ or when there exists a $t^* \in \{\mathbf{2}, \mathbf{3}, \mathbf{4}, ...\}$ such that for $t \geq t^*, s_t = 1$ (all firms locate in the low-earnings neighborhood) and for $1 \leq t < t^*, s_t > s_{t-1}$. The law of motion described in equation 7 determines the presence of policy snowballing. Our first result establishes the necessary and sufficient conditions for policy snowballing. The proof for this result, as well as later results, is in the Appendix.

Result 1: Policy snowballing. *Suppose that $s_1 < 1$.*[8] *There is policy snowballing if and only if $\theta_3 > 0$. See Appendix for proof.*

According to Result 1, the key ingredient for policy snowballing is θ_3, the feedback effect. Policy snowballing occurs and, given the restrictions on θ_1, θ_2, and θ_3 in the model setup, can only occur when pollution today decreases the marginal harm from pollution that occurs next period. The result establishes that if the harm process is one in which pollution acts on the same base earnings $\overline{e}_H$ irrespective of past pollution, then there can be no policy snowballing. Policy snowballing requires and is in essence caused by the feedback effect. All else equal, if a higher share of firms located in a neighborhood in the previous period, then the damages from locating in that neighborhood are lower in this period. Without this effect, the adjustment to the change in legal regime would occur in one period.

To see how this leads to perpetual policy snowballing, consider how θ_3 affects the location decision across multiple time periods. At time period t, firms calculate the damage differences (6) between the neighborhoods and compare it to their idiosyncratic preferences to choose in which neighborhood to locate. Suppose that the balance of these decisions means that

[8] Having all firms immediately locate in the low-earnings neighborhood is uninteresting as the number of firms can no longer increase, and there is no scope for θ_3 to play a role in dynamics.

more firms have moved into the low-earnings neighborhood. Then, $s_t > s_{t-1}$. This means that when firms are calculating the damages from neighborhood choice at period $t+1$, the damages to be paid from locating in the low-earnings neighborhood now will have decreased precisely because $s_t > s_{t-1}$. Conversely, the damages to be paid from locating in the high-earnings neighborhood will increase because the share of firms locating there decreased. So the damage differences (6) increases, and the share of firms in the low-earnings neighborhood will increase in time period $t+1$. The legal regime induces a state dependence in the marginal impact of pollution, leading to policy snowballing.

Due to the importance of $\theta_3 > 0$ for our results, we discuss its empirical relevance. $\theta_3 > 0$ means that past pollution decreases the marginal impact of contemporaneous pollution. In other words, it produces concavity in the pollution damage function. There is some evidence that pollution can have concave effects on health. For example, Chay and Greenstone (2003) found evidence for nonlinear effects of TSPs (total suspended particulates) on infant mortality with larger impacts of a one-unit drop of TSPs occurring in counties with lower initial levels. More recently, Jbaily et al. (2022) found larger impacts on mortality among the Medicare population from a given increase in $PM_{2.5}$ when starting at lower levels of pollution. Lee et al. (2010) found a cross-sectional inverted U-shaped dose-response correlation between Persistent Organic Pollutants and Type 2 Diabetes. They described a theory as to why some bodily responses could display such a concave U-shape rather than the traditional linear dose-response model for cellular toxicity.[9]

Other studies do not find concave effects. Some studies have found linear effects from pollution (Chang et al., 2019; Bharadwaj et al., 2017). Others have found convex effects, especially threshold effects, below which no effect is detected and above which effects are linear (Chang et al., 2016; Lichter et al., 2017). In the case where $\theta_3 < 0$ (convexity), the share of firms in the low-income neighborhood will fluctuate: it will first increase after the switch to the strict liability regime, then fall as some firms find it profitable to move back to

[9] Their theory drew from Daston et al. (2003), Welshons et al. (2003), and Vasseur and Cossu-Leguille (2006).

the high-earnings neighborhood, then increase again, and so on. As such, policy snowballing with a nondecreasing share of firms in the low-earnings neighborhood over time does not occur. In the rest of the paper, we will focus on the case with $\theta_3 > 0$.

3.2 Income Snowballing

Next, we examine when we would see a repeated increase in income inequality and its relation to policy snowballing. To do so, we first define income. Formally, we define income in the low-earnings neighborhood at time t as

$$I_{Lt} = I_{Lt}(s_{t-1}, s_t) = e_{Lt} + \Omega\left(\left(\theta_1 - \theta_3 s_{t-1}\right) s_t + \theta_2 s_{t-1}\right) \overline{e}_L \tag{8}$$

where $0 \leq \Omega \leq 1$. This new parameter represents the wedge between damages paid by the firms (which would make the harmed individuals whole) and the payments received by the harmed individuals. These can be thought of as lawyers' fees that plaintiffs have to pay to take the firms to court. This assumption is consistent with the "American Rule," the standard U.S. practice by which each party pays for its own legal fees (Derfner and Wolf, 2018, ¶1.01). As a result of having to pay lawyers for the service of bringing suit against the polluters, parties are unable to obtain full compensation for the harm they experience. That the lawyers' fees would be a percentage of the settlement is typically how such arrangements work, and so we are left with the harmed parties only obtaining a percentage in damages of the total harm they suffered.

The first term in the income equation is the earnings of individuals in the low-earnings neighborhood at time t. The second term in the expression is the amount of income received from damage payments. It includes payments for harms from two different periods: harm today $\left(\left(\theta_1 - \theta_3 s_{t-1}\right) s_t\right)$ and harm from pollution in the previous period $\left(\theta_2 s_{t-1}\right)$. Damage payments are timed so that individuals receive payments today for harm realized today. At time t, a firm located in the low-earnings neighborhood will be ordered to pay $\left(\theta_1 - \theta_3 s_{t-1}\right) \overline{e}_L$

today and $\theta_2 \bar{e}_L$ tomorrow. Summing up over the number of firms that located in the low-earnings neighborhood gives the s_t and s_{t-1} in Equation 8. This definition of income allows for an easy comparison of today's income with a counterfactual income absent pollution or absent a change in pollution.

Define the level of income inequality Q_t at time t as the ratio of high-earnings neighborhood earnings to low-earnings neighborhood earnings: $Q_t = \frac{I_{Ht}}{I_{Lt}}$. Furthermore, we formally define *income snowballing* analogously to *policy snowballing.* Define *income snowballing* as occurring when $Q_t > Q_{t-1}$ for all $t \geq 1$ or when there exists a $t^* \in \{\mathbf{2}, \mathbf{3}, \mathbf{4}, ...\}$ such that for all $Q_t = c \in \mathbb{R}$ for all $t \geq t^*$ (after some time income inequality is constant) and for $1 \leq t < t^*, Q_t > Q_{t-1}$. Then we have the following result.

Result 2: Income snowballing. *If there is policy snowballing, then a necessary and sufficient condition for income snowballing is* $\Omega < 1$. *See Appendix for proof.*

Although policy snowballing will occur as long as there is a feedback effect, income snowballing requires an extra ingredient: that damages received by harmed individuals are incomplete. If there are no frictions preventing harmed individuals from receiving the full damages that firms pay, the income of individuals in both neighborhoods will be the same as their counterfactual income absent the change in regime, keeping income inequality the same. Distributionally, therefore, there are no impacts from policy snowballing if compensation for harms is complete. But a change in the legal regime to a more efficient rule and a wedge between harm and damage payments received combine to increase income inequality over time, introducing impacts from insufficient compensation and growing inequality.

4 Model Dynamics

In this section, we describe the model's steady-state outcome, how its parameters affect the steady state, and how quickly the steady state is arrived at. We also analyze the quantitative importance of snowballing and describe optimal policy. To do so, we specify a uniform distribution for firm preferences.

In subsection 4.1, we establish that a steady state always exists (excepting one knife-edge condition), and we characterize when there is an interior solution. We also discuss how changes in the model parameters affect the location of the final steady state, providing additional intuition about the forces at play.

Moreover, defining "speed" of convergence as (roughly) the number of time periods needed to get close to the steady state, we show in subsection 4.2 that the speed of convergence is governed by a push-and-pull between firm preference dispersion and the feedback parameter.

In subsection 4.3, we illustrate the importance of the snowballing described here by asking how much needed redistribution—from the perspective of the counterfactual with no legal change—would be missed if a legislature were trying to return inequality to its original level but ignored the dynamic evolution of pollution brought about by the change in the legal regime. We characterize the amount of transfers to the poor missed. And we provide an example showing the large amount that state-dependence dynamics can impact the harm of a change in legal rules on the poor.

Finally, in subsection 4.4, we turn to welfare and characterize the welfare weights required to prefer the inefficient regime over the efficient one if legal compensation is incomplete.

4.1 Steady State Existence and Comparative Statics

Throughout this section we assume that firm preferences are uniform. Specifically, we assume that the cdf of firm preferences is given by

$$F_U \sim U\left[-M \cdot \left(\overline{e}_H + \overline{e}_L\right), M \cdot \left(\overline{e}_H + \overline{e}_L\right)\right] \tag{9}$$

where $M > 0$ is a constant that controls the dispersion of firm preferences. The uniform preferences allow for a more detailed analysis of the model dynamics.

The uniform preferences assumption means that the law of motion becomes a first-order linear difference function, for which we can use established mathematics results to characterize the process and the steady state. With the uniform distribution assumption (9), the law of motion (7) becomes

$$s_t = \frac{1}{2} + \frac{(\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H \theta_3}{2M(\overline{e}_H + \overline{e}_L)} + \frac{\theta_3}{2M} s_{t-1} \tag{10}$$

except for in corner cases.[10] With this formula, it is easier to see why $\theta_3 > 0$ implies policy snowballing (and why $\theta_3 < 0$ implies alternatively increasing and decreasing s_t): when $\theta_3 > 0$, past firm shares in the low-earnings neighborhood enter positively into the law of motion for s_t (alternatively, they enter negatively when $\theta_3 < 0$).

We can establish, with a simple technical assumption, that the process set in motion by the change in legal regime will eventually reach a steady state. Formally, we establish that the process defined by (7) will converge to a steady state. Additionally, we derive an equation for the steady state that applies in non-corner cases of steady state. Result 3 below summarizes the main conclusions, proven in the Appendix.

Result 3: Steady state: *Let $\theta_3 \neq 2M$. Suppose that $\theta_3 > 0$ and hence there is policy snowballing when the new legal regime is implemented. Then the process converges to a steady-state share of firms in the low-earnings neighborhood, $s^* > \frac{1}{2}$. Generally, the steady state is given by the equation*

$$s^* = \frac{M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H \theta_3}{(2M - \theta_3)(\overline{e}_H + \overline{e}_L)}$$

However, if the above expression is greater than one or less one-half, then $s^* = 1$. In that case, all of the firms will eventually locate in the low-earnings neighborhood. See Appendix for proof.

