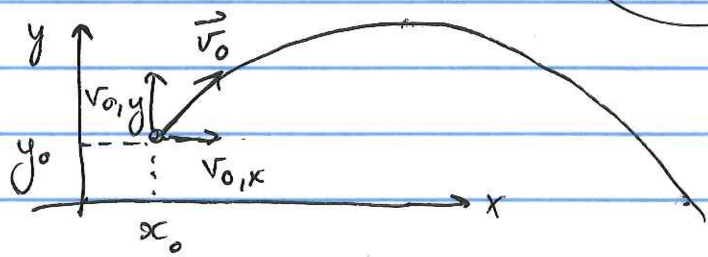
Ballistic motion

↳ only gravity works on objects
(ignore air resistance)

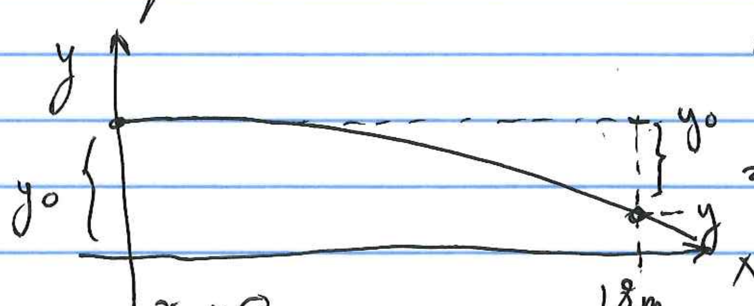
$$a_x = 0 \quad , a_y = -g = -9.80\ m/s^2 \approx -10\ m/s^2$$

$$\begin{cases} x = x_0 + v_{0,x}\, t \\ y = y_0 + v_{0,y}\, t - \frac{1}{2} g t^2 \end{cases}$$



$$\begin{cases} v_x = v_{0,x} + \cancel{a_x}\, t = v_{0,x} = \text{constant} \\ v_y = v_{0,y} + a_y\, t = v_{0,y} - g t \end{cases}$$

Joe throws baseball horizontally at 38 m/s ($\approx$ 85 mph). The ball crosses home plate 18 m away. How far has this ball fallen ~~b~~ ~~a~~ when it passes home plate?



1) use x - equation to determine t
2) use t in the y equation to determine how much ball has dropped

1) $x = \underset{=0}{x_0} + v_{0,x}\, t \rightarrow t = \frac{18\,m}{38\,m/s} = \underline{0.47\,s}$

2) $t = 0.47\,s$ in $y = y_0 + \overset{0}{\cancel{v_{0,y}}}\, t - \underbrace{\frac{1}{2} g t^2}$

$-(y - y_0) = (y_0 - y)$

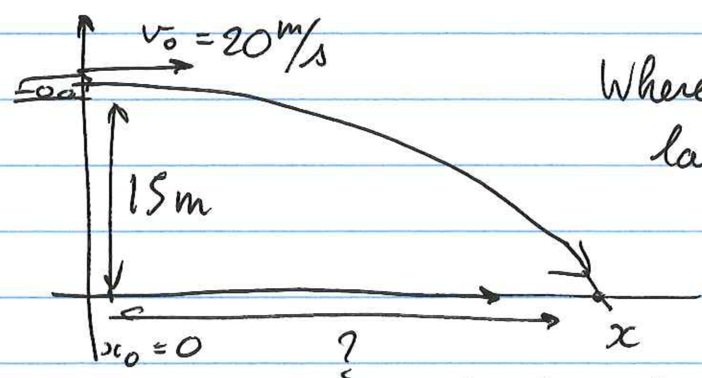
how much the ball has dropped

$-\frac{1}{2} g t^2 = -\frac{1}{2}(9.80\,m/s^2)(0.47\,s)^2 = \underline{-1.1\,m}$

$y - y_0 = -\frac{1}{2} g t^2$

$y < y_0 \rightarrow y - y_0 < 0$

Example:



Where does the car land/crash?

