ss_solar_system_1

October 21, 2025

1 Solar System I: Structure

The solar system consists of the titular star, the 8 planets **Mercury**, **Venus**, **Earth**, **Mars**, **Jupiter**, **Saturn**, **Uranus**, and **Neptune**. In between Mars and Jupiter we find the **asteroid belt**, and beyond Neptune there is the **Kuiper belt**.

1. The mean distance between the **Sun** and the Earth is 1 AU by definition. This is about 150 million kilometers. How many seconds does it take for the light emitted by the sun to reach us?

2 BEGIN ANSWER

```
[1]: from aa_tools.astroc import sf, AU, km, c
     from math import floor
     Delta_t=AU/c

     minute=60.0

     minutes=floor(Delta_t/minute)
     seconds=floor(Delta_t-minutes*minute)

     print(f"At the speed c = {sf(c)} m/s it takes a time of distance/c =␣
       ↪{round(Delta_t)} seconds.")
     print(f"That's about {minutes} minutes and {seconds} seconds.")
```

```
/Users/wdlinch3/Documents/GitHub/aa-tjhsst/.venv/lib/python3.12/site-
packages/vpython/__init__.py:1: UserWarning: pkg_resources is deprecated as an
API. See https://setuptools.pypa.io/en/latest/pkg_resources.html. The
pkg_resources package is slated for removal as early as 2025-11-30. Refrain from
using this package or pin to Setuptools<81.
  from pkg_resources import get_distribution, DistributionNotFound

<IPython.core.display.HTML object>

<IPython.core.display.Javascript object>

At the speed c = 3.0e+08 m/s it takes a time of distance/c = 499 seconds.
That's about 8 minutes and 19 seconds.
```

3 END ANSWER

Table 6.1 on page 138 of the textbook has a table with information about the objects in the solar system. Part of it is reproduced here:

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
M $(M_\oplus)$	0.055	0.82	1	0.11	318	95	15	17
R (R)	0.38	0.95	1	0.53	11.2	9.5	4.0	3.9
ρ (g/cm^3)	5.4	5.2	5.5	3.9	1.3	0.7	1.3	1.6
a (AU)	0.39	0.72	1	1.52	5.2	9.5	19.2	30.1
e	0.21	0.01	0.02	0.09	0.05	0.05	0.05	0.01
P (yr)	0.25	0.62	1	1.9	11.9	29.4	84	164

Table 1: Physical data of the planets relative to the Earth's mass $M_\oplus = 6.0 \times 10^{24}$ kg, radius $R_\oplus = 6400$ km, and orbital radius 1 AU $= 1.5 \times 10^8$ km.

2. Make a scale model showing the distances of the planets from the Sun.

4 BEGIN ANSWER

5 END ANSWER

3. Make a scale model of the sizes of the planets.

6 BEGIN ANSWER

7 END ANSWER

4. Is it feasible to make a model that accurately represents the scales of the sizes and the distances simultaneously? If so, please make one. If not, why not?

8 BEGIN ANSWER

9 END ANSWER

5. The 8 planets split into two groups using the physical features described in the table. Describe your splitting by referring explicitly to the concepts of size, mass, density ρ, and proximity to the Sun. The members of these two groups are referred to as **terrestrial** or **Jovian**. Explain why you think that is.

10   BEGIN ANSWER

11   END ANSWER

The terrestrial planets have some further similarities and differences. For example, they all have **atmospheres** although they are either much thinner than Earth's (Mercury and Mars) or thicker and hotter (Venus). Only the Earth has oxygen and liquid water on its surface. Earth and Mars have moons, but Mercury and Venus do not. Earth and Mercury have (weak) **magnetospheres**, but Venus and Mars do not. Earth and Mars spin at (almost) the same rate, but Mercury and Venus spin much more slowly, and Venus even spins in the opposite direction (**retrograde**).

