Phys 772: Week 1 Thursday

Quigg 1–2

Standard Model = current best description of observed interactions and phenomenology

$\updownarrow$

known to be incomplete on theoretical grounds
- fine-tuning
- structure
- gravity
- parameters, masses

6 quarks: u, d, c, s, t, b

6 leptons: $e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau$ → spin $\frac{1}{2}$

4 gauge bosons: γ, Z, W, g → spin integer

All particles are pointlike, treated as fields

$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$ → color charge in QCD $SU(3)_c$

$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$ → colorless

left-handed components, $SU(2)_L$ doublets

Parity : $\vec{x} \to -\vec{x}$ — but $\vec{j} \to \vec{j}$

Charge : $\varphi \to \varphi^+$ — CPT invariant

Time : $t \to -t$

→ weak interaction decay $^{60}Co \to {}^{60}Ni\ e^- \bar{\nu}_e$

$n \to p\ e^- \bar{\nu}_e$

→ only left-handed neutrino, electron !
(right-handed $\bar{\nu}$, positron)

Symmetry structure :

$SU(3)_c \times SU(2)_L \times U(1)_Y$

↓ Y = Hyperchange in electroweak theory

Lie groups → focus for several lectures

Scalar fields : $\varphi(x)$ with x_μ position

Spinor fields : $\psi(x)$ ∂_μ derivative

↳ we will expand upon this by placing fields of e, ν_e in specific $SU(2)$ doublets

Vector fields : $A^\mu(x)$

Lagrangian density $\mathcal{L}(\varphi, \psi, \partial_\mu\varphi, \partial_\mu\psi, A^\mu)$

$$\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} = 0 \qquad (ELE)$$

$$S = \int dt \, L(t) \quad \text{with} \quad L(t) = \int d^3x \, \mathcal{L}(\varphi, \partial_\mu \varphi)$$

1) For scalar fields, $\varphi(x)$

$$\mathcal{L}(\varphi, \partial_\mu\varphi) = \frac{1}{2}\left((\partial_\mu \varphi)^2 - m^2\varphi^2 \right) - V(\varphi)$$

$$V(\varphi) = k\frac{\varphi^3}{3!} + \lambda \frac{\varphi^4}{4!} \qquad \text{self interaction}$$

$$ELE: \quad \partial_\mu \partial^\mu \varphi + m^2\varphi + k\frac{\varphi^2}{2!} + \lambda\frac{\varphi^3}{3!} = 0$$

$$(\Box + m^2)\varphi \longrightarrow \text{Klein-Gordon eqn}$$

2) For spinor fields: $\psi(x)$

$$\mathcal{L}(\psi, \partial_\mu \psi) = \bar{\psi}(i\not{\partial} - m)\psi = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

ELE: $(i\not{\partial} - m)\psi = 0 \rightarrow$ Dirac eqn

3) For vector fields: $A(x)$ (massless)

$$\mathcal{L}(A) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2)$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

ELE: $\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = 0$

4) For massive vector fields: $V(x)$

$$\mathcal{L}(V) = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} m^2 V_\mu V^\mu$$

5) Interaction terms, for example

$$\mathcal{L}_{\psi A} = -q\bar{\psi}\gamma^\mu \psi A_\mu \quad \text{with} \quad j^\mu = q\bar{\psi}\gamma^\mu\psi$$

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