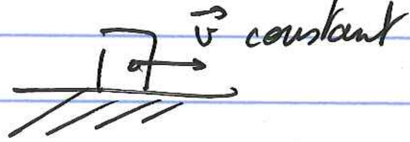
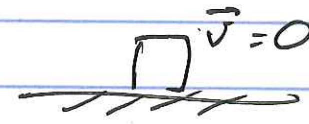
* Static equilibrium

- constant velocity : dynamic equilibrium
- zero velocity : static equilibrium (special case)

dynamic equilibrium

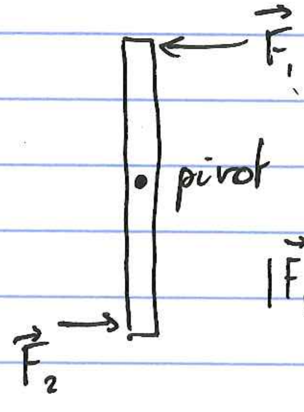


static equilibrium



$\vec{v}$ constant $\rightarrow$ $\vec{a}$ is zero $\rightarrow$ $\vec{F}_{net}$ is zero

But : is $\vec{F}_{net} = 0$ a sufficient condition ?



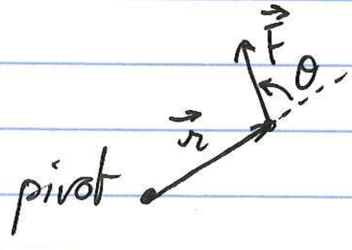
$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = 0$

but not in equilibrium

$|\vec{F}_1| = |\vec{F}_2|$

It matters <u>where</u> the forces apply!

* Torque of a force $\left\{ \begin{array}{l} \text{around} \\ \text{about} \end{array} \right.$ a pivot point

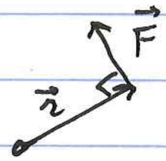


$\tau = \underline{r}\, \underline{F} \sin\theta$ = torque

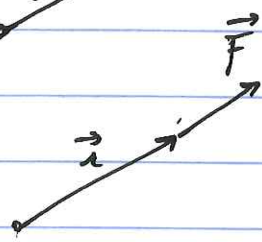
units : $N \cdot m$

$\tau = rF\sin\theta \rightarrow$ dependence on θ

$\theta = 90° \rightarrow \sin\theta = 1 \rightarrow \tau$ maximal
for constant r, F

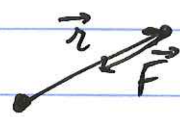


$\theta = 0°$



$\sin\theta = 0 \rightarrow \tau = 0$

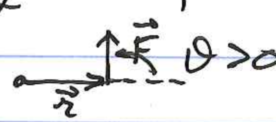
$\theta = 180°$



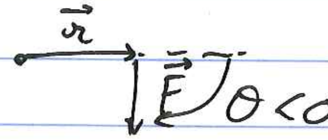
$\sin\theta = 0 \rightarrow \tau = 0$

* τ is technically a 1D vector quantity : +, −

+ : CCW $\longrightarrow$



− : CW



* Theorem for zero torque :

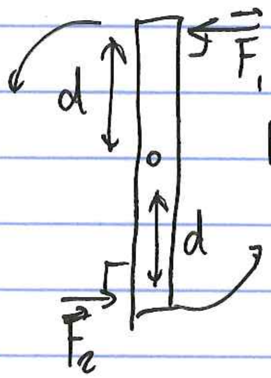
If $\tau_{net} = 0$ around one point of a rigid object, then $\tau_{net} = 0$ around every point of the object

Conditions for static equilibrium

If $\vec{F}_{net} = 0$ AND $\tau_{net} = 0$,

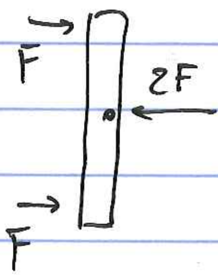
then $\vec{a} = 0$ AND there will be no rotation

$\Rightarrow$ the object will be in equilibrium



$|\vec{F}_1| = |\vec{F}_2| = F$

$$\tau_{net} = \vec{F}\vec{d} + \vec{F}\vec{d} = 2\vec{F}\vec{d} \neq 0$$

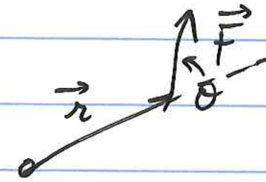


$$\tau_{net} = Fd - Fd = 0$$

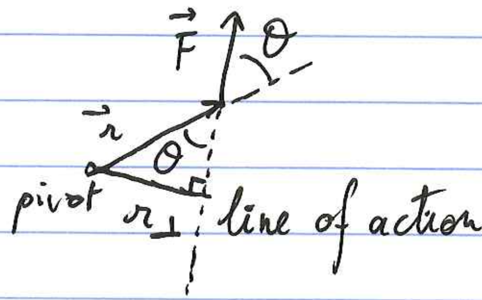
$\rightarrow$ equilibrium

$$F_{net} = 2F - 2F = 0$$

$\tau = r F \sin\theta$



1)

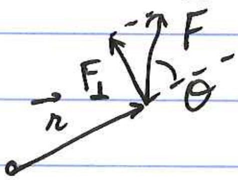


$r_\perp = r \sin\theta$

$\tau = r_\perp F$

torque is product of the magnitude of the force and the ~~pe~~ perpen-dicular distance from the pivot to the line of action