While we leave the technical details of the proof to the Appendix, before moving on we briefly describe the determination of steady state, the basics of the transitional dynamics,

[10]See the proof of Result 3 in the Appendix for the complete characterization of the law of motion.

and the occurrence of corner cases. A candidate for steady state is the unique point at which the law of motion (10) intersects with the line $s_t = s_{t-1}$. This point is given by the formula for s^* in Result 3. It is a candidate, not necessarily the actual, steady state, for two reasons.

First, the point of intersection is not necessarily the steady state because it may not be *stable*. Stability is an issue for any difference process. With an unstable steady state, unless the process begins exactly at the steady state, the process will diverge to a corner solution. With the first-order linear difference equation (10), it is easy to characterize the stability of the point. If $\theta_3 > 2M$, then the steady state is unstable.[11] In that case, unless s^* is exactly one-half (that is, equal to the initial condition), then the process will not converge to the candidate steady state.

The second reason why the intersection of (10) and $s_t = s_{t-1}$ may not be the steady state of the law of motion is that s_t is restricted to be between 0 and 1. If the candidate steady state is above 1, then the process stops at $s^* = 1$. Less intuitive is the reason why if the candidate steady state is less than one-half, then the actual steady state is also $s^* = 1$. The reason is that, as we demonstrate in the proof in the Appendix, the candidate steady state is only less than one-half when $\theta_3 > 2M$. Hence, it is an unstable steady state. Because the share of firms begins at one-half, the instability of the steady state will cause the share of firms to move off towards infinity. The process is, however, stopped at $s^* = 1$.

After the switch to the efficient legal regime, if $\theta_3 > 0$, then the share of polluting firms in the low-earnings neighborhood will increase every period. With the uniform preferences for the firms, we can say that in many cases the increase in subsequent periods will become small enough so as to never lead all firms to locate in the low-earnings neighborhood. Instead, the share of firms in the low-earnings neighborhood will approach a steady state s^*. We can also know exactly what the steady state is using the model parameters.

Furthermore, note that, if $\theta_3 = 0$ so that there is no policy snowballing, the steady state is reached immediately, as there are no dynamics. Thus, with the exception of the knife-edge

[11] Recall that in Result 3 we do not consider $\theta_3 < 0$. If we did, then the condition for stability would be $|\theta_3/2M| < 0$.

condition, there is always a steady state.

Comparative Statics

We now examine how changes in the model parameters affect the final share of firms that will locate in the low-earnings neighborhood. To do so, we focus on the cases in which the steady state is less than 1, so that we can simply analyze the expression for s^* in Result 3. Note, too, that this assumption implies that $\theta_3 < 2M$ (see the Appendix proof for Result 3), an inequality useful for signing the comparative statics.

The first comparative statics results are

$$\frac{\partial s^*}{\partial \theta_1} > 0, \qquad \frac{\partial s^*}{\partial \theta_2} > 0.$$

Increasing the main effects from pollution, either the effects from pollution this period or the effects from pollution emitted in the previous period, increases the steady-state share of firms in the low-earnings neighborhood. This is because increasing the harm from pollution, which works as a percentage of base earnings level, leads to a bigger absolute increase in damages to pay in the high-earnings neighborhood than the low-earnings neighborhood since high-earnings individuals have a higher base earnings level. So, the steady state sustains a higher share of firms in the low-earnings neighborhood when either main effect increases.

Next, we have that

$$\frac{\partial s^*}{\partial M} < 0.$$

The intuition for this result is that a larger M means that firms have more dispersed preferences for location, which means that a larger share of firms will have a strong preference to locate in the rich neighborhood. This reduces the share that locate in the low-earnings neighborhood in equilibrium.

We deal with how changes in base earnings affect s^* together. The comparative static results are

$$\frac{\partial s^*}{\partial \overline{e}_H} > 0 \qquad\qquad \frac{\partial s^*}{\partial \overline{e}_L} < 0.$$

As the inequality in the initial earnings grows, the low-earnings neighborhood becomes increasingly attractive. Because firms care only about absolute damages, and harm is a percentage of base earnings, an increase in the gap between the base earnings increases the absolute damages difference between locating in the high-earnings neighborhood and locating in the low-earnings neighborhood. In short, the higher the level of initial inequality, the more unequal the equilibrium pollution will be.

The final comparative static to examine is how s^* changes with changes in θ_3.

$$\frac{\partial s^*}{\partial \theta_3} = \frac{(\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2 - M)}{(2M - \theta_3)^2 (\overline{e}_H + \overline{e}_L)}.$$

The effect of a change in θ_3 on the steady state is ambiguous. Its sign depends on the sign of $\theta_1 + \theta_2 - M$. If $\theta_1 + \theta_2 > M$, then $\frac{\partial s^*}{\partial \theta_3} > 0$: a higher feedback parameter increases the share of polluting firms in the poor neighborhood. If $\theta_1 + \theta_2 < M$, then the reverse is true: $\frac{\partial s^*}{\partial \theta_3} < 0$. Note that, intriguingly, a larger feedback parameter—which mitigates the harm from pollution—ultimately ends up causing *more* pollution in the poor areas in the former case: something that seems like it would make things better for the poor actually can make them worse.

The ambiguity of its impact on the steady state occurs because an increase in θ_3 has two effects. First, it increases the state dependence of the price of harm. As a result, a given increase in the level of pollution will cause a bigger increase in the relative attractiveness of the low-earnings neighborhood, pushing firms to the low-earnings neighborhood. How much this increase in relative attractiveness matters is mediated by the dispersion of firm references, M. With a high M, firm preferences are more dispersed, and hence the share of firms in the low-earnings neighborhood is less sensitive to changes in the damages difference between the neighborhoods.

Second, an increase in θ_3 decreases the harm from pollution. Because the decrease in harm from pollution is a percentage of base earnings, a higher θ_3 will lead to a bigger absolute fall in damages from locating in the high-earnings neighborhood. The importance of this effect is mediated by the main effects, θ_1 and θ_2. If θ_1 and θ_2 are high, then the fall from an increase in θ_3 of the *relative* damages for locating in the high-earnings neighborhood will be small because most of the damages difference will come from the main effects θ_1 and θ_2.

In sum, there are two effects that come out of an increase in θ_3. In the first effect, a higher θ_3 pushes firms towards the low-earnings neighborhood, and the intensity of the effect is inversely related to the size of M. In the second effect, a higher θ_3 will push firms towards the high-earnings neighborhood, and the intensity of the effect is inversely related to the size of $\theta_1 + \theta_2$. What the comparative statics results show is that we can sign $\frac{\partial s^*}{\partial \theta_3}$ by simply signing the sum of the mediators: $-M + (-1)(-1)(\theta_1 + \theta_2) = \theta_1 + \theta_2 - M$.[12]

We can think of this as a simple comparison of the two effects. If $\theta_1 + \theta_2$ is bigger than M, then the second effect is weak relative to the first; the main effects are large so an increase in θ_3 barely increases the relative attractiveness of the high-earnings neighborhood. By contrast, the firm preference dispersion is low, so the increase in state dependence from an increase in θ_3 is likely to move many firms into the low-earnings neighborhood. Hence, the first effect dominates and the steady-state share of firms in the low-earnings neighborhood increases with an increase in θ_3. If $\theta_1 + \theta_2$ is smaller than M, then the reverse is true.

4.2 Speed of Convergence

With uniform firm preferences, we can also evaluate how fast the process converges to steady state. We measure speed of convergence by the rate of change of distance from the steady

[12]Note double use of (-1) for the second effect to translate the intuition into mathematics. The first negative comes from the fact that the second effect pushes firms towards the high-earnings neighborhoods. The second negative comes from the fact that intensity of the effect is inversely proportional to $\theta_1 + \theta_2$.

state; formally, we express this as $\frac{s_t - s^*}{s_{t+1} - s^*}$ (see, e.g., Süli and Mayers, 2003). Loosely, the speed of convergence relates to the number of periods it takes to get "close" to the steady state. Assuming the steady state is an interior point, we can apply the law of motion and the formula for the steady state to measure the speed of convergence in terms of the parameters.

Result 4: Speed of Convergence: *With firm preferences (9) and $s^* < s$, then the speed of convergence is*

$$\frac{s_t - s^*}{s_{t+1} - s^*} = \frac{2M}{\theta_3}$$

See Appendix for proof.

A small $\frac{2M}{\theta_3}$ means that $s_{t+1} - s^*$ is close to $s_t - s^*$ and hence the distance from steady state changed little between period t and $t+1$.

The primary intuition for why the ratio $\frac{2M}{\theta_3}$ determines the speed of the convergence process is that the more sensitive that firm location choice is to changes in neighborhood firm shares, the more steps the process needs to converge to steady state; hence, the slower the process is.

The logic for why a higher M increases the speed of convergence is similar to why a higher M leads to fewer firms in the poor neighborhood in equilibrium. M governs the dispersion of firm preferences. A higher M means that aggregate firm location decisions are less sensitive to the changes in relative damages between the neighborhoods. If firm preferences are more dispersed, an increase in the share of firms in the low-earnings neighborhood leads to fewer firms choosing to move into the low-earnings neighborhood compared to a scenario with less dispersed firm preferences. A higher M, therefore, means that the process of converging to steady state is faster since firms are not very sensitive to changes in damage differences between the neighborhoods. The process needs fewer steps before settling down.

In contrast to its ambiguous effect on the equilibrium share of firms in the poor neighborhood, however, θ_3 has an unambiguously negative effect on the speed of convergence. Indeed, this result provides useful intuition into the dynamics of policy snowballing. Recall that θ_3 governs how sensitive the relative damages between neighborhoods are to changes in the share of firms in the neighborhoods. A higher θ_3 means that a given increase in the share of firms in the low-earnings neighborhood leads to a larger decrease in the relative cost of locating in the low-earnings neighborhood. Thus, firm location decisions are more sensitive to changes in the share of firms in the low-earnings neighborhood. Because of this increased sensitivity, the process takes more steps before it settles down, and thus it is slower.

As a final note on speed of convergence, we note that the actual time represented by the periods in the model is also of practical importance. As of now we have been agnostic about the amount of time represented by a time period in our model. Though we leave it unmodeled, we think of it as the time horizon at which firms can feasibly decide to move. If firms can make moving decisions on a relatively small time scale, then the process converges more quickly and snowballing is less important.

