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GEMS
DIAMOND**

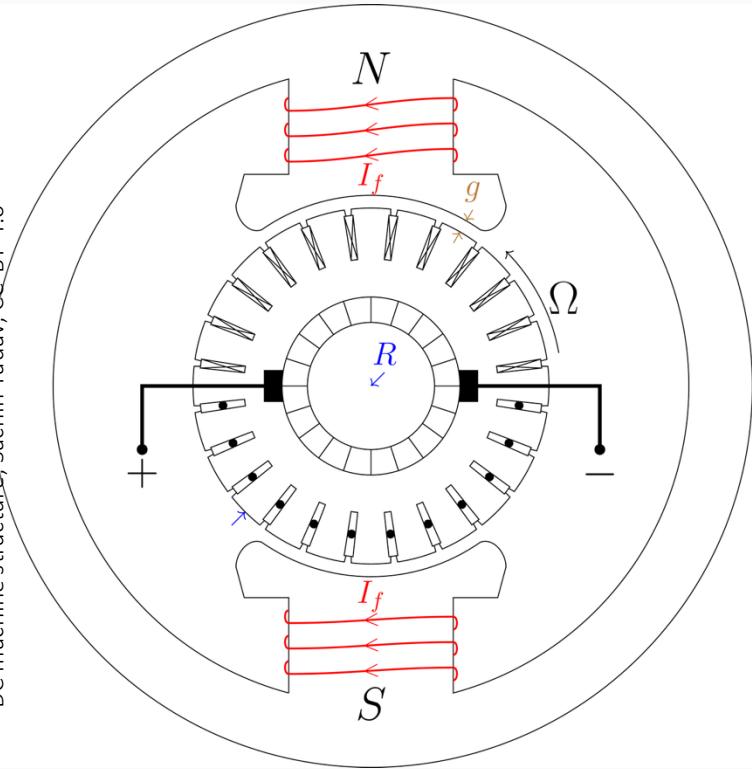


DC Motors

Structure, principle and models

Dr. Jianning Dong

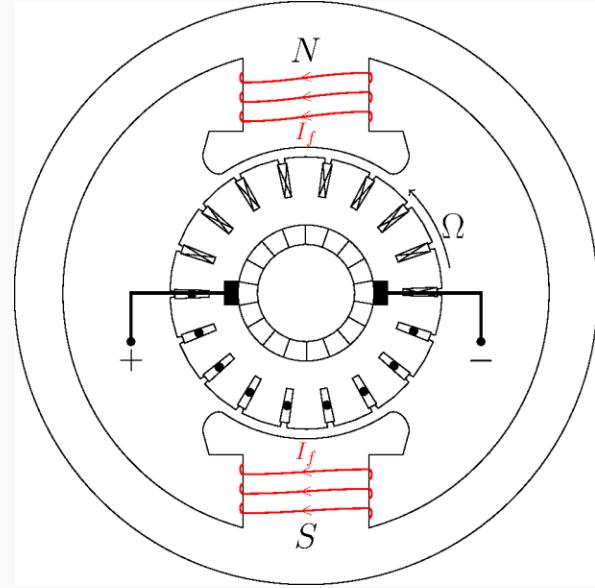
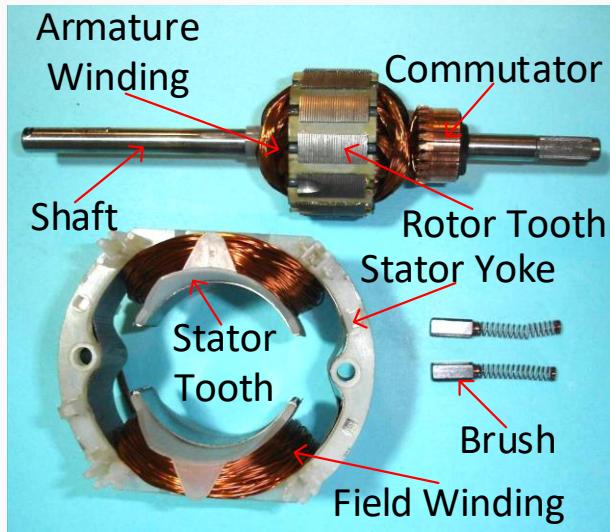




