PHYS 107 - Week 13 - Wednesday

* Simple harmonic motion : any oscillation that follows Hooke's law, like a mass & spring system, pendulum, clamped ruler, but much more broadly applicable.

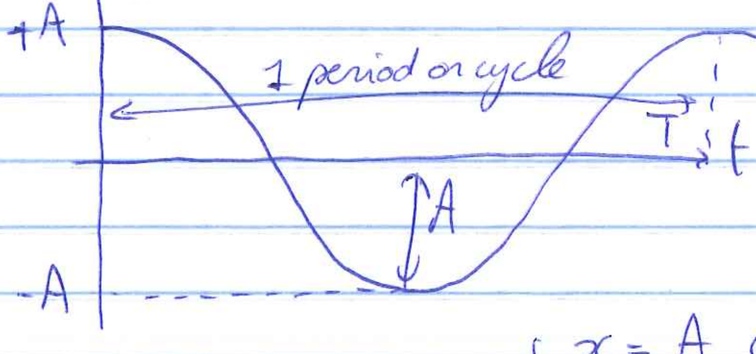
A = amplitude = maximum displacement from equilibrium

$f = \frac{1}{T}$ = frequency independent of A

* Mass & spring : $T = 2\pi\sqrt{\frac{m}{k}}$

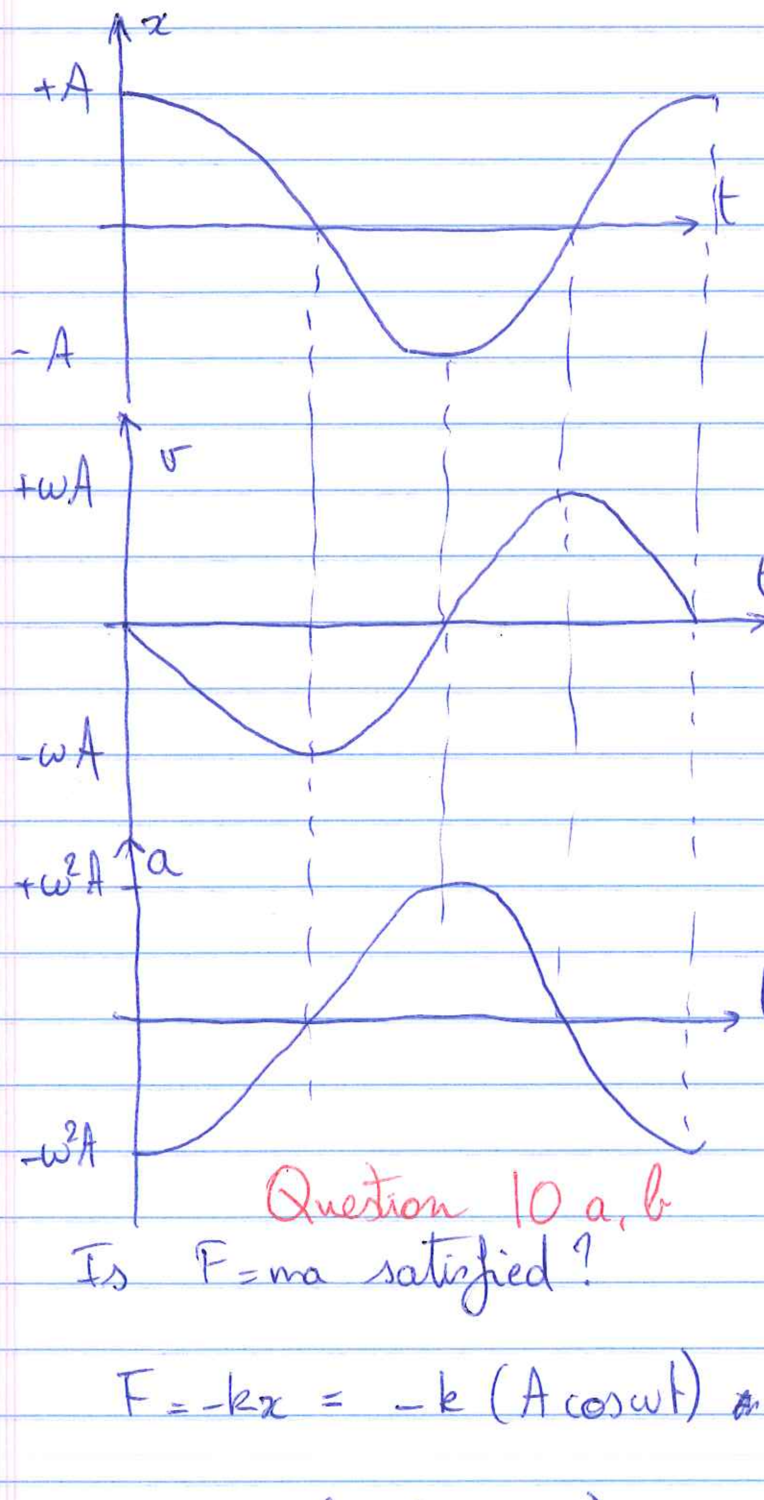
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \omega = \sqrt{\frac{k}{m}}$$

* How does displacement from equilibrium change over time?



$$\begin{cases} x = A\cos(\omega t) \\ x = A\cos(2\pi f t) \\ x = A\cos(2\pi \frac{t}{T}) \end{cases}$$

* Connection between displacement and velocity/acceleration



$x = A\cos\omega t$

$v = -\omega A \sin\omega t$
$= \text{slope of } x \text{ versus } t$

$v_{max} = \omega A = \sqrt{\frac{k}{m}}\, A$

$a = -\omega^2 A \cos\omega t$
$= \text{slope of } v \text{ versus } t$

$a_{max} = \omega^2 A = \frac{k}{m} A$

Question 10 a, b

Is $F = ma$ satisfied?

