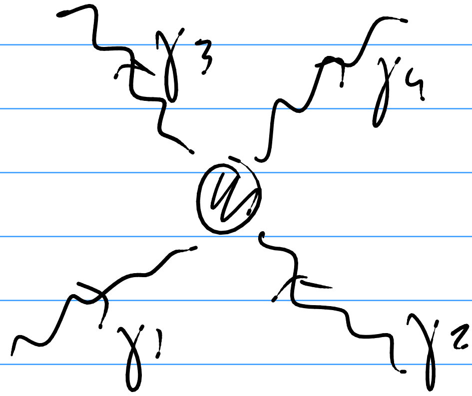
Phys 772: Week 6 Tuesday

* Loops and perturbation theory

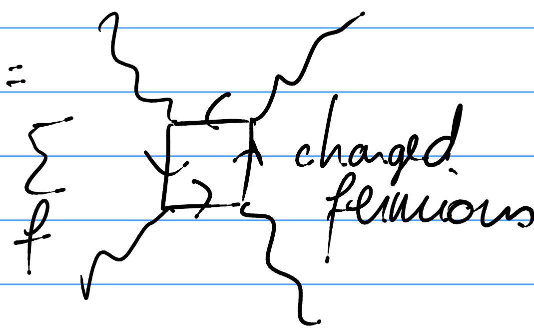
In Standard Model, only limited benefit without loop diagrams (tree-level). Need higher order loop diagrams for precision or for processes that cannot be described other-wise

LBL



$\gamma_1 \gamma_2 \to \gamma_3 \gamma_4$

=



$\to \quad M = M_{\text{tree-level}} + M_{\text{one-loop}} + M_{\text{two-loop}} + \cdots$

- with hopefully decreasing effect
- without divergences

Divergences must ultimately cancel when full perturbative series with all diagrams is calculated for physical observables.

* Running couplings in QED (Langacker 2.12)

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ + e A\!\!\!/ - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\downarrow$$

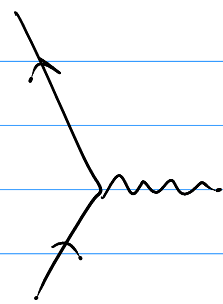
$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ + e_0 A\!\!\!/ - m_0\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

with e_0, m_0 bare coupling and bare quark mass

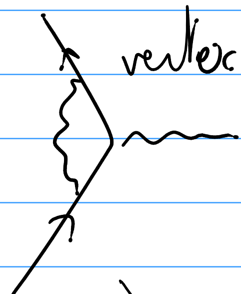
e, m are observable coupling / mass

Tree level:

$ie_0\gamma^\mu$

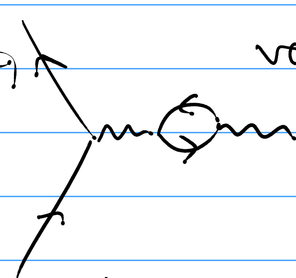


One loop:



vertex corr Z_1

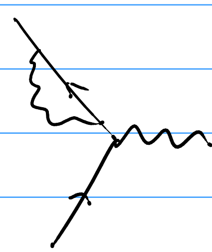
+



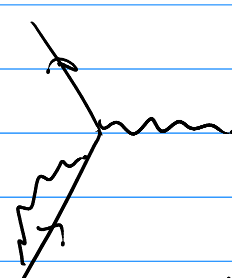
vacuum polarization photon self-energy Z_3

$$ie_0 \frac{\Gamma^\mu}{Z_1} Z_3^{1/2} Z_2$$

$\Gamma^\mu = \gamma^\mu + O(e^2) +$



+



electron self-energy, Z_2

Renormalization factors Z_1, Z_2, Z_3 are divergent

Due to Ward-Takahashi identity $Z_1 = Z_2 \rightarrow$ only dependence on Z_3 left:

$$e = e_0 Z_3^{1/2}, \quad e^2 = e_0^2 Z_3$$

Photon propagator with vacuum polarization

$$-\frac{i g_{\mu\nu}}{q^2} e_0^2 \longrightarrow -\frac{i g_{\mu\nu}}{q^2} e_0^2 \Big(\underbrace{1 - \frac{e_0^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2}}_{\text{vacuum polarization } Z_3^{1/2}} + e_0^2 \underbrace{\Pi(q^2)}_{\to 0 \text{ for } q^2 \to 0}\Big)$$