4.3 Accounting for Dynamics: What Would it Take to Return Inequality to its Original Level?

Typical analysis of legal rules is static, considering costs and benefits in one period. In this subsection, we consider what static accounting misses given the snowballing dynamics. As we have described above, we suppose the legislature changed the legal regime from the more equitable, inefficient regime to the more efficient one. It wishes to keep inequality the same as before the regime change, but fails to take snowballing into account. Specifically, it looks only at the change in polluter location after one period and calculates transfers to the poor based on this one period. We call this the "static" transfer. We illustrate how much the legislature can miss by failing to account for the dynamic process of firm movement into poor areas first by measuring the gap between the one-period statically calculated transfers and the transfers needed to make up for the growth of income inequality in steady state.[13] Then, we present a numerical example showing how this gap can grow over time. Finally,

[13]Note that this failure to take into account the dynamic effects of a policy is not a snowballing-specific problem for the legislature. It can come about in many policy changes where the dynamic effects of the policy are not well understood or difficult to foresee.

we discuss an alternative policy option for the government and why that may not be more fruitful than the transfers explored here. Throughout, we suppress the formal expressions defining the different lump-sum taxes and transfers as well as derivations of results, leaving these instead for the Appendix.

4.3.1 The Gap Between Static and Dynamic Transfers

The government wishes to transfer money to the poor to undo the growth in inequality that comes about from moving to the more efficient regime. To do so, it observes how many firms moved into the low-earnings neighborhood after the first period of the new regime. Then, it calculates the appropriate lump-sum transfer needed to return income inequality to its original level.

Specifically, after the first period of the new regime, the government observes the new share of firms in the low-earnings neighborhood, s_1, and calculates a lump-sum transfer $\tau_{1,s}$ to return inequality to its original level. Having observed the share of firms in the low-earnings neighborhood, for the rest of time the legislature assumes that the share of polluting firms in the low-earnings neighborhood will remain at s_1. So for the second period and beyond, it picks a lump-sum transfer, τ_s, to keep inequality at its original level, assuming that the share of firms in the low-earnings neighborhood will remain at s_1. We call τ_s the *static transfer*. This taxes-and-transfers scheme will fail to fully compensate for the change in income inequality, as it will not take into account the continuing policy and income snowballing that will occur period after period. Let $\tau_{d,t}$ be the transfer at time t that reproduces the original level of inequality. Call this the *dynamic transfer.*

We are interested in the *transfer gap*, which we define as the difference between the static and the dynamic transfer as a share of the dynamic transfer: $\frac{\tau_{d,t}-\tau_s}{\tau_{d,t}}$. Because from Result 2 we know that income inequality is increasing over time, it must also be that $\tau_{d,t}$ increases over time. As a result, the transfer gap will also increase over time. With the uniform firm preferences assumption, we can be more precise and compare what the transfer should be at steady state to preserve the original income inequality to the static transfer calculated above. Let τ_d be the steady-state lump-sum tax that produces income inequality in steady state equal to the original income inequality. Assuming that $\theta_3 > 0$, $\Omega < 1$, and that $s^* < 1$,[14] we calculate that the transfer gap as a fraction of the steady-state transfer is

$$\frac{\tau_d - \tau_s}{\tau_d} = \frac{2(s^* - s_1)}{2s^* - 1}.$$

A simple takeaway from this expression is that the further away s_1 is from s^*, the larger that gap is between the static transfers and correct dynamic transfers. Since both are determined by model primitives, however, it is better to understand how the model parameters themselves determine this gap. In the Appendix, we show that this expression in terms of primitives is

Result 5: Static Transfer Gap:

$$\frac{\tau_d - \tau_s}{\tau_d} = \frac{\theta_3}{2M}.$$

See Appendix for Proof.

This expression demonstrates that the transfer gap as a share of the steady-state transfer is equal to the inverse of the speed of snowballing. A faster speed of convergence (i.e. a lower $\frac{\theta_3}{2M}$) will tend to produce smaller transfer gaps. Intuitively, this is because when convergence is faster there are fewer steps needed to get close to the steady state and, hence, s_1 will not be as far from s^*. As a result, the myopic legislature will miss the correct transfer by less when the convergence process is faster. Note that again, like with the equilibrium comparative statics, the feedback parameter also has a perverse effect here: though θ_3 mitigates the harm from pollution in the short run, it ends up causing a larger compensation gap because it also increases the uncompensated feedback drawing more firms into the poor neighborhood over

[14]The first two assumptions ensure that there is policy and income snowballing, guaranteeing that $\tau_d \neq 0$. The last assumption allows us to substitute in the expression for s^* from Result 3.

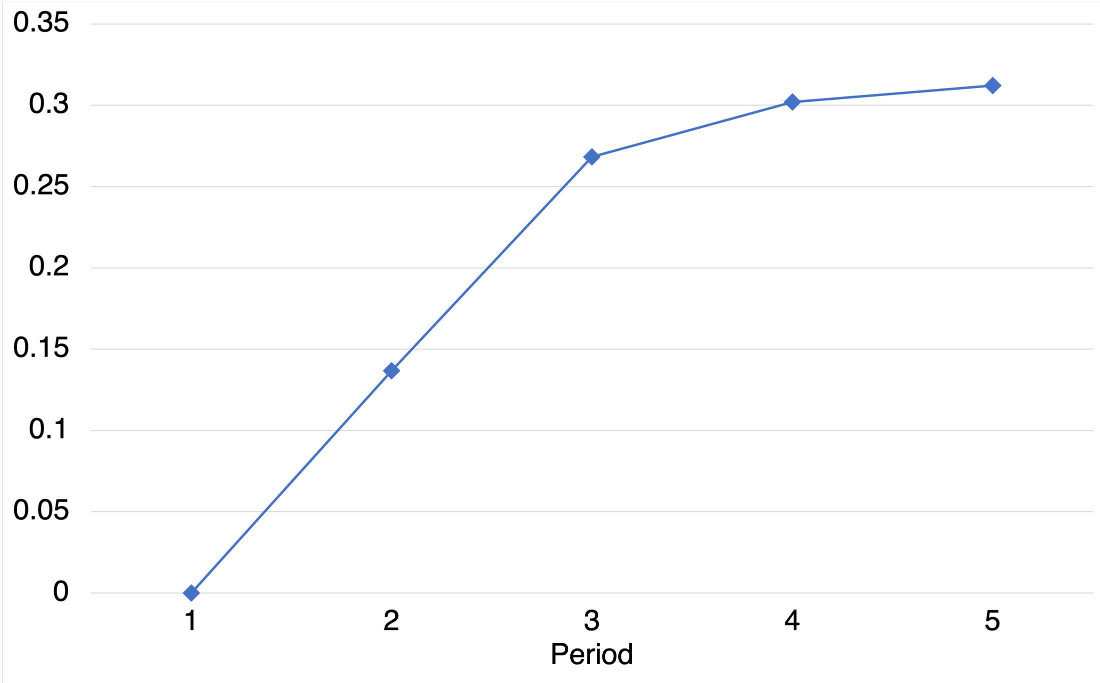
time.

The transfer gap demonstrates the chief political economy implication of policy and income snowballing. A major tenet of the law and economics approach is to recommend the implementation all Kaldor-Hicks improvements whenever they are found; distributional concerns and preferences can be addressed through the taxes and transfer system. Supposing that political economy frictions can be overcome to implement redistribution after the adoption of a more efficient but less equitable legal rule (Fennell and McAdams, 2016; Liscow, 2018a), if the change in legal regime produces policy and income snowballing, the legislature will either need to have perfect foresight the first time around on the dynamic path of inequality resulting from the change in legal regime, or it will need to repeatedly return to update the transfers as earnings inequality grows as a result of the change in legal rule.

4.3.2 An Example

We present an example economy, showing how the gap between the static transfer and the transfer that correctly returns inequality to its original level evolve over a few periods. The example shows the large compensation gap that can arise due to ignoring snowballing. We use the following parameters: $\bar{e}_H = 100,000, \bar{e}_L = 25,000, \theta_1 = 0.2, \theta_2 = 0.2, \theta_3 = 0.19, \Omega = 0.9$ and $M = 0.3$. The ratio of rich income to poor income, our measure of income inequality, in this economy at time $t = 0$, when the share of firms in each neighborhood has been set to $\frac{1}{2}$, is 4. After the government institutes strict liability for damages, the steady state produces a level of income inequality of 4.83. When the government introduces the statically-calculated transfer, τ_s, the level of inequality is only 4.28. Figure 1 plots the evolution of $\frac{\tau_{d,t}-\tau_s}{\tau_{d,t}}$. over five periods. In the second period, the statically calculated transfer misses 15% of the correct transfer, and this number keeps increasing over time until it reaches 31.7% by the fifth period, at which point the share of firms in the low-earnings neighborhood is close to its steady-state value and so the gap will not grow much after that. So, in the end, a large share—about a third—of the correct transfer is missed by ignoring snowballing.

Figure 1: Transfer Gap As a Share of the Dynamic Transfer Over Time



4.3.3 The Government's Policy Toolkit

Our analysis here focused on one particular kind of action the government could take to ameliorate the increased inequality from the switch to the more efficient legal regime, both because taxes-and-transfers are a commonly hypothesized response to legal changes and because of the analytical tractability. But, in principle, one could imagine other kinds of action the government could take. In particular, the government policy could set $\Omega = 1$.

A full understanding of the tradeoffs of adopting such a policy is beyond the scope of this paper, but we briefly speculate. On the one hand, if available, this policy lever would allow the government to avoid calculating the dynamic path of inequality that results from the change in legal regime and still yield no change in inequality.

On the other hand, there are possible downsides to the policy. Defenders of the American Rule, in which each party pays for its legal fees, point to problems with having the loser pay all fees. One of the main arguments in its favor, and against a system in which the loser pays all legal fees, is that the American Rule opens up the judicial system to more people as it reduces the cost stemming from the uncertainty and risk of potentially having to pay the burdensome loser's fees (Vargo, 1993; Root, 2004; Karsten and Bateman, 2016). Alternatively, the government could decide to pay for all lawyers' fees for the winning party in torts cases. But that would introduce distortion through the taxes required to fund the fees and possibly problematic incentives for lawyers whose fees were paid by the government.

4.4 Welfare

In this subsection, we further quantify the importance of the snowballing and the failure to compensate by characterizing the welfare weights that would lead a social planner to prefer one regime to another. We consider two regimes we have discussed: the inefficient regime in which half of the firms locate in each neighborhood and the efficient regime without government transfers. This inefficient regime is essentially what Sharkey (2021) proposes with her suggestion that torts use a constant level of earnings for economic damages, regardless of actual income. We again assume here that $\theta_3 > 0$, meaning that there is policy snowballing, and that $\Omega < 1$, meaning that there is income snowballing and damages fail to fully compensate harmed individuals. Importantly, we find that it can be social-welfare-maximizing to adopt the *inefficient* policy, and we specify when that is the case.

