ss_jupiter_1

January 5, 2026

1 Jupiter I

This worksheet is intended as a starting point for a set of notes intended to build our understanding of Jupiter. In this worksheet, we explore in detail the following topics: * General physical parameters of Jupiter and comparison with Sun, Earth and other planets: * mass, size, density * rotation rate * oblateness * Rotating frames of reference and effects on Jupiter * pseudo-forces: Coriolis and centrifugal * zonal flow * flattening * Structure of interior of Jupiter * Convection * Phases of Hydrogen * Magnetic field of Jupiter * Satellites of Jupiter * Galilean moons * Io, Europa, Ganymede, Callisto * Tidal locking and libration * Orbital and Laplace resonance * Comparison to Solar System * Inner group * Metis, Adrastea, Amathena, Thebe * Jovio-stationary orbit

1.1 Astrophysical characteristics

1. Look up the mass, radius, and rotational period of the Sun, Jupiter, and the Earth. Compare these physical characteristics to one another by taking ratios.

[1]:
```python
from aa_tools import astroconst
from astropy.units import kg, km, hr

bodies1=["sun", "jupiter", "earth"]

masses={ i : getattr(astroconst, f"M_{i}") for i in bodies1 }
radii={ i : getattr(astroconst, f"R_{i}") for i in bodies1 }
periods={ i : getattr(astroconst, f"P_rot_{i}") for i in bodies1 }

print(f" {"body":^15} |  {"M (kg)"}  |  {"R (km)"}  | {"P_rot (hr)":^15} ")
print(f"-------------------------------------------------------")
for b in bodies1:
    print(f" {b.capitalize():^15} | {masses[b].to_value(kg):.2e} | {radii[b].
↪to_value(km):.2e} | {round(periods[b].to_value(hr)):^12}")

print("\n")

for b1 in bodies1:
    for b2 in bodies1:
        if b1 > b2:
            print(f"The mass of {b1.capitalize()} is {round(masses[b1]/
↪masses[b2])} times that of {b2.capitalize()}.")
```

```
        print(f"The radius of {b1.capitalize()} is {round(radii[b1]/
↪radii[b2])} times that of {b2.capitalize()}.")
        print(f"The rotational period of {b1.capitalize()} is {periods[b1]/
↪periods[b2]:.1f} times that of {b2.capitalize()}.\n")
```

```
      body       |  M (kg)   |  R (km)   |   P_rot (hr)
-------------------------------------------------------
       Sun       | 1.99e+30 | 6.96e+05 |     609
     Jupiter     | 1.90e+27 | 6.99e+04 |      10
      Earth      | 5.97e+24 | 6.37e+03 |      24


The mass of Sun is 1048.0 times that of Jupiter.
The radius of Sun is 10.0 times that of Jupiter.
The rotational period of Sun is 61.4 times that of Jupiter.

The mass of Sun is 332946.0 times that of Earth.
The radius of Sun is 109.0 times that of Earth.
The rotational period of Sun is 25.4 times that of Earth.

The mass of Jupiter is 318.0 times that of Earth.
The radius of Jupiter is 11.0 times that of Earth.
The rotational period of Jupiter is 0.4 times that of Earth.
```

2. Calculate the average density of Jupiter. To which solar system body (bodies) is this similar?

[2]:
```
from math import pi
from astropy.units import Quantity, g, cm

def rho(b: str)-> Quantity:
    return 3*masses[b]/(4*pi*radii[b]**3)

rho_jupiter = rho("jupiter")
rho_sun = rho("sun")

print(f"The mass density of Jupiter is {rho_jupiter.to(g / cm**3):.1f}.")
print(f"This is close to the solar density {rho_sun.to(g / cm**3):.1f}.")
```

```
The mass density of Jupiter is 1.3 g / cm3.
This is close to the solar density 1.4 g / cm3.
```

The **oblateness** or **flattening** of a spherical celestial body is a measure of how far it deviates from being perfectly spherical. If c is defined as the center-to-pole radius (a.k.a. the polar radius) and a is defined to be the center-to-equator radius (a.k.a. the equatorial radius), then ba would be close to 1 for a spherical object. The flattening is therefore defined to be

$$f = 1 - \frac{c}{a} \qquad \text{(oblateness)}$$

(Note the similarity and difference with the definition of eccentricity of an ellipse.)

3. For Jupiter, the equatorial radius $a = 71,492$ `km` and the polar radius is $c = 66,854$ `km`. Use this to calculate its oblateness. For comparison, the flattening of the Sun is 9×10^{-6}. Compare Jupiter's oblateness to that of other solar system planets (found in a table on this page).

[3]:
```
def oblateness(b: str):
    a, b, c = astroconst.radii_axes(b)
    R_equatorial=(a+b)/2
    R_polar = c
    return 1 - R_polar/R_equatorial

bodies= ["sun"] + astroconst.planets

#print("   body    |     oblateness  ")
#print("----------------------------")
#for b in bodies:
#    print(f"   {b.capitalize()[:3]}    |    {oblateness(b):.5f}  ")
#print("----------------------------")

print(f"\n{"body":^10} {"oblateness":^20}")
print("---------------------------------")
for b in bodies:
    o=f"{oblateness(b):.4f}"
    print(f"{b.capitalize():^10} {o:^20}")
print("")
```

```
   body         oblateness
---------------------------------
   Sun            0.0000
 Mercury          0.0009
  Venus           0.0000
  Earth           0.0034
   Mars           0.0059
 Jupiter          0.0649
  Saturn          0.0980
  Uranus          0.0229
 Neptune          0.0171
```

1.2 Rotating frames of reference

There are (at least) two approaches to describing Jupiter's oblateness. Intuitively, it's puffed up because it rotates so quickly and it's made up of a non-rigid material. Let us describe it in this approach by going to a frame of reference in which we spin with Jupiter.

