PHYS 107 - Week 05 - Friday

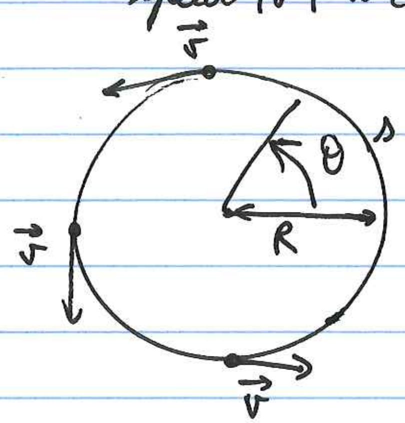
Q Acceleration with constant speed

* Uniform circular motion

~~Q Velocity and acceleration~~

speed $|\vec{v}|$ is constant, but direction of $\vec{v}$ changes

$\longrightarrow$ velocity is <u>not</u> constant



$|\vec{v}| = v$ is constant

radius R is constant

In time Δt, the object crosses an angle θ and a distance s

$\hookrightarrow$ $\theta = \dfrac{s}{R}$ or $s = \theta R$

One full rotation: $s = 2\pi R \rightarrow \underline{\theta = 2\pi}$

degrees	vs.	radians
$360°$		2π
$180°$		π
$90°$		$\dfrac{\pi}{2}$

Period T for 1 full revolution

$\hookrightarrow$ $v = \dfrac{d}{\Delta t} = \dfrac{2\pi R}{T}$

Frequency $f = \dfrac{1}{T}$ in units $\dfrac{1}{s} = \underline{Hz}$

* Example: earth rotates with a period of 1 day, and has radius of 6400 km
What is the speed with which we are rotating right now? (assume equator)

$$v = \frac{2\pi R}{T} = \frac{2\pi\,(6400\times 10^3\,m)}{(24\,hrs)\left(\frac{3600\,s}{hr}\right)} = 460\,m/s = 920\,mph$$

* Angular velocity

$$\omega = \frac{\Delta\theta}{\Delta t} \quad \left(\text{units: } \frac{rad}{s}\right)$$

$$\Delta t = \Delta s / v = \Delta s \Big/ \frac{2\pi R}{T}$$

$$\Delta\theta = \frac{\Delta s}{R}$$

$$\hookrightarrow \omega = \frac{\Delta\theta}{\Delta t} = \frac{\frac{\Delta s}{R}}{\Delta s / \frac{2\pi R}{T}} = \frac{1}{R}\cdot\frac{2\pi R}{T} = \frac{2\pi}{T}$$

$\omega = \frac{2\pi}{T}$: angular velocity or frequency

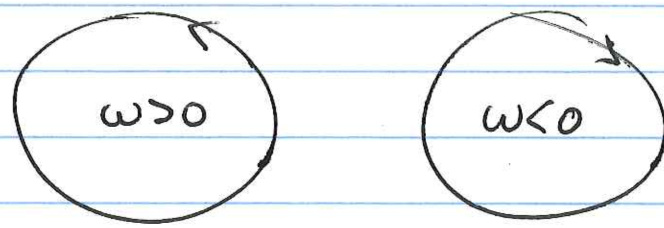
$f = \frac{1}{T}$: frequency

$$\hookrightarrow \omega = 2\pi f$$

Since $v = \frac{2\pi R}{T} \rightarrow \underline{v = \omega R}$

* Signs of ω

$\omega > 0$: CCW rotation
$\omega < 0$: CW rotation

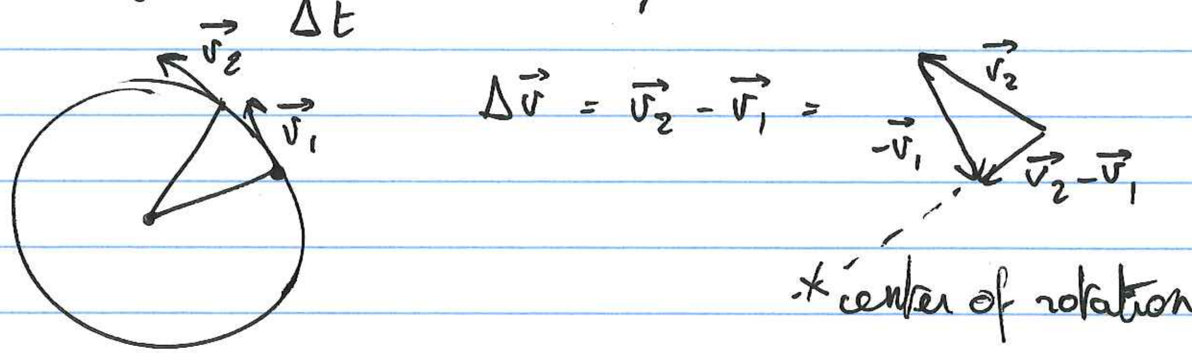


Q Velocity and acceleration

* Angular acceleration : kinematics

Since $\vec{v}$ is changing (direction), there must be an acceleration!

$$\vec{a}_c = \frac{\Delta\vec{v}}{\Delta t} = \text{centripetal acceleration}$$



It turns out that $a_c = \frac{v^2}{R}$ or $a_c = \frac{(R\omega)^2}{R} = R\omega^2$

* Angular acceleration: dynamics

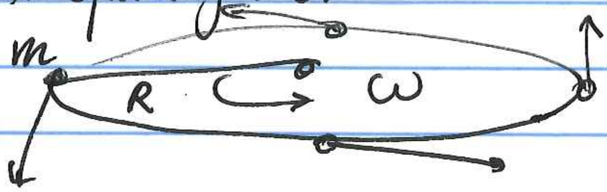
There must be a force that generates the acceleration $\vec{a}_c$ based on Newton's 2nd law:

$$\vec{F}_{net} = m\vec{a}_c$$

Since $\vec{a}_c$ points to the center of rotation there must be a net force pointing to the center of rotation as well.

~~Q Hill~~ Q Roller Coaster

* Spinning mass

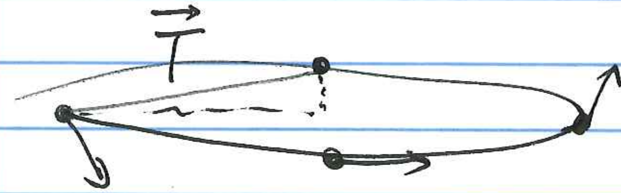


$$a_c = \frac{v^2}{R} = R\omega^2$$

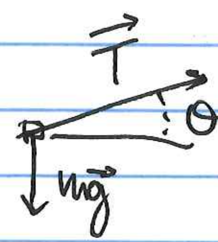
force is tension in string

$$T = ma_c = \frac{mv^2}{R} \text{ inwards}$$

* With gravity



FBD



$$\vec{F}_{net} = m\vec{a}_c$$

$x: \; -T\cos\theta = -ma_c = -m\frac{v^2}{R} \Rightarrow mg\frac{\cos\theta}{\sin\theta} = m\frac{v^2}{R}$

$y: \; T\sin\theta - mg = 0$

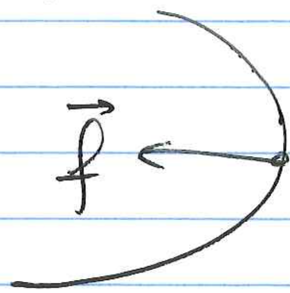
$\hookrightarrow T = \frac{mg}{\sin\theta}$

Q Hill

* Car on a flat road in a curve:

what provides the force that generates $\vec{a_c}$?

friction! $\qquad \vec{f} = m\vec{a}_C$, both pointing inwards



Q Car in curve