Phys 772 : Week 4 Thursday

* Symmetry breaking and the Higgs mechanism

Start from Lagrangian with a symmetry,
e.g.
1) real scalar: $\mathcal{L} = \frac{1}{2}(\partial^\mu \varphi)^2 - V(\varphi)$
hermitian

$$V(\varphi) = \frac{1}{2}\mu^2 \varphi^2 + \frac{1}{4}\lambda \varphi^4$$

with $\varphi \to -\varphi$, Z_2 symmetry (rotation over $\{0, 180°\}$)

2) complex scalar: $\mathcal{L} = (\partial^\mu \varphi)^\dagger (\partial_\mu \varphi) - V(\varphi)$

$$V(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2$$

with $\varphi \to e^{i\beta}\varphi$, $U(1)$ symmetry (rotation over β)

redefining $\varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$

with φ_1, φ_2 real scalars

$$V(\varphi_i) = \frac{1}{2}\mu^2(\varphi_1^2 + \varphi_2^2) + \frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2$$

with $\begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix} \to O \begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix}$, $O(2)$ symmetry

3) two complex scalars . $\mathcal{L} = (\partial^\mu \Phi)^\dagger (\partial_\mu \Phi) - V(\Phi)$

redefining $\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$

with φ_i real scalars

$$V(\varphi_i) = \frac{1}{2}\mu^2 \sum_i \varphi_i^2 + \frac{1}{4}\lambda \left(\sum_i \varphi_i^2\right)^2$$

with $\begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_4 \end{pmatrix} \to O \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_4 \end{pmatrix}$, $O(4)$ symmetry

$O(4) \sim SU(2) \times SU(2)$

The classical ground state occurs at lowest potential energy $\to \langle 0|\varphi|0\rangle = \langle\varphi\rangle$ = vacuum expectation value of φ (vev)

with $\left.\frac{\partial V}{\partial \varphi_i}\right|_{\langle\varphi\rangle} = 0$ and $\left.\frac{\partial^2 V}{\partial \varphi_i \varphi_j}\right|_{\langle\varphi\rangle} \geq 0$

Always $\lambda > 0$, otherwise $V(\varphi \to \infty) \to -\infty$

For $\mu^2 > 0$: $\langle\varphi\rangle = 0$ or $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

For $\mu^2 < 0$: $\langle\varphi\rangle \neq 0$ $\to$ ground state has non-zero vacuum expectation value

Vacuum does NOT respect original symmetry

$\to$ spontaneous symmetry breaking (SSB)

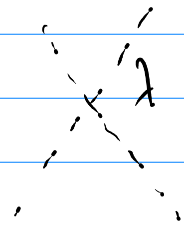
Transform to excitations around vacuum : $\varphi = \langle\varphi\rangle + \varphi'$

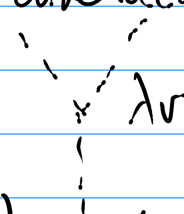
1) real scalar : $\mathcal{L} = \frac{1}{2}(\partial^\mu\varphi')^2 - V(\varphi')$

$\frac{\partial V}{\partial \varphi} = 0$ for $\langle\varphi\rangle = \pm v$ with $\langle\varphi\rangle^2 = v^2 = \frac{-\mu^2}{\lambda} > 0$ → random choice

$\to \varphi = -\frac{\mu^2}{\lambda} + \varphi'$

$$\to V(\varphi') = \underbrace{-\frac{\mu^4}{4\lambda}}_{\text{vacuum energy}} \underbrace{-\mu^2\varphi'^2}_{\frac{1}{2}m^2\varphi'^2 \text{ mass term for } \varphi'^2 \text{ with } m^2 = -2\mu^2} + \underbrace{\lambda v\varphi'^3 + \frac{\lambda}{4}\varphi'^4}_{\text{3 and 4 leg interactions}}$$