Suppose all workers share an increasing, twice-differentiable utility function $u(y_i)$, where y_i is the income of person i. To think about welfare, we will consider the welfare weight on the low-earnings individuals, g_L, and the welfare weight placed on the high-earnings individuals, g_H, that would justify choosing one regime over the other. Because all that matters is the size of the weights relative to each other, we set $g_H = 1$. We can then interpret g_L as how much more the social planner weights the utility of the low-earnings individuals relative to the high-earnings individuals. The social planner chooses the regime to maximize

$$g_L u(I_L) + u(I_H).$$

To simplify matters for characterizing the set of welfare weights that would lead the social planner to choose one regime over another, further assume that the individuals' utility function is linear in income. This focuses relative weighting of dollars in the hands of the poor versus the rich on only the welfare weights, rather than also mixing in the curvature of a utility function. One can think of this as a reduced-form way of comparing the welfare value of income in the hands of the rich and poor. Then we obtain the following result:

Result 6: Welfare-Maximizing Regime: *With a utility function that is linear in income, the social planner is indifferent between the efficient and inefficient regimes when the welfare weight $\hat{g}_L$ satisfies*

$$\hat{g}_L = \frac{\left[\theta_1 + \theta_2 - (\frac{3}{2} - s^*)\theta_3\right] \bar{e}_H}{\left[\theta_1 + \theta_2 - (s^* + \frac{1}{2})\theta_3\right] \bar{e}_L}.$$

Furthermore, the social planner would choose the efficient regime without transfers over the inefficient regime if

$$g_L < \frac{\overline{w}_H}{\overline{w}_L},$$

and she would choose the inefficient regime over the efficient regime without transfers if

$$g_L > \left(\frac{\theta_1 + \theta_2 - \frac{1}{2}\theta_3}{\theta_1 + \theta_2 - \frac{3}{2}\theta_3}\right)\left(\frac{\bar{e}_H}{\bar{e}_L}\right).$$

See Appendix for proof.

With a linear utility function, we can obtain a simple expression for the welfare weight that makes the social planner indifferent between the inefficient regime and the efficient regime without transfers. The expression for $\hat{g}_L$ shows that as the steady-state share of firms in the low-earnings neighborhood increases, the indifference weight also increases. This follows simply from more pollution leading to lower earnings. A higher steady-state share of firms in the low-earnings neighborhood means a lower income in the low-earnings neighborhood and a higher income in the high-earnings neighborhood, increasing steady-state inequality. Because a higher level of steady-state inequality reflects a higher level of inefficiency in the initial regime, it takes a higher concern for the low-earnings neighborhood to justify sustaining the inefficient regime.

Result 6 also shows a couple of cases in which one does not need to know the steady-state share of firms s^* to know which regime the social planner would prefer. If the preference for the low-earnings neighborhood is smaller than the ratio of the high-earnings base to the low-earnings base, then in no circumstance would the social planner prefer the inefficient regime. In that case, the total utility gains from switching to the efficient regime always overcome any preference for the low-earnings individuals. The other case shows that if the weight on the low-earnings individual is high enough, greater than $\left(\frac{\theta_1+\theta_2-\frac{1}{2}\theta_3}{\theta_1+\theta_2-\frac{3}{2}\theta_3}\right)\left(\frac{\bar{e}_H}{\bar{e}_L}\right)$, then there is no circumstance in which the social planner would choose the efficient regime. No total utility gains are high enough to overcome the planner's objection to the rising inequality.

Finally, we can characterize the relationship among the indifference weights that make each of three regimes optimal in comparison to the inefficient regime: the efficient regime without transfer, the efficient regime with myopic transfer (transfer τ_s), and the efficient regime with fully compensatory steady-state transfer (transfer τ_d).

Result 7: *Let $\hat{g}_{no-transfer}$ be the weight that makes the social planner indifferent between the inefficient regime and the efficient regime without transfers. Let $\hat{g}_{myopic}$ be the weight that makes the social planner indifferent between the inefficient regime and the efficient regime with the myopic transfer τ_s. Suppose that these exist.*[15] *Then we have the following relationship*

$$\hat{g}_{no-transfer} < \hat{g}_{myopic}$$

Moreover, there is no welfare weight that would lead the social planner to prefer the inefficient

[15]Note that $\hat{g}_{myopic}$ may not exist if the static transfer to the low-earnings individuals makes the total income of the low-earnings in steady state higher than the income of the low-earnings in the inefficient regime. In that case, the efficient regime with myopic transfer Pareto-dominates the inefficient regime.

regime over the efficient regime with the correct transfer. See Appendix for proof.

Intuitively, because the transfer lowers steady-state inequality, the social planner would have to put a much bigger weight on the utility of the low-earnings individuals to still prefer the inefficient regime compared to the weight she would need to prefer the inefficient regime to the efficient regime without a transfer. And, because the dynamic transfer that keeps inequality at the same level as before means that both low-earnings and high-earnings individuals see a higher income than in the inefficient regime, the efficient regime with the dynamic transfer Pareto-dominates the inefficient regime. No social planner would prefer the latter to the former.[16]

5 Extension: Labor Demand Impacts

The model so far has assumed that, when firms move into an area, workers are harmed because of pollution, but do not benefit because of increased labor demand. In this section, we relax this assumption. For simplicity (to avoid the complication of polluting firms paying higher earnings because of increased labor demand), we still assume that the polluting firms do not employ anybody from the neighborhoods. Rather we imagine that the presence of the firms can lead to investment, local government revenue, or the transit of people from outside the neighborhoods that could positively impact labor demand. Of course, if workers benefitted because of increased labor demand, then that would partially offset the extent of inequality snowballing. We explain how our results change as a result of labor demand impacts.[17]

Suppose that earnings increase by parameter ϕ such that the next period's earnings for the low-earnings neighborhood are

[16] In the same vein, the social planner would also strictly prefer the efficient regime with transfers calculated to perfectly compensate the low earners for their income loss from the switch to the inefficient, initial regime. This is because the income of the high earning would increase, while the income of the low earners would stay the same.

[17] The proofs for the result changes are available upon request. Since they just replace one parameter and follow the same math that is already in the Appendix, we do not include them to economize on space.

$$e_{Lt} = \left[1 - (\theta_1 - \phi)\, s_t - \theta_2 s_{t-1} + \theta_3 s_t s_{t-1}\right] \overline{e}_L.$$

Furthermore, damages do not account for the firm's effect on labor demand. The damages to be paid from locating in the low-earnings neighborhood are still:

$$D_{Lt} = (\theta_1 + \theta_2 - \theta_3 s_{t-1})\, \overline{e}_L.$$

Results 1, 3 and 4 are unchanged. These results depend only on firm decisions; since damages are the same as without the labor demand effects, these results are the same. Result 5 and 7 also remain the same. Even though both results involve income, which is affected by labor demand impacts, the labor demand impacts get cancelled out of the expressions.

Results 2 and 6 do change because they involve the income and do not have labor demand effects that are cancelled out. Below, we begin with the change to Result 2:

Result 2LD: Income snowballing with labor demand impacts. *If there is policy snowballing and $\phi < (1 - \Omega)(\theta_1 - \theta_3)$, then a necessary and sufficient condition for income snowballing is $\Omega < 1$.*

The second result now needs the additional condition that $\phi < (1 - \Omega)(\theta_1 - \theta_3)$. This guarantees that income always decreases with an increase in pollution. Note that the conditions that previously guaranteed that income decreased with increased pollution were $\theta_1 > \theta_3$ and $\theta_2 > \theta_3$. However, because an increase in current period pollution, n_t, now can also increase income by ϕ, we need to ensure that this increase is less than the decrease in income from current pollution: $(1 - \Omega)(\theta_1 - \theta_3)$. Hence, high enough labor demand effects may break the connection between policy snowballing and income snowballing.

Result 6 is modified in a similar way:

Result 6LD: *With a utility function that is linear in income, the social planner is* *indifferent between the regimes when the welfare weight $\hat{g}_L$ satisfies*

$$\hat{g}_L = \frac{\left(\phi - (1-\Omega)\left[\theta_1 + \theta_2 - (\frac{3}{2} - l^*)\theta_3\right]\right)\overline{e}_H}{\left(\phi - (1-\Omega)\left[\theta_1 + \theta_2 - (l^* + \frac{1}{2})\theta_3\right]\right)\overline{e}_L}.$$

Furthermore, we can give simpler characterizations of when the social planner would choose one regime over the other. She would choose the efficient regime without transfers over the inefficient if

$$g_L < \frac{\overline{e}_H}{\overline{e}_L},$$

and she would choose the inefficient regime over the efficient regime without transfers if

$$g_L > \frac{\phi - (1-\Omega)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{\phi - (1-\Omega)\left(\theta_1 + \theta_2 - \frac{3}{2}\theta_3\right)}\left(\frac{\overline{e}_H}{\overline{e}_L}\right).$$

There are two differences in the expression for the welfare weight. First, the expression now includes ϕ in the denominator and numerator. The within-neighborhood income differences across the two regimes still depend on the uncompensated damages, but the effect is now attenuated by labor demand impacts. For the low-income, the increase in pollution and attendant damages are dampened by the increase in labor demand from more firms moving into the neighborhood. Similarly, the boon to the high-income neighborhood from polluting firms moving out is dampened by the reduced labor demand. A higher labor demand effect increases the indifference welfare weight, since labor demand reduces the inequality effect of the efficient regime. The second difference is that the term for incomplete compensation, $(1-\Omega)$, is now in the welfare weight expression. Because the impact of labor demand does not depend on incomplete compensation, $(1-\Omega)$ can no longer be factored out of the numerator and denominator.

6 Factors Outside the Model

In the interest of parsimony, we did not include many features of reality in the model. These other factors could change the results. For example, we did not include the firms' profits in the model. Including a model for the firms' profits would allow us to give people in the neighborhoods ownership in the companies and see how the distribution of profits affects inequality snowballing. For example, if profits are generally positive and the people in the rich neighborhood own the majority of the companies, then that might exacerbate inequality snowballing as the rich would have another source of income advantage over the poor. Alternatively, if the poor owned most of the firms, the profits could compensate for the losses in earnings or fewer firms would locate in the poor neighborhood as the firms would internalize the losses to earnings from locating in the poor neighborhood.

Except for the policy change hypothetical, the model does not consider the availability of taxes and transfers, which could undo the income snowballing inequality, but not the policy snowballing. Another factor that is not modeled is mobility, which could mitigate the results by allowing people to escape the harm. The model also ignores the possibility of a correlation between firm preferences and local earnings. And, the functional form of proportional harm to earnings is important; if pollution caused the same dollar harm to rich and poor, the results would be different. This section is not intended to provide a comprehensive list of all the assumptions in the model (e.g., the model also assumes the same number of people live in rich and poor places), but rather to suggest that—while the model is quite general—more work remains to be done to determine the scope of that generality.

