$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t} = \text{slope of position vs. time}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \text{slope of velocity vs. time}$$

We only consider constant acceleration $\vec{a}$

$$\underbrace{x}_{\text{at time } t} = \underbrace{x_0}_{\text{at time } 0} + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + a t$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

* Gravity and free fall

Empirical observations:

- near the Earth's surface all objects released above the ground experience the same acceleration

$$\vec{a} = -\underbrace{9.80\,\text{m/s}^2}_{} = -g \quad \text{(vector)}$$

$$|\vec{a}| = g = 9.80\,\text{m/s}^2 \quad \text{(scalar)}$$

$$\vec{a} \approx -10\,\text{m/s}^2$$

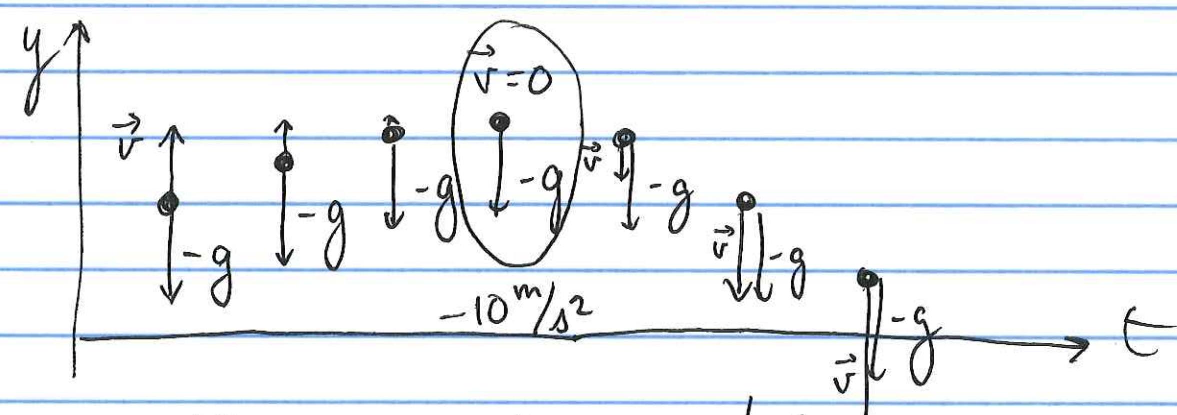
- this is independent of material, mass, density, etc ...

- this is true as long as air resistance, other friction can be neglected

Pennies fall just as quickly as feathers in vacuum

same acceleration

* Vertical kinematics



$\vec{a} = -9.80 \, m/s^2$ is constant

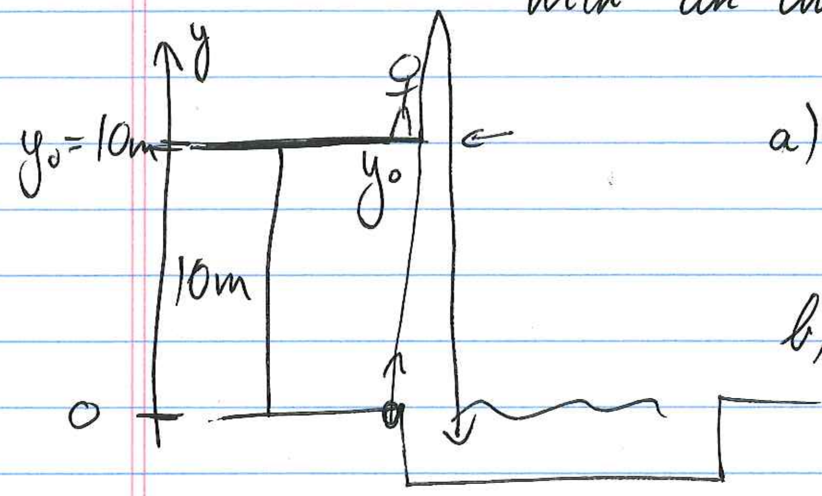
but $\vec{v}$ changes from upward to downward
$\quad + \qquad -$

$$y = y_0 + v_0 t + \frac{1}{2} \underset{-g}{\underset{\shortparallel}{a}} t^2$$

$$\boxed{y = y_0 + v_0 t - \frac{1}{2} g t^2}$$

$v = v_0 - gt$ / $v^2 = v_0^2 - 2g(y - y_0)$

Example: diver jumps off a 10m high board with an initial velocity of $v_0 = +2\,m/s$



a) when does the diver return to the same height as the board?