Λ = cut-off , $Z_3 = \left(1 - \frac{e_0^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2}\right)$ indeed diverges

$$-\frac{i g_{\mu\nu}}{q^2} e_0^2 \to -\frac{i g_{\mu\nu}}{q^2} e_0^2 \left(Z_3 + e_0^2 \Pi(q^2)\right)$$

$$= -\frac{i g_{\mu\nu}}{q^2} e^2 \left(1 + \frac{e_0^2}{Z_3} \Pi(q^2)\right)$$

$$= -\frac{i g_{\mu\nu}}{q^2} e^2 \left(\frac{1}{1 - \frac{e_0^2}{Z_3} \Pi(q^2)}\right)$$

$$\approx -\frac{i g_{\mu\nu}}{q^2} \left(\frac{e^2}{1 - e^2 \Pi(q^2)}\right)$$

for $\Pi(q^2)$ in terms of e^2, m without divergences

In QED : Λ = Planck scale , 10^{19} GeV

$$Z_3 = 1 - \underbrace{\frac{\alpha}{3\pi} \ln \frac{\Lambda^2}{m^2}}_{8\% \text{ effect}} \qquad \left(\alpha = \frac{e^2}{4\pi}\right)$$

* Renormalization group equation for QED

$\Pi(q^2)$ is correction due to one electron loop,

$$\frac{1}{1-e^2\Pi(q^2)} = 1 + e^2\,\Pi(q^2) + e^4\,\Pi(q^2)^2 + \dots$$

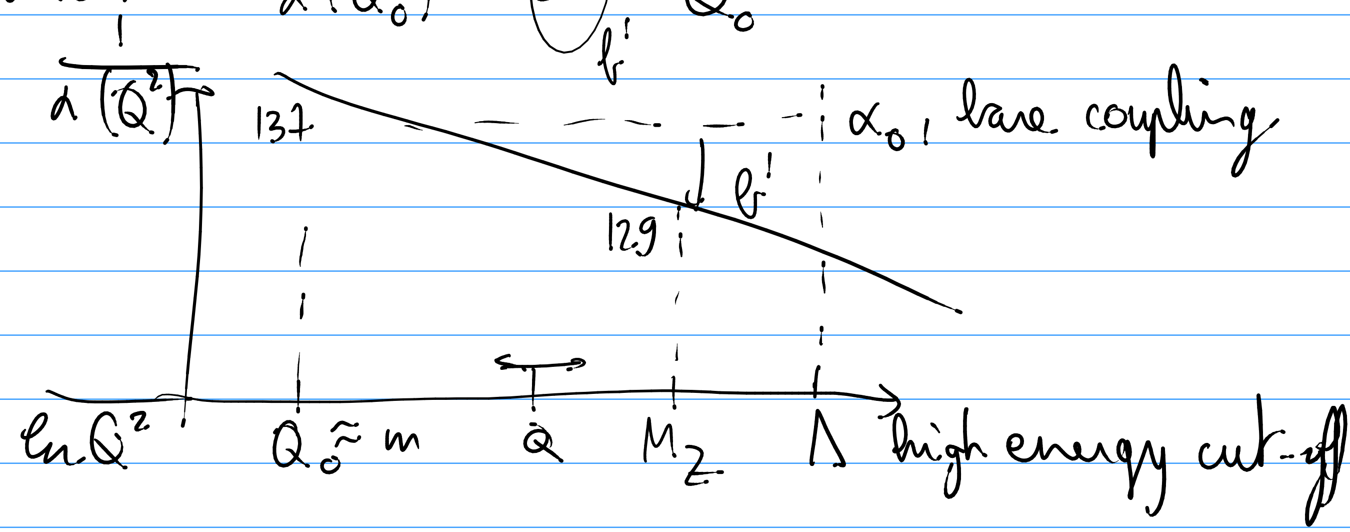
one-loop two-loop

$$\Pi(q^2) = \frac{1}{12\pi^2}\ln\frac{Q^2}{m^2} \quad \text{for } Q^2 > 4m^2 \text{ (above threshold)}$$

$$\left.\begin{aligned} \rightarrow \quad e^2(Q^2) &= \frac{e^2(Q_0^2)}{1-e^2(Q_0^2)\,\Pi(Q^2)} \\ \text{or} \quad \alpha(Q^2) &= \frac{\alpha(Q_0^2)}{1-\frac{\alpha(Q_0^2)}{3\pi}\ln\frac{Q^2}{Q_0^2}} \end{aligned}\right\} = \text{effective coupling}$$

$$\downarrow$$

$$\frac{1}{\alpha(Q^2)} = \frac{1}{\alpha(Q_0^2)} - \underbrace{\frac{1}{3\pi}}_{b'}\ln\frac{Q^2}{Q_0^2}$$