Recall that in our unit on astrodynamics, an emphasis was placed on Newton's first law. This law established that there are frames of reference that absolutely at rest: They do not move over and they do not spin. Inertial frames of reference were ones that are moving at a constant velocity with respect to this at-rest frame.

This was important because Newton's second law was the statement that *in and **inertial** frame of reference*, the net force on a particle $\vec{F}_{net}$ is related to that particle's acceleration $\vec{a}$ by $\vec{F}_{net} = m\vec{a}$.

Suppose instead, that we go to a frame of reference that is rotating around the origin of an inertial frame. To specify such a rotation, we need an axis around which it is rotating and the angular speed with which it does so. In other words, we need to specify a vector $\vec{\omega}$, say, the size ω of which gives the rotational speed in radians per second. This vector will lie along the axis of rotation with the direction assigned by the right-hand-rule.

This is not an inertial frame of reference, and therefore, we have no right to expect that $\vec{F}_{net} = m\vec{a}$ in this frame. In fact, we can start with $\vec{F}_{net} = m\vec{a}$ in the inertial frame and then transform to coordinates that rotate as implied by the anguar velocity vector $\vec{\omega}$. If you do this carefully using multi-variable calculus, you will find two corrections to Newton's second law. To express them clearly, we first rewrite Newton II as $\vec{a} = \frac{\vec{F}_{net}}{m}$, and then switch to the rotating frame to find:

$$\vec{a} = \frac{\vec{F}_{net}}{m} - 2\vec{\omega} \times \vec{v} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) \tag{1}$$

The first term is the usual one from Newton's Second Law. The second term is called the **Coriolis term**. It is responsible for redirecting motion into unexpected directions. The third term is called the **centrifugal term**. It is responsible for the apparent outward-pushing effect when you are in a spinning frame of reference. We will discuss these effects more in class, but for now just know that ω (lower-case omega) is the rotational rate (in radians/second) and $\vec{\omega}$ points along the axis around which the rotation is happening. $\vec{r}$ is the position in space of the rotating particle, and $\vec{v}$ is its velocity vector.

The operation $\times$ between two vectors gives another vector that points in a direction perpendicular to the two vectors and assigned by the right-hand-rule; it is called the **cross product**: For example, the term $\vec{\omega} \times \vec{v}$ is the cross produce of the vector $\vec{\omega}$ pointing along the axis of rotation and the vector $\vec{v}$ is the velocity vector of a particle. To find the direction of $\vec{\omega} \times \vec{v}$, you are supposed to use your right hand with your fingers pointing along $\vec{\omega}$ and curl them so that they point toward the direction of $\vec{v}$. Then $\vec{\omega} \times \vec{v}$ will be in the direction of your thumb. This is called the **right-hand rule**.

4. Try to apply the right-hand rule to the third term $-\vec{\omega} \times (\vec{\omega} \times \vec{r})$ to show that the effect of it is to accelerate particles in a direction outward and parallel to the equator. (So not radially outward.) This is the **centrifugal effect**. Hint: the placement of the parenthesis is important; you must the direction of find $\vec{\omega} \times \vec{r}$ first, then repeat the construction to find the direction of $\vec{\omega} \times (\cdots)$, and finally, reverse the result because of the $-$ sign.

1.2.1 Answer 4

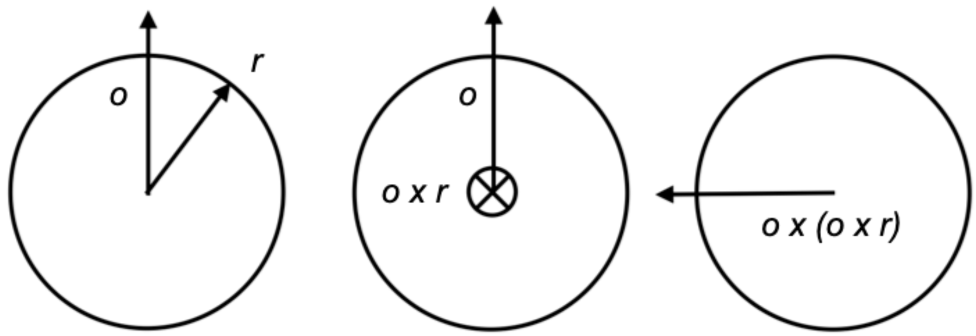


Fig A4 Left: Vectors $\vec{o}$ and $\vec{r}$. Middle: $\vec{o} \times \vec{r}$ points into the page and is indicated with a $\otimes$. Right: The vector constructed by taking $\vec{o}$ and crossing it with $\vec{o} \times \vec{r}$ to give $\vec{o} \times (\vec{o} \times \vec{r})$.

