PHYS 107 - Week 06 - Wednesday

* Midterm exam 1: evaluation

* Gravitational force and uniform circular motion

$$\vec{a}_c = r\omega^2 \rightarrow F_c = F_g = G\frac{M_1 M_2}{r^2} = ma_c$$

towards center of rotation

a_c of mass m on surface of earth:

$$a_c = \frac{F_c}{m} = G\frac{M_{Earth}}{r^2_{Earth}}$$

with $G = 6.673 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

$M_{Earth} = 5.98 \times 10^{24}\ kg$

$r_{Earth} = 6380\ km$

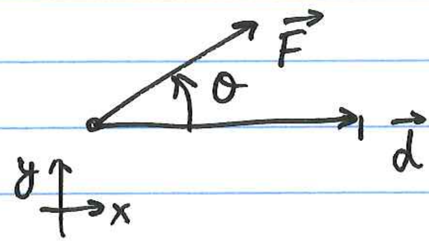
$\downarrow$

$a_c = 9.80\ m/s^2 = g$!

* Work and Energy

Q 1 sliding mass

Work = force times distance of motion <u>parallel</u> to the force



$\vec{d}$ = displacement
$\vec{F}$ = force

$$W = |\vec{F}| \cdot |\vec{d}| \cdot \cos\theta$$

θ = angle between force and displacement

Also: $W = F_x\, d$ = force component along the direction of motion

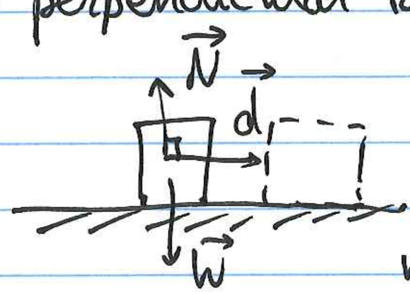
$$F_x = |\vec{F}| \cos\theta$$

$W = F\, d_F$ = distance component along direction of force

$$d_F = |\vec{d}| \cos\theta$$

If $\vec{F} \perp \vec{d} \rightarrow W = 0$ because $\theta = 90°$ and $\cos\theta = 0$

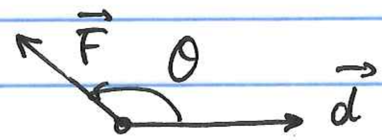
↳ perpendicular to displacement → no work is done



no work is done by normal forces (NEVER!)

no work is done by weight (IN THIS CASE)

What happens if $\theta > 90° \rightarrow W < 0$



Intuitively: $W > 0$: energy is being added to the system by the force
work is done on the system

$W < 0$: energy is removed from the system by the force
work is done by the system

$W = 0$: energy of system is unchanged

* Units of work and energy

$W = Fd\cos\theta \rightarrow$ units of force $\times$ length

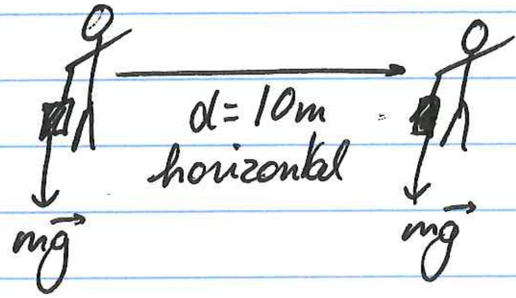
$1\ N \cdot m = 1\ J$, Joule

(other units of energy : $1\ cal = 4.186\ J$
$1\ Cal = 1\ kcal$)

(Granola bar nutritional information)

Examples of work done by gravity on $\underline{m = 10\,kg}$

1)

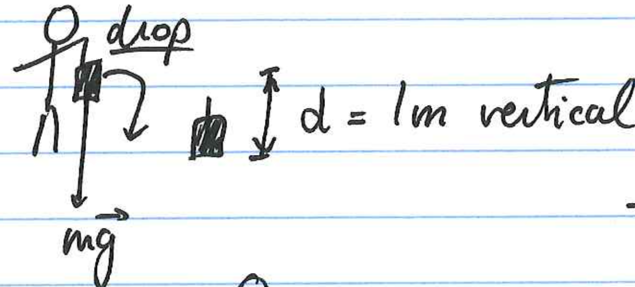


$W = mg\,d\cos\theta$

$\theta = 90°$

$\rightarrow W = 0$

2)



$W = mg\,d\cos\theta$

$\theta = 0°$

$\rightarrow W \cong 100\,J$

Q 2-a, b, c

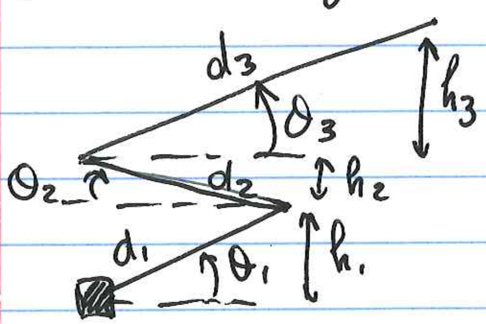
3)

$W = mg\,d\cos\theta$

$\theta = 180°$

$\rightarrow W \cong -100\,J$

4) at an angle → only vertical distance (height) counts

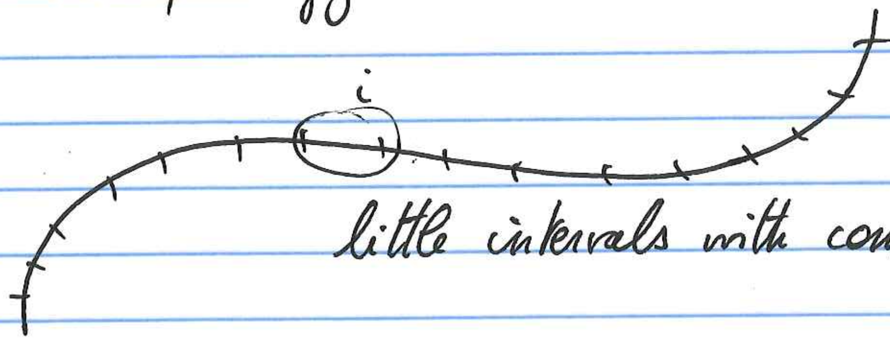


$$W = mg\left(d_1\cos\theta + d_2\cos\theta_2 + d_3\cos\theta_3\right)$$

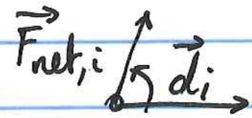
$$= mg\left(h_1 + h_2 + h_3\right)$$

$$W_{tot} = mg\,h$$

* Work / Energy theorem



little intervals with constant force $\vec{F}_{net,i}$



$\rightarrow \quad W_i = F_{net,i}\, d_i \cos\theta_i$

$$W_{tot} = \sum W_i = \sum F_{net,i}\, d_i \cos\theta_i$$

Take x along $\vec{d}_i$ for interval i : $d_y = 0$

$$\hookrightarrow \begin{cases} v_x^2 = v_{x,0}^2 + 2a_x d_x \\ v_y^2 = v_{y,0}^2 + 2a_y \cancel{d_y} \end{cases} \quad d_y = 0$$

$$\hookrightarrow \begin{cases} v_x^2 \cancel{m} = v_{x,0}^2 + 2\,\dfrac{F_{net,i}}{m}\, d_x \\ v_y^2 = v_{y,0}^2 \end{cases}$$

$$\hookrightarrow v^2 = v_x^2 + v_y^2 = v_0^2 + \frac{2}{m} W_i$$

$\hookrightarrow$ for a longer trajectory

$$W_{net} = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$$

Kinetic energy $= \frac{1}{2} m v^2$