Phys 772: Week 5 Tuesday

* Completion of non-abelian Higgs mechanism

* Strong isospin → SU(2) symmetry in strong interaction
  ⊥
  Weak isospin → SU(2) symmetry in EW

$m_n = 939.57$ MeV } almost identical mass,
$m_p = 938.27$ MeV } interact / behave very similarly under strong interaction

(electric charges different, but considering only strong interaction and at scales where electromagnetism is weak)

→ consider p, n as two states of nucleon N (ψ_p, ψ_n)

doublet $N = \begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} \text{strong isospin up} \\ \text{strong isospin down} \end{pmatrix}$

similar to spin $\begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$ for spin $\frac{1}{2}$ particle

→ isospin $I_z \begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$ for $I = \frac{1}{2}$ particle

Transformations of $N = \begin{pmatrix} p \\ n \end{pmatrix}$ under SU(2) 2-dimensional representation leave Lagrangian invariant
effective, no Standard Model

$m_{\pi^0} = 134.96$ MeV
$m_{\pi^\pm} = 139.57$ MeV

→ consider π^+, π^0, π^- as three states of pion π

triplet $\pi = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$ with $\left.\begin{matrix} \varphi_{\pi_1} \\ \varphi_{\pi_2} \\ \varphi_{\pi_2} \end{matrix}\right\}$ real scalars

with $\begin{cases} \pi^\pm = \frac{1}{\sqrt{2}}(\pi_1 \mp i\pi_2) \\ \pi^0 = \pi_3 \end{cases}$ charge eigenstates

Transformations of π under SU(2) 3-dimensional representation leave Lagrangian invariant

⇒ most generally (charge / fermion conservation)

$\mathcal{L}_{int} = g_{pn}\, p^+ n \pi^+ + g_{np}\, n^+ p \pi^- + g_{pp} p^+ p \pi^0 + g_{nn} n^+ n \pi^0$

but with relations between g's to impose isospin symmetry, e.g. $n \leftrightarrow p$ requires $g_{pp} = \pm g_{nn}$

π = vector in SU(2) isospin space
N = doublet

$$\rightarrow \mathcal{L}_{int} = g \underbrace{(N^+ \sigma^i N)}_{\text{vector}} \underbrace{\pi_i}_{\text{vector}}$$

→ $\mathcal{L}_{int}$ is scalar, invariant under SU(2) rotations

$$\bar{\sigma}\cdot\bar{\pi} = \sigma^i\pi_i = \begin{pmatrix} \pi^0 & -\sqrt{2}\,\pi^+ \\ -\sqrt{2}\,\pi^- & -\pi^0 \end{pmatrix}$$

$$\rightarrow \mathcal{L}_{int} = g\, N^+(\bar{\sigma}\cdot\bar{\pi})N$$

$$= p^+p\,\pi^0 - \sqrt{2}\,p^+n\,\pi^+ - \sqrt{2}\,n^+p\,\pi^- - n^+n\,\pi^0$$

$\hookrightarrow$ connections between g's imposed by symmetry observed

In reality, $G_\pi \approx 13.06$ after correction
$\rightarrow$ not perturbative

Symmetries under $SU(2)$

$$\begin{pmatrix} p' \\ n' \end{pmatrix} = e^{i\bar{\beta}\cdot\frac{\bar{\sigma}}{2}}\begin{pmatrix} p \\ n \end{pmatrix}$$

generators $\frac{\bar{\sigma}}{2}$ in 2×2 defining representation

$$\begin{pmatrix} \pi_1' \\ \pi_2' \\ \pi_3' \end{pmatrix} = e^{i\bar{\beta}\cdot\bar{T}}\begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

generators $(T_i)_{jk} = -i\varepsilon_{ijk}$
3×3 adjoint (real) representation

$\hookrightarrow$ adjoint: $\begin{pmatrix} \pi_1' \\ \pi_2' \\ \pi_3' \end{pmatrix} = e^{i\bar{\beta}\cdot\frac{\bar{\sigma}}{2}}\begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} e^{-i\bar{\beta}\cdot\frac{\bar{\sigma}}{2}}$

