DC drive close-loop control

How to design and tune a DC drive close-loop controller?

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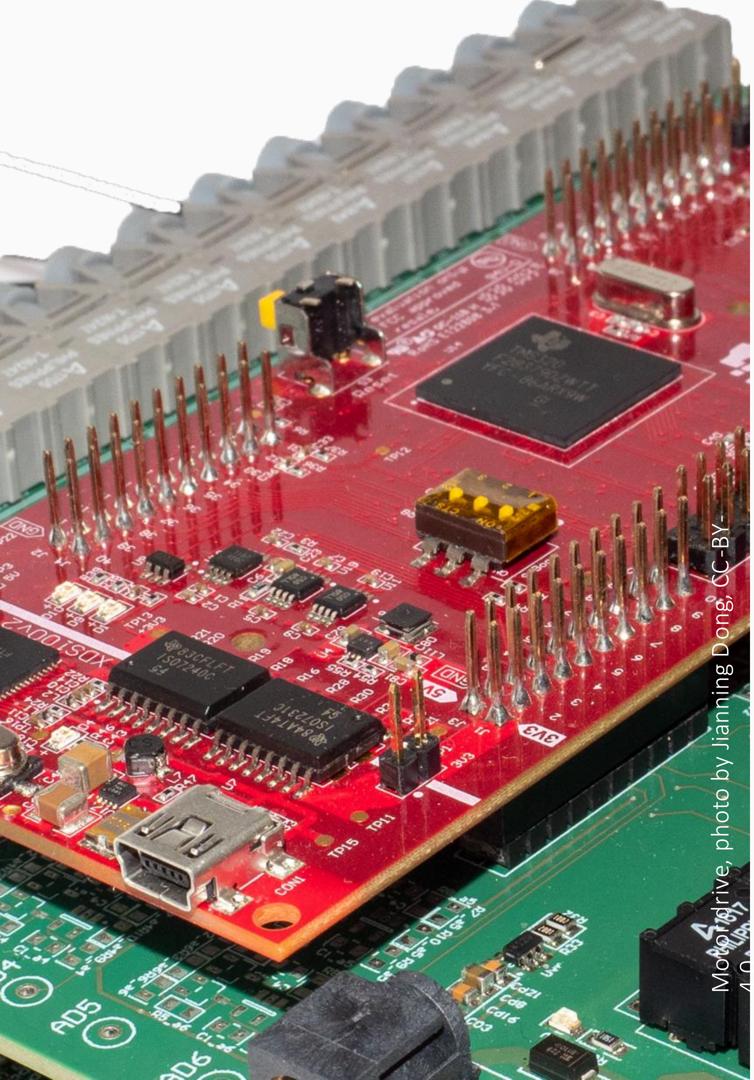
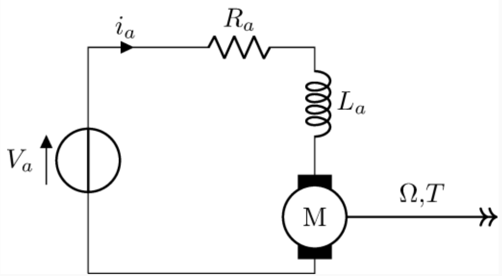


Lecture Outline

1. DC drive system dynamics
2. PI control for current control
3. Cascaded speed/position control
4. Discrete controller implementation

How to describe the dynamics of a DC drive system?

