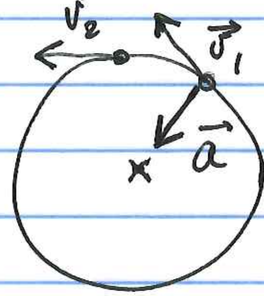
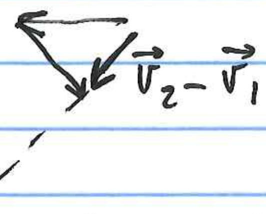
* Uniform circular motion

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

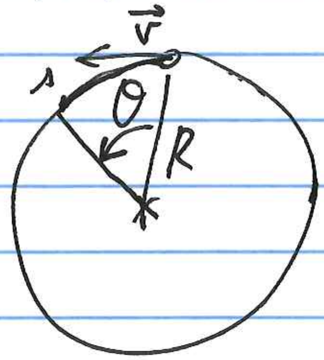


$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$$



* Uniform circular motion

$|\vec{v}| = v$ = constant, but direction of $\vec{v}$ changes



Δt time interval, object describes an angle θ, and crosses distance s

$$\theta = \frac{s}{R} \rightarrow s = \theta R$$

One full rotation: $\theta = 360°$, $s = 2\pi R$

$$\underline{\theta} = \frac{s}{R} = \frac{2\pi R}{R} = \underline{2\pi} \text{ for 1 full rotation}$$

radians

radians	degree
2π	$360°$
π	$180°$
$\pi/2$	$90°$

angle in radians $= (\text{angle in degrees}) \dfrac{\pi}{180°}$

Period of rotation T : one full rotation

$$\underline{v} = |\vec{v}| = \frac{d}{\Delta t} = \underline{\frac{2\pi R}{T}}$$

Frequency of rotation $f = \frac{1}{T}$

T in units of s → f in units $\frac{1}{s} = Hz$

* Example: earth rotates with a period $T = 1$ day.
radius of 6400 km = R
what is the speed with which everyone (on the equator) is rotating around the earth's center?

$$v = \frac{2\pi R}{T} = \frac{2\pi(6400 \times 10^3\,m)}{(1\,day)\left(\frac{3600\,s}{1\,hour}\right)\left(\frac{24\,hours}{day}\right)}$$

$$= 460\,m/s \text{ or } 920\,mph$$

$v = 0$

$R < 6400km$ → v is smaller

equator : $R = 6400km$

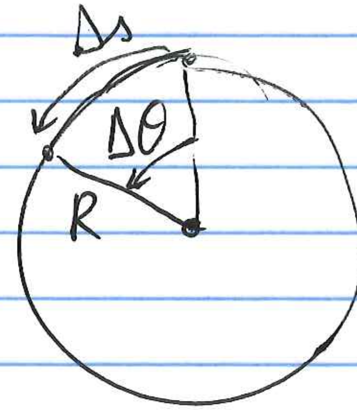
↳ v depends on radius

* ~~Ma~~ Angular velocity

linear
"regular" velocity

$$\underline{\omega} = \frac{\Delta\theta}{\Delta t} \qquad\qquad \underline{v} = \frac{\Delta x}{\Delta t}$$

$$\underline{\Delta\theta} = \frac{\Delta s}{R}$$



$s = R\theta$

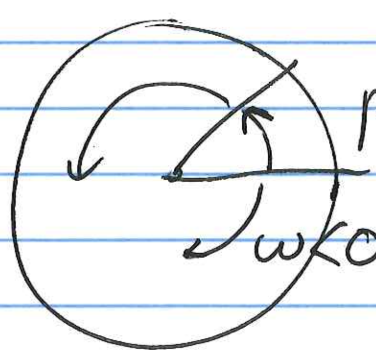
$$\underline{\Delta t} = \frac{\Delta s}{v} = \frac{\Delta s}{\left(\frac{2\pi R}{T}\right)}$$

$$\underline{\omega} = \frac{\Delta s / R}{\Delta s / \left(\frac{2\pi R}{T}\right)} = \frac{1}{R}\left(\frac{2\pi R}{T}\right) = \frac{2\pi}{T} = \underline{2\pi f}$$

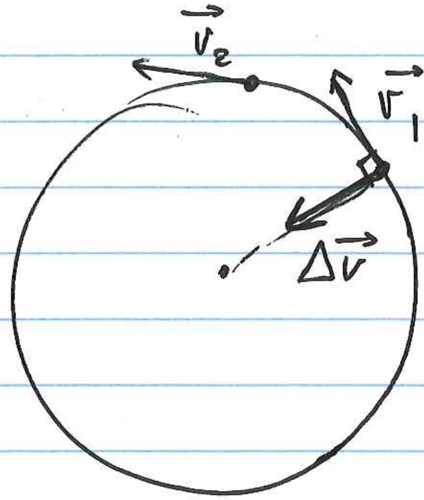
$$\underline{v} = \frac{2\pi R}{T} = \left(\frac{2\pi}{T}\right) R = \underline{\omega} R$$

