PHYS 107 - Week 2 - Monday

Reminder : average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{d}{\Delta t}$

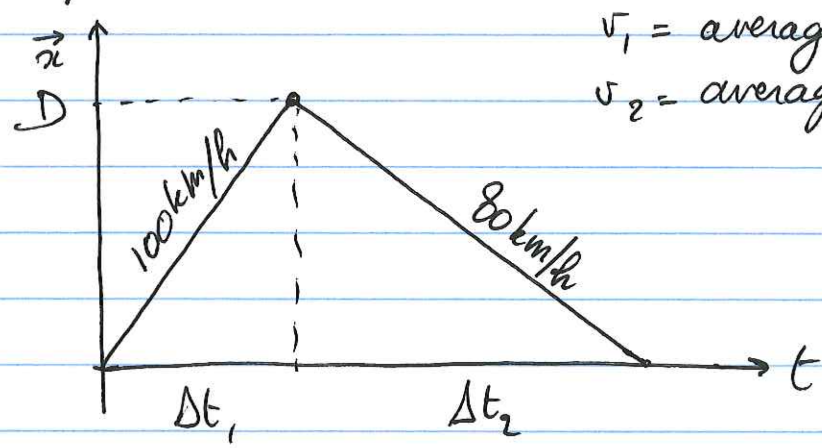
= scalar

average velocity = $\frac{\text{displacement}}{\text{total time}} = \frac{\Delta \vec{x}}{\Delta t}$

Example in 1 dimension

v_1 = average speed 1 = 100 km/h

v_2 = average speed 2 = 80 km/h



what is the average speed of the entire trip?

Q 1D-kinematics

average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{2D}{\Delta t_1 + \Delta t_2}$

$\Delta t_1 = \frac{D}{100 km/h}$ (e.g. $50 km = D \rightarrow \Delta t = 0.5 h$)

$\Delta t_2 = \frac{D}{80 km/h}$

$\rightarrow$ average speed = $\frac{2D}{\frac{D}{100 km/h} + \frac{D}{80 km/h}} = 75 \, km/h$

* Acceleration = rate of change of velocity, $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

(velocity = rate of change of displacement, $\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$)

strictly speaking this is average acceleration ...

Example: accelerating car

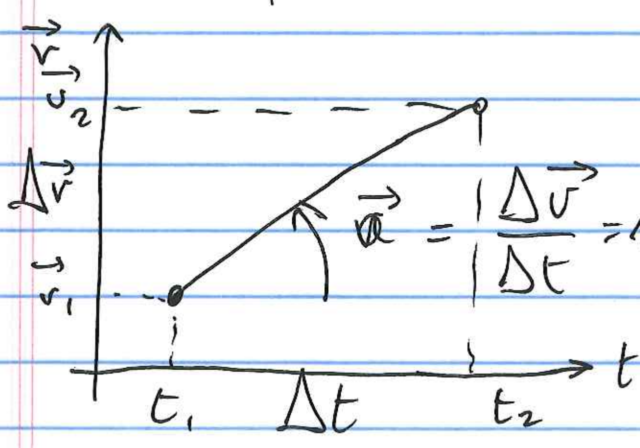
t [s]	$\vec{v}$ [m/s]
0	0
10	+30
20	+60
30	+90

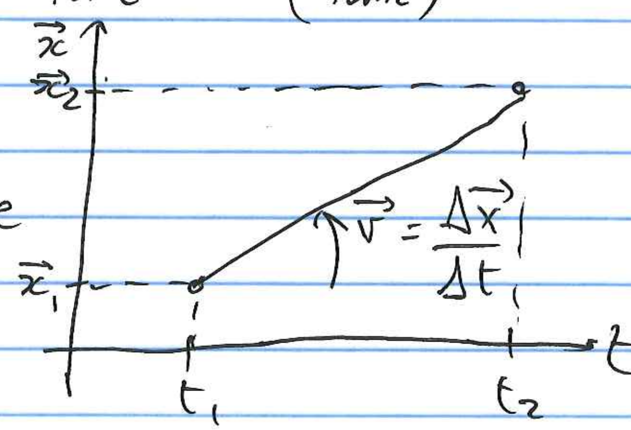
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{+30 \text{ m/s}}{\Delta t} = 3 \text{ m/s}^2$$

Q PE 2.16

Note: $\Delta \vec{v} = v_{\underline{later}} - v_{\underline{earlier}}$

Units of $\vec{a}$: $\text{m/s}^2 = \frac{\text{Velocity}}{\text{Time}} = \frac{\text{Length}}{(\text{Time})^2}$





$\longrightarrow$ $\vec{a}$ is the slope of the graph of $\vec{v}$ versus t

AND $\vec{v}$ is the slope of the graph of $\vec{x}$ versus t

$$\text{slope} = \tan(\text{angle}) = \frac{\text{opposite}}{\text{adjacent}}$$

Q 7-a, b

Note: in this course we will only deal with constant acceleration $\vec{a}_{avg} \equiv \vec{a}$

In PHYS 101 we need calculus for variable acceleration

* All necessary formulas for contant 1D acceleration can be easily derived.

$$\left\{\begin{array}{l} t_1 = 0 \\ v_1 = v_0 \\ x_1 = x_0 \end{array}\right. \longrightarrow \left\{\begin{array}{l} t_2 = t \\ v_2 = v \\ x_2 = x \end{array}\right. \qquad \vec{v}_{avg} = \frac{\Delta \vec{x}}{\Delta t} = \frac{x - x_0}{t}$$