DC drive system dynamic models: electrical dynamics



V_a: armature voltage
R_a: armature resistance
L_a: armature inductance
i_a: armature current
e_b: back electromotive force (emf)

$$\begin{aligned} v_a &= R_a i_a + L_a \frac{\mathrm{d}i_a}{\mathrm{d}t} + e_b \\ &= R_a i_a + L_a \frac{\mathrm{d}i_a}{\mathrm{d}t} + K\Omega \end{aligned}$$

$$e_b = K\Omega$$

K: motor constant
Ω: angular velocity

Taking Laplace transformation

$$V_a(s) = (R_a + sL_a)I_a(s) + K\Omega(s)$$

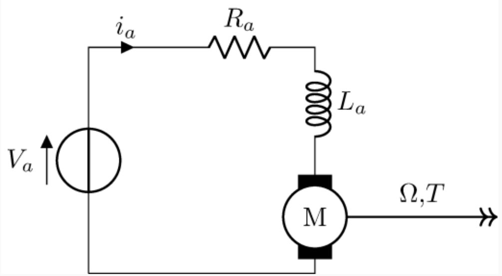
Now solve for $I_a(s)$:

$$I_a(s) = G_e(s)V_a(s) - G_e(s)K\Omega(s)$$

where

$$G_e(s) = \frac{1}{R_a + sL_a}$$

DC drive system dynamic models: mechanical dynamics



$$T = J\frac{\mathrm{d}\Omega}{\mathrm{d}t} + b\Omega$$

T: torque
J: moment of inertia
b: damping coefficient

Taking Laplace transformation

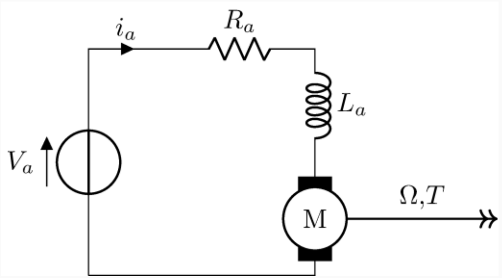
$$T(s) = sJ\Omega(s) + b\Omega(s)$$

Convert to transfer function

$$\frac{\Omega(s)}{T(s)} = \frac{1}{sJ + b} = G_m(s)$$

V_a: armature voltage
R_a: armature resistance
L_a: armature inductance
i_a: armature current
e_b: back electromotive force (emf)

DC drive system dynamic models: electromechanical model



V_a: armature voltage
R_a: armature resistance
L_a: armature inductance
i_a: armature current
e_b: back electromotive force (emf)

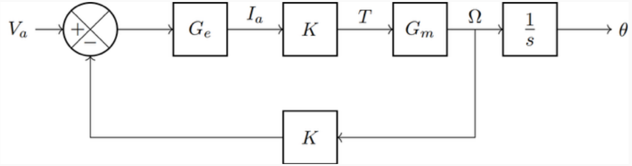
Substitute $I_a(s)$ from torque equation to electrical model

$$V_a(s) = (R_a + sL_a)\frac{T(s)}{K} + K\Omega(s)$$

Substitute the mechanical model

$$V_a(s) = \frac{(R_a + sL_a)(sJ + b)}{K}\Omega(s) + K\Omega(s)$$

Resulted block diagram



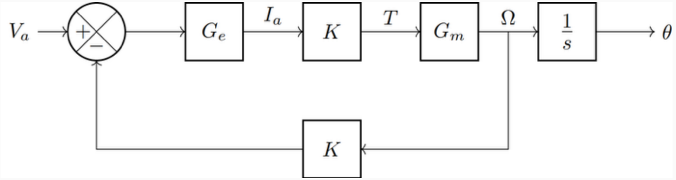
Final transfer functions

$$\frac{\Omega(s)}{V_a(s)} = \frac{K}{(R_a + sL_a)(sJ + b) + K^2}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{1}{s}\frac{\Omega(s)}{V_a(s)}$$