1) y-equation will give us the time t
2) use time t in the x-equation

1) $y = y_0 + \cancel{v_{0,y}}\, t - \frac{1}{2} g t^2$

$0 = 15\,m - \frac{1}{2} g t^2 \longrightarrow t = \sqrt{\frac{2(15\,m)}{(9.80\,m/s^2)}} = \underline{1.75\,s}$

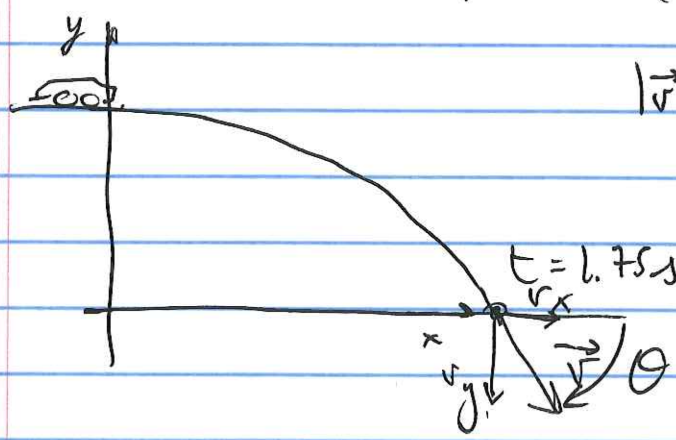
$0 = y_0 - \frac{1}{2} g t^2 \rightarrow t^2 = \frac{2y_0}{g}$

$t = \sqrt{\frac{2y_0}{g}}$

e) ~~xx~~ use $t = 1.75\,s$ in ~~y~~ x-equation

$$x = \underbrace{x_0}_{=0} + v_{0,x}\, t$$

$$x = v_{0,x}\, t = (20\,m/s)(1.75\,s) = \underline{35\,m}$$



$|\vec{v}|,\ \theta$

$$v_x = v_{0,x} = \text{constant} = 20\,m/s$$

$$v_y = \cancelto{0}{v_{0,y}} - g t$$

$$v_y = -g t$$

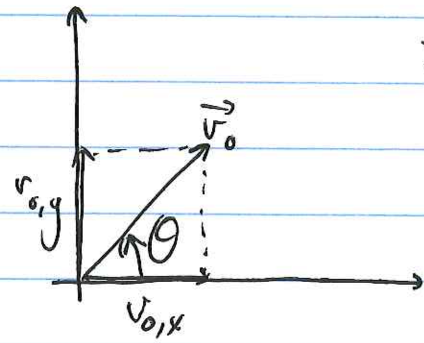
$$\begin{cases} v_x = 20\,m/s \\ v_y = -(1.75\,s)(9.80\,m/s^2) = -17.2\,m/s \end{cases}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(20\,m/s)^2 + (-17.2\,m/s)^2}$$

$$= 26.4\,m/s$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{-17.2\,m/s}{20\,m/s} \rightarrow \theta = -40.7^\circ$$

* General case $(x_0 = 0, y_0 = 0)$



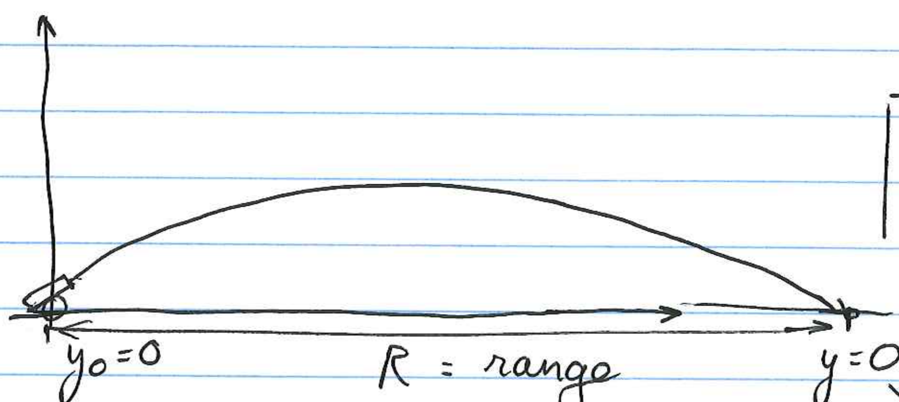
$$\vec{v}_0 = (v_0 \cos\theta, v_0 \sin\theta)$$

