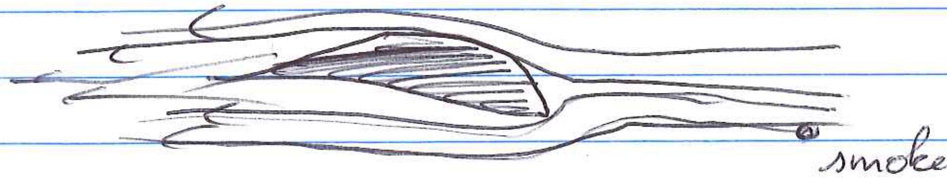
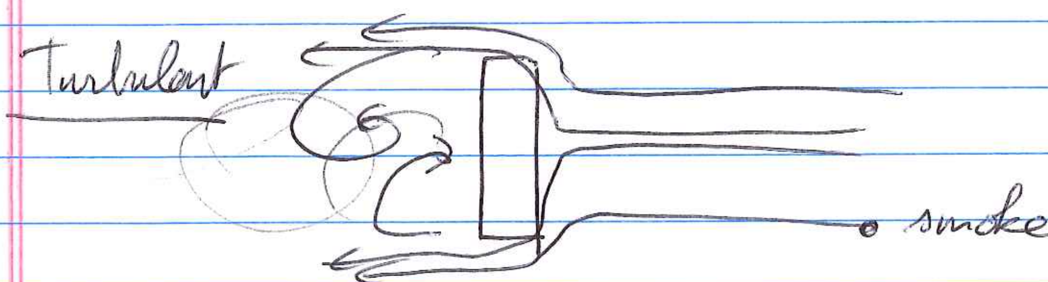
Laminar: no mixing of fluid layers, streamlines




Turbulent




eddies, swirls, caused by - sharp corners
- swift flow

Reynolds number $\underline{N_R} = \dfrac{2 \rho v r}{\eta}$ for tube of radius $\underline{r}$

unit-less

$N_R > 3000 \rightarrow$ turbulence
$N_R < 2000 \rightarrow$ laminar
in between $\rightarrow$ unstable, ill-determined

Reynolds can be used to scale up or scale down systems

Turbulence is more likely:
- higher density
- higher velocity
- higher radius
- lower viscosity

Example: ~~tt~~ At what speed will blood flow turn turbulent in artery with $r = 2$ mm ?

$$\rho = 1060\ \mathrm{kg/m^3}, \quad \eta = 2.08 \times 10^{-3}\ \mathrm{Pa \cdot s}$$

$$N_R = 3000 = \boxed{\frac{2\rho r v}{\eta}} \longrightarrow v = \frac{N_R\,\eta}{2\rho r}$$

$$v = \frac{(3000)(2.08\times10^{-3}\ \mathrm{Pa\cdot s})}{2(1060\ \mathrm{kg/m^3})(0.002\ \mathrm{m})} = \underline{3.0\ \mathrm{m/s}}$$

Typical blood flow speed in pulmonary artery is $\underline{1.9\ \mathrm{m/s}}$

* General expression $N_R = \dfrac{\rho v L}{\eta}$ $\quad 2r = L$ for tube

L is any characteristic length scale

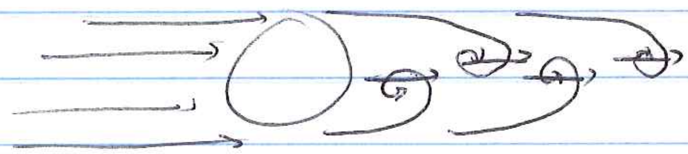
Example: Tacoma ~~Nt~~ Narrows bridge:

At~~w~~ what speed will a cable (21' diameter) cause turbulence?

$$N_R = \frac{\rho v L}{\eta} \longrightarrow v = \frac{N_R\,\eta}{\rho L} = \frac{(3000)(10^{-3}\ \mathrm{Pa\cdot s})}{(1.29\ \mathrm{kg/m^3})(21\ \mathrm{inch})}$$

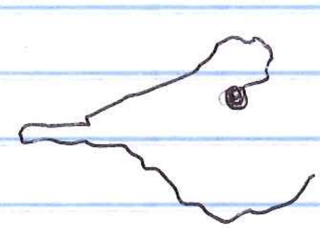
$$v = 4\ \mathrm{m/s},\quad 9\ \mathrm{mph}$$

