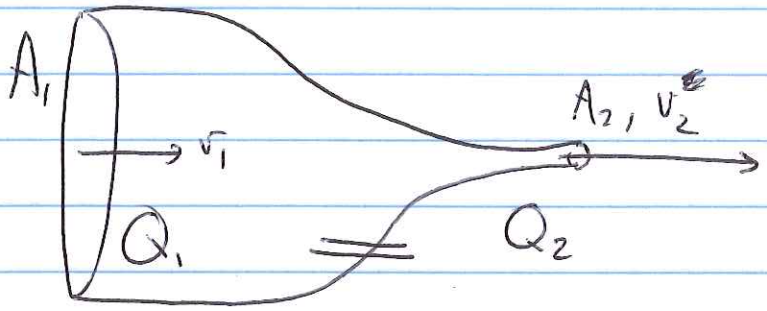
Week 11 - Friday

* Continuity equation

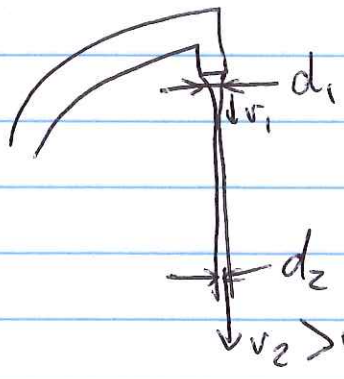


$$Q_1 = A_1 v_1 = Q_2 = A_2 v_2$$

$$\hookrightarrow \quad v_2 = \frac{A_1}{A_2} v_1 >> v_1$$

Example: water from a faucet

Q water faucet



$A_1 v_1 = Q_1 = Q$

$\|$

$A_2 v_2 = Q_2 = Q$

$\rightarrow A_2$ will be smaller than A_1

Example: blockages in arteries: how much faster will the blood flow if only half the diameter is available?

$$A_1 v_1 = A_2 v_2 = Q = \text{constant}$$

$$v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{\pi r_1^2}{\pi r_2^2} = v_1 \left(\frac{r_1}{r_2}\right)^2$$

but $r_1 = 2 r_2 \rightarrow v_2 = 2^2 v_1 = 4 v_1$

4 times faster blood flow → Doppler echo-cardiography measure local blood velocity to find obstructions.

* How does pressure influence velocity?

assumptions:
- incompressible fluid → liquids only
- low viscosity of fluid (see later)
- flow is slow / laminar / non-turbulent

Bernoulli's equation

$$\boxed{P + \rho g h + \frac{1}{2} \rho v^2 = \text{constant}}$$

or $P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$

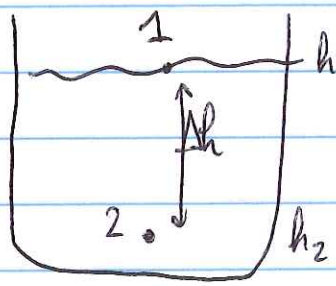
* Connection with conservation of energy:

