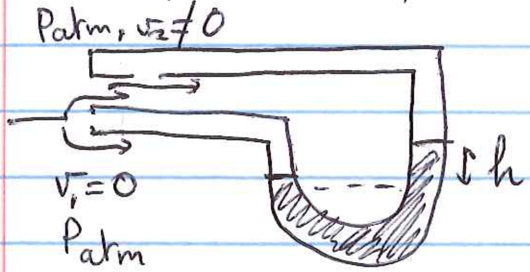
Bernouilli's equation

$$\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$$

$\uparrow$ OE $\qquad \uparrow$ PE $\qquad \uparrow$ KE

* Venturi effect : $v \rightarrow$ decreased P

Air speed sensor, water speed sensors on sail boats

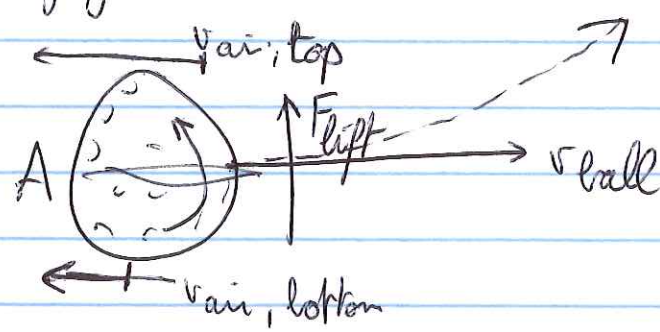


$$P_{atm} + \rho g h = P_{atm} + \frac{1}{2}\rho v^2$$

$$\hookrightarrow \cancel{\rho} g h = \frac{1}{2}\cancel{\rho} v^2$$

$$v = \sqrt{2gh}$$

* Golf ball, baseball .



$$v_{air,top} > v_{air,bottom}$$

$$\downarrow$$

$$P_{top} < P_{bottom}$$

$$F_{lift} = A \cancel{P_{top}} (P_{bottom} - P_{top})$$

* Vascular flutter : patients with arteriosclerosis, plaque buildups in arteries

restriction in diameter

↓

v increase due to $Q = Av = \text{constant}$

↓

P decreases

↓

artery walls collapse

↓

v zero

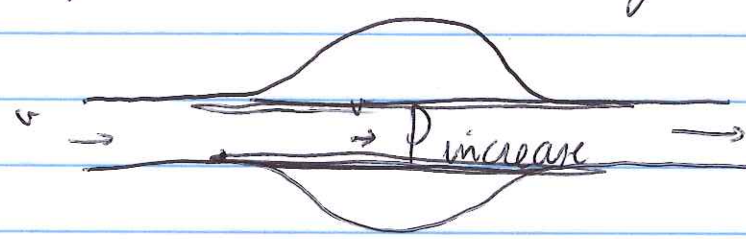
↓

P increases

↓

artery opens again

* Aneurysm : weaker spot in artery → walls balloon outward



v decreases due to continuity equation

→ P increases → ruptures the wall

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

$\hookrightarrow$ conservation of energy

$$KE = \frac{1}{2} m v^2 \rightarrow \frac{1}{2}\rho v^2 = \frac{KE}{V}$$

$$\frac{\text{energy}}{\text{time}} = \frac{\text{energy}}{\text{volume}} \cdot \frac{\text{volume}}{\text{time}} = \text{power}$$

$$\text{Power} = \left(P + \rho g h + \frac{1}{2}\rho v^2\right) \underline{Q}$$

Example: How much power does the heart use to supply blood?

$$Q = 83 \ \text{cm}^3/\text{s}$$
$$\Delta P = 110 \ \text{mm Hg}$$
$$v = 30 \ \text{cm/s}$$
$$h = 5 \ \text{cm}$$

$$PQ = (110 \ \text{mm Hg})\left(\frac{10^5 \ \text{Pa}}{760 \ \text{mm Hg}}\right)\left(83 \times 10^{-6} \ \text{m}^3/\text{s}\right) = 1.2 \ \text{W}$$

