Phys 772: Week 11 Tuesday

* CKM Matrix and CP Violation

- space reflection: $\mathcal{P}: \vec{x} \to -\vec{x}$, $\vec{p} \to -\vec{p}$
  = vectors
  $t \to t$ $\quad x^\mu \to x_\mu$
  = scalars $\quad x \to x' = (t, -\vec{x})$
  $\varphi \to \varphi$, $\psi \to \psi$

angular momentum: $\vec{L} = \vec{x} \times \vec{p} \to \vec{L}$, $\vec{J}$, $\vec{S}$
= axial vectors

helicity $h = \vec{S} \cdot \frac{\vec{p}}{|\vec{p}|} \to -h$ = pseudoscalars

four-vectors: s^μ = axial four-vectors
x^μ, p^μ = four-vectors

{intrinsic} parity quantum number of state under transformation

$$P\,|a(\vec{p},\vec{s})\rangle = (\pm 1)\,|a(-\vec{p},\vec{s})\rangle$$

$$P\,|a(\vec{p},h)\rangle = \underbrace{(\pm 1)}_{\eta_P = \text{intrinsic parity}}|a(-\vec{p},-h)\rangle$$

$P^2 = \mathbb{I}$
$\downarrow$
$\eta_P^2 = 1$

invariance of Lagrangian:

$$P\mathcal{L}(t,\vec{x})P^{-1} = \mathcal{L}(t,-\vec{x})$$

transformed $\mathcal{L}$ = $\mathcal{L}$ at transformed point e.g. $P\varphi(t,\vec{x})P^{-1} = \eta_P\,\varphi(t,-\vec{x})$

$$\rightarrow \begin{cases} \varphi(x) \rightarrow \eta_P\,\varphi(x') \\ \partial_\mu\varphi(x) \rightarrow \eta_P\,\partial'^\mu\varphi(x') \end{cases}$$

$$\rightarrow \partial_\mu\varphi(x)^\dagger\,\partial^\mu\varphi(x) \rightarrow \partial'^\mu\varphi(x')^\dagger\,\partial'_\mu\varphi(x')$$

$$\varphi(x)^\dagger\varphi(x) \longrightarrow \varphi(x')^\dagger\varphi(x')$$

because $|\eta_P|^2 = 1$

$$\rightarrow \mathcal{L} = (\partial_\mu\varphi)^\dagger(\partial^\mu\varphi) - m^2\varphi^\dagger\varphi$$

is invariant, regardless of η_P

e.g. $P\psi(t,\vec{x})P^{-1} = \gamma^0\psi(t,-\vec{x})$

$$P\bar{\psi}(t,\vec{x})P^{-1} = \bar{\psi}(t,-\vec{x})\gamma^0$$

$$P\bar{\psi}_1(x)\Gamma\psi_2(x)P^{-1} = \eta_1\eta_2\bar{\psi}_1(x')\gamma^0\Gamma\gamma^0\psi_2(x')$$

$\hookrightarrow \Gamma = 1 \rightarrow \gamma^0\Gamma\gamma^0 = 1$ scalar

$\Gamma = \gamma^5 \rightarrow \gamma^0\Gamma\gamma^0 = -\gamma^5$ pseudoscalar

$\Gamma = \gamma^\mu \rightarrow \gamma^0\Gamma\gamma^0 = \gamma_\mu$ vector

$\Gamma = \gamma^5\gamma^\mu \rightarrow \gamma^0\Gamma\gamma^0 = -\gamma^5\gamma_\mu$ axial vector

e.g. $PA^\mu(t,\vec{x})P^{-1} = A_\mu(t,-\vec{x})$

- charge conjugation:

$$C\,|a(\vec{p},\vec{s})\rangle = \underbrace{(\pm 1)}_{\eta_C}\,|a^c(\vec{p},\vec{s})\rangle$$

η_C = intrinsic charge conjugation phase

$$\begin{cases} C\varphi^\dagger C^{-1} = \eta_C\,\varphi \\ C\varphi C^{-1} = \eta_C^*\,\varphi^\dagger \end{cases} \xrightarrow{\varphi' = \sqrt{\pm\eta_C}\,\varphi} \begin{cases} C\varphi'^\dagger C^{-1} = \pm\varphi' \\ C\varphi' C^{-1} = \pm\varphi'^\dagger \end{cases}$$