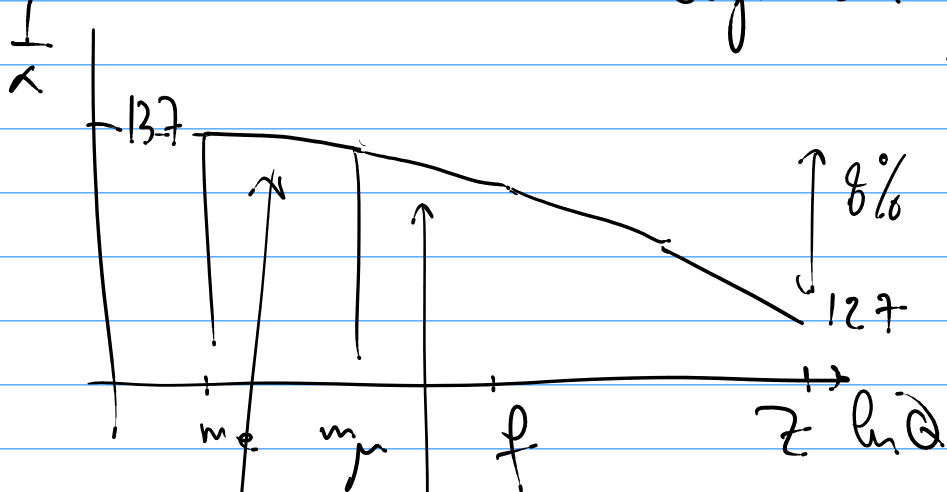


$$\frac{d\alpha(Q^2)}{d\ln Q^2} = \beta(q^2) = \frac{1}{3\pi}\alpha^2(Q^2) + O(\alpha^3)$$

generally: $\beta(q^2) = b'\alpha^2(Q^2) + O(\alpha^3)$

$$b' = \frac{1}{3\pi}\sum_{m_f<Q} \underbrace{C_r q_r^2}$$

e.g. $3\times\frac{q_u^2}{3\pi}$, $1\times\frac{q_\mu^2}{3\pi}$



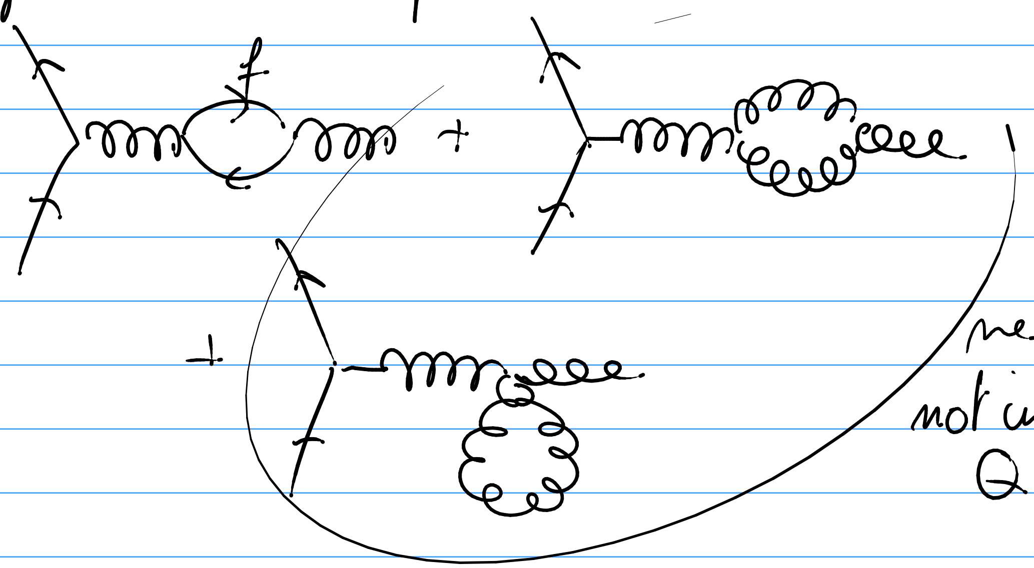
$$\frac{1}{\alpha(Q^2)} = \frac{1}{\alpha(Q_0^2)} - \frac{1}{3\pi}\ln\frac{Q^2}{Q_0^2}$$

$$\frac{1}{\alpha(Q^2)} = \frac{1}{\alpha(m_\mu^2)} - \frac{2}{3\pi}\ln\frac{Q^2}{m_\mu^2}$$

* Running coupling in QCD (Langacker 5.4)

$\alpha_s = \frac{g_s^2}{4\pi} \to \alpha_s(Q^2)$ depends on scale Q^2

gluon vacuum polarization



$$\frac{dg^2}{d\ln Q^2} = 4\pi\,\beta(g^2) \quad \text{or} \quad \frac{d\alpha}{d\ln Q^2} = \beta(\alpha) = b'\alpha^2(Q^2) + O(\alpha^3)$$

$$= \underbrace{b g^4(Q^2)}_{\text{one-loop diagrams}} + O(g^6)$$

One-loop beta function is

$$b = -\frac{1}{(4\pi)^2}\left(\frac{11}{3}C_2(G) - \frac{4}{3}T_F - \frac{1}{3}T_{\varphi c} - \frac{1}{6}T_{\varphi h}\right)$$

