* Uniform circular motion

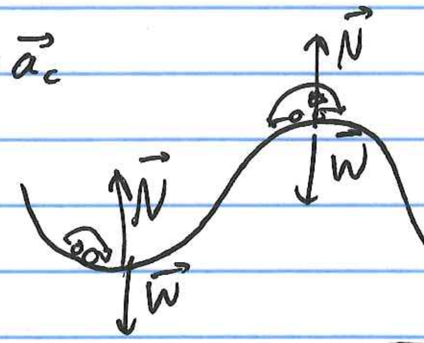
speed $|\vec{v}|$ is constant, but direction of $\vec{v}$ varies

↳ centripetal acceleration $\vec{a}_c = \frac{v^2}{r}$ towards center of rotation

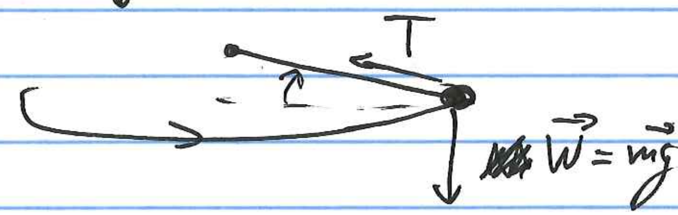
Acceleration must be caused by a net force:

$$\vec{F}_{net,c} = m\vec{a}_c$$

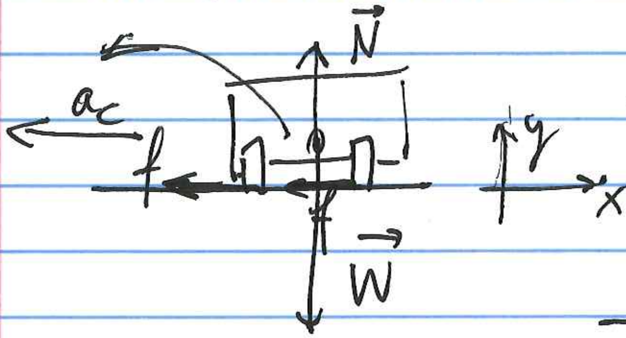
Examples: - normal



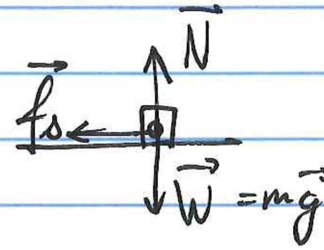
- tension in a string

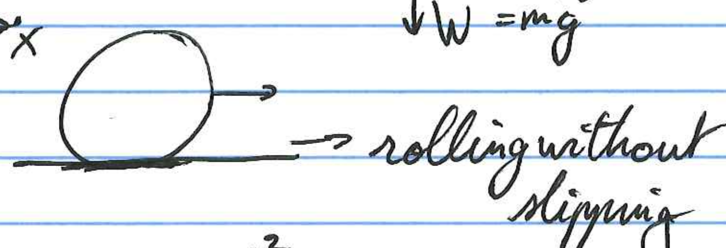


* Car on an unbanked curve



FBD :





$$\begin{cases} x: -f_s = -ma_c = -m\frac{v^2}{r} \\ y: N - mg = 0 \end{cases}$$

↳ $N = mg$

$$f_s = \cancel{m}\frac{v^2}{r} \leq \underbrace{\mu_s N}_{f_{s,max}} = \mu_s \cancel{m} g$$