→ can redefine fields to get rid of phases such that $\eta_C = \pm 1$

e.g.
$$\begin{cases} C\psi C^{-1} = \eta_C^*\,\psi^c \\ C\bar{\psi} C^{-1} = \eta_C\,\bar{\psi}^c \end{cases}$$

$\psi^c = C\bar{\psi}^T$ with $C = i\gamma^2\gamma^0$

$\bar{\psi}$: electrons $\qquad$ $\bar{\psi}^c$: positrons

- CP as an accidental symmetry for small theories

overall phases of fields are not observable

$\rightarrow$ for N fields there are N phases that can be chosen

Example : two complex scalars, φ_1 and φ_2

$$\mathcal{L} = \sum_{i=1}^{2} \left(\partial_\mu \varphi_i \partial^\mu \varphi_i^+ - m_i^2 \varphi_i^+ \varphi_i \right) - V(\varphi_1, \varphi_2)$$

terms in $V(\varphi_1, \varphi_2)$ of type $\varphi_i^+ \varphi_i$ or $(\varphi_i^+ \varphi_i)^2$ are automatically CP invariant

when $\varphi_i(x) \xrightarrow[P]{} \eta_{P_i} \varphi(x') \xrightarrow[C]{} \underbrace{\eta_{P_i} \eta_{C_i}^*}_{\text{product of phases}} \varphi^+(x')$

if $V(\varphi_1, \varphi_2) = g_{122} \varphi_1 \varphi_2^2 + g_{122}^* \varphi_1^+ \varphi_2^{+2}$

then $V(\varphi_1, \varphi_2) \xrightarrow[CP]{} g_{122} \eta_1^* \eta_2^{*2} \varphi_1 \varphi_2^2 + g_{122}^* \eta_1 \eta_2^2 \varphi_1^+ \varphi_2^{+2}$

if $g_{122} \neq g_{122}^* \rightarrow$ complex phase $e^{i\alpha}$ in $g_{122} = |g_{122}| e^{i\alpha}$

$\downarrow$

redefine $\begin{cases} \hat{\varphi}_2 = \varphi_2 \\ \hat{\varphi}_1 = \varphi_1 e^{i\alpha} \end{cases}$ then

$$V(\hat{\varphi}_1, \hat{\varphi}_2) = |g_{122}| \left(\hat{\varphi}_1 \hat{\varphi}_2^2 + \hat{\varphi}_1^+ \hat{\varphi}_2^{+2} \right)$$

$\hookrightarrow$ invariant under CP

but could add more interactions, e.g

$$V(\varphi_1, \varphi_2) = \cdots \varphi_1 \varphi_2^2 + \cdots \varphi_1^2 \varphi_2$$
$$+ \cdots \varphi_1^3 + \cdots \varphi_2^3$$

→ too many possible vertices that can break CP invariance, but only two possible fields to redefine phases

Corolary: extensions of SM → more vertices → more opportunities for remaining CP violation

* CKM matrix :

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \text{with } V_{CKM} \text{ unitary}$$

$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13} e^{-i\delta} \\ & 1 & \\ -s_{13} e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

1 CP violating phase in V_{CKM} for $N=3$

$\hookrightarrow e^{i\delta}$ in s_{13}

Cabibbo angle $s_{12} = \sin\theta_{12} = 0.23$

Wolfenstein parametrization : $\lambda = \sin\theta_{12}$

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$\tan\delta = \frac{\eta}{\rho}$, η and δ are CP violating

Look at specific combinations of $V_{q_1 q_2}$ to measure λ, η, ρ, etc

$$\bar{\rho} + i\bar{\eta} = (\rho + i\eta)(1 - \lambda^2/2) + O(\lambda^4) = -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}$$