Lecture Outline

- 1 Structure
- 2 Operation principles
- 3 Equivalent circuit
- 4 Performance calculation
- 5 Dynamic model

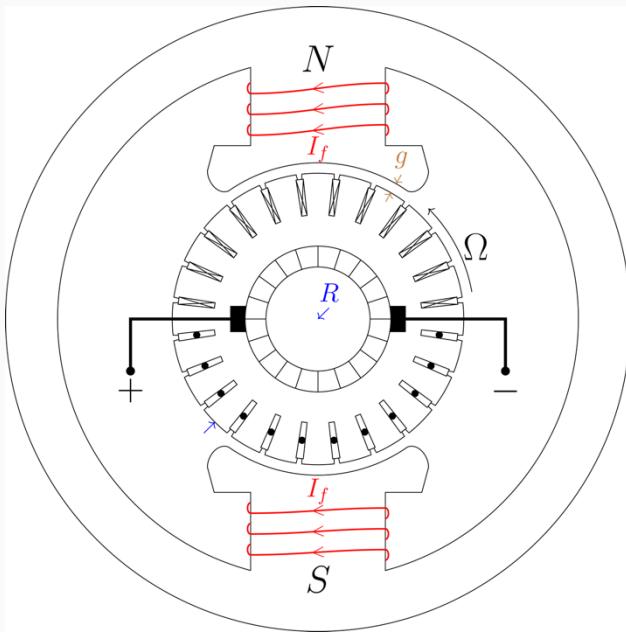
How a DC motor is constructed?

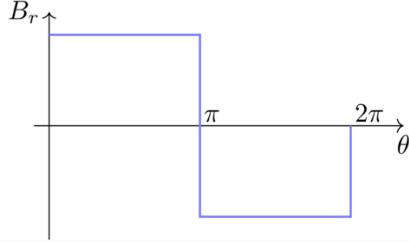


- **Field winding and magnetic core (stator and rotor):** source and path for magnetic field
- **Armature winding:** conductors to bear torque-producing current
- **Brush-commutators:** reverse current direction according to polarity (mechanical rectification)

How magnetic field is generated?

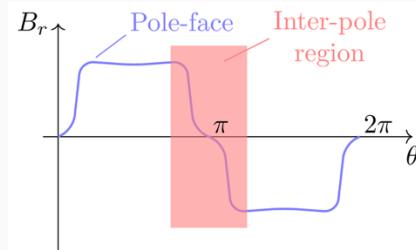
Operation principle: air-gap magnetic field



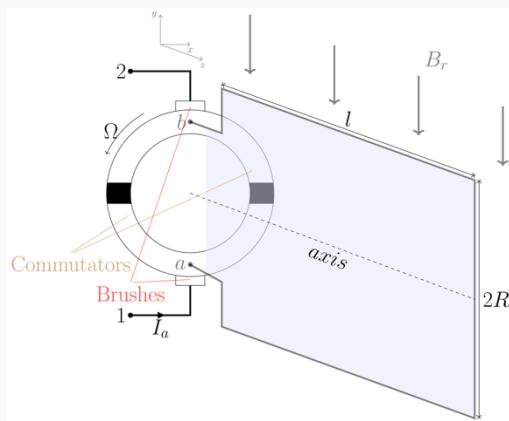
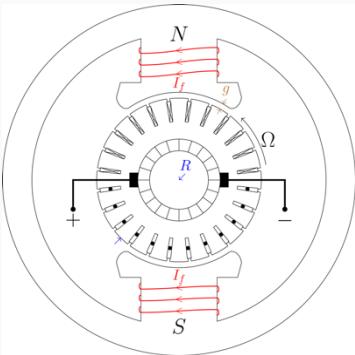
- Ideal air-gap magnetic field (from Ampère's Law)
- 

