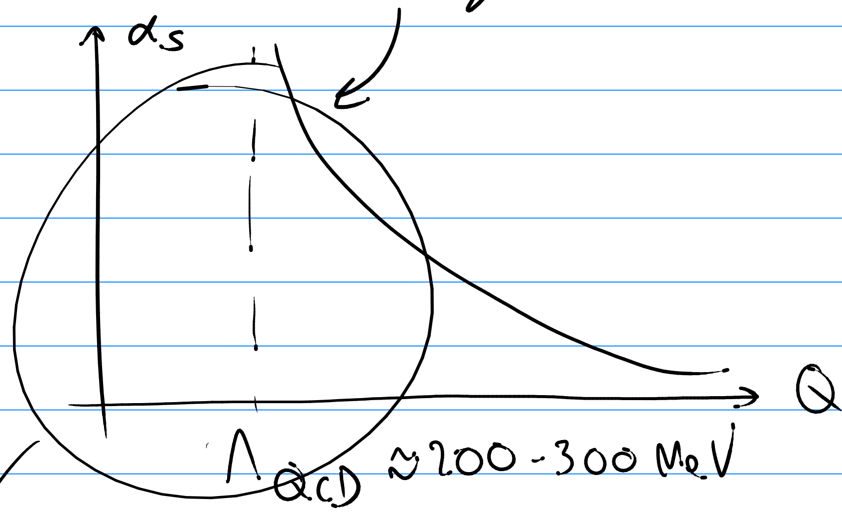
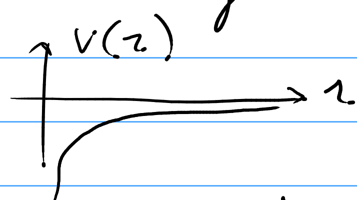
Phys 772: Week 7 Thursday

* Long distance QCD physics:
  non-perturbative physics
  spectroscopy



↳ Confinement: only colorless objects can be isolated

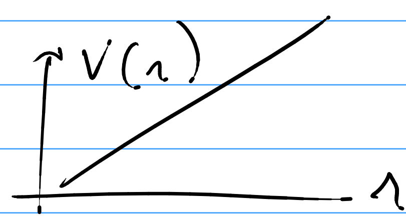
QED: $V(r) \sim -\frac{\alpha}{r}$



due to the perturbative nature of QED only the 1^{st} order term is important, all subsequent terms are small

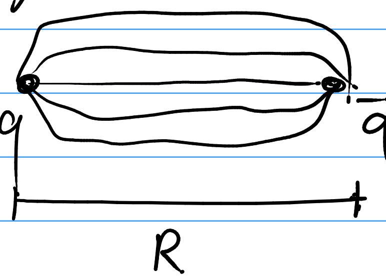
↳ given sufficient finite energy, can separate two charged particles over arbitrary distance

QCD: $V(r) \sim \alpha_s r$



higher order effects play crucial role

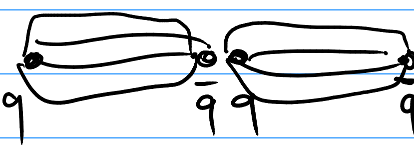
→ flux tube model



→ energy ~ R for constant energy density

→ $V(r) \sim \alpha_s r$

when energy > $2m_\pi$ →



→ only $SU(3)_c$ singlet states allowed

$\frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$ for $q\bar{q}$

$\frac{1}{\sqrt{6}}(rgb - rbg + brg - bgr + gbr - grb)$ for qqq

↓

color singlet, antisymmetric in exchange of any 2 colors

* Symmetries in QCD

Accidental SU(2) or SU(3) symmetries

SU(2) : $m_u \cong m_d \approx 0$ ← good symmetry

SU(3) : $m_u \approx m_d \approx m_s < \Lambda_{QCD}$

broken to 25%

Combining 2 fermions with SU(2) ~~iso~~spin

$$2 \otimes 2 = \left(\pm\frac{1}{2}\right) \otimes \left(\pm\frac{1}{2}\right)$$

$$j_1 = \frac{1}{2} \quad j_2 = \frac{1}{2} \longrightarrow = |j_1 - j_2|, \dots, j_1 + j_2$$

with notation $2j+1$ $\qquad$ $|j_1 - j_2| = 0$ $\qquad$ $j_1 + j_2 = 1$

$$\rightarrow \quad 2 \otimes 2 = 1 \oplus 3$$

$$1 ; \quad \frac{1}{\sqrt{2}}\left(|+\tfrac{1}{2}, -\tfrac{1}{2}\rangle - |-\tfrac{1}{2}, +\tfrac{1}{2}\rangle \right) \quad \text{with } j_z = 0$$