2) complex scalar : $\mathcal{L} = (\partial^\mu\varphi')^\dagger(\partial_\mu\varphi') - V(\varphi')$

$\frac{\partial V}{\partial \varphi_i} = 0$ for $\langle\varphi\rangle^2 = \varphi_1^2 + \varphi_2^2 = v^2 = \frac{-\mu^2}{\lambda} > 0$

$$\varphi = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_1'\\ \varphi_2'\end{pmatrix}$$

$$\to \mathcal{L} = \frac{1}{2}(\partial^\mu\varphi_1')^2 + \frac{1}{2}(\partial^\mu\varphi_2')^2 - V(\varphi_i)$$

$$\rightarrow V(\varphi') = -\frac{\mu^4}{4\lambda} - \mu^2 \varphi_1'^2 + \lambda v \varphi_1' (\varphi_1'^2 + \varphi_2'^2) + \frac{\lambda}{4}(\varphi_1'^2 + \varphi_2'^2)^2$$

φ_1' : $\frac{1}{2} m^2 \varphi_1'^2$ mass term for real scalar φ_1' with $m^2 = -2\mu^2$

φ_2' : no mass term $\rightarrow$ massless Goldstone boson

No observed massless scalar (spin-0) particles $\rightarrow$ so why do we care?

3) two complex scalars:

$$\langle \varphi \rangle = \left\langle \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \right\rangle = \left\langle \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \right\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

choose non-zero vacuum in neutral field

$\varphi = \langle \varphi \rangle + \varphi'$ with $\langle \varphi' \rangle = 0$

$\varphi_3' = H$ will become real scalar field, Higgs field

$\hookrightarrow m^2 = -2\mu^2$, $m_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v$

$\varphi_1', \varphi_2', \varphi_4'$ are 3 massless Goldstone bosons, not observed in nature

$v = 246$ GeV, $m_H = 125$ GeV observed
$\hookrightarrow$ from M_W observations

What with the massless Goldstone bosons?
They combine with gauge invariance to give mass to the gauge bosons!

* Consider U(1) global gauge invariance complex scalar field which will introduce 1 massless gauge boson:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\varphi)^+(D_\mu\varphi) - \mu^2\varphi^+\varphi - \lambda(\varphi^+\varphi)^2$$

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2) = \frac{1}{\sqrt{2}}\left(v + (\sigma + i\chi)\right)$$

$$v = \sqrt{-\frac{\mu^2}{\lambda}}, \quad \langle\varphi\rangle = v$$

$\rightarrow \left\{ \begin{array}{l} \text{massive physical scalar } \sigma \\ \text{massless Goldstone boson } \chi \end{array} \right.$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\mu^2(v+\sigma+i\chi)^+(v+\sigma+i\chi)$$
$$-\frac{1}{4}\lambda\left((v+\sigma+i\chi)^+(v+\sigma+i\chi)\right)^2$$
$$+\frac{1}{2}\left[(\partial^\mu + igA^\mu)(v+\sigma+i\chi)\right]^+\left[(\partial_\mu + igA_\mu)(v+\sigma+i\chi)\right]$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial^\mu\sigma)^2 - \frac{1}{2}(-2\mu^2)\sigma^2 + \frac{1}{2}(\partial^\mu\chi)^2$$
$$-\frac{\mu^4}{2\lambda} + \text{many interaction terms}$$

Since this is all symmetric, by design, under local $U(1)$ transformations, we write

$$\varphi = \frac{1}{2}(v + \sigma + i\chi) = e^{i\xi/v} \frac{1}{\sqrt{2}}(v + \eta)$$ polar basis, Kibble transf.

