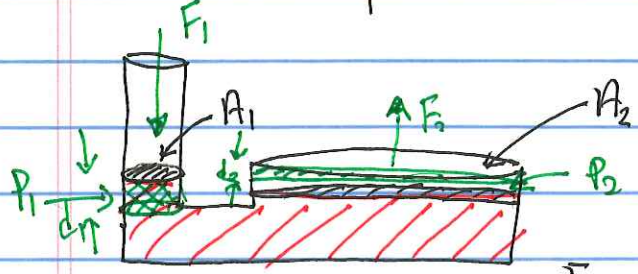
Hydraulics

Pascal's Principle



liquid: incompressible
$\rho \simeq$ constant

↳ extra pressure $P_1 = \frac{F_1}{A_1} \rightarrow P_2 = \frac{F_2}{A_2}$ $P_1 = P_2$ (Pascal's Principle)

$$\therefore F_2 = F_1\left(\frac{A_2}{A_1}\right) \gg F_1$$

"Force amplifiers"
↳ Force is not conserved!

Volume of fluid is conserved

$$V_1 = V_2$$
$$A_1 d_1 = A_2 d_2 \qquad \therefore d_2 = \left(\frac{A_1}{A_2}\right) d_1 \leq d_1$$

$\therefore W_{in} = W_{out}$ } Energy is conserved
$F_1 d_1 = F_2 d_2$ }

Consider Diver @ 10m depth in water

$$P_0 \quad P = \rho g h = \left(1000 \tfrac{Kg}{m^3}\right)\left(10 \tfrac{m}{s^2}\right)(10\,m) = 10^5\,Pa \simeq 1\,atm$$

$$P = P_0 + \rho g h = 2\,atm$$

What matters is Difference between P and inner ear ($\simeq P_0$)

$$\therefore P_{gauge} = P - P_0$$
↳ $P_{atmosphere}$

P = "absolute" pressure

Car tires @ 32 psi → $P_{gauge} = 32\,psi$

$P = P_{atm} + P_{gauge}$

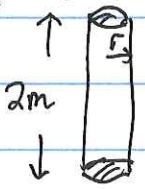
$= 14.7\,psi + 32\,psi \simeq 47\,psi$

Manometer → pressure gauge

$\Delta P = \rho g h$

↳ Pressure units "mm of Hg" 760 mm of Hg ≃ 1 atm

What is total force on your body due to P_{atm}?



$r \simeq 12\,cm$

surface Area $\simeq (2\pi r)(h)$

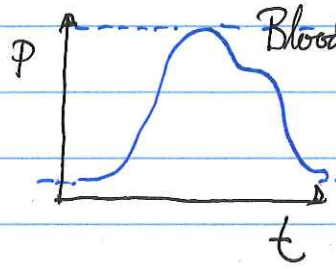
$\simeq (2\pi)(0.12\,m)(2\,m)$

$\simeq 1.5\,m^2$

$F = P_{atm} A$

$= (10^5\,Pa)(1.5\,m^2) = 150{,}000\,N!$

Pressures in Human Body



Measure Blood Pressure @ feet (relative to heart)

$\Delta P = \rho g \Delta h = \left(1060\,\frac{kg}{m^3}\right)\left(10\,\frac{m}{s^2}\right)(1.4\,m) = 14{,}840\,Pa$

$= 110$ mm of Hg!

Other pressures in Body: intraocular pressure } 12 → 24 mm of Hg

↳ glaucoma ≈ 85 mm of Hg

↳ $\Delta F_{on\ optic\ nerve} \rightarrow \Delta P A \rightarrow (85-12)(mm\ of\ Hg)\left(\frac{10^5\,Pa}{760\,mm\ of\ Hg}\right)\left(6\,cm^2\right)$ $\left(10^{-4}\,\frac{m^2}{cm^2}\right)$

$\simeq 6\,N$

Bouyancy : Archimedes Principle

$$F_B = W_{displaced}$$

F_B: Bouyant Force

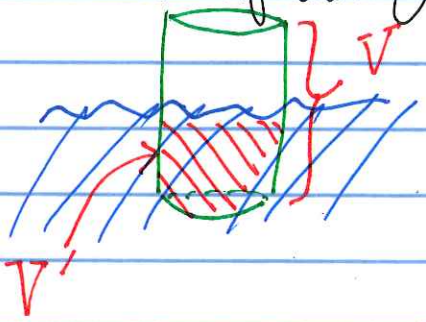
$W_{displaced}$: Weight of fluid displaced

if. $W_{disp} \geq W_{object}$ → float

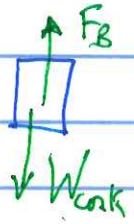
if " $\leq$ " → sinks

if $\rho_{object} < \rho_{fluid}$ → float

Example A cork of density $\simeq 200\ Kg/m^3$; floats in water. What fraction of cork is underwater?



FBD



$\vec{a} = 0$

$\Sigma F = 0$

$F_B - W_c = 0$

$\therefore F_B = W_c$

$$W_{cork} = V \rho_{cork} g$$

$$F_B = \rho_{H_2O} V' g$$

$$\rho_{cork} g V = \rho_{H_2O} V' g$$

$$\frac{V'}{V} = \frac{\rho_{cork}}{\rho_{H_2O}} = \frac{200\ Kg/m^3}{10^3\ Kg/m^3} = 0.2$$

$\therefore$ 20% of cork is submerged

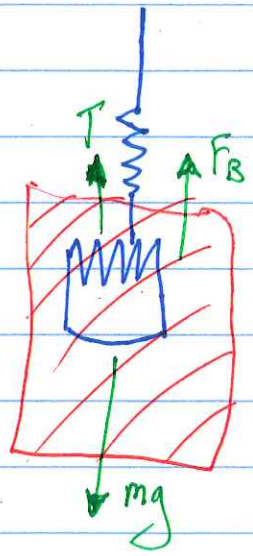
Example Au crown S.g. = 19.3 $= \rho/\rho_{H_2O}$

Volume = $50\,cm^3$

weight in air → $W = mg = \rho V g = (19.3)\left(\frac{1gm}{cm^3}\right)\left(10\frac{m}{s^2}\right)(50\,cm^3)$

$= 9.46\,N$

weight in H_2O



$F_B + T - mg = 0$

T = weight scale reads

$\therefore T = mg - F_B$

$= mg - W_{disp}$

$= 9.46N - \rho_{H_2O} V g$

$= " - \left(\frac{1gm}{cm^3}\right)(50\,cm^3)\left(10\frac{m}{s^2}\right)$

$= 8.97N$