CS 533 (Spring 2019)
Assignment 02: *Implementing the Nagel-Schrekenberg Traffic Model*

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(*also see accompanying MATLAB .mlx files titled Craft_CS533_Assignment_02.mlx*)

Introduction

This document summarizes the principal (example) results for Assignment 1, which asked students to complete Exercise 1.1 from Owen [3], implementing a form of Nagel & Schrekenberg's (1994) traffic model [2]. Code and computations were completed using MATLAB [1]; see the accompanying MATLAB file `Craft_CS533_Assignment02.mlx` for computational details, algorithmic code, and original figures.

For this implementation of the Nagel-Schrekenberg traffic model, we used a (circular) track having M = 1,000 "slots" (*i.e.*, possible locations for the cars), and burn-in period B of 1,000 time steps, $v_{max} = 5$, and $p = 1/3$ for the braking function. Car positions and velocities were updated synchronously from one time step to the next using the following step-by-step algorithm (where v indicates velocity, x indicates position, and d indicates the distance from one car to the next, all in "slot" units) [Owen, pg 4]:

$$v \leftarrow min\ (v + 1, v_{max}); \qquad [1]$$

$$v \leftarrow min\ (v, d - 1); \qquad [2]$$

$$v \leftarrow max\ (0, v - 1),\ \textit{with probability } p \qquad [3]$$

$$x \leftarrow x + v \qquad [4]$$

This ordering of updates for each car prevents collisions, but produces interesting traffic-jam behavior as the cars become more densely distributed across the M possible locations on the roadway.

(*a*) Owen (2013), Exercise 1.1, Part (*a*)

Using M = 1000 slots, a burn-in period of B = 1000 time steps, $v_{max} = 5$, and $p = 1/3$, we produce a flow trace image for k = 50 cars and 1000 time steps post-burn-in period as shown in *Figure* 1 below. Dots along each row indicate car positions for that time step.

The total travel distance for all k = 50 cars combined was 232,399 units (which will vary a bit from replication to replication, due to the probabilistic nature of the braking function).

The traces for three specific but arbitrary cars are shown in red, green, and blue, to help make the trace of some cars more obvious. At this density of cars, we see mostly parallel traces, with the distances between cars varying very little and no significant traffic jams occurring. At this density of cars, there is plenty of room between cars to allow the probabilistic braking behavior (equation [3]) without generally interfering with the car(s) following behind.

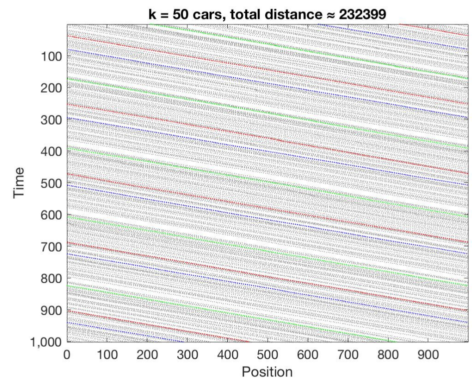

Figure 1. Flow trace image for k = 50 cars for 1,000 time steps post-burn-in period. Red, green, and blue dots are three specificbut arbitrary cars color-coded to make their progression easier to trace. The density of cars is low enough to avoid any indications of traffic congestion.

(*b*) Owen (2013), Exercise 1.1, Part (*b*)

The same process used in Part (*a*) above was then used to compute the total distance traveled for all k cars, for k = 55, 60, … 500. The resulting *fundamental diagram* [2], [3] is shown in *Figure* 2 below, plotting total distance traveled *vs.* number of cars k.

There we see a peak flow (as measured by total distance traveled across all cars) corresponding to approximately k = 115 cars. The structure of this fundamental diagram is very similar to the diagram presented in [2], and our peak flow is similar to the ρ = 0.08 reported in [2] (which would correspond to k = 80 in our simulation).

