Phys 772: Week 7 Tuesday

* Structure functions in $ep \to eX$ inelastic scattering

$$\frac{d\sigma}{dk'\,d\Omega} = \frac{\alpha^2}{4k^2 \sin^4\frac{\theta}{2}}\left[W_2(\nu,Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,Q^2)\sin^2\frac{\theta}{2}\right]$$

with $W_1(\nu,Q^2)$ and $W_2(\nu,Q^2)$ structure functions that describe inelastic scattering

* Generalization of $ep \to ep$ elastic scattering $\left(\tau = \frac{Q^2}{4M^2}\right)$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4k^2\sin^4\frac{\theta}{2}}\,\frac{k'}{k}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right]$$

$$\begin{cases} G_E = F_1 + \kappa\tau F_2 \\ G_M = F_1 + \kappa F_2 \end{cases} \xrightarrow{Q^2\to 0} \begin{cases} G_E = 1 + \kappa\tau \\ G_M = 1+\kappa \end{cases} \xrightarrow{\kappa\to 0} \begin{cases} G_E = 1 \\ G_M = 1 \end{cases}$$

Magnetic moment $= \frac{e}{2M}(1+\kappa) \to \kappa =$ anomalous magnetic moment

If $\kappa = 0 \to G_E = G_M = G = F_1(Q^2)$ for all Q^2

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4k^2\sin^4\frac{\theta}{2}}\,\frac{k'}{k}\,F(Q^2)^2\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

Compare to $e\mu \to e\mu$ elastic scattering off a fundamental spin-$\frac{1}{2}$ particle

$\sim$ $ep \to ep$ elastic scattering ignoring QCD structure of p

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4k^2 \sin^4\frac{\theta}{2}} \frac{k'}{k}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

Compare to $e\pi \to e\pi$ elastic scattering off a fundamental spin-0 particle, ignoring QCD structure

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4k^2 \sin^4\frac{\theta}{2}} \frac{k'}{k}\left[\cos^2\frac{\theta}{2}\right]$$

$\rightarrow$ $\sin^2\frac{\theta}{2}$ term connected to magnetic moment of the target (spin-$\frac{1}{2}$, internal structure)

$\rightarrow$ $\cos^2\frac{\theta}{2}$ term connected to electric charge distribution of the target

* Back to structure functions: all elastic scattering cross sections include $\delta\left(\nu - \frac{Q^2}{2M}\right)$ in $\frac{d\sigma}{dk' d\Omega}$

$$\begin{cases} W_2(\nu, Q^2) = \delta\left(\nu - \frac{Q^2}{2M}\right) = \frac{1}{\nu}\delta\left(1 - \frac{Q^2}{2M\nu}\right) \\ W_1(\nu, Q^2) = \frac{Q^2}{2M^2}\delta\left(\nu - \frac{Q^2}{2M}\right) = \frac{Q^2}{2M\nu}\delta\left(1 - \frac{Q^2}{2M\nu}\right) \end{cases}$$

$$\begin{cases} F_1(\nu, Q^2) = M W_1(\nu, Q^2) \\ F_2(\nu, Q^2) = \nu W_2(\nu, Q^2) \end{cases}$$

Now include form factor $F(Q^2)$ with $\kappa = 0$

$$\begin{cases} F_1(\nu, Q^2) = \frac{1}{2}\frac{Q^2}{\nu} F(Q^2)^2 \delta\left(1 - \frac{Q^2}{2M\nu}\right) \\ F_2(\nu, Q^2) = F(Q^2)^2 \delta\left(1 - \frac{Q^2}{2M\nu}\right) \end{cases}$$

$\downarrow$

elastic scattering off partons

$$\begin{cases} F_1(x) = \frac{1}{2x} F_2(x) \\ F_2(x) = \sum_f e_f^2 \, x \, q(x) \end{cases}$$

with $q(x)$ = parton distribution function

= probability density function to find parton with momentum fraction x

Callan Gross relation:

$2xF_1(x) = F_2(x)$ for point-like partons

In proton: $u_v(x)$, $d_v(x)$ valence, $u_s(x)$, $d_s(x)$, $\bar{u}_s(x)$, $\bar{d}_s(x)$ } sea

$$u_s(x) = d_s(x) = \bar{u}_s(x) = \bar{d}_s(x)$$

$$s_s(x) = \bar{s}_s(x)$$

In neutron: $u_V^n(x) = d_V^p(x)$
$d_V^n(x) = u_V^p(x)$

$$\frac{1}{x}\left(F_2^{ep}(x) - F_2^{en}(x)\right) = \frac{1}{3}\left(u_v(x) - d_v(x)\right)$$

