

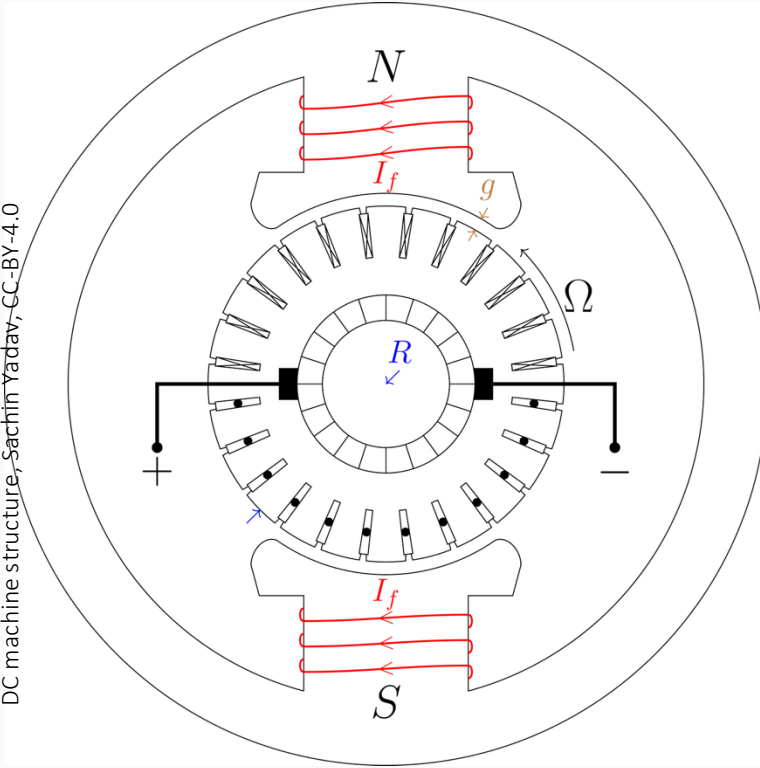


DC Motors

Structure, principle and models

Dr. Jianning Dong

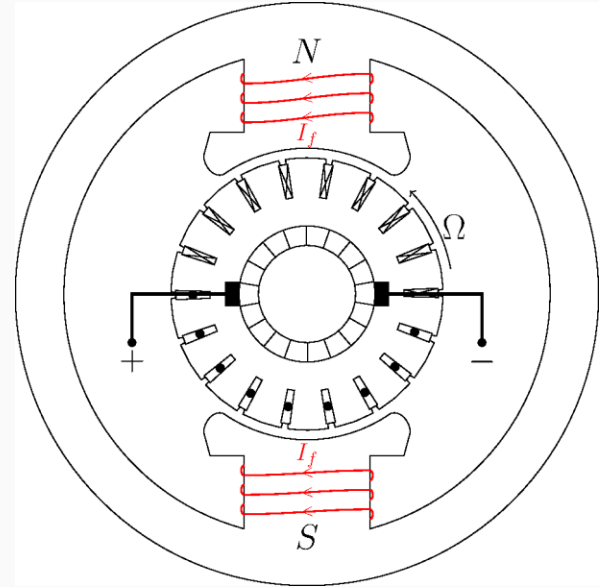
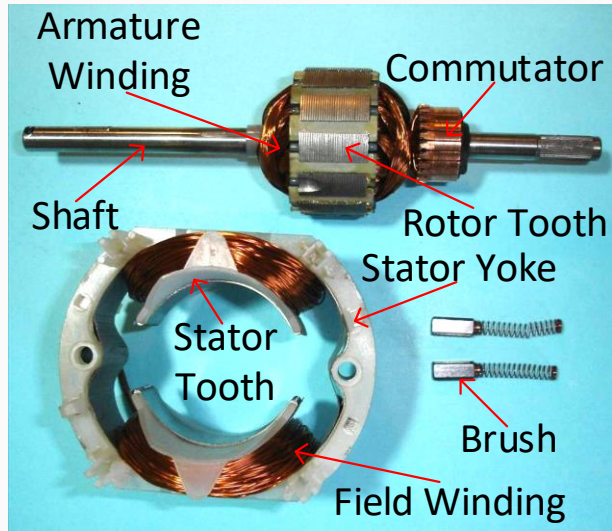