"Vortex shedding"



* Diffusion, Brownian motion, random walk

Brownian motion : nano-particles in water get bombarded from all sides by water molecules
→ nano-particles start moving



position $x$ is a random variable that changes randomly : $\langle x \rangle = 0$ = average position

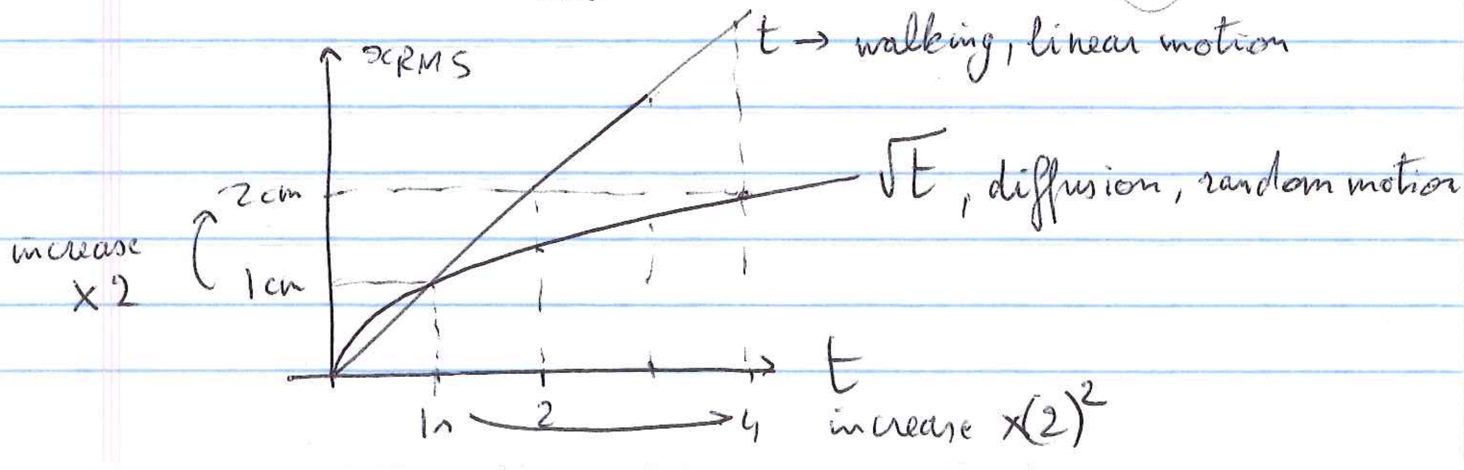
$x_{RMS} = \sqrt{\langle x^2 \rangle}$ = root-mean-square of position

* Diffusion law : $x_{RMS} = \sqrt{2Dt}$

D = diffusion constant of a molecule in a particular solution, units $\frac{m^2}{s}$


$t=0$ $\qquad t_1$ $\qquad t_2$

$x_{RMS} = \sqrt{2Dt_1}$ $\qquad x_{RMS} = \sqrt{2Dt_2}$



$D$ for $O_2$ in $H_2O$ : $1.0 \times 10^{-9} \frac{m^2}{s}$ ) smaller

$C_6H_{12}O_6$ in $H_2O$ : $0.7 \times 10^{-9} \frac{m^2}{s}$ ↓ for larger

(glucose) molecule

hemoglobin in $H_2O$ $0.07 \times 10^{-9} \frac{m^2}{s}$

DNA in $H_2O$ $0.0013 \times 10^{-9} \frac{m^2}{s}$

Breathing cycle : how far will $O_2$ diffuse in $H_2O$ for 1 second breathing cycle?

$$x_{RMS} = \sqrt{2Dt} = \sqrt{2\left(1.0 \times 10^{-9} \frac{m^2}{s}\right)(1s)}$$
$$= 45 \mu m$$

Red blood cell : 8 μm

$$t = \frac{x_{RMS}^2}{2D_{hemoglobin}} = 0.5 s$$

Amoeba : 500 μm

$$t = \frac{x_{RMS}^2}{2D_{O_2 \, in \, H_2O}} = \frac{(500 \times 10^{-6} m)^2}{2\left(1.0 \times 10^{-9} \frac{m^2}{s}\right)} = 100 s$$

Short distances are ruled by diffusion

Longer distance require an active transport mechanism

→ pumps, molecular motors, ATP flagella