PHYS 107 - Week 7 - Friday

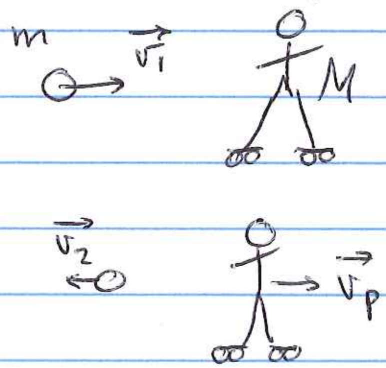
* Collisions : <u>conservation of momentum</u> : $\vec{p}_{tot,i} = \vec{p}_{tot,f}$

$\hookrightarrow \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = 0 \rightarrow \vec{p}$ constant

Q Collision 4-a elastic

What about kinetic energy?

Example: catching & throwing a ball on roller skates



$m = 2kg \quad |\vec{v}_1| = 8\,m/s$

$M = 80kg \quad |\vec{v}_2| = 2\,m/s$

1) what is the recoil speed?
2) is mechanical energy conserved?

1) $\vec{p}_i = m\vec{v}_1$

$\vec{p}_f = m\vec{v}_2 + M\vec{v}_p$

$\vec{p}_i = \vec{p}_f \longrightarrow m\vec{v}_1 = m\vec{v}_2 + M\vec{v}_p$

$$v_p = \frac{m}{M}\left(\vec{v}_1 - \vec{v}_2\right)$$

$$= \frac{2kg}{80kg}\left(+8\,m/s - (-2\,m/s)\right)$$

$$= +0.25\,m/s$$

2) $KE_i = \frac{1}{2} m v_1^2 = \frac{1}{2}(2kg)(8\,m/s)^2 = 64\,J$

$$KE_f = \frac{1}{2} m v_e^2 + \frac{1}{2} M v_p^2 = \frac{1}{2}(2kg)(2\,m/s)^2 + \frac{1}{2}(80kg)(0.25\,m/s)^2$$
$$= 4J + 2.5J = 6.5J$$

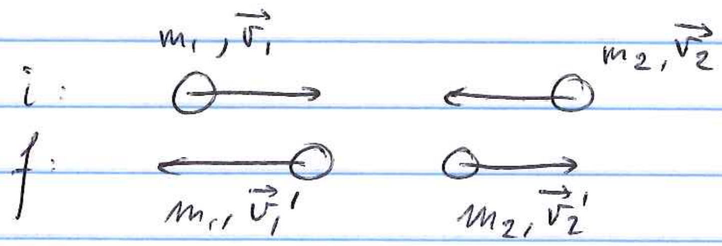
→ nearly 60 J was converted into other energy OE_f !

Mechanical energy (KE + PE) is <u>not</u> conserved in the general case

* Classification of collisions :

- Elastic collisions : $KE_i = KE_f$
- Inelastic collisions : $KE_i \neq KE_f$
- → perfectly inelastic collisions : KE_f is as low as possible

* 1D elastic collisions



$$\begin{cases} \text{momentum constant: } m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2' \\ \text{kinetic energy constant: } \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 \end{cases}$$

If we know m_1, m_2, v_1, v_2 → can solve for v_1', v_2'

Further manipulation:

$$\begin{cases} m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2 \\ m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \end{cases}$$

$$\hookrightarrow \begin{cases} m_1 (v_1^2 - v_1'^2) = m_2 (v_2'^2 - v_2^2) \\ m_1 (v_1 - v_1') = m_2 (v_2' - v_2) \end{cases}$$

$$\hookrightarrow \begin{cases} m_1 (v_1 - v_1')(v_1 + v_1') = m_2 (v_2' - v_2)(v_2' + v_2) \\ m_1 (v_1 - v_1') = m_2 (v_2' - v_2) \end{cases}$$

$$\hookrightarrow \quad v_1 + v_1' = \cancel{m}\, v_2' + v_2$$

or $\underline{v_1 - v_2 = v_2' - v_1'}$ (always valid)

* If one object is initially at rest: $v_2 = 0$

$$\hookrightarrow v_1 = v_2' - v_1'$$

momentum constant: $m_1 v_1 = m_1 v_1' + m_2 v_2'$

$$\hookrightarrow m_1 v_1 = m_1 v_1' + m_2 (v_1 + v_1')$$

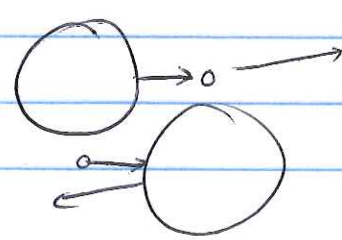
$$\hookrightarrow (m_1 - m_2) v_1 = (m_1 + m_2) v_1'$$

$$\hookrightarrow v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$m_1 = m_2 \rightarrow v_1' = 0$

$m_1 \gg m_2 \rightarrow v_1' = v_1$

$m_1 \ll m_2 \rightarrow v_1' = -v_1$



* Ballistic pendulum as an example of inelastic collision

Q Collisions 5-a, b

If we can't use $KE_i = KE_f$, then we need something else:

