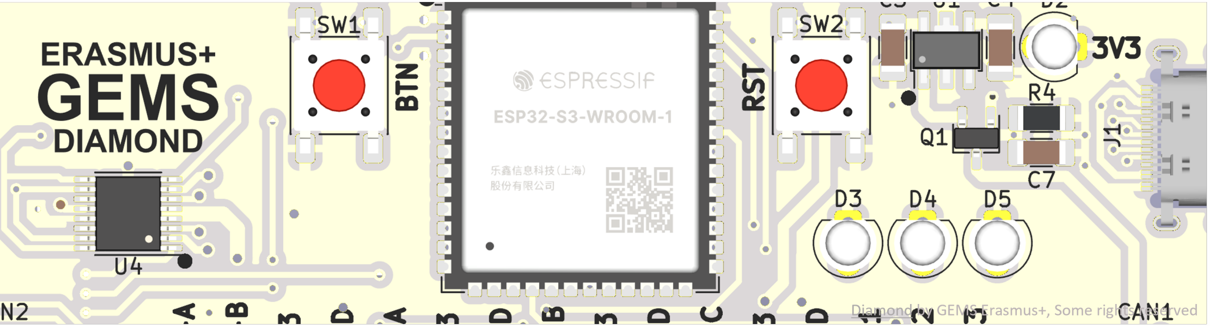

Tutorial 5

Implementation of Motor Model in a Microcontroller

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1. Motor model
2. Continuous to Discrete time domain conversion
3. Implementation in ESP32 microcontroller

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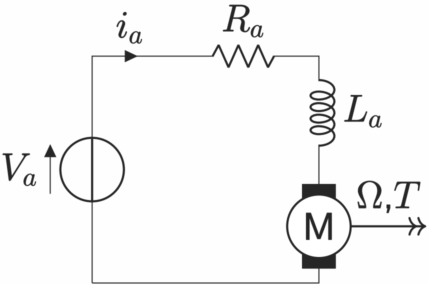
Motor model



$$\frac{\Omega(s)}{V_a(s)} = \frac{K}{(sL_a + R_a)(sJ + b) + K^2} \qquad (1)$$

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Conversion methods

Table: Continuous to discrete conversion methods [1].

Method	Use case/advantage
Zero-order hold	For staircase inputs
First-order hold	For piecewise linear inputs
Impulse-invariant mapping	For impulse train inputs
Tustin approximation	Frequency domain match, capture dynamics at particular frequencies
Zero-pole matching equivalents	SISO model, good frequency match
Least-squares	SISO model, good frequency match, capture fast dynamics

Tustin approximation

$$z = e^{sT_s} \tag{2}$$

$$= \frac{e^{\frac{sT_s}{2}}}{e^{\frac{-sT_s}{2}}} \tag{3}$$

$$\approx \frac{1 + sT_s/2}{1 - sT_s/2} \tag{4}$$

where, T_s is the sample time of the controller. Inverting, we get

$$s = \frac{2}{T_s}\frac{1 - z^{-1}}{1 + z^{-1}} \tag{5}$$

Motor model in discrete time domain

Substituting (5) in (1), we get:

$$\frac{\Omega(z)}{V_a(z)} = KT_s^2 \frac{1 + 2z^{-1} + z^{-2}}{a + bz^{-1} + cz^{-2}} \tag{6}$$

where,

$$\begin{aligned}
a =& (K^2 + bR_a)T_s^2 + 2T_s(JR_a + bL_a) + 4JL_a \\
b =& 2(K^2 + bR_a)T_s^2 - 4JL_a \\
c =& (K^2 + bR_a)T_s^2 - 2T_s(JR_a + bL_a) + 4JL_a
\end{aligned}$$

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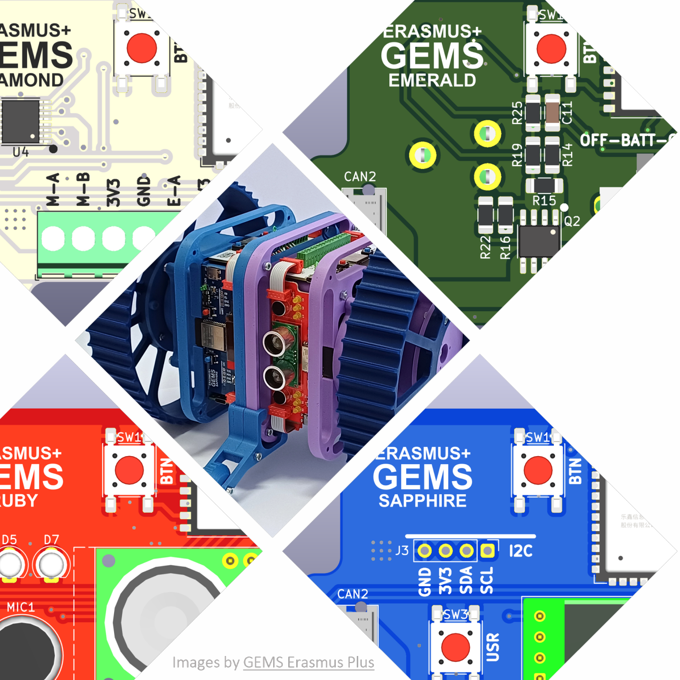
Difference equation

To implement the model in a microcontroller, a difference equation can be formed with z^{-1} as the one-sample delay operator.

$$\begin{aligned}\Omega[n] =& a_1 V_a[n] + a_2 V_a[n-1] + a_3 V_a[n-2] \\ & - b_1 \Omega[n-1] - b_2 \Omega[n-2]\end{aligned}$$

Conclusion

- Review of motor model.
- Review of voltage to speed transfer function.
- Continuous time domain to discrete time domain conversion.
- Creation of the difference equation.
- Implementation of difference equation into an ESP32 microcontroller.



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References I

[1] Mathworks. *Continuous-Discrete Conversion Methods*. Oct. 2025. URL: https://www.mathworks.com/help/control/ug/continuous-discrete-conversion-methods.html.