PHYS 107 - Week 3 - Monday

* Ballistic motion or projectile motion

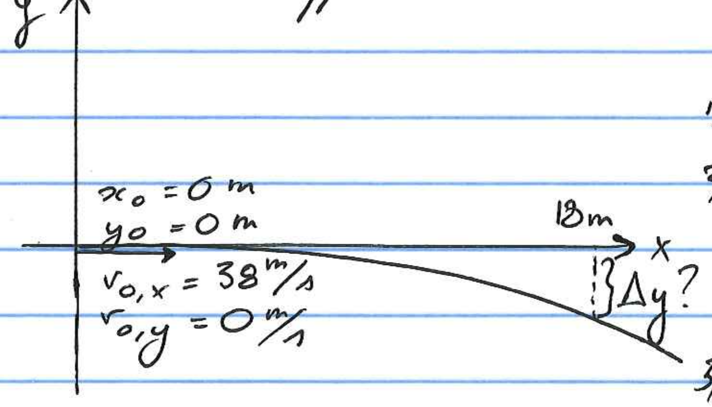
↳ only gravity works on objects
→ ignore air resistance

$$\underline{a_x = 0 \quad , \quad a_y = -g}$$

$$\hookrightarrow \begin{cases} x = x_0 + v_{0,x} t \\ y = y_0 + v_{0,y} t - \frac{1}{2} g t^2 \end{cases} \qquad \begin{cases} v_x = v_{0,x} = \text{constant} \\ v_y = v_{0,y} - g t \end{cases}$$

X and Y motion can be treated independently

Example: Tribe pitcher throws baseball horizontally at 38 m/s (≈ 85 mph). The ball crosses home plate 18m away. How far has it dropped when it crosses home plate?



1) choose origin at $x_0 = 0, y_0 = 0$

2) get time from x equation:

$$x = v_{0,x} t \rightarrow t = \frac{18\,m}{38\,m/s}$$

$$\rightarrow t = 0.47\,s$$

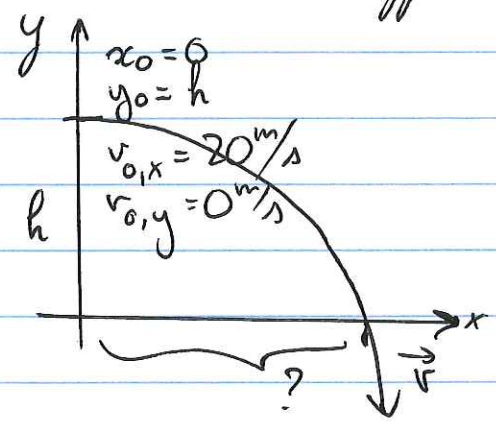
3) get vertical distance using t

$$y = -\frac{1}{2} g t^2 = \frac{1}{2}(9.80\,m/s^2)(0.47\,s)^2$$

$$\underline{= -1.1\,m}$$

Q 2D-kin B2

Example : A car drives off a cliff at 20 m/s (horizontally). The cliff is 15 m high. Where does the car land?



1) get time from y equation :

$$y = 0 = h - \frac{1}{2} g t^2$$

$$\longrightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(15\,m)}{20\,m/s^2}}$$

$$\underline{t = 1.75\,s}$$

2) get horizontal distance using t

$$x = v_{0,x} t = (20\,m/s)(1.75\,s)$$

$$x \; \underline{= 35\,m}$$

Follow-up question : what is the velocity upon impact?

$$v_x = v_{0,x} = 20\,m/s = \text{constant}$$

$$v_y = v_{0,y} - gt = -gt = -(9.80\,m/s^2)(1.75\,s)$$

$$= -17.2\,m/s$$

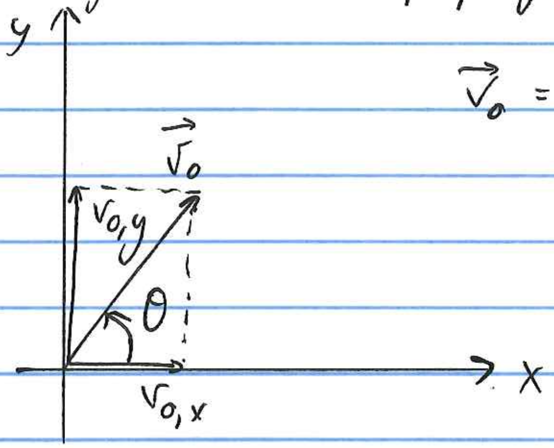
$$\hookrightarrow |\vec{v}| = \sqrt{(20\,m/s)^2 + (-17.2\,m/s)^2}$$

$$= \underline{26.4\,m/s}$$

$$\tan\theta = \left(\frac{-17.2\,m/s}{20\,m/s}\right) \rightarrow \underline{\theta = -40.7^\circ}$$

Q 2D-kin 3

* General case of projectile motion : $\vec{v}_0$ not horizontal



$$\vec{v}_0 = (\underbrace{v_0 \cos\theta}_{v_{0,x}}, \underbrace{v_0 \sin\theta}_{v_{0,y}})$$

$$\cancel{x = v_0 \cos\theta}$$

$$x = x_0 + v_{0,x}\, t = x_0 + (v_0 \cos\theta)\, t$$

$$y = y_0 + v_{0,y}\, t - \frac{1}{2} g t$$

$$= y_0 + (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

Starting from $x_0 = 0, y_0 = 0$

$$\rightarrow \begin{cases} x = v_0 \cos\theta \; t \\ y = v_0 \sin\theta \; t - \frac{1}{2} g t^2 \end{cases}$$

1) when does the object hit the ground?

$y = 0$ when $0 = v_0 \sin\theta \; t - \frac{1}{2} g t^2$

$$\rightarrow t = 0 \quad \text{or} \quad t = \frac{2 v_0 \sin\theta}{g}$$

2) where does the object hit the ground?

$$x = v_0 \cos\theta \; t = 2 \frac{v_0^2}{g} \cos\theta \sin\theta$$

$$x = \frac{v_0^2 \sin 2\theta}{g} \quad \text{using} \quad 2\cos\theta\sin\theta = \sin 2\theta$$

$$\hookrightarrow R = \frac{v_0^2 \sin 2\theta}{g} = \text{range}$$

Q Maximal range

Maximum range when $\sin 2\theta$ is largest
$\rightarrow \theta = 45°$

3) what is the maximum height?
↳ when is $v_y = 0$? what is the height then?

$$\cancel{y}\ v_y = \cancel{y}\ v_{0,y} - gt = v_0 \sin\theta - gt = 0$$

$$\rightarrow t = \frac{v_0 \sin\theta}{g} \quad \text{(half of total flight time)}$$

$$y_{max} = v_0 \sin\theta\ t - \frac{1}{2} g t^2$$

$$= \frac{v_0^2 \sin^2\theta}{g} - \frac{1}{2} \frac{v_0^2 \sin^2\theta}{g}$$

$$= \frac{1}{2} \frac{v_0^2 \sin^2\theta}{g}$$

Largest height when $\sin\theta$ is largest
$\rightarrow \theta = 90°$

Q Maximal height