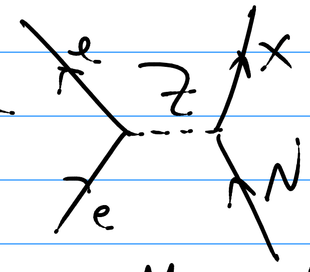
Phys 772: Week 10 Tuesday

* Interference effects in WNC

$eN \to eX$

+



$|\ \ |^2$

$M_\gamma \sim V \qquad M_Z \sim V-A$

$\hookrightarrow$ interference term: $M_\gamma^* M_Z \sim V(V-A)$

$\downarrow$ VA } $\gamma^\mu \gamma_\mu \gamma^5$ effects

$$\mathcal{L}_I^{eN} = \frac{G_F}{\sqrt{2}}\left(C_{1q}\bar{e}\gamma_\mu\gamma^5 e \cdot \bar{q}\gamma^\mu q + C_{2q}\bar{e}\gamma_\mu e \cdot \bar{q}\gamma^\mu\gamma^5 q\right)$$

$C_{1q} = 2g_A^e g_V^q$ = weak vector charge of quark

$C_{2q} = 2g_V^e g_A^q$ = weak axial charge of quark

Asymmetry $A_{PV} = \dfrac{M_\gamma^* M_Z}{|M_\gamma|^2} \sim \dfrac{M_Z}{M_\gamma} \sim \dfrac{\frac{1}{M_Z^2}}{\frac{1}{Q^2}}$

$$A_{PV} = Q^2\left[a_1 + a_2\left(\frac{1-(1-y)^2}{1+(1-y)^2}\right)\right]$$

$a_1 \sim \left(C_{1u} - \frac{1}{2}C_{1d}\right) = \left(-\frac{3}{4} + \frac{5}{3}\sin^2\theta_W\right)$

$a_2 \sim \left(C_{2u} - \frac{1}{2}C_{2d}\right) = \left(\sin^2\theta_W - \frac{1}{4}\right)$

y = normalized energy transfer

$\rightarrow$ $y = 0$ for elastic scattering $\rightarrow$ only a_1

$y > 0$ for DIS scattering $\rightarrow$ also a_2

* Atomic parity violation:

mixing of distinct parity electron wavefunctions

S has + parity } transition requires
P has − parity } violation of parity

$\downarrow$

AMO experiments measuring disallowed transitions

Cs $6S \rightarrow 7S$ with weak charge

$$Q_W = -2\left[\left(2C_{1u} + C_{1d}\right)Z + \left(C_{1u} + 2C_{1d}\right)N\right]$$

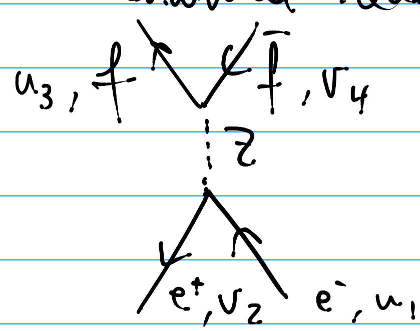
$$\sim Z\left(1 - 4\sin^2\theta_W\right) - N$$

$\rightarrow$ measurement of $\sin^2\theta_W$ at very low energy

* Forward-backward asymmetry

$$\underset{R\;\;\;L}{\overset{\uparrow\;\;\uparrow}{e^+ e^-}} \xrightarrow{Z} \underset{L\;\;\;R}{\overset{\uparrow\;\;\uparrow}{f \bar{f}}}$$

s-channel reaction (t-channel for $f = e$)



$$\mathcal{M} = -i\varepsilon_{LL}(\bar{u}_3 \gamma_\mu P_L v_4)(\bar{v}_2 \gamma^\mu P_L u_1)$$

$$+ \varepsilon_{LR} \cdots + \varepsilon_{RL} \cdots + \varepsilon_{RR} \cdots$$

Midterm: Z boson decay $\rightarrow (g_V^2 + g_A^2)$ dependence

$$+ (g_V^2 - g_A^2)\, m_f^2$$

If $m_f = 0 \rightarrow$ terms drop out, no interference between L and R particles since uniquely defined helicity

$$\begin{cases} M(-+,-+) = i\varepsilon_{LL}\, s\,(1+\cos\theta) \\ M(-+,+-) = i\varepsilon_{LR}\, s\,(1-\cos\theta) \\ M(+-,+-) = i\varepsilon_{RR}\, s\,(1+\cos\theta) \\ M(+-,-+) = i\varepsilon_{RL}\, s\,(1-\cos\theta) \end{cases}$$

