

Rotational kinematics  $\rightarrow$  description of the motion only



$\downarrow$   
nothing or what causes the motion

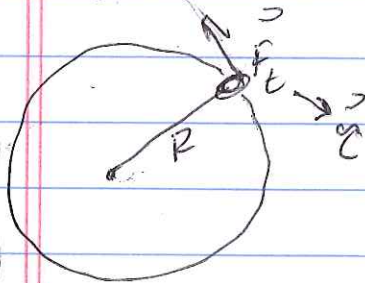
Rotational Dynamics  $\rightarrow$  causes the angular motion, ~~velocity~~ motion, velocity, acceleration

$$\vec{F} = m\vec{a}$$

$\nearrow$   $\uparrow$   $\nwarrow$   
dynamics mass (inertial) kinematics

day dynamics

mass (inertial)



$F \rightarrow$  Torque  $\tau = F_t R$

$$\boxed{F_t = F_{\perp}}$$

acceleration  $\vec{a} \rightarrow$  angular acceleration  $\alpha$

$$a = \frac{a_t}{R}, \quad a_t = \alpha R$$

mass  $m \Rightarrow I$  moment of inertia

$$\vec{F} = m \vec{a}_t$$

$$\tau = F_t R$$

$$\vec{\tau} = F_t R = m \vec{a}_t R = m (\vec{\alpha} R) R$$

$$= (m R^2) \vec{\alpha} = I \vec{\alpha}$$

$$I = m R^2$$

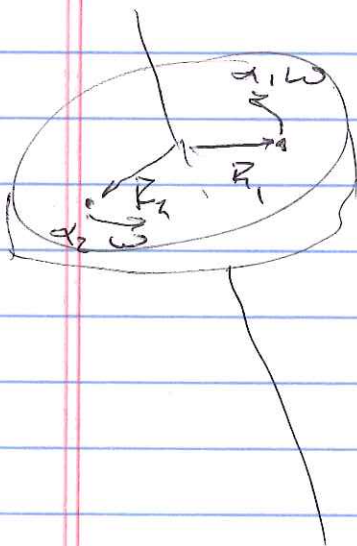
CCW  
CW

$\tau$  - positive

$\tau$  - negative

$\alpha$  positive

$\alpha$  negative



disc to consist

of many points  
all with the same  $\alpha, \omega$

$$\begin{aligned}\tau_1 &= m_1 R_1^2 \alpha \\ \tau_2 &= m_2 R_2^2 \alpha \\ &\vdots\end{aligned}$$

$$\begin{aligned}\vec{\tau}_{\text{net}} &= (m_1 R_1^2 + m_2 R_2^2 + \dots) \alpha \\ &= (I) \alpha\end{aligned}$$

$I$  is known for common geometries  
Pg 328 in text Book

$$I_{\text{Disk}} = I_{\text{CYLINDER}} = \frac{1}{2} MR^2$$

$M$  = TOTAL MASS

$R$  = OUTER RADIUS

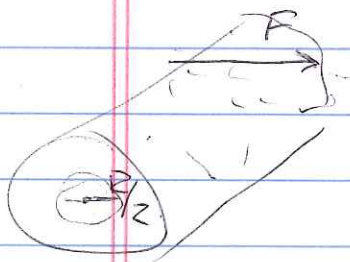
$$I_{\text{hoop}} = MR^2$$

EXAMPLE: CYLINDER

$$M = 2 \text{ kg}$$

$$R = 10 \text{ cm}$$

$F = 10 \text{ N}$  ON THE OUTER  
DIAMETER



WHAT WILL  $\alpha$  BE?

$$F \text{ is } F_{\perp}$$

$$I_{\text{cm}} = \frac{1}{2} MR^2 = \frac{1}{2} (2 \text{ kg})(10 \text{ cm})^2 = 0.01 \text{ kg} \cdot \text{m}^2$$

$$\tau = F_{\perp} R = (10 \text{ N})(10 \text{ cm}) = 1 \text{ N} \cdot \text{m}$$

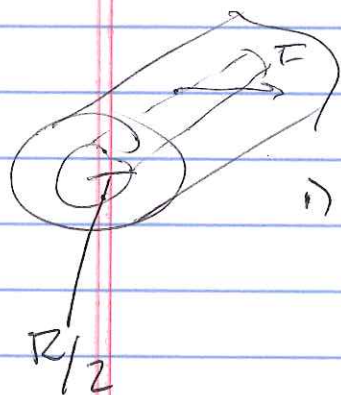
$$\tau = I \alpha \rightarrow \alpha = \frac{\tau}{I} = 100 \frac{\text{rad}}{\text{sec}^2}$$

$$\frac{\text{rad}}{\text{sec}^2} = \frac{1}{\text{sec}^2}$$



~~What is the force applied~~  
 ~~$\frac{R}{2}$~~

$F = 10 \text{ N}$  now is applied  
at a radius of  $\frac{R}{2}$



WHAT IS  $\alpha$  NOW

1)  $I = \frac{1}{2} MR^2$

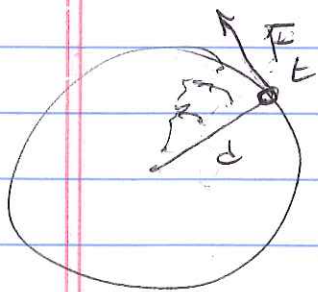
cylinder of same shape  
size  $I$  doesn't change.

$$\tau = F_t R \rightarrow F_t \frac{R}{2}$$

Radius where the force  
is applied.

$$\alpha = \frac{\tau}{I} = \frac{F_t \frac{R}{2}}{\frac{1}{2} MR^2} = 50 \frac{\text{RAD}}{\text{SEC}^2}$$

# Work for Rotational Dynamics



$$W = F_t d \cos \theta$$

$$\theta = 0 \Rightarrow \cos \theta = 1$$

$$W = F_t d \quad \text{multiple by } 1 = \frac{R}{R}$$

$$= (F_t R) \left( \frac{d}{R} \right)$$

$$W = \tau \theta$$

$$KE_{\text{ROT}} = \frac{1}{2} m v_t^2 = \left( \frac{1}{2} m R^2 \right) \frac{v^2}{R^2} = \frac{1}{2} I \frac{v^2}{R^2}$$

$$\text{multiply by } R \frac{R}{R^2} = 1$$

$$I = m R^2$$

$$= \frac{1}{2} I \frac{v^2}{R^2} = \frac{1}{2} I \omega^2$$

$$\frac{v}{R} = \omega$$

$$W = F_t d \longrightarrow W_{\text{ROT}} = \tau \theta$$

$$KE = \frac{1}{2} m (v^2) \longrightarrow KE_{\text{ROT}} = \frac{1}{2} I (\omega^2)$$