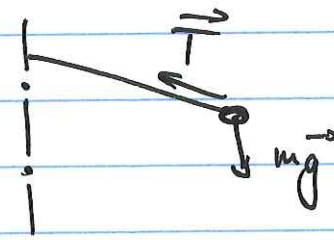
PHYS 107 - Week 6 - Monday

* Uniform circular motion
  - $|\vec{v}|$, speed is constant around circle
  - $\vec{v}$, direction changes continuously

centripetal acceleration $= \vec{a}_c = \frac{v^2}{r}$ towards center

Some external net force $\vec{F}_{net,c}$ on the object has to cause this acceleration to exist : centripetal force

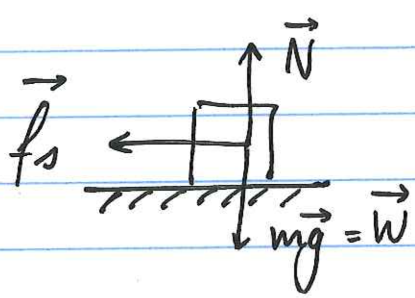
Examples : - tension in string



Q steel ball on string

- normal force on road as a car goes over a hill or through valley

* Car on an unbanked curve



static friction because wheels are rolling, not slipping

$a_c = \frac{v^2}{r}$

$$\begin{cases} x: & -\overset{f_s}{\cancel{(\mu_s N)}} = -ma_c = -m\frac{v^2}{r} \\ y: & N - W = N - mg = 0 \end{cases}$$

$$\rightarrow \quad N = mg \rightarrow f_s = m\frac{v^2}{r} \leq \mu_s mg$$

Q Car in curve

→ maximum speed such that

$$\frac{v^2}{r} \leq \mu_s g \quad \text{or} \quad \underline{\mu_s \geq \frac{v^2}{rg}}$$

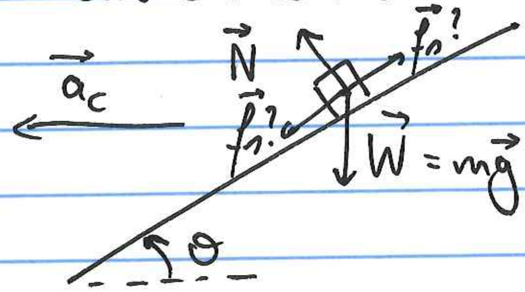
minimum coefficient of static friction required to prevent slipping

