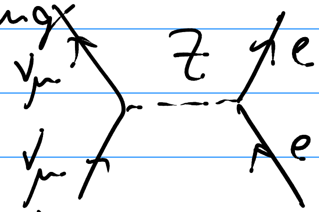
# Phys 772: Week 9 Tuesday

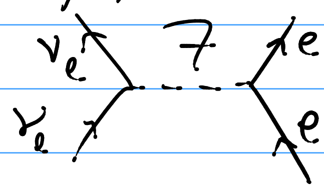
* Weak scattering versus electromagnetic scattering

1) Elastic scattering

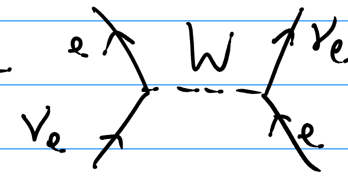
$\nu_\mu e^- \to \nu_\mu e^-$




$\nu_e e^- \to \nu_e e^-$




$+$




WNC: $\mathcal{L}_{NC}^{\nu_\mu e} = -\frac{G_F}{\sqrt{2}}\, \bar{\nu}_\mu \gamma^\mu (1-\gamma^5)\nu_\mu \;\; \bar{e}\gamma_\mu (g_V - g_A\gamma^5) e$

WCC: $\mathcal{L}_{CC}^{\nu_e e} = -\frac{G_F}{\sqrt{2}}\, \bar{\nu}_e \gamma^\mu (1-\gamma^5) e \;\; \bar{e}\gamma_\mu (1-\gamma^5)\nu_e$

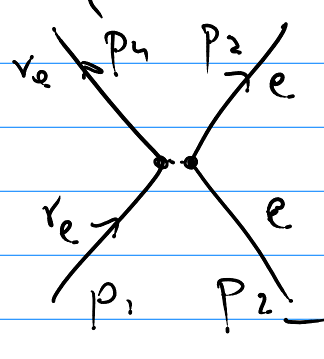
$\rightarrow$ sum equivalent to $\quad g_V \to g_V + 1$
$\qquad\qquad\qquad\qquad\quad\;\; g_A \to g_A + 1$

$$\begin{cases} g_V^{e\nu} = -\frac{1}{2} + 2\sin^2\theta_W \\ g_A^{e\nu} = -\frac{1}{2} \end{cases}$$

+ HOT in SM
+ BTSM contributions

Kinematic variable $y = \frac{p_1 \cdot p_4}{p_1 \cdot p_2} = \frac{T_e}{E_\nu}$ in lab frame

$\quad$ undetected




at rest

$= \frac{\text{kinetic energy final } e}{\text{total energy initial } \nu_e}$

$=$ fractional energy transfer to $e$

$= \frac{(s - m_e^2)(1+\cos\theta)}{2s}$

eDIS : $y = \frac{\nu}{E}$, $0 \leq y \leq \frac{1}{1+\frac{xM}{2k}}$ | $\nu$DIS : $0 \leq y \leq \frac{1}{1+\frac{m_e}{2E_\nu}}$

$y$ = Lorentz invariant, between 0 and ≈ 1 for $m_e \ll E_\nu$

$$\frac{d\sigma^{\nu e}}{dy} = \frac{g_F^2 m_e E_\nu}{2\pi}\left[(g_V+g_A)^2 + (g_V-g_A)^2(1-y)^2 - (g_V^2-g_A^2)\frac{m_e}{E_\nu}y\right]$$

$$\frac{d\sigma^{\bar\nu e}}{dy} = \frac{g_F^2 m_e E_\nu}{2\pi}\left[(g_V+g_A)^2(1-y)^2 + (g_V-g_A)^2 - (g_V^2-g_A^2)\frac{m_e}{E_\nu}y\right]$$

$\downarrow$

comparison of $\frac{d\sigma^{\nu_\mu e}}{dy}$ and $\frac{d\sigma^{\bar\nu_\mu e}}{dy}$ allows to determine $g_V$ and $g_A$.

