Phys 772 : Week 8 Thursday

* Yukawa sector, weak vs. mass eigenstates
* Weak NC
* Electroweak sector of the Standard Model

$SU(2)_L \times U(1)_Y$ in $\begin{cases} L \text{ doublets of weak isospin} \\ R \text{ singlets} \end{cases}$

$$q_L = \begin{array}{c} t_3 \\ +\frac{1}{2} \\ -\frac{1}{2} \end{array} \begin{pmatrix} u \\ d \end{pmatrix}_L \quad , \quad \overset{t_3=0}{u_R, \, d_R} \qquad + \begin{pmatrix} c \\ s \end{pmatrix} + \begin{pmatrix} t \\ b \end{pmatrix} : m = 1,2,3 \quad F=3$$

$$l_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad , \quad \nu_{eR}, \, e^-_R \qquad + \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} + \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} : m = 1,2,3 \quad F=3$$

→ these are weak eigenstates with definitive weak isospin (3^{rd} component) quantum numbers

Higgs mechanism introduces Yukawa coupling terms between $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ and $\tilde{\varphi} = i\sigma^2 \varphi = \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix}$ and q, l.

$$\mathcal{L}_{Yuk} = -\sum_{m,n=1}^{F} \left(\Gamma^u_{mn} \bar{q}_L \tilde{\varphi} u_{nR} + \Gamma^d_{mn} \bar{q}_L \varphi d_{nR} + \Gamma^\nu_{mn} \bar{l}_L \tilde{\varphi} \nu_{nR} + \Gamma^l_{mn} \bar{l}_L \varphi e_{nR} \right) + h.c.$$

$\Gamma^u, \Gamma^d, \Gamma^\ell, \Gamma^\nu$ are full of arbitrary parameters $F \times F$

($\Gamma^\nu = 0$ if no ν_R are included)

Example of term for d quarks:

$$\Gamma^d_{mn} (\bar{u}_m \; \bar{d}_m)_L \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{nR} = \Gamma^d_{mn} \left[\bar{u}_{mL} \varphi^+ d_{nR} + \bar{d}_{mL} \varphi^0 d_{nR} \right]$$

$$\Gamma^{d+}_{mn} \bar{d}_{nR} \varphi^+ \begin{pmatrix} u_m \\ d_m \end{pmatrix}_L = \Gamma^{d*}_{mn} \left[\bar{d}_{nR} \varphi^- u_{mL} + \bar{d}_{nR} \varphi^{0+} d_{mL} \right]$$

$$\rightarrow \text{for } \langle \varphi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} \langle \varphi^+ \rangle \\ \langle \varphi^0 \rangle \end{pmatrix}$$

$\langle \hat{\varphi} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$ and Higgs field H in φ^0 direction:

$$\mathcal{L}_{Yuk} = -\sum_{m,n=1}^{F} \bar{u}_{mL} \Gamma^u_{mn} \frac{1}{\sqrt{2}} (v + H) u_{nR}$$

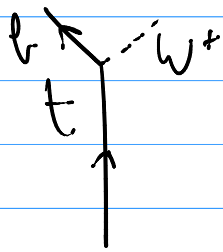
$$+ \; d, \ell, \nu \text{ terms } + h.c.$$

$M^u_{mn} = \Gamma^u_{mn} \frac{v}{\sqrt{2}}$ for generated fermion masses

$h^u_{mn} = \Gamma^u_{mn} \frac{1}{\sqrt{2}}$ for Yukawa couplings with H

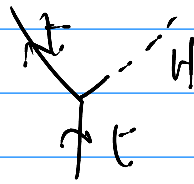
$$\rightarrow \mathcal{L}_{Yuk} = -\sum_{m,n=1}^{F} \bar{u}_{mL} (M^u_{mn} + h^u H) u_{nR}$$

$$+ \; d, \ell, \nu \text{ terms } + h.c.$$



for t decay

but large Yukawa coupling with H



$$h^u_{33} = \Gamma^u_{33}\frac{1}{\sqrt{2}} = \frac{m_t}{v} = \frac{173\,GeV}{245\,GeV} \simeq 1$$