If $v \to 2v$ then μ_s needs to be 4 times larger

If $r \to \frac{1}{2} r$ then μ_s needs to be 2 times larger

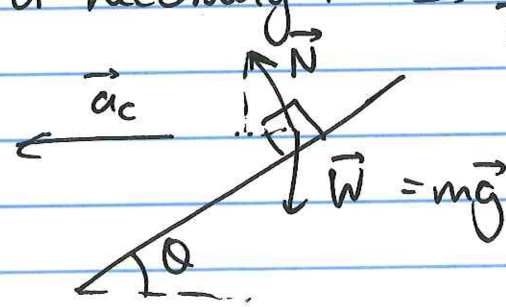
Q Car vs. truck on unbanked curve

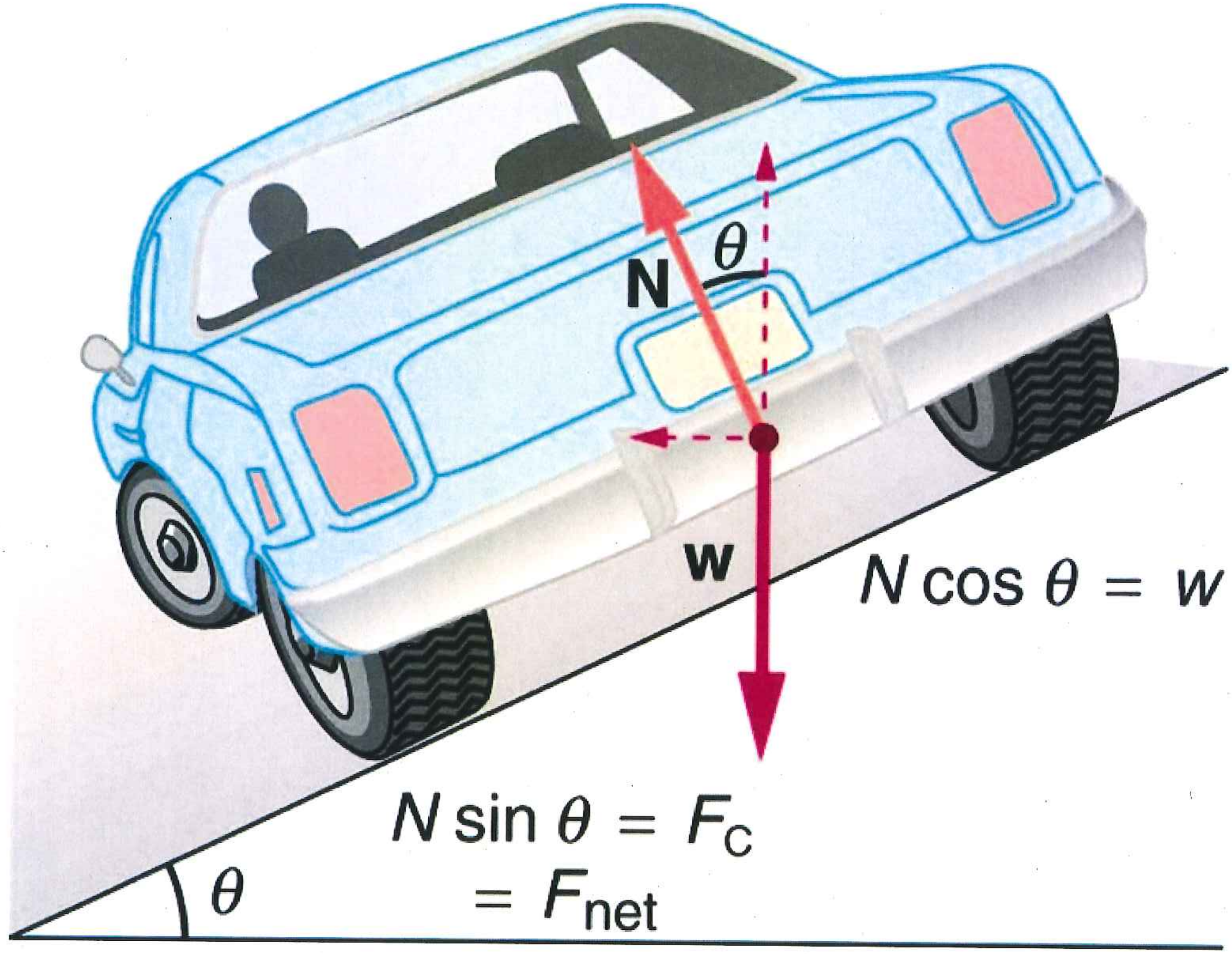
* Car on banked curve



depending on the speed, the curve's slope, etc the direction of $\vec{f}_s$ can change

What is the "ideal bank angle" when no friction is necessary? → $\vec{f}_s = 0$





N
θ
w
$N \cos\theta = w$
$N \sin\theta = F_c$
$= F_{net}$
θ

$$m\vec{a}_c = \vec{F}_{net,c} = \vec{N} + \vec{W}$$

$$\begin{cases} x: & -m\frac{v^2}{r} = -N\sin\theta \\ y: & 0 = N\cos\theta - mg \end{cases} \rightarrow N = \frac{mg}{\cos\theta}$$

$$\rightarrow \quad -m\frac{v^2}{r} = -\frac{mg}{\cos\theta}\sin\theta$$

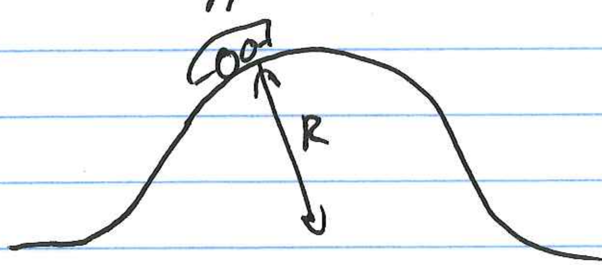
or $\tan\theta = \frac{v^2}{rg} \rightarrow$ $r = 40\,m$, $v = 20\,m/s$

$\hookrightarrow \tan\theta = \underline{45°} = \frac{v^2}{rg}$

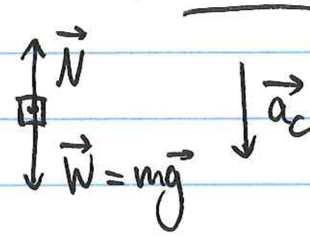
Video Victoria Pendleton

* Car on a hill:
  How fast does a car need to go before the wheels leave the ground?

