Rotational kinematics → description of motion only
↳ nothing about causes

↓

Rotational dynamics → causes of angular motion
velocity
acceleration

$\vec{F} = m\vec{a}$

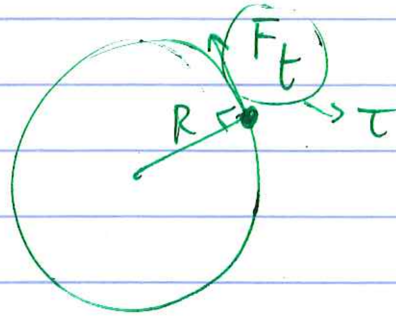
dynamics ($\vec{F}$), mass (inertial) (m), kinematics ($\vec{a}$)

force $\vec{F}$ → torque $\vec{\tau} = F_{\perp} R$

acceleration $\vec{a}$ → angular acceleration

$\frac{a_t}{R} = \alpha$

$a_t = \alpha R$



mass m → I, moment of inertia

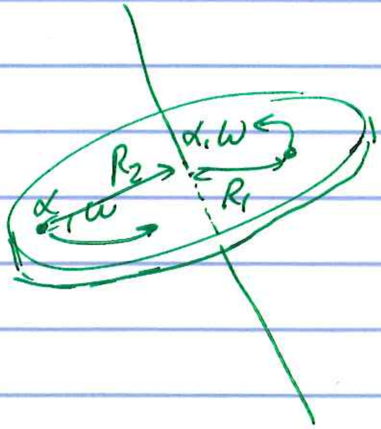
$\vec{F}_t = m\vec{a}_t \qquad F_t R = \tau$

$\tau = F_t R = m a_t R = mR^2 \alpha$

$\vec{\tau} = (mR^2)\vec{\alpha} = I\alpha \qquad$ with $mR^2 = I$

τ CCW, τ positive → α positive

τ CW, τ negative → α negative



disc consists of many points with all same α, ω

$$\tau_1 = m_1 R_1^2 \alpha$$
$$\tau_2 = m_2 R_2^2 \alpha$$
$$\vdots$$
$$+ \overline{\qquad\qquad\qquad}$$
$$\vec{\tau}_{net} = \underbrace{(m_1 R_1^2 + m_2 R_2^2 + \dots)}_{I} \vec{\alpha}$$

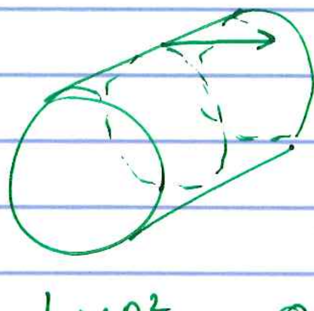
I is known for simple geometries (tabulated on pg. 328 of text)

$$I_{disk} = I_{cylinder} = \frac{1}{2} M R^2$$

M = total mass
R = outer radius

$$I_{hoop} = M R^2$$

Example: cylinder of mass 2kg, $R = 10$ cm



$F = 10N$ on outer diameter

$\downarrow$

what will α be?

$$I = \frac{1}{2} M R^2 = 0.01 \ kg \cdot m^2$$

$$\tau = F_\perp R = (10N)(10cm) = 1 \ N \cdot m$$

$$\tau = I\alpha \longrightarrow \alpha = \frac{\tau}{I} = 100 \frac{rad}{s^2}$$

force is applied at $\frac{R}{2}$

what will α be now?

A: $\alpha = 50 \frac{rad}{s^2}$

$$\alpha = \frac{\tau}{I} = \frac{F_{\perp} \frac{R}{2}}{\frac{1}{2} M R^2} = 50 \frac{rad}{s^2}$$

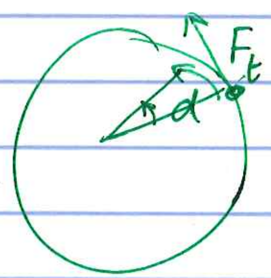
B: $\alpha = 100 \frac{rad}{s^2}$

C: $\alpha = 200 \frac{rad}{s^2}$

D: 

Work for ~~d~~ rotational dynamics

$$W = F_t\, d \cos \underbrace{\theta}_{=0} = F_t\, d = (F_t R)\left(\frac{d}{R}\right)$$



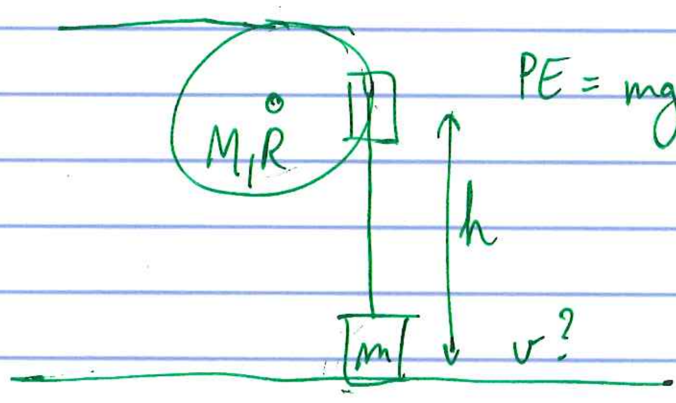
$$\underline{W = \tau \theta}$$

$$KE_{rot} = \frac{1}{2} m v_t^2 = \frac{1}{2} m R^2 \frac{v^2}{R^2} = \frac{1}{2} I \omega^2$$

$$\frac{v}{R} = \omega$$

$$W = F d \longrightarrow W_{rot} = \tau \theta$$

$$KE = \frac{1}{2} m v^2 \longrightarrow \underline{KE_{rot}} = \frac{1}{2} I \omega^2$$



$PE = mgh$