Forward angles θ between e^- and f: $\cos\theta > 0$

$$\sigma_F = \int_0^1 \frac{d\sigma}{d\cos\theta}\, d\cos\theta$$

Backward angles: $\cos\theta < 0$

$$\sigma_B = \int_{-1}^0 \frac{d\sigma}{d\cos\theta}\, d\cos\theta$$

$$\Rightarrow A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4}\left(\frac{|\varepsilon_{LL}|^2 + |\varepsilon_{RR}|^2 - |\varepsilon_{LR}|^2 - |\varepsilon_{RL}|^2}{\quad " \quad " \quad + \quad " \quad + \quad "}\right)$$

ε_{LL} etc are coupling coefficients that include both γ and Z s-channel component

The γ exchange does not depend on L or R :

$$\varepsilon^{\gamma}_{AB} = \frac{Q_f e^2}{s}$$ ← vertex and propagator

The Z exchange depends on L and R :

$$\varepsilon^{Z}_{AB} = -4\sqrt{2}\, G_F M_Z^2 \frac{1}{s - M_Z^2 + i M_Z \Gamma_Z} \varepsilon_A(f)\, \varepsilon_B(e)$$

↳ remember that G_F has a $\frac{1}{M_Z^2}$, so $G_F M_Z^2$ is little g

Breit-Wigner form for unstable Z boson with decay width Γ_Z → phase shift angle δ

$\varepsilon_L(f), \varepsilon_R(f)$ from g_V^f, g_A^f

$\Rightarrow$ A_{FB} is function of $g_V^f, g_A^f, g_V^e, g_A^e$

For $f = \mu$ → $g_V^\mu = g_V^e$, $g_A^\mu = g_A^e = -\frac{1}{2}$

$$g_V^e = -\frac{1}{2} + 2\sin^2\theta_W$$

↳ $A_{FB}(e^+e^- \to \mu^+\mu^-)$ function of $\sin^2\theta_W$

one of 2 best measurements of $\sin^2\theta_W$

* Radiative corrections

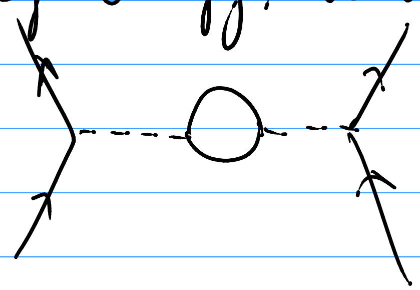
For precision electroweak measurements: need to include higher order terms $\rightarrow$ divergences

To cancel divergences: use renormalized quantities that are finite and physical, and absorb infinities in counter-terms

e.g. $e(\mu) = e_0 + \delta e$ at scale μ

$\hookrightarrow$ for example, use $\overline{MS}$ scheme for this
= prescription of renormalization procedure and connecting formulas between $\sin^2\theta_W$, M_W, M_Z, g, g', G_F
because only at tree level do the exact relations hold

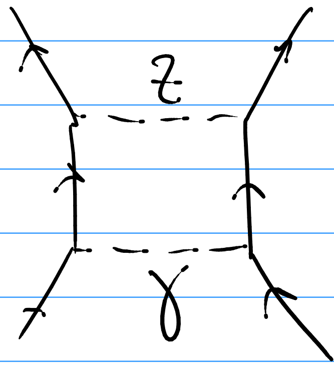
- self-energy, vacuum polarization corrections



can contain f, W, H in loop
$\downarrow$
dependence of M_W, M_Z on the m_t and M_H

$\longrightarrow O(\alpha m_t^2)$, $O(\alpha \ln M_H)$

- box diagrams :



integral over loop from QED scale up to M_Z scale → no simplifications possible

* Z boson decay :

$$\sigma(e^+e^- \to f\bar{f}) \sim \frac{s\Gamma_Z^2}{(s-M_Z^2)^2 + \frac{s^2\Gamma_Z^2}{M_Z^2}}$$ Breit-Wigner shape

$$\Gamma(Z \to f\bar{f}) \sim (g_V^{f2} + g_A^{f2}) + m_f^2 (g_V^{f2} - g_A^{f2})$$

Invisible width $= \Gamma(Z \to \nu\bar{\nu})$ for which $m_\nu = 0$

$$\Gamma_{inv} = \Gamma_Z - \Gamma(Z \to had) - \sum_i \Gamma(Z \to l_i \bar{l}_i)$$

↳ confirmation of $N_\nu = 3$: no (light) neutrinos beyond ν_e, ν_μ, ν_τ