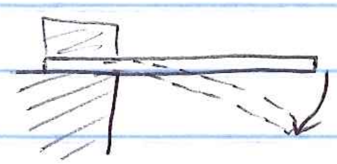
PHYS 107 - Week 12 - Friday

* Waves and periodic motion, oscillations

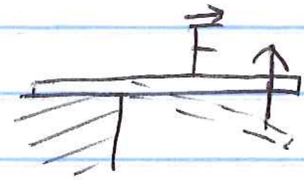
What is periodic motion? - grandfather clock with pendulum,
- guitar string, water waves,
- sound, light, electrical phenomena etc
- vibrations in molecules
- earthquakes, waves on earth
- oscillations in brightness of stars

Simplest oscillations and waves: caused by <u>restoring force</u> around equilibrium

Example: meter stick



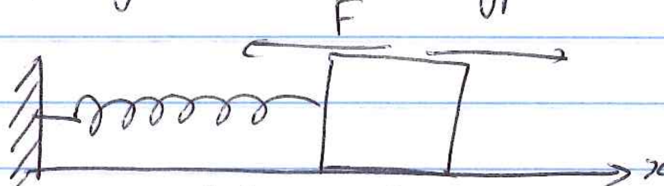
give stick some deviation from equilibrium
→ force wants to bring system back to equilibrium



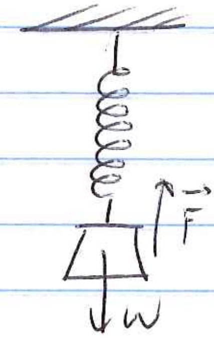
Features: - stable equilibrium position
- restoring force tries to bring system back to equilibrium
- system builds up kinetic energy and overshoots
- now the restoring force is on the opposite side

Q 8 Restoring force

* Spring as the prototypical oscillating system



equilibrium when
spring not extended



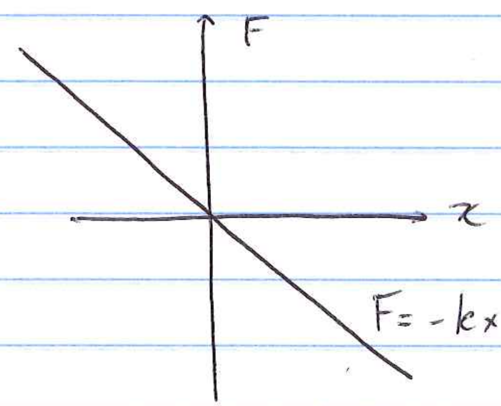
equilibrium when $F = W$

Hooke's law $F = -k\Delta L$ is the restoring force
equilibrium at $x = 0 \rightarrow F = -kx$

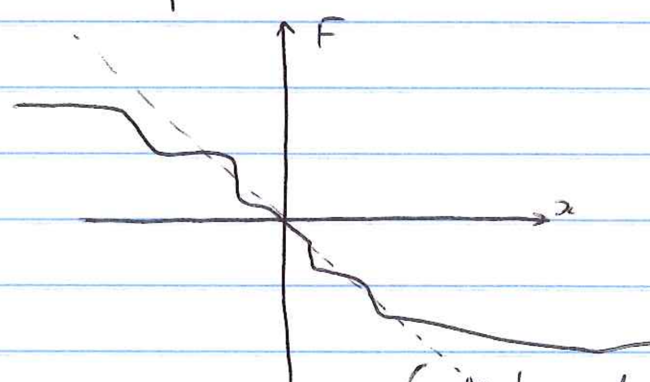
Q 9a,b Spring Max Acceleration/Energy

* Other types of springs: compression/expansion, shearing

Ideal:



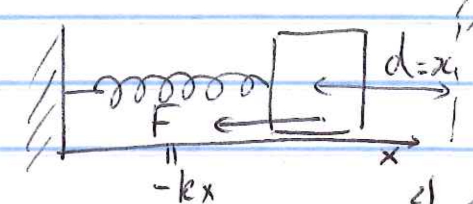
General.



↳ first order approximation
→ there is always a small enough region where $F = -kx$

* Energy   PE $\rightleftharpoons$ KE $\rightleftharpoons$ PE

Potential energy for elastic system:

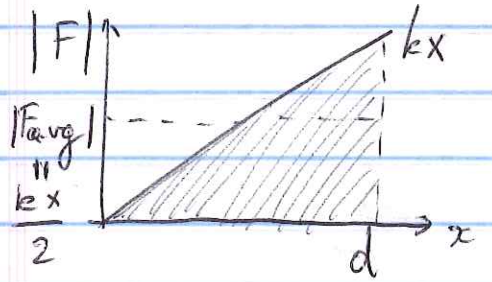


i) naive approximation:

$W = F \cdot d \cos\theta = Fx \cos 180°$

$= -kx \cancel{x} \cos 180° = kx^2$

2) but F is not constant



$\rightarrow W = F_{avg}\, x = \frac{1}{2} kx^2$

$PE = \frac{1}{2} k X^2$, with X = maximum value of x

* Frequency and period

T = period = time needed for 1 oscillation (unit: s)

$f = \frac{1}{T}$ = frequency = number of oscillations in 1 second (units: s^{-1} = Hz)

piano playing note A: 440 Hz $\rightarrow T = \frac{1}{f} = 2.27$ ms

WCWM: 90.3 MHz $\rightarrow T = 1.1 \times 10^{-8}$ s

$= 11$ ns

* Simple harmonic motion (SHM)
↳ name for any oscillation that follows Hooke's law like a mass and spring system, pendulum, ruler, but much more broadly applicable

