Phys 772: Week 5 Thursday

* SU(2) weak isospin

* $SU(2)_L \times U(1)_Y$ $\quad \rightarrow \quad$: $\langle\varphi\rangle = v = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$

$\downarrow$

$L^i = \frac{\sigma^i}{2}$ $\qquad\qquad L = Y$

$L^i v = \frac{\sigma^i}{2}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix} \neq 0 \qquad \rightarrow \qquad L v = Y \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix} \neq 0$

but $(\underbrace{L^3 + Y}_{Q}) v = 0$ symmetry is not broken

$\rightarrow$ QED $U(1)_Q$

$\hookrightarrow$ $SU(3)_C \times SU(2)_L \times U(1)_Y$ in $\{n_3, n_2, n_1\}$ rep

q_{mL} in $\{3, 2, \frac{1}{6}\}$ $\rightarrow$ $Q = \pm\frac{1}{2} + \frac{1}{6} = \begin{pmatrix}2/3\\-1/3\end{pmatrix}$

l_{mL} in $\{1, 2, -\frac{1}{2}\}$ $\rightarrow$ $Q = \pm\frac{1}{2} - \frac{1}{2} = \begin{pmatrix}0\\-1\end{pmatrix}$

u_{mR} in $\{3, 1, \frac{2}{3}\}$ $\rightarrow$ $Q = \frac{2}{3}$

d_{mR} in $\{3, 1, -\frac{1}{3}\}$ $\rightarrow$ $Q = -\frac{1}{3}$

l_{mR} in $\{1, 1, -1\}$ $\rightarrow$ $Q = -1$

ν_{mR} in $\{1, 1, 0\}$ $\rightarrow$ $Q = 0$

Q is equal for L and R by design

$m = 1, \dots, 3$ families

* SU(2) Higgs doublet and SSB

$(D^\mu \varphi)^\dagger (D_\mu \varphi)$ becomes

$$= \frac{1}{2}\left(\frac{1}{2} v g\right)^2 \left(W_\mu^{1\,2} + W_\mu^{2\,2}\right) + \frac{1}{8} v^2 \left(g' B_\mu - g W_\mu^3\right)^2$$

Define: $W_\mu^{\pm} = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)$

Define
$$\begin{cases} Z_\mu = \dfrac{-g' B_\mu + g W_\mu^3}{\sqrt{g^2 + g'^2}} = -\sin\theta_W \cdot B_\mu + \cos\theta_W \cdot W_\mu^3 \\ A_\mu = \cos\theta_W \cdot B_\mu + \sin\theta_W \cdot W_\mu^3 \end{cases}$$

$$\to (D^\mu \varphi)^\dagger (D_\mu \varphi) = M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$
$$\left(+ 0 \cdot A_\mu A^\mu \right)$$

$M_W = g \frac{v}{2}$, $M_Z = \sqrt{g^2 + g'^2}\, \frac{v}{2}$, $M_A = 0$ (photon)

$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$, $\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$

$\to \tan\theta_W = \frac{g'}{g}$ $\to$ θ_W = electroweak mixing angle

$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$

$\approx 0{,}2397$

Since we know that A_μ interacts with strength e given by electron charge

$$\rightarrow \quad A_\mu = \cos\theta_W \underset{\underset{g'}{\uparrow}}{B_\mu} + \sin\theta_W \underset{\underset{g}{\uparrow}}{W^3_\mu}$$

$$\rightarrow \quad e = g \sin\theta_W = g' \cos\theta_W$$

$$\mathcal{L}_{Higgs} = (D^\mu \varphi)^\dagger (D_\mu \varphi) - \mu^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^4$$

$$= M_W^2 W^+_\mu W^{-\mu} \left(1 + \frac{H}{v}\right)^2 + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \left(1 + \frac{H}{v}\right)^2$$

$$+ \frac{1}{2}(\partial^\mu H)^2 + \mu^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 + \frac{\mu^4}{4\lambda}$$

* Standard Model Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{fermions} + \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{fermions} = \sum_{m=1}^{3} \Big(\bar{q}_{mL}\, i\not{D}\, q_{mL} + \bar{\ell}_{mL}\, i\not{D}\, \ell_{mL} + \bar{u}_{mR}\, i\not{D}\, u_{mR} + \bar{d}_{mR}\, i\not{D}\, d_{mR} + \bar{\ell}_{mR}\, i\not{D}\, \ell_{mR} + \bar{\nu}_{mR}\, i\not{D}\, \nu_{mR} \Big)$$

$$\mathcal{L}_{gauge} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^{i}_{\mu\nu} W^{\mu\nu}_{i} - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