DC drive system dynamic models: frequency response

Block diagram and transfer functions



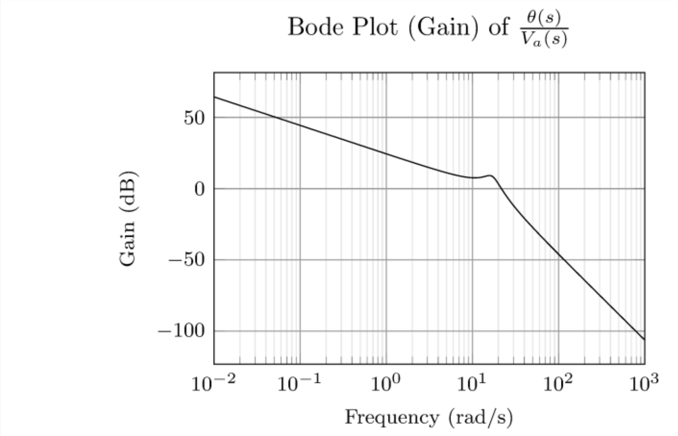
$$G_e(s) = \frac{1}{R_a + sL_a}$$

$$\frac{\Omega(s)}{T(s)} = \frac{1}{sJ + b} = G_m(s)$$

$$\frac{\Omega(s)}{V_a(s)} = \frac{K}{(R_a + sL_a)(sJ + b) + K^2}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{1}{s}\frac{\Omega(s)}{V_a(s)}$$

Frequency response example



Problems

- Slow response and non-controlled current
- Steady state error
- To be solved by close-loop control

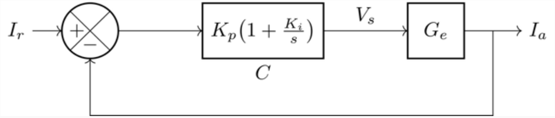
How to regulate the torque and speed of a DC drive system?

Torque control and current loop

Transfer functions

Electrical system $$G_e(s) = \frac{1}{R_a + sL}$$

PI controller $$C(s) = K_p\left(1 + \frac{K_i}{s}\right)$$



Open-loop transfer function

$$C(s)G_e(s) = \frac{K_p s + K_i}{s}\frac{1}{R_a + sL_a}$$

Set equal to design target

$$C(s)G_e(s) = \frac{\omega_c}{s} \Rightarrow \begin{array}{l} K_p s + K_i = \omega_c(R_a + sL_a) \\ K_i = \omega_c R_a, \quad K_p = \omega_c L_a \end{array}$$

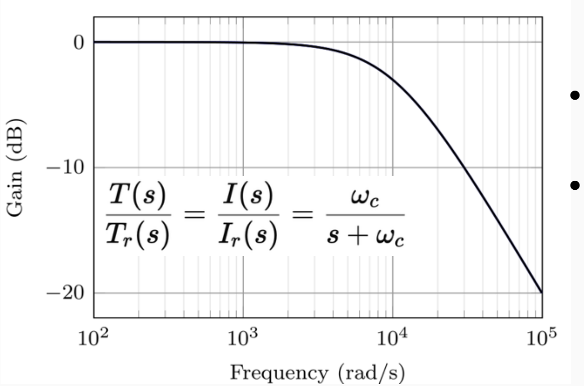
Design goals

$$\frac{T}{T_r} = \frac{KI_a}{KI_r} \approx 1$$ ➡ Current *I* closely follows reference *Ir*

Target open loop transfer function ➡ $$C(s)G_e(s) = \frac{\omega_c}{s}$$

ω_c is a design parameter called bandwidth.

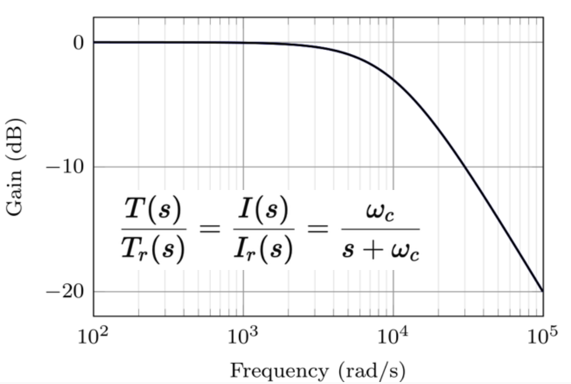
Close-loop response



- ~0 dB at low frequency (reference following)
- Attenuation at high frequency (noise rejection)