$$3 ; \quad |+\tfrac{1}{2}, +\tfrac{1}{2}\rangle \qquad j_z = +1$$

$$\frac{1}{\sqrt{2}}\left(|+\tfrac{1}{2}, -\tfrac{1}{2}\rangle + |-\tfrac{1}{2}, +\tfrac{1}{2}\rangle \right) \qquad j_z = 0$$

$$|-\tfrac{1}{2}, -\tfrac{1}{2}\rangle \qquad j_z = -1$$

anti-symmetric singlet of states

symmetric triplet of states

Similar for 2 nucleons, e.g. ^6He, ^6Li, ^6Be $= {}^4He + NN$

$|+\frac{1}{2}\rangle = p$, $|-\frac{1}{2}\rangle = n$

$1: \frac{1}{\sqrt{2}}(pn - np)$

$3: \begin{matrix} pp \\ \frac{1}{\sqrt{2}}(pn + np) \\ nn \end{matrix}$

mass ↑

^6He	^6Li	^6Be
$I_3 = -1$	$I_3 = 0$	$I_3 = +1$
nn	$I = 1$	pp
	$I = 0$	
	$\frac{1}{\sqrt{2}}(pn - np)$	

Extend to 3 nucleons: $2 \otimes 2 \otimes 2 = (1 \oplus 3) \otimes 2$

$j^1 = 0$, $j^2 = 1$, $j = 1$

$= 1 \otimes 2 \oplus 3 \otimes 2$

$j^2 = \frac{1}{2}$, $j = 1$

$= 2 \oplus 2 \oplus 4$

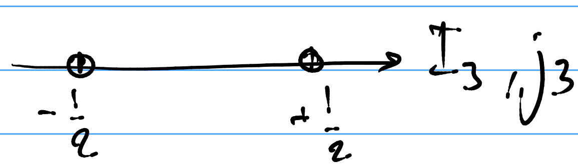
$j = \frac{1}{2}$, $j = \frac{3}{2}$

↓

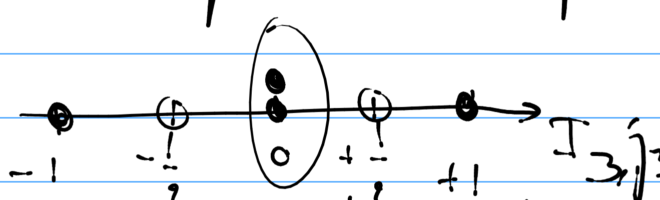
3 spin $\frac{1}{2}$ fermions or
3 isospin nucleons combine into
2 doublets of spin $\frac{1}{2}$
1 quartet of spin $\frac{3}{2}$

Graphically

single particle in 2 rep

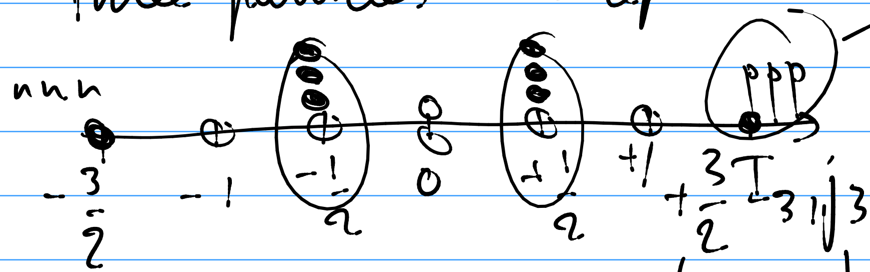


two particles in 2 rep



superposition in I_3, j_3
but different I, j

Three particles in 2 rep



4 is symmetric under exchange of any particles

again superposition states

→ 2 spin states for $j=\frac{1}{2}$, $j_3=\pm\frac{1}{2}$ is the fundamental representation of SU(2)

3 spin states for $j=1$, $j_3=-1,0,+1$

→ 3 representation of SU(2)

$\begin{bmatrix} +1 \\ 0 \\ -1 \end{bmatrix}$ and 3x3 unitary transformations

3x3 hermitian generators

* Isospin of $\begin{pmatrix} p \\ n \end{pmatrix}$ and $\begin{pmatrix} -\bar{n} \\ \bar{p} \end{pmatrix}$ anti-particles