2) Neutrino scattering from hadrons

$\nu N \to \{\nu N, \ell\}$ , $\nu N \to \{\nu N\pi, \ell\}$ , $\nu N \to \{\nu X, \ell\}$

elastic — resonance inelastic — deep inelastic

$\nu N \to \nu\Delta \to \nu N\pi$

→ consider deep inelastic scattering in parallel with electromagnetic deep inelastic scattering

isoscalar targets with $\#n = \#p$

WCC : $\nu_\mu N \to \mu^- X$ $\quad$ $\bar\nu_\mu N \to \mu^+ X$

WNC : $\nu_\mu N \to \nu_\mu X$ $\quad$ $\bar\nu_\mu N \to \bar\nu_\mu X$

* Deep inelastic neutrino scattering, WCC: $\nu N \to \ell X$

$$\frac{d\sigma^{\nu DIS}}{dk' d\Omega} = \frac{G_F^2}{32\pi^2} \frac{k'}{k} L_\nu^{\mu\nu} W_{\mu\nu}^\nu$$

→ no $\frac{1}{M_Z^2}$ for $\gamma$ exchange → $\left(\frac{1}{q^2}\right)^2$ dep.

cf. $$\frac{d\sigma^{eDIS}}{dk' d\Omega} = \frac{\alpha^2}{q^4} \frac{k'}{k} L_e^{\mu\nu} W_{\mu\nu}^e$$

Leptonic tensor: $L_e^{\mu\nu} = 2\left[k^\mu k'^\nu + k'^\mu k^\nu + g^{\mu\nu}\frac{q^2}{2}\right]$

$\nu$DIS: $L_\nu^{\mu\nu} = 8\left[k^\mu k'^\nu + k'^\mu k^\nu + g^{\mu\nu}\frac{q^2}{2} + \underbrace{i\varepsilon^{\mu\nu\rho\sigma} k'_\rho k_\sigma}_{\text{from } Tr[\gamma^\mu \gamma^5 \not{k} \gamma^\nu \gamma^5 \not{k}']}\right]$

↓ due to $\gamma^5$ PV nature

Hadronic tensor: $W_{\mu\nu}^e = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1^e(Q^2,\nu)$

$$+ \frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) W_2^e$$

$\nu$DIS: $W_{\mu\nu}^\nu = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1^\nu(Q^2,\nu)$

$$+ \frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) W_2^\nu(Q^2,\nu)$$

$$+ i\varepsilon_{\mu\nu\rho\sigma} \frac{p^\rho q^\sigma}{2M^2} W_3^\nu(Q^2,\nu)$$

PV due to V-A nature

Introduce $\begin{cases} F_1(x,Q^2) = M W_1(Q^2,\nu) \\ F_{2,3}(x,Q^2) = \nu W_{2,3}(Q^2,\nu) \end{cases}$

Scaling arguments indicated that $F_{1,2,3}(x,Q^2) \to F_{1,2,3}(x)$

Change from $dk'\,d\Omega = 2\pi\,dk'\,d\cos\theta = 2\pi M \frac{k}{k'}\,y\,dx\,dy$

$$x = \frac{kk'(1-\cos\theta)}{M\nu}$$

$$y = \frac{k-k'}{k} = \frac{\nu}{k'}$$

eDIS : $$\frac{d\sigma^e}{dx\,dy} = \frac{8\pi\alpha^2 Mk}{Q^4}\left(F_2^e(x)\left(1-y-\frac{Mxy}{2k}\right) + 2xF_1^e(x)\frac{y^2}{2}\right)$$

primary dependence on kinematics in $\sigma$ total

$\nu$DIS : $$\frac{d\sigma^{\overset{(-)}{\nu}}}{dx\,dy} = \frac{G_F^2 M E_\nu}{\pi}\left(F_2^\nu(x)(1-y) + 2xF_1^\nu(x)\frac{y^2}{2} \pm F_3^\nu(x)\cdot xy\left(1-\frac{y}{2}\right)\right)$$

Previously: $F_2^e(x) = 2xF_1^e(x) = \sum_i e_i^2\, x q_i(x)$

$\hookrightarrow$ $F_2^\nu(x) = 2xF_1^\nu(x) = \sum_i q_W^2\, x q_i(x)$ depends on convention $\to 1$

For u,d only: $= 2x\left(d(x) + \bar{u}(x)\right)$
$F_3^\nu(x) = 2x\left(d(x) - \bar{u}(x)\right)$ } for $\nu$ ($u \leftrightarrow d$ for $\bar{\nu}$)

sign change due to A terms

$\Downarrow$ allows for distinguishing $u, \bar{u}, d, \bar{d}$

For u,d,c,s : need to use flavor mixing:
$d \to d'$, $s \to s'$

$$F_2^{\nu}(x) = 2x\Big( \overset{d\to u}{d(x)\,(|V_{ud}|^2} + \overset{d\to c}{|V_{cd}|^2)} \simeq 1$$

$$+ \underset{s\to u}{s(x)\,(|V_{us}|^2} + \underset{s\to c}{|V_{cs}|^2)} \simeq 1$$

$$\underset{\bar{u}\to \bar{d}/\bar{s}}{\pm\, \bar{u}(x)} \underset{\bar{c}\to \bar{d}/\bar{s}}{\pm\, \bar{c}(x)} \Big) \quad \text{for } Q^2 \gg m_c^2$$