The graph shows the ideal air-gap magnetic field B_r as a function of the angle θ . The field is constant at a value B_{r0} from $\theta = 0$ to $\theta = \pi$, and then drops to zero until $\theta = 2\pi$, where it returns to B_{r0} .

$$B_r = \mu_0 \frac{NI_f}{g}$$
- Real air-gap magnetic flux density considering inter-poles



Operation principle: induced voltage and force



- Consider a single loop of wire

- Induced voltage on wire side

$$E = B_r l v = \pm B_r l \Omega R$$

- Lorentz force on wire side

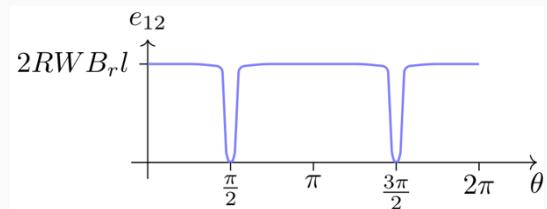
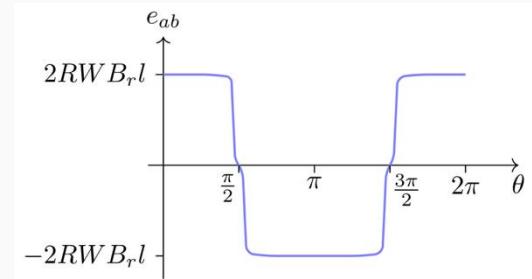
$$F = \pm B_r l I_a$$

- Total induced voltage at the brush terminal

$$E_t = 2 |B_r l v| = 2 B_r l \Omega R$$

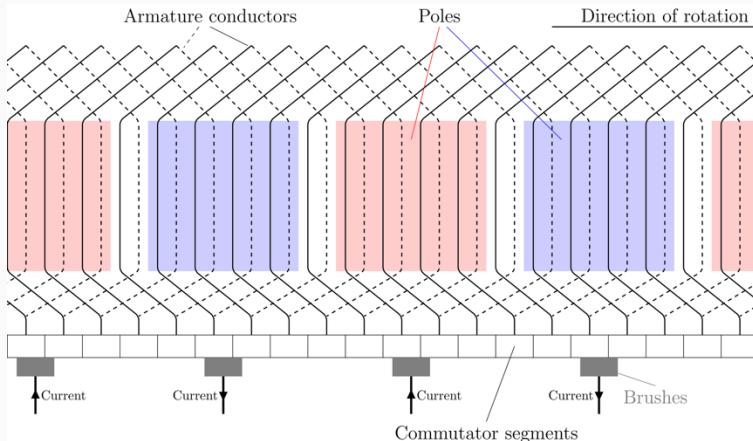
- Total torque on one wire loop

$$T_t = 2 |B_r l I_a| R = 2 B_r l I_a R$$



Operation principle: armature winding and brush-commutator

Characterize a DC machine armature winding



A "lap" winding featuring two parallel paths

- Number of active conductors underneath the poles C_a
- Number of current parallel paths m

