Phys 772: Week 12 Thursday

* Multiple families of mass-mixed neutrinos:

$$\mathcal{L} = -\frac{1}{2}\begin{pmatrix} \bar{\nu}_L & \bar{\nu}_L^c \end{pmatrix}\begin{pmatrix} M_T & M_D \\ M_D^T & M_S \end{pmatrix}\begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} + h.c.$$

with $\nu_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$, etc for ν_L^c, ν_R, ν_R^c

↳ weak eigenstates

- For Dirac masses only this reverts to

$$\mathcal{L}_D = -\frac{1}{2}\begin{pmatrix} \bar{\nu}_L & \bar{\nu}_L^c \end{pmatrix}\begin{pmatrix} 0 & M_D \\ M_D^T & 0 \end{pmatrix}\begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} + h.c.$$

$$= -\left(\bar{\nu}_L M_D \nu_R + \bar{\nu}_R M_D^\dagger \nu_L \right) + (\text{sterile } \nu_R^c, \bar{\nu}_L^c)$$

→ A_L^ν and A_R^ν diagonalize M_D:

$$A_L^{\nu\dagger} M_D A_R^\nu = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}$$

such that $\nu_L' = A_L^{\nu\dagger}\nu_L$, $\nu_R' = A_R^{\nu\dagger}\nu_R$

↳ mass eigenstates

PMNS mass mixing matrix $V_{PMNS} = A_L^{\nu\dagger} A_L^e$

$$\mathcal{L}_{WCC} = \bar{\nu}_L \gamma^\mu V_{PMNS}\, e_L W_\mu$$

- For Majorana masses only.

$$\mathcal{L}_M = -\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \bar{\nu}_L^c\end{pmatrix}\begin{pmatrix}M_T & 0\\ 0 & M_S\end{pmatrix}\begin{pmatrix}\nu_R^c\\ \nu_R\end{pmatrix} + h.c.$$

$$= -\frac{1}{2}\left(\bar{\nu}_L M_T \nu_R^c + \bar{\nu}_R^c M_T^\dagger \nu_L\right) \quad \text{(active, triplet)}$$

$$-\frac{1}{2}\left(\bar{\nu}_L^c M_S \nu_R + \bar{\nu}_R M_S^\dagger \nu_L^c\right) \quad \text{(sterile, singlet)}$$

$\rightarrow$ A_L^ν and A_R^ν diagonalize M_S but because Majorana fermions connect L and R through

$$\Psi_R^c = i\sigma^2 \Psi_L^*$$

$\downarrow$

$A_L^\nu = A_R^{\nu *} K$ with K composed of phases $\rightarrow$ choose such that $K = \mathbb{1}$

$\downarrow$

leaves less freedom in V_{PMNS} to remove phases (since already picked in $K = \mathbb{1}$)

$\Rightarrow$ V_{PMNS} has additional phases:

- 3 mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$
- 1 CP-violating phase δ
- 2 Majorana phases α_1, α_2

$\hookrightarrow$ $\begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & 1\end{pmatrix}$

since only phase differences are observable

- General multi-family mass mixing matrix:

$$\mathcal{L} = -\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \bar{\nu}^c_L\end{pmatrix}\begin{pmatrix}M_T & M_D \\ M_D^T & M_S\end{pmatrix}\begin{pmatrix}\nu_R^c \\ \nu_R\end{pmatrix}$$

Now no separation of active and sterile sector due to Dirac mass terms mixing:

$\rightarrow$ 6 eigenvalues m_i, $i = 1, \dots, 6$
6 eigenstates:

$$\nu_i'^L = A_L^{\nu +}\begin{pmatrix}\nu_L \\ \nu_L^c\end{pmatrix} \rightarrow 6\text{x}6 \text{ matrix } A_L^\nu$$

Can again choose Majorana phases such that

$$A_R^\nu = A_L^{\nu *}$$

- Seesaw mechanism for multi-families

for 1-family case: $m_S \gg m_T, m_D$
$m_D \approx O(m_e)$
$m_T \approx 0$

$\downarrow$

for 3-family case: eigenvalues of $M_S \gg$ eigenvalues of M_D

mass mixing matrix $\begin{pmatrix}\overset{\text{small}}{M_T} & M_D \\ M_D^T & \underset{\text{large}}{M_S}\end{pmatrix}$

