Phys 772 : Week 8 Tuesday

* History of the weak interaction

β- decay : $n \to p e^- \bar{\nu}_e$ at nucleon level

$(N,Z) \to (N-1, Z+1)\ e^-$ in nuclei

$\downarrow$

violation of energy conservation when $\bar{\nu}_e$ is not taken into account

$\to$ Pauli postulate of neutral, weakly interacting particle

= little neutral one

= neutrino

detection by Reines & Cowan in $\bar{\nu}_e p \to n e^+$

similar detection of $\nu_\mu \to \mu$

and $\nu_\tau \to \tau$

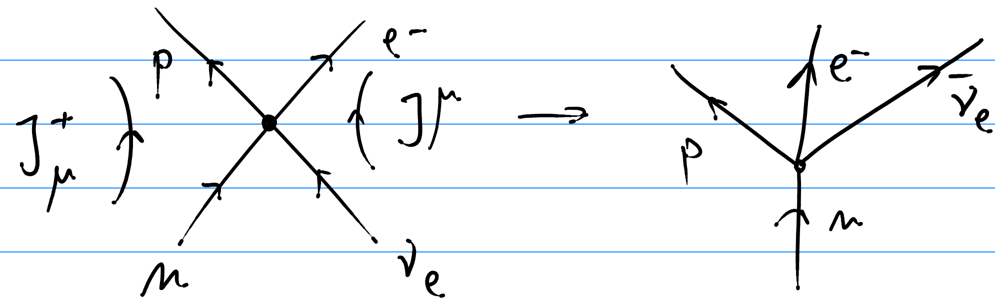
Various theories of weak interaction

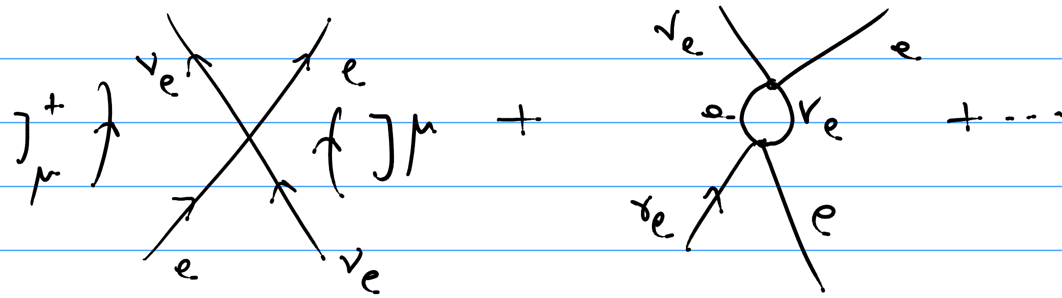
1) Fermi theory of weak interaction

$$\mathcal{L} = G_F J^+_\mu J^\mu \quad , \text{ four fermion interaction}$$

with $J^+_\mu = \bar{p}\gamma_\mu n + \bar{\nu}_e \gamma_\mu e \quad (\Delta Q = +1)$

and $J_\mu = \bar{n}\gamma_\mu p + \bar{e}\gamma_\mu \nu_e \quad (\Delta Q = -1)$





with $G_F = 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$ $\left(\propto \frac{1}{M_W^2}\right)$

$\downarrow$ non-renormalizable, effective theory only valid at low energy $E_{CM} < 500\,\mathrm{GeV}$

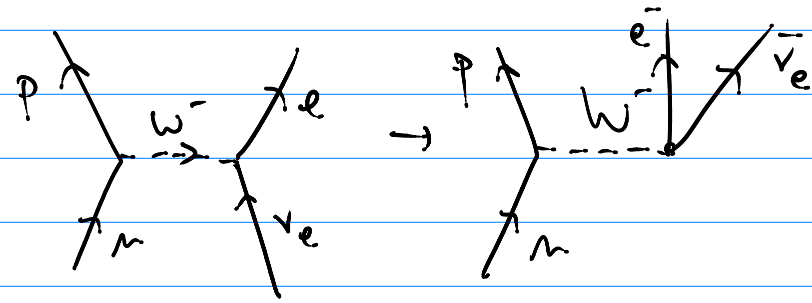
Extensions to include $\nu_\mu, \mu, \nu_\tau, \tau$
$d \to u$, $s \to u$

