Comparison of the exponential distribution and the Central Limit Theorem

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Overview

In this document we will investigate the exponential distribution in R and compare it with the Central Limit Theorem. We will compare:

1. The sample mean and the theoretical mean of the exponential distribution.
2. The sample variance of sample means of draws of the exponential distribution and the theoretical variance.
3. The distribution of sample means of the exponential distribution and the normal distribution implied by the Central Limit Theorem.

Sample mean vs. Theoretical mean

We draw 1000 samples of 40 draws from an exponential distribution with $\lambda = 0.2$. The mean of this distribution is $\frac{1}{0.2} = 5$. According to the Law of Large Numbers we expect the mean of the sample means of 40 draws to be very close to 5.

Let's compare them now:

```
set.seed(42)

sample_means <- vector(, 1000)
for(i in 1:1000) {
    sample_means[i] <- mean(rexp(40, 0.2))
}
mean(sample_means)
```

```
## [1] 4.986508
```

The sample mean, 4.987, is very close to the theoretical mean, 5.

Sample variance vs. Theoretical variance

The variance of the exponential distribution with $\lambda = 0.2$ is $\frac{1}{0.2^2} = 25$. The theoretical variance of the mean of 40 draws from the exponential distribution is $25/40 = 0.625$. We test this in R:

```
var(sample_means)
```

```
## [1] 0.6344405
```

The result, 0.634, is close the theoretical variance, 0.625.

Distribution of sample means vs. Normal distribution

According to the Central Limit Theorem, we expect the distribution of sample means of 40 draws from $Exp(0.2)$ to be approximately normally distributed with mean 5 and variance 0.625.

```r
means <- data.frame(sample_means)

ggplot(means, aes(x = sample_means)) +
    geom_histogram(binwidth = 0.4, color = 'black', fill = 'white', aes(y = ..density..)) +
    stat_function(fun = dnorm, args = list(mean = 5, sd = sqrt(0.625)), color = "purple") +
    xlab('Sample mean') +
    ylab('Density') +
    ggtitle('Sample distribution approximates \n normal distribution')
```