For each active conductor

Back-emf and torque

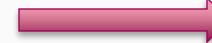
$$E_{bc} = B_r l \Omega R$$

$$T_c = B_r l I_a R$$

Total induced back-emf and torque

$$E_b = \frac{C_a}{m} B_r l \Omega R \quad \text{Machine coefficient } G$$

$$T_e = \frac{C_a}{m} B_r l I_a R$$



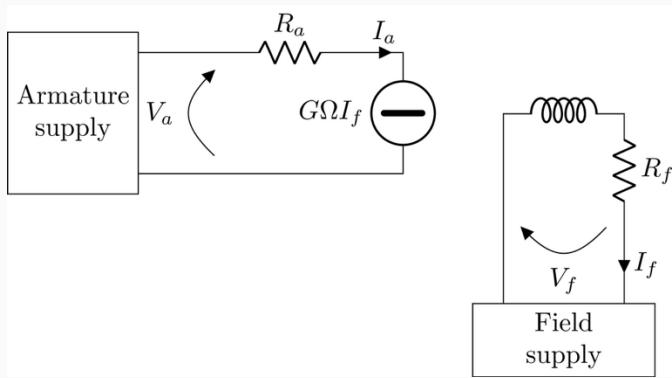
$$E_b = GI_f \Omega$$

$$T_e = GI_f I_a$$

How to calculate DC motor performance?

DC machine steady state performance: equivalent circuit

Back-emf and torque



$$E_b = GI_f \Omega$$

$$T_e = GI_f I_a$$

KVL armature circuit steady state

$$V_a = R_a I_a + G\Omega I_f$$

Armature current steady state

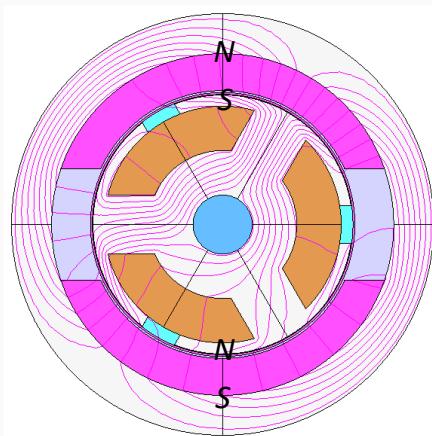
$$I_a = \frac{V_a - G\Omega I_f}{R_a}$$

Mechanical power output

$$P_{em} = T_e \Omega = E_b I_a$$

Another way to provide the magnetic field

Permanent magnet DC motor: an alternative way



Permanent magnet (PM) provides a fixed air-gap magnetic field

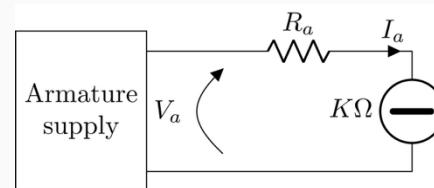
Stator

- Magnetic core and permanent magnets
- No field winding
- Field excited by permanent magnets

Rotor

- Armature winding
- Commutator and brushes

Equivalent circuit of PM DC machine

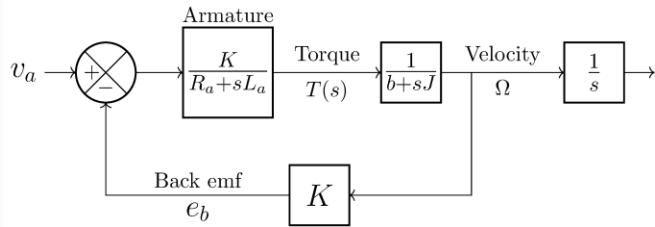


$$E_b = GI_f\Omega = K\Omega$$

$$T_e = GI_f I_a = KI_a$$

Dynamic model of DC motors

DC motor dynamics: described by ODEs and block diagrams



Block diagram describing dynamics
of a DC machine

Armature voltage equation considering inductance

$$\begin{aligned} v_a &= R_a i_a + L_a \frac{di_a}{dt} + e_b \\ &= R_a i_a + L_a \frac{di_a}{dt} + K\Omega \end{aligned}$$

