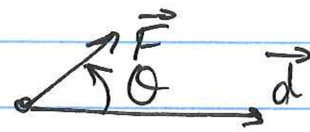
PHYS 107 - Week 06 - Friday

* Units of work and energy

$$\text{Work} = W = |\vec{F}| \cdot |\vec{d}| \cos\theta = F d \cos\theta$$



Work in units force × length

N × m

$$1\ \text{N}\cdot\text{m} = 1\ \text{J}, \text{ Joule}$$

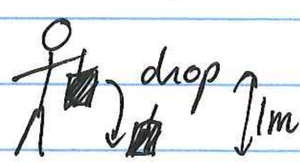
Other units of energy : $1\ \text{cal} = 4.186\ \text{J}$

$1\ \text{Cal} = 1\ \text{kcal}$

Granola bar : nutritional information

Work $W > 0$: energy being added to system by the force $\vec{F}$

e.g.



$W = mgd\cos\theta$, $\theta = 0°$

$= 100\ \text{J}$

$m = 10\ \text{kg}$

Work $W < 0$: energy being removed from system by force $\vec{F}$

work done <u>against</u> force $\vec{F}$

e.g.

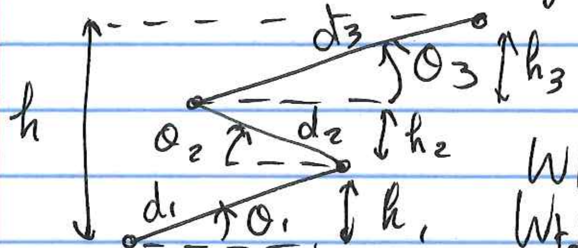
$m = 10\,kg$

$\theta = 180°$

↳ $W = -100\,J$

Q 2a, b, c

* Motions at an angle:



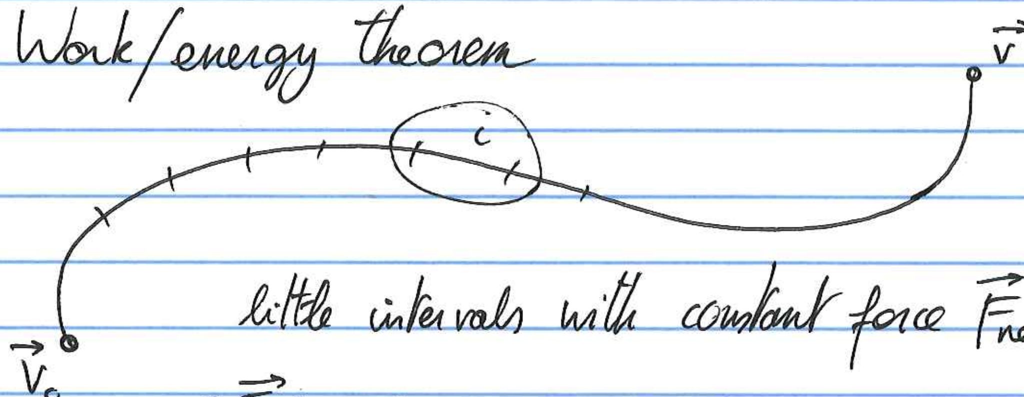
$W_{tot} = mg\,(d_1\cos\theta_1 + d_2\cos\theta_2 + d_3\cos\theta_3)$