b) how long does it take the diver to reach the water?

a) $y = y_0 + v_0 t - \frac{1}{2} g t^2$

given

$\rightarrow \quad y = y_0 = \underbrace{y_0 + v_0 t - \frac{1}{2} g t^2}$

$0 = v_0 t - \frac{1}{2} g t^2$

$\cancel{t = 0\,s}$ or $\left(v_0 - \frac{1}{2} g t\right) = 0$

$\hookrightarrow t = \frac{2 v_0}{g} = \frac{2(2\,m/s)}{10\,m/s^2} = 0.4\,s$

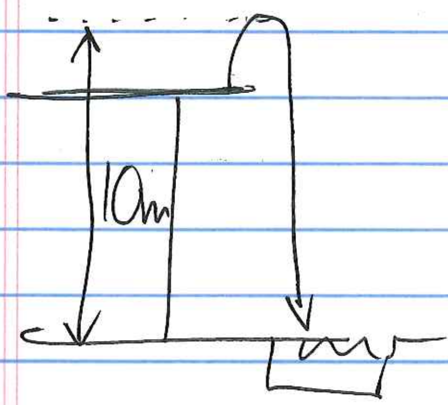
$$\boxed{\begin{gathered} a x^2 + b x + c = 0 \\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{gathered}}$$

b) $y = 0 = y_0 + v_0 \underline{t} - \frac{1}{2} g \underline{t^2}$

$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4 y_0 \left(-\frac{1}{2} g\right)}}{2\left(-\frac{1}{2} g\right)}$

$\cancel{t = -1.23\,s}$ OR $t = +1.63\,s$

c) what was the highest point she reached?



$v = 0$ at the highest point

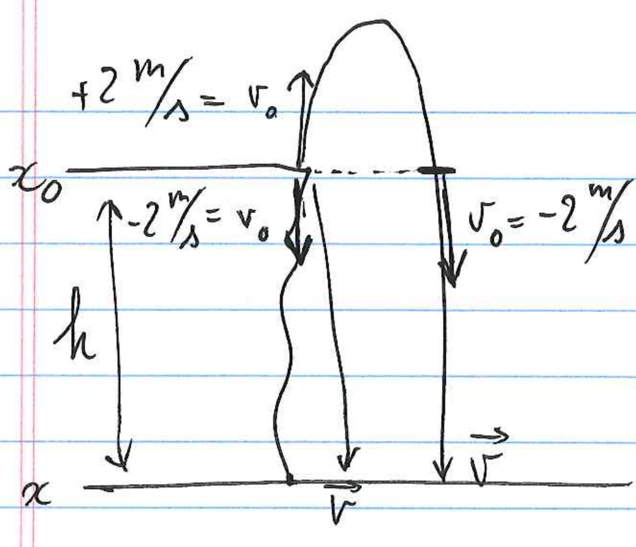
$$v^2 = v_0^2 - 2g(x - x_0)$$

$$x - x_0 = -\frac{v^2 - v_0^2}{2g}$$

$$x = x_0 - \frac{\cancel{v^2} - v_0^2}{2g}$$

$$= x_0 + \frac{v_0^2}{2g} = 10\,m + \frac{(2\,m/s)^2}{2(10\,m/s^2)}$$

$$= 10m + 0.2m = \underline{10.2\,m}$$



$$v = v_0 - gt$$

$$\boxed{v^2 = v_0^2 - 2g\cancel{t}\underbrace{(x - x_0)}_{h}}$$

$$v^2 = (\pm 2\,m/s)^2 - 2gh$$