Transformation under SU(2) of $\begin{pmatrix} p \\ n \end{pmatrix}$:

$$\begin{pmatrix} p' \\ n' \end{pmatrix} = e^{-i\vec{\beta}\cdot\frac{\vec{\sigma}}{2}} \begin{pmatrix} p \\ n \end{pmatrix}$$

e.g. $\beta = (0,\pi,0)$: $e^{-i\theta\frac{\sigma_2}{2}} = \cos\frac{\theta}{2} - i\sigma_2 \sin\frac{\theta}{2}$

$$\begin{pmatrix} p' \\ n' \end{pmatrix} = -i\sigma_2 \begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} p \\ n \end{pmatrix}$$

Now apply charge conjugation operator C:

$$\begin{cases} Cp = \bar{p} \\ Cn = \bar{n} \end{cases}$$

$$\begin{pmatrix} \bar{p}' \\ \bar{n}' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{p} \\ \bar{n} \end{pmatrix}$$

Would like to have $\begin{pmatrix} \bar{p} \\ \bar{n} \end{pmatrix}$ transform similarly to $\begin{pmatrix} p \\ n \end{pmatrix}$:

- rearrange $\begin{pmatrix} \bar{p} \\ \bar{n} \end{pmatrix}$ to $\begin{pmatrix} \bar{n} \\ \bar{p} \end{pmatrix}$ so again $I_3 = +\frac{1}{2}$ corresponds with most positively charged particle
  $\hookrightarrow \begin{pmatrix} \bar{n}' \\ \bar{p}' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \bar{n} \\ \bar{p} \end{pmatrix}$
- introduce a minus sign so matrix is unchanged:

$$\begin{pmatrix} -\bar{n}' \\ \bar{p}' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -\bar{n} \\ \bar{p} \end{pmatrix}$$

$\Rightarrow \begin{pmatrix} -\bar{n} \\ \bar{p} \end{pmatrix}$ transforms as $\begin{pmatrix} p \\ n \end{pmatrix}$ under $SU(2)$

$\rightarrow$ nucleon / anti-nucleon pairs are then (based on nucleon / nucleon pairs):

$1: \quad \frac{1}{\sqrt{2}}(p\bar{p} + n\bar{n}) \rightarrow I = 0$

$3: \quad \underbrace{n\bar{p}}_{I_3 = -1}, \quad \frac{1}{\sqrt{2}}(p\bar{p} - n\bar{n}), \quad \underbrace{-p\bar{n}}_{I_3 = +1} \rightarrow I = 1$

→ From isospin SU(2) to light quark SU(3), u, d, s

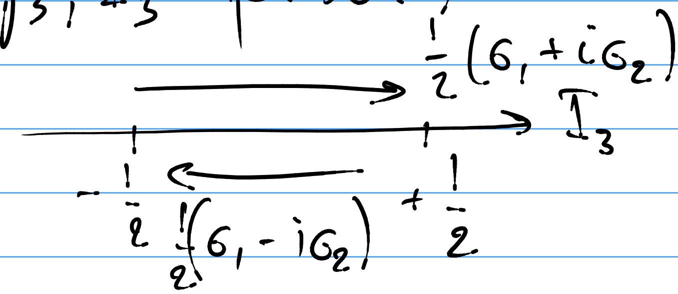
SU(2) : 1 diagonal generator $\frac{\sigma_3}{2} = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

↓

1 good quantum number since diagonal matrix commutes with H

↓

j_3, I_3 for SU(2)



Other two generators form step-up, step-down operators between I_3 eigenstates

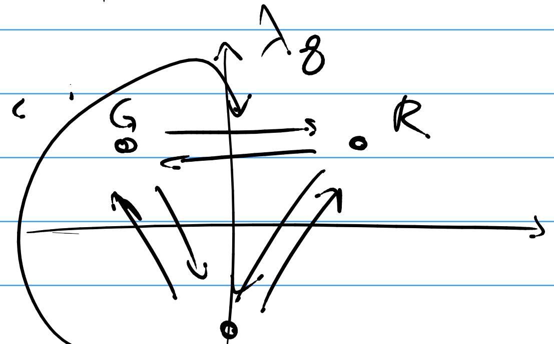
SU(3) : 2 diagonal generators $\lambda_3 = \begin{pmatrix} \sigma_3 & \begin{matrix}0\\0\end{matrix} \\ 0\ 0 & 1\end{pmatrix}$