7 Other Possible Settings with Inequality Snowballing

Commenting on the degree to which the dynamics discussed in this article contribute to increasing inequality is beyond the article's scope. Nevertheless, it is worth discussing other policies that have features similar to the model here and could thus lead to inequality snowballing. Recall that there are three key policy features that can lead to inequality snowballing. In the specific model here, (1) earnings affect the application of a legal rule, (2) the legal rule in turn affects earnings in the future, creating a feedback loop ($\theta_3 > 0$), and (3) compensation is incomplete ($\Omega < 1$). More generally, what is needed is that (1) parties' willingness to pay affects the application of a legal rule, (2) the legal rule in turn affects willingness to pay in the future, creating state dependence and a feedback loop, and (3) compensation is incomplete. And, of course, rich and poor people need to be differentiated somehow: by geography (rich vs. poor neighborhoods), by means of consumption (flying on airplanes vs. riding public busses), or otherwise. For example, a policy wherein (1) more resources are given to the rich because they are willing to pay more for them, (2) those resources in turn increase willingness to pay in the future, and (3) compensation is incomplete would satisfy all three conditions.

One example is cost-benefit analysis of transportation infrastructure. When allocating funding for competitive grant programs, the federal Department of Transportation requires cost-benefit analysis that places a higher value on time saved in high-speed rail and airports than in bus lines because the former are used largely by richer people and the latter largely by poorer people.[18] The analysis follows federal requirements for producing efficient regulations by measuring individuals' willingness to pay using their wages because of the time value of money. Consider how such policymaking follows the analysis in this paper. If an individual has low wages, then her willingness to pay for transportation is lower. As a result, the government neglects transportation that benefits the poor and invests, all else equal, in more transportation that benefits the rich. This results in longer commutes for the poor, which may increase fatigue or limit their ability to pursue training or educational opportunities. In turn, this reduces productivity and lowers earnings (and, in any case, arguably reduces willingness to pay because of the relatively reduced wealth of the poor). The analysis then repeats in the next period, leading to snowballing inequality in the absence of compensation

[18]See explanation in Liscow (2018a), p. 1688-91.

to the poor for the smaller amount of transportation spending that they receive. This possibility is consistent with recent research suggesting the importance of transportation for income mobility (Chetty and Hendren, 2018).

Another set of policies that is promising for exhibiting snowballing are those in which wealth alone drives the vicious cycle, since greater wealth tends to increase willingness to pay. Consider, for example, pollution that harms housing values, leading to a tort (Chay and Greenstone, 2005; Currie et al., 2015). To the extent that housing values reflect willingness to pay to avoid pollution, the rich will be able to recover a larger amount than the poor. This would deter other polluters from locating nearby, thereby forcing more polluters onto poor neighborhoods, disproportionately reducing the property values—and therefore—the wealth of the poor. In subsequent periods, the poor would be even further immiserated relative to the rich. Similar analysis applies outside of torts, to the panoply of regulatory areas (e.g., zoning and administrative approvals) in which governments decide on the siting of polluting facilities. And, indeed, many argue that such facilities are disproportionately sited in poor areas (Been, 1993).

Similar analysis could also be applied to eminent domain and redevelopment efforts. Poor people may be more likely to be subjected to eminent domain because their homes are worth less, and eminent domain may in turn result in uncompensated income losses as people's lives are disrupted through displacement.

This analysis can be flipped as well: When cities are analyzing where to build parks, they may look at the economic benefits as reflected by increases in housing values. As a result, richer areas are more likely to have parks built in them, generating increases in property values and thus wealth that would in turn drive greater demand for amenities in the future—and thus yet more spending on amenities in the richer areas.

One could imagine a whole host of other mechanisms: Poorer people are—all else equal—willing to pay less for road safety, effective policing, good hospitals, and communications infrastructure, which could affect willingness to pay in subsequent periods through earnings or wealth. But the point here is not to lay out the range of areas in which such a mechanism could be at play, but rather to suggest that the range could be significant.

At the same time, there are many circumstances in which there would not be state dependence, and, therefore, policy snowballing, because of the lack of a feedback effect. We note here some cases where we would not expect policy snowballing. Doing so helps emphasize that the presence of snowballing is not obvious and that the setting developed here can help shed light on where it occurs. Most basically, there are efficient legal rules that do not disproportionally benefit the rich—what Liscow (2018a) calls "neutral" efficient legal rules. For example, consider a tort in which a polluter causes damages to a laundromat, requiring that the laundromat purchase an air purifier for \$10,000 and thereby causing economic losses of \$10,000. It does not matter whether the laundromat is owned by a poor person or a rich person; the damages are \$10,000. Since there is no bias, there is no policy snowballing.

Likewise, for there to be policy snowballing, there must be a particular kind of feedback loop, in which the legal rule leads to harms to the thing (like earnings) that determines the application of the legal rule in the next period. Consider, for example, a modified case of the main example in the article; here, regulators are deciding where to locate power plants producing pollution that reduces life expectancy but does not reduce one's working life or productivity. In this case, there would be no positive feedback loop because, even as the poor had more reduction in their lifespan because of the pollution, there would not be a disproportionate reduction in the poor's willingness to pay for additional life since their financial resources are constant. If anything, there may be a negative feedback as the years of life for the poor become scarcer and therefore more valuable.

8 Discussion and Conclusion

This article is a proof of possibility for and exploration of how efficient policies can lead to a vicious cycle by harming the earnings of the poor over time solely through the operation of the efficient legal rule. The model has three essential features: First, more of a disamenity like pollution (or, equivalently, less of an amenity) is allocated to the poor because they are willing to pay less than the rich to avoid it. Second, that disproportionate allocation in turn disproportionately reduces the willingness to pay of the poor to avoid the disamenity, setting up state dependence and a feedback loop in the price of harm. These two together lead to what we call "policy snowballing," or spiraling disproportionate reductions in earnings for the poor, leading to more pollution on them, and so on. With complete compensation, policy snowballing does not pose distributional issues. However, with the third feature of the model, incomplete compensation that does not offset the distributional impacts of policy snowballing, income snowballing can result. At least in principle, many legal rules may satisfy these conditions. In any case, it is important to know the size of the taxes and transfers needed to compensate for the distributional impacts of efficient legal rules. And static compensation may miss considerable dynamic harm.

More generally, as the welfare analysis showed, the dynamics improve the case for having earnings-blind legal rules that would thwart the perverse cycle. Analysis of other rules changes too. For example, punitive damages, which have been seen as a way to help those with lower earnings who receive little in torts awards (Finley 2004), look more favorable with these dynamics.

Whether such dynamics are at play in the real world is a question beyond the scope here, though the article does suggest an important mechanism for increasing disparities by income, as well as race and gender (e.g., Yuracko and Avraham, 2018). More research should be done on this question. Theoretical work on other policy settings would be valuable for pinpointing the most credible settings for snowballing, determining what empirical parameters are most important to measure, and developing testable empirical implications. Empirically, qualitative legal research on when in fact efficient legal rules are used in relevant settings would be very valuable. Quantitative empirical work would also be useful. For example, there may be natural experiments available for testing for the presence of snowballing. One possible setting is how changes in state law (or federal Circuit splits) relate to subsequent changes in either the allocation of the amenity or disamenity (for policy snowballing) or income inequality (for income snowballing).

In the meantime, these results raise the stakes of such work, by showing that a common policy goal can have such perverse distributional consequences. It is helpful for policymakers to know the distributional impacts from the adoption of efficient legal rules so that they can appropriately compensate the various parties. And compensating for only the static harm may not nearly compensate for the actual, dynamic harm over time to the poor that results from the policy snowballing demonstrated here.

Finally, this article also opens the door for a research agenda beyond policy snowballing and increasing income inequality. We have demonstrated how legal rules can induce state dependence in the legal costs of exerting harm. One could conceive of legal rules outside of the realm of torts that also beget state dependence as legal rules commonly condition on past behavior. For example, the punishment for traffic violations commonly depends on an individual's past record. The dynamics and consequences of state dependence produced by such legal rules remain to be explored.

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Appendix

For all proofs, we introduce a new object to simplify the presentation of the proofs. Let $G(s_{t-1})$ be the difference in damage payments across the neighborhoods when the share of firms in the low-earnings neighborhood in the previous period was s_{t-1}. That is:

$$G(s_{t-1}) = D_{Ht} - D_{Lt} = (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \theta_3 \overline{e}_H + \theta_3 s_{t-1}(\overline{e}_H + \overline{e}_L).$$

Proof of Result 1: Conditions for Policy Snowballing

To prove Result 1, first we establish that there is always an increase of firms in the low-earnings neighborhood in the first period. Because the share of firms in either neighborhood is evenly split at $t = 0$, i.e. $s_0 = \frac{1}{2}$, there is an increase in the share of firms in the low-earnings neighborhood in the first period when $s_1 > \frac{1}{2}$. Using the law of motion, this means there will be policy snowballing in the first period if $F\left((\overline{e}_H - \overline{e}_L)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)\right) > \frac{1}{2}$. Since the firm preference distribution has median zero by assumption, the condition for policy snowballing in the first period amounts to

$$(\overline{e}_H - \overline{e}_L)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right) > 0.$$

So long as there is a difference in permanent earnings level between the two neighborhoods ($\bar{e}_H > \bar{e}_L$), the term $\theta_1 + \theta_2 - \frac{1}{2}\theta_3$ determines whether or not there is policy snowballing in the first period. Note that the assumptions in (3) imply that $\theta_1 + \theta_2 - \theta_3 > 0$. This inequality and the assumption $\theta_3 \geq 0$ further imply that $\theta_1 + \theta_2 - \frac{1}{2}\theta_3 > 0$. So after the first period, there is always policy snowballing.

Next, we show that if there is an increase in the share of firms in the low-earnings neighborhood at period $t \geq 2$ i.e. $s_t > s_{t-1}$, then it must be that $\theta_3 > 0$. Applying the law of motion (Equation 7) and the expression for the damage difference (Equation 6) to $s_t > s_{t-1}$ implies that:

$$
\begin{aligned}
& F\left(G\left(s_{t-1}\right)\right) > F\left(G\left(s_{t-2}\right)\right) \\
\Longrightarrow\ & G\left(s_{t-1}\right) > G\left(s_{t-2}\right) \\
\Longleftrightarrow\ & \left(\bar{e}_H + \bar{e}_L\right)\theta_3\left(s_{t-1} - s_{t-2}\right) > 0 \\
\Longrightarrow\ & \theta_3 > 0
\end{aligned}
$$

Finally we show that if $\theta_3 > 0$, then there is policy snowballing. We proceed first by induction. We have already demonstrated that $s_1 > s_0$. Now, suppose that $s_{t-1} > s_{t-2}$. We will show that this implies that either $s_t > s_{t-1}$ or, if $s_{t-1} = 1$, then $s_t = 1$. In either case, we have that $\theta_3 > 0 \implies \left(\bar{e}_H + \bar{e}_L\right)\theta_3\left(s_{t-1} - s_{t-2}\right) > 0$. From above, we know that $\left(\bar{e}_H + \bar{e}_L\right)\theta_3\left(s_{t-1} - s_{t-2}\right) > 0 \iff G\left(s_{t-1}\right) > G\left(s_{t-2}\right)$. Consider now the first case, $s_{t-1} < 1$. This means that, because the pdf of F has compact support, $F\left(G\left(s_{t-1}\right)\right) > F\left(G\left(s_{t-2}\right)\right)$. Hence, by the law of motion, $s_t > s_{t-1}$. Now suppose that $s_{t-1} = 1$. Then $G\left(s_{t-1}\right) > G\left(s_{t-2}\right) \implies F\left(G\left(s_{t-1}\right)\right) = F\left(G\left(s_{t-2}\right)\right)$ by the properties of cdfs. So, $s_t = s_{t-1} = 1$. Putting it all together, by the principle of induction we have that for all t where $s_{t-1} > s_{t-2}$, either $s_t > s_{t-1}$ or $s_t = s_{t-1} = 1$. To fully conclude the proof for policy snowballing, note that if $s_{t-1} = s_{t-2} = 1$, then $G\left(s_{t-1}\right) = G\left(s_{t-2}\right) \implies s_t = s_{t-1} = 1$. But since we know that

$s_1 > s_0$ the first t such that $s_t = 1$ must have that $t \geq 2$.