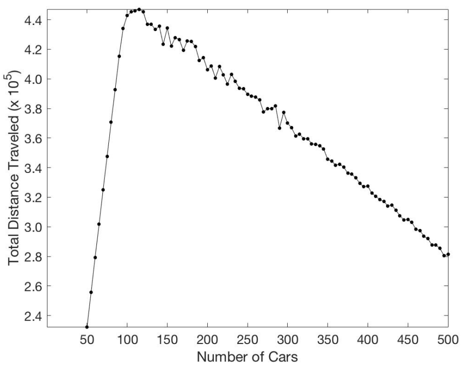

Figure 2. A fundamental diagram [3], plotting total distance traveled for all *k* cars, for each value of *k* = 50, 60, 70 … 500. Peak traffic flow occurs at approximately *k* = 115 cars.The shape of this function is very similar to that reported in [2], and the peak is very similar to their peak at a density rho = 0.08 which would be equivalent to *k* = 80 in our simulation.

Apparently, while the traffic density is small enough, we see overall flow increasing as we increase the number of cars, but as we approach $k \approx 115$ or so (corresponding to $\rho \approx 0.12$), the cars become densely enough distributed to begin causing traffic delays, after which point increasing the number of cars introduces further delays and a degradation in the overall distance traveled. There are more and more cars, but each car cannot travel as fast, and thus not as far, as the cars could when there were fewer cars.

We can see this effect when we compare the flow trace images for *k* = 100 (just as we approach the peak flow), *k* = 115 (at peak flow), and *k* = 150 (well after the peak flow). These trace images are shown in *Figure* 3 below. Mild traffic jams appear even in the *k* = 100 case and continue to accumulate for the *k* =115, and we see more dramatic increases as we increase *k* beyond 115 to *k* = 150 and beyond. For *k* = 100, we see a few short-lived traffic back-ups, lasting 100–300 time steps or so. These are just barely large enough that some cars encounter the same traffic jam twice in their trip around the track.

By *k* = 115, we also see some small traffic jams but now also a large traffic jam lasting through the entire simulation. All three of the example cars hit this same traffic jam four different times. And then by *k* = 150 we see multiple long-lasting traffic jams, many occurring simultaneously across the 1000-slot roadway.

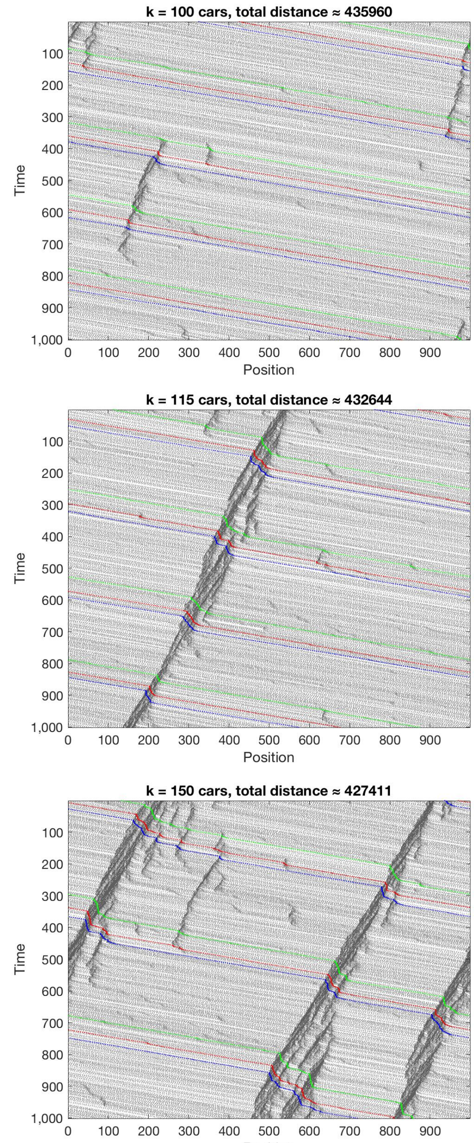

Figure 3. Trace flow images for the Nagel-Schrekenberg traffic simulation for *k* = 100, 115, and 150 cars (top, middle, and bottom panel, respectively). Red, blue, and green traces show trajectories or three particular but arbitrary cars, to help illustrate individual car behavior. The traffic "jams" increase considerably in number and severity as we increase *k* past the peak flow value in the *k* = 100–120 region.