* Signs of ω

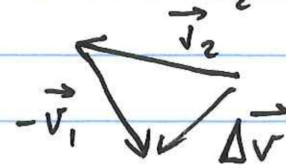
$\omega > 0$: CCW rotation
$\omega < 0$: CW rotation



positive angle $\rightarrow \omega > 0$



$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$



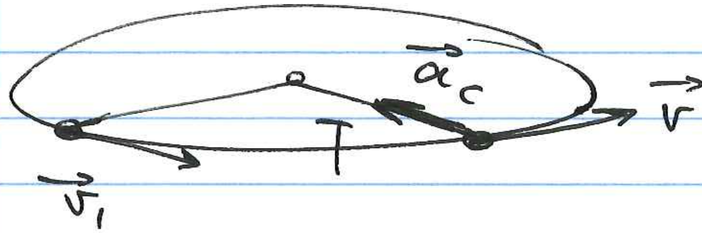
$$\vec{a}_c = \frac{\Delta\vec{v}}{\Delta t}$$: centripetal acceleration

$$\vec{a}_c = \frac{v^2}{R}$$ towards the center of rotation

$$\underline{a_c} = \underline{\frac{v^2}{R}} = \frac{(\omega R)^2}{R} = \underline{\omega^2 R}$$

Dynamics : $\vec{F}_{net.} = m\vec{a}_c$

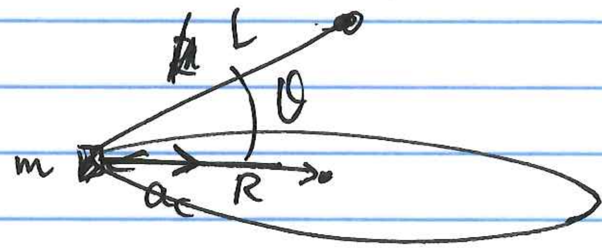
$\vec{F}_{net,c}$ points towards the center



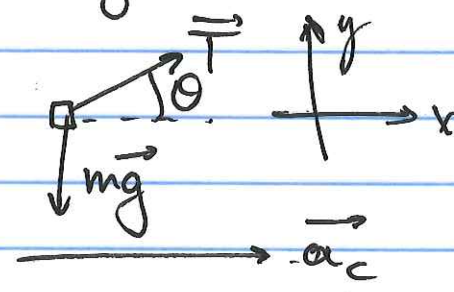
$$a_c = \frac{v^2}{R}$$

$$T = F_{net,c} = ma_c = m\frac{v^2}{R}$$

What is the angle at which the string mass spins



FBD



$$\vec{F}_{net} = m\vec{a} : \begin{cases} x : T\cos\theta = m\dfrac{v^2}{R} \\ y : T\sin\theta - mg = 0 \end{cases}$$

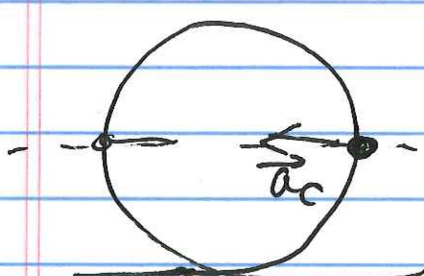
$$T = \frac{mg}{\sin\theta}$$

$$\cancel{m}g\,\frac{\cos\theta}{\sin\theta} = \cancel{m}\frac{v^2}{R}$$

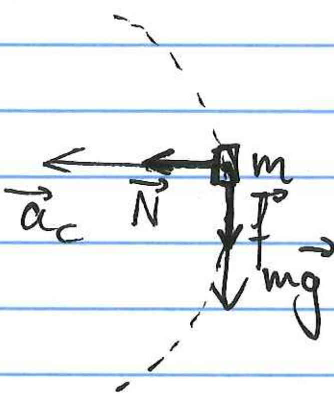
$$\frac{1}{\tan\theta} = \frac{v^2}{Rg} = \frac{1}{10}$$

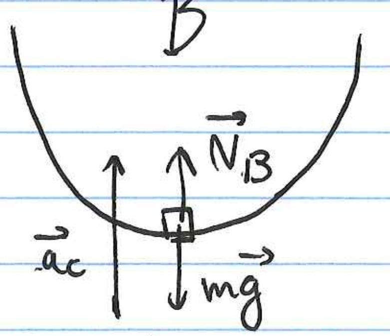
$v \rightarrow 1\ m/s$

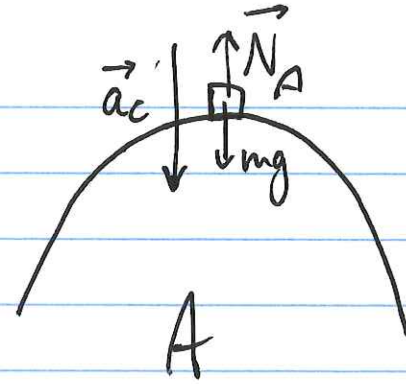
$R \rightarrow 1\ m$



FBD







$N_B - mg > 0$ $\quad a_c > 0$

$N_A - mg < 0$ $\quad a_c < 0$

$N_B > N_A$