Simplified drive dynamics

Close-loop response



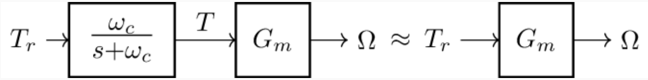
- ~0 dB at low frequency (reference following)
- Attenuation at high frequency (noise rejection)

Simplified drive model with closed current loop

- Below bandwidth frequency ω_c, any reference torque is follow, i.e. for sufficiently large bandwidth,

$$\frac{T}{T_r} = \frac{KI_a}{KI_r} \approx 1$$

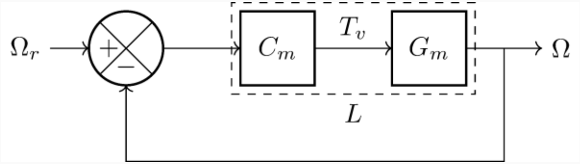
- Back emf acts as a disturbance to the loop and is rejected in the closed loop below bandwidth frequency.



$$\frac{\Omega(s)}{T_r(s)} = G_m(s) = \frac{1}{sJ+b}$$

Cascaded speed control: loop shaping and tuning

Speed close-loop control



Mechanical system $G_m(s) = \dfrac{1}{sJ + b}$

Speed PI controller $C_m(s) = K_{pm}\left(1 + \dfrac{\omega_{im}}{s}\right)$

Close-loop transfer function

$$L(s) = C_m(s)G_m(s) = \frac{K_{pm}(\omega_{im} + s)}{s(Js + b)}$$

$$\frac{\Omega(s)}{\Omega_r(s)} = \frac{L(s)}{1 + L(s)}$$

Design target: design C_m so that

$$\frac{\Omega(s)}{\Omega_r(s)} \approx \frac{\omega_m}{s + \omega_m}$$

ω_m is the desired speed loop bandwidth

Controller tuning

To achieve the desired bandwidth ω_m

- Choose $\omega_{im} = \omega_m < \omega_c / 10$ so that simplification applies
- Then compute the speed loop gain

$$K_{Pm} = \frac{1}{|G_m(j\omega_m)|} = |Js + b|\Big|_{s=j\omega_m} = \sqrt{(J\omega_m)^2 + b^2}$$

Cascaded speed control: theory vs. reality

Design target: design C_m so that

$$\frac{\Omega(s)}{\Omega_r(s)} \approx \frac{\omega_m}{s+\omega_m}$$

ω_m is the desired speed loop bandwidth

Assumption in theory

The drive is simplified under the condition

$$\omega_c \gg \omega_m \quad (\text{e.g.}, \omega_c > 10\omega_m)$$

Then the inner torque control loop behaves as a unity gain

Reality check

- If current loop bandwidth is not significantly larger, inner loop dynamics no longer negligible, system becomes second-order
- Stability and performance affected

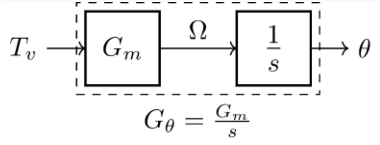
Practical recommendation

- Maintain stability
- Always choose sufficiently low speed loop bandwidth, even if inner loop bandwidth is not high

How to realize position tracking in a DC drive system?

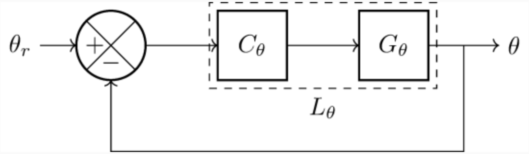
Cascaded position control

Plant model simplified



$$G_\theta(s) = \frac{G_m(s)}{s} = \frac{1}{s(sJ+b)}$$

Position close-loop



PD controller

$$C_\theta(s) = K_{dp}s + K_{pp}$$

Close-loop transfer function

$$\frac{\theta(s)}{\theta_r(s)} = \frac{L_\theta(s)}{1 + L_\theta(s)}$$

$$= \frac{\frac{K_{dp}s+K_{pp}}{s(Js+b)}}{1 + \frac{K_{dp}s+K_{pp}}{s(Js+b)}} = \frac{K_{dp}s + K_{pp}}{s(Js+b) + K_{dp}s + K_{pp}}$$