$\rightarrow$ determination needs $V_{ud}$, $V_{cd}$, $V_{us}$, $V_{cs}$

$$\rightarrow \frac{d\sigma^{\nu}}{dx\,dy} = \frac{2G_F^2 M E_\nu}{\pi}\left[ x(d+s) + x(\bar{u}+\bar{c})(1-y^2) \right]$$

$$\frac{d\sigma^{\bar{\nu}}}{dx\,dy} = \frac{2G_F^2 M E_\nu}{\pi}\left[ x(u+c)(1-y^2) + x(\bar{d}+\bar{s}) \right]$$

Combination of eDIS and $\nu$DIS on "isoscalar" target

$$\frac{F_{2N}}{F_{2N}^{\nu}} = \frac{\frac{1}{2}(F_{2n}^{e} + F_{2p}^{e})}{\frac{1}{2}(F_{2n}^{\nu} + F_{2p}^{e})} = \frac{\left(\frac{4}{9}+\frac{1}{9}\right)\left(\frac{1}{2}x(u+d+\bar{u}+\bar{d})\right)}{x(u+d+\bar{u}+\bar{d})}$$

$$= \frac{5}{18}$$ determined solely by the fractional quark charges

$*$ Deep inelastic neutrino scattering, WNC : $\nu N \to \nu X$

$$\mathcal{L}_{NC}^{\nu N} = -\frac{G_F}{\sqrt{2}}\, \bar{\nu}\gamma^{\mu}(1-\gamma^5)\nu$$

$$\times \Big( \sum_i \varepsilon_{Li}^{\nu N}\, \bar{q}_i \gamma_\mu \overbrace{(1-\gamma^5)}^{P_L} q_i + \varepsilon_{Ri}^{\nu N}\, \bar{q}_i \gamma_\mu \overbrace{(1+\gamma^5)}^{P_R} q_i \Big)$$

$\varepsilon_L^{\nu N}$ and $\varepsilon_R^{\nu N}$ are L and R weak couplings

$$\varepsilon_L^{\nu N} = \frac{1}{2}\left(g_V^{\nu N} + g_A^{\nu N}\right) = +\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \quad \Big| \quad -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$$

$$\varepsilon_R^{\nu N} = \frac{1}{2}\left(g_V^{\nu N} - g_A^{\nu N}\right) = -\frac{2}{3}\sin^2\theta_W \quad \Big| \quad +\frac{1}{3}\sin^2\theta_W$$

u-type, c | s, d type

$$\Rightarrow \frac{d\sigma}{dx\,dy} = \frac{2G_F^2 M E_\nu}{\pi}\Big[\left(|\varepsilon_L(u)|^2 + |\varepsilon_R(u)|^2(1-y^2)\right)\left(xu(x) + x\,c(x)\right)$$

$$+ \quad (u \to d) \quad \left(x\,d(x) + x\,s(x)\right)$$

$$+ \quad (L \leftrightarrow R) \quad \left(x\,\bar{u}(x) + x\,\bar{c}(x)\right)$$

$$+ \quad (u \to d) \quad \left(x\,\bar{d}(x) + x\,\bar{s}(x)\right)\Big]$$

Evaluate integrals in $\int_0^1 (1-y)^2 dy = \frac{1}{3}$

$$q(x) = \frac{u+d}{2} \qquad \bar{q}(x) = \frac{\bar{u}+\bar{d}}{2}$$

$$\sigma_{cc}^{\nu} = \frac{2G_F^2 M E_\nu}{\pi}\int_0^1 dx \left(x\,q(x) + \frac{1}{3}x\,\bar{q}(x)\right)$$

$$\sigma_{cc}^{\bar{\nu}} = \left(\frac{1}{3}x\,q(x) + x\,\bar{q}(x)\right)$$

$$\sigma_{nc}^{\nu} = g_L^2(\quad) + g_R^2(\quad)$$

$$\sigma_{nc}^{\bar{\nu}} = g_R^2(\quad) + g_L^2(\quad)$$

→ ratios cancel out unknown $E_\nu$ effects etc

$$\begin{cases} g_L^2 = |\varepsilon_L(u)|^2 + |\varepsilon_L(d)|^2 = \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W \\ g_R^2 = \qquad R \qquad\qquad R \qquad = \frac{5}{9}\sin^4\theta_W \end{cases}$$

$$\rightarrow \begin{cases} R_\nu = \frac{\sigma_{nc}^\nu}{\sigma_{cc}^\nu} = g_L^2 + g_R^2 \\ R_{\bar\nu} = \frac{\sigma_{nc}^{\bar\nu}}{\sigma_{cc}^{\bar\nu}} = g_L^2 + \frac{1}{r} g_R^2 \end{cases}$$

with $r = \frac{\sigma_{cc}^\nu}{\sigma_{cc}^{\bar\nu}} = \frac{\frac{1}{3} + \varepsilon}{1 + \frac{\varepsilon}{3}}$ , $\varepsilon = \frac{\int_0^1 dx\, x\bar q(x)\, dx}{\int_0^1 dx\, x\, q(x)\, dx}$

