Physics 107: Physics for Life-Sciences

Midterm Exam: November 16, 2015

This test is administered under the rules and regulations of the honor code of the College of William & Mary.

Name: _____

Signature: _____

Instructions:

- This is a closed book, closed notes test.

- Calculators are NOT needed and NOT allowed. Devices with wireless connections are NOT allowed.

- Start your work from the fundamental equations on the formula sheet, and derive any additional expressions that you may need.

- Circle your answer for each part of each problem.

- Clearly mark out any work that you wish the grader to disregard. Do not waste your time erasing.

- Your work will be graded based on your ability to write down a logical and organized solution grounded in the correct assessment of the physics of a situation. No credit will be given for an answer that is not justified by a logical solution or where that justification is not organized or readable. Partial credit will be given up to the point where your solution departs from a correct analysis of the physics involved for any given part of a problem.

Question	Points	Score
1	5	
2	5	
3	10	
4	10	
5	20	
6	20	
Total:	70	

1. (5 points) A container, closed on the right side but open to atmosphere on the left side, is almost completely filled with water as shown on the figure. Three points in the container are marked on the figure. Rank the points according to their pressures, from highest to lowest.
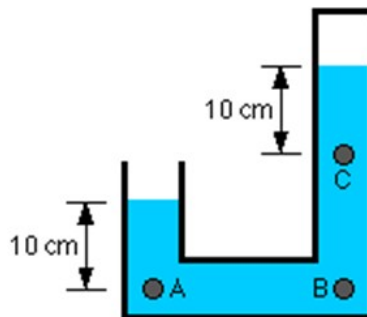
   $\checkmark$ $P_A = P_B > P_C$
   $\bigcirc$ $P_B > P_A > P_C$
   $\bigcirc$ $P_B > P_A = P_C$
   $\bigcirc$ $P_C > P_B > P_A$
   $\bigcirc$ $P_C > P_A = P_B$

2. (5 points) A wooden block with a weight of 100 N floats exactly 50% submerged in a liquid. The upward buoyant force exerted on the block by the liquid...

   $\bigcirc$ is exactly 50 N.

   $\checkmark$ **is exactly 100 N.**

   $\bigcirc$ depends on the density of the block.

   $\bigcirc$ depends on the density of the liquid.

   $\bigcirc$ depends on both the density of the block and the liquid.

3. You have an axe to grind. The grindstone has a mass M of 90 kg, a radius R of $\frac{1}{3}$ m and is initially turning at 90 rpm. It can be modeled as a disk with moment of inertia $I = \frac{1}{2}MR^2$.

   You press the steel axe against the grindstone with a radial force of 30 N. The coefficient of kinetic friction between steel and stone is $\mu_k = 0.2$.

   (a) (5 points) What is the torque on the grindstone?
   $\bigcirc$ $1\,\text{N} \cdot \text{m}$
   $\checkmark$ $2\,\text{N} \cdot \text{m}$
   $\bigcirc$ $6\,\text{N} \cdot \text{m}$
   $\bigcirc$ $10\,\text{N} \cdot \text{m}$
   $\bigcirc$ $18\,\text{N} \cdot \text{m}$

   > **Solution:** If the radial force N is 30 N and the coefficient of friction $\mu_k = 0.2$, then the force of friction will be $f = 6\,\text{N}$. This means that the torque will be $\tau = fR = 6\,\text{N} \cdot \frac{1}{3}\,\text{m} = 2\,\text{N} \cdot \text{m}$.

   (b) (5 points) What will be the angular acceleration as the stone grinds to a halt?
   $\bigcirc$ $0.2\,\text{rad/s}^2$
   $\bigcirc$ $0.5\,\text{rad/s}^2$
   $\bigcirc$ $2.0\,\text{rad/s}^2$
   $\bigcirc$ $3.0\,\text{rad/s}^2$
   $\checkmark$ **Another value, namely:** —————————————— rad/s^2

   > **Solution:** Together with a moment of inertia $I = \frac{1}{2}MR^2 = 5\,\text{kg} \cdot \text{m}^2$ the torque gives us $\alpha = \tau/I = fR/I = 2\,\text{N} \cdot \text{m}/5\,\text{kg} \cdot \text{m}^2 = 0.4\,\text{rad/s}^2$.

4. Even when the head is held erect, as in the figure below, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles in the back of the neck must therefore exert a force to keep it erect. That is why your head falls forward when you fall asleep in class.