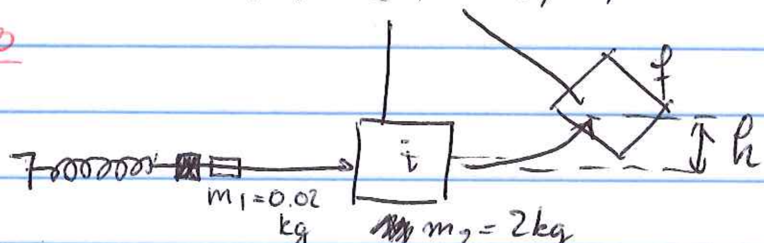
$$v_1' = v_2' \quad : \text{both objects stick together}$$

$$m_1 v_1 + \cancel{m_2 v_2}^{\,0} = m_1 v_1' + m_2 \underset{v_1'}{\underset{\parallel}{v_2'}} \quad \text{(always valid)}$$

$$\downarrow$$

$$m_1 v_1 = (m_1 + m_2) v_1'$$

Demo



$$PE_f = (m_1+m_2)gh$$
$$KE_i = \frac{1}{2}(m_1+m_2)v_1'^2$$

$$h = 0.20\,m \rightarrow PE_f = (m_1+m_2)gh \rightarrow KE_i = \frac{1}{2}(m_1+m_2)v_1'^2$$

$$\hookrightarrow h = \frac{v_1'^2}{2g} = \left(\frac{m_1}{m_1+m_2}\right)^2 v_1^2 \frac{1}{2g}$$

$$v_1 = \sqrt{2gh}\left(\frac{m_1+m_2}{m_1}\right) = 200\,m/s$$

$$v_1' = \frac{m_1}{m_1+m_2} v_1 = 2\,m/s$$

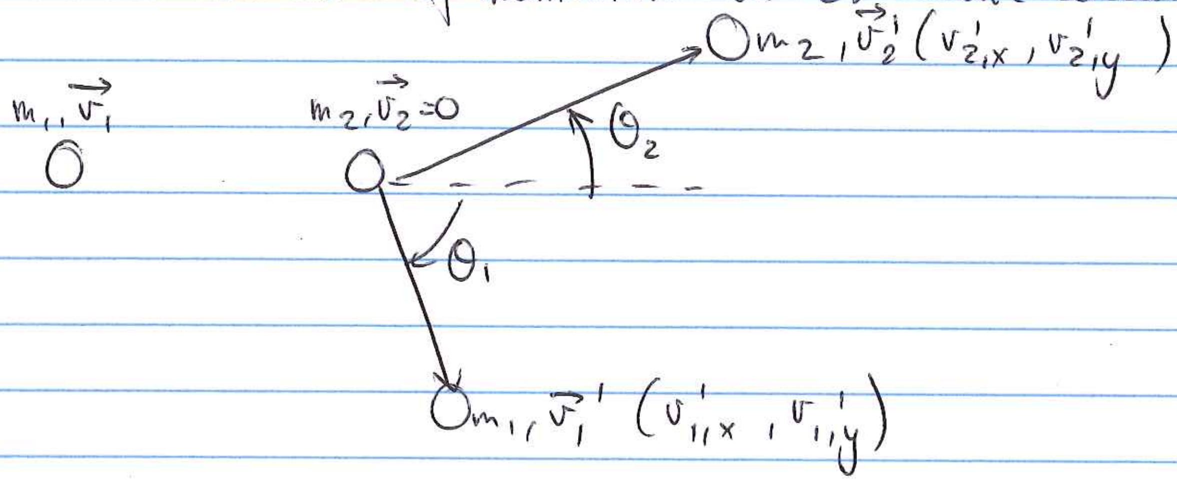
* Change in kinetic energy:

$$KE_i = \frac{1}{2} m_1 v_i^2 = 400\,J$$

$$KE_f = \frac{1}{2}(m_1 + m_2)\, v_f'^2 = 4J \qquad PE_f = (m_1+m_2)gh = 0.4\,J$$

→ 396 J converted into heat

* 2D conservation of momentum in 2D elastic collisions



$$m_1 \vec{v}_1 \rightarrow \begin{cases} x: & m_1 v_1 = m_1 v'_{1,x} + m_2 v'_{2,x} \\ y: & 0 = m_1 v'_{1,y} + m_2 v'_{2,y} \end{cases}$$

$$\begin{cases} m_1 v_1 = m_1 v_1' \cos\theta_1 + m_2 v_2' \cos\theta_2 \\ 0 = m_1 v_1' \sin\theta_1 + m_2 v_2' \sin\theta_2 \end{cases}$$

Along with $KE_i = KE_f$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$(p_x)^2 \rightarrow \quad mv_1^2 = mv_1'^2\cos^2\theta_1 + mv_2'^2\cos^2\theta_2$

$m_1 = m_2 \qquad\qquad + mv_1'v_2'\cos\theta_1\cos\theta_2$

$$\underline{mv_1^2} = \underline{mv_1'^2}(1 - \cancel{\sin^2\theta_1}) + \underline{mv_2'^2}(1 - \cancel{\sin^2\theta_2})$$

$KE_i = KE_f \qquad + mv_1'v_2'\cos\theta_1\cos\theta_2$

$$\hookrightarrow \quad mv_1'v_2'\cos(\theta_1 - \theta_2) = 0$$

$$\downarrow$$

$$\underline{\theta_1 - \theta_2 = 90^\circ}$$