- Step 1: In the diagram above at left, the angular velocity $\vec{\omega}$ is represented by $\vec{o}$. This is the appropriate direction if, as viewed from the top, Jupiter is spinning counter-clockwise. (Just like seeing Earth from the side with north pointing up: The angular momentum vector of Earth points from the center to the North Pole.) Consider a point on the surface and the vector $\vec{r}$ that points to it and starts at the center of Jupiter.
- Step 2: Using your right hand, point your fingers in the direction of $\vec{o}$ (*i.e.* straight up). Keeping your palm fixed, bend your fingers so that they form an L shape relative to your palm. your fingers are probably pointing into the page. This is not the direction of $\vec{r}$, so to line your fingers up correctly, you will need to rotate your arm and wrist clockwise (as seen from above) about 90 degrees. If you do the correctly, your fingers are curling to the right, and your thumb should be pointing into the page. This is the direction of the vector $\vec{o} \times \vec{r}$, let's call it $\vec{c}$. Thus, we have found that $\vec{c} = \vec{o} \times \vec{r}$ points into the page. It is customary to indicate such a 3D vector in a 2D projection with the symbol $\bigotimes$.
- Step 3: Next, we determine the direction of $\vec{o} \times (\vec{o} \times \vec{r}) = \vec{o} \times \vec{c}$. We, again, begin with our fingers up and palm facing into the page (right hand only!) because that this the direction of $\vec{o}$. Again, if we curl our fingers so our (right!) hand is in the shape of an L, they are pointing into the board. This time, that is already the correct direction for $\vec{c}$ so that extending our thumb gives the direction of $\vec{o} \times \vec{c}$. If you do it correctly, your thumb should be pointing to the left, so $\vec{o} \times \vec{c}$ points to the left/inward as indicated in the figure at right.
- Step 4: We have determined the direction of $\vec{o} \times \vec{c} = \vec{o} \times (\vec{o} \times \vec{r})$, but the third term in equation (1) is $-\vec{\omega} \times (\vec{\omega} \times \vec{r})$ which points in the opposite direction. Thus, we have derived that the "centrifugal" term is indeed centrifugal (*i.e.* away from the center). Note that what we really mean here is that it points away from the *axis of rotation* not way from the center of the ball.

1.3 Jupiter's interior

5. Construct a quantitative model of the cross section of Jupiter similar to the one you made for the Sun in `ss_sol`. Your model should include the basic structure

of Jupiter's interior in terms of sizes, densities, temperatures, and composition. Even if you do this correctly, you may come across conflicting reports and large uncertainties. Many of these have been resolved by the recent Juno mission. (See also this video about Juno cam for fun and inspiration.)

1.3.1 Answer 5

...

6. Inspired by the Sun's story, try to construct a narrative that ties together Jupiter's composition (as a gas giant), the temperatures in its layers, convection, its differential rotation rates, and its oblateness. What does your model suggest about any potential magnetic field? Is there a dynamo at work?

1.3.2 Answer 6

...

The second term in equation (1) can be used to understand Jupiter's iconic atmospheric bands. From your comparisons to the interior of the Sun, you have probably concluded that there are strong convective currents responsible for heat transfer from the 25,000 K core to 300 K atmosphere. Similarly to the Sun there are regions in which the hotter material rises and others in which it is sinking.

7. Use the right-hand rule applied to the second term in equation (1) to argue that currents rising to the surface (the **zones**) will experience a deflection counter to the surface rotation and sinking currents (the **bands**) will deflect into the direction of the rotation. Use this to explain the band structure of the surface currents. This is called the **zonal flow**. Compare your understanding to figures 11.5 and 11.6 in the textbook.

1.3.3 Answer 7

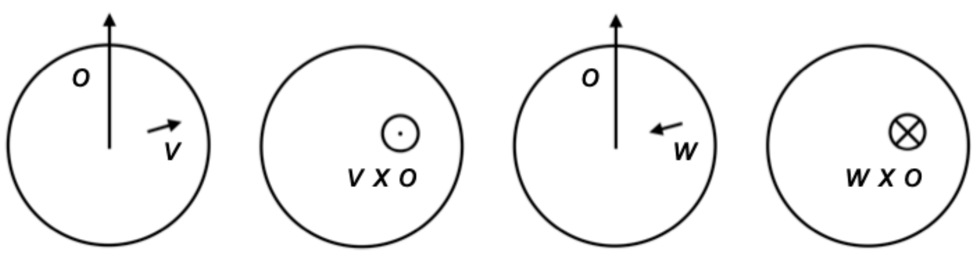


Fig A7:
Left: The vector $\vec{o}$ stands for $\vec{\omega}$ and represents the angular velocity of Jupiter. The vector and $\vec{v}$ represents the velocity of a particle flowing from the center to the surface. Middle left: The cross product $\vec{v} \times \vec{o}$ points out of the page and is denoted by $\odot$. Middle right: The vector and $\vec{w}$ represents the velocity of a particle sinking from the surface to the center. Right: The cross product $\vec{w} \times \vec{o}$ points into the page and is denoted by $\otimes$.

- Outward flow: The left two images in the diagram above represent the generic situation with a particle flowing out from the center to the surface as is the situation with the **zones**. The Coriolis term (second term) in equation (1) is $-2\vec{\omega} \times \vec{v}$. We want the direction of this term, so we drop the 2 and consider only $-\vec{\omega} \times \vec{v}$. As we discussed in class $\vec{\omega} \times \vec{v}$ and $\vec{v} \times \vec{\omega}$ point in opposite directions and have the same magnitude, so $-\vec{\omega} \times \vec{v} = \vec{v} \times \vec{\omega}$. Therefore, to find the direction of the acceleration of the particle, it suffices to determine the direction of $\vec{v} \times \vec{\omega}$.