The perpendicular distance between the line of action for the weight of the head and the pivot point is $r_{W\perp} = 2.0\,\text{cm}$ and the perpendicular distance between the line of action for the force the muscles exert on the head and the pivot point is $r_{M\perp} = 5.0\,\text{cm}$. Assume the weight of the head is $W = 50\,\text{N}$.
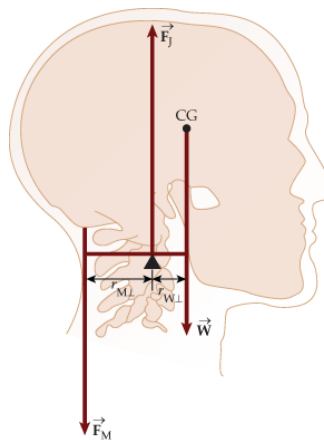
(a) (5 points) What is the force exerted by the muscles?

> **Solution:** The net torque around the point of support must be zero, or $\tau_{net} = r_{M\perp}F_M - r_{W\perp}W = 0$. This leads to
> $$F_M = \frac{r_{W\perp}}{r_{M\perp}}W = \frac{2.0\,\text{cm}}{5.0\,\text{cm}}50\,\text{N} = 20\,\text{N}.$$

(b) (5 points) What is the force F_J on the point of support?

> **Solution:** The net force on the head must be zero, or $F_{net} = F_j - F_M - W = 0$. This leads to
> $$F_j = F_M + W = 20\,\text{N} + 50\,\text{N} = 70\,\text{N}.$$

5. You place a beaker that is two-thirds full of water with a density of $\rho_{H_2O} = 1000\,\text{kg/m}^3$ on a laboratory scale. You then use a light-weight cord to suspend an aluminum object with density $\rho_{Al} = 2700\,\text{kg/m}^3$ in the water. The object is completely submerged, and none of the water spills out of the beaker. You observe that the reading on the scale increases by $\Delta m = 0.0027\,\text{kg}$. You may approximate the magnitude of the gravitational acceleration g as $10.0\,\text{m/s}^2$.
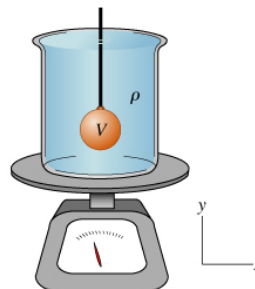
(a) (5 points) In words, what is the reason the scale reads a larger weight?

Solution: The force of buoyancy pushes the aluminum object upwards, but Newton's 3rd law requires that the object will push down on the water (and the scale) with the same magnitude.

(b) (10 points) What is the volume of the object?

Solution: Because the buoyant force F_B is equal to the weight of the displace liquid $\rho_{H_2O}Vg$, this will also be the additional force on the scale. That will result in an additional reading $\Delta m = \rho_{H_2O}V$. The volume is therefore

$$V = \frac{\Delta m}{\rho_{H_2O}} = \frac{0.0027\,\text{kg}}{1000\,\text{kg/m}^3} = 2.7 \times 10^{-6}\,\text{m}^3 = 2.7\,\text{cm}^3.$$

(c) (5 points) How much does the tension in the cord change when the object is submerged?

Solution: The tension decreases by the buoyant force, $\rho_{H_2O}Vg = \Delta mg = 0.027\,\text{N}$.

6. A 10 m-long horizontal garden hose with a diameter 0.016 m has at the end a nozzle with a diameter 0.008 m. Water flows through the hose at a velocity of $v_h = 2\,\text{m/s}$. Assume for parts a and b that the water has no viscosity or other form of energy dissipation, and that the density of water is $1000\,\text{kg/m}^3$.

(a) (5 points) What is the velocity of the water as it comes out of the nozzle?

> **Solution:** Using the continuity equation we can determine the velocity in the hose: $A_h v_h = A_n v_n$. This gives use
> $$v_n = \frac{A_h}{A_n} v_h = \frac{r_h^2}{r_n^2} v_h = \left(\frac{r_h}{r_n}\right)^2 v_h = 2^2 \cdot 2\,\text{m/s} = 4 \cdot 8\,\text{m/s} = 8\,\text{m/s}.$$

(b) (5 points) What is the pressure difference between the water in the hose and water in the nozzle?

> **Solution:** Using Bernoulli's equation we can determine the difference between P_h and P_n:
> $$P_h - P_n = \frac{1}{2}\rho v_n^2 - \frac{1}{2}\rho v_h^2 = \frac{1}{2}\rho(v_n^2 - v_h^2) = \frac{1}{2}1000\,\text{kg/m}^3(64-4)(\text{m/s})^2 = 30 \cdot 10^3\,\text{Pa}.$$

(c) (10 points) The viscosity of the water ($\eta = 10^{-3}\,\text{Pa} \cdot \text{s}$) will cause a pressure drop ΔP as the water flows through the garden hose. What will be the magnitude of this pressure drop? How does this compare to your answer in part b? *Hint:* Allow factors of π to drop out in the algebra before starting your numerical calculations.