(*c*) Owen (2013), Exercise 1.1, Part (*c*)

The same process used in Part (*b*) above was then used to compute a collection of 5 fundamental diagrams, all presented on the same graph in different colors in *Figure* 4 below. Notice the close agreement in values for *k* values ranging from *k* = 50 to *k* = 90, and increased variability evident in the region for *k* = 100–200, where the probabilistic braking behavior is having quite varied effects, and then the decreasing variability thereafter as k continues to increase. As k continues to increase, the contraints on the individual cars from all the other cars on the road forces more and more similar overall distance-travel behavior at any particular k value.

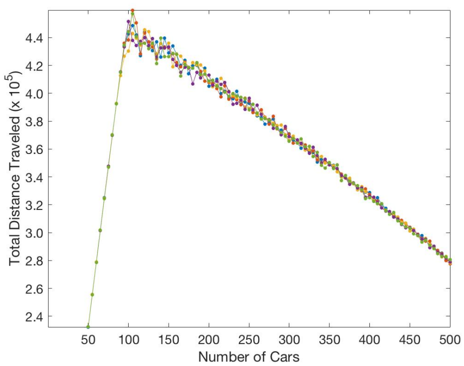

Figure 4. Five replications of a fundamental diagram [Owen], plotting total distance traveled for all k cars, for each value of $k = 50, 60, 70 \ldots 500$. Here the peak traffic flow occurs at approximately $k = 105$ cars.Variability is quite low for low-density car configurations ($k \approx 50$–90), then increases dramatically as we reach peak flow. Variability later decreases considerably again as the car density strongly constrains the total distance outcomes.

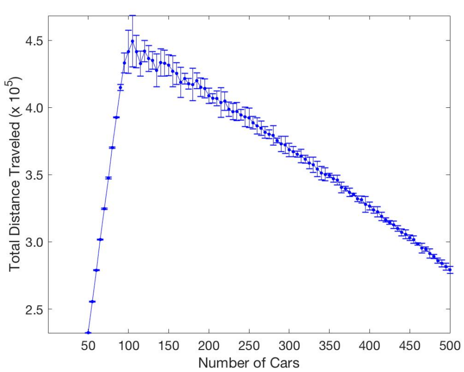

Figure 5. The mean distances traveled for the 5 fundamental diagram replications shown in *Figure* 4. the peak flow appears to occur for $k \approx 105$, but could really be anywhere in the $k \approx 90$–150 range.

In *Figure* 5 above we plot the mean total distances from *Figure* 4, along with 99% confidence intervals based on the t distribution ($df = 4$ for each point). The results are consistent with our earlier description: at around $k \approx 100$, the car density is sufficient to begin producing degradations in the overall distance accumulated due to the probabilistic braking behavior of cars; after the peak flow region, the cars have decreasing space to avoid being negatively impacted by the braking behavior of their neighbors.

(*d*) Owen (2013), Exercise 1.1, Part (*d*)

The same process used in Parts (b) and (c) above was used to compute another collection of 5 fundamental diagrams, for the number of cars $k = 50, 55, 60, \ldots 500$ as before, but with the stipulation that the initial positions (before the "burn-in" period) of the k cars each time are at $1, 2, 3, \ldots k$. In other words, instead of being roughly equally distributed across the entire length of the road as before, each set of cars is "piled up" at the beginning, much like a car race.

