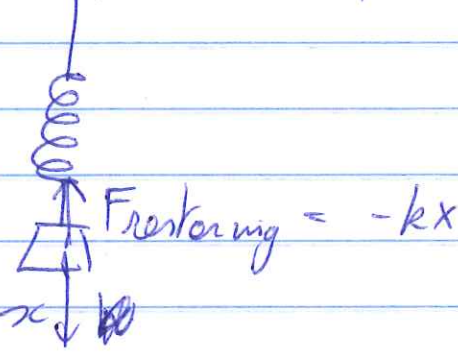
* Simple harmonic motion :

any osillation that follows Hooke's law

equilibrium → displacements, restorings force

mass & spring



pendulum, clamped ruler

A = amplitude = maximum displacement from equilibrium

f = frequence = $\frac{1}{\text{period}} = \frac{1}{T}$

independent of A for SHM

*Mass & spring

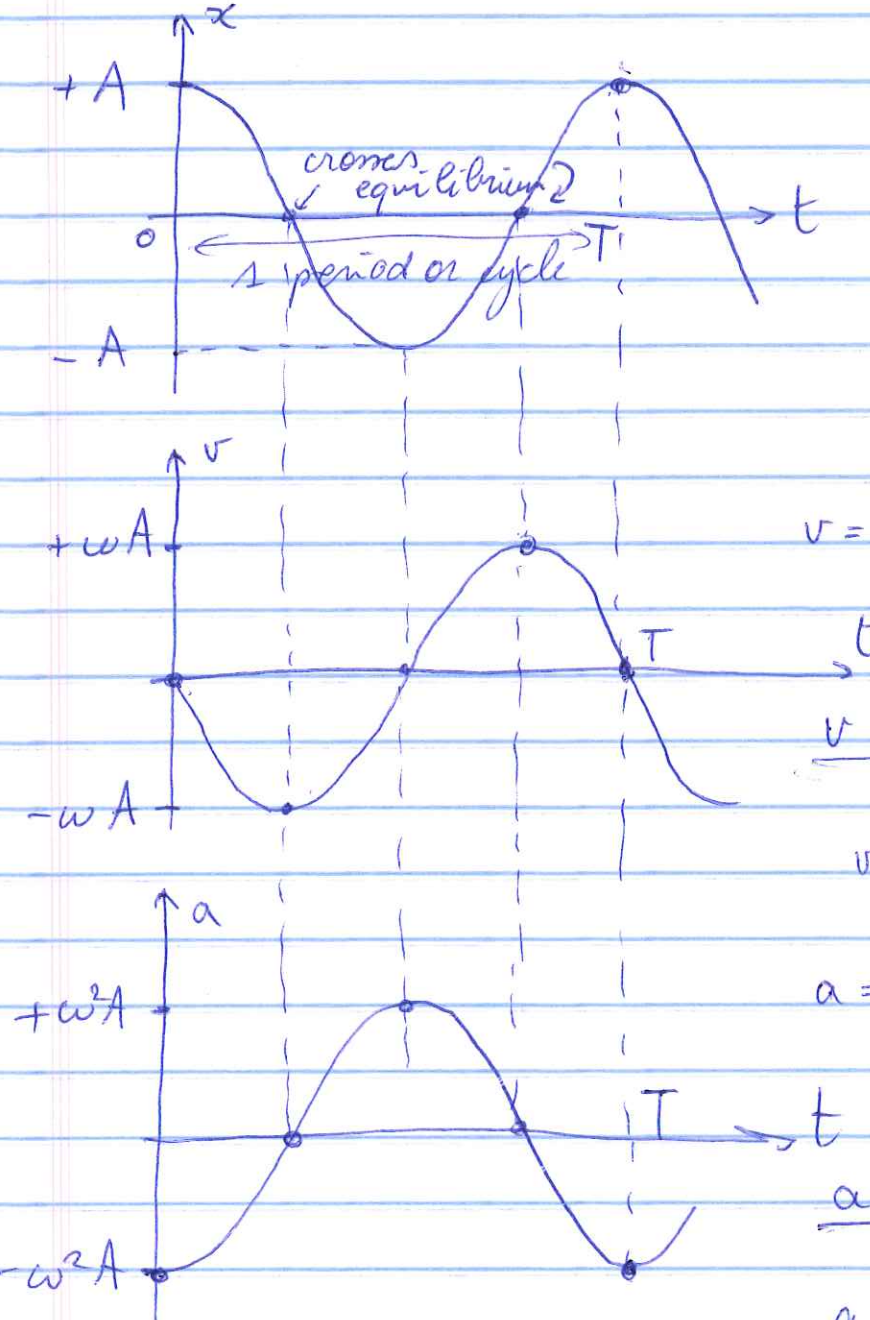
$$T = 2\pi\sqrt{\frac{m}{k}} \quad (s)$$