$\lambda_8 = \frac{1}{\sqrt{3}}\,\mathrm{diag}(1, 1, -2)$

↓

2 good quantum numbers

For SU(3),



λ_3, λ_8 quantum numbers

$\frac{1}{2}(\lambda_1 \pm i\lambda_2)$, $\frac{1}{2}(\lambda_4 \pm i\lambda_5)$, $\frac{1}{2}(\lambda_6 \pm i\lambda_7)$ step up step down

For SU(3) u, d, s : I_3 and Y

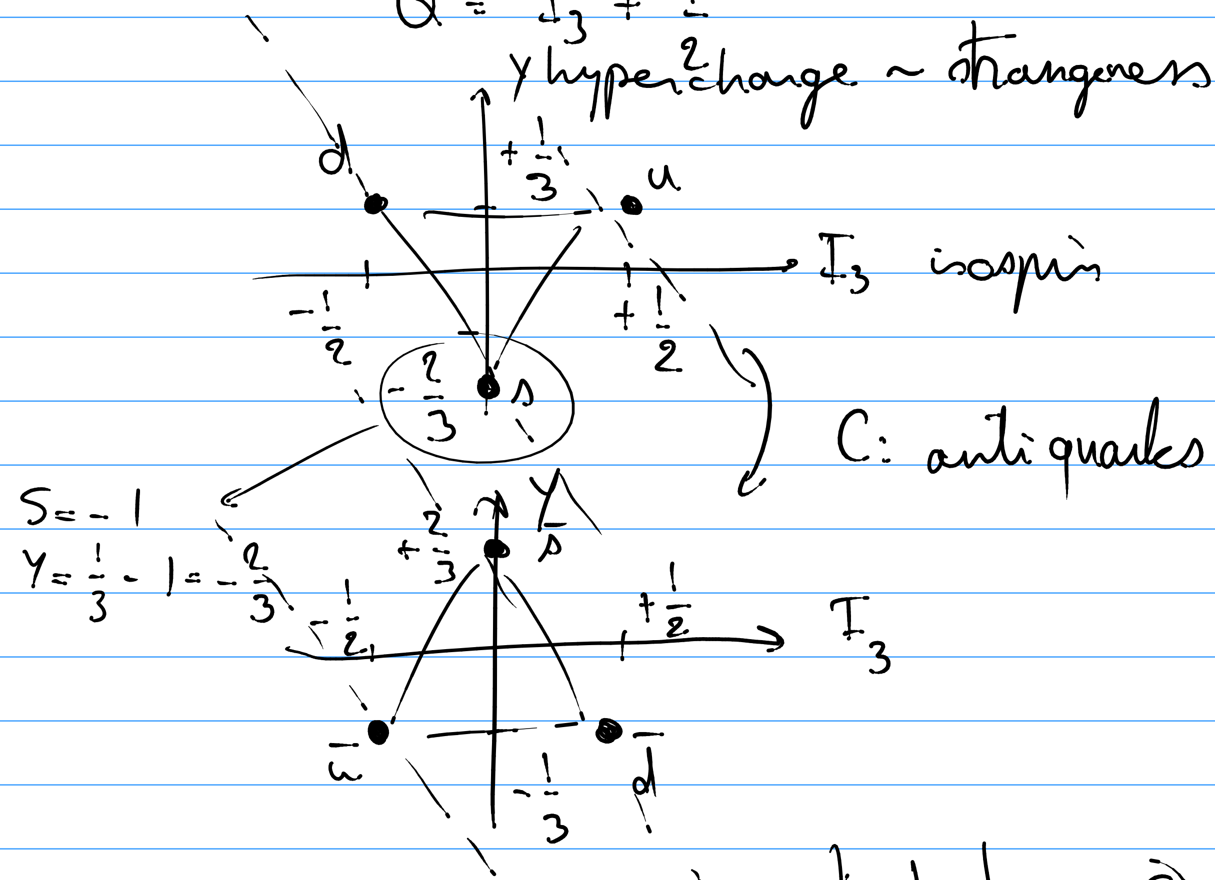
Y = <u>strong</u> hypercharge ($\neq$ weak isospin for SM doublets)

$= B + S$

- $B = 1$ for baryon $\rightarrow \frac{1}{3}$ for quarks
- S = strangeness $= -1$ for every s quark

