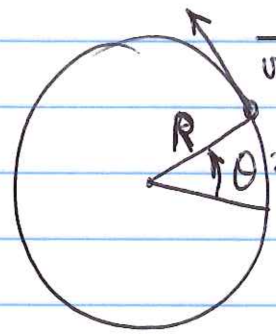
PHYS 107 - Week 9 - Friday

* Rotational kinematics



$\vec{v}_t$ = tangential velocity

$$\theta\,(\text{radians}) = \frac{x}{R} = \text{angular coordinate}$$

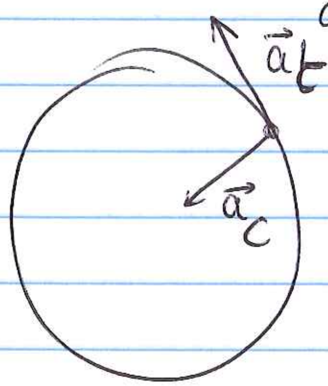
$$\frac{\Delta\theta}{\Delta t}\left(\frac{\text{radians}}{s}\right) = \frac{v_t}{R} = \text{angular velocity}$$

$$\frac{\Delta\omega}{\Delta t}\left(\frac{\text{radians}}{s^2}\right) = \frac{a_t}{R} = \text{angular acceleration}$$

$$v_t = \frac{\Delta x}{\Delta t}\,, \quad a_t = \frac{\Delta v_t}{\Delta t} = \text{tangential acceleration}$$

$\neq$

a_c = centripetal acceleration



a_t = rate of change of v_t (speed)

a_c = rate of change of <u>direction</u> of $\vec{v}$

a_t = rate of change of <u>magnitude</u> of $\vec{v}$

* Connection to 1D linear kinematics :

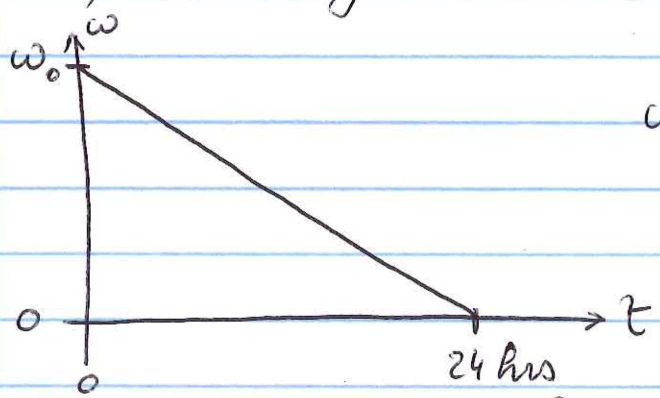
$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \xrightarrow{\frac{1}{R}} \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t$$

$$v = v_0 + at \longrightarrow \omega = \omega_0 + \alpha t$$

$$v^2 = v_0^2 + 2a(x - x_0) \longrightarrow \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Example: A clock starts by keeping time perfectly, but slows down uniformly to a stop after 24 hours.

1) what is α for the second hand ?
2) what is ω after 80 revolutions ?
3) how many revolutions does the clock complete in 2 hours?



$$\omega_0 = \frac{1\ \text{rev}}{60\ s} \cdot \frac{2\pi\ \text{rad}}{1\ \text{rev}} = 0.105\ \frac{\text{rad}}{s}$$

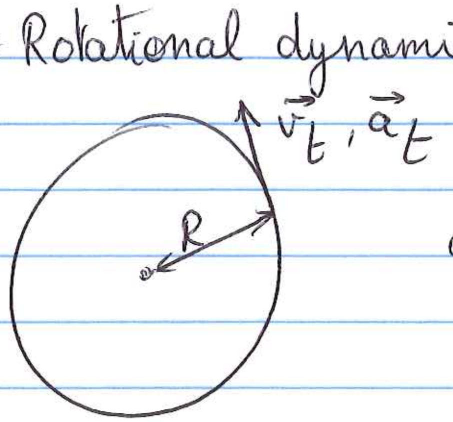
1) $$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 0.105\ \frac{\text{rad}}{s}}{(24\ \text{hrs})\left(\frac{3600\ s}{\text{hr}}\right)} = -1.21 \times 10^{-6}\ \frac{\text{rad}}{s^2}$$