* Unitarity tests of V_{CKM} : $V_{CKM}^{\dagger} V_{CKM} = 1$

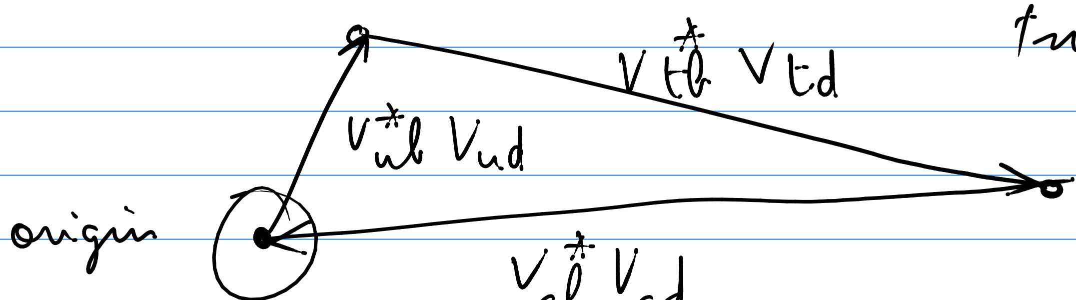
→ diagonal $(V_{CKM}^{\dagger} V_{CKM})_{11} = 1$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

→ off-diagonal $(V_{CKM}^{\dagger} V_{CKM})_{31} = 0$

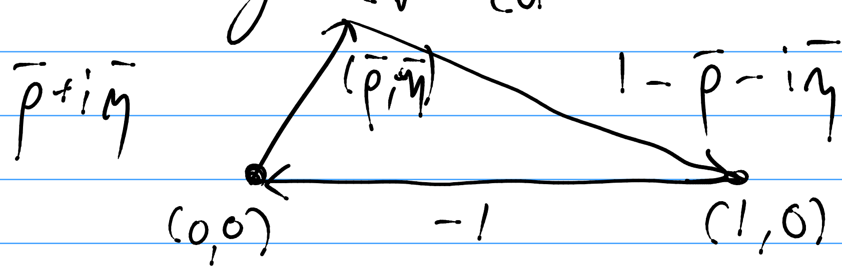
$$V_{ub}^{*} V_{ud} + V_{cb}^{*} V_{cd} + V_{tb}^{*} V_{td} = 0$$

sum of 3 complex number : triangle
(six unitarity triangles)



should sum back to origin

Scale by $V_{cb}^{*} V_{cd} = -1$:



V_{cd}, V_{cb} well known from D, B decays

measurements of $\left| \dfrac{V_{ub}^{*} V_{ud}}{V_{cb}^{*} V_{cd}} \right|$ give $|\bar{\rho} + i\bar{\eta}|$

* Neutral kaon mixing :

Remember $SU(3)_{u,d,s}$ structure :

$$
\begin{array}{ccccc}
 & K^0 & & K^+ & \\
 & |d\bar{s}\rangle & & |u\bar{s}\rangle & \\
\pi^- & & \pi^0 & & \pi^+ \\
 & |s\bar{u}\rangle & & |s\bar{d}\rangle & \\
 & K^- & & \bar{K}^0 &
\end{array}
$$

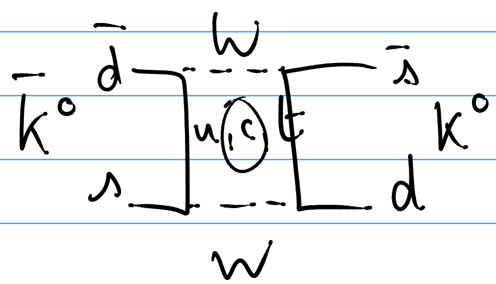
$$CP\,\underbrace{|d\bar{s}\rangle}_{|K^0\rangle} = \eta_K \underbrace{|s\bar{d}\rangle}_{|\bar{K}^0\rangle}$$

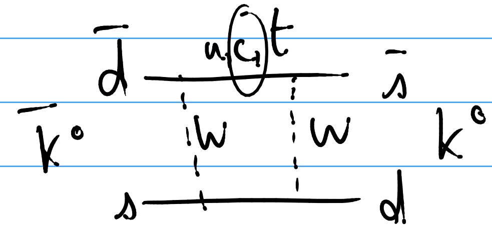
$$CP\,|\bar{K}^0\rangle = \eta_K^*\,|K^0\rangle$$