$F = -kx = -k(A\cos\omega t)$

$ma = m(-\omega^2 A \cos\omega) = -m\frac{k}{m}A\cos\omega t = -kA\cos\omega t$

↳ yes, $F = ma$

* Energy conservation:

$$E_{total} = \underbrace{\frac{1}{2}mv^2}_{KE} + \underbrace{\frac{1}{2}kx^2}_{PE}$$

$$= \frac{1}{2}m(-\omega A\sin\omega t)^2 + \frac{1}{2}k(A\cos\omega t)^2$$

$$= \frac{1}{2}m\underbrace{\omega^2}_{k/m}A^2\sin^2\omega t + \frac{1}{2}kA^2\cos^2\omega t$$

$$= \frac{1}{2}kA^2(\underbrace{\sin^2\omega t + \cos^2\omega t}_{=1})$$

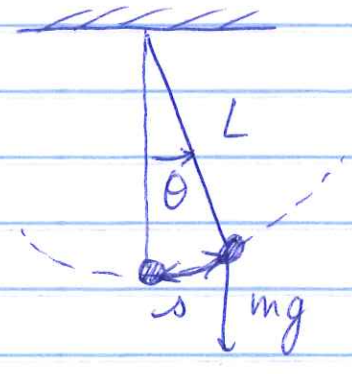
$$= \frac{1}{2}kA^2 = \underline{\text{constant}}$$

$$E_{total} = \frac{1}{2}kA^2 = \frac{1}{2}kx_{max}^2 = \frac{1}{2}mv_{max}^2$$

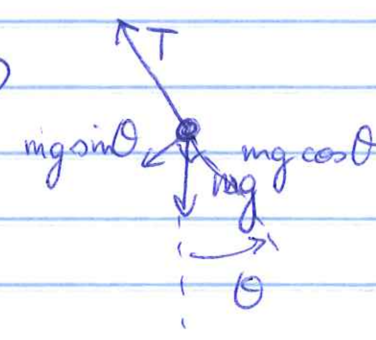
$$PE_{max} = KE_{max}$$

Question 10 c

* Simple pendulum



FBD



↳ radial: $T - mg\cos\theta = 0$

tangential: $F_{restoring} = -mg\sin\theta$

For small angles: $\sin\theta \approx \theta$ (in radians)

$\rightarrow \quad F_{restoring} = -mg\,\theta$

Distance from equilibrium $s = L\theta$

$$F = -mg\frac{s}{L} = -\left(\frac{mg}{L}\right)s$$

$\rightsquigarrow F = -kx$ with $\underline{k = \frac{mg}{L}}$

$\Rightarrow$ Simple Harmonic motion with

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left(\frac{mg}{L}\right)}} = \underline{2\pi\sqrt{\frac{L}{g}}}$$

again independent of A! (as found by
even independent of m! Galileo in church)

$T = 2\pi\sqrt{\frac{L}{g}}$ depends on g $\rightarrow$ longitude matters

Question 13 a, 15

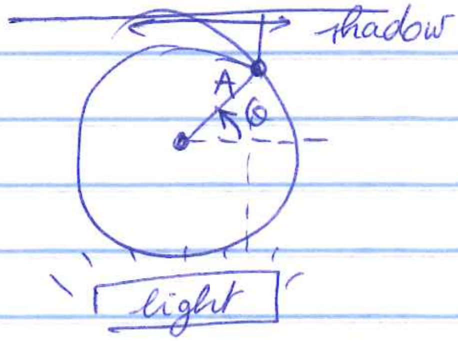
When does SHM break down? when $\sin\theta = \theta$ is not valid anymore

$\rightarrow$ large θ

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \rightarrow \omega = 2\pi f = \sqrt{\frac{g}{L}}$$

~~Question~~

* SHM and rotation



$x = A\cos\theta$

$\theta = \theta_0 + \omega t$ with $\theta_0 = 0$

$\downarrow$

$x = A\cos\omega t = A\cos(2\pi f t)$

* Energy of the pendulum:

$E_{total} = \frac{1}{2} k A^2$ for mass & spring

for pendulum with small θ: $k = \frac{mg}{L}$

$\Rightarrow E_{total} = \frac{1}{2}\frac{mg}{L}A^2 = \frac{1}{2} m\omega^2 A^2$

$E_{total} \propto A^2$ is a general feature of SHM

* Physical pendulum $T = 2\pi\sqrt{\frac{I}{mgd}}$

SIMPLE HARMONIC MOTION