$W_{tot} = mg\,(h_1 + h_2 + h_3)$

$W_{tot} = mg\,h$

↳ work done is independent of path details, only depends on total height

* Work/energy theorem



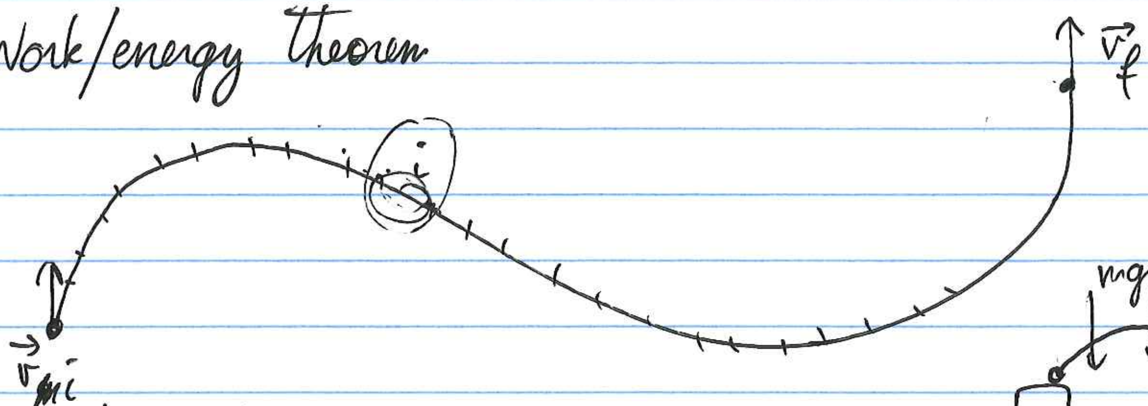
little intervals with constant force $\vec{F}_{net,i}$

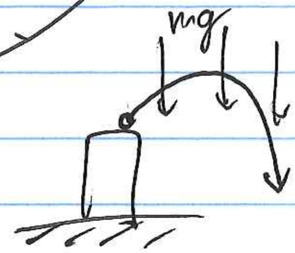
$W_i = F_{net,i}\, d_i \cos\theta_i$

$W_{tot} = \sum_i W_i$

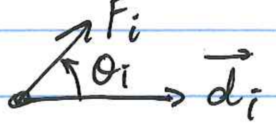
Along each interval: pick x in direction of $\vec{d}_i$

* Work/energy theorem





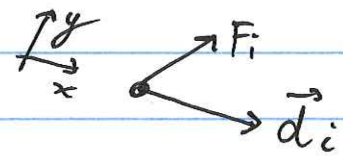
little interval i :



$$\underline{W_i = F_i d_i \cos\theta_i}$$

$$W_{total} = \sum_{i=0}^{f} W_i$$

in the little interval: x-direction along $\vec{d}_i$



$\vec{d}_i$ only has x-component

$d_{i,y} = 0$

$$\begin{cases} v_x^2 = v_{0,x}^2 + 2a_x d_x \\ v_y^2 = v_{0,y}^2 + 2a_y \cancel{d_y} = v_{0,y}^2 \end{cases}$$

$$a_x = \frac{F_{net,i,x}}{m} \qquad \underline{W_i = F_d\, d}$$

↑ dynamics

$$v^2 = v_x^2 + v_y^2 = v_0^2 + 2\frac{F_{net,i}}{m} d_x$$

$$v^2 = v_0^2 + \frac{2}{m} W_i$$

$$\underline{v_f^2} = \underline{v_i^2} + \frac{2}{m} \underline{W_{total}}$$

↑ kinematics

$$\begin{cases} v_x^2 = v_{x,0}^2 + 2a_x \underbrace{d_x}^{=d} = v_{x,0}^2 + 2\dfrac{F_{net,i}}{m} d_x \\ v_y^2 = v_{y,0}^2 + 2a_y \underbrace{d_y}_{=0} = v_{y,0}^2 \end{cases}$$

$$\hookrightarrow v^2 = v_0^2 + \frac{2}{m} \underbrace{\left(F_{net,i}\, d_x\right)}_{W_i}$$

→ for the full trajectory: $W_{net} = \sum W_i$
$= \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$