↳ peaks at $\frac{1}{3}$ ($\underbrace{u_v(x) - d_v(x)}_{=1}$)

Momentum balance (ignoring strange quarks)

p: $$\int dx\, F_2^{ep}(x) = \left(\frac{2}{3}\right)^2 \int_0^1 dx\, x(u+\bar{u}) + \left(-\frac{1}{3}\right)^2 \int_0^1 dx\, x(d+\bar{d})$$

$$= \frac{4}{9}\varepsilon_u + \frac{1}{9}\varepsilon_d \overset{\text{(data)}}{=} 0.18$$

ε_u = fraction of momentum carried by u quarks
ε_d = ... by d quarks

n: $$\int dx\, F_2^{en}(x) = \frac{1}{9}\varepsilon_u + \frac{4}{9}\varepsilon_d \overset{\text{(data)}}{=} 0.12$$

→ $\varepsilon_u = 0.36$ $\quad\varepsilon_d = 0.18$

→ $\varepsilon_g = 0.46$: ~50% of nucleon momentum carried by gluons

Polarized pdfs : $\Delta q(x) = q^+(x) - q^-(x)$

↳ $q^+(x)$ = probability of finding quark with spin along proton spin

$q^-(x)$ = ... against

* DGLAP (see previous lecture, end)

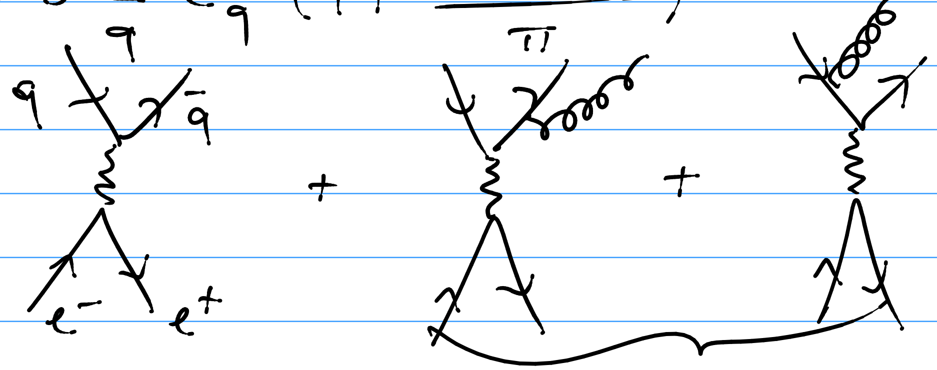
* Instead of pp, $p\bar{p}$; $e^-e^+ \to$ hadrons

introduced already

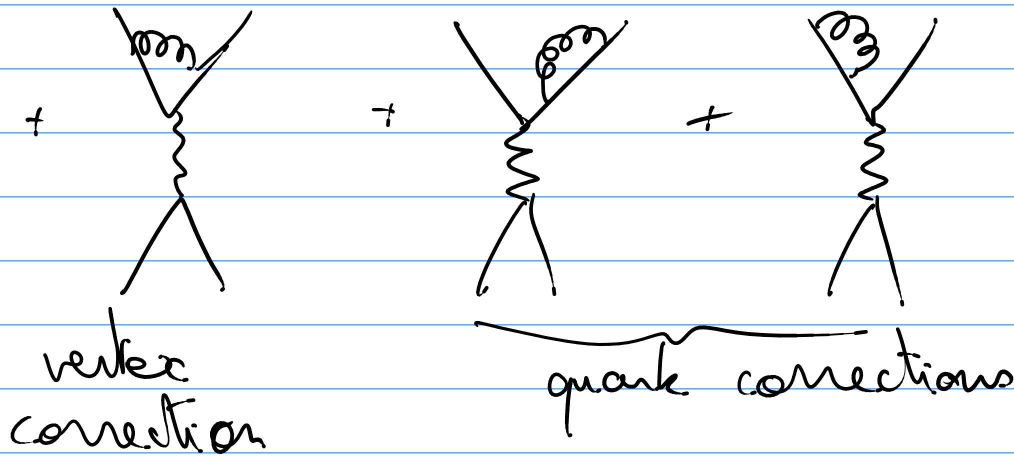
$$R = \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^+\mu^-)} = 3 \sum_q e_q^2$$

Corrections due to short distance QCD:

$$R = 3 \sum_q e_q^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$



gluon radiation, each with $\frac{\alpha_s}{2\pi}$



→ total contribution, after cancellation of $m_g \to 0$ divergences, is finite and