$C_2(G)$ = quadratic casimir $= n$ for $SU(n)$
$= 0$ for $U(1)$

$$\text{QED}: \quad b = -\frac{1}{(4\pi)^2}\left(-\frac{4}{3}\sum_{m_i<Q} q_i^2\right) > 0$$

$$b' = 4\pi b = \frac{1}{3\pi}\sum_{m_i<Q} q_i^2 \rightarrow \frac{1}{\alpha(Q^2)} = \frac{1}{\alpha(Q_0^2)} - 4\pi b \ln\frac{Q^2}{Q_0^2}$$

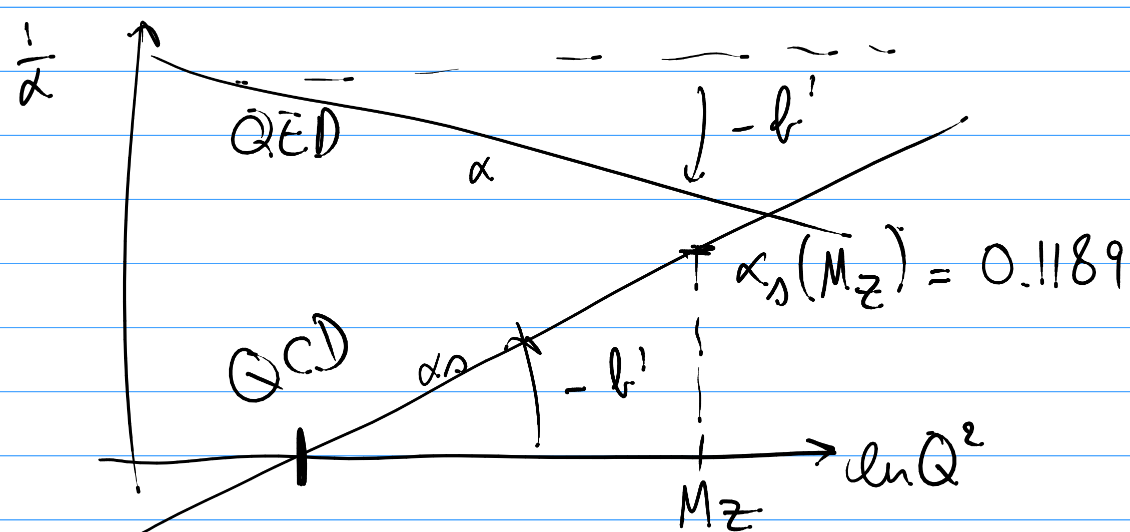
$$\text{QCD}: \quad b = -\frac{1}{(4\pi)^2}\left(\frac{11}{3} n_c - \frac{4}{3}\sum_{m_i<Q}\frac{1}{2}\right)$$

N_f = flavors

$$b' = 4\pi b = -\frac{1}{12\pi}\left(\underbrace{11\, n_c - 2\, n_q}\right) < 0$$

$n_c = 3$, $n_q = 3, 4, 5$

$$\rightarrow \frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s(Q_0^2)} + \frac{33 - 2n_q}{12\pi}\ln\frac{Q^2}{Q_0^2}$$



$$\Lambda_{QCD} \approx 100 - 300 \text{ MeV}$$

↳ dimensional transmutation: $\alpha_s \to \infty$

* Asymptotic freedom and confinement

- At $Q \lesssim 1\,GeV$, $Q \approx \Lambda_{QCD} \rightarrow g_s \approx O(1)$

Not possible to treat QCD perturbatively; must take all orders into account.

→ only colorless states can be free
color singlets; symmetric under $SU(3)_c$ transformations (qqq, $q\bar{q}$)

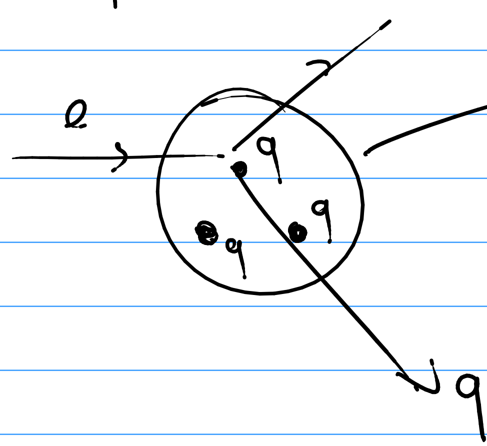
= confinement

- At $Q \gg 1\,GeV$, $Q \gg \Lambda_{QCD} \rightarrow g_s \approx 0$

Quarks behave as free particles

= asymptotic freedom

* Deep inelastic scattering:



hadronization

"color string breaking"

→ hadronization