$$\frac{v^2}{r} \leq \mu_s g$$

$$\mu_s \geq \frac{v^2}{rg}$$

$$\boxed{\mu_{s,min} = \frac{v^2}{rg}}$$

$$v^2 \leq \underline{\mu_s r g}$$

$$v_{max} = \sqrt{\mu_s r g}$$

$$a_c = \frac{v^2}{r}$$

$v \to \frac{1}{2}v$ then $a_c \to \frac{1}{4}a_c$

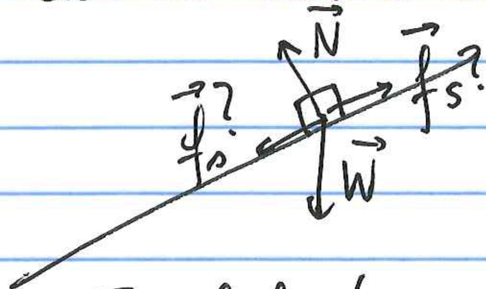
$F_{net,c} \to \frac{1}{4}F_{net,c}$

If $v \to 2v$ then μ_s must be 4 times as large

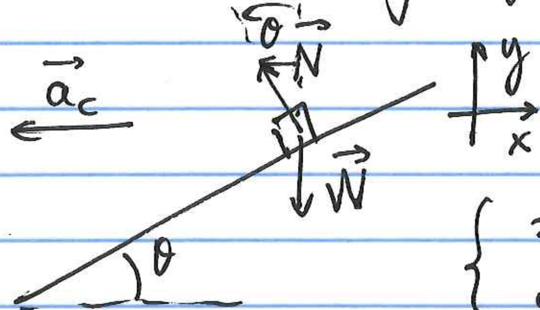
If $r \to \frac{1}{2}r$ then μ_s must be twice as large

$$\vec{F}_{net,c} = \underline{m}\vec{a}_c$$

* Cars on banked curve



Ideal banking angle (for a certain r, and v)



$a_c = \frac{v^2}{r}$

$$\begin{cases} x: -ma_c = -m\frac{v^2}{r} = -N\sin\theta \\ y: 0 = N\cos\theta - mg \end{cases}$$

$\hookrightarrow N = \frac{mg}{\cos\theta}$

$$-m\frac{v^2}{r} = -\left(\frac{mg}{\cos\theta}\right)\sin\theta = -mg\tan\theta$$

$$\tan\theta = \frac{v^2}{rg} \rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$v = 20\,m/s \approx 50\,mph$
$r = 40\,m$

$$\theta = \tan^{-1}\left(\frac{(20)^2}{(40)(10)}\right) = \tan^{-1}(1) = 45°$$

* Gravitational force

Moon in circular trajectory around Earth

$$a_c = \frac{v^2}{r} = r\,\omega^2$$

$$r = 384000 \text{ km} = 3.84 \times 10^8 \text{ m}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{27.3 \text{ day}} = \frac{2\pi}{(27.3 \text{day})\left(\frac{24 \text{ hrs}}{1 \text{ day}}\right)\left(\frac{3600 \text{ s}}{1 \text{ hr}}\right)}$$

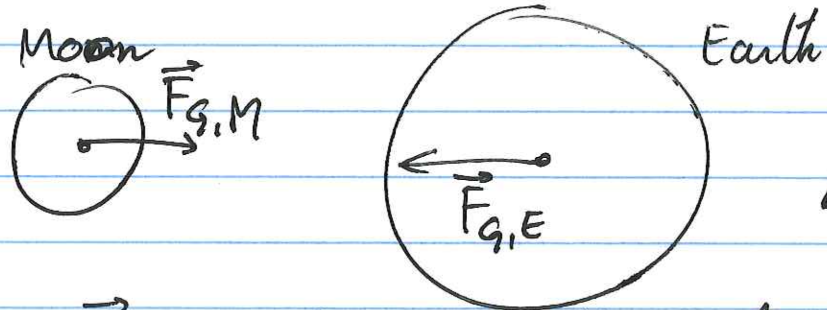
$$= 2.66 \times 10^{-6} \left(\frac{\text{rad}}{\text{s}}\right) \approx \frac{1}{\text{s}}$$

$$\underline{a_c = 2.72 \times 10^{-3} \text{ m/s}^2}$$

this caused by a force : $\vec{F}_G = m_{moon}\,\vec{a}_c$

$$F_G = G \frac{M_{Earth} M_{moon}}{r^2}$$

with $G = \underline{6.673 \times 10^{-11}} \frac{\text{N m}^2}{\text{kg}^2}$



~~Earth Moon~~

$M_{Earth} = 5.98 \times 10^{24}$ kg

$$\vec{F}_{G,M} = -\vec{F}_{G,E} = G \frac{M_{Earth} M_{Moon}}{r^2}$$

$$a_c = \frac{F_{G,M}}{m_{Moon}} = G \frac{M_{Earth}}{r^2} = 2.70 \times 10^{-3} \text{ m/s}^2$$