Phys 772 : Week 12 Tuesday

* Neutrinos and their masses

Remember spinors :

Dirac spinor $\psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \rightarrow$ 4-component

Weyl spinors $\Psi_L, \Psi_R \rightarrow$ 2-components

$$\psi_L = \begin{pmatrix} \Psi_L \\ 0 \end{pmatrix} = P_L \psi = \frac{1-\gamma^5}{2}\psi = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\psi$$

Dirac fermion field and Lagrangian :

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi$$

$$\bar{\psi}\psi = \psi^+\gamma^0\psi = (\Psi_L^+ \;\; \Psi_R^+)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix}$$

$$= \Psi_L^+\Psi_R + \Psi_R^+\Psi_L \quad (\text{Weyl})$$

$$\bar{\psi}(\psi_L + \psi_R) = \bar{\psi}P_L\psi + \bar{\psi}P_R\psi = \bar{\psi}P_L P_L\psi + \bar{\psi}P_R P_R\psi$$

$$= \overline{P_R\psi}\,P_L\psi + \overline{P_L\psi}\,P_R\psi$$

$$= \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R \quad (\text{Dirac})$$

$$= \Psi_R^+\Psi_L + \Psi_L^+\Psi_R \quad (\text{Weyl})$$

* Weyl spinors

- Parity transformation:

$$\begin{cases} P\psi(x)P^{-1} = \gamma^0\psi(x') & \text{with } x' = (t, -\vec{x}) \\ P\bar{\psi}(x)P^{-1} = \bar{\psi}(x')\gamma^0 \end{cases}$$

$$\rightarrow \begin{cases} P\psi_{L,R}(x)P^{-1} = \gamma^0\psi_{R,L}(x') \\ P\bar{\psi}_{L,R}(x)P^{-1} = \bar{\psi}_{R,L}(x')\gamma^0 \end{cases}$$

$$\rightarrow \begin{cases} P\Psi_L(x)P^{-1} = \Psi_R(x') \\ P\Psi_R(x)P^{-1} = \Psi_L(x') \end{cases}$$

- Charge conjugation:

$$\begin{cases} C\psi C^{-1} = \psi^c = \mathcal{C}\bar{\psi}^T = \mathcal{C}\gamma^{0T}\psi^{+T} \\ C\bar{\psi}C^{-1} = \bar{\psi}^c = (\psi^c)^+\gamma^0 = -\psi^T\mathcal{C}^{-1} \end{cases}$$

$$\text{with } \mathcal{C} = -i\gamma^2\gamma^0 = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix}$$

$$\rightarrow C\begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix}C^{-1} = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \Psi_L^* \\ \Psi_R^* \end{pmatrix} = \begin{pmatrix} \Psi_L^c \\ \Psi_R^c \end{pmatrix}$$

$$\rightarrow \begin{cases} C\Psi_L C^{-1} = \Psi_L^c = -i\sigma^2\Psi_R^* \\ C\Psi_R C^{-1} = \Psi_R^c = i\sigma^2\Psi_L^* \end{cases}$$

- CP transformation:

P: maps L, R onto R, L
C: maps L, R onto R^*, L^*

$$(CP)\,\Psi_L\,(CP)^{-1} = \Psi_R^c(x') = i\sigma^2\,\Psi_L^*(x')$$

↳ only single Weyl spinor: can still have CP transformation

* Left-handed and right-handed neutrinos

ν_L : active neutrinos in $SU(2)_L$ gauge interactions

ν_R : sterile neutrinos, singlet in $SU(2)_L$
↓
no interactions except:
possible Higgs Yukawa
neutrino mass mixing
BTSM physics

As spinors:
$$\begin{cases} \nu_L \xrightarrow{CP} \nu_R^c & \text{are active neutrinos} \\ \nu_R \xrightarrow{CP} \nu_L^c & \text{are sterile neutrinos} \end{cases}$$

Now consider how we can build mass terms beyond Dirac mass terms $\bar{\psi} m \psi$ which are chirally symmetric.

Dirac chiral spinors : $\psi_L, \psi_R \to \nu_L, \nu_R, \bar{\nu}_L, \bar{\nu}_R$

Weyl spinors : $\Psi_L, \Psi_R \to N_L, N_R$

* Dirac mass : (as before)

$$\mathcal{L}_D = -m_D(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) = -m_D \bar{\nu}_D \nu_D$$

think: ψ_L, ψ_R and $\psi = \psi_L + \psi_R$

$\to \nu_D = \nu_L + \nu_R$ = Dirac spinor

$$\mathcal{L}_D = -m_D(N_L^\dagger N_R + N_R^\dagger N_L), \quad \text{Weyl spinors}$$

Global symmetry $\begin{cases} \nu_L \to e^{i\beta}\nu_L \\ \nu_R \to e^{i\beta}\nu_R \end{cases}$ $\to$ conserved quantity L = lepton number

Where could this term come from?