Again: connection to electric charge:

$$Q = I_3 + \frac{Y}{2}$$



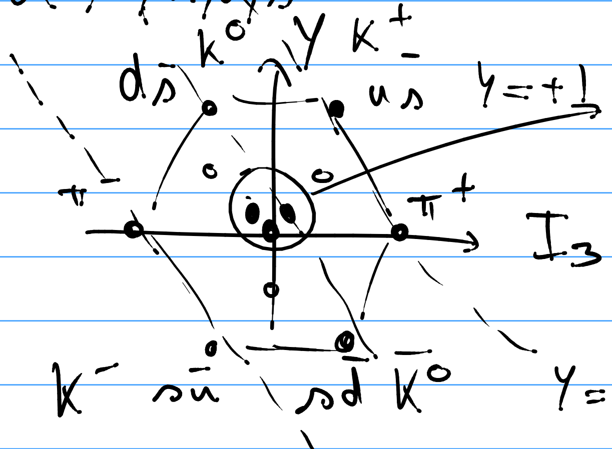
constant charge Q along dashed diagonals

* Mesons: SU(2), u, d:

$1 : \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, $I=0, I_3=0$ (η)

$3 : -u\bar{d}\ (I_3=+1)$, $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$, $d\bar{u}$
(π^+, π^0, π^-)

SU(3), u, d, s



$3 \otimes 3^* = 1 \oplus 8$
1 singlet, 2 in octet at $I_3=0$, $Y=0$

→ singlet: $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) = \eta'$

2 $I_3=0, Y=0$ states: $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) = \pi^0$ as above

→ orthogonality: $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ "η"

π^+, π^0, π^- : $I = 1$, $K^+, K^0 / K^-, \bar{K}^0$: $I = \frac{1}{2}$

Orbital angular momentum $L = 0, 1, \ldots$
Spin of $q\bar{q}$ pair $S = 0, 1$

$$J = L + S = \underbrace{0, 1}_{L=0}, \underbrace{0, 1, 2}_{L=1}, \ldots$$

J^{PC} for mesons with

$P = -(-1)^L$ (from Y_{Lm} spatial wave functions)

$C = (-1)^{L+S}$

$\to J^{PC}$ for $L=0,\ S=0$: 0^{-+}

pseudo scalar mesons nonet :

$\underbrace{\pi}_{I=0},\ \underbrace{K}_{I=\frac{1}{2}},\ \underbrace{\eta, \eta'}_{I=0}$

$L=0,\ S=1$: 1^{--}

vector mesons nonet :

$\underbrace{\rho}_{I=1},\ \underbrace{K^*}_{I=\frac{1}{2}},\ \underbrace{\omega, \phi}_{I=0}$

etc

many, many states

Mass degeneracy lifted by $m_s \neq 0$

$\longrightarrow \sim 150$ MeV for each strangeness

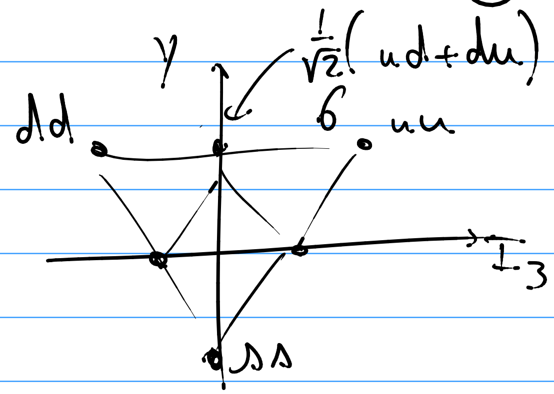
A. Baryons : qqq

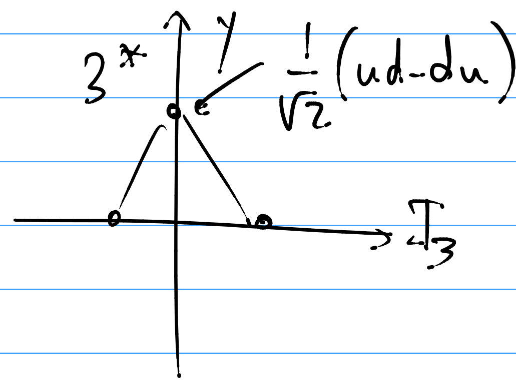
$SU(2)$, u, d : $2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$