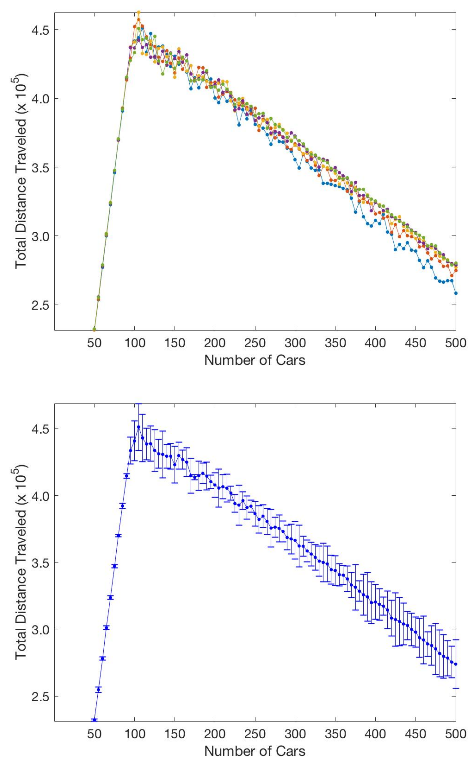

Figure 6. Five replications of a fundamental diagram (top panel) and the resulting means and 99% confidence intervals (bottom panel), when the initial k cars occupy positions $1, 2, 3, \ldots, k$ instead of being roughly equally distributed for initial positions. The overall shape of the graphs is the same as those in part (c) above, but now we see considerable variability remaining for the high k values.

The individual fundamental diagrams and the means with 99% confidence intervals are shown above in *Figure* 6. The final values and graph shapes are basically the same as those seen in part (c) earlier, but now the variability remains quite high for the larger values of *k*. This is understandable. For small values of *k*, the 1000-iteration burn-in period is more than sufficient to get the cars spread out along the road. With high values of *k*, though, even 1,000 iterations is insufficient to overcome the initial cramming-together of the cars. For example, when $k = 500$, only the 500th car gets to move anywhere in the 1st iteration; the 499th and 500th cars get to move during the 2nd iteration, etc. Only by the 500th iteration does the 1st car get to go anywhere (probably one unit forward). Thus after 1000 iterations we still have considerable piling up of the cars and considerable variable results based on each car's negative response to the probabilistic braking of the car in front of it.

The flow trace image of the 1000 time steps *after* the 1000-step burn-in period is shown below in *Figure* 7. Notice that even at the beginning, we still have a massive traffic jam covering about half the road space.

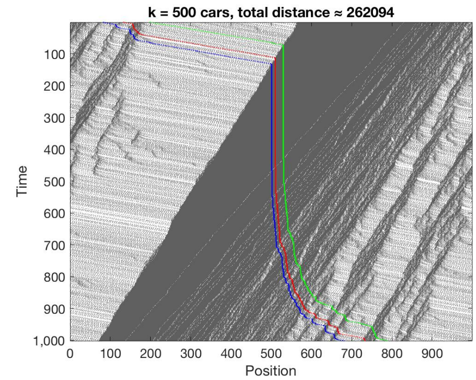

Figure 7. Flow trace diagram for $k = 500$ cars, starting *after* a 1,000-time step burin period but before the burn-in period using initial car positions 1, 2, 3, … 500. Although a few cars have escaped the traffic, a massive traffic jam remains in place throughout the remaining simulation time. We see how the marked cars rejoin the traffic jam around time step ≈ 100, then basically sit still for 400–500 time steps before gradually accelerating.

Summary & Conclusions

This has been a brief exercise-based investigation of the Nagel-Schrekenberg traffic model [2]. We were able to replicate the basic results (in both flow trace diagram format and fundamental diagram format) of Nagel & Schrekenberg's original paper [2], finding that inverted *V*-shaped function in which maximal traffic flow occurs for some intermediate value of the number of cars *k* experiencing a simple probabilistic braking behavior. It also turns out that the eventual shape and magnitude of the fundamental diagram is robust under the two options we explored for the initial positions of the cars.

References

[1] MATLAB. *version 9.0.5.9 (R2018b)*. The MathWorks Inc., Natick, Massachusetts, 2018.

[2] Kai Nagel & Michael Schreckenberg (1992). A cellular automaton model for freeway traffic. *Journal de Physique I, EDP Sciences*, 2(12), 2221–2229. <10.1051/jp1:1992277>. <jpa-00246697>

[3] Art B. Owen (2013). *Monte Carlo Theory, Methods, and Examples*. Pre–publisher version available online at http://statweb.stanford.edu/~owen/mc/