(start) finish

$$\boxed{x = x_0 + \vec{v}_{avg}\, t}$$

$\vec{v}$ itself could change

$$\vec{a}_{avg} = \vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{t}$$

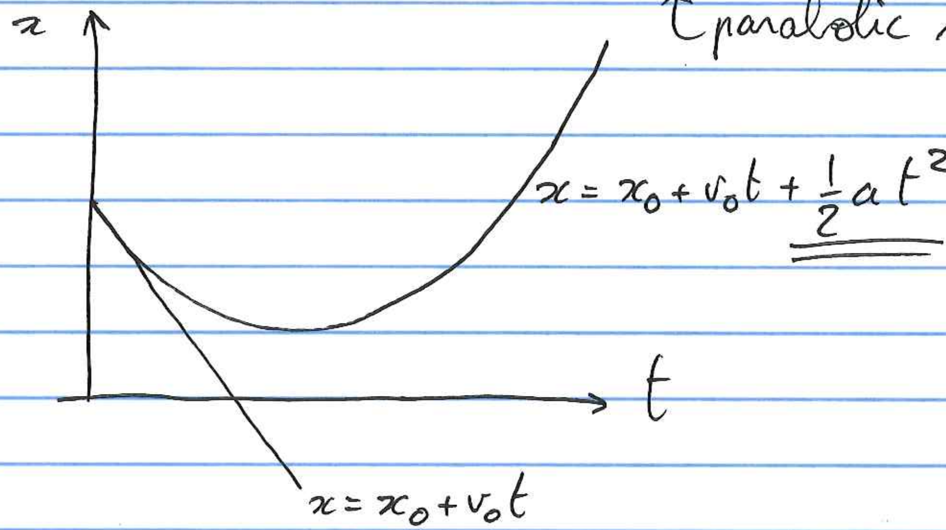
$$\boxed{\vec{v} = \vec{v}_0 + \vec{a}\, t}$$

$$\vec{v}_{avg} = \frac{\vec{v}_0 + \vec{v}}{2} \rightarrow x = x_0 + \left(\frac{\vec{v}_0 + \vec{v}}{2}\right) t$$

$$\rightarrow x = x_0 + \left(\frac{\vec{v}_0 + \vec{v}_0 + \vec{a}\, t}{2}\right) t$$

$$\longrightarrow \boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2} \quad \text{(from } t \text{ to } x\text{)}$$

↑ parabolic shape



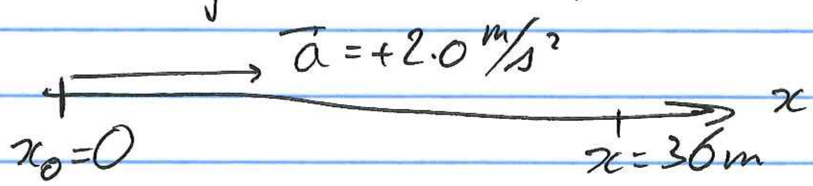
Note: x is not always vt since we start at x_0 and with non-zero initial speed v_0

$$x = x_0 + \left(\frac{v_0 + v}{2}\right) t = x_0 + \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right)$$

$$= x_0 + \frac{v^2 - v_0^2}{2a}$$

$$\longrightarrow \boxed{v^2 = v_0^2 + 2a(x - x_0)} \quad \text{(no } t\text{)}$$

Example: How long does it take a car to travel 36m if it accelerates from rest with $\vec{a} = +2.0\,m/s^2$?



$x_0 = 0\,m$ $\quad x = 36\,m$

$v_0 = 0\,m/s$ $\quad \longrightarrow$ unknown t, v

$a = +2.0\,m/s^2$

When does car reach 36m?

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$36m = 0m + 0\,m/s \cdot t + \frac{1}{2} a t^2$$

$$\longrightarrow t^2 = \frac{2(36m)}{(2.0\,m/s^2)} = 36\,s^2$$

$$t = \sqrt{36}\,s = 6\,s$$

What is its velocity then?

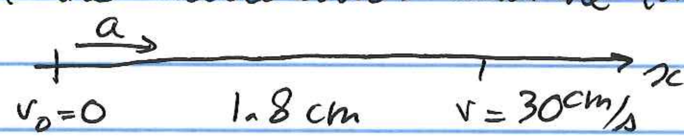
$$v = v_0 + a t = 0\,m/s + 2.0\,m/s \cdot 6s$$

$$= 12\,m/s$$

* Problem solving strategies:

- draw a picture or diagram
- choose a coordinate system, what directions are important, where is zero?
- write down the knowns and unknowns
- determine which equations are applicable and will give you the unknowns in terms of the knowns
- plug in the numbers (units!)
- check whether the response is reasonable.

Example: The left ventricle of the heart accelerates blood from rest to 30 cm/s in a distance of 1.8 cm. Find the acceleration and the time required.



$$v^2 = \underset{=0}{v_0^2} + 2a(x - \underset{=0}{x_0})$$

$$v^2 = 2a(1.8\,\text{cm}) = (30\,\text{cm/s})^2$$

$$a = \frac{v^2}{2\Delta x} = \frac{(30\,\text{cm/s})^2}{2(1.8\,\text{cm})} = 250\,\text{cm/s}^2 = 2.5\,\text{m/s}^2$$

$$v = v_0 + at \rightarrow t = \frac{v - v_0}{a} = \frac{30\,\text{cm/s}}{2.5\,\text{m/s}^2} = 0.12\,\text{s}$$