↳ Kinetic energy $KE = \frac{1}{2} m v^2$

Work/energy theorem : $W_{net} = KE_f - KE_i$

difference in kinetic energy is work done on system

* Example: picking up a 10 kg mass ; $v_i = 0$, $v_f = 0$

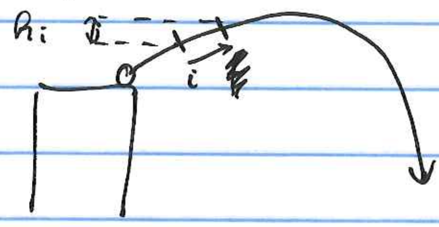
$W_g = -mgd = -100\,J$

$W_{net} = KE_f - KE_i = 0$

$\hookrightarrow W_{mechanical\ energy} + W_g = W_{net} = 0$

$\hookrightarrow W_{mechanical\ energy} = -W_g = 100\,J$

* Objects in free fall



$$W_1 = -mg\, d_i \cos\theta_i = -mgh_i = -mg(y_f - y_i)$$

$$W_2 = \cdots$$

$$\vdots$$

$$W_{total} = -mg(y_f - y_i)$$

Define Potential Energy $PE = mgy$

↳ Work done by gravity is difference between PE in initial and final states

→ $W_{net} = -(PE_f - PE_i)$

* Recap :

Work $W = Fd\cos\theta$

$W > 0$ is work done by force on the system to increase energy in the system

Kinetic energy $KE = \frac{1}{2}mv^2$

energy of motion of object with mass m

Potential energy PE in <u>gravity</u> $= mgh$

capacity of object to do work based on its position relative to Earth

Work/energy theorem : for <u>any</u> force the work done is

$$W_{net} = KE_f - KE_i$$

(independent of path)

For <u>gravity</u> : $W_{net} = -(PE_f - PE_i)$

$\Rightarrow$ Conservation of energy for gravity :

$$W_{net} = KE_f - KE_i = -(PE_f - PE_i)$$

$$\hookrightarrow PE_i + KE_i = PE_f + KE_f$$

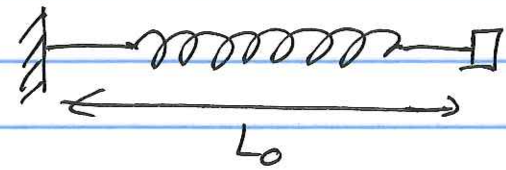
Q8 bowling ball

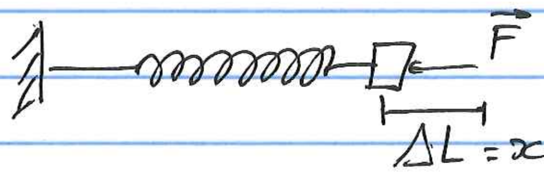
* Other examples of potential energy: <u>springs</u>

Hooke's law $F = k\Delta L$

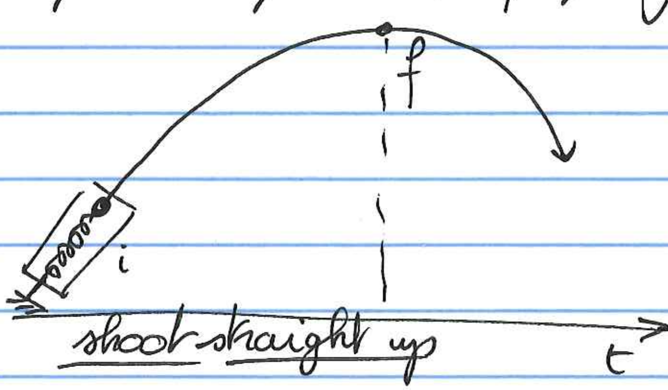
$\downarrow$

$PE = \frac{1}{2}k(\Delta L)^2 = \frac{1}{2}kx^2$





Example: compression of spring for launcher



$PE_i = mg\cancel{h_i}^{\,0} + \frac{1}{2}kx_i^2$

$KE_i = 0$

$PE_f = mgh_f + \cancel{\frac{1}{2}kx_f^2}^{\,0}$

$KE_f = 0$

$\rightarrow \frac{1}{2}kx^2 = mgh \rightarrow h = \frac{kx^2}{2mg}$

* Conservative forces: work only depends on the initial and final point, not on the path taken $\rightarrow$ PE exists