Decay of $H \to t\bar{t}$ only allowed for $m_H > 140\,GeV$

For lighter H: $H \to b\bar{b}$ (dijet, backgrounds)

Since Γ^u, M^u are arbitrary, not symmetric: must apply unitary transformations to L, R such that

$$A^{u+}_L M^u A^u_R = M^u_D = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

$$\to \mathcal{L}_{Yuk} = -\sum_{m,n} \overline{u'_{mL}} \left(A^{u+}_L M^u A^u_R\right) u'_{mR}$$

$$\to \mathcal{L}_{Yuk} = -\sum_m \underbrace{\left(\overline{A^{u+}_L u_{mL}}\right)} M^u_D \underbrace{\left(A^{u+}_R u_{mR}\right)}$$

mass eigenstates u'_{mL}, u'_{mR}

$$\left(A^{u+}_L u_{mL}\right) = u'_{mL}$$

(i.e. $d' = d\cos\theta_c + s\sin\theta_c$ etc, s', ...)

How do we determine A^u_L, A^u_R etc ?

$M^u_D M^{u\dagger}_D$ is hermitian, as is $M^{u\dagger}_D M^u_D$

$$= \left(A^{u\dagger}_L M^u A^u_R\right)\left(A^{u\dagger}_R M^{u\dagger} A^u_L\right) = M^{u\,2}_D = \begin{pmatrix} m_u^2 & & \\ & m_c^2 & \\ & & m_t^2 \end{pmatrix}$$

$$= A^{u\dagger}_L M^u M^{u\dagger} A^u_L = A^{u\dagger}_R M^{u\dagger} M^u A^u_R$$

$\rightarrow$ eigenvalues problem for A^u_L and A^u_R
orthogonal eigenvectors with eigenvalues m_u^2, m_c^2, m_t^2

Freedom in phase of $A^u_{L,R}$ can be used to make sure that m_u is real and positive.

Kinetic terms: $\bar{q}_L i\not{\partial} q_L + \bar{q}_R i\not{\partial} q_R$

$$\downarrow$$

$$\bar{q}'_L i\not{\partial} q'_L + \bar{q}'_R i\not{\partial} q'_R$$

since $A^{u\dagger}_L A^u_L = 1 = A^{u\dagger}_R A^u_R$

$$\rightarrow \mathcal{L}_{\text{fermions}} = \sum_{m=1}^{F} \bar{q}\left(i\not{\partial} - m_m\left(1 + \frac{H}{v}\right)\right) q$$

in terms of mass eigenstates

BUT: other terms in $\not{D}$ that describe interaction with $W_{1,2,3}$ are still in terms of weak eigenstates ! $\rightarrow V_{CKM}$

* Charged currents : $A_L^{u+} A_L^d \neq 1 \rightarrow V_{CKM}$

$\updownarrow$

Neutral currents : $A_L^{u+} A_L^u = 1 \rightarrow$ no effect from V_{CKM}

Charged currents :

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} \left(J_W^\mu W_\mu^- + J_W^{\mu +} W_\mu^+ \right)$$

$$- \frac{2gg'}{2\sqrt{g^2+g'^2}} J_Q^\mu A_\mu \qquad \frac{gg'}{\sqrt{g^2+g'^2}} = e = g \sin\theta_W$$

$$- \frac{g^2+g'^2}{2\sqrt{g^2+g'^2}} J_Z^\mu Z_\mu \qquad \frac{g^2+g'^2}{\sqrt{g^2+g'^2}} = \frac{g}{\cos\theta_W}$$

with $J_W^\mu = \sum_{m=1}^{F} \left(\bar{e} \gamma^\mu \underbrace{(1-\gamma^5)}_{2P_L} \nu + \bar{d} \gamma^\mu \underbrace{(1-\gamma^5)}_{2P_L} u \right)$

$$J_W^{\mu +} = \sum_{m=1}^{F} \left(\bar{\nu} \gamma^\mu (1-\gamma^5) e + \bar{u} \gamma^\mu (1-\gamma^5) d \right)$$

$$J_Q^\mu = \sum_{m=1}^{F} \left(\underbrace{\tfrac{2}{3}}_{q_u} \bar{u} \gamma^\mu u \underbrace{- \tfrac{1}{3}}_{q_d} \bar{d} \gamma^\mu d \underbrace{-}_{q_e} \bar{e} \gamma^\mu e \right)$$