(V-A) to introduce parity-violation

2) Intermediate Vector Boson Theory

$$\mathcal{L} = g W_\mu^+ J^{+\mu} + g W_\mu^- J^\mu$$

with intermediate vector boson $W^\pm$



If $Q^2 \ll M_W^2 \rightarrow$ propagator reduces to

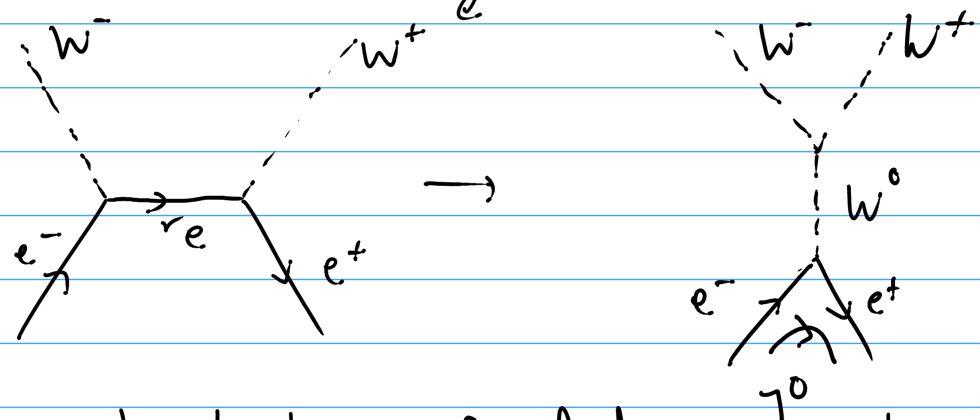
$$\frac{1}{q^2 - M_W^2} \rightarrow -\frac{1}{M_W^2} \propto G_F$$

In particular:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \quad \text{for } Q^2 \ll M_W^2$$

- No renormalizable mechanism to introduce mass term for W^+, W^- in Lagrangian

- Issues with diagram lead to divergences



$\rightarrow$ must introduce W^0 which couples to neutral current $J^0 = \bar{e}\gamma^\mu e$

↳ J^+, J^-, J^0 satisfy $SU(2)$ gauge theory vertex relations (Glashow)

3) Standard Model electroweak SU(2) gauge symmetry

Mass can be introduced by Higgs mechanism (Salam, Weinberg)

Results in renormalizable theory ('t Hooft Veltman)

$\Rightarrow$ Fermi theory & Standard Model both useful

* Fermi theory

V-A = vector - axial vector

$$J^+_\mu = \underbrace{J^{l+}_\mu}_{\text{leptonic}} + \underbrace{J^{h+}_\mu}_{\text{hadronic}}$$

i) $$J^{l+}_\mu = \bar{\nu}_e \gamma_\mu (1-\gamma^5) e + \bar{\nu}_\mu \gamma_\mu (1-\gamma^5) \mu + \bar{\nu}_\tau \gamma_\mu (1-\gamma^5) \tau$$

$$= 2 \left(\bar{\nu}_{eL} \gamma_\mu e_L + \bar{\nu}_{\mu L} \gamma_\mu \mu_L + \bar{\nu}_{\tau L} \gamma_\mu \tau_L \right)$$

with $e_L = P_L e = \frac{1-\gamma^5}{2} e$ etc

$\Rightarrow$ $1 - c\gamma^5$ introduces difference between $V_\mu = \gamma_\mu$ and $A_\mu = \gamma_\mu \gamma^5$ interaction

for $c = 1$ $\rightarrow$ $V - A$, only 1 chirality interacts

2) $J^{h+}_\mu = \bar{p}\, \gamma_\mu (1-\gamma^5)\, n$ as direct extension from β decay