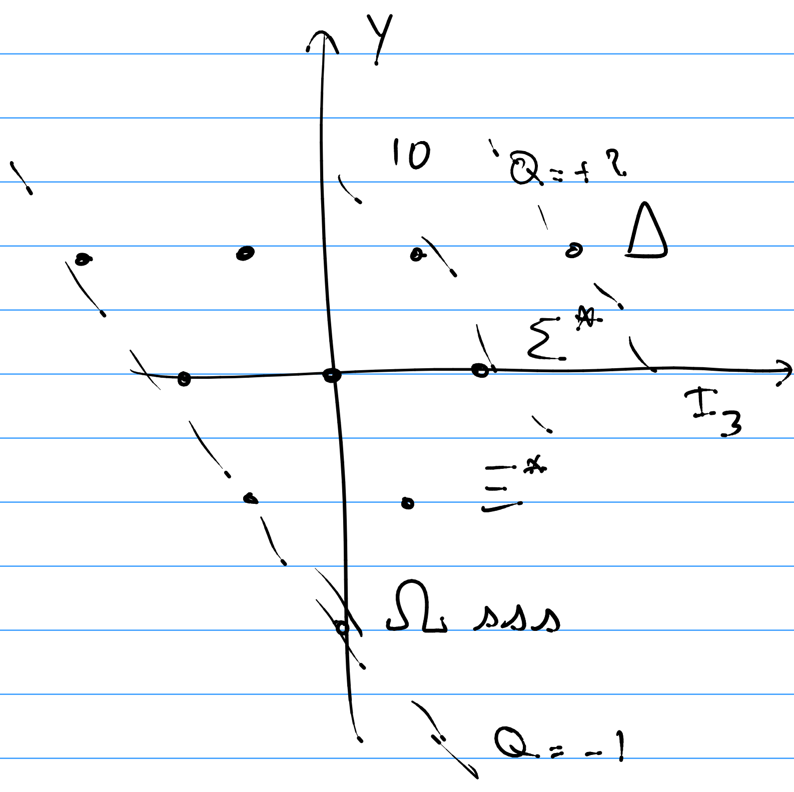
$SU(3)$, u, d, s : $3 \otimes 3 \otimes 3 = (6 \oplus 3^*) \otimes 3$

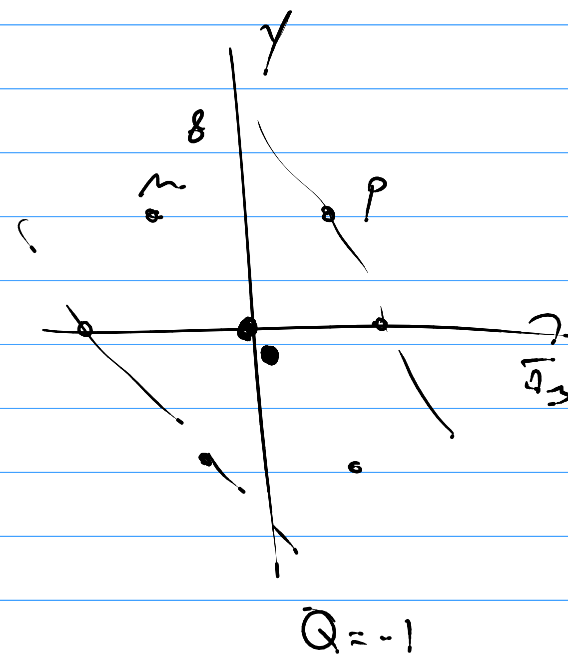
$= (6 \otimes 3) \oplus (3^* \otimes 3)$

$= 1 \oplus 8 \oplus 8 \oplus 10$









→ look at "Δ^+" state: uud combinations

$$\Delta^+ = \frac{1}{\sqrt{3}}(uud + udu + duu) \quad \text{symmetric}$$

$$p_A = \frac{1}{\sqrt{2}}(ud - du)u \quad \text{anti-symmetric in first two quarks}$$

↓ orthogonality

$$p_S = \frac{1}{\sqrt{6}}\left[(ud + du)u - 2uud\right]$$

→ six uds combinations

$$\text{singlet} = \frac{1}{\sqrt{6}}(uds - usd + sud - sdu + dsu - dus)$$

always in singlet antisymmetric

Combining spin, flavor, color, space

$$2 \otimes 2 \otimes 2 = \underset{S}{4} \oplus \underset{M_S}{2} \oplus \underset{M_A}{2}$$

$$3 \otimes 3^* = \underset{S}{1} \oplus \underset{A}{8}$$

$$3 \otimes 3 \otimes 3 = \underset{A}{1} \oplus \underset{M_S}{8} \oplus \underset{M_A}{8} \oplus \underset{S}{10}$$

Ω^- : sss symmetric flavor in 10

symmetric spin $\frac{3}{2}$ in 4

↓

needs antisymmetric color wavefunction