$$P \cdot V = \frac{F}{A} \cdot Ad = Fd = \text{work}$$

$$\rho g h \cdot V = mgh = \text{gravitational potential energy}$$

$$\frac{1}{2}\rho v^2 \cdot V = \frac{1}{2}mv^2 = \text{kinetic energy}$$

* Special case: $v_1 = v_2 = 0$, $h_2 = 0$



$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

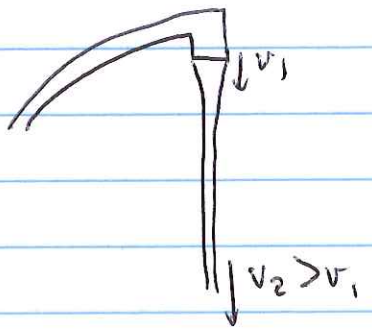
$$\Delta h = h_1 - h_2$$

$$P_2 = P_1 + \rho g \Delta h$$

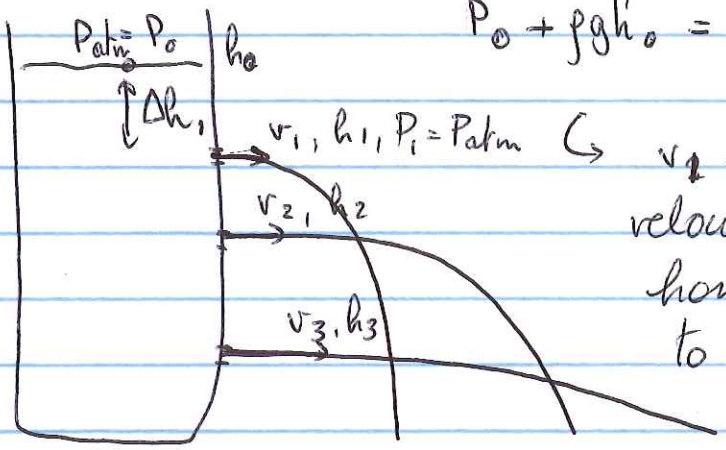
* Special case: $P_1 = P_2$

$$\frac{1}{2}\rho v_2^2 = \frac{1}{2}\rho v_1^2 + \rho g h_1 \rightarrow v_2^2 = v_1^2 + 2gh_1$$

ballistic motion of fluids



* Torricelli's theorem



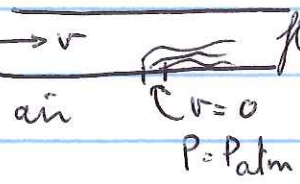
$$P_0 + \rho g h_0 = P_0 + \frac{1}{2}\rho v_1^2$$

↳ $v_1 = \sqrt{2g\,\Delta h_1}$

velocity of fluid out of horizontal opening is equal to vertical velocity of dropped mass

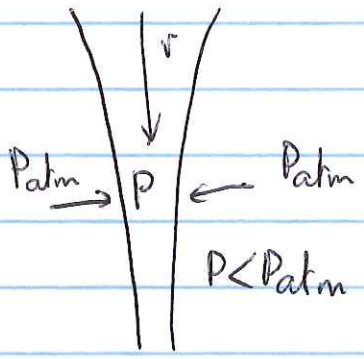
* Special case: $h_1 = h_2$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Venturi effect: 

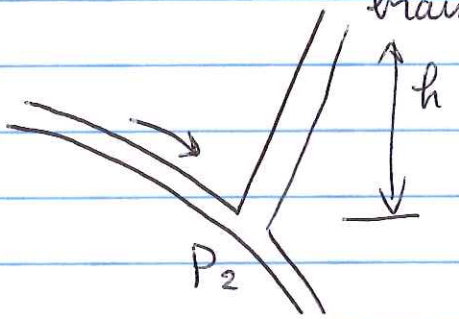
$$P_{atm} = P + \frac{1}{2}\rho v^2 \rightarrow P = P_{atm} - \frac{1}{2}\rho v^2 < P_{atm}$$

Example: blow between sheets of paper



* Transient ischemic attack, symptoms of a stroke, but transient

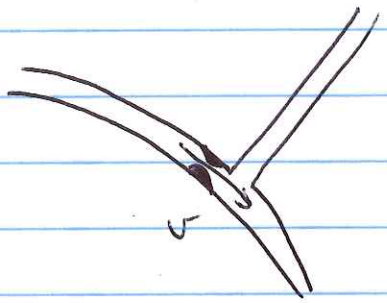
Normal



$$P_2 = P_1 + \rho g h$$

If P_2 is too low, then $P_1 < 0$ and no blood flows to the brain

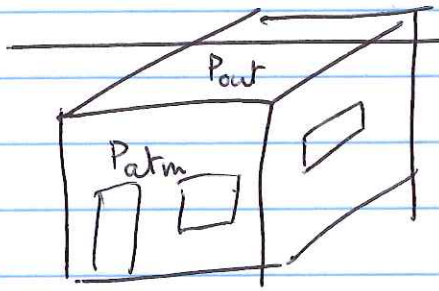
Obstruction



v increases → P_2 decreases → blood does not flow to brain

Treatment: lie down → $h = 0$ → lower blood pressure is still sufficient

* Hurricanes lift roof of house



$$P_{out} + \frac{1}{2}\rho v_{wind}^2 = P_{atm} = P_{in}$$

$$F_{lift} = (P_{out} - P_{in})A = \left(\frac{1}{2}\rho v_{wind}^2\right)A = \underline{1.3 \times 10^5 N}$$

For $A = 15\,m \times 15\,m$

$v_{wind} = 30\,m/s \approx 60$ mph