- To find the direction of $\vec{v} \times \vec{\omega}$, we put out our hand with our fingers pointing in the direction of $\vec{v}$. We want to curl our fingers toward the upward direction, so we need to rotate our arm and wrist until that is possible. When we extend our thumb, we find it pointing out of the page. Therefore, a zonal flow at the point where the particle is will experience an acceleration out of the page.
- Imagine changing our vantage point so that this particle is coming outward and toward us. Then you should convince yourself that the acceleration will be to the left: This is a westward acceleration. Thus zones flow westward.
- You can repeat the whole argument for the sinking particle appropriate to the **bands**, but that is unnecessary because a sinking particle's vector differs from a rising one by reversing the direction. Thus, we can take $\vec{v} \mapsto -\vec{v}$. If we do this, we see that $\vec{v} \times \vec{\omega} \to -\vec{v} \times \vec{\omega}$ so the direction of the resulting vector is reversed. Thus, the bands flow eastward.

For a better understanding of the bizarre physics of rotating frames of reference, see this video on the Coriolis and centrifugal effects in rotating frames of reference.

1.3.4 Phases of hydrogen

It is difficult to understand the interior of Jupiter without talking in more detail about the phases of matter. In particular the phases of hydrogen. In class we will discuss phase diagrams in general and that of hydrogen (shown below) in particular. The general idea is that if we lower the temperature, then vapors condense to liquids and liquids freeze into solids. Similarly, if we increase the pressure, a vapor can be made to condense to a liquid or a liquid can be forced to solidify. In general then, the state of matter depends on the pressure and temperature.

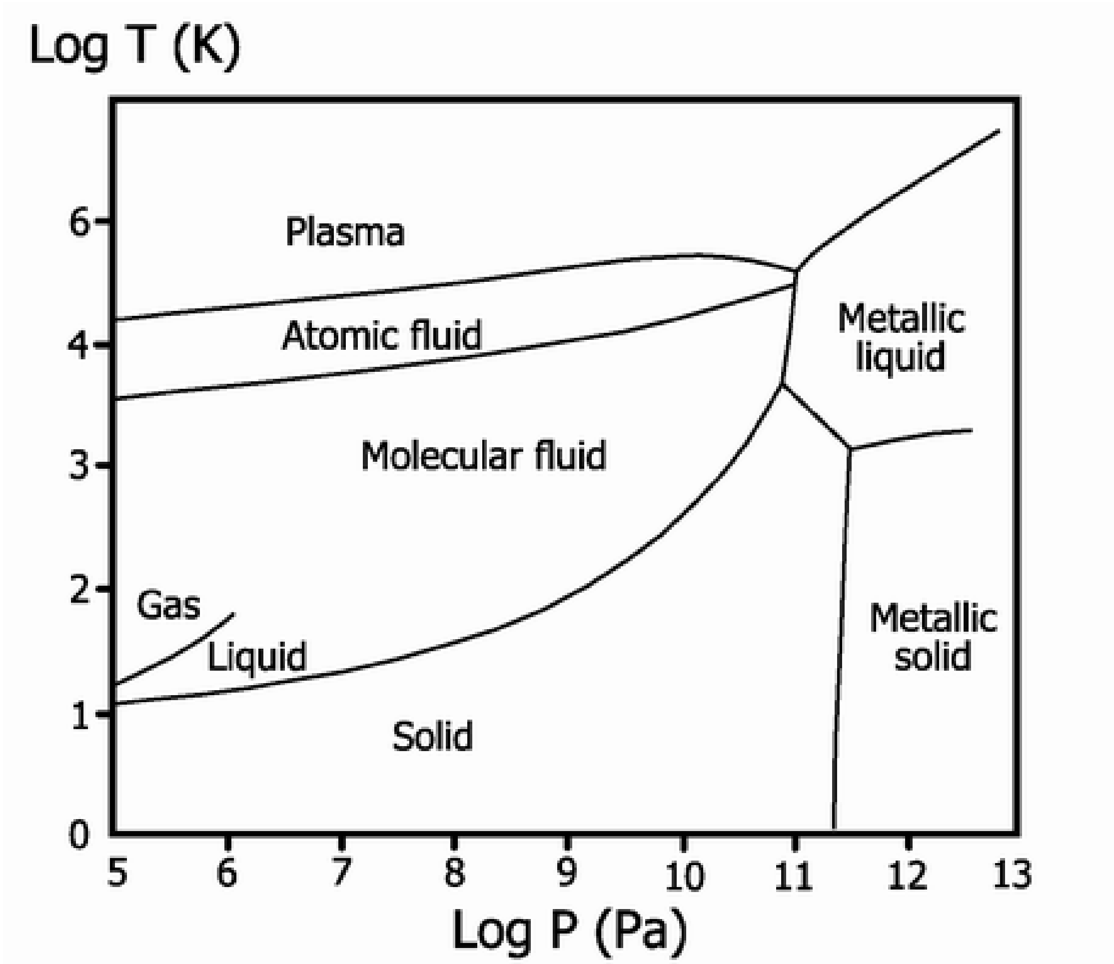


Fig1: The phase diagram of hydrogen

Note that in the plot above, it is the logarithm (base 10) of the temperature and pressure that are plotted. For example the point where the Gas/Liquid line ends has coordinates approximately $(\log(P), \log(T)) = (6, 2)$. This means that at that point in the diagram the pressure is $P = 10^6$ `Pa` $= 1$ `MPa` and the temperature is $T = 10^2 = 100$ `K`. Recall also from chemistry that the **pressure**

$$P = \frac{\text{Force}}{\text{Area}} \qquad \text{(pressure)}$$

is the force exerted per unit area. The standard units for pressure are **Pascals** with 1 `Pa` = 1 `N/m2`.