6. Add up the masses of all the terrestrial planets except Earth and compare the result to the mass of the Earth.

12   BEGIN ANSWER

```
[2]: from aa_tools import astroc
     planets=["mercury", "venus", "earth", "mars", "jupiter", "saturn", "uranus",
     ↪"neptune"]

     masses={ i : getattr(astroc, f"M_{i}") for i in planets }
     M_earth=masses["earth"]

     print(f"\n{"planet":^10} {"mass (kg)":^15} {"mass (M_earth)":^15}")
     print("------------------------------------")
     for key, value in masses.items():
         #print(masses.items())
         print(f"{key.capitalize():<10} {value:^15.2e} {round(value/M_earth,2):^15}")
     print("")

     mass_terrestrial=sum(list(masses.values())[:4])

     print(f"The sum of the masses of all the terrestrial planets (other than Earth)
     ↪is {sf(mass_terrestrial)} kg.")
     print(f"This is almost exactly {round(mass_terrestrial/M_earth)} Earth masses.
     ↪\n")
```

```
  planet        mass (kg)      mass (M_earth)
------------------------------------------
Mercury       3.30e+23            0.06
Venus         4.87e+24            0.81
Earth         5.98e+24             1.0
Mars          6.42e+23            0.11
Jupiter       1.90e+27          317.94
Saturn        5.68e+26           95.05
Uranus        8.68e+25           14.52
Neptune       1.02e+26           17.07
```

The sum of the masses of all the terrestrial planets (other than Earth) is
1.2e+25 kg.
This is almost exactly 2 Earth masses.

13 END ANSWER

7. Add up the masses of all the planets except Jupiter and compare it to the mass of Jupiter.

14 BEGIN ANSWER

```
[3]: M_jupiter=masses["jupiter"]
     mass_njupiter = sum( v for k, v in masses.items() if k != "jupiter" )

     print(f"The mass of all the planets except Jupiter is {mass_njupiter:.1e} kg,␣
       ↪whereas Jupiter has a mass of {M_jupiter:.1e} kg.")
     print(f"Thus, Jupiter is {round(M_jupiter/mass_njupiter, 1)} times more massive␣
       ↪than all the other planets combined.")
```

The mass of all the planets except Jupiter is 7.7e+26 kg, whereas Jupiter has a
mass of 1.9e+27 kg.
Thus, Jupiter is 2.5 times more massive than all the other planets combined.

15 END ANSWER

8. By what factor is the mass of the Sun larger than the sum of the masses of all the planets?
What percentage of the mass of the solar system is accounted for by the Sun alone?

16 BEGIN ANSWER

```
[4]: from aa_tools.astroc import M_sun

     #print(f"The mass of all the planets is {sf(mass_njupiter+M_jupiter)} kg.")
     print(f"The mass of the Sun is {sf(M_sun)} kg, whereas the mass of all the␣
       ↪planets is {sf(mass_njupiter+M_jupiter)} kg.")
     print(f"Thus mass of the Sun is {round( M_sun/(mass_njupiter+M_jupiter) )}␣
       ↪times larger than the mass of the planets combined.")
     print(f"In other words, the Sun accounts for {round(M_sun/
       ↪(mass_njupiter+M_jupiter+M_sun)*100, 2)}% of the mass of the solar system.")
```

The mass of the Sun is 2.0e+30 kg, whereas the mass of all the planets is
2.7e+27 kg.
Thus mass of the Sun is 745 times larger than the mass of the planets combined.
In other words, the Sun accounts for 99.87% of the mass of the solar system.

17   END ANSWER

As you just calculated, 99.9% of the mass of the solar system resides in the Sun. The 8 planets orbiting the Sun are evenly split between the terrestrial planets (a.k.a. the **rocky planets**) and the Jovian planets (a.k.a. the **gas giants**). The rocky planets all orbit within 1.5 AU. The gas giants inhabit the space between roughly 5 AU and 30 AU.

17.1   Asteroids

Between Mars and Jupiter there