Physics 107: Physics for Life-Sciences

Midterm Exam: November 17, 2014

This test is administered under the rules and regulations of the honor code of the College of William & Mary.

Name: Solutions

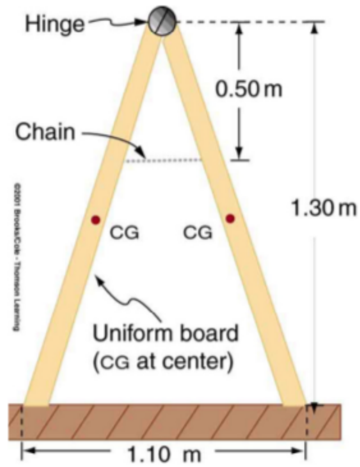
Signature: ______________________

Instructions:

- This is a closed book, closed notes test.
- Calculators are permitted, but not laptops or cell phones. Devices with wireless connections are not allowed.
- Start your work from the fundamental equations on the formula sheet, and derive any additional expressions that you may need.
- Circle your answer for each part of each problem.
- Clearly mark out any work that you wish the grader to disregard. Do not waste your time erasing.
- Your work will be graded based on your ability to write down a logical and organized solution grounded in the correct assessment of the physics of a situation. No credit will be given for an answer that is not justified by a logical solution or where that justification is not organized or readable. Partial credit will be given up to the point where your solution departs from a correct analysis of the physics involved for any given part of a problem.

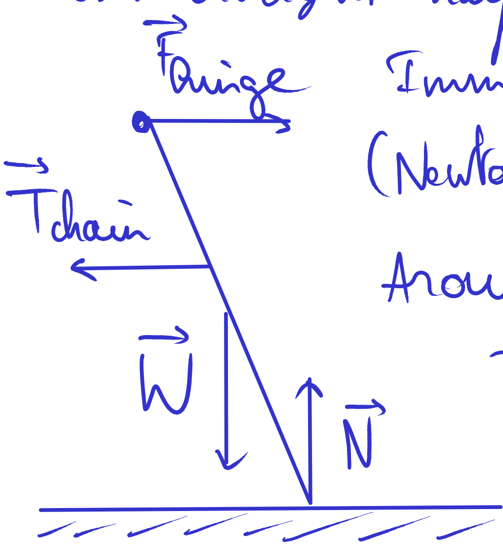
| Question | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 10 | |
| Total: | 70 | |

1. A deli owner creates a lunch special display board by taking a uniform board which has a mass of 8.00 kg and positioning it with the dimensions as given in the figure (the chain is massless). The sign is standing in equilibrium on the sidewalk. Assume that there is no friction between the legs and the sidewalk.




(a) (10 points) What is the tension in the chain?

Consider right half of the board.




Immediately: (Newton 2nd law) $F_{hinge} = T_{chain}$

$W = N = (4.00kg)\,g$

Around hinge: $\tau_{net} = 0$

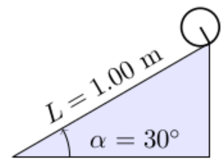
$$\tau_{net} = (0.50m)\,T_{chain} + \left(\frac{1.10m}{4}\right) W - \left(\frac{1.10m}{2}\right) N = 0$$

$\longrightarrow$ $T_{chain} = 21.6\ N$

(b) (10 points) What is the force exerted on each side of the hinge?

$F_{hinge} = T_{chain} = 21.6\ N$

2. Three objects are released from rest at the top of a ramp: a solid ball, a solid cylinder and a hollow cylinder. They are all made out of aluminum with a density of 2700 kg/m$^3$ and have the same radius of 10 cm. Both cylinders have a length of 50 cm, and the hollow cylinder has a wall thickness of 2 mm (which you can ignore where appropriate). The 1.00 m long ramp is at an angle of 30° with the horizontal.




The moment of inertia of a solid sphere around its center is $I_{solid\,sphere} = \frac{2}{5}MR^2$, of a solid cylinder $I_{solid\,cylinder} = \frac{1}{2}MR^2$, and of a hollow cylinder $I_{hollow\,cylinder} = \frac{1}{2}M(R^2_{inner} + R^2_{outer})$,

(a) (5 points) Which of the three objects will have the highest velocity at the bottom of the ramp? Justify your answer without resorting to calculations.