Proof of Result 2: Conditions for Income Snowballing:

Using the definition of income (8) and the formula for earnings at time t from (1) gives that the level of inequality at time $t = 0$ is

$$\begin{aligned} Q_0 = \frac{I_{H0}}{I_{L0}} &= \frac{\left[1 - \frac{1}{2}\theta_1 - \frac{1}{2}\theta_2 + \frac{1}{4}\theta_3\right]\overline{e}_H + \Omega\left(\left(\theta_1 - \frac{1}{2}\theta_3\right)\frac{1}{2} + \frac{1}{2}\theta_2\right)\overline{e}_H}{\left[1 - \frac{1}{2}\theta_1 - \frac{1}{2}\theta_2 + \frac{1}{4}\theta_3\right]\overline{e}_L + \Omega\left(\left(\theta_1 - \frac{1}{2}\theta_3\right)\frac{1}{2} + \frac{1}{2}\theta_2\right)\overline{e}_L} \\ &= \frac{\overline{e}_H}{\overline{e}_L}. \end{aligned}$$

The initial inequality is just the ratio of the permanent earnings. This means that the polluting firms have no effect on income inequality (relative to the no pollution baseline) when half of the firms are located in each neighborhood for at least two consecutive periods.

In subsequent periods, inequality is

$$Q_t = \frac{1 - (1 - \Omega)\left((1 - s_t)\theta_1 + (1 - s_{t-1})\theta_2 - (1 - s_t)(1 - s_{t-1})\theta_3\right)}{1 - (1 - \Omega)\left(s_t\theta_1 + s_{t-1}\theta_2 - s_t s_{t-1}\theta_3\right)} \cdot \frac{\overline{e}_H}{\overline{e}_L}.$$

Note that $\Omega = 1 \implies Q_t = \frac{\overline{e}_H}{\overline{e}_L}$. Hence, $Q_t > Q_{t-1} \implies \Omega < 1$; $\Omega < 1$ is necessary for income snowballing.

Suppose that $\Omega < 1$, $l_1 < 1$, and there is policy snowballing. We will first show that when $s_{t-1} < s_t$, then $Q_{t-1} < Q_t$. We begin by establishing that $s_{t-2} < s_{t-1} < s_t$ implies both that $I_{L,t-1} > I_{Lt}$ (the income of the low-earnings individuals decreases from $t-1$ to t) and $I_{H,t-1} < I_{Ht}$ (the income of the high-earnings individuals decreases from $t-1$ to t). The expression $I_{L,t-1} - I_{Lt}$ can be expressed as:

$$\begin{aligned} I_{L,t-1} - I_{Lt} &= 1 - (1 - \Omega)\left(s_{t-1}\theta_1 + s_{t-2}\theta_2 - s_{t-1}s_{t-2}\theta_3\right) - 1 + (1 - \Omega)\left(s_t\theta_1 + s_{t-1}\theta_2 - s_t s_{t-1}\theta_3\right) \\ &= (1 - \Omega)\left((s_t - s_{t-1})\theta_1 + (s_{t-1} - s_{t-2})\theta_2 - (s_t - s_{t-2})s_{t-1}\theta_3\right) \end{aligned}$$

The $1-\Omega$ is positive since Ω <1. We focus on the other term. Re-arrange it as

$$\left(\frac{(s_t - s_{t-1})}{(s_t - s_{t-2})}\theta_1 + \frac{(s_{t-1} - s_{t-2})}{(s_t - s_{t-2})}\theta_2 - s_{t-1}\theta_3\right)(s_t - s_{t-2}).$$

The term $\frac{(s_t - s_{t-1})}{(s_t - s_{t-2})}\theta_1 + \frac{(s_{t-1} - s_{t-2})}{(s_t - s_{t-2})}\theta_2$ is a convex combination of θ_1 and θ_2. Since both are larger than θ_3 and $s_{t-1}\theta_3 < \theta_3$, we can conclude that $I_{L,t-1} - I_{Lt} > 0$ and hence that the income of the low-earnings individuals decreased from $t-1$ to t.

One can show $I_{H,t-1} < I_{Ht}$ through similar logic. $s_{t-2} < s_{t-1} < s_t$ implies that $1 - s_{t-2} > 1 - s_{t-1} > 1 - s_t$. We can use this to sign $I_{Ht} - I_{Ht-1}$:

$$\begin{aligned}
I_{Ht} - I_{Ht-1} &= 1 - (1-\Omega)\left((1-s_t)\theta_1 + (1-s_{t-1})\theta_2 - (1-s_t)(1-s_{t-1})\theta_3\right) \\
&\quad - 1 + (1-\Omega)\left((1-s_{t-1})\theta_1 + (1-s_{t-2})\theta_2 - (1-s_{t-1})(1-s_{t-2})\theta_3\right) \\
&= (1-\Omega)\left((s_t - s_{t-1})\theta_1 + (s_{t-1} - s_{t-2})\theta_2 - (s_t - s_{t-2})(1-s_{t-1})\theta_3\right) \\
&= (1-\Omega)(s_t - s_{t-2})\left(\frac{(s_t - s_{t-1})}{(s_t - s_{t-2})}\theta_1 + \frac{(s_{t-1} - s_{t-2})}{(s_t - s_{t-2})}\theta_2 - (1-s_{t-1})\theta_3\right) \\
&> 0.
\end{aligned}$$

Knowing that $l_{t-2} < l_{t-1} < l_t \implies I_{L,t-1} > I_{Lt}$ and $I_{H,t-1} < I_{Ht}$ allows us to conclude that $Q_t > Q_{t-1}$, income inequality increased from $t-1$ to t, since $Q_t = \frac{I_{Ht}}{I_{Lt}}$ and income is always positive (since earnings are always at least zero).

Next, we show that $Q_0 < Q_1$. The critical term for signing $I_{L0} - I_{L1}$ is

$$\left(\frac{\left(s_1 - \frac{1}{2}\right)}{\left(s_1 - \frac{1}{2}\right)}\theta_1 + \frac{\left(\frac{1}{2} - \frac{1}{2}\right)}{\left(s_1 - \frac{1}{2}\right)}\theta_2 - \frac{1}{2}\theta_3\right)\left(s_1 - \frac{1}{2}\right) = \left(\theta_1 - \frac{1}{2}\theta_3\right)\left(s_1 - \frac{1}{2}\right).$$

Since $\theta_1 > \theta_3$ and we know that $l_1 > l_0 = \frac{1}{2}$, we can conclude that $I_{L0} > I_{L1}$. Similar reasoning allows us to conclude that $I_{H0} < I_{H1}$ and therefore that $Q_0 < Q_{1.}$. Thus, we have shown that whenever $l_{t-1} < l_t$, then $Q_{t-1} < Q_t$.

To finish the proof, we deal with the possibility that the policy snowballing is of the kind in which for some $\hat{t} \geq 2$, $l_t = 1$ for all $t \geq \hat{t}$. We thus have that $s_{\hat{t}-1} < s_{\hat{t}} = s_{\hat{t}+1} = 1$. To sign $Q_{\hat{t}+1} - Q_{\hat{t}}$, we need to sign $I_{L\hat{t}} - I_{L,\hat{t}+1}$, which depends on the sign of:

$$\left(\frac{(1-1)}{\left(1-s_{\hat{t}-1}\right)}\theta_1 + \frac{\left(1-s_{\hat{t}-1}\right)}{\left(1-s_{\hat{t}-1}\right)}\theta_2 - \theta_3\right)\left(1-s_{\hat{t}-1}\right) = \left(1-s_{\hat{t}-1}\right)(\theta_2 - \theta_3).$$

Since $1 > l_{\hat{t}-1}$ and $\theta_2 > \theta_3$, we can conclude that the income of the low-earnings neighborhood decreased from $\hat{t}$ to $\hat{t}+1$, while the income of those in the high-earnings neighborhood increased. Hence, $Q_{\hat{t}+1} > Q_{\hat{t}}$: inequality also increased. For all $t > \hat{t}+1$, $s_{t-2} = s_{t-1} = s_t = 1$; for those t, income inequality will not be changing.

Proof of Result 3: Expression for the Law of Motion and Proof of Convergence to Steady State.

Applying the CDF of the uniform distribution gives the following form for the law of motion:

$$s_t = \begin{cases} 0 & \text{if } G(s_{t-1}) < -M \cdot (\overline{e}_H + \overline{e}_L) \\ T(s_{t-1}) & \text{if } -M \cdot (\overline{e}_H + \overline{e}_L) \leq G(s_{t-1}) \leq M \cdot (\overline{e}_H + \overline{e}_L) \\ 1 & \text{if } G(s_{t-1}) > M \cdot (\overline{e}_H + \overline{e}_L) \end{cases} \tag{11}$$

where

$$T(s_{t-1}) = \frac{1}{2} + \frac{(\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H \theta_3}{2M(\overline{e}_H + \overline{e}_L)} + \frac{\theta_3}{2M} s_{t-1}. \tag{12}$$

Let s^* denote the steady state. Let $\hat{s}$ be the value such that $T(\hat{s}) = \hat{s}$. This proof will proceed in multiple steps. First, we will argue that $\hat{s}$ such that $T(\hat{s}) = \hat{s}$ exists and is unique when $\theta \neq 2M$. Second, we will show that $\hat{s} > \frac{1}{2}$ implies that $\theta_3 < 2M$. Third, we will use last fact to show that if $\hat{s} \in \left(\frac{1}{2}, 1\right)$, then $\hat{s} = s^*$. Fourth, we will show that if $\hat{s} > 1$,then $s^* = 1$. Fifth, we end by showing that because $\hat{s} < \frac{1}{2}$ implies that $\theta_3 > 2M$, then $s^* = 1$.