Integrator (1/s) in the plant model

- PI controller not needed (improper phase margin)
- A PD controller necessary to ensure stability

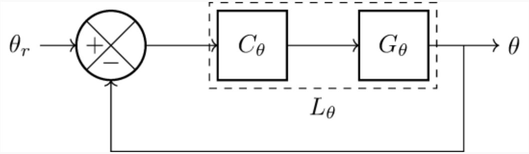
Open-loop transfer function

$$L_\theta(s) = C_\theta(s)G_\theta(s) = (K_{dp}s + K_{pp})\frac{1}{s(Js+b)}$$

$$= \frac{K_{dp}s + K_{pp}}{s(Js+b)}$$

Cascaded position control: tuning

Position close-loop



$$\frac{\theta(s)}{\theta_r(s)} = \frac{L_\theta(s)}{1+L_\theta(s)}$$
$$= \frac{\frac{K_{dp}s+K_{pp}}{s(Js+b)}}{1+\frac{K_{dp}s+K_{pp}}{s(Js+b)}} = \frac{K_{dp}s+K_{pp}}{s(Js+b)+K_{dp}s+K_{pp}}$$

Shape the closed-loop system to match a standard second-order response

$$\frac{\theta(s)}{\theta_r(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Controller parameters for loop shaping

$$K_{pp} = \omega_n^2 \cdot J$$
$$K_{dp} = 2\zeta\omega_n \cdot J - b$$

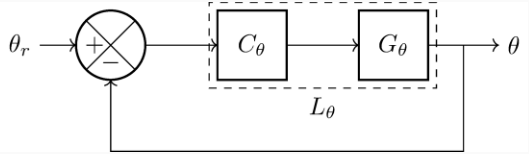
ω_n is the desired position tracking bandwidth

ς is the damping coefficient of the closed loop

If $b << 2\varsigma\omega_n J$, then $K_{dp} \approx 2\zeta\omega_n \cdot J$

Cascaded position control: problem with PD controller

PD controller at high frequencies



$$C_\theta(s) = K_{dp}s + K_{pp}$$

$$\frac{\theta(s)}{\theta_r(s)} = \frac{K_{dp}/Js + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Results in infinite gain at high frequency ➡ Noise amplification

Solution: Tamed PD or lead compensator

Apply a low pass filter to a PD controller

$$C_\theta^{\mathrm{new}}(s) = \frac{K_{dp}s + K_{pp}}{\frac{s}{\omega_l} + 1}$$

A lead compensator

ω_l is usually chosen to be 3-10 times the bandwidth ω_n

How to implement the controller in a digital system?

Digital implementation: Tustin (Bilinear Transform)

Discretization: micro-controllers (MCUs) operate in accurate steps

- Convert a continuous controller to a discretized controller
- From Laplace domain $C(s)$ to z domain $C(z)$