with $q = \underbrace{y}_{U(1)_Y} + \underbrace{t_3}_{SU(2)_L}$

$$J^\mu_Z = \sum_{m=1}^{F} t_3 \left(\bar{\ell}\, \gamma^\mu (1-\gamma^5)\, \ell\right) - 2 \sin^2\theta_W \, J^\mu_Q$$

$$g \frac{\sin^2\theta_W}{\cos\theta_W} \rightarrow g' \sin\theta_W$$

For J^μ_Q and J^μ_Z: only like fermions combine
$\rightarrow$ mass eigenstates can be used

For J^μ_W, $J^{\mu\,+}_W$:

$$J^{\mu\,+}_W = 2 \sum_{m=1}^{F} \left(\bar{\nu}_L \gamma^\mu \ell_L + \bar{u}_L \gamma^\mu d_L \right)$$

$$= 2 \sum_{m=1}^{F} \left(\bar{\nu}'_L \underline{A^{\nu +}_L \gamma^\mu A^\ell_L} \ell'_L + \bar{u}'_L \underline{A^{u+}_L \gamma^\mu A^d_L} d_L \right)$$

$$A^{\nu +}_L A^\ell_L = V_\ell \approx 1 \qquad A^{u+}_L A^d_L = V_q$$

$V_q = V_{CKM}$ = product of unitary transformations

$\rightarrow V_q$ = unitary

$2F^2$ variables, F^2 unitarity constraints

$\rightarrow F^2$ degrees of freedom

Also have phases for each of the $2F$ fermions
$\rightarrow$ set all to same phase, reduces $2F-1$ d.o.f.

$\rightarrow F^2 - 2F - 1 = (F-1)^2$ degrees of freedom

$F=2$ (u,d,c,s) : 1 degree of freedom $= \theta_c$

$F=3$: 4 degrees of freedom

$$(F-1)^2 = \underbrace{\frac{F(F-1)}{2}}_{\text{angles}} + \underbrace{\frac{(F-1)(F-2)}{2}}_{\text{phases}}$$

angles: $F=2 \to 1$, $F=3 \to 3$

phases, CP violation: $F=2 \to 0$, $F=3 \to 1$

Presence of V_{CKM} in J^μ_W explains why there are transitions possible between the families of qs

Not included in J^μ_Z → FCNC are sign of physics BTSM

A Weak neutral current:

$$J^\mu_Z = \sum_f \bar{\psi}\, \gamma^\mu \left(g^f_V - g^f_A \gamma^5 \right) \psi$$

$$\begin{cases} g^f_V = t_{3f} - 2 q_f \sin^2\theta_W & \text{(mixture of weak and EM)} \\ g^f_A = t_{3f} & \text{(pure weak isospin)} \end{cases}$$

$$= \sum_f \bar{\psi}\, \gamma^\mu \Big(\underbrace{(g^f_V + g^f_A)}_{\varepsilon_L} P_L + \underbrace{(g^f_V - g^f_A)}_{\varepsilon_R} P_R \Big) \psi$$

→ Z bosons interact with R chirality but with different strength

$$= \underbrace{\sum_f t_{3f}\bar{\psi}\gamma^\mu(1-\gamma^5)\psi}_{\psi_L \text{ only}} + \sum_f 2q_f \sin^2\theta_W \underbrace{\bar{\psi}\gamma^\mu\psi}_{L \text{ and } R}$$

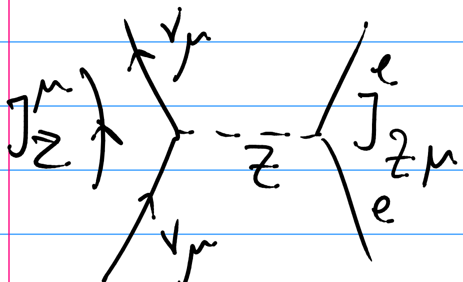
$$Z = -\sin\theta_W \underset{g'}{B} + \cos\theta_W \underset{g}{W^3}$$

one factor $\sin\theta_W$ due to mixing

other factor $\sin\theta_W$ because q_f in reference to $e = g\sin\theta_W$

$g\, q_f \sin\theta_W$ is the actual interaction strength

* $\nu_\mu e$ scattering (elastic): neutral current reaction, not present in G_F CC theory



can be described by effective theory that dominates M_Z in denominator

$$\mathcal{L}_Z^{NC} = -\frac{G_F}{\sqrt{2}} J^Z_\mu J_{Z\mu}$$

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\cos\theta_W}\right)^2 \frac{1}{M_Z^2} \quad \text{with } \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2}{8M_Z^2\cos^2\theta_W}$$