- in Lagrangian explicitly $\to$ violates weak isospin conservation

$$\overset{\nu_L}{\longleftarrow}\overset{\times}{\underset{m_D}{}}\overset{\nu_R}{\longleftarrow}$$

$t_{3\nu_L} = \frac{1}{2}$ $\quad m_D \quad$ $t_{3\nu_R} = 0$ $\quad \to \Delta t_{3\nu} = \frac{1}{2}$

- Higgs mechanism, Yukawa coupling
  $\to$ Higgs vev $\langle\varphi^0\rangle$ has $T_{3\varphi} = -\frac{1}{2}$ (doublet)

$$\hookrightarrow h_\nu = \frac{m_\nu}{v} = \frac{O(1\,\text{eV})}{246\,\text{GeV}} \sim 10^{-11} \ll h_t \approx 1$$

$$\ll h_e \approx 10^{-5}$$

→ requires "fine-tuning" of Yukawa couplings if the same mechanism as for other leptons

* Majorana mass terms:

$$\psi_a = \psi_{aL} + \psi_{aR} = \begin{pmatrix} \chi_{aL} \\ \chi_{aR} \end{pmatrix}, \quad \psi_b = \psi_{bL} + \psi_{bR} = \begin{pmatrix} \chi_{bL} \\ \chi_{bR} \end{pmatrix}$$

$$\hookrightarrow \mathcal{L} = -m\,\bar\psi_a \psi_b = -m\left(\bar\psi_{aL}\psi_{bR} + \bar\psi_{bR}\psi_{aL}\right)$$

$$\text{Dirac: } \psi_b = \begin{pmatrix} \chi_{aL} \\ \chi_{aR} \end{pmatrix} \Longrightarrow \text{Majorana: } \psi_b = \begin{pmatrix} \chi_{aL}^c \\ \chi_{aR}^c \end{pmatrix}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\mathcal{L}_D = -m_D\left(\bar\nu_L \nu_R + \bar\nu_R \nu_L\right) \Longrightarrow \mathcal{L}_T = -\frac{m_T}{2}\left(\bar\nu_L \nu_R^c + \bar\nu_R^c \nu_L\right)$$

$$\Rightarrow \mathcal{L}_T = -\frac{m_T}{2}\,\bar\nu_M \nu_M \quad \text{with} \quad \nu_M = \nu_L + \nu_R^c$$

ν_R for Dirac

$$\hookrightarrow \nu_M^c = C\bar\nu_M^T = C\bar\nu_L^T + C\overline{\nu_R^c}^T$$

$$= \nu_L^c + \nu_R$$

$$\nu_L \xrightarrow{\ \times\ m_T\ } \nu_L \leftarrow \nu_L^c$$

$$t_{3\nu_L} = \frac{1}{2} \qquad t_{3\nu_L^c} = -\frac{1}{2} \longrightarrow \Delta t_{3\nu_L} = 1$$

↳ requires coupling to two vevs of $\langle\varphi^0\rangle$ $\longrightarrow \Delta t_{3\varphi^2} = 1$

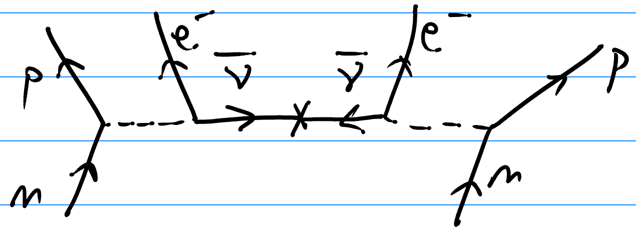
or coupling to Higgs $\langle\varphi_T^0\rangle$ with $t_{3\varphi} = 1$

↳ e.g. $\begin{pmatrix} \varphi_T^0 \\ \varphi_T^+ \\ \varphi_T^{++} \end{pmatrix} \begin{matrix} +1 \\ 0 \\ -1 \end{matrix}$ triplet

$2\nu\beta\beta$: $2\times(n\to p\, e^- \bar\nu) \longrightarrow {}^{A}_{N}Z \to {}^{A}_{N-2}Z+2,\ 2e^-,\ 2\bar\nu$

$0\nu\beta\beta$: first $\bar\nu$ reabsorbed by second decay as ν !

$${}^{A}_{N}Z \to {}^{A}_{N-2}Z+2,\ 2e^-,\ 0\,\bar\nu$$



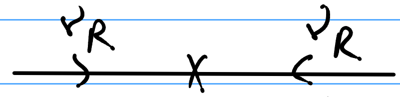
Previous discussion: $\underbrace{\nu_L, \bar\nu_L}_{\text{active neutrinos (L-handed)}},\ \underbrace{\nu_R^c, \bar\nu_R^c}_{\text{active anti-neutrinos (R-handed)}}$