$$\downarrow$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad (Hz)$$

$$\downarrow$$

$$\omega = 2\pi f = \sqrt{\frac{k}{m}} \quad (rad/s)$$



$x = A\cos(\omega t)$

$\text{rad/s} \cdot \text{s} = \text{rad}$

$x = A\cos(2\pi f t)$

$x = A\cos\left(2\pi \frac{t}{T}\right)$

v = slope of x curve versus t

$v = -\omega A \sin(\omega t)$

$v_{max} = \omega A = \sqrt{\frac{k}{m}}\, A$

a = slope of v curve versus t

$a = -\omega^2 A \cos(\omega t)$

$a_{max} = \omega^2 A$

$F = ma$?

$F = -kx = -kA\cos(\omega t)$

$$\begin{aligned} ma &= m(-\omega^2 A\cos(\omega t)) \\ &= -\omega^2 m A\cos(\omega t) \\ &= -\frac{k}{m} m A\cos(\omega t) \\ &= -kA\cos(\omega t) \end{aligned}$$

$\omega^2 = \frac{k}{m}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

* Energy conservation in SHM

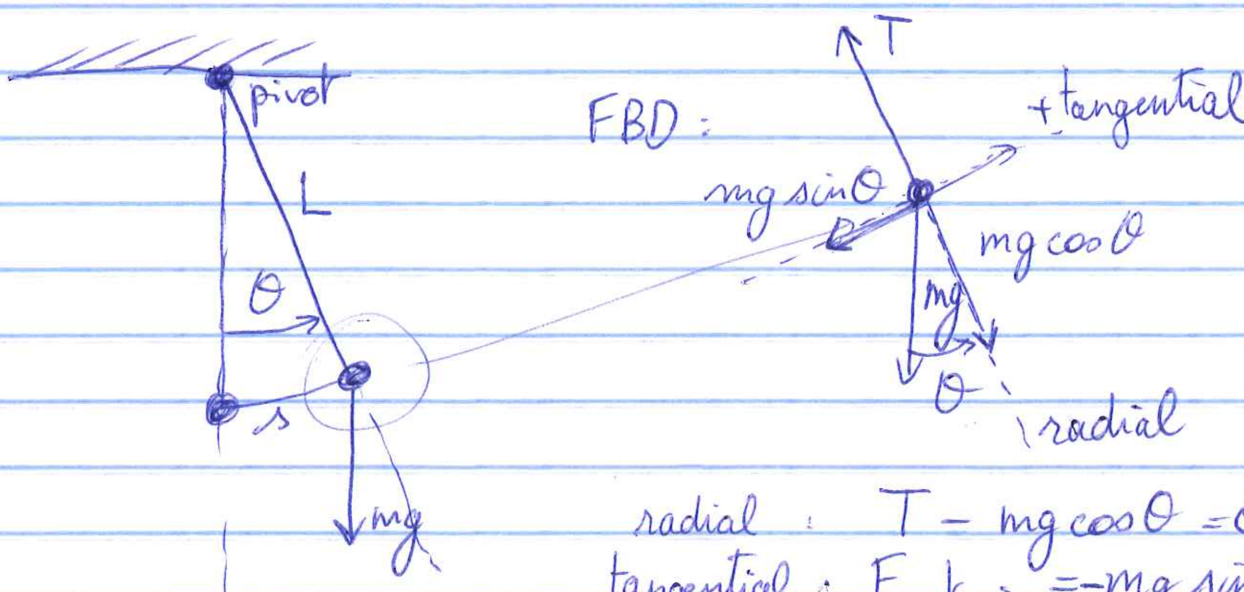
$$E_{total} = PE + KE$$
$$= \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$
$$= \frac{1}{2}k(A\cos\omega t)^2 + \frac{1}{2}m(-\omega A\sin\omega t)^2$$
$$= \frac{1}{2}kA^2\cos^2\omega t + \frac{1}{2}\underbrace{m\omega^2}A^2\sin^2\omega t$$

