Physics 107: Physics for Life-Sciences

Final Exam: December 8, 2014

This test is administered under the rules and regulations of the honor code of the College of William & Mary.

Name: Solutions

Signature: ____________________

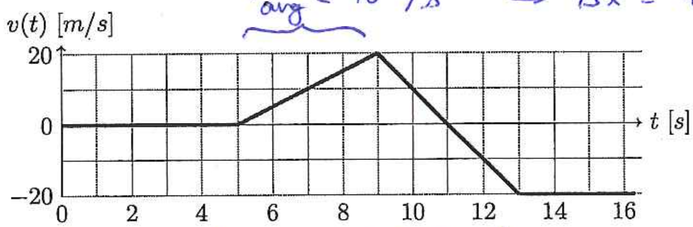
Instructions:

- This is a closed book, closed notes test.
- Calculators are permitted, but not laptops or cell phones. Devices with wireless connections are not allowed.
- Start your work from the fundamental equations on the formula sheet, and derive any additional expressions that you may need.
- Circle your answer for each part of each problem.
- Clearly mark out any work that you wish the grader to disregard. Do not waste your time erasing.
- Your work will be graded based on your ability to write down a logical and organized solution grounded in the correct assessment of the physics of a situation. No credit will be given for an answer that is not justified by a logical solution or where that justification is not organized or readable.
- Partial credit will be given up to the point where your solution departs from a correct analysis of the physics involved for any given part of a problem.

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	20	
7	15	
8	20	
9	20	
10	10	
11	10	
12	10	
13	20	
Total:	150	

1. (5 points) You are operating a small remote controlled toy car on the sidewalk along Jamestown Road. As you are standing on the sidewalk, the speed of the toy car relative to you is plotted below. What is the displacement of the toy car between 0 s and 13 s?

$v_{avg} = 10\,m/s \rightarrow \Delta x = 40\,m$



$v_{avg} = 0\,m/s$ $\quad$ $v_{avg} = 0\,m/s$

- ○ -20 m
- ○ 4 m
- ☑ 40 m
- ○ 80 m
- ○ 300 m

2. (5 points) A ball is thrown straight up, reaches a maximum height, and then falls back downward. Which of the following are true when it reaches its maximum height?

x

t

v

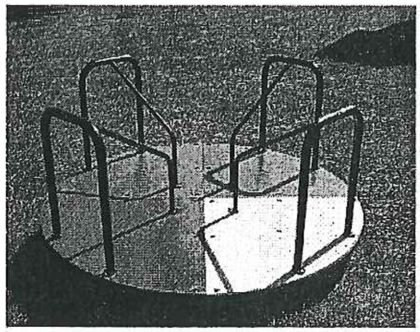
a

- ○ Its velocity and its acceleration are zero.
- ○ Its velocity is zero and its acceleration points upward.
- ☑ Its velocity is zero and its acceleration points downward.
- ○ Its velocity points downward and its acceleration points upward.
- ○ Its velocity and its acceleration point downward.

3. (5 points) A 2.2 kg pumpkin is going to be dropped from the top of the Wren building to the sidewalk below. The height is 15.5 m above the sidewalk. You want to set a delay timer on your camera for taking a picture upon impact. You want to determine how long it takes for the pumpkin to hit the ground after it was dropped. Select the relationship that will most directly lead you to the result using the given information?

- ○ $v = v_0 + a\Delta t$
- ○ $\Delta KE + \Delta PE = W_{nc}$
- ☑ $\Delta y = v_0 \Delta t + \frac{1}{2} a \Delta t^2$ $\qquad$ $\Delta y = 15.5\,m$, $v_0 = 0\,m/s$, $a = -9.8\,m/s$ → solve for Δt
- ○ $W = Fd\cos\theta$
- ○ $v^2 = v_0^2 + 2a\Delta y$
- ○ $\Delta PE_g = mg\Delta y$

4. (5 points) A merry-go-round on a playground can be thought of as a rotating disk of 52 kg with a diameter of 2.52 m. A girl on the playground starts from rest and has the merry-go-round spinning at 0.2 revolutions per second when she lets go after a quarter turn. She is standing next to it and no one is on the merry-go-round while it spins. What is the velocity of a point at the outer edge of the merry-go-round after it gets to its final speed?