$\downarrow$

$J^{h+}_\mu = \bar{u}\, \gamma_\mu (1-\gamma^5)\, d$

but both d and s quarks decay to u

$\rightarrow$ weak d eigenstate $d' = d\cos\theta_c + s\sin\theta_c$

$\sin\theta_c \approx 0.23$

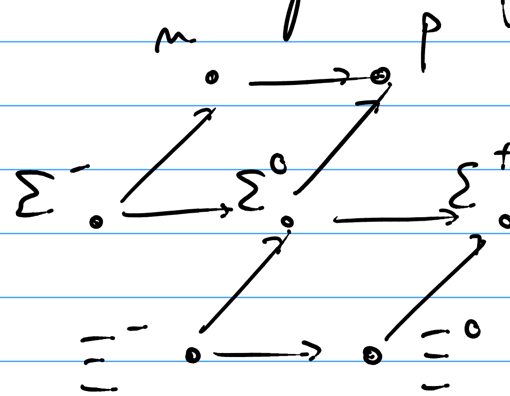
Cabibbo angle

$\hookrightarrow \sin\theta_c \approx 0.97$

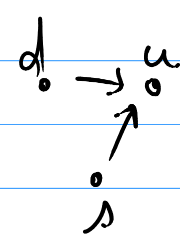
$\downarrow$

$J^{h+}_\mu = \bar{u}\, \gamma_\mu (1-\gamma^5)\, d'$

Observed decays through weak interaction

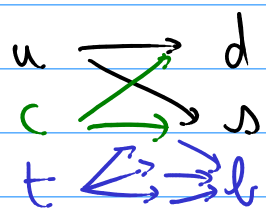


$\rightarrow$ Allowed quark decays



charged current reactions (CC)
can change flavor ($d \to u$, $s \to u$)
(W^+, W^- exchange)

neutral current reactions (NC)
can NOT change flavor (no $d \to s$)
(W^0, Z^0 exchange)



- With just u, d, s:

$$J_\mu^{h+} = \bar{u}\, \gamma_\mu (1-\gamma^5) (d \cos\theta_c + s \sin\theta_c)$$

- With u, d, c, s:

$$J_\mu^{h+} = \bar{u}\, \gamma_\mu (1-\gamma^5) \overbrace{(d \cos\theta_c + s \sin\theta_c)}^{d'}$$
$$+\ \bar{c}\, \gamma_\mu (1-\gamma^5) \underbrace{(-d \sin\theta_c + s \cos\theta_c)}_{s'}$$

- With u, d, c, s, t, b:

$$J_\mu^{h+} = (\bar{u}\ \ \bar{c}\ \ \bar{t})\, \gamma_\mu (1-\gamma^5) V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Unitarity of V_{CKM}:

- With just u, d, c, s: d' and s' must be orthogonal eigenstates for the weak interaction $\rightarrow \cos\theta_c, \sin\theta_c$ dependence

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

$$M(n \rightarrow p e^- \bar{\nu}_e) \; : \; M(\Sigma^- \rightarrow p e^- \bar{\nu}_e)$$

as $\quad \cos\theta_c \; : \; \sin\theta_c$

$$M(\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e) \; : \; 1$$

Or $|M(\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e)|^2 =$

$$|M(n \rightarrow p e^- \bar{\nu}_e)|^2 + |M(\Sigma^- \rightarrow p e^- \bar{\nu}_e)|^2$$