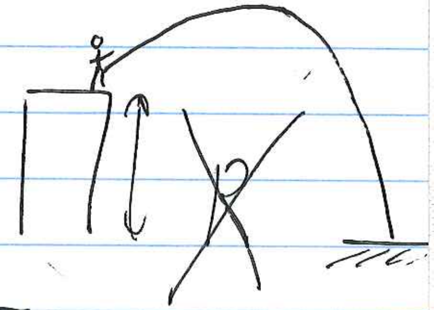
$$|\vec{v}| = v_0 = \sqrt{v_{0,x}^2 + v_{0,y}^2}$$

$$\tan\theta = \frac{v_{0,y}}{v_{0,x}}$$

Starting from $x_0 = 0$, $y_0 = 0$:

$$\begin{cases} x = v_{0,x} t = v_0 \cos\theta . t \\ y = v_{0,y} t - \frac{1}{2} g t^2 = \underline{v_0 \sin\theta . t} - \frac{1}{2} g t^2 \end{cases}$$





1) y-equation to determine t
2) plug in t in our x equation

$$y = v_0 \sin\theta . t - \frac{1}{2} g t^2 = 0$$

$t = 0\,s$ OR $v_0 \sin\theta - \frac{1}{2} g t = 0$

$$t = \frac{2 v_0 \sin\theta}{g}$$

$t = \frac{2 v_0 \sin\theta}{g}$ into x-equation

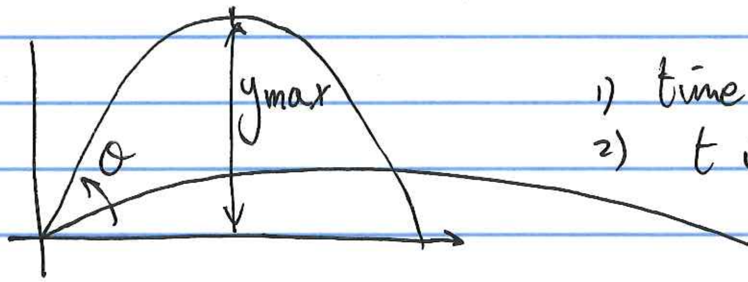
$$x = \overset{0}{x_0} + v_0 \cos\theta \cdot t = v_0 \cos\theta \cdot t$$

$$x = v_0 \cos\theta \, \frac{2 v_0 \sin\theta}{g} = \frac{v_0^2 \, 2\cos\theta \sin\theta}{g}$$

$$2\cos\theta \sin\theta = \sin 2\theta$$

$$\hookrightarrow R = x = \frac{v_0^2 \sin 2\theta}{g} = \frac{(10\,m/s) \sin 2(30^\circ)}{(10\,m/s^2)}$$

$\sin 2\theta$ is largest when $\underline{\theta = 45^\circ}$



1) time of highest point
2) t in y-equation

1) $v_y = 0 = v_{0,y} - gt = v_0 \sin\theta - gt = 0$

$$t = \frac{v_0 \sin\theta}{g}$$

2) $t = \frac{v_0 \sin\theta}{g}$ in y-equation

$$y_{max} = v_0 \sin\theta \cdot t - \frac{1}{2} g t^2$$

$$= v_0 \sin\theta \, \frac{v_0 \sin\theta}{g} - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g}\right)^2$$

$$= \frac{v_0^2 \sin^2\theta}{g} - \frac{1}{2} \frac{v_0^2 \sin^2\theta}{g}$$

$$y_{max} = \frac{1}{2} \frac{v_0^2 \sin^2 \theta}{g}$$

$\sin^2 \theta$ is largest when $\theta = 90°$

$\rightarrow$ y_{max} is largest when $90°$