> **Solution:** Using Poisseuille's law we can relate the pressure drop ΔP to the given parameters of the garden hose:
> $$\Delta P = \frac{8\eta \ell Q}{\pi r_h^4} = \frac{8\eta \ell A_h v_h}{\pi r_h^4} = \frac{8\eta \ell \pi r_h^2 v_h}{\pi r_h^4} = \frac{8\eta \ell v_h}{r_h^2} = \frac{8 \cdot 10^{-3}\,\text{Pa} \cdot \text{s} \cdot 10\,\text{m} \cdot 2\,\text{m/s}}{(8 \cdot 10^{-3}\,\text{m})^2} = 2.5 \cdot 10^3\,\text{Pa}$$
> .

Possibly useful relations (feel free to detach this page):

$\vec{v}_{avg} = \Delta\vec{x}/\Delta t$

$\vec{a}_{avg} = \Delta\vec{v}/\Delta t$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v = v_0 + at$

$v_{avg} = \frac{v_0 + v}{2}$

$v^2 = v_0^2 + 2a(x - x_0)$

$\vec{F}_{net} = m\vec{a}$

$\vec{F}_{BA} = -\vec{F}_{AB}$

$W = Fd\cos\theta$

$W_{net} = -\Delta PE = \Delta KE$

$P = \frac{W}{\Delta t}$

$\text{Eff} = \frac{W_{out}}{E_{in}}$

$KE_i + PE_i + W_{nc} = KE_f + PE_f$

$PE_k = \frac{1}{2}kx^2$

$PE_g = mgh$

$KE_{trans} = \frac{1}{2}mv^2$

$KE_{rot} = \frac{1}{2}I\omega^2$

$I_{point} = MR^2$

$I_{disk} = \frac{1}{2}MR^2$

$I_{sphere} = \frac{2}{5}MR^2$

$P = \frac{F}{A}$

$P_{gauge} = P - P_{atm}$

$\rho = \frac{M}{V}$

$\rho_{water} = 10^3\,\text{kg/m}^3$

$Q = \frac{\Delta V}{\Delta t} = Av$

$Q = \frac{\Delta P \pi r^4}{8\eta L}$

$\text{Power} = PQ$

$A \cdot v = \text{constant}$

$P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$

$F_B = \rho g V_{displaced}$

$F_{ST} = \gamma L$

$P = \frac{4\gamma}{r}$

$h = \frac{2\gamma}{\rho g r}$

$N_R = \frac{\rho v L}{\eta} = \frac{2\rho v r}{\eta}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$R = \frac{v_0^2}{g}\sin 2\theta$

$h = \frac{v_0^2}{2g}\sin^2\theta$

$0 \le f_s \le \mu_s N$

$f_k = \mu_k N$

$\frac{F}{A} = Y\frac{\Delta L}{L}$

$F_k = -kx$

$\vec{W} = m\vec{g}$

$\vec{g} = 9.80\,\text{m/s}^2 \text{ downward}$

$F_G = G\frac{mM}{r^2}$

$G = 6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2$

$\vec{I} = \vec{F}_{avg}\Delta t$

$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$

$v_1 - v_2 = v_2' - v_1'$

$\theta = \frac{s}{r}$

$v = r\omega$

$f = \frac{1}{T} \text{ and } \omega = 2\pi f = \frac{2\pi}{T}$

$a_c = \frac{v^2}{r} = r\omega^2$

$F_c = m\frac{v^2}{r} = mr\omega^2$

$\tau = rF\sin\theta = r_{\perp}F$

$\omega = \frac{\Delta\theta}{\Delta t}$

$\alpha = \frac{\Delta\omega}{\Delta t}$

$\tau = I\alpha$

$L = I\omega$

$\tau = \frac{\Delta L}{\Delta t}$

$x_{rms} = \sqrt{2Dt}$

$1\,\text{atm} = 10^5\,\text{Pa} = 760\,\text{mm}\cdot\text{Hg}$

$1\,\text{cal} = 4.186\,\text{J} \text{ and } 1\,\text{Cal} = 1000\,\text{cal}$

$\cos\theta = \text{adjacent/hypotenuse}$

$\sin\theta = \text{opposite/hypotenuse}$

$\tan\theta = \sin\theta/\cos\theta$

$\sin 30° = \cos 60° = \frac{1}{2}$

$\cos 30° = \sin 60° = \frac{\sqrt{3}}{2}$

$\tan 45° = 1$