Lecture Outline

- 1 Structure
- 2 Operation principles
- 3 Equivalent circuit
- 4 Performance calculation
- 5 Dynamic model

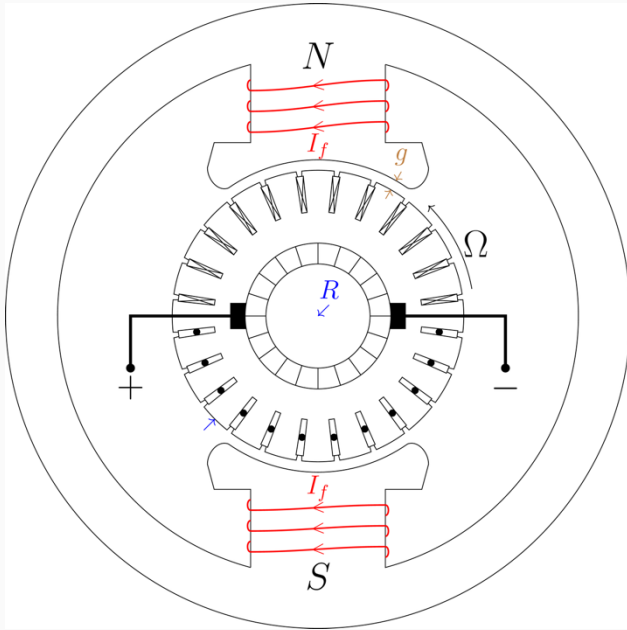
How a DC motor is constructed?



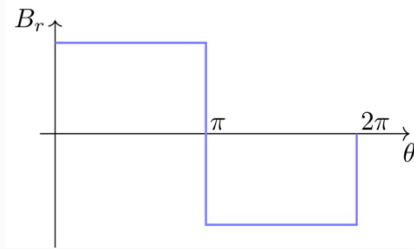
- **Field winding and magnetic core** (stator and rotor): source and path for magnetic field
- **Armature winding**: conductors to bear torque-producing current
- **Brush-commutators**: reverse current direction according to polarity (mechanical rectification)

**How magnetic field is
generated?**

Operation principle: air-gap magnetic field

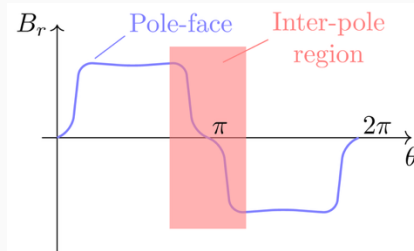


- Ideal air-gap magnetic field (from Ampère's Law)

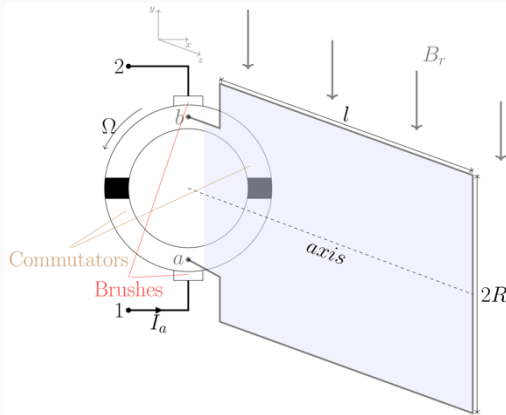
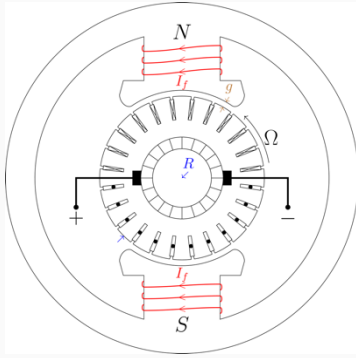