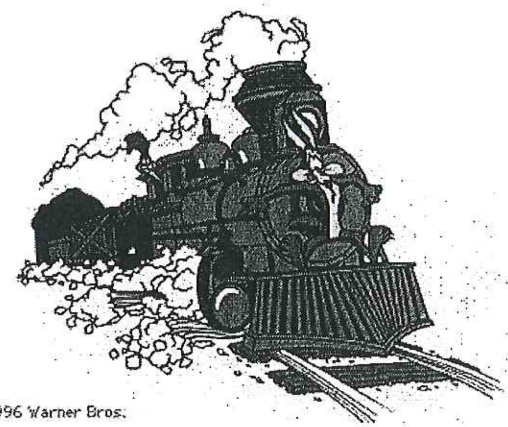
- ○ 3.17 m/s
- ☑ 1.58 m/s
- ○ 0.79 m/s
- ○ 0.50 m/s
- ○ 0.25 m/s

$\omega = (0.2)\frac{2\pi}{s} \rightarrow v = r\omega = \left(\frac{2.52\,m}{2}\right)(0.2)\frac{2\pi}{s}$

5. (5 points) Explain **in words only** the difference between a stable and an unstable equilibrium. Limit yourself to at most three sentences.

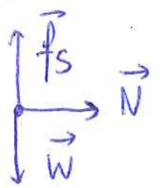
In a stable equilibrium there is a restoring force which will bring the system back to equilibrium. In an unstable equilibrium there is no restoring force and the system will move further and further from equilibrium.

6. Road Runner's revenge. Wile E. Coyote is on the vertical front of a train that is accelerating. He is not standing on anything and is not holding on to anything. He is being held up solely by the frictional force between him and the front of the accelerating train. Coyote has a mass of 20.0 kg, and the coefficient of static friction is 0.45 for this situation.



© 1996 Warner Bros.

(a) (5 points) Draw the free body diagram for the coyote.



(b) (5 points) Write down the components of the important vectors on the coyote in terms of their *magnitudes and angles*. If the magnitudes and angles of these vectors are unknown, then define variables to break down the components in the format $(F\cos\theta, F\sin\theta)$.

$\vec{W} = m\vec{g}$: magnitude = 196 N, direction = vertically downwards

$\vec{N} = m\vec{a}$: magnitude depends on solution of c, direction = horizontally to the right

$\vec{f_s} = -\vec{W} \leq \mu_s N$: magnitude $\leq \mu_s N$, direction = vertically upwards

(c) (10 points) Using Newton's laws determine the minimum acceleration required for the coyote to not slip down the front of the train?

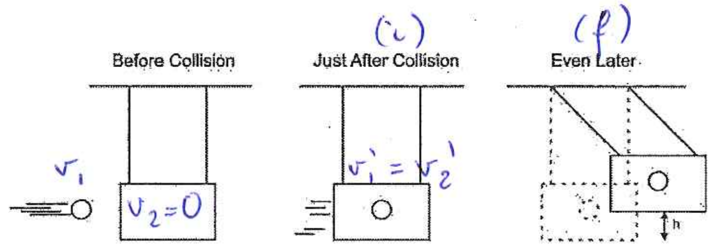
$\vec{F}_{net} = m\vec{a}$

$x: \vec{N} = m\vec{a} \rightarrow a = \frac{N}{m}$

$y: \vec{f_s} + \vec{W} = 0 \rightarrow f_s = mg$

$f_s \leq \mu_s N \rightarrow mg \leq \mu_s ma \rightarrow a \geq \frac{g}{\mu_s} = 21.78\ m/s^2$

7. You have been hired by the Police Department to examine their new pistols. They have asked you to determine the velocity of the bullets when they leave the handgun. To test this you fire a standard 5.17 g bullet toward a wood block hanging from a rope (called a ballistic pendulum). When the bullet hits the block is embeds itself in the wood. You observe that the block swings upward after the collision. When you do your experiment you note that the 2.27 kg block swings to a height of 0.268 m above its original hanging position.



(a) (5 points) What is the potential energy of the block and bullet when it is at its highest position?

$$PE_f = (m_1 + m_2) g h = 5.98 \text{ J}$$

(b) (5 points) What is the speed of the block and the bullet just after the bullet hits the block?

$$PE_f = KE_i \rightarrow \frac{1}{2}(m_1 + m_2) v^2 = (m_1 + m_2) g h$$

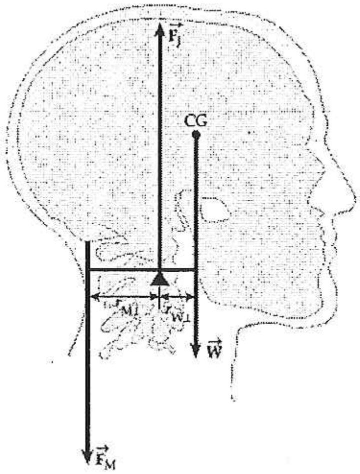
$$\rightarrow v = \sqrt{2gh} = 2.29 \text{ m/s}$$