$\nearrow$ nearly $\mathbb{1}$ matrix

With $B_L^{\nu +} = \begin{pmatrix} \mathbb{1} & -M_D M_S^{-1} \\ (M_S^{-1} M_D)^{+} & \mathbb{1} \end{pmatrix}$ this makes the mass mixing matrix block-diagonal

$$B_L^{\nu +} \begin{pmatrix} M_T & M_D \\ M_D^T & M_S \end{pmatrix} B_L^{\nu} = \begin{pmatrix} M_T - M_D M_S^{-1} M_D^T & 0 \\ 0 & M_S \end{pmatrix}$$

$\Rightarrow \left\{ \begin{array}{l} M_T - M_D M_S^{-1} M_D^T \text{ diagonalized by } A_L^{\nu} \\ \quad \hookrightarrow \text{ light } (\frac{1}{m_S} \text{ suppressed) eigenvalues} \\ M_S \text{ diagonalized by } A_L^{\nu_S} \\ \quad \hookrightarrow \text{ heavy } m_S \text{ eigenvalues} \end{array} \right.$

mass eigenstates for both blocks are independent to the extent that approximation is valid

$\downarrow$

lowest mass eigenstates have masses $\sim m_D \frac{m_D}{m_S} \simeq O(eV)$

- Most general case $\rightarrow$ sterile components mixed into low mass eigenstates, and low sterile eigenstates

$$\rightarrow V_{PMNS} = \begin{array}{c} \begin{array}{cccccc} \nu_1^A & \nu_2^A & \nu_3^A & \nu_1^S & \nu_2^S & \nu_3^S \end{array} \\ \begin{pmatrix} & & & & & \\ & & & & & \\ & & & & & \end{pmatrix} \end{array} \begin{array}{c} e \\ \mu \\ \tau \end{array} \quad (6\times 3)$$

$$= \underset{6\times 6}{A_L^{\nu +}} \underset{6\times 3}{\begin{pmatrix} \mathbb{1} & 0 \end{pmatrix}} \underset{3\times 3}{A_L^{e}}$$

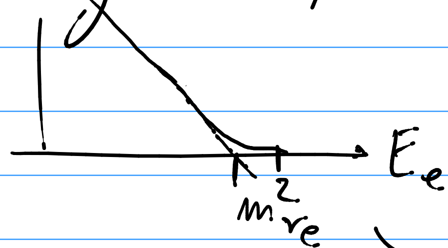
* Experimental neutrino constraints

- number of light families : $\Gamma^{Z}_{\text{invisible}} \to n = 3$

  active neutrinos with $m_\nu \lesssim M_Z/2$

- masses of neutrinos :

  β-decay end-point measurements with KATRIN



$${}^3H \to {}^3He + e^- + \bar{\nu}_e$$

$$\to m^2_{\nu_e} = \sum_i |V_{ei}|^2 m_i^2$$

↳ currently running

SN1987A : delay time between light and neutrinos and spread of arrival times

$0\nu\beta\beta$: spectrum of decay should have peak without missing energy since no neutrinos carry away energy

↓

Klapdor-Kleingrothaus hypothesis of observation of $0\nu\beta\beta$ in Ge (HDMS → Dark Matter search)

↳ not confirmed by other Ge detectors

★ Neutrino oscillations :

2-family example: $\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$

weak eigenstate $\qquad\qquad$ mass eigenstate

at $t=0$: $|\nu\rangle = |\nu_\mu\rangle$ from e.g. $\pi \to \mu\nu$ in flight

at $t>0$: $|\nu(t)\rangle = -|\nu_1\rangle \sin\theta_{12}\, e^{-iE_1 t} + |\nu_2\rangle \cos\theta_{12}\, e^{-iE_2 t}$