$$B_r = \mu_0 \frac{NI_f}{g}$$

- Real air-gap magnetic flux density considering inter-poles



Operation principle: induced voltage and force



- Consider a single loop of wire

- Induced voltage on wire side

$$E = B_r l v = \pm B_r l \Omega R$$

- Lorentz force on wire side

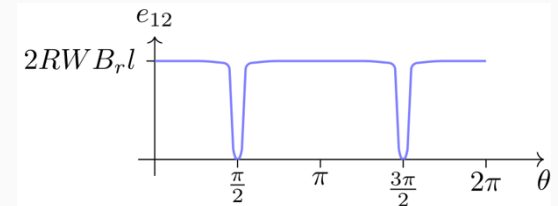
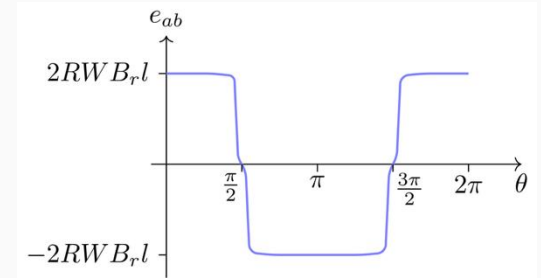
$$F = \pm B_r l I_a$$

- Total induced voltage at the brush terminal

$$E_t = 2 |B_r l v| = 2 B_r l \Omega R$$

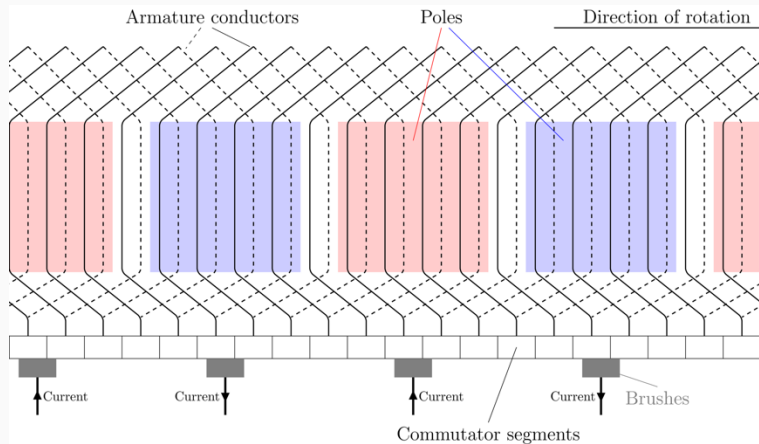
- Total torque on one wire loop

$$T_t = 2 |B_r l I_a| R = 2 B_r l I_a R$$



Operation principle: armature winding and brush-commutator

Characterize a DC machine armature winding



A "lap" winding featuring two parallel paths

- Number of **active conductors** underneath the poles C_a
- Number of current **parallel paths** m