Sphere: Equal PE is available to all objects (per unit mass). This energy is converted into $KE_{trans} + KE_{rot}$. The smallest $I$ will result in smallest $KE_{rot}$ and largest $KE_{trans}$, and thus highest velocity.

(b) (15 points) Using conservation of energy, calculate the velocity at the bottom of the ramp for each of the objects.

$PE = Mgh$ for all three objects

$$KE_{trans} = \frac{1}{2}Mv^2$$

$$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}\frac{I}{R^2}v^2$$

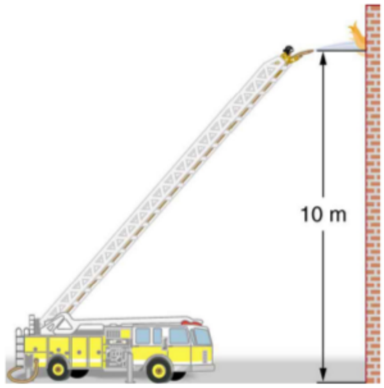
$$\rightarrow Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{I}{R^2}v^2 = \frac{1}{2}\left(M + \frac{I}{R^2}\right)v^2$$

or $v^2 = \dfrac{2gh}{1+\frac{I}{MR^2}}$ : $v_{sphere} = \sqrt{\dfrac{2gh}{7/5}} = 2.65\ m/s$

$$v_{sol.\,cyl} = \sqrt{\frac{2gh}{3/2}} = 2.56\ m/s$$

$$v_{hol.\,cyl} \simeq \sqrt{\frac{2gh}{2}} = 2.21\ m/s$$

3. A fire hose has a diameter of 6.40 cm. Suppose the hose carries a flow of 40.0 l/s (1 liter is a volume of 1 $dm^3$). The hose goes 10.0 m up a ladder to a nozzle with an inside diameter of 3.00 cm. The water has a density of 1.00 $kg/dm^3$ and a viscosity of 1.00 mPa · s.




(a) (5 points) What is the velocity with which the water will flow out of the nozzle?

$$Q = Av = 40.0\, \ell/s = 40.0\, dm^3/s = 40.0 \times 10^{-3}\, m^3/s$$

$$A = \pi \left(\frac{d}{2}\right)^2 = 0.00071\ m^2$$

$\hookrightarrow v = 56.6\ m/s$ in the nozzle

(b) (5 points) Will the flow in the fire hose be turbulent?

$v = 12.4\ m/s$ in the hose (similar calc.)

$$N_R = \frac{2\rho v r}{\eta} = 796 \times 10^3 \gg 3000$$

$\longrightarrow$ turbulent

(c) (5 points) Will the flow in the nozzle be turbulent?

$$N_R = \frac{2\rho v r}{\eta} = 1.7 \times 10^6 \gg 3000$$

$\longrightarrow$ turbulent

(d) (5 points) What must the gauge pressure in the fire engine be to sustain this flow?

$$P_{engine} + \frac{1}{2}\rho v_{hose}^2 = P_{atm} + \rho g h + \frac{1}{2}\rho v_{nozzle}^2$$

$$P_{engine} - P_{atm} = \rho g h + \frac{1}{2}\rho\left(v_{nozzle}^2 - v_{hose}^2\right)$$

$$= 1.62 \times 10^6 \text{ Pa}$$

4. (10 points) Atmospheric pressure is sometimes expressed as 760 mm Hg. This means that atmospheric pressure can cause a column of mercury (Hg) to raise over 760 mm in a glass vacuum tube (pressure at the mercury surface is zero). How high can atmospheric pressure cause a column of water to rise in a similar vacuum tube? The density of mercury is 13.6 kg/dm$^3$ and the density of water is 1.00 kg/dm$^3$.