$$\approx (1 + i\xi/v)\left(\frac{1}{\sqrt{2}}(v+\eta)\right)$$

$$= \frac{1}{\sqrt{2}}(v + \eta + i\xi)$$

$\rightarrow \sigma \approx \eta$ and $\chi \approx \xi$

and pick a gauge transformation $e^{i\beta(x)}$ such that

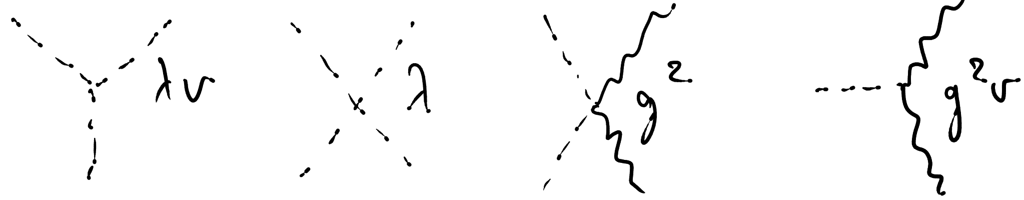
$$\varphi \rightarrow e^{i\beta(x)}\varphi = \frac{1}{\sqrt{2}}(v+\eta)$$

$$\hookrightarrow \beta(x) = -\xi(x)/v$$

$$A_\mu \rightarrow A_\mu - \frac{1}{g}\partial_\mu \beta(x) = A_\mu + \frac{1}{gv}\partial_\mu \xi$$

I.e. we pick a specific gauge such that only $\eta \approx \sigma \approx$ the massive scalar boson remains:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\mu^2 (v+\eta)^\dagger (v+\eta) - \frac{\lambda}{4}\left((v+\eta)^\dagger (v+\eta)\right)^2$$
$$+ \frac{1}{2}\left[(\partial^\mu + igA^\mu)(v+\eta)\right]^\dagger \left[(\partial_\mu + igA_\mu)(v+\eta)\right]$$
$$= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial^\mu \eta)^2 - \frac{1}{2}(-2\mu^2)\eta^2 - \frac{\mu^4}{2\lambda}$$
$$- \lambda v \eta^3 - \frac{\lambda}{4}\eta^4 + \frac{1}{2}g^2 v^2 A_\mu A^\mu$$



$$+ \; g^2 v \eta A_\mu A^\mu + \frac{1}{2} g^2 \eta^2 A_\mu A^\mu$$

$\Rightarrow$ We get a $\frac{1}{2} g^2 v^2 A_\mu A^\mu$ gauge boson mass term with $M_A = gv$

We started with $U(1)$ gauge symmetry $\longrightarrow$ covariant derivatives $\longrightarrow$ non-zero mass term $\longleftarrow$ spontaneous symmetry breaking (SSB)

Notice: in the absence of SSB: $v = 0$, $M_A = 0$

Theory is still gauge invariant, but we had to pick a specific vacuum, so vacuum is not.

Degrees of freedom: $\left\{ \begin{array}{l} 2 \text{ d.o.f. in complex scalar} \\ 2 \text{ d.o.f. in massless } A_\mu \\ \quad (2 \text{ polarization states}) \end{array} \right.$

$\downarrow$

$\left\{ \begin{array}{l} 1 \text{ d.o.f. in massive scalar} \\ 3 \text{ d.o.f. in massive } A_\mu \\ \quad (\text{also longitudinal pol.}) \end{array} \right.$

Could explicitly keep χ field $\rightarrow$ would have included term

$$g v A_\mu \partial^\mu \chi$$ $\rightarrow$ A_μ can turn into χ as it propagates

gv

$\downarrow$

eigenstate diagonali- which we have performed here through gauge transformation

* Fermion mass terms:

chiral fermion spinors (massless): $\psi = \psi_L + \psi_R$

$$\begin{cases} \psi_L = P_L \psi, & \psi_R = P_R \psi \\ \bar{\psi}_L = \bar{\psi} P_R, & \bar{\psi}_R = \bar{\psi} P_L \end{cases}$$