For each active conductor

Back-emf and torque

$$E_{bc} = B_r l \Omega R$$

$$T_c = B_r l I_a R$$

Total induced back-emf and torque

$$E_b = \frac{C_a}{m} B_r l \Omega R$$

$$T_e = \frac{C_a}{m} B_r l I_a R$$

Machine coefficient G



$$E_b = G I_f \Omega$$

$$T_e = G I_f I_a$$

**How to calculate DC
motor performance?**

DC machine steady state performance: equivalent circuit

Back-emf and torque

$$E_b = G I_f \Omega$$

$$T_e = G I_f I_a$$

KVL armature circuit steady state

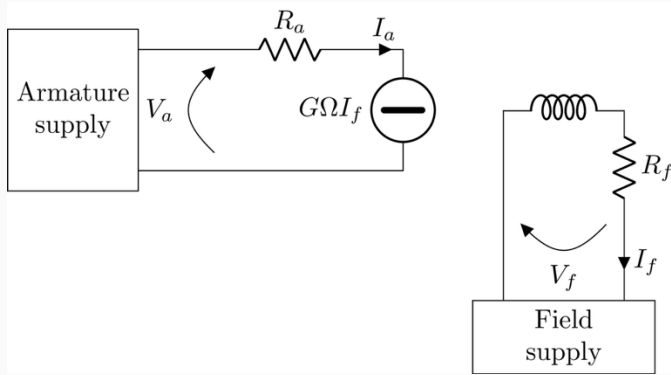
$$V_a = R_a I_a + G \Omega I_f$$

Armature current steady state

$$I_a = \frac{V_a - G \Omega I_f}{R_a}$$

Mechanical power output

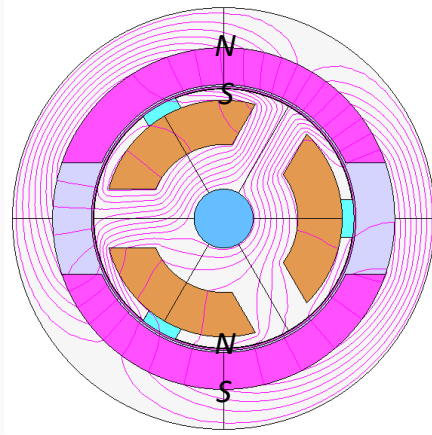
$$P_{em} = T_e \Omega = E_b I_a$$



**Another way to provide
the magnetic field**

—

Permanent magnet DC motor: an alternative way



Permanent magnet (PM) provides a fixed air-gap magnetic field

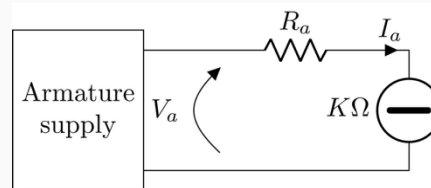
Stator

- Magnetic core and permanent magnets
- No field winding
- Field excited by permanent magnets

Rotor

- Armature winding
- Commutator and brushes

Equivalent circuit of PM DC machine

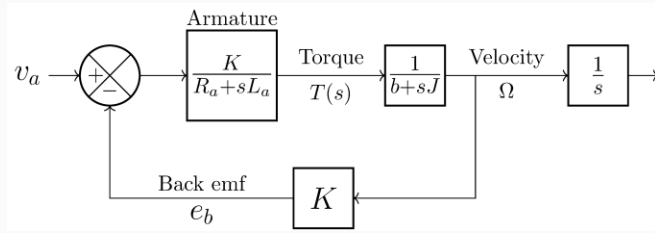


$$E_b = G I_f \Omega = K \Omega$$

$$T_e = G I_f I_a = K I_a$$

Dynamic model of DC motors

DC motor dynamics: described by ODEs and block diagrams



Block diagram describing dynamics of a DC machine

Armature voltage equation considering inductance

$$\begin{aligned} v_a &= R_a i_a + L_a \frac{di_a}{dt} + e_b \\ &= R_a i_a + L_a \frac{di_a}{dt} + K\Omega \end{aligned}$$

