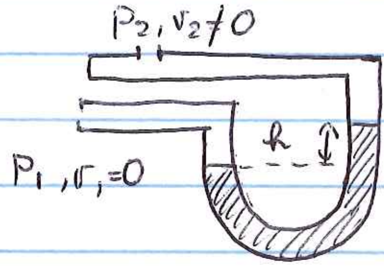
PHYS 107 - Week 12 - Monday

* Bernouilli's equation for dynamics of liquids

$$\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$$

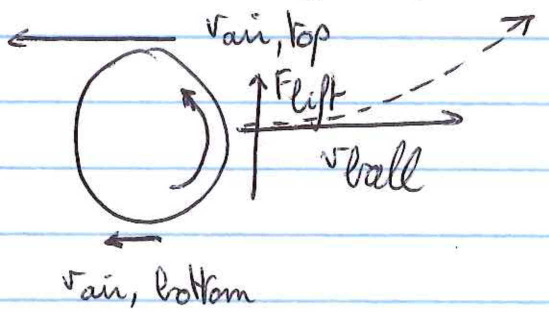
* Air speed measurement, sail boat water speed measurement based on Venturi effect



$$P_1 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2$$

$$\hookrightarrow v_2 \propto \sqrt{h}$$

* Lift of a golf ball (or other ball) with dimples
~~vel~~ viscosity traps air in the dimples



$$v_{air,top} > v_{air,bottom}$$
$$\downarrow$$
$$P_{top} < P_{bottom}$$
$$\downarrow$$
$$F_{lift}$$

* "Vascular flutter" in patient with arteriosclerosis, plaque in arteries obstructing blood flow

restriction in diameter

↓

v increases because $Q = Av$ is constant

↓

P decreases due to Venturi effect

↓

artery walls collapse completely

↓

v zero, flow stops

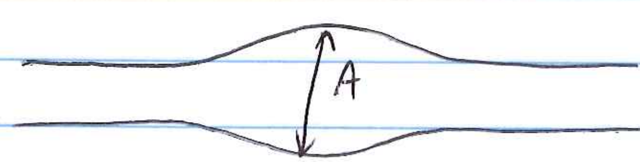
↓

P increases again

↓

artery open again

* Aneurysm : weaker spot in artery → wall balloons outward



v decreases → P increases → can cause already extended artery wall to rupture

* Power in fluid flow: if $P + \rho g h + \frac{1}{2}\rho v^2$ is energy/volume

then $\frac{\text{energy}}{\text{volume}} \times \frac{\text{volume}}{\text{time}} = \frac{\text{energy}}{\text{time}} = \text{power}$

↳ $(P + \rho g h + \frac{1}{2}\rho v^2)Q = \text{Power}$

Example: how much power does the heart need to supply the blood flow?

$Q = 83\ \text{cm}^3/\text{s}$

$\Delta P = 110\ \text{mm Hg}$

$v = 30\ \text{cm/s}$ } across heart

$h = 5\ \text{cm}$

↳ $PQ = (110\ \text{mmHg})\left(\frac{10^5\ \text{Pa}}{760\ \text{mmHg}}\right)(83 \times 10^{-6}\ \text{m}^3/\text{s}) = 1.2\ \text{W}$

↳ $\left(\frac{1}{2}\rho v^2\right)Q = \frac{1}{2}(1050\ \text{kg/m}^3)(0.3\ \text{m/s})^2(83 \times 10^{-6}\ \text{m}^3/\text{s}) = 0.004\ \text{W}$

↳ $(\rho g h)Q = (1050\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.05\ \text{m})(83 \times 10^{-6}\ \text{m}^3/\text{s})$
$= 0.04\ \text{W}$

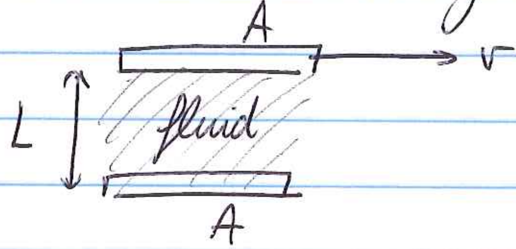
↳ total power of 1.244 W, most of the power goes to increasing the pressure

* Viscosity: non-conservative effects, energy loss in fluid

We consider laminar flow:  v large in center

$v=0$ at the edges due to <u>friction, adhesive forces</u>

Viscosity says how easy it is to move 1 layer of fluid when another close-by layer is at rest



what force is required on the top layer such that it will have a velocity v?
(energy loss; constant velocity should not require a force if there was no energy loss)

$$F = \eta \frac{vA}{L}$$, η = viscosity coefficient in units Pa·s

Examples: $\eta = 1.005 \times 10^{-3}$ Pa·s for water at 20°C
0.284×10^{-3} Pa·s for water at 100°C

$\eta = 2.084 \times 10^{-3}$ Pa·s for blood at 37°C

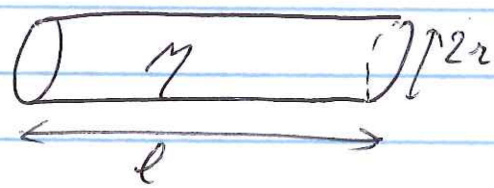
$\eta = 10$ Pa·s for honey

* How does viscosity affect the flow of a fluid?

$$Q = \frac{P_2 - P_1}{R} = \frac{\text{pressure difference}}{\text{resistance to flow}}$$

R depends on η, length of tube, radius of tube, geometry

$$Q = \frac{(P_2 - P_1)\ \pi r^4}{8 \eta l}$$



Example: what is the speed of blood in the pulmonary artery? if $\Delta P = 3.5\ mm\ Hg = 450\ Pa = P_1 - P_2$

$l = 8.5\ cm$

$r = 2.4\ cm$

$$\hookrightarrow Q = Av \rightarrow v = \frac{Q}{A} = \frac{\Delta P \pi r^4}{8 \eta l} \frac{1}{\pi r^2} = \frac{r^2 \Delta P}{8 \eta l} = 1.9\ m/s$$

Example: Inject a patient with a 3.2 cm long syringe, diameter of 0.28 mm, mass flow rate of 1.5 g/s, liquid has ρ and η of water at 20°C

What is the pressure difference needed across the syringe?

$$Q = \frac{1.5 g}{s} \times \frac{1\ cm^3}{g} = 1.5\ cm^3/s = 1.5 \times 10^{-6}\ m^3/s$$

$$\Delta P = \frac{Q\, 8 \eta l}{\pi r^4} = \frac{(1.5 \times 10^{-6} m^3/s)\ 8 (1.005 \times 10^{-3} Pa \cdot s)}{\pi (0.14 \times 10^{-3} m)^4} = 3.2\ atm$$

$\rightarrow F = P \cdot A = (3.2 \times 10^{5} Pa)(1 \times 10^{-4} m^2) = 32 N$