Amplitude A = maximum displacement from equilibrium

In SHM : f, T are independent from A
→ same frequency for large and small amplitude (example: guitar string has same frequency regardless of amplitude)

What does the frequency depend on in SHM? $\underline{m, k}$

$$T = 2\pi \sqrt{\frac{m}{k}} \quad \text{or} \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \text{frequency}$$

$$\omega = 2\pi f = \sqrt{\frac{k}{m}} = \text{angular frequency}$$

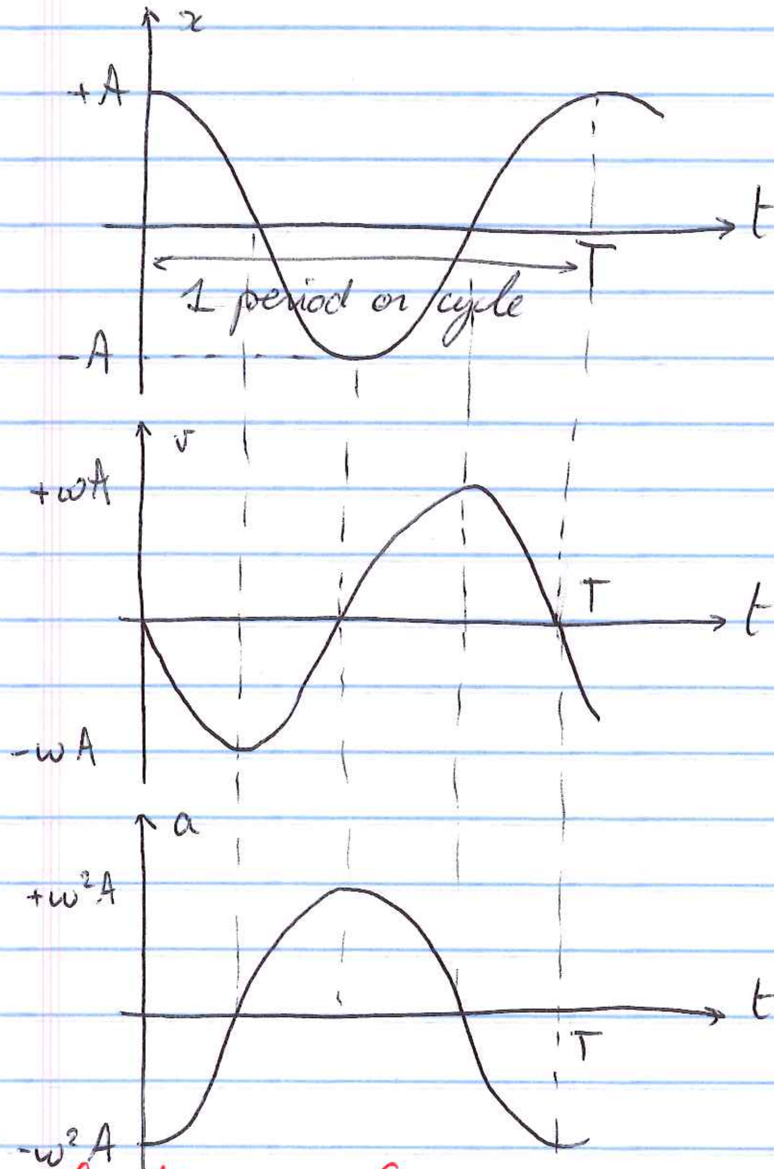
Why? $\left.\begin{array}{l} m \text{ in units kg} \\ k \text{ in units } \frac{N}{m} = \frac{kg}{s^2} \end{array}\right\}$ only way to get units of s^{-1} is as $\sqrt{\frac{k}{m}}$

Example: shock absorbers on a car with $m = 2500\,kg$
$k = 10^6\,N/m$

↳ $T = 2\pi\sqrt{\frac{m}{k}} = 0.3\,s$

* How does x change over time?

$$x = A\cos(\omega t) = A\cos(2\pi f t) = A\cos\left(2\pi \frac{t}{T}\right)$$



$$v = -\sqrt{\frac{k}{m}}\, A\cancel{\cos}\sin(2\pi f t)$$

$$= -\omega A \sin(\omega t)$$

$$a = -\omega^2 A\cos(\omega t)$$

Q 10a, b Mass on a Spring

$$\rightarrow \quad v_{max} = \omega A = \sqrt{\frac{k}{m}}\, A$$

$$a_{max} = \omega^2 A = \frac{k}{m} A$$

F = ma? yes: $F = -kx = -k(A\cos(\omega t)) = -m\omega^2 A\cos(\omega t)$

+ Energy conservation : KE + PE = constant

$$E_{total} = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$= \frac{1}{2} m (-\omega A \sin \omega t)^2 + \frac{1}{2} k (A \cos \omega t)^2$$

$$= \frac{1}{2} m (\underbrace{\omega^2}_{= k/m} A^2 \sin^2 \omega t) + \frac{1}{2} k (A^2 \cos^2 \omega t)$$

$$= \frac{1}{2} k A^2 (\sin^2 \omega t + \cos^2 \omega t) = \underline{\frac{1}{2} k A^2}$$

$$= \text{constant}$$

$$E_{total} = \frac{1}{2} k A^2 = \frac{1}{2} k x_{max}^2 = \frac{1}{2} m v_{max}^2$$

$$PE_{max} = KE_{max}$$