2) $$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) = \left(0.105\frac{\text{rad}}{s}\right)^2 + 2\left(-1.21 \times 10^{-6}\frac{\text{rad}}{s^2}\right) \cdot (80 \times 2\pi\ \text{rad})$$

$$\hookrightarrow \omega = 9.8 \times 10^{-3}\ \frac{\text{rad}}{s}$$

3) $$\theta = \theta_0 + \omega_0 (2\ \text{hrs}) + \frac{1}{2}\left(-1.21 \times 10^{-6}\frac{\text{rad}}{s^2}\right)(2\ \text{hrs})^2 = 115 \times 2\pi$$

* Rotational dynamics : what causes the motion?



analogy to $\vec{F} = m\vec{a}$ for rotational motion

force $\vec{F}$ $\longrightarrow$ torque $\tau = F_\perp \cdot R$

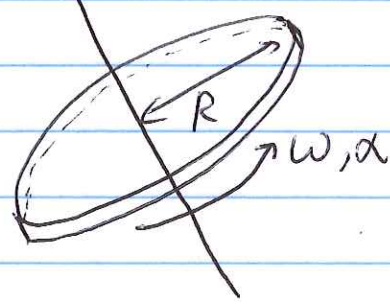
acceleration $\vec{a}_t$ $\longrightarrow$ angular acceleration $\alpha = \frac{a_t}{R}$

mass m $\longrightarrow$ moment of inertia I

$\vec{F} = m\vec{a}$ $\longrightarrow$ $\tau = I\alpha$

$\hookrightarrow F_t = m a_t \rightarrow \underbrace{F_t R}_{\tau} = m a_t R = \underbrace{m R^2}_{I} \alpha$

* Moment of inertia depends on shape, mass, size of object that is rotating $\rightarrow$ rigid body



rotating disk : sum of many points with $I_i = m r_i^2$ each

$\downarrow$

$\tau = I_{total}\, \alpha$

I for simple shapes

$I = \frac{1}{2} M R^2$ for disks or cylinders

M: total mass

R: outer radius

$I = MR^2$ for hoop of radius R

* Example: You apply a force of 10N tangentially on the outer radius of a 2kg cylinder with $R = 10$ cm. What is α?

$$I = \frac{1}{2} M R^2 = 0.01 \ kg \cdot m^2$$

$$\tau = F_t \cdot R = (10N)(0.1 \ m) = 1 \ N \cdot m$$

$$\hookrightarrow \alpha = \frac{\tau}{I} = 100 \ \frac{rad}{s^2}$$

What if F_t is applied at $\frac{R}{2}$ instead?

$$\alpha = 50 \ \frac{rad}{s^2}$$

* Rotational work and energy

Work $W = F_t d = (F_t R)\frac{d}{R} = \tau\theta$

$\rightarrow \boxed{W = \tau\theta}$ (compare with $W = Fx$)

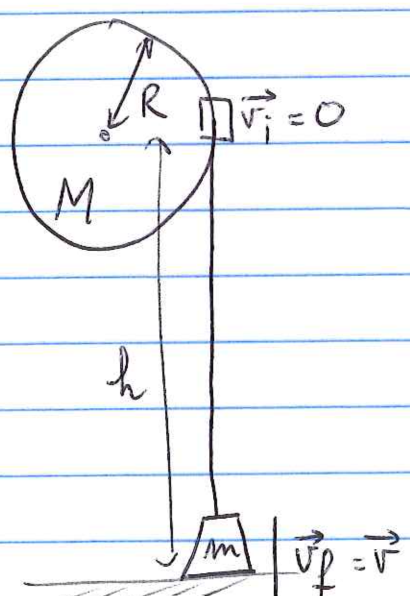
$$KE_{rot} = \frac{1}{2} m v_t^2 = \frac{1}{2}(mR^2)\left(\frac{v_t}{R}\right)^2 = \frac{1}{2} I\omega^2$$

$$\boxed{KE_{rot} = \frac{1}{2} I\omega^2}$$

$$\hookrightarrow KE = \frac{1}{2} m v^2 + \frac{1}{2} I\omega^2$$

velocity of the axis of rotation

Example: Mass attached to pulley is dropped from rest. What is the velocity at the lowest point?



$PE_i = mgh$, $PE_f = 0$

$KE_i = 0$, $KE_f = \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{1}{2} MR^2\right)\omega^2$

$\omega = \frac{v}{R}$

$\downarrow$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{1}{2} MR^2\right)\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{1}{2}\left(m + \frac{M}{2}\right) v^2 \rightarrow v \, !$$