$+$

complex scalar and SSB

$$\mathcal{L} = \bar{\psi}_L i \not{\partial} \psi_L + \bar{\psi}_R i \not{\partial} \psi_R \cancel{- \bar{\psi}_L m \psi_L - \bar{\psi}_R m \psi_R}$$

$$+ (\partial^\mu \varphi)^\dagger (\partial_\mu \varphi) - \mu^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^2 \quad (V(\varphi) \text{ terms})$$

$$- h \bar{\psi}_L \psi_R \varphi - h^* \bar{\psi}_R \psi_L \varphi^\dagger$$

$\mathcal{L} \xrightarrow[L \leftrightarrow R]{} \mathcal{L}' = \mathcal{L}$ requires h, h^* and φ, φ^*

This has $U(1)$ chiral symmetry when $m = 0$ only:

$$\begin{cases} \varphi \rightarrow e^{i\beta} \varphi \\ \bar{\psi}_L \psi_R \rightarrow e^{-i\beta} \bar{\psi}_L \psi_R \end{cases}$$

Spontaneous symmetry breaking for $\mu^2 < 0$

$\rightarrow \varphi = \frac{1}{\sqrt{2}}(v + \varphi_1' + i\varphi_2')$

Interaction terms for h = real

$$\mathcal{L}_{int} = -h\,\bar{\psi} P_R P_R \psi \frac{1}{\sqrt{2}}(v + \varphi_1' + i\varphi_2') - h\,\bar{\psi} P_L P_L \psi \frac{1}{\sqrt{2}}(v + \varphi_1' - i\varphi_2')$$

$$= -h\,\bar{\psi}\left(\frac{1+\gamma^5}{2}\right)\psi \frac{1}{\sqrt{2}}(v + \varphi_1' + i\varphi_2')$$

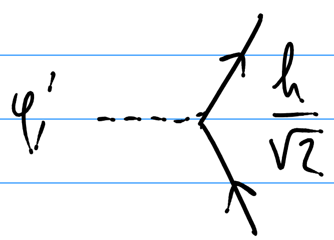
$$- h\,\bar{\psi}\left(\frac{1-\gamma^5}{2}\right)\psi \frac{1}{\sqrt{2}}(v + \varphi_1' - i\varphi_2')$$

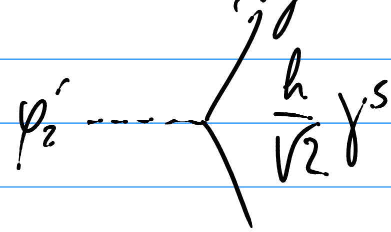
$$= -h\left(\bar{\psi}_L \psi_R \varphi + \bar{\psi}_R \psi_L \varphi^*\right)$$

$$= -h\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)\frac{1}{\sqrt{2}}(v + \varphi_1') - h\left(\bar{\psi}_L \psi_R - \bar{\psi}_R \psi_L\right)\frac{1}{\sqrt{2}} i\varphi_2'$$

$$= -\frac{hv}{\sqrt{2}}\bar{\psi}\psi\left(1 + \frac{\varphi_1'}{v}\right) - \frac{h}{\sqrt{2}} i\bar{\psi}\gamma^5\psi\,\varphi_2'$$

$m_\psi = \frac{hv}{\sqrt{2}}$ generated from spontaneous symmetry breaking





Recall polar basis $\varphi = \frac{1}{\sqrt{2}}(v + \varphi_1' + i\varphi_2') = \frac{1}{\sqrt{2}}(v+\eta)e^{i\chi/v}$

$$\rightarrow \mathcal{L}_{int} = -\frac{hv}{\sqrt{2}}\bar{\psi}_L \psi_R \left(1 + \frac{\eta}{v}\right) e^{i\chi/v}$$

* For non-abelian, e.g. SU(2), gauge symmetry and for $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ → similar procedure, but more complex math