1) Existence and Uniqueness of $\hat{s}$: Because $\theta \neq 2M$, $T(x) = \frac{1}{2} + \frac{(\bar{e}_H - \bar{e}_L)(\theta_1+\theta_2) - \bar{e}_H\theta_3}{2M(\bar{e}_H+\bar{e}_L)} + \frac{\theta_3}{2M}x$ is a line with slope not equal to one. Therefore, it will intersect with $f(x) = x$ at one and only one point, meaning that an $\hat{s}$ such that $T(\hat{s}) = \hat{s}$ exists and is unique. Specifically, the expression for $\hat{s}$ is

$$\hat{s} = \frac{M(\bar{e}_H + \bar{e}_L) + (\bar{e}_H - \bar{e}_L)(\theta_1 + \theta_2) - \bar{e}_H\theta_3}{(2M - \theta_3)(\bar{e}_H + e_L)}.$$

2) $\hat{s} > \frac{1}{2} \implies \theta_3 < 2M$: Using the expression from above, if $\hat{s} > \frac{1}{2}$ then

$$\begin{aligned}
& \frac{M(\bar{e}_H + \bar{e}_L) + (\bar{e}_H - \bar{e}_L)(\theta_1 + \theta_2) - \bar{e}_H\theta_3}{(2M - \theta_3)(\bar{e}_H + \bar{e}_L)} > \frac{1}{2} \\
\implies \quad & \frac{M(\bar{e}_H + \bar{e}_L) + (\bar{e}_H - \bar{e}_L)(\theta_1 + \theta_2) - \bar{e}_H\theta_3}{(2M - \theta_3)(\bar{e}_H + \bar{e}_L)} - \frac{1}{2} > 0 \\
\implies \quad & \frac{2(\bar{e}_H - \bar{e}_L)(\theta_1 + \theta_2) - \theta_3(\bar{e}_H - \bar{e}_L)}{2(2M - \theta_3)(\bar{e}_H + \bar{e}_L)} > 0 \\
\implies \quad & \frac{2(\bar{e}_H - \bar{e}_L)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{2(2M - \theta_3)(\bar{e}_H + \bar{e}_L)} > 0 \\
\implies \quad & 2M - \theta_3 > 0.
\end{aligned}$$

All but the last of lines above follow from simple algebra. The last line follows from the constraint $\theta_1 + \theta_2 - \theta_3 > 0$. That constraint implies that $\theta_1 + \theta_2 - \frac{1}{2}\theta_3 > 0$. So, in the second-to-last line, for the left-hand expression to be positive, it must be that $2M - \theta_3 > 0$. Thus $\hat{s} > \frac{1}{2} \implies \theta_3 < 2M$.

3) $\hat{s} \in \left(\frac{1}{2}, 1\right) \implies \hat{s} = s^*$: To show this result, we first argue that the linear first-order difference equation $s_t = T(s_{t-1}) = \frac{1}{2} + \frac{(\bar{e}_H - \bar{e}_L)(\theta_1+\theta_2) - \bar{e}_H\theta_3}{2M(\bar{e}_H+\bar{e}_L)} + \frac{\theta_3}{2M}s_{t-1}$ has a unique, globally stable steady state. Formally, a steady state $\widehat{s}$ of a first-order difference equation is globally stable if for all possible initial conditions, $s_0 \in (-\infty, \infty)$, the associated sequence s_t induced by the first-order difference equation system converges to $\hat{s}$. The steady state is globally unique because the linear process has slope not equal to one. By the second result in this proof, we know that $\theta_3 < 2M$. This means that the slope of $s_t = T(s_{t-1})$ is less than one. By theorems from the mathematics of linear difference equations, this means that the steady

state $\hat{s}$ is globally stable.[19] In particular, we know that from a starting point $s_0 = \frac{1}{2}$, the first-order difference system $s_t = T(s_{t-1})$ will produce a sequence that converges to $\hat{s}$.

All that is left to show is that because $s_t = T(s_{t-1})$ converges to $\hat{s}$, the first-order difference equation described in (11) must also converge to $\hat{s}$. On the interval $(0, 1)$, the two systems are identical. All we have to do is show that an edge case of $s_t = 0$ or $s_t = 1$ never occurs. This follows from Result 1 because it implies that s_t must be strictly increasing. Because $s_0 > \frac{1}{2}$ and $\hat{s} < 1$, this means that $s_t \in (\frac{1}{2}, 1)$ and so an edge case is never reached. Therefore, the steady state of the difference equation (11) is equal that of $s_t = T(s_{t-1})$ i.e $\hat{s} = s^*$.

4) $\hat{s} \geq 1 \implies s^* = 1$: Using the logic from the proof of part 3, if $\hat{s} \geq 1$, then the sequence produced by $s_0 = \frac{1}{2}$ and the linear first-order difference equation (11) is strictly increasing until $s_t = 1$. At that point, the share of firms can no longer change and the sequence remains there for all future time periods. Hence, $s^* = 1$.

5) $\hat{s} < \frac{1}{2} \implies s^* = 1$: Using the same argument as in part 2 of this proof, if $\hat{s} < \frac{1}{2}$ then it must be that $2M < \theta_3$. Thus, the slope of $s_t = T\left(s_{t-1}\right)$ will be greater than one in absolute value. By the theorems of first-order difference equations, this means that the steady state $\hat{s}$ will be unstable; all sequences with starting points $s_0 \neq \hat{s}$ will move away from the steady state. Since $\hat{s} < \frac{1}{2}$ and $s_0 = \frac{1}{2}$, this means that once again the sequence s_t produced by first-order difference equation (11) will be strictly increasing until it hits the edge case $s_t = 1$, where it will subsequently remain. Thus, $\hat{s} < \frac{1}{2} \implies s^* = 1$.

Expressions for Comparative Statics

The expression for the steady-state share of firms in the low-earnings neighborhood is

$$s^* = \frac{M\left(\bar{e}_H + \bar{e}_L\right) + \left(\bar{e}_H - \bar{e}_L\right)\left(\theta_1 + \theta_2\right) - \bar{e}_H\theta_3}{\left(2M - \theta_3\right)\left(\bar{e}_H + \bar{e}_L\right)}.$$

[19] For example, see Theorem 2.2 in Acemoglu (2009).

The partial derivatives of s^* are

$$
\begin{aligned}
\frac{\partial s^*}{\partial \theta_1} &= \frac{(\overline{e}_H - \overline{e}_L)}{(2M - \theta_3)(\overline{e}_H + \overline{e}_L)} \\
\frac{\partial s^*}{\partial \theta_2} &= \frac{(\overline{e}_H - \overline{e}_L)}{(2M - \theta_3)(\overline{e}_H + \overline{e}_L)} \\
\frac{\partial s^*}{\partial M} &= -\frac{2(\overline{e}_H - \overline{e}_L)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{(2M - \theta_3)^2(\overline{e}_H + \overline{e}_L)} \\
\frac{\partial s^*}{\partial \overline{e}_H} &= \frac{2\overline{e}_L\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{(2M - \theta_3)(\overline{e}_H + \overline{e}_L)^2} \\
\frac{\partial s^*}{\partial \overline{e}_L} &= -\frac{2\overline{e}_H\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{(2M - \theta_3)(\overline{e}_H + \overline{e}_L)^2} \\
\frac{\partial s^*}{\partial \theta_3} &= \frac{(\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2 - M)}{(2M - \theta_3)^2(\overline{e}_H + \overline{e}_L)}.
\end{aligned}
$$

Proof of Result 4: Speed of Convergence

We solve for the speed of snowballing in terms of model primitives using the equation for l^* when it is an interior solution and the law of motion.

$$
\begin{aligned}
\frac{s_t - s^*}{s_{t+1} - s^*} &= \frac{s_t - \frac{M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H\theta_3}{(2M - \theta_3)(\overline{e}_H + \overline{e}_L)}}{\frac{M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_h\theta_3 + \theta_3(\overline{e}_H + \overline{e}_L)l_t}{2M(\overline{e}_H + \overline{e}_L)} - \frac{M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_h\theta_3}{(2M - \theta_3)(\overline{e}_H + \overline{e}_L)}} \\
&= \frac{(\overline{e}_H + \overline{e}_L)s_t - \frac{M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H\theta_3}{(2M - \theta_3)}}{\frac{M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_h\theta_3 + \theta_3(\overline{e}_H + \overline{e}_L)s_t}{2M} - \frac{M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H\theta_3}{(2M - \theta_3)}} \\
&= \frac{2M(2M - \theta_3)(\overline{e}_H + \overline{e}_L)s_t - 2M\left[M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H\theta_3\right]}{(2M - \theta_3)\theta_3(\overline{e}_H + \overline{e}_L)s_t - \theta_3\left[M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H\theta_3\right]} \\
&= \frac{2M\left[(2M - \theta_3)(\overline{e}_H + \overline{e}_L)s_t - \left[M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H\theta_3\right]\right]}{\theta_3\left[(2M - \theta_3)(\overline{e}_H + \overline{e}_L)s_t - \left[M(\overline{e}_H + \overline{e}_L) + (\overline{e}_H - \overline{e}_L)(\theta_1 + \theta_2) - \overline{e}_H\theta_3\right]\right]} \\
&= \frac{2M}{\theta_3}
\end{aligned}
$$

Proof of Result 5: Expressions for Lump-Sum Transfers and the Transfer Gap

The legislature selects the transfer in the first period, $\tau_{1,s}$ to satisfy the equation

$$\frac{I_{H1}\left(\frac{1}{2}, 1-s_1\right)-\tau_{1,s}}{I_{L1}\left(\frac{1}{2}, s_1\right)+\tau_{1,s}}=\frac{\overline{e}_H}{\overline{e}_L}.$$

Here, the I_{N1} functions come from Equation 8 and reflect the fact that in the previous period the share of firms in each neighborhood was $\frac{1}{2}$. $\tau_{1,s}$ returns the level of income inequality to its original level, $\frac{\overline{e}_H}{\overline{e}_L}$.

The myopic legislature picks τ_s in the second period and beyond to satisfy

$$\frac{I_{Ht}\left(1-s_1, 1-s_1\right)-\tau_s}{I_{Lt}\left(s_1, s_1\right)+\tau_s}=\frac{\overline{e}_H}{\overline{e}_L}.$$

Here, the legislature assumes that the share of firms in the low-earnings neighborhood will remain at s_1 for the rest of time. Operating under that assumption, it chooses the lump-sum tax-and-transfer scheme to return the world to its original level of inequality.