$$P_{atm} = \rho_{Hg}\, g\, h_{Hg} = \rho_{H_2O}\, g\, h_{H_2O} = \underbrace{760\text{mm}}_{h_{Hg}}\ \text{Hg}$$

$$\hookrightarrow \quad h_{H_2O} = \frac{\rho_{Hg}}{\rho_{H_2O}}\, h_{Hg} = 10.3 \text{ m}$$

Possibly useful relations (feel free to detach this page):

$\vec{v}_{avg} = \Delta\vec{x}/\Delta t$
$\vec{a}_{avg} = \Delta\vec{v}/\Delta t$
$v = v_0 + at$
$v_{avg} = \frac{v_0+v}{2}$
$x = x_0 + v_0 t + \frac{1}{2}at^2$
$v^2 = v_0^2 + 2a(x - x_0)$
$R = \frac{v_0^2}{g}\sin 2\theta$
$h = \frac{v_0^2}{2g}\sin^2\theta$
$\vec{F}_{net} = m\vec{a}$
$\vec{F}_{BA} = -\vec{F}_{AB}$
$\vec{W} = m\vec{g}$
$\vec{g} = 9.80\,\mathrm{m/s^2}$ downward
$0 \le f_s \le \mu_s N$
$f_k = \mu_k N$
$\frac{F}{A} = Y\frac{\Delta L}{L}$
$F_k = -kx$
$KE = \frac{1}{2}mv^2$
$W = Fd\cos\theta$
$W_{\mathrm{net}} = -\Delta PE$
$W_{\mathrm{net}} = \Delta KE$
$PE_k = \frac{1}{2}kx^2$
$PE_g = mgh$
$KE_i + PE_i + W_{nc} = KE_f + PE_f$
$P = \frac{W}{\Delta t}$
$\mathrm{Eff} = \frac{W_{out}}{E_{in}}$
$F_G = G\frac{mM}{r^2}$
$G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2\ /\ kg^2}$
$\vec{I} = \vec{F}_{avg}\Delta t$
$\vec{p} = m\vec{v}$
$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$
$v_1 - v_2 = v_2' - v_1'$
$\theta = \frac{s}{r}$
$v = r\omega$
$f = \frac{1}{T}$
$\omega = 2\pi f$

$a_c = \frac{v^2}{r} = r\omega^2$
$F_c = m\frac{v^2}{r} = mr\omega^2$
$KE_{rot} = \frac{1}{2}I\omega^2$
$I_{point} = MR^2$
$I_{disk} = \frac{1}{2}MR^2$
$I_{sphere} = \frac{2}{5}MR^2$
$\tau = rF\sin\theta = r_\perp F$
$\omega = \frac{\Delta\theta}{\Delta t}$
$\alpha = \frac{\Delta\omega}{\Delta t}$
$\tau = I\alpha$
$L = I\omega$
$\tau = \frac{\Delta L}{\Delta t}$
$P = \frac{F}{A}$
$P_{gauge} = P - P_{atm}$
$\rho = \frac{M}{V}$
$Q = \frac{\Delta V}{\Delta t} = Av$
$Q = \frac{\Delta P \pi r^4}{8\eta L}$
Power $= PQ$
$A \cdot v =$ constant
$P + \rho g y + \frac{1}{2}\rho v^2 =$ constant
$F_B = \rho g V_{displaced}$
$F_{ST} = \gamma L$
$P = \frac{4\gamma}{r}$
$h = \frac{2\gamma}{\rho g r}$
$N_R = \frac{\rho v L}{\eta} = \frac{2\rho v r}{\eta}$
$x_{rms} = \sqrt{2Dt}$
$1\,\mathrm{atm} = 10^5\,\mathrm{Pa} = 760\,\mathrm{mm \cdot Hg}$
$\rho_{water} = 10^3\,\mathrm{kg/m^3}$
$1\,\mathrm{cal} = 4.186\,\mathrm{J}$
$1\,\mathrm{Cal} = 1000\,\mathrm{cal}$
$\cos\theta =$ adjacent/hypotenuse
$\sin\theta =$ opposite/hypotenuse
$\tan\theta = \sin\theta/\cos\theta$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$