Comments:

1) pions are pseudoscalars $\rightarrow$ $N^{+}\gamma^{5}(\bar{\sigma}\cdot\bar{\pi})N$

2) can be extended now to $SU(3)$ for u,d,s or $\pi^{0}, \pi^{\pm}, \eta, K^{0}, \bar{K}^{0}, K^{\pm}$

* Now different approach: impose $\binom{p}{n}$ strong isospin $SU(2)$ symmetry as gauge symmetry

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi \quad \text{with} \quad \psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$$

$SU(2)$ symmetry $\rightarrow N^2 - 1 = 3$ gauge bosons (massless in absence of Higgs)

should have used ρ instead of π

$\hookrightarrow$ gauge bosons $\pi_1^{\mu}, \pi_2^{\mu}, \pi_3^{\mu}$ with generators $\frac{\sigma^i}{2}$

$$D^{\mu} = \partial^{\mu} - ig\frac{\bar{\sigma}}{2}\cdot\bar{\pi}^{\mu}$$

$$(D'^{\mu}\psi') = e^{i\bar{\beta}\cdot\frac{\bar{\sigma}}{2}} D^{\mu}\psi$$

when $\psi' = e^{i\bar{\beta}\cdot\frac{\bar{\sigma}}{2}}\psi$

and $\pi_i' = \left(e^{i\bar{\beta}\cdot\bar{T}}\right)_{ij}\pi_j$ or $\bar{\pi}' = e^{i\bar{\beta}\cdot\bar{T}}\bar{\pi}$

$$\pi_i'^{\mu} = \pi_i^{\mu} + \delta\pi_i^{\mu}$$

We already know that $(T^i)_{jk} = -i\varepsilon_{ijk}$ is the real adjoint representation

Explicitly:

$$D'^\mu \psi' = \left(\partial^\mu - ig\frac{\sigma^i}{2}\pi_i'^\mu\right)\left(1 + i\beta_j\frac{\sigma_j}{2}\right)\psi$$

$$= \left(\partial^\mu - ig\frac{\sigma_i}{2}\pi_i{}^\mu - ig\frac{\sigma_i}{2}\delta\pi_i{}^\mu\right)\left(1 + i\beta_j\frac{\sigma_j}{2}\right)\psi$$

$$= \Big(\partial^\mu - ig\frac{\sigma_i}{2}\pi_i{}^\mu - ig\frac{\sigma_i}{2}\delta\pi_i{}^\mu + i\frac{\sigma_j}{2}(\partial^\mu\beta_j)$$

$$+ g\frac{\sigma_i}{2}\pi_i{}^\mu\beta_j\frac{\sigma_j}{2}\Big)\psi + i\frac{\sigma_j}{2}\beta_j(\partial^\mu\psi)$$

$$e^{i\vec\beta\cdot\frac{\vec\sigma}{2}}D^\mu\psi = \left(1 + i\frac{\sigma_i}{2}\beta_i\right)\left(\partial^\mu - ig\frac{\sigma_j}{2}\pi_j{}^\mu\right)\psi$$

$$= \left(\partial^\mu - ig\frac{\sigma_j}{2}\pi_j{}^\mu + g\frac{\sigma_i}{2}\beta_i\frac{\sigma_j}{2}\pi_j{}^\mu\right)\psi$$

$$+ i\frac{\sigma_i}{2}\beta_i(\partial^\mu\psi)$$

$$\Rightarrow \sigma_i\delta\pi_i{}^\mu = \frac{1}{g}(\partial^\mu\beta_i)\sigma_i + \frac{i}{2}\beta_i\pi_j{}^\mu\underbrace{[\sigma_i, \sigma_j]}_{=2i\varepsilon_{ijk}\sigma_k}$$

