Phys 772: Week 13 Tuesday

* Standard Model: $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\mathcal{L}_{SM} = -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ \frac{1}{2} (\partial_\mu H)^2 + \frac{\mu^4}{4\lambda} + \mu^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4$$
$$+ M_W^2 W^+_\mu W^{-\mu} \left(1 + \frac{H}{v}\right)^2 + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \left(1 + \frac{H}{v}\right)^2$$
$$+ \sum_r \bar{\psi}_r \left[i\not{\partial} - m_r \left(1 + \frac{H}{v}\right)\right] \psi_r$$
$$- \frac{g}{2\sqrt{2}} \left(J^\mu_W W^-_\mu + J^{\mu +}_W W^+_\mu \right) - \frac{g}{2\cos\theta_W} J^\mu_Z Z_\mu$$
$$- e J^\mu_Q A_\mu$$

$\rightarrow$ renormalizable theory (Higgs mechanism)

SM agrees with all experimental observations

But: considered to be an effective low-energy theory that has an underlying, more fundamental description.

- Parameter counting:

12 fermion Yukawa couplings → masses
  6 q, 3 l, 3 ν
6 mixing angles
  3 in V_{CKM}, 3 in V_{PMNS}
2 CP-violating phases
  1 in V_{CKM}, 1 in V_{PMNS}
  2 Majorana phases in V_{PMNS}
3 coupling constants:
  g_s, g, g'
2 Higgs potential parameters
  $\mu, \lambda \to M_H, v$
1 θ_{QCD}
2 M_{Planck}, cosmological constant ("$\frac{\mu}{4\lambda}$"-like term)

28 parameters

+ assignments of Y, T_3 for all fermions

+ 3 generations

⇒ seems like a lot of freedom to choose parameters, with "periodic table" like repetition

- Gauge group structure

$$SU(3)_c \times \underbrace{SU(2)_L \times U(1)_Y}$$

g, g' combine to form electric charge coupling, e

→ quantization of $e/3$ seems arbitrary without an underlying group that breaks into $SU(2)_L \times U(1)$

→ some underlying symmetry group would explain connection between quark electric charges and lepton electric charges

Some arguments are anthropic: if the charges were different, atoms would not be neutral and we would not be here to ask the question

→ most physicists still prefer a more minimalist explanation involving a simpler theory

- Fermion mass hierarchy:

$m_e = 10^{-5} m_t$ and $m_{u\text{-type}} > m_{d\text{-type}}$

↳ large range of mass scales for fermions, even larger if m_ν included (10^{-11})

⇒ are the generations of fermions consecutive excitations or power terms of underlying physics?

Similar to seesaw to suppress $m_D \approx m_e$ to $m_\nu = 10^{-6} m_D = m_D \frac{m_D}{m_S}$

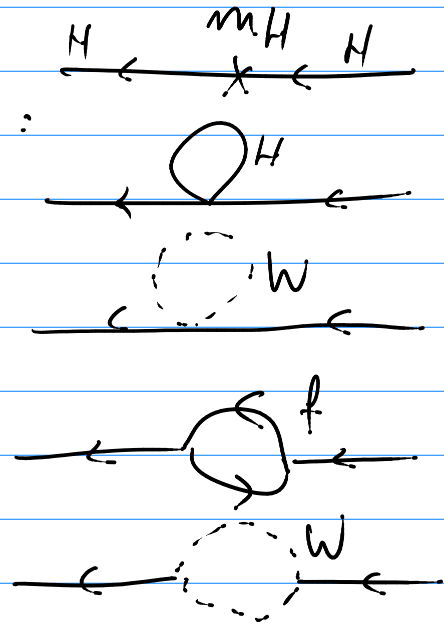
E.g. $m_e \propto \left(\frac{m'}{m_S}\right)^3$, $m_\mu \propto \left(\frac{m'}{m_S}\right)^2$, $m_\tau \propto \left(\frac{m'}{m_S}\right)^1$

- Higgs mass hierarchy:

$$\left.\begin{aligned} m_H &= \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v \\ m_W &= \frac{g}{2} v \end{aligned}\right\}$$ both H and W(Z) must have masses that are of $O(1)\cdot v$

Theoretical arguments limit m_H to band between $110 \text{ GeV} < m_H < 700 \text{ GeV}$

However,
loop diagrams:



$+ \quad \lambda$

$+ \quad g^2$

$+ \quad h_f^2$

$+ \quad g^2$

$$\rightarrow \quad m_H^2 = (m_H)_0^2 + \mathcal{O}(\lambda, g^2, h_f^2)\, \Lambda^2$$

where Λ = ultraviolet cutoff
and $\mathcal{O}(\lambda, g^2, h_f^2) \approx \mathcal{O}(1)$

If UV-cutoff is at M_{Planck} $\rightarrow$ $m_H \approx M_{Planck}$

$\Rightarrow$ must explain why $\underbrace{\mathcal{O}(\lambda, g^2, h_f^2)}_{= 10^{-17}} \ll 1$

The symmetries of the underlying theory can cause $\mathcal{O}(\lambda, g^2, h_f^2, \ldots) = 0$ exactly, such that m_H is determined by finite effects

$\hookrightarrow$ supersymmetry

Or m_H could be receiving a zero mass as result of SSB, similar to how $m_\pi \approx 0$ in QCD due to L/R chiral symmetry breaking

- Strong CP problem:

$SU(2)_L$ breaks CP explicitly → phases in V_{CKM}
$\downarrow$ V_{PMNS}

nature does not mind a bit of CP violation

In $SU(3)_c$ sector → $\theta_{QCD}\, G^i_{\mu\nu} \tilde{G}^{\mu\nu}_i$

but $\theta_{QCD} < 10^{-11}$ instead of natural size $O(1)$

⟹ what suppresses θ_{QCD} to such small values?

Possible explanations: - θ_{QCD} turns out to be unobservable for some reason (phases can be rotated away) → $\theta_{QCD} = O(1)$ but zero in experiments

- Or there is an underlying symmetry group $U(1)$ which undergoes spontaneous symmetry breaking and θ_{QCD} acts as first order derivative at the equilibrium → zero by definition

- Absence of gravity in Standard Model
→ general relativity is a classical (non quantum) field theory → needs quantum gravity, string theory, or similar

Related: cosmological constant, Higgs field potential energy expectation value at the equilibrium:

$$\langle 0 | V(\varphi) | 0 \rangle = -\frac{\mu^4}{4\lambda}$$

generates cosmological constant due to gravitational pull from energy density:

$$\Lambda_{cosm} = \Lambda_{SSB} = 8\pi G_N |\langle 0 | V(\varphi) | 0 \rangle|$$

but this is 10^{56} times larger than the observed cosmological constant, and has the wrong sign as well!

↳ again fine-tuning, at $O(10^{56})$

- Dark matter, dark energy

rotation of galaxies
→ requires $O(\text{TeV})$ particles

energy that contributes $\frac{3}{4}$ of universe
→ defines why the expansion is accelerating

* Possible solutions to these problems:

- neutrino masses $\leftarrow$ seesaw mechanism
- baryon asymmetry, large CP violation
  $\rightarrow$ sterile neutrino decays, initial conditions
- dark matter could be lightest supersymmetric particle, which can't decay into SM particles (neutralino)
- higher gauge groups: $SU(5)$ (cf. $SU(3)$ includes $SU(2)$)