Homework 5 - CS 146 . William De Bruce

Problem 1:

1.) $T(N) = 2T(N-1) + 1$

$T(N-1) = 2T(N-2) + 1$

$T(N-2) = 2T(N-3) + 1$

$T(N-3) = 2T(N-4) + 1$

$$\begin{aligned}
T(N) &= 2T(N-1) + 1 \\
&= 2(2T(N-2)+1)+1 \\
&= 4T(N-2) + 3 \\
&= 4(2T(N-3)+1) + 3 \\
&= 8T(N-3) + 7 \\
&= 8(2T(N-4)+1) + 7 \\
&= 16T(N-4) + 15 \\
&\approx N-K = 0, \quad N = K \\
&\approx 2^K T(N-K) + 2^K - 1 \\
&\approx 2^K (T(0) - 1) \\
&= O(2^n)
\end{aligned}$$

2^N

$a > 1$, Master Theorem: $a = 2$, $b = 1$,

$T(n) = O(2^n)$

2.) $T(N) = 3T(N-1) + n$

$T(N-1) = 3T(N-2) + n$

$T(N-2) = 3T(N-3) + n$

$T(N) = 3(3T(N-2) + n) + n$

$= 9T(N-2) + 3n + n$

$= \cancel{9T}$

$= 9(3T(N-3) + n) + 4n$

$= 27T(N-3) + \cancel{5n}\,13n$

$N - k = 0$

$T(N) = 3^{k-1}T(N-k) + (3k+1)n$

$= 3^{N-1}T(\cancel{N0}) + \cancel{(3N+1)n}$

$= 3^{N-1}T(\emptyset) + 3N^2 + N$

$\cancel{O(3^N)}$

$= O(N3^N)$

$a \geq 1$, $a = 3$, $k = 1$, $b = 1$

$T(n) = O(n3^n)$

3.) $T(n) = 9\,T(N/2) + n^2$

$a = 9$, $b = 2$ $f(n) = n^2$

$\cancel{f(n)}$

$9 > 2^2$

$T(n) = \Theta(n^{\log_b a}) = \Theta(n^{\log_2 9}) \approx \boxed{\Theta(n^3)}$

4.) $100\,T(N/2) + n^{\log_2 cn + 1}$

$a = 100$, $b = 2$, $f(n) = \log_2 cn + 1$

100 $\cancel{2^{\log_2 cn + 1}}$

$2^{c \log_2 n}$

$\log_2 cn = k$

$2^k = cn$

$d = c \log_2 n$ $c \log_2 n$

$f(n) = n^{\log_2 cn + 1}$

$= n^c = n^2$ $c = 2$?

100 $2^{c \log_2 n}$

5.) $T(n) = 4\,T(N/2) + n^2 \log n$

$=$ $a = 4$, $b = 2$ $d = 2$ $f(n) = n^2 \log n$

$4 = 2^2$ $T(n) = \Theta(n^d \log n)$

$\boxed{= \Theta(n^2 \log n)}$

6.) $T(N) = 5\,T(n/2) + n^2 \cancel{\log n}$

$a = 5$ $b = 2$ $d = 2$

$5 > 2^2$ $T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 5})$

$\approx \boxed{\Theta(n^2)}$

b) Prob #2.

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(N/2) + 10n & n > 1 \end{cases}$$

$a = 2 \quad b = 2 \quad d = 1 \quad f(n) = 10n$

$\cancel{2} = 2$

$\cancel{2} \; 2 = 2, \quad T(n) = \theta(n^d \log n)$

$$\boxed{= \theta(n \log n)}$$