$\downarrow$

$r$ can be determined from $R_\nu, R_{\bar\nu}$

$\downarrow$

$r = 44\%$ , $\varepsilon = 12.5\%$ of momentum carried by anti-quarks

Other ratio : Paschos-Wolfenstein ratio for $N = Z$

$$R^- = \frac{\sigma_{nc}^\nu - \sigma_{nc}^{\bar\nu}}{\sigma_{cc}^\nu - \sigma_{cc}^{\bar\nu}} = g_L^2 - g_R^2 = \frac{1}{2} - \sin^2\theta_W$$

$\rightarrow$ NuTeV anomaly : $\sin^2\theta_W$ $3\sigma$ high compared to precision Z-pole LEP/SLAC data

$\downarrow$

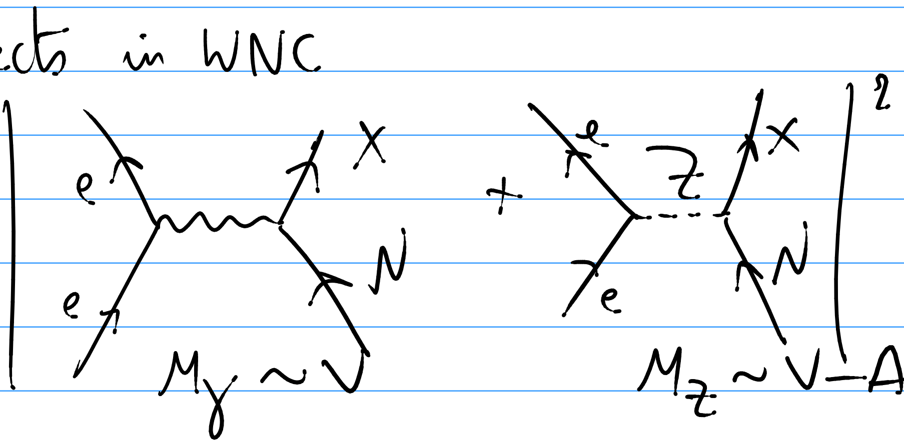
possible explanation (Ian Cloët) due to charge symmetry violation of $m_u \neq m_d$

$\hookrightarrow$ $u_p(x) \neq d_n(x)$

$\hookrightarrow$ explain anomaly

* Interference effects in WNC

$eN \to eX$




$\hookrightarrow$ interference term: $M_\gamma^* M_Z \sim V(V-A)$

$\downarrow$

$\left.\begin{matrix} VA \\ \gamma^\mu \gamma_\mu \gamma^5 \end{matrix}\right\}$ effects

$$\mathcal{L}_I^{eN} = \frac{G_F}{\sqrt{2}} \Big( C_{1q}\, \bar{e} \gamma_\mu \gamma^5 e \cdot \bar{q} \gamma^\mu q$$
$$+ C_{2q}\, \bar{e} \gamma_\mu e \cdot \bar{q} \gamma^\mu \gamma^5 q \Big)$$

$C_{1q} = 2 g_A^e g_V^q$ = weak vector charge of quark

$C_{2q} = 2 g_V^e g_A^q$ = weak axial charge of quark

Asymmetry $A_{PV} = \dfrac{M_\gamma^* M_Z}{|M_\gamma|^2} \sim \dfrac{M_Z}{M_\gamma} \sim \dfrac{\frac{1}{M_Z^2}}{\frac{1}{Q^2}}$

$$A_{PV} = Q^2 \left[ a_1 + a_2 \left( \frac{1-(1-y)^2}{1+(1-y)^2} \right) \right]$$

$a_1 \sim \left( C_{1u} - \frac{1}{2} C_{1d} \right) = \left( -\frac{3}{4} + \frac{5}{3} \sin^2 \theta_W \right)$

$a_2 \sim \left( C_{2u} - \frac{1}{2} C_{2d} \right) = \left( \sin^2 \theta_W - \frac{1}{4} \right)$