8. What is the pressure of our own atmosphere in these units? Show that this corresponds to a weight of 10 Newton for every square centimeter. Estimate the weight of the atmosphere on your body. Why are you not crushed under this enormous weight?

1.3.5 Answer 8

8. The pressure of our own atmosphere is 1 bar $= 10^5$ `Pa` = `N/m**2`. Converting to centimeters, 1 `m` = 100 `cm`, so 1 bar corresponds to $10^5/(100)^2 = 10$ `N/cm**2`. Assuming a body surface area ~ 2 `m**2`, we find a weight of $200,000$ `N`! We are

not crushed under this pressure because we are not experiencing it only from the outside. The air in our lungs is also at this pressure, so pressures are acting throughout the body uniformly.

Note: I always thought that his meant that if we suddenly depressurized from 1 bar to the 0 bar of space, that we would explode, unlike what is represented in some sci-fi movies. It turns out that I was wrong about that: I looked around for an explanation, and the consensus seems to be that muscle and skin tissue is strong enough to keep the body from exploding!

9. Go back to your cross section and include the pressure ranges in megapascals (MPa) for the different layers. Together with the temperatures, check which phase of matter hydrogen is in for your various regions using the phase diagram for hydrogen given above. Do they match the reported phases?

1.3.6 Answer 9

9. The cross-sectional diagram has the following regions: …

Here is a video about Jupiter's interior.

1.4 Jupiter's satellites

Recall from your Kepler practice that Jupiter has four large moons (a.k.a. the **Galilean moons**) called **Io**, **Europa**, **Ganymede**, and **Callisto**. In standard natural satellite terminology, these are also known as Jupiter I – IV. (Yes, the Moon is a.k.a Earth I.)

	Io	Europa	Ganymede	Callisto
Orbital radius (km)	421,800	671,100	1,070,400	1,882,700
Orbital radius (R_jupiter)				
Orbital period (days)		3.55		
Rotational period (days)	1.77	3.55	7.15	16.7
Mass (kg)				
Diameter (km)				
Density (g/cm3)				

10. Use what you know about Jupiter and Kepler's laws to finish filling out the second and third rows.

[4]:
```python
from astropy.constants import G
from astropy.units import km, d
from aa_tools.astroconst import galileans, a_orb, P_orb

t="2026-01-03T00:00:00" # set epoch to today

mu_jupiter = G*masses["jupiter"]

a_galilean = { b: a_orb( b, "jupiter", t, False ) for b in galileans }
```

```
P_galilean = { b: P_orb( b, "jupiter", t, False ) for b in galileans }

print(f"\n{"moon":^10} {"a (km)":^15} {"a (R_jup)":^5} {"P (dy)":^15}")
print(f"------------------------------------------------------------")
for b in a_galilean:
    print(f"{b.capitalize():^10} {round(a_galilean[b].to_value(km)):^15}␣
↪{round(a_galilean[b]/radii["jupiter"],1):^10} {round(P_galilean[b].to(d)/
↪d,2):^10}")
```

```
   moon         a (km)       a (R_jup)      P (dy)
-----------------------------------------------------------
    Io          422038          6.0         1.77
  Europa        671264          9.6         3.55
 Ganymede       1070859         15.3        7.16
 Callisto       1883676         26.9        16.7
```

11. What do you notice about the orbital and rotational periods? Explain what this says about the moons as they orbit. Explain how this relates to the motion of our own Moon.

1.4.1 Answer 11

The rotational periods are equal to the orbital periods. Thus, the same side of each moon always faces Jupiter. This is also true for our moon. More on this **tidal locking** below.

12. Calculate the *ratios* of the orbital periods of the moons. What do you notice?

[5]:
```
it = iter(P_galilean.items())
b_prev, P_prev = next(it)   # first pair

for b_cur, P_cur in it:
    r = (P_cur/P_prev).decompose()
    print(f"The ratio of the period of {b_cur.capitalize()} and {b_prev.
↪capitalize()} is {r:.2}.")
    b_prev, P_prev = b_cur, P_cur

print(f"We see that Io goes around twice in the time it takes Europa to orbit␣
↪once. Similarly, Europa goes around twice while Europa orbits once and Io␣
↪orbits 4 times. Callisto is not participating in this dance.")
```

```
The ratio of the period of Europa and Io is 2.0.
The ratio of the period of Ganymede and Europa is 2.0.
The ratio of the period of Callisto and Ganymede is 2.3.
We see that Io goes around twice in the time it takes Europa to orbit once.
Similarly, Europa goes around twice while Europa orbits once and Io orbits 4
times. Callisto is not participating in this dance.
```

The phenomenon that the moons' orbital periods are multiples of one another is called **orbital**

resonance (see this page). The specific 1:2:4 resonance is called a Laplace resonance. The basic idea behind this is that if energy is pumped into a system periodically, that system tends to also become periodic with a period that is related to the period of the driving force.

The phenomenon that the Moon rotates on its axis with the same period as its revolution around the Earth (and in the same sense) is called **tidal locking** (see this page).