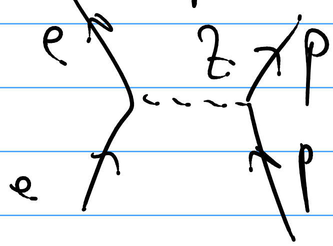
$$\Rightarrow \mathcal{L} = -\frac{G_F}{\sqrt{2}}\left(\bar{\nu}_\mu\, t_{3\nu}\,(1-\gamma^5)\,\nu_\mu\right)\left(\bar{e}\,\gamma^\mu\,(\underbrace{g_V^e - g_A^e}_{\text{because } q_e \neq 0}\gamma^5)\,e\right)$$

with $\begin{cases} g_V^e = -\frac{1}{2} + 2\sin^2\theta_W \\ g_A^e = -\frac{1}{2} \end{cases}$ with $t_{3\nu} = +\frac{1}{2} = g_V^\nu = g_A^\nu$

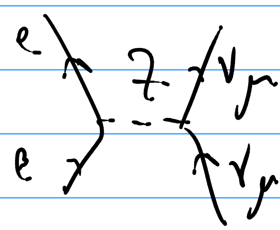
Common practice in experiments to write 4 fermion vertex couplings:

$\nu_\mu e \xrightarrow{Z} \nu_\mu e \longrightarrow \begin{cases} g_V^{e\nu} = 2\,t_{3\nu}\,g_V^e = -\frac{1}{2} + 2\sin^2\theta_W \\ g_A^{e\nu} = 2\,t_{3\nu}\,g_A^e = -\frac{1}{2} \end{cases}$

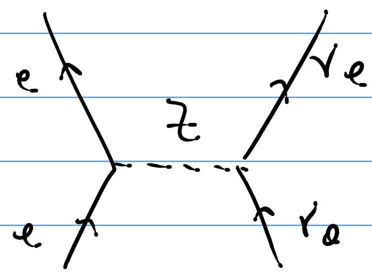
$ep \xrightarrow{Z} ep \longrightarrow \begin{cases} C_{1q} = 2\,g_A^e\,g_V^q \overset{u}{=} -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W \\ C_{2q} = 2\,g_V^e\,g_A^q = -\frac{1}{2} + 2\sin^2\theta_W \end{cases}$

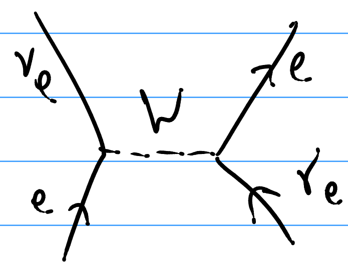


For $\nu_\mu e \to \nu_\mu e$: only NC



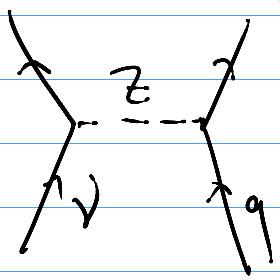
For $\nu_e e \to \nu_e e$: also CC



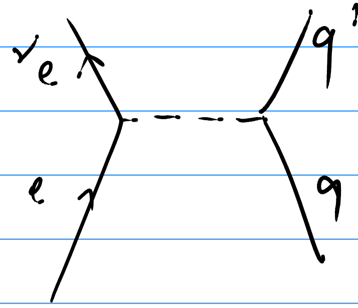


$\longrightarrow$ can be used to measure $g_A, g_V \to \sin^2\theta_W$

* νq scattering (DIS)



NC



CC

$V_{qq'}$ CKM measurements

↓ use NC to access $q(x)$ PDFs