$i = 1, 2, 3$

$a = 1, \dots, 8$

with
$$\begin{cases} B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^{i}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g\, \varepsilon_{ijk} W^{j}_{\mu} W^{k}_{\nu} \\ G^{a}_{\mu\nu} = \partial_\mu G^{a}_{\nu} - \partial_\nu G^{a}_{\mu} - g_s f_{abc} G^{b}_{\mu} G^{c}_{\nu} \end{cases}$$

with $D_\mu = \partial_\mu + i g' Y B_\mu + i g \frac{\vec{\sigma}}{2} \cdot \vec{W}_\mu + i g_s \frac{\vec{\lambda}}{2} \cdot \vec{G}_\mu$

$$\rightarrow \quad \bar{q}_{mL} i \not{D} q_{mL} =$$

$$i \left(\bar{u}^{\alpha}_{mL} \quad \bar{d}^{\alpha}_{mL} \right) \left(\partial_\mu + i g' \overset{\frac{1}{6}}{Y} B_\mu + i g \frac{\sigma_i}{2} W^i_\mu + i g_s \frac{\lambda^{\alpha\beta}_a}{2} G^a_\mu \right) \begin{pmatrix} u^\beta_{mL} \\ d^\beta_{mL} \end{pmatrix}$$

$i = 1, \dots, 3$

$\alpha, \beta = r, g, b$

$a = 1, \dots, 8$

$SU(3)_c$ commutes with $SU(2) \times U(1)$ subgroups

$$\mathcal{L}_{Higgs} = (D^\mu \varphi)^\dagger (D_\mu \varphi) - \mu^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^2$$

with $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ as $\{1, 2, \frac{1}{2}\} \rightarrow Q = \begin{pmatrix} +1 \\ 0 \end{pmatrix}$

with $\mu^2 < 0$, $\lambda > 0$, $v = -\frac{\mu^2}{\lambda}$, $\langle \varphi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\mathcal{L}_{Yukawa} = -\sum_{m,n}^{3} \left(g_{um} \bar{q}_{mL} \varphi u_{mR} + g_{dm} \bar{q}_{mL} \varphi d_{mR} + g_{em} \bar{l}_{mL} \varphi e_{mR} + g_{\nu m} \bar{l}_{mL} \varphi \nu_{mR} \right) + h.c.$$

with $g_{em} = \sqrt{2} \frac{m_{em}}{v}$ coupling of l_m to Higgs

$\bar{e}_{mL} \varphi = \bar{\nu}_L \varphi^+ + \bar{e}_L \varphi^0$ is $SU(2)$ invariant

$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix} \rightarrow$ also $\bar{l}_{mL} \eta\, l_{mR}$ vertex

The last term $g_{\nu m} \bar{\ell}_{mL} \varphi \nu_{mR}$ is absent if we do not include ν_R in the Standard Model $\rightarrow$ then m_ν must be zero

If φ is a weak isospin doublet under SU(2) then

$$\tilde{\varphi} = i \sigma_2 \varphi^+ = \begin{pmatrix} \varphi^{0+} \\ -\varphi^- \end{pmatrix}$$

is also a SU(2) doublet with $\{1, 2^*, -\frac{1}{2}\}$

with $\tilde{\varphi} = \frac{1}{\sqrt{2}} \begin{pmatrix} v + \eta \\ 0 \end{pmatrix}$

$\rightarrow$ $\tilde{\varphi}$ operates on upper weak isospin components, u_{mR}, ν_{mR}

$\downarrow$

slight variation to $\mathcal{L}_{Yukawa}$

Finally: $g_{em}; g_{um}; g_{dm}; g_{\nu m}$ could be a general 3×3 matrix $\Gamma_\ell, \Gamma_u, \Gamma_d, \Gamma_\nu$ if the weak eigenstates and mass eigenstates are different

$$\Rightarrow \mathcal{L}_{Yukawa} = -\sum_{m,n}^{3} \left(\bar{q}_{mL} \Gamma^u_{mn} \tilde{\varphi} u_{nR} + \text{etc} \right) + \text{h.c.}$$