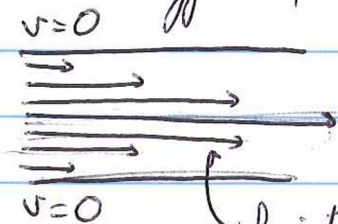
$$(\rho g h) Q = (1050 \ \text{kg/m}^3)(9.8 \ \text{m/s}^2)(0.05 \ \text{m})\left(83 \times 10^{-6} \ \frac{\text{m}^3}{\text{s}}\right) = 0.04 \ \text{W}$$

$$\left(\frac{1}{2}\rho v^2\right) Q = \frac{1}{2}(1050 \ \text{kg/m}^3)(0.3 \ \text{m/s})^2\left(83 \times 10^{-6} \ \text{m}^3/\text{s}\right) = 0.004 \ \text{W}$$

total power is $\underline{1.244 \ \text{W}}$

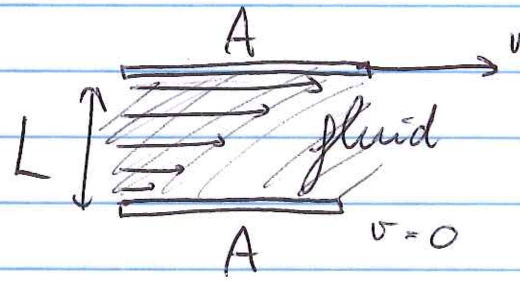
* Viscosity : non-conservative effects, causes energy loss

Laminar flow :



v largest at center

friction internally to the fluid
adhesive and cohesive forces



What force do I need to apply to move the top layer with a constant velocity v?
(energy loss)

$$F = \eta \frac{vA}{L}$$, η = viscosity coefficient in units Pa·s

units of $\frac{FL}{vA} = \frac{N \cdot m}{m/s \; m^2} = \frac{N}{m^2} s$

η, ξ water $\eta = 1.005 \times 10^{-3}$ Pa·s (at 20°C)

0.284×10^{-3} Pa·s (at 100°C)

blood $\eta = 2.084 \times 10^{-3}$ Pa·s (at 37°C)

honey $\eta =$ 10.0 Pa·s

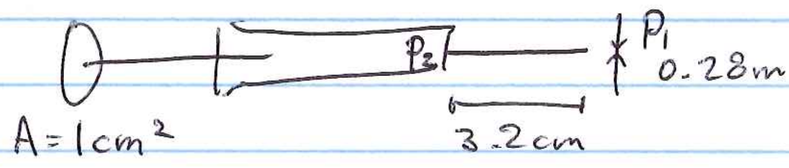
Flow of fluid through a pipe

$$Q = \frac{P_2 - P_1}{R} = \frac{\text{pressure difference}}{\text{resistance to flow}}$$

cylindrical pipe, with radius r, length l, fluid with a viscosity η:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}$$

Example: What is the force needed to inject a patient with a syringe that is 3.2 cm long, 0.28 mm diameter, mass flow of 1.5 g/s, liquid has η, ρ of water.
Plunger has surface area of 1 cm²



$$Q = \frac{1.5\,g}{s} \cdot \rho_{H_2O} = \frac{1.5\,g}{s} \cdot \frac{1\,cm^3}{g} = 1.5\,\frac{cm^3}{g} = 1.5 \times 10^{-6}\ m^3/s$$

1.005×10^{-3} Pa-s

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l} \Rightarrow (P_2 - P_1) = \frac{8\eta l Q}{\pi r^4} = 3.2\ atm$$

$$P_2 - P_1 = 3.2\,atm \rightarrow F = A(P_2 - P_1) = (1\,cm^2)(3.2\,atm)$$
$$= \underline{32\ N}$$