NETWORK SCIENCE

Optimal network topology for responsive collective behavior

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Animals, humans, and multi-robot systems operate in dynamic environments, where the ability to respond to changing circumstances is paramount. An effective collective response requires suitable information transfer among agents and thus critically depends on the interaction network. To investigate the influence of the network topology on collective response, we consider an archetypal model of distributed decision-making and study the capacity of the system to follow a driving signal for varying topologies and system sizes. Experiments with a swarm of robots reveal a nontrivial relationship between frequency of the driving signal and optimal network topology. The emergent collective response to slow-changing perturbations increases with the degree of the interaction network, but the opposite is true for the response to fast-changing ones. These results have far-reaching implications for the design and understanding of distributed systems: a dynamic rewiring of the interaction network is essential to effective collective operations at different time scales.

INTRODUCTION

A wide range of complex systems are characterized by relatively simple dynamical rules while still producing excessively complex emergent collective behaviors. Examples abound in the natural world [e.g., a flock of birds, a school of fish, a swarm of insects (*1–9*)], in social systems [e.g., social networks (*10–12*)], and in engineered multi-agent systems [e.g., self-organized networks of mobile sensors, multi-vehicle coordination, and swarm robotics systems (*13–16*)].

Historically, particular attention has been directed toward investigating varieties of collective behaviors obtained by testing a wide range of local agent-to-agent interaction rules (*6*, *9*). Collective behaviors have also been investigated from the network-theoretic perspective (*4*, *8*, *17–21*). It is now clear that such rich collective behaviors are the outcome of a complex interplay between network topology—characteristic of the group-level organization—and the dynamical laws at the agent's level (*4*, *8*, *20–22*).

Many collective behaviors can be studied through the lens of distributed consensus problems, including collective motion in animal groups and multi-robot systems. Over the past decade, the number of studies on decentralized consensus and cooperation in networked multi-agent systems has experienced a spectacular growth, with concomitant developments in various fields of engineering and science (*2*, *3*, *23–26*). Consensus dynamics is the cornerstone of cooperative control strategies for vehicular formation (*13*, *16*, *23*), swarm robotics (*14*, *15*), and synchronization of coupled oscillators (*23*, *27*). Decentralized consensus is also at the core of collective opinion dynamics and complex contagion processes in social networks (*10–12*), as well as complex collective responses in biological swarms (*3–8*).

Previous studies focused on establishing the influence of the interaction network topology on (i) the capacity of the collective to reach consensus in the presence of noise, communication constraints, and time delays (*21*, *23*); (ii) the speed of consensus (i.e., its convergence rate) (*18*, *25*, *28*); (iii) the stability and stabilization of consensus (*23*); and (iv) the ability to steer the system toward a particular consensus value by means of various control techniques such as pinning control, cooperative tracking control, or model reference consensus (*19*, *20*). However, the effects of the network topology on other dynamical properties of distributed multi-agent systems such as their adaptivity or responsiveness to external perturbations have received considerably less attention (*4*).

It is important to emphasize that a capacity for fast consensus is not necessarily indicative of a responsive collective behavior. For instance, ferromagnets at low temperature exhibit a global spontaneous magnetization—a process that can be described by a distributed consensus protocol. It is known that both the degree of consensus (i.e., magnetization) and the speed at which it is reached increase with decreasing temperature, but the capacity of the system to respond to external perturbations is maximized at a finite critical temperature.

Similarly, in the context of animal collective motion, it has been observed that midges exhibit low levels of ordering while maintaining large connected correlations, thus having a high collective response (*5*). With these observations, the authors eloquently argued that one must be careful in relating collective order (i.e., degree of consensus) with the collective responsiveness. The collective response of the animal group was obtained experimentally by measuring the correlations in the fluctuations of their behavior. While inferring a collective response to external perturbations from these fluctuations is not formally justified for out-of-equilibrium systems, extensive numerical studies (*29*) have shown that this equivalence holds in the context of collective motion based on distributed heading consensus. Moreover, simulations have shown that this measure of susceptibility is a good indicator of the group's performance in biologically relevant functions such as predator avoidance (*8*). These facts along with other empirical evidence have led to the conclusion that responsiveness, rather than high consensus or order, is the true hallmark of collective behavior (*3*).

To study how the responsiveness of a collective is affected by its interaction network topology, we consider an elementary example of distributed decision-making: a linear time-invariant (LTI) system of agents performing consensus over a scalar state variable. The agents—i.e., the nodes of the interaction network—are all identical, except for one "leader" [also known as "stubborn" agent in some contexts (*12*, *25*)] with some arbitrary predefined dynamics. From the control-theoretic perspective, this leader introduces a time-varying control input signal into the system. In the biological context, this dynamical leader represents a member of a swarm with access to privileged information about a food

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