$\downarrow$

Could also make sterile neutrinos ν_R, ν_L^c into Majorana particles:

$$\mathcal{L}_S = -\frac{m_S}{2}\left(\overline{\nu_L^c}\,\nu_R + \bar\nu_R\,\nu_L^c\right) = -\frac{m_S}{2}\,\bar\nu_{M_S}\nu_{M_S}$$

with $\nu_{M_S} = \nu_R + \nu_L^c = \nu_{M_S}^c$



$t_{3\nu_R}=0$ $\quad m_S \quad$ $t_{3\nu_R}=0$ $\to \Delta t_{3\nu_R}=0 \to \langle\varphi_S^0\rangle$ with $t_{3\varphi_S}=0$ singlet

In summary, for single neutrino flavor:

Dirac mass neutrino four-component spinor

$$\nu_D = \nu_L + \nu_R = \begin{pmatrix} N_L \\ N_R \end{pmatrix} \leftarrow \text{2 Weyl spinor degrees of freedom}$$

Active neutrino Majorana spinor (requires Higgs Triplet)

$$\nu_M = \nu_L + \nu_R^c = \begin{pmatrix} N_L \\ N_R^c \end{pmatrix} = \begin{pmatrix} N_L \\ i\sigma^2 N_L^* \end{pmatrix} \leftarrow \text{dependent Weyl spinors}$$

Sterile neutrino Majorana spinor (requires Higgs singlet)

$$\nu_S = \nu_R + \nu_L^c = \begin{pmatrix} N_L^c \\ N_R \end{pmatrix} = \begin{pmatrix} N_L^c \\ i\sigma^2 N_L^{c*} \end{pmatrix}$$

$$\Rightarrow \mathcal{L} = -\frac{1}{2}\begin{pmatrix} \bar{\nu}_L & \bar{\nu}_L^c \end{pmatrix}\begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix}\begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} + h.c.$$

weak eigenstates + mass mixing matrix

$\Downarrow$

2 mass eigenstates $\neq$ weak eigenstates

$$\Rightarrow \nu_{iM} = \nu_{iL} + \nu_{iR}^c \ , \ i = 1, 2$$

with Majorana mass eigenvalues m_i

Transformation : $\begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix} = A_L^{\nu +} \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix}$

$$\begin{pmatrix} \nu_{1R}^c \\ \nu_{2R}^c \end{pmatrix} = A_R^{\nu +} \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix}$$

→ similar to treatment of V_{CKM}, but less constraints on V_{PMNS} being close to diagonal (little mixing between families)

Limits for single neutrino flavor:

a) Pure Majorana: $m_D = 0$ → $m_1 = m_S$, $m_2 = m_T$
diagonal mass matrix

b) Pure Dirac: $m_S = 0$, $m_T = 0$ → recover Dirac mass term

c) Seesaw mechanism: heavy sterile neutrinos
Majorana

$m_S \approx O(10^{14}\ GeV)$
m_T small, m_D of typical SM Yukawa coupling size

↳ $\begin{cases} \nu_{1L} \approx \nu_L - \frac{m_D}{m_S} \nu_L^c \approx \nu_L & \text{with } m_1 \approx m_T - \frac{m_D^2}{m_S} \text{ light} \\ \nu_{2L} \approx \frac{m_D}{m_S} \nu_L + \nu_L^c \approx \nu_L^c & \text{with } m_2 \approx m_S \text{ heavy} \end{cases}$

→ at low energies only ν_{1L} is left

* Multiple families : $\nu_L \to \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}, \nu_L^c \to \begin{pmatrix} \nu_{1L}^c \\ \nu_{2L}^c \\ \nu_{3L}^c \end{pmatrix}$

$$\to \mathcal{L} = -\left(\bar{\nu}_{1L} \; \bar{\nu}_{2L} \; \bar{\nu}_{3L} \; \bar{\nu}_{1L}^c \; \bar{\nu}_{2L}^c \; \bar{\nu}_{3L}^c \right) \times \begin{pmatrix} M_T & M_D \\ M_D^T & M_S \end{pmatrix} \begin{pmatrix} \nu_{1R}^c \\ \nu_{2R}^c \\ \nu_{3R}^c \\ \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$$

↳ ν_L and ν_R are mass eigenstates in terms of weak eigenstates

$$\begin{cases} \nu_L = A_L^{\nu +} \nu_L^0 \\ \nu_R = A_R^{\nu +} \nu_R^0 \end{cases}$$

↳ $V_{PMNS} = A_L^{\nu +} A_L^e$ in $\mathcal{L} \bar{\nu}_L \gamma^\mu V e_L$