Torque equation

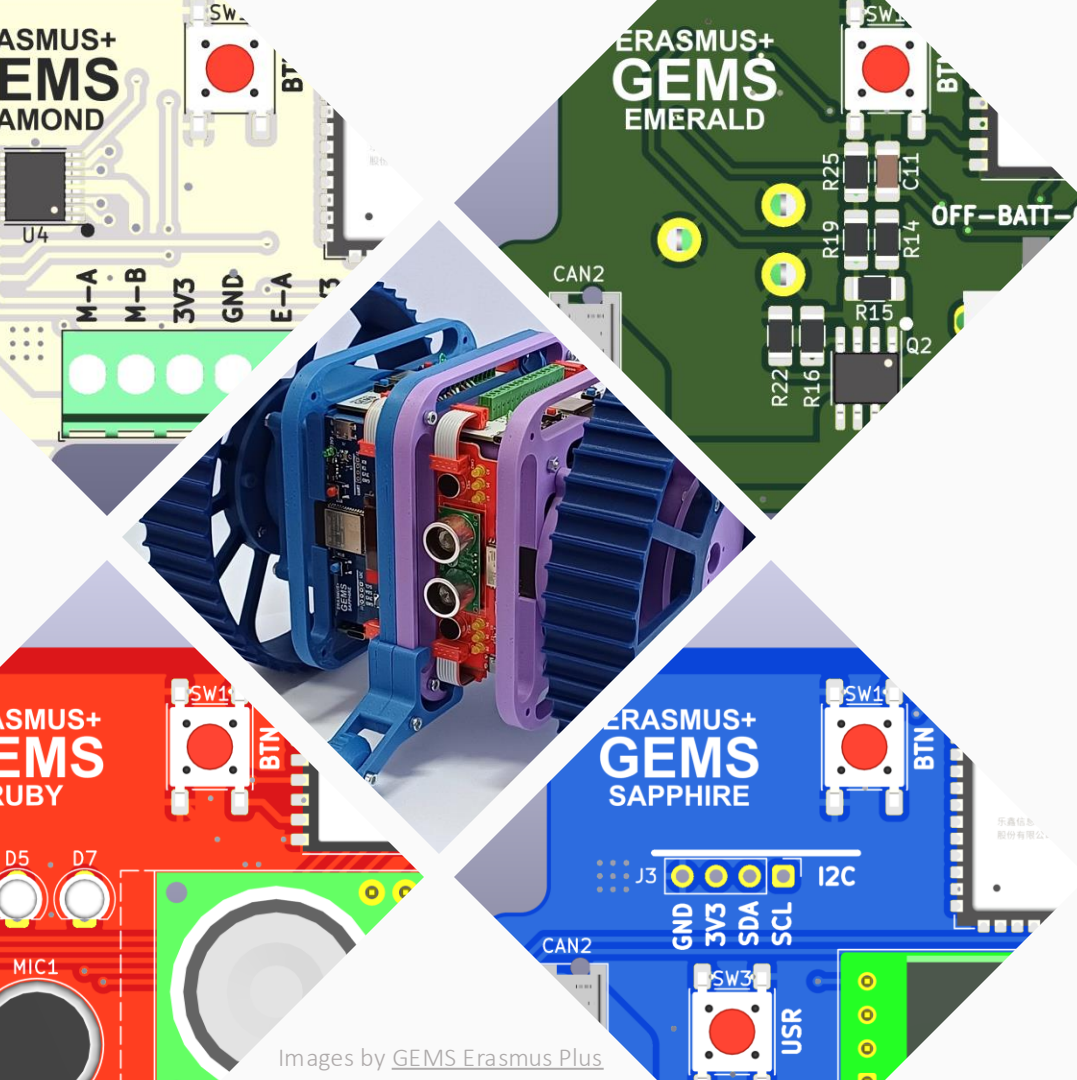
$$T = K i_a$$

Mechanical dynamics

$$T - T_L = J \frac{d\Omega}{dt} + b\Omega$$

Conclusions

- DC motor has linear torque/back-emf characteristics.
- Steady state performance analyzed based on equivalent circuit and Kirchhoff's Laws.
- Dynamics modelled by differential equations or transfer functions.



Images by [GEMS Erasmus Plus](#)

Thank you for watching!

This video was created by Delft University of Technology for the GEMS Erasmus+ project (a collaboration between University of Ljubljana, University of Alcalá, Teaching Factory Competence Center, and Delft University of Technology). It is released under a [Creative Commons Attribution Non Commercial Share Alike 4.0 International License](#)



**Co-funded by
the European Union**

Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or CMEPIUS. Neither the European Union nor the granting authority can be held responsible for them.