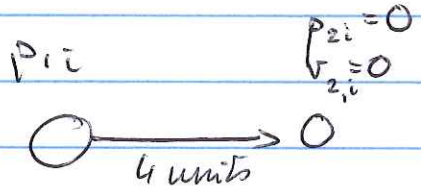
* Collisions : momentum $\vec{p} = m\vec{v}$ (vector)

↳ conservation of momentum

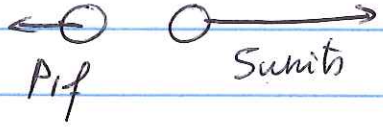
$\vec{p}_{i,total} = \vec{p}_{f,total}$

i : $\vec{p}_{total,i} = m_1\vec{v}_1 + m_2\vec{v}_2$

f : $\vec{p}_{total,f} = m_1\vec{v}_1' + m_2\vec{v}_2'$



$p_{total,i} = p_{1i} = 4$ units

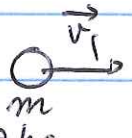


$p_{total,f} = p_{2f} + p_{1f}$

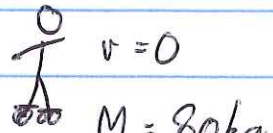
$5 \quad -1 \quad = 4$ units

* Kinetic energy : catch & throw ball on rollerskates

i :



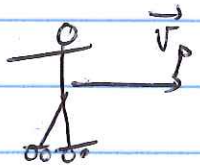
$|\vec{v}_1| = 8\ m/s$



1) what is the recoil speed v_p of the person?
2) is the kinetic energy conserved/constant?

f :

$|\vec{v}_2| = 2\ m/s$



i) $\vec{p}_{i,\text{total}} = m\vec{v}_1$

$\vec{p}_{f,\text{total}} = m\vec{v}_2 + M\vec{v}_p$

$\Big\} \quad m\vec{v}_1 = m\vec{v}_2 + M\vec{v}_p$

$$\vec{v}_p = \frac{m}{M}\left(\vec{v}_1 - \vec{v}_2\right)$$

$$= \frac{2\,kg}{80\,kg}\left(+8\,m/s - (-2\,m/s)\right)$$

$$= +0.25\,m/s$$

2) kinetic energy:

$$KE_i = \frac{1}{2}mv_1^2 = \frac{1}{2}(2\,kg)(+8\,m/s)^2 = 64\,J$$

$$KE_f = \frac{1}{2}mv_2^2 + \frac{1}{2}Mv_p^2$$

$$= \frac{1}{2}(2\,kg)(-2\,m/s)^2 + \frac{1}{2}(80\,kg)(+0.25\,m/s)^2$$

$$= 4\,J + 2.5\,J = 6.5\,J$$

→ nearly 60 J turned into OE_f

→ KE is not generally conserved

*Classification of collisions

$KE_i = KE_f$: elastic collision

$KE_i \neq KE_f$: inelastic collision

↳ perfectly inelastic collision: KE_f is as low as possible

* 1D elastic collisions

i: $m_1, \vec{v}_1$ $\quad$ $m_2, \vec{v}_2$

f: $m_1, \vec{v}_1'$ $\quad$ $m_2, \vec{v}_2'$

$$\begin{cases} KE_i = KE_f \\ \vec{p}_i = \vec{p}_f \end{cases}$$

If you know $m_1, m_2, \vec{v}_1, \vec{v}_2 \rightarrow$ calculate $\vec{v}_1'$ and $\vec{v}_2'$

$$\frac{1}{2} m_1 \vec{v}_1^{\,2} + \frac{1}{2} m_2 \vec{v}_2^{\,2} = \frac{1}{2} m_1 \vec{v}_1'^{\,2} + \frac{1}{2} m_2 \vec{v}_2'^{\,2}$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

$$m_1 (v_1^2 - v_1'^2) = m_2 (v_2'^2 - v_2^2)$$

$$m_1 (v_1 - v_1') = m_2 (v_2' - v_2)$$

$$\underline{m_1 (v_1 - v_1')(v_1 + v_1')} = \underline{m_2 (v_2' - v_2)(v_2' + v_2)}$$

$$\underline{m_1 (v_1 - v_1')} = \underline{m_2 (v_2' - v_2)}$$

$$v_1 + v_1' = v_2' + v_2$$

$$v_1 - v_2 = v_2' - v_1' \quad \text{(always true)}$$

If m_2 is initially at rest: $v_2 = 0$

$$v_2 - v_1 = v_1' - v_2'$$

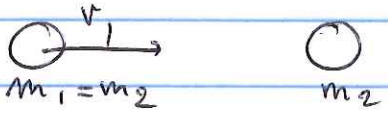
$$v_1 = v_2' - v_1' \longrightarrow v_2' = v_1 + v_1'$$