For U(1): 1 generator only → SSB breaks entire symmetry

For SU(2): 3 generators: SSB could break subset of generators

Generally: $\varphi = \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_n \end{pmatrix} \rightarrow \langle\varphi\rangle = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ (v_i could be zero)

$$\varphi = \langle\varphi\rangle + \varphi'$$

$\langle\varphi\rangle$ determined from $\frac{\partial V}{\partial \varphi_i} = 0$

Most general Lagrangian (dimension ≤ 4)

$$\mathcal{L} = V_0 + \frac{1}{2!}\mu_{ab}\varphi'_a\varphi'_b + \frac{1}{3!}K_{abc}\varphi'_a\varphi'_b\varphi'_c + \frac{1}{4!}\lambda_{abcd}\varphi'_a\varphi'_b\varphi'_c\varphi'_d$$

with $\mu_{ab} = \left.\frac{\partial^2 V}{\partial\varphi_a\partial\varphi_b}\right|_{\langle\varphi\rangle}$ etc

mass matrix

If $V(\varphi)$ is invariant $\to$ $\delta V(\varphi) = 0$

$$0 = \delta V(\varphi) = \frac{\partial V}{\partial \varphi_a} \delta \varphi_a = \frac{\partial V}{\partial \varphi_a} (i\beta \cdot L)_{ab} \varphi_b$$

$$\downarrow$$

$$\frac{\partial V}{\partial \varphi_a} L^i_{ab} \varphi_b = 0 \text{ for all } i$$

$$\downarrow \left.\frac{\partial}{\partial \varphi_c}\right|_{\langle\varphi\rangle}$$

$$\left.\frac{\partial^2 V}{\partial \varphi_a \partial \varphi_c}\right|_{\langle\varphi\rangle} L^i_{ab} \varphi_b = 0$$

$$\to \mu_{ac} (L^i v)_c = 0 \text{ for all } i$$

If $L^i v = 0$ for $i = 1, \ldots, M$: $\langle\varphi\rangle$ symmetric

$L^i v \neq 0$ for $i = M+1, \ldots, N$: $\langle\varphi\rangle$ broken

then $L^i v$, $i = M+1, \ldots, N$ must be $N-M$ eigenvectors of μ with eigenvalues zero

$\downarrow$

$N-M$ massless Goldstone bosons, one for each broken generator

remaining $p = n - (N-M)$ eigenvalues non-zero $\to$ massive scalars

$\uparrow$

dimension of representation = number of fields

Similarly for SSB to give gauge bosons mass

$$\varphi = \begin{pmatrix} v_1 + \sigma_1 \\ \vdots \\ v_p + \sigma_p \\ \chi_{p+1} \\ \vdots \\ \chi_n \end{pmatrix} = e^{i\bar{\xi}\cdot\bar{L}}(v+\eta) = e^{i\bar{\xi}\cdot\bar{L}} \begin{pmatrix} v_1 + \eta_1 \\ \vdots \\ v_p + \eta_p \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$\left(\varphi = v + \sigma + i\chi \rightarrow e^{i\xi}(v+\sigma) \right)$$

Again gauge transformation $e^{-i\bar{\beta}\cdot\bar{L}}$ with $\bar{\beta} = -\bar{\xi}$ will remove the exponential term, leaving

$$\frac{1}{2}(D^\mu\varphi)^\dagger(D_\mu\varphi) = \frac{1}{2}\left[\left(\partial_\mu + ig\bar{A}_\mu\cdot\bar{L}\right)(v+\eta)\right]^\dagger \left[\left(\partial^\mu + ig\bar{A}^\mu\cdot\bar{L}\right)(v+\eta)\right]$$

The quadratic terms in $A^i_\mu A^{j\mu}$ give the mass matrix:

$$M^2_{ij} = g^2 v^T L_i L_j v \rightarrow N-M \text{ non-zero eigenvalues}$$

while the gauge boson interactions with the Higgs fields are given by:

$$g^2 v^T L_i L_j \eta A^{i\mu} A^j_\mu$$