$$\downarrow$$

$$\sigma_i\left(\delta\pi_i{}^\mu - \frac{1}{g}(\partial^\mu\beta_i) + \varepsilon_{ijk}\beta_j\pi_k^\mu\right) = 0$$

or $\quad \delta\pi_i{}^\mu = \frac{1}{g}(\partial^\mu\beta_i) - \varepsilon_{ijk}\beta_j\pi_k$

or $\quad \vec\pi' = e^{i\vec\beta\cdot\vec T}\vec\pi \quad$ with $(T_i)_{jk} = -i\varepsilon_{ijk}$

the adjoint representation

Electromagnetic charges of $\vec{\pi}$:

$$\pi^{\pm} = \frac{1}{\sqrt{2}}\left(\pi_1 \mp i\pi_2\right) \qquad \pi^0 = \pi_3$$

Do we observe these $\vec{\pi}$ <u>vector</u> particles?
$\vec{\rho}$ mesons, ρ^+, ρ^0, ρ^-, $m_\rho \approx 775$ MeV
can be thought as the gauge bosons of strong isospin gauge invariance, which is spontaneously broken to give ρ mass

* Strong isospin symmetry breaking due to $m_p \neq m_n$

$$\mathcal{L}_{SB} = -\varepsilon \bar{\psi}\, \sigma_3\, \psi = -\varepsilon\left(\bar{\psi}_p \psi_p - \bar{\psi}_n \psi_n\right)$$

$$\rightarrow m_p = m_N + \varepsilon, \quad m_n = m_N - \varepsilon$$

$\mathcal{L}_{SB}$ still symmetric under generator $\frac{\sigma_3}{2}$

but not $\frac{\sigma_1}{2}, \frac{\sigma_2}{2}$: $\psi \to e^{i\beta\frac{\sigma_3}{2}}\psi$: $\mathcal{L}_{SB} \to \mathcal{L}_{SB}$

$\downarrow$

Symmetry $SU(2)$ broken to $U(1)_{T_3}$ = 3rd comp isospin

$$T_3(p) = +\frac{1}{2}, \quad T_3(n) = -\frac{1}{2}$$

$$T_3(\pi^0) = 0, \quad T_3(\pi^{\pm}) = \pm 1.$$

Also symmetric under $U(1)_B$ = baryon number

$B(p) = B(n) = 1$
$B(\pi) = 0$

Conservation of $U(1)_{T_3} \times U(1)_B$ is equivalent

of $\begin{cases} \text{conservation of } T_3 \text{ and } B \\ \text{conservation of } Q \text{ and } B \end{cases}$

$$Q = T_3 + \frac{B}{2} \qquad (\text{indeed true for } p, n, \pi)$$

* Weak interactions : weak isospin

In electroweak theory: particles are classified in SU(2) singlets and doublets:

example: $\psi_{e^-_R}$ is an SU(2) singlet

$\begin{pmatrix} \psi_{\nu_\mu L} \\ \psi_{\mu^-_L} \end{pmatrix}$ is an SU(2) doublet

Just as $m_p \approx m_n$ leads to SU(2) strong isospin being a "good" symmetry, but is observational and not a "fundamental" symmetry of QCD, also SU(2) "weak isospin" is a symmetry that can't be "explained".

Reminder $\begin{cases} \psi_L = P_L \psi \\ \psi_R = P_R \psi \end{cases}$

ψ_L and ψ_R can be treated as independent fields if there is no $m_\psi \bar{\psi}\psi$ term, which has $m_\psi(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$.