- With all quarks:

$$V_{ud} \gg V_{us} \gg V_{ub}$$

$$\sum_i V_{ik} = \sum_k V_{ik} = 1 \quad \text{(normalization)}$$

$$\sum_k V_{ik} V_{jk}^* = 0 \quad \text{(orthogonality)}$$

If $V_{ij} \neq V_{ij}^{*}$ → charge conjugation leads to $V^{*}_{CKM} \neq V_{CKM}$

and parity P does not cancel effect of C on $V_\mu - A_\mu$

↓

CP violation in $N \geq 3$ quarks per family

⇓

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \left(J^{l+}_{\mu} + J^{h+}_{\mu} \right) \left(J^{l\mu} + J^{h\mu} \right)$$

$$= \mathcal{L}_{ll} + \mathcal{L}_{hl} + \mathcal{L}_{hh}$$

leptonic: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

semileptonic: $n \to p e^- \bar{\nu}_e$, $K^+ \to \mu^+ \nu_\mu$

hadronic: $\Theta (K^+) \to \pi^+ \pi^0$, $\tau (K^+) \to \pi^+ \pi^+ \pi^0$ — parity violation

* Electroweak tests:

e.g. $\mu^- \to \nu_\mu e^- \bar{\nu}_e$ with

$$\mathcal{M} = -i \frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\mu (1-\gamma^5) \mu \; \bar{e} \gamma^\mu (1-\gamma^5) \nu_e$$

$$\to |\mathcal{M}|^2 \to d\Gamma \to \Gamma \to \tau = \frac{1}{\Gamma}$$

$$\tau^{-1} = \Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} \longrightarrow \text{measurement of } G_F$$

$$d\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} F(\text{kinematics})\, dE\, d\cos\theta$$

Allow for other terms in relevant Lagrangian,

$$\mathcal{L} = 4\frac{G_F}{\sqrt{2}}\Big(g^V_{LL}\, \bar{e}_L \gamma_\mu \nu_{eL}\, \bar{\nu}_{\mu L} \gamma^\mu \mu_L$$
$$+ g^V_{RR} \cdots + g^V_{LR} \cdots + g^V_{RL} \cdots$$
$$+ g^S_{LL}\, \bar{e}_L \nu_{eL} \bar{\nu}_{\mu L} \mu_L + \cdots$$
$$+ g^T_{LR}\, \bar{e}_L \frac{\sigma_{\mu\nu}}{\sqrt{2}} \nu_{eR}\, \bar{\nu}_{\mu L} \frac{\sigma^{\mu\nu}}{\sqrt{2}} \mu_R + \cdots \Big) + \text{h.c.}$$

$$\longrightarrow \quad d\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} F(\text{Michel parameters})\, dE\, d\cos\theta$$

* Back to Standard Model description:

$$\mathcal{L} = -\frac{g}{2\sqrt{2}}\left(J^\mu_W W^-_\mu + J^{\mu +}_W W^+_\mu \right)$$

$$-\frac{g g'}{\sqrt{g^2+g'^2}} J^\mu_Q A_\mu - \frac{\sqrt{g^2+g'^2}}{2} J^\mu_Z Z_\mu$$

with $$J^\mu_W = \sum_f \left(\bar{e}\gamma^\mu(1-\gamma^5)\nu + \bar{d}\gamma^\mu(1-\gamma^5)u \right)$$

$$J^\mu_Q = \sum_f q_f \left(\bar{e}\gamma^\mu \nu + \bar{d}\gamma^\mu u \right)$$

$$J^\mu_Z = \sum_f \Big(\bar{e}\gamma^\mu(g_V - \gamma_A\gamma^5)\nu$$
$$+ \bar{d}\gamma^\mu(g_V - g_A\gamma^5)u \Big)$$

with $\begin{cases} g_V = t_3 - 2\sin^2\theta_W \, q \\ g_A = t_3 \end{cases}$ $\Big\}$ weak isospin 3^{rd} component

At $Q^2 \ll M_W^2 \rightarrow \mathcal{L} = \frac{G_F}{\sqrt{2}} J^+_{W\mu} J^\mu_W$

with $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$

with $v = 246$ GeV

$$\mathcal{M} = \left(-i\frac{g}{2\sqrt{2}}\right)^2 J_W^{+\nu} \left(g_{\nu\mu} - \frac{q_\nu q_\mu}{M_W^2}\right)\left(\frac{-i}{q^2 - M_W^2}\right) J_W^\mu$$

$$M_W = \frac{1}{2} g v$$