$$\omega^2 = \frac{k}{m}$$

$$= \frac{1}{2}kA^2\cos^2\omega t + \frac{1}{2}kA^2\sin^2\omega t$$
$$= \frac{1}{2}kA^2\underbrace{(\cos^2\omega t + \sin^2\omega t)}_{1} = \frac{1}{2}kA^2$$

$$E_{total} = \frac{1}{2}kA^2 = \underline{\frac{1}{2}kx^2_{max}} = \frac{1}{2}mv^2_{max}$$

* Simple pendulum



radial: $T - mg\cos\theta = 0$

tangential: $\underline{F_{restoring} = -mg\sin\theta}$

$\underline{s = \theta L}$

$F = -kx$ small angles θ: $\sin\theta \approx \theta$ (in radians)

$F_{restoring} = -mg\sin\theta \approx -mg\theta$

s = distance from equilibrium $= \theta L$

$\rightarrow \theta = \dfrac{s}{L}$

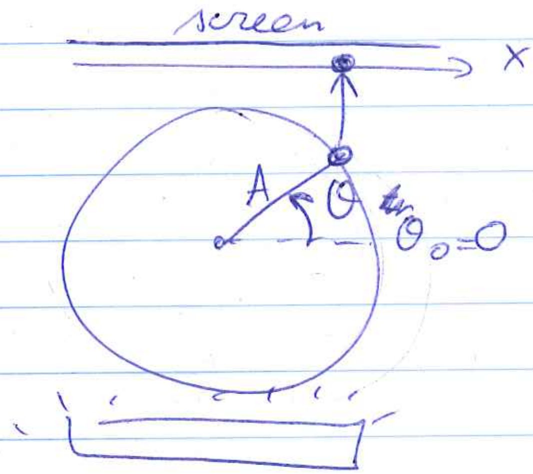
$$F_{restoring} = -mg\frac{s}{L} = -\underbrace{\frac{mg}{L}}_{k} s$$

$$\underline{T} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left(\frac{mg}{L}\right)}} = \underline{2\pi\sqrt{\frac{L}{g}}}$$

SHM: does not depend on amplitude $\underline{A, m}$

$$T = 2\pi\sqrt{\frac{L}{g}} \longrightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$\omega = 2\pi f = \sqrt{\frac{g}{L}}$$



$$x = A\cos\theta$$

$$\theta = \theta_0 + \omega t = \omega t \text{ with } \theta_0 = 0$$

$$x = A\cos\omega t$$

* Energy of pendulum

$E_{total} = \frac{1}{2} k A^2$ for mass & spring

pendulum: $k = \frac{mg}{L}$ $\qquad \frac{g}{L} = \omega^2$

$\rightarrow E_{total} = \frac{1}{2}\left(\frac{mg}{L}\right)A^2 = \frac{1}{2} m\omega^2 A^2$

depends on A^2, typical feature of SHM