→ ψ_L and ψ_R can have different SU(2) weak isospin behavior

Mass will be generated through Higgs mechanism

$\Rightarrow$ left-handed doublets : $L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L$, $Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$ ($T_3 = \pm\frac{1}{2}$)

right-handed singlets : $\nu_{\ell R}$, ℓ_R , u_R, d_R ($T_3 = 0$)

in SM: $\nu_{\ell R}$ has no color, no charge, no weak isospin $\longrightarrow$ can't interact $\rightarrow$ drop

$$\mathcal{L}_{\text{fermions}} = \sum_{\substack{L,\, \ell_R,\, Q, \\ u_R,\, d_R}} i \bar{\psi} \not{\partial} \psi$$

of all 3 families

Invariant under $SU(2)_L$ $\begin{cases} L \to e^{i\vec{\beta}\cdot\frac{\vec{\sigma}}{2}} L & \text{(doublet)} \\ \ell_R \to \ell_R & \text{(singlet)} \end{cases}$

and under $U(1)_Y$ $\begin{cases} L \to e^{i\beta} L \\ \ell_R \to e^{i\beta'} \ell_R \end{cases}$

$\Rightarrow$ $SU(2) \times U(1)$ symmetry

and under $SU(3)_c$ $Q_a \to \left(e^{i\vec{\beta}\cdot\vec{\lambda}}\right)_{ab} Q_b$

$\Rightarrow$ $SU(3)_c \times SU(2)_L \times U(1)_Y$

Now we impose all these symmetries as gauge symmetries:

$$\mathcal{L} = \sum i\bar{\psi}\not{D}\psi \quad \text{with } D_\mu = \partial_\mu - ig'\frac{Y}{2}B_\mu - ig\frac{\sigma^i}{2}W^i_\mu - ig_s\frac{\lambda^a}{2}G^a_\mu$$

with Y the generator of $U(1)_Y$ (constant) and B_μ its gauge boson

$\frac{\sigma^i}{2}$ the 3 generators of $SU(2)_L$ and W_μ its gauge bosons (W_1, W_2, W_3)

$\frac{\lambda^a}{2}$ the 8 generators of $SU(3)_C$ and G^a_μ its gauge bosons (gluons)

$SU(2)_L$ gauge bosons W_1, W_2, W_3 here are similar to the π_1, π_2, π_3 gauge bosons in strong isospin

$$\begin{cases} W^0 = W_3 \\ W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2) \end{cases}$$

Also: $Q = T_3 + \frac{Y}{2}$ with $Y = 1$ for $\begin{pmatrix}\nu \\ \ell\end{pmatrix}_L$

Y = $U(1)$ hypercharge $\qquad Y = -2$ for ℓ_R

* Higgs mechanism with SU(2) doublet

$$\mathcal{L}_{Higgs} = (D^\mu \varphi)^\dagger (D_\mu \varphi) - \mu^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^2$$

with $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$, $Y = 1$

and $\langle \varphi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$, $v^2 = -\frac{\mu^2}{\lambda}$

(Note that φ^+ component must have $\langle \varphi^+ \rangle = 0$ because otherwise electric charge could be absorbed by the vacuum)

Now, for $SU(2)_L \times U(1)_Y$ (since φ does not carry color)

$$\left[\left(\partial^\mu + i g' \frac{Y}{2} B^\mu + i g \frac{\vec{\sigma}}{2} \cdot \vec{W}^\mu \right) \varphi \right]^\dagger \cdot$$

$$\cdot \left[\left(\partial_\mu + i g' \frac{Y}{2} B_\mu + i g \frac{\vec{\sigma}}{2} \cdot \vec{W}_\mu \right) \varphi \right]$$

becomes at $\langle \varphi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\frac{1}{8} v^2 g^2 \left((W^1_\mu)^2 + (W^2_\mu)^2 \right) + \frac{1}{8} v^2 (g' B_\mu - g W^3_\mu)^2$$

$$= \left(\frac{1}{2} v g\right)^2 W^+_\mu W^{-\mu} + \frac{1}{2} \left(\frac{1}{2} v \sqrt{g^2 + g'^2} \right) Z_\mu Z^\mu$$