Torque equation

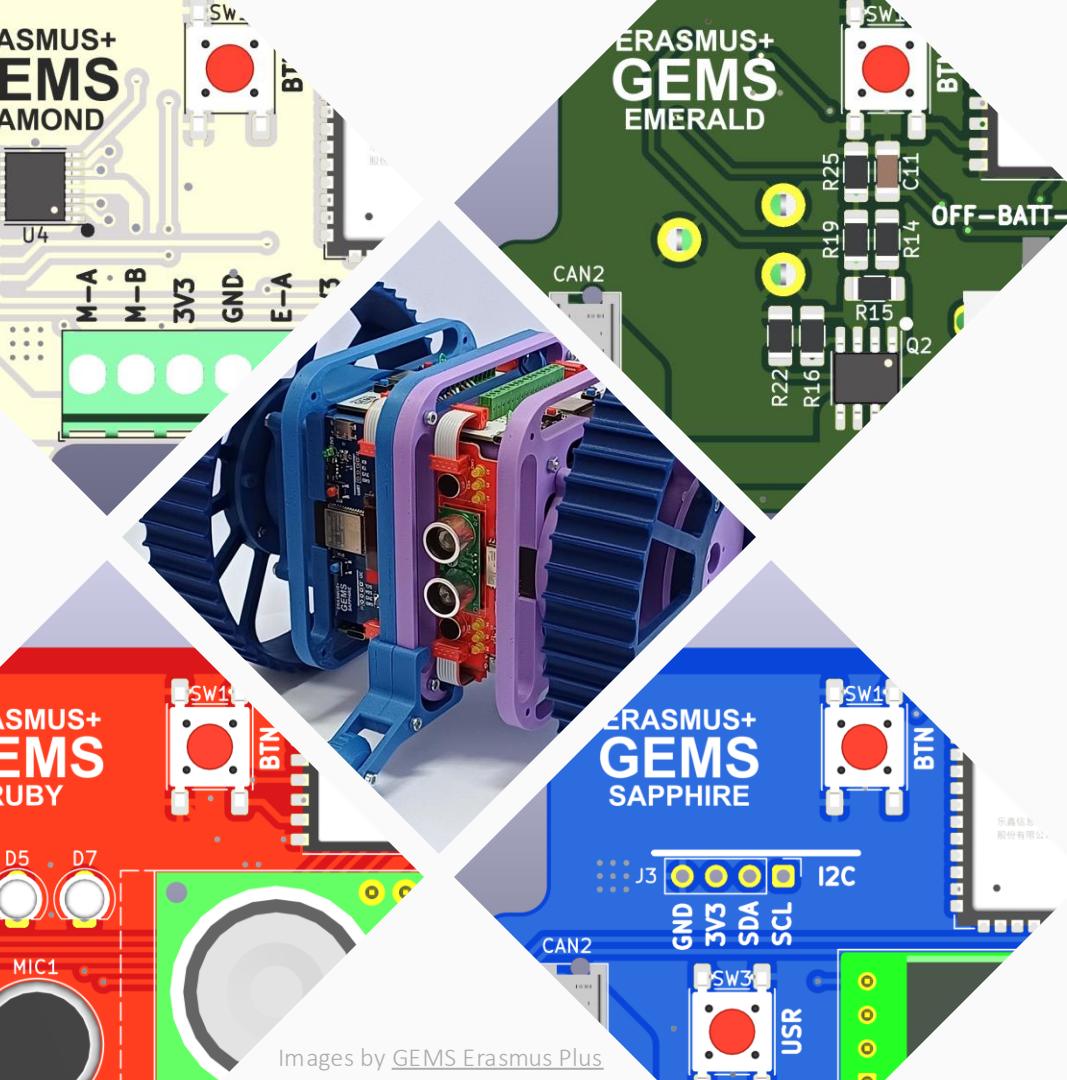
$$T = Ki_a$$

Mechanical dynamics

$$T - T_L = J \frac{d\Omega}{dt} + b\Omega$$

Conclusions

- DC motor has linear torque/back-emf characteristics.
- Steady state performance analyzed based on equivalent circuit and Kirchhoff's Laws.
- Dynamics modelled by differential equations or transfer functions.



Images by [GEMS Erasmus Plus](#)

Thank you for watching!

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