13. The diagram below is a sketch representing the gravitational acceleration on the surface of a spherical body caused by a very massive object (the dot on the right) in close proximity. It is an exaggeration of the gravitational field on the surface of a secondary (*e.g.* a moon) co-orbiting a primary (*e.g.* a planet). In the accompanying diagram draw the analogous representation of the gravitational field as it appears to someone on the secondary. To do this, take the vector in the middle labeled C, and subtract it from all the other vectors. You should get a field that looks symmetrical across the vertical axis through the center of the body.

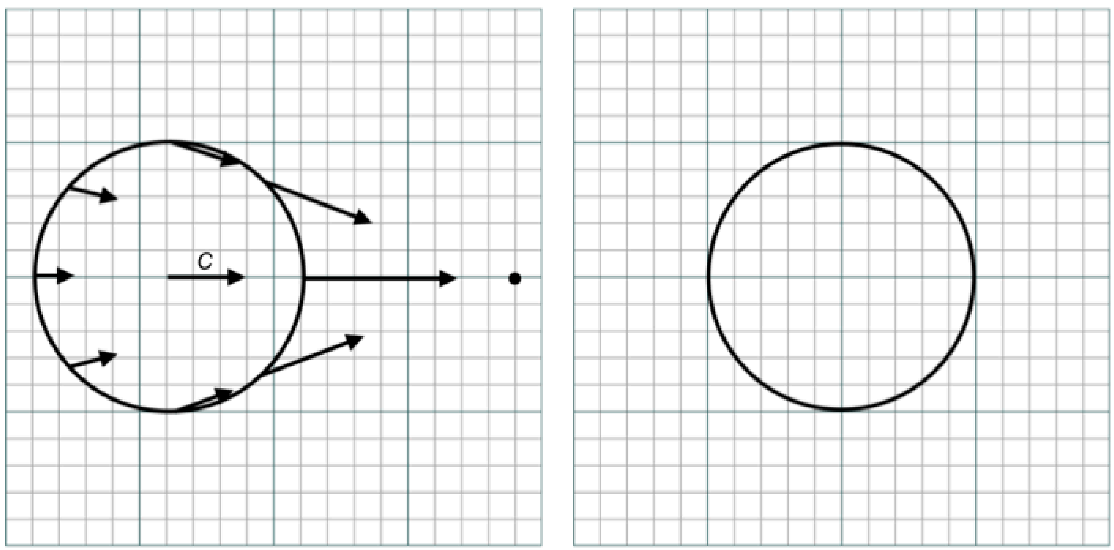
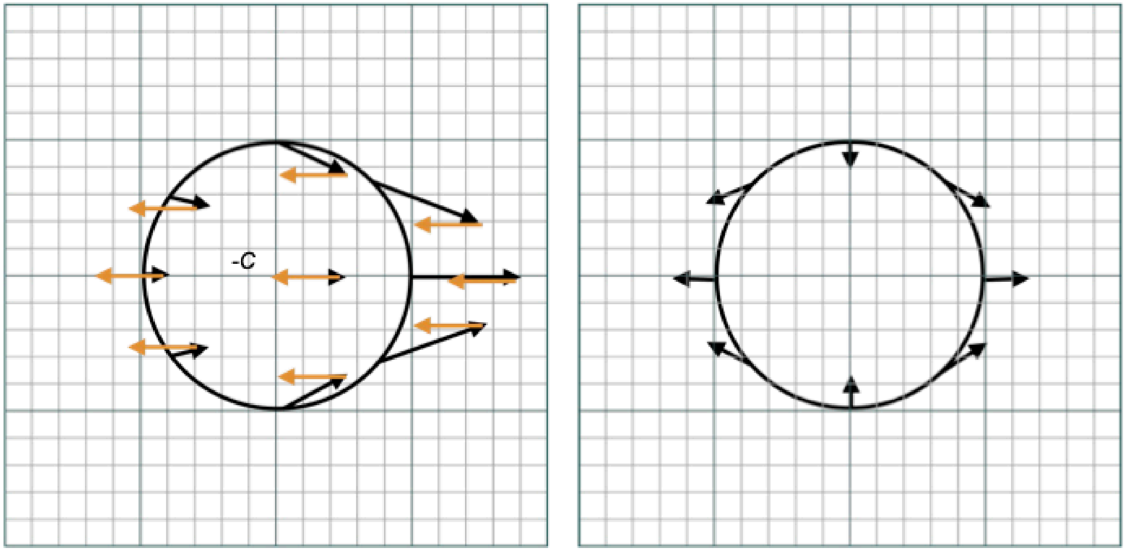


Fig2 Left: Tidal field as seen in an inertial frame. Right: Construct the tidal field as seen from the (non-inertial) revolving body's frame by subtracting the middle vector C in the figure on the left from all the other vectors in that figure.

1.4.2 Answer 13



Left: Subtraction of C is vector addtion with $-C$ (shown in orange). Right: The resultant of the vector additions gives the field in the corotating frame. This represents what the body experiences in its own rest frame.

This is called the **tidal field**. It shows that a secondary body is stretched in the direction of the primary and squeezed in the perpendicular direction. If a body is rotating on its axis, this tidal field will drag on the body, causing its rotation to slow down. This is called **tidal friction**. Eventually, its rotational period will match its orbital period, resulting in **tidal locking**.