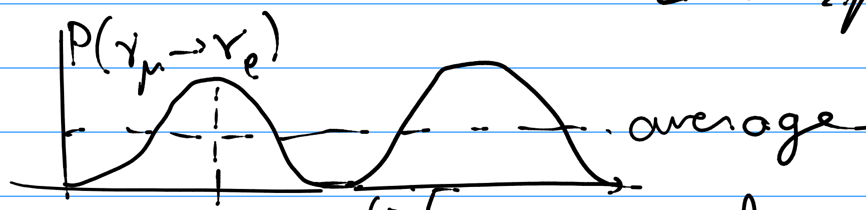
$$= \left(-|\nu_1\rangle \sin\theta_{12}\, \underbrace{e^{-i\frac{m_1^2}{2E}t}}_{\text{rest mass}} + |\nu_2\rangle \cos\theta_{12}\, e^{-i\frac{m_2^2}{2E}t}\right) \underbrace{e^{-iEt}}_{\text{kinetic energy}}$$

$$\downarrow$$

$$P(\nu_\mu \to \nu_e) = (\sin\theta_{12}\cos\theta_{12})^2 \left(-e^{-i\frac{m_1^2}{2E}t} + e^{-i\frac{m_2^2}{2E}t}\right)^2$$

$$= (\sin 2\theta_{12})^2 \left(\sin \frac{\Delta m^2}{4E} t\right)^2$$

$t \to L$ at speed of light

$\Rightarrow$ 

$L_{osc} = \dfrac{4\pi E}{\Delta m^2}$: all ν_μ have oscillated into ν_e

detection $\nu_e n \to e^- p$: detect e^- and p and reconstruct n at rest

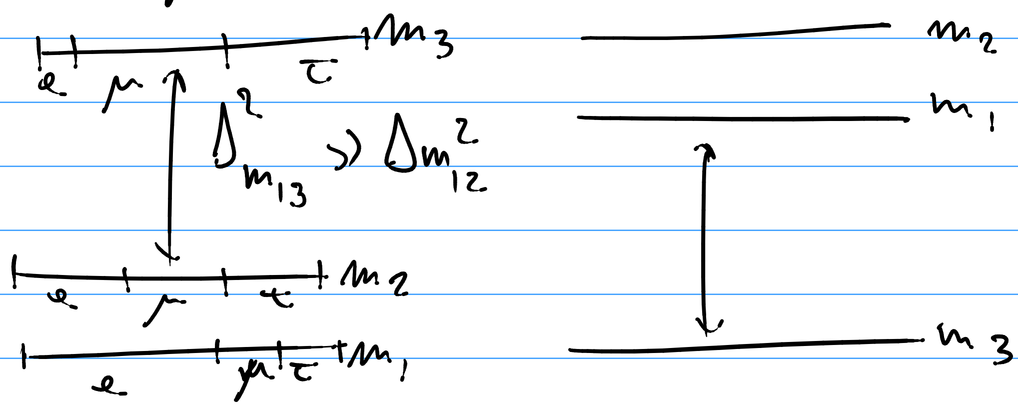
→ appearance experiments (measure ν_e)
→ disappearance experiments (measure ν_μ)

If E unknown/spread out → $\sin^2\left(\frac{\Delta m^2}{4E} L\right) \approx \frac{1}{2}$

⇒ oscillations can be used to measure Δm_{ij}^2 only

if $\Delta m_{ij}^2 \gg \Delta m_{jk}^2$ then oscillations decouple

↳ regular vs. inverted hierarchy



contributions of weak eigenstates in mass eigenstates given by $\theta_{12}, \theta_{23}, \theta_{13}$

Experimental difficulties:

- E spread
- only $\bar{\nu}_e$ at reactors (or in atmosphere), ν_e in sun fusion (→ detection of only $\frac{1}{3}$ of flux → SNO)
- $\nu_\mu, \bar{\nu}_\mu$ at pion beams (allows $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ and $P(\nu_\mu \to \nu_e)$ comparison)
- ν_e or ν_μ detection in inverse β

- MSW (Mikheyev - Smirnov - Wolfenstein) effect:
  ↳ impact from matter on propagation of neutrinos
  ⇒ scattering goes like G_F with number density n

in vacuum: $\frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$ oscillation magnitudes

$\downarrow$

in matter: $\frac{\Delta m^2}{4E}\begin{pmatrix} & \\ & \end{pmatrix} + \frac{G_F n}{\sqrt{2}}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

↳ can modify transition amplitude constructively or destructively

e.g.: $\frac{\Delta m^2}{4E}\cos 2\theta = \frac{G_F n}{\sqrt{2}}$ ⟶ only off-diagonal elements → maximal transition probability

⟶ solar neutrinos ν_e disappearance

atmospheric neutrinos ν_μ: azimuthal dependence

reactor neutrinos $\nu_e, \bar{\nu}_e$: short baseline oscillation

accelerator neutrinos $\nu_\mu, \bar{\nu}_\mu$: long baseline