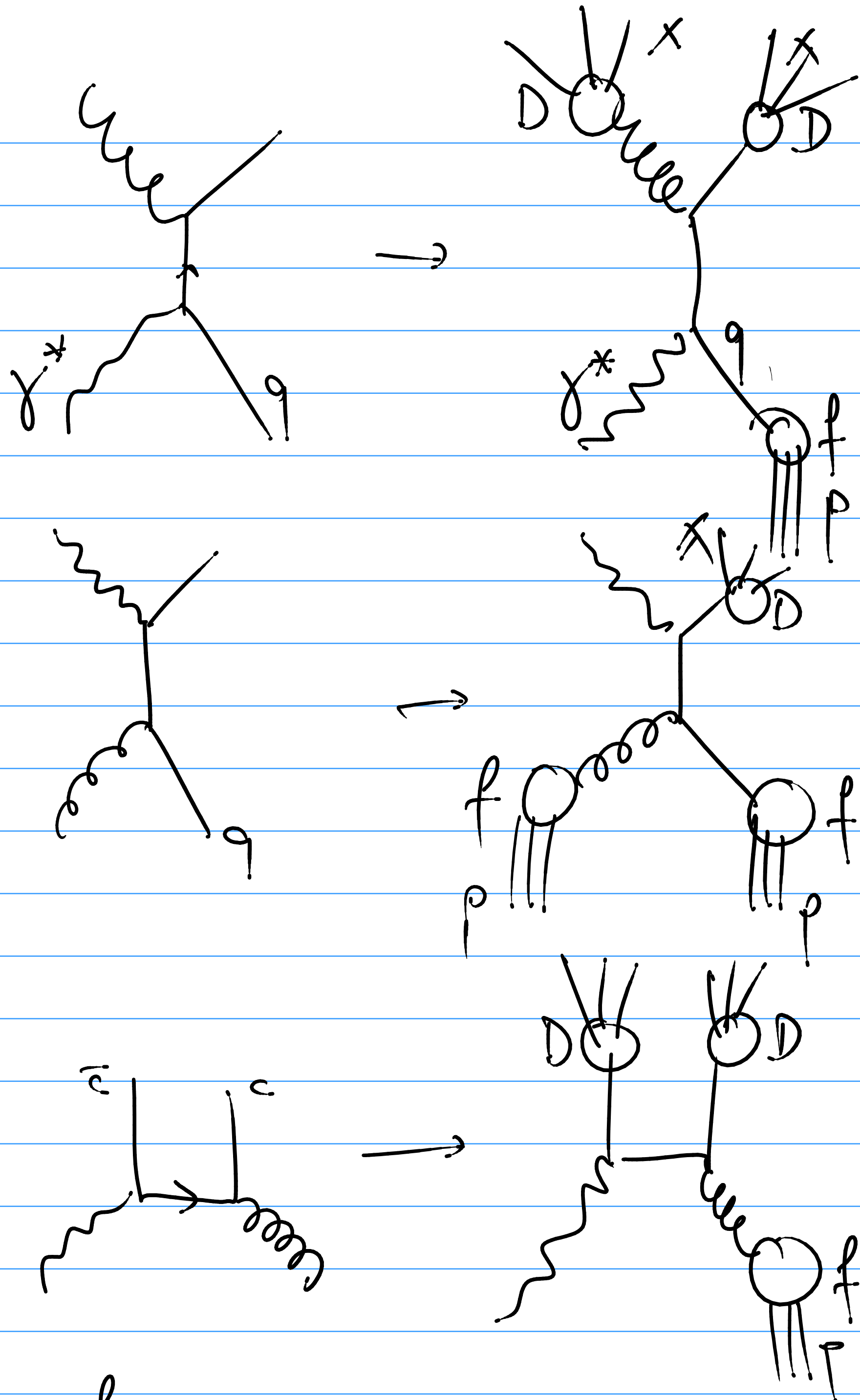
$$\left(1 + \frac{\alpha_S(Q^2)}{\pi}\right)$$

→ R was constant before adding $\ln Q^2$ corrections → now R is variable ! with Q^2

→ scaling violation due to strong QCD

* Factorization, structure functions, fragmentation function

All short distance processes have to be convoluted with structure functions / pdfs, and/or fragmentation functions D e.g



$D_q^h(z)$, $D_{\bar q}^h(z)$

$$\frac{d\sigma}{dz}(e^-e^+ \to hX) = \sum_q \sigma(e^-e^+ \to q\bar q)\left[D_q^h(z) + D_{\bar q}^h(z)\right]$$

* Drell-Yan : $pp \to \ell^+ \ell^-$