14. If the orbit of the secondary is elliptical, then Kepler's Second Laws implies that the near side must **librate**: It cannot stay perfectly aligned and seems to wobble. Here's a gif of the libration of the Moon; what do you see happening? Explain this libration from the constant rotational speed of the Moon on its axis and the implications of Kepler's second law concerning the speed of the Moon on its elliptical orbit.

1.4.3 Answer 14

If the orbit is elliptical, then the speed of the body in its orbit will be faster than average at perigee and slower at apogee. Since the rotational speed is not changing, this implies that at perigee, the rotation is too slow to keep up and therefore slightly behind and at apogee it is too fast and slightly ahead. This slight lag/advance repeats over one orbital period, resulting in the libration. Furthermore, since the Moon is closer at perigee it looks a bit bigger than average and at apogee it is smaller.

15. Previously, we have discussed that angular momentum must be conserved and that this causes rotational spin-up when mass contracts toward a center. If tidal friction spins an object down instead of up, what do you expect should happen to the orbit of that object?

1.4.4 Answer 15

To conserve angular momentum, the object losing rotational angular momentum must somehow be gaining orbital angular momentum and/or the primary must be spinning up. In the former case, then the primary must be moving further out. In the case of the Earth and the Moon, the Earth's angular speed is higher than the Moon's orbital speed, so the tidal force of the Moon on the Earth will cause the rotation to slow. This torque on the Earth much have an equal and opposite torque acting on the Moon's orbit causing it to gain angular momentum, which it does my moving further away.

Io is tidally locked to Jupiter but it is in orbital resonance with Europa. This resonance causes periodic tugs on Io that keep its orbit shaped as an ellipse. Because of this, Io is constantly undergoing libration that deforms its shape. This perpetual deformation heats Io just like how repeatedly bending metal heats up the metal, so Io presumably has a sea of molten lava under its crust. Indeed, although Io should be a long-dead space rock, it is instead the most geologically active object in the solar system. Its hottest volcanoes spew heavy ions (mostly sulfur) into the magnetosphere you speculated that Jupiter should have in problem 6.

16. Let's assume Jupiter's magnetosphere rotates at the same rate as Jupiter. (In fact, we get an accurate measurement of Jupiter's rotation rate precisely by looking at this magnetosphere.) Together with the Alvén theorem (a.k.a. frozen-in theorem), argue that the heavy ions from Io are dragged around and smeared out into a toroidal tube with a radius equal to that of Io's orbit. This is called Io's **plasma torus** (see here). What is the relative speed of the plasma wind that you would experience if you were standing on the surface of Io? What do you think the radiation levels are like on Io?

1.4.5 Answer 16

The speed of the wind is the difference between Io's orbital speed and speed of the plasma being dragged around with Jupiter's rotation. The angular speed of Jupiter's rotation is $\omega_J = \frac{2\pi}{P_{rot,J}}$ and orbital angular speed (a.k.a. the mean motion) of Io $n_{Io} = \frac{2\pi}{P_{Io}}$. The speed of the plasma at Io's orbital distance is $v_{plasma} = a_{Io}\omega_J$ and Io's speed is $v_{Io} = a_{Io}n_{Io}$. So the difference $v_{rel} = v_{plasma} - v_{Io} = a_{Io}(\omega_J - n_{Io}) = ...$

[6]:
```
from astropy.units import s

omega_J = (2*pi/periods["jupiter"]).to(1/s)

n_io = 2*pi/P_galilean["io"]

v_plasma= a_galilean["io"]*omega_J

v_io=a_galilean["io"]*n_io

print(f"Io's orbital speed is {round(v_io)} and the speed of the plasma in that␣
↪location is {round(v_plasma)} in the same direction. Therefore the trailing␣
↪side of Io is experiencing a wind of damaging ions with a speed of␣
↪{round(v_plasma-v_io)}.")
```

```
Io's orbital speed is 17.0 km / s and the speed of the plasma in that location
is 74.0 km / s in the same direction. Therefore the trailing side of Io is
experiencing a wind of damaging ions with a speed of 57.0 km / s.
```

17. Go back to the table above and look up the masses and diameters of the Galilean moons. Use these numbers to calculate the densities in g/cm^3 and fill in the last row. What do you notice? Thinking back to the nebular hypothesis, formulate a hypothesis explaining this observation.

[7]:
```python
import aa_tools.astroconst as ac
from astropy.units import kg

masses_galileans = { b: getattr(ac, f"M_{b}") for b in galileans }
radii_galileans = { b: getattr(ac, f"R_{b}") for b in galileans }

print(f"\n{"body":^10} | {"mass(kg)"}| {"diam(km)"}| {"rho(g/cm^3)"} ")
print("---------------------------------------------")
for b in galileans:
    M=masses_galileans[b]
    R=radii_galileans[b]
    rho=3*M/(4*pi*R**3)
    print(f"{b.capitalize():^10} | {M/kg:.1e} | {(2*R).to_value(km):.0f} | {rho.
↪to_value(g/cm**3):.1f} ")
```

```
   body    | mass(kg)| diam(km)| rho(g/cm^3)
---------------------------------------------
    Io     | 8.9e+22 | 3643 | 3.5
  Europa   | 4.8e+22 | 3122 | 3.0
 Ganymede  | 1.5e+23 | 5262 | 1.9
 Callisto  | 1.1e+23 | 4821 | 1.8
```