The steady-state level transfer required to return the world to its original level of inequality is τ_d and solves:

$$\frac{I_{Ht}\left(1-s^*, 1-s^*\right)-\tau_d}{I_{Lt}\left(s^*, s^*\right)+\tau_d}=\frac{\overline{e}_H}{\overline{e}_L}.$$

We begin calculating the transfer gap by first calculating the ratio of myopic transfer to the steady-state transfer:

$$\begin{aligned}
\frac{\tau_s}{\tau_d} &= \frac{\left[I_{Ht}\left(1-s_1, 1-s_1\right)-\frac{\overline{e}_H}{\overline{e}_L} I_{Lt}\left(s_1, s_1\right)\right]\left(1+\frac{\overline{e}_H}{\overline{e}_L}\right)^{-1}}{\left[I_{Ht}\left(1-s^*, 1-s^*\right)-\frac{\overline{e}_H}{\overline{e}_L} I_{Lt}\left(s^*, s^*\right)\right]\left(1+\frac{\overline{e}_H}{\overline{e}_L}\right)^{-1}} \\
&= \frac{\left(1-2s_1\right)\left(\theta_1+\theta_2-\theta_3\right)(1-\Omega)}{\left(1-2s^*\right)\left(\theta_1+\theta_2-\theta_3\right)(1-\Omega)} \\
&= \frac{2s_1-1}{2s^*-1}.
\end{aligned}$$

From the expression for s^* from Result 3, which we can freely apply since we assume $s^* < 1$, we obtain an expression for the denominator:

$$\begin{aligned} 2s^* - 1 &= \frac{2M\left(\overline{e}_H + \overline{e}_L\right) + 2\left(\overline{e}_H - \overline{e}_L\right)\left(\theta_1 + \theta_2\right) - 2\overline{e}_H\theta_3}{\left(2M - \theta_3\right)\left(\overline{e}_H + \overline{e}_L\right)} - \frac{\left(2M - \theta_3\right)\left(\overline{e}_H + \overline{e}_L\right)}{\left(2M - \theta_3\right)\left(\overline{e}_H + \overline{e}_L\right)} \\ &= \frac{2\left(\overline{e}_H - \overline{e}_L\right)\left(\theta_1 + \theta_2\right) - 2\overline{e}_H\theta_3 + \theta_3\left(\overline{e}_H + \overline{e}_L\right)}{\left(2M - \theta_3\right)\left(\overline{e}_H + \overline{e}_L\right)}. \end{aligned}$$

From the law of motion, we obtain the expression for the numerator:

$$\begin{aligned} 2s_1 - 1 &= 2\left(\frac{1}{2} + \frac{\left(\overline{e}_H - \overline{e}_L\right)\left(\theta_1 + \theta_2\right) - \overline{e}_H\theta_3}{2M\left(\overline{e}_H + \overline{e}_L\right)} + \frac{\theta_3}{2M}s_0\right) - 1 \\ &= \frac{2\left(\overline{e}_H - \overline{e}_L\right)\left(\theta_1 + \theta_2\right) - 2\overline{e}_H\theta_3}{2M\left(\overline{e}_H + \overline{e}_L\right)} + \frac{\theta_3}{2M} \\ &= \frac{2\left(\overline{e}_H - \overline{e}_L\right)\left(\theta_1 + \theta_2\right) - 2\overline{e}_H\theta_3 + \theta_3\left(\overline{e}_H + \overline{e}_L\right)}{2M\left(\overline{e}_H + \overline{e}_L\right)}. \end{aligned}$$

Combining the two expression gives

$$\begin{aligned} \frac{\tau_s}{\tau_d} &= 1 - \frac{\theta_3}{2M} \\ \Longrightarrow \quad \frac{\tau_d - \tau_s}{\tau_d} &= \frac{\theta_3}{2M} \end{aligned}$$

Proof of Result 6: Social Planner's Preference for the Efficient Regime Over the Inefficient Regime

The income in the low-earnings neighborhood under the inefficient regime is

$$I_{L0} = \left[1 - (1 - \Omega)\left(\frac{1}{2}\theta_1 + \frac{1}{2}\theta_2 - \frac{1}{4}\theta_3\right)\right]\overline{e}_L.$$

Under the strict liability regime without transfers, the steady-state income level in the low-earnings neighborhood [20] is

$$I_L^* = \left[1 - (1 - \Omega)\left(s^*\theta_1 + s^*\theta_2 - (s^*)^2\,\theta_3\right)\right]\bar{e}_L.$$

The welfare weight $\hat{g}$ that makes the social planner indifferent between the inefficient regime and the efficient regime without transfers satisfies

$$\hat{g}_L u\left(I_{L0}\right) + u\left(I_{H0}\right) = \hat{g}_L u\left(I_L^*\right) + u\left(I_H^*\right).$$

Re-arranging gives:

$$\hat{g}_L = \frac{u\left(I_H^*\right) - u\left(I_{H0}\right)}{u\left(I_{L0}\right) - u\left(I_L^*\right)}.$$

Assuming that utility is linear in income. Let u' be the slope of utility with respect to income. Then the condition becomes:

$$\hat{g}_L = \frac{u' \cdot \left(I_H^* - I_{H0}\right)}{u' \cdot \left(I_{L0} - I_L^*\right)} = \frac{I_H^* - I_{H0}}{I_{L0} - I_L^*}.$$

Plugging in the corresponding values for the incomes gives

$$\hat{g}_L = \frac{(1-\Omega)\left[\theta_1 + \theta_2 - (\frac{3}{2} - s^*)\theta_3\right]\bar{e}_H}{(1-\Omega)\left[\theta_1 + \theta_2 - (s^* + \frac{1}{2})\theta_3\right]\bar{e}_L}.$$

This proves the first claim of the result.

Next, note that following the logic from above, the social planner prefers the efficient regime when she has welfare weight g_L such that

$$g_L < \frac{\left[\theta_1 + \theta_2 - (\frac{3}{2} - s^*)\theta_3\right]}{\left[\theta_1 + \theta_2 - (s^* + \frac{1}{2})\theta_3\right]}\frac{\bar{e}_H}{\bar{e}_L}.$$

[20] The analogous equation for the high-earnings neighborhood can be obtained by substituting "$1 - s$" for "s"

Since $s^* > \frac{1}{2}$, then it must be that

$$1 < \frac{\left[\theta_1 + \theta_2 - (\frac{3}{2} - s^*)\theta_3\right]}{\left[\theta_1 + \theta_2 - (s^* + \frac{1}{2})\theta_3\right]}.$$

So, if

$$g_L < \frac{\overline{e}_H}{\overline{e}_L},$$

then it must follow that $g_L < \frac{\left[\theta_1+\theta_2-(\frac{3}{2}-s^*)\theta_3\right]}{\left[\theta_1+\theta_2-(s^*+\frac{1}{2})\theta_3\right]}\frac{\overline{e}_H}{\overline{e}_L}$; and the social planner would prefer the efficient regime regardless of the value of s^*.

For the last result, observe that the indifference expression $\frac{\left[\theta_1+\theta_2-(\frac{3}{2}-s^*)\theta_3\right]}{\left[\theta_1+\theta_2-(s^*+\frac{1}{2})\theta_3\right]}\frac{\overline{e}_H}{\overline{e}_L}$ is increasing in s^*. Because $s^* \leq 1$, if the welfare weight satisfies

$$g_L > \frac{\left[\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right]}{\left[\theta_1 + \theta_2 - \frac{3}{2}\theta_3\right]}\frac{\overline{e}_H}{\overline{e}_L}$$

then regardless of the steady-state share of firms in the low-earnings neighborhood the social planner will always prefer the inefficient regime without transfers to the inefficient regime.

Result 7: Comparing the Different Regimes

Recall from above that the weight that makes the social planner indifferent between the inefficient regime and the efficient regime without transfer is

$$\hat{g}_{no-transfer} = \frac{I_H^* - I_{H0}}{I_{L0} - I_L^*}.$$

The income expression with the transfer simply adds the transfer to the income of the low-earnings individuals and subtracts it from the income of the high-earnings people. So, the corresponding expression for the welfare weight that makes the social planner indifferent between the inefficient regime and the efficient regime with the myopic transfer is

$$\hat{g}_{myopic} = \frac{I_H^* - \tau_s - I_{H0}}{I_{L0} - I_L^* - \tau_s}.$$

The assumption that $\hat{g}_{myopic}$ exists means the low-earnings do not make a higher income in the steady state with the myopic transfer than in the inefficient regime. Hence, $I_{L0} - I_L^* - \tau_s > 0$. Now, evaluate the difference between the two:

$$\begin{aligned}
\hat{g}_{myopic} - \hat{g}_{no-transfer} &= \frac{\left(I_H^* - \tau_s - I_{H0}\right)\left(I_{L0} - I_L^*\right)}{\left(I_{L0} - I_L^* - \tau_s\right)\left(I_{L0} - I_L^*\right)} - \frac{\left(I_H^* - I_{H0}\right)\left(I_{L0} - I_L^* - \tau_s\right)}{\left(I_{L0} - I_L^*\right)\left(I_{L0} - I_L^* - \tau_s\right)} \\
&= \frac{\tau_s\left(I_H^* - I_{H0}\right) - \tau_s\left(I_{L0} - I_L^*\right)}{\left(I_{L0} - I_L^* - \tau_s\right)\left(I_{L0} - I_L^*\right)} \\
&= \frac{\tau_s\left(1-\Omega\right)\left(l^* - \frac{1}{2}\right)\left(\left[\theta_1 + \theta_2 - (\frac{3}{2} - s^*)\theta_3\right]\bar{e}_H - \left[\theta_1 + \theta_2 - (s^* + \frac{1}{2})\theta_3\right]\bar{e}_L\right)}{\left(I_{L0} - I_L^* - \tau_s\right)\left(I_{L0} - I_L^*\right)} \\
&= \frac{\tau_s\left(1-\Omega\right)\left(s^* - \frac{1}{2}\right)\left(\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)\left(\bar{e}_H - \bar{e}_L\right) - \bar{e}_H\theta_3 + s^*\left(\bar{e}_H + \bar{e}_L\right)\theta_3\right)}{\left(I_{L0} - I_L^* - \tau_s\right)\left(I_{L0} - I_L^*\right)}.
\end{aligned}$$

Note that the expression in the last line is increasing in s^*. Because $s^* > \frac{1}{2}$, this means that if the expression $\left(\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)\left(\bar{e}_H - \bar{e}_L\right) - \bar{e}_H\theta_3 + s^*\left(\bar{e}_H + \bar{e}_L\right)\theta_3\right)$ is greater than 0 when evaluated at $s^* = \frac{1}{2}$, then it is always greater than 0. Evaluated at $\frac{1}{2}$, this expression is $\left(\theta_1 + \theta_2 - \theta_3\right)\left(\bar{e}_H - \bar{e}_L\right)$, which is greater than 0 since $\theta_1 > \theta_3$. Thus, $\hat{g}_{myopic} - \hat{g}_{no-transfer} > 0$.