What happens when wheels leave the ground? $\underline{N=0}$



FBD



$$F_{net,y} = ma_c = N - mg = -m\frac{v^2}{r}$$

$$\rightarrow N = 0 \text{ if } mg < m\frac{v^2}{r}$$

$$\text{or } v > \sqrt{gr}$$

if $r = 100\,m \rightarrow v > \sqrt{1000}\,m/s = 31.6\,m/s$

≈ 70 mph

* Gravitational force : <u>circular motion of Moon around the Earth</u>

$$a_c = \frac{v^2}{r} = R\omega^2$$

$$r = 3.84 \times 10^6\ m = 384\,000\ km$$

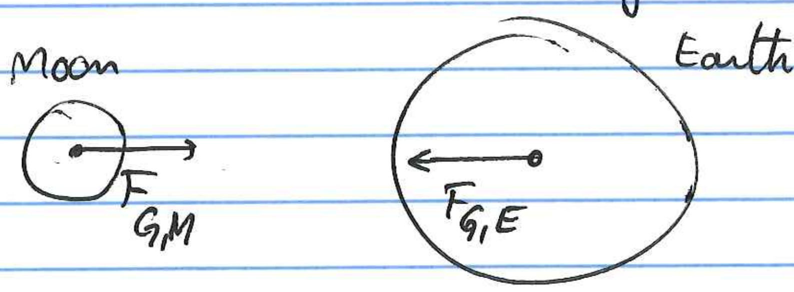
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(27.3\,d)\left(\frac{24\,hr}{1\,d}\right)\left(\frac{3600\,s}{1\,hr}\right)} = 2.66 \times 10^{-6}\ \frac{rad}{s}$$

$$\rightarrow a_c = 2.72 \times 10^{-3}\ m/s^2$$

What force causes this acceleration?

$$F_G = G\frac{m_1 m_2}{r^2} = G\frac{M_{Earth} M_{Moon}}{r^2}$$

with $G = 6.673 \times 10^{-11}\ \frac{N m^2}{kg^2}$



3^{rd} law : $F_{G,M} = -F_{G,E}$

$$M_{Earth} = 5.98 \times 10^{24}\ kg$$

$$\rightarrow F_G = G\frac{M_{Earth} M_{moon}}{r^2} = M_{moon}\, a_c \rightarrow a_c = 2.70 \times 10^{-3}\ m/s^2$$

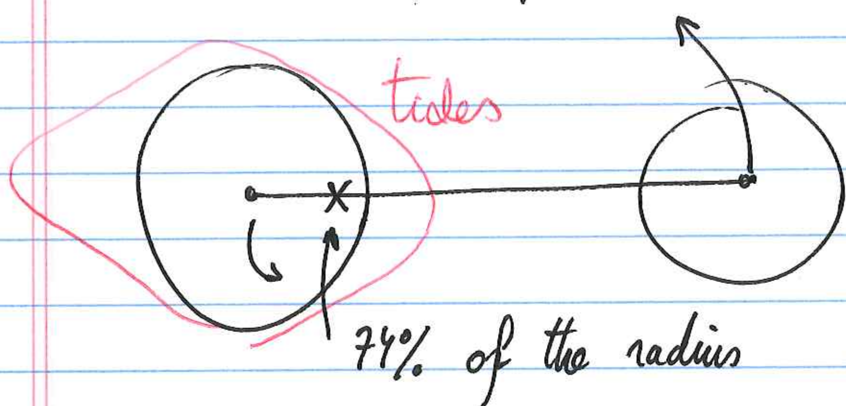
Notice how M_{moon} dropped out

* g from gravitational force at $r_E = 6380\,km$

$$F_G = G\frac{M_{Earth}\, m}{r_E^2} = m a_c$$

$$\hookrightarrow a_c = g = G\frac{M_{Earth}}{r_E^2} = 9.80\,m/s^2$$

* Common center of mass



The COM is what goes around the sun on its trajectory