momentum conservation: $m_1 v_1 = m_1 v_1' + m_2 v_2'$

$$m_1 v_1 = m_1 v_1' + m_2 (v_1 + v_1')$$

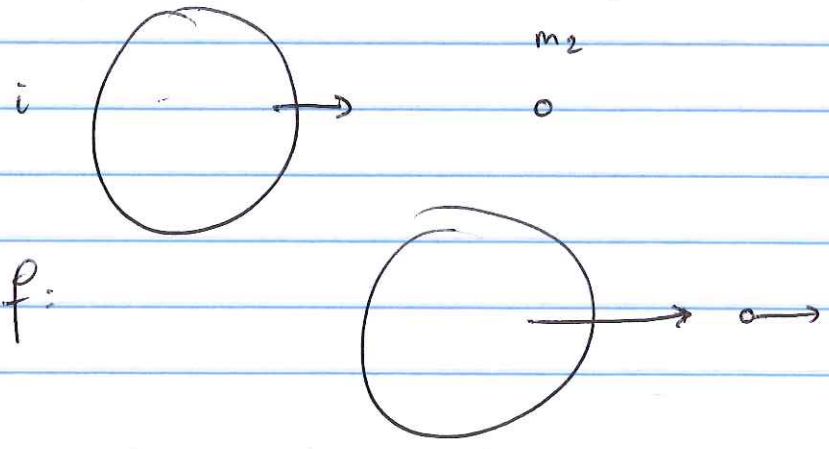
$$\boxed{v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1} \quad \text{if } v_2 = 0$$

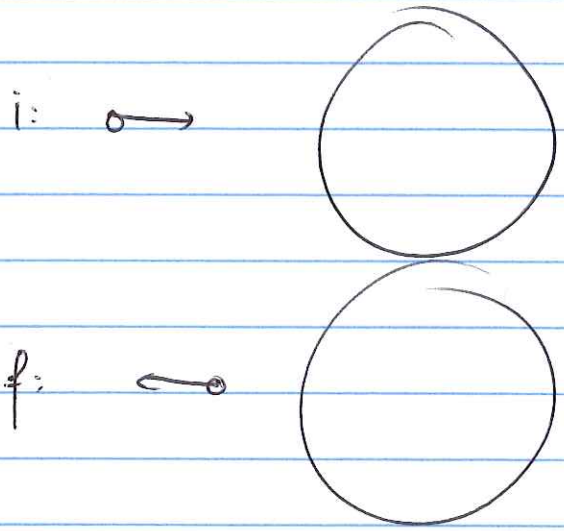
$m_1 = m_2$: $v_1' = 0$

i: 

f: 

$m_1 >> m_2 \longrightarrow v_1' = v_1$





$m_2 \gg m_1 \rightarrow v_1' = -v_1$

* Inelastic collisions

~~We can't use~~ $KE_i = KE_f$

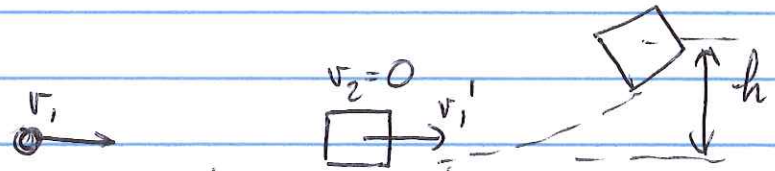
$$\begin{cases} v_1' = v_2' & \text{(sticking)} \\ m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \end{cases}$$

Special $v_2 = 0$:

$$\begin{cases} v_1' = v_2' \\ m_1 v_1 = m_1 v_1' + m_2 v_2' = (m_1 + m_2) v_1' \end{cases}$$

$$m_1 v_1 = (m_1 + m_2) v_1'$$

$$v_1' = \frac{m_1}{m_1 + m_2} v_1$$



$m_1 = 0.02\,kg$ $\quad m_2 = 2\,kg$

$KE = 0$

$PE = (m_1 + m_2)gh$

$KE = \frac{1}{2} m v_1'^2$ $\quad h = 0.20\,m$

$KE = \frac{1}{2} m v_1'^2 \longleftarrow PE = (2.02\,kg)(10\,m/s^2)(0.20\,m)$

$\hookrightarrow (m_1 + m_2)$

$\underline{v_1'^2 = \sqrt{2gh}} = \frac{m_1}{m_1 + m_2} v_1 = \underline{2\,m/s}$

$v_1 = \left(\frac{m_2 + m_2}{m_1}\right)\sqrt{2gh} = \underline{200\,m/s}$

$\frac{2\,kg}{0.02\,kg} \approx 100$

$KE_i = \frac{1}{2} m_1 v_1^2 = 400\,J$ $\quad (v_1 = 200\,m/s)$

$KE_f = \frac{1}{2}(m_1 + m_2) v_1'^2 = 4\,J$ $\quad (v_1' = 2\,m/s)$