Solution: Tustin approximation

Substitute Laplace variable *s* with

$$s \approx \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$$

T: sampling period

z: z-transform variable

Advantages of Tustin approximation

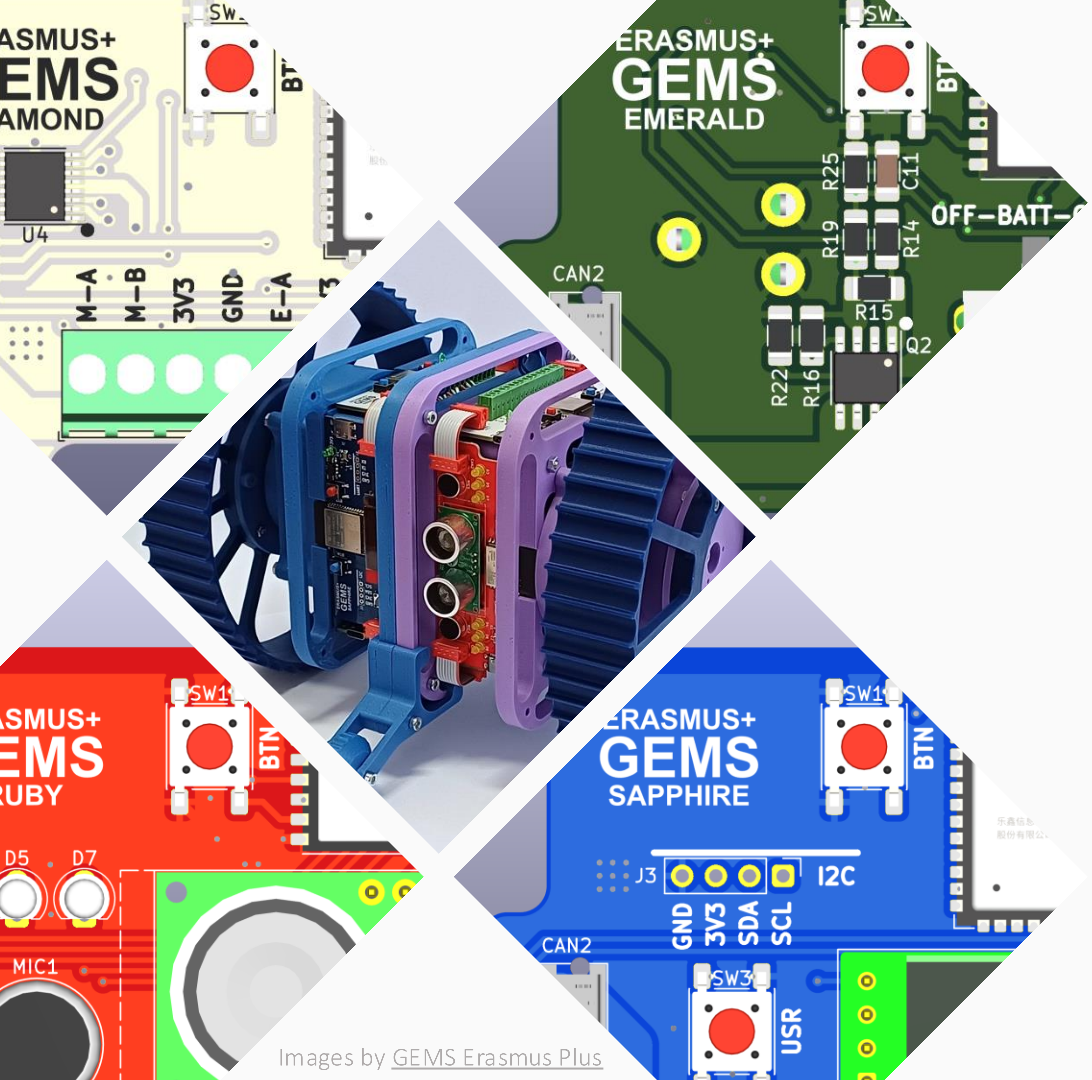
Compared to other methods (e.g. Euler)

- Preserves stability
- Approximates response accurately near Nyquist
- Common and reliable for implementing analog controllers digitally

You can use Scilab function cls2dls() or Matlab function c2d() for bilinear transform based discretization.

Conclusions

- Dynamics of DC drive system can be modelled in transfer functions
- Torque control loop can be realized with a PI controller
- In cascaded speed control, the external speed loop should have sufficiently lower bandwidth
- A tamed PD controller is suitable for position tracking
- Controller designed based on transfer functions can be digitalized by Tustin transformations.



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