2)



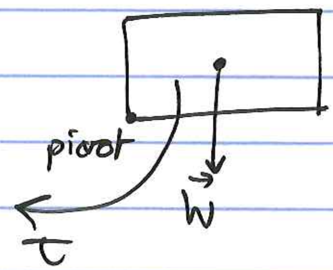
$F_\perp = F \sin\theta$

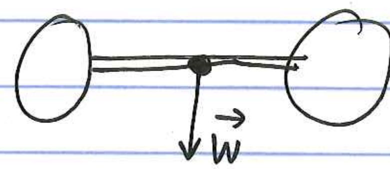
$\tau = r F_\perp$

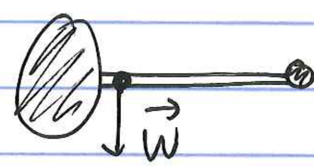
torque is the product of the distance between the pivot and the point where the force applies, and the perpendicular component of the force.

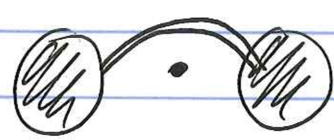
* Where do forces apply?

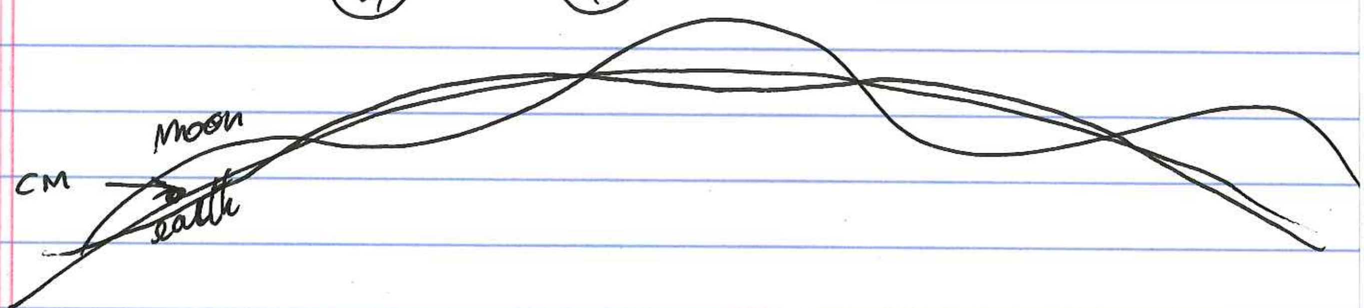
Extended force : $\vec{r}_{CM} = \dfrac{\vec{r}_1 m_1 + \vec{r}_2 m_2 + \cdots}{m_1 + m_2 + \cdots}$





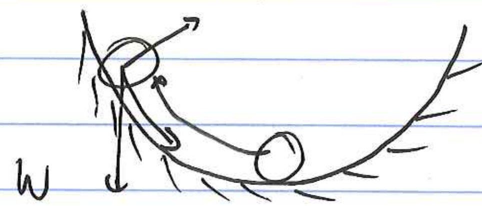




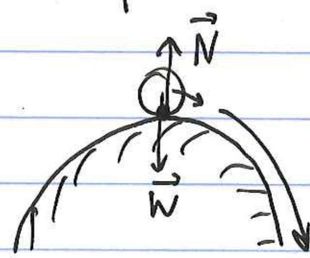


Stability of equilibrium ($\vec{F}_{net} = 0$, $\tau_{net} = 0$)

Stable equilibrium: small change in the position results in a restoring force that brings object back to equilibrium

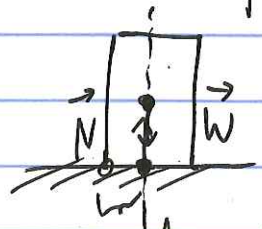


Unstable equilibrium: small change leads to a force that brings the object further from equilibrium → runaway behavior



Neutral equilibrium:

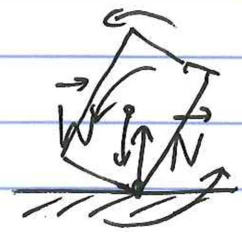
* Conditions for stable equilibrium under gravity



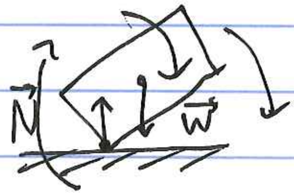
$F_{net} = 0$ } equilibrium
$\tau_{net} = 0$ } stable

$r_\perp$ line of action : $\tau = F r_\perp$

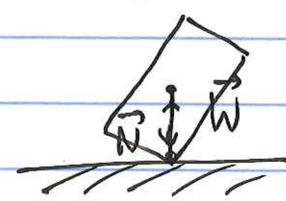
$\tau_{net} = 0$

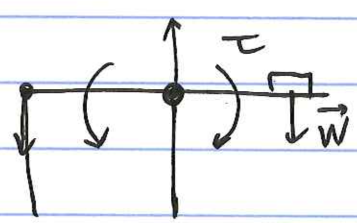


restoring force



unstable equilibrium





$\tau_{net} = 0$