$\rightarrow$ redefine $|K^0\rangle$, $|\bar{K}^0\rangle$ such that $\eta_K = 1$

All this is valid in QCD only picture $\rightarrow$ no explicit CP violation

With weak interaction : K^0 and $\bar{K}^0$ can mix





$\rightarrow$ mass matrix $\begin{pmatrix} M_{K^0} & M_{K\bar{K}} \\ M^*_{K\bar{K}} & M_{\bar{K}^0} \end{pmatrix}$

$M_{K\bar{K}}$ = mixing term, small but relevant because $M_{K^0} - M_{\bar{K}^0}$ is also small (zero)

No CP violation:

$M_{K\bar{K}} = M_{K\bar{K}}^* =$ real ;

$\longrightarrow \quad K^0_{1,2} = \dfrac{K^0 \mp \bar{K}^0}{\sqrt{2}}$ = K-short, K-long

eigenvalues ± 1 under CP

$CP|K^0_1\rangle = +|K^0_1\rangle \qquad CP|\pi^+\pi^-\rangle = |\pi^+\pi^-\rangle$

$CP|K^0_2\rangle = -|K^0_2\rangle \qquad CP|\pi^0\pi^0\rangle = |\pi^0\pi^0\rangle$

↳ only $K^0_1 \to \pi^+\pi^-$ or $K^0_1 \to \pi^0\pi^0$

↳ strong decay, small Γ, short track $\longrightarrow K^0_S$

K^0_L decays instead to $\pi^+\pi^-\pi^0$, $\pi^0\pi^0\pi^0$ with much smaller phase space

$M(K^0) \gtrsim 3\pi$

K^0_L is even more likely to decay weakly, $s \to u$, $l\nu$

$\Delta m_K = m_{K^0_2} - m_{K^0_1} = 2M_{K\bar{K}} \sim |V_{cd}|^2 |V_{cs}|^2 m_c^2$

very small, can't be measured directly in mass spectra

$\rightarrow$ use oscillations to measure Δm_K

Start with pure K^0 beam, e.g. $\pi^- p \to K^0 \Lambda$

$$|\psi\rangle = |K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_S\rangle + |K_L\rangle\right)$$

At some later time t:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|K_S\rangle e^{-im_S t} + |K_L\rangle e^{-im_L t}\right)$$

$$= \left(\cos\left(\frac{\Delta m_K t}{2}\right)|K^0\rangle - i\sin\left(\frac{\Delta m_K t}{2}\right)|\bar{K}^0\rangle\right) e^{im_{K_0} t}$$

oscillation probability $|\psi\rangle|^2$

$$\sin^2\left(\frac{\Delta m_K t}{2}\right)$$

Oscillation time $\frac{2\pi}{\Delta m_K} \approx 10^{-9}\,s$, $\frac{1}{\Gamma_{K_S}} \simeq 10^{-10}\,s$

$\rightarrow$ decay important $\left(\frac{1}{\Gamma_{K_L}} \approx 10^{-8}\,s\right)$

$\rightarrow$ replace $e^{-im_S t} \rightarrow e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t}$

CP violation in $K^0\bar{K}^0$:

$$M_{K\bar{K}} \neq M^*_{K\bar{K}} \rightarrow K_S \neq K_1^0$$

$$K_L \neq K_2^0$$

Now:
$$\begin{cases} |K_S\rangle = \dfrac{|K_1^0\rangle + \tilde{\varepsilon}|K_2^0\rangle}{\sqrt{1+\tilde{\varepsilon}^2}} \\[2ex] |K_L\rangle = \dfrac{\tilde{\varepsilon}|K_1^0\rangle + |K_2^0\rangle}{\sqrt{1+\tilde{\varepsilon}^2}} \end{cases}$$

with small $\tilde{\varepsilon}$ by $\dfrac{1-\tilde{\varepsilon}}{1+\tilde{\varepsilon}} = \sqrt{\dfrac{M^*_{K\bar{K}} - i\Gamma^*_{K\bar{K}}/2}{M_{K\bar{K}} - i\Gamma_{K\bar{K}}/2}}$

$\tilde{\varepsilon} \neq 0$ if $M^*_{K\bar{K}} \neq M_{K\bar{K}}$

and/or $\Gamma^*_{K\bar{K}} \neq \Gamma_{K\bar{K}}$