1.4.6 Answer 17

The whole system resembles a small solar system. Jupiter has a density that is only slightly higher than that of the Sun, while Io has a density only slightly lower than the rocky planet Mars. The density of the moons decreases as we go further out which is also (roughly) the case for the planets of the solar system. Although it may be incorrect, it is natural to postulate that the Jovian system formed very similarly to the solar system itself. To carry on with the analogy, not that…

[8]:
```python
print(f"... Jupiter is roughly {round(masses["jupiter"]/
↪masses_galileans["ganymede"],-4)} times more massive than the Ganymede, and␣
↪{round(radii["jupiter"]/radii_galileans["ganymede"],0)} times larger in␣
↪diameter.\nThese numbers should be compared to the Sun/Jupiter ratios of␣
↪1000 and 10 that we calculated in question 1.")
```

```
… Jupiter is roughly 10000.0 times more massive than the Ganymede, and 27.0
times larger in diameter.
```

```
These numbers should be compared to the Sun/Jupiter ratios of 1000 and 10 that
we calculated in question 1.
```

Besides the 4 Galilean Moons, there are four smaller satellites within Io's orbit called the **Amalthea group** or the **inner moons** consisting of **Metis**, **Adrastea**, **Amalthea**, and **Thebe**. All are tidally locked with Jupiter.

18. Make another table of the orbital radii of the Amalthean group. Write the radii in multiples of Jupiter's radius, and check that their periods obey Kepler's third law.

[9]:
```python
from aa_tools.astroconst import amaltheans, a_orb, R_jupiter, P_orb
from astropy.units import hr

#ac.a_orb("io", "jupiter", t)

a_amaltheans = { b: a_orb(b, "jupiter", t) for b in amaltheans }
P_amaltheans = { b: P_orb(b, "jupiter", t, False) for b in amaltheans }

print(f"\n {"body":^10} | {"orbital radius (R_J)"} | {"orbital radius (R_J)"} |␣
↪{"orbital period (hr)"} | {"a^3/P^2":^13} ")
print(f"--------------------------------------------------------------------------------
for b in amaltheans:
    #check kepler (circular reasoning really, since kepler was used to get␣
↪periods)
    kepler=a_amaltheans[b].to_value(km)**3/P_amaltheans[b].to_value(hr)**2
    #
    print(f" {b.capitalize():^10} | {int(round(a_amaltheans[b].to_value(km),␣
↪-2)):^20} | {round(a_amaltheans[b]/R_jupiter,2):^20} |␣
↪{round(P_amaltheans[b].to_value(hr),1):^20}|   {kepler:.3e} ")

print(f"\nRecall that Jupiter's radius is {R_jupiter:.2e}, and that it rotates␣
↪with a period of 10 hours.\nThe quantity a^3/P^2 has been represented in the␣
↪units of R_J^3/hr^2.")
```

```
    body     | orbital radius (R_J) | orbital radius (R_J) | orbital period (hr)
|    a^3/P^2
--------------------------------------------------------------------------------
---------------
   Metis     |        128900        |         1.84         |         7.2
|   4.159e+13
  Adrastea   |        129900        |         1.86         |         7.3
|   4.159e+13
  Amalthea   |        182000        |         2.6          |         12.0
|   4.159e+13
   Thebe     |        222400        |         3.18         |         16.3
|   4.159e+13

Recall that Jupiter's radius is 6.99e+04 km, and that it rotates with a period
```

```
of 10 hours.
The quantity a^3/P^2 has been represented in the units of R_J^3/hr^2.
```

19. Calculate the radius of a Jovio-stationary orbit. (Hint: Use the formula for the orbital radius in our equation sheet.) Analogously to geostationary orbits, this is an orbit in the equatorial plane with a period that is equal to the rotational period of Jupiter. Compare the radius of this orbit to those of the Amalthean moons. What do your findings indicate about Metis and Adrastea?

1.4.7 Answer 19

The orbital radius formula can be recalled by remembering that the gravitational paremeter $\mu = GM$ has dimensions L^3/T^2. Therefore μP^2 has dimensions of L^3 and its cube-root is a length. We must now simply remember that there was a factor of 2π accompanying P. Therefore,

$$r = \sqrt[3]{\frac{GMP^2}{4\pi^2}}$$

For a jovio-synchronous orbit, we want $P = P_{rot,J}$ to be Jupiter's rotation period, which we found already in problem ??. Plugging this into our formula, we find a the jovio-synchronous orbit radius…

[10]:
```python
P_rot_jupiter = periods["jupiter"]

r_joviostat = ( (mu_jupiter*P_rot_jupiter**2/4/pi**2)**(1/3) ).decompose()

rat_joviostat = (r_joviostat/R_jupiter).decompose()

print(f"The jovio-sychronous orbit radius is {r_joviostat.to(km):.2e}. This translates to  {round(rat_joviostat,2)} Jupiter radii.")
```

```
The jovio-sychronous orbit radius is 1.60e+05 km. This translates to  2.29
Jupiter radii.
```

We see that Metis and Adrastea orbit faster than Jupiter's rotation, whereas Amalthea and Thebe orbit more slowly. This implies that if it were possible to see all four moons from the surface in the Northern Hemisphere (it's not: they are way too small and there is no "surface" to speak of), the two inner moons would slowly move eastward as the two outer moons drift westward.

[11]:
```python
    """
    import importlib
    import aa_tools.astroconst as ac
    importlib.reload(ac)
    """
```

[11]: `'\nimport importlib\nimport aa_tools.astroconst as ac\nimportlib.reload(ac)\n'`