(c) (5 points) What is the speed of the bullet before the collision?

$$m_1 v_1 + \cancel{m_2 v_2} = m_1 v_1' + m_2 v_2' = (m_1 + m_2) v_1'$$
$$(v_2 = 0)$$

$$\rightarrow v_1 = \frac{m_1 + m_2}{m_1} v_1' = \frac{m_1 + m_2}{m_1} (2.29 \text{ m/s}) = 1008 \text{ m/s}$$

8. Even when the head is held erect, as in the figure below, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles in the back of the neck must therefore exert a force to keep it erect. That is why your head falls forward when you fall asleep in class. The perpendicular distance between the line of action for the weight of the head and the pivot point is $r_{W\perp} = 2.1\,\mathrm{cm}$ and the perpendicular distance between the line of action for the force the muscles exert on the head and the pivot point is $r_{M\perp} = 5.6\,\mathrm{cm}$. Assume the weight of the head is 50 N.



(a) (10 points) What is the force exerted by the muscles?

$$\vec{\tau}_{net} = 0 = \vec{\tau}_M + \vec{\tau}_W$$

$$\rightarrow \quad F_M \cdot r_{M\perp} = F_W \cdot r_{W\perp}$$

$$\rightarrow \quad F_M = W \frac{r_{W\perp}}{r_{M\perp}} = 18.75\ \mathrm{N}$$

(b) (10 points) What is the force on the point of support?

$$\vec{F}_{net} = 0 = \vec{F}_M + \vec{F}_J + \vec{F}_W$$

$$\rightarrow \quad F_J = F_M + F_W = 68.75\ \mathrm{N}$$

9. You place a beaker that is two-thirds full of water with a density of $\rho_{H_2O} = 1000\,\mathrm{kg/m^3}$ on a laboratory scale. You then use a light-weight cord to suspend an aluminum object with density $\rho_{Al} = 2700\,\mathrm{kg/m^3}$ in the water. The object is completely submerged, and none of the water spills out of the beaker. The reading on the scale changes by 0.054 kg.

   (a) (10 points) What is the volume of the object?

extra force on the scale $= F_B = \rho_{H_2O} V_{disp}\, g = (0.054\,\mathrm{kg})g$

$$\rightarrow V = V_{disp} = \frac{0.054\,\mathrm{kg}}{\rho_{H_2O}} = 5.4 \times 10^{-5}\,\mathrm{m}^3$$

   (b) (10 points) What is the tension in the cord while the object is submerged?

$$\vec{F}_{net} = 0 = \vec{F}_B + \vec{T} + \vec{W}$$

$\vec{T}$ $\vec{F}_B$ $\vec{W}$

$$\rightarrow T = W - F_B = V \rho_{Al}\, g - V \rho_{H_2O}\, g$$

$$= V(\rho_{Al} - \rho_{H_2O})g = 0.9\,\mathrm{N}$$

10. A long horizontal hose of diameter 0.016 m is connected to a faucet. At the other end, there is a nozzle of diameter 0.004 m. Water squirts from the nozzle at a velocity of 38 m/s. Assume that the water has no viscosity or other form of energy dissipation, and that the density of water is $1000\,\mathrm{kg/m^3}$.

   (a) (5 points) What is the velocity of the water within the hose?

$$A_1 v_1 = A_2 v_2 \rightarrow v_2 = \frac{A_1}{A_2} v_1 = \frac{r_1^2}{r_2^2} v_1 = 2.375\,\mathrm{m/s}$$

   (b) (5 points) What is the pressure differential between the water in the hose and water in the nozzle?

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = 7.2 \times 10^5\,\mathrm{Pa}$$

11. A spring has a length of 0.333 m when a 0.300 kg mass hangs from it, and a length of 0.750 m when a 3.00 kg mass hangs from it. With the mass of 3.00 kg attached it is set in simple harmonic motion by pulling it down an additional 0.125 m and letting it go from there.

(a) (5 points) What is the force constant of the spring?

$$m_1 g = k(\ell_1 - \ell_0)$$

$$m_2 g = k(\ell_2 - \ell_0)$$

$$\rightarrow (m_1 - m_2) g = k(\ell_1 - \ell_2)$$

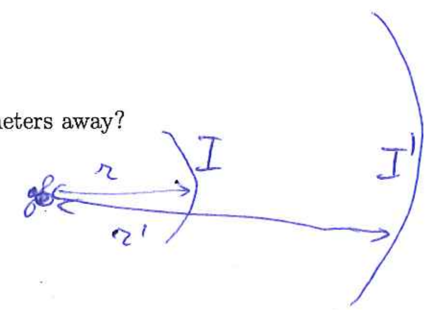
$$\rightarrow \quad k = \frac{(m_1 - m_2) g}{(\ell_1 - \ell_2)} = 63 \ N/m$$

(b) (5 points) What will be the period of oscillations?

$$T = 2\pi \sqrt{\frac{m}{k}} = 1.37 \ s$$

12. A large housefly 3.0 meters away from you makes a noise of 40 dB.

(a) (5 points) How many decibels would the housefly make if it were 30 centimeters away?



$$I' = \frac{r^2}{r'^2} I = 100 \ I$$

$$40 \ dB = 10 \log\left(\frac{I}{10^{-12} \frac{W}{m^2}}\right)$$

$$\rightarrow 10 \log\left(\frac{I'}{10^{-12} \ W/m^2}\right) = 10 \log\left(\frac{100 \ I}{10^{-12} \ W/m^2}\right) = 10 \log 100 + 40 dB = 60 dB$$

(b) (5 points) With a noise level of 40 dB, how much energy falls on your eardrum, which has a diameter of 1.0 cm, in one minute?

$$I = \frac{P}{A} \quad , \quad P = \frac{E}{\Delta t} \quad \rightarrow \quad E = I \cdot A \cdot \Delta t$$

$$= \left(10^{-8} W/m^2\right) \pi \left(\frac{1.0 cm}{2}\right)^2 \cdot 60 s$$

$$= 4.7 \times 10^{-11} \ J$$

$$40 \ dB = 10 \log\left(\frac{I}{10^{-12} \ W/m^2}\right)$$

$$\downarrow$$

$$I = 10^{-12} \frac{W}{m^2} \cdot 10^4 = 10^{-8} \ W/m^2$$

13. The William & Mary Pep Band is moving towards you at 2 m/s. A clarinet player is playing a perfect 185.0 Hz note. Assume the speed of sound is 331 m/s.

(a) (5 points) What frequency do you hear?

$$f_{obs} = f_{src} \frac{v}{v - v_{src}} = 186 \text{ Hz}.$$

(b) (5 points) What is the wavelength of the sound?

$$\lambda = \frac{v}{f} = 1.79 \text{ m}$$

(c) (5 points) Another clarinet player is standing still (*i.e.* not marching) by the first player, and also plays a perfect 185.0 Hz note. What is the beat frequency that you hear?

$$f_{beat} = |f_1 - f_2| = 1 \text{ Hz}$$

(d) (5 points) A clarinet is a tube that is open at both ends. If 185.0 Hz is the fundamental frequency, what it the length of the clarinet?

$n = 1$

$$f_1 = 185 \text{ Hz} = \frac{nv}{2L} \rightarrow L = \frac{v}{2f_1} = 0.89 \text{ m}$$

Possibly useful relations (please detach this page):

$\vec{v}_{avg} = \Delta\vec{x}/\Delta t$
$\vec{a}_{avg} = \Delta\vec{v}/\Delta t$
$v = v_0 + at$
$v_{avg} = \frac{v_0+v}{2}$
$x = x_0 + v_0 t + \frac{1}{2}at^2$
$v^2 = v_0^2 + 2a(x - x_0)$
$R = \frac{v_0^2}{g}\sin 2\theta$
$h = \frac{v_0^2}{2g}\sin^2\theta$
$\vec{F}_{net} = m\vec{a}$
$\vec{F}_{BA} = -\vec{F}_{AB}$
$\vec{W} = m\vec{g}$
$\vec{g} = 9.80\,\mathrm{m/s^2}$ downward
$0 \le f_s \le \mu_s N$
$f_k = \mu_k N$
$\frac{F}{A} = Y\frac{\Delta L}{L}$
$F_k = -kx$
$W = Fd\cos\theta$
$W_{net} = -\Delta PE = \Delta KE$
$KE = \frac{1}{2}mv^2$
$PE_k = \frac{1}{2}kx^2$
$PE_g = mgh$
$KE_i + PE_i + W_{nc} = KE_f + PE_f$
$P = \frac{W}{\Delta t}$
$\mathrm{Eff} = \frac{W_{out}}{E_{in}}$
$F_G = G\frac{mM}{r^2}$
$G = 6.67 \times 10^{-11}\,\mathrm{N\cdot m^2\,/\,kg^2}$
$\vec{I} = \vec{F}_{avg}\Delta t$
$\vec{p} = m\vec{v}$
$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$
$v_1 - v_2 = v_2' - v_1'$
$\theta = \frac{s}{r}$
$v = r\omega$
$f = \frac{1}{T}$ and $\omega = 2\pi f = \frac{2\pi}{T}$
$a_c = \frac{v^2}{r} = r\omega^2$
$F_c = m\frac{v^2}{r} = mr\omega^2$
$KE_{trans} = \frac{1}{2}mv^2$
$KE_{rot} = \frac{1}{2}I\omega^2$
$I_{point} = MR^2$
$I_{disk} = \frac{1}{2}MR^2$
$I_{sphere} = \frac{2}{5}MR^2$
$\tau = rF\sin\theta = r_\perp F$
$\omega = \Delta\theta/\Delta t$
$\alpha = \Delta\omega/\Delta t$
$\tau = I\alpha$

$L = I\omega$
$\tau = \frac{\Delta L}{\Delta t}$
$P = \frac{F}{A}$
$P_{gauge} = P - P_{atm}$
$\rho = \frac{M}{V}$
$Q = \frac{\Delta V}{\Delta t} = Av$
$Q = \frac{\Delta P \pi r^4}{8\eta L}$
Power $= PQ$
$A \cdot v =$ constant
$P + \rho g y + \frac{1}{2}\rho v^2 =$ constant
$F_B = \rho g V_{displaced}$
$F_{ST} = \gamma L$
$P = \frac{4\gamma}{r}$
$h = \frac{2\gamma}{\rho g r}$
$N_R = \frac{\rho v L}{\eta} = \frac{2\rho v r}{\eta}$
$x_{rms} = \sqrt{2Dt}$
$x(t) = A\cos\omega t$ and $x_{max} = A$
$v(t) = -A\omega\sin\omega t$ and $v_{max} = A\omega$
$a(t) = -A\omega^2\cos\omega t$ and $a_{max} = A\omega^2$
$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx_{max}^2 = \frac{1}{2}mv_{max}^2$
$v = \sqrt{T/\mu}$ with $\mu = \frac{m}{L}$
spring: $\omega = \sqrt{k/m}$
pendulum: $\omega = \sqrt{g/\ell}$
$y(x,t) = A\cos(\omega t \pm kx)$ with $-$ for left-moving wave
$\omega = \frac{2\pi}{T}$ and $k = \frac{2\pi}{\lambda}$
$v = \lambda f = \frac{\omega}{k}$
$v_{sound} = 331\,\mathrm{m/s}\sqrt{\frac{T}{273\,K}}$
string: $\lambda_n = \frac{2L}{n}$, $f_n = \frac{nv}{2L}$ with $n = 1, 2, 3, \ldots$
open-open: $\lambda_n = \frac{2L}{n}$, $f_n = \frac{nv}{2L}$ with $n = 1, 2, 3, \ldots$
open-closed: $\lambda_n = \frac{4L}{n}$, $f_n = \frac{nv}{4L}$ with $n = 1, 3, 5, \ldots$
$f_{obs} = f_{src}\frac{v}{v \pm v_{src}}$ with $-$ for src moving towards obs
$f_{obs} = f_{src}\frac{v \pm v_{obs}}{v}$ with $+$ for obs moving towards src
$f_{beat} = |f_1 - f_2|$
$I = \frac{P}{A} = \frac{P}{4\pi r^2}$
$\beta = 10\log\frac{I}{I_0}$ in dB with $I_0 = 10^{-12}\,\mathrm{W/m^2}$
$1\,\mathrm{atm} = 10^5\,\mathrm{Pa} = 760\,\mathrm{mm\cdot Hg}$
$\rho_{water} = 10^3\,\mathrm{kg/m^3}$
$1\,\mathrm{cal} = 4.186\,\mathrm{J}$ and $1\,\mathrm{Cal} = 1000\,\mathrm{cal}$
$\cos\theta =$ adjacent/hypotenuse
$\sin\theta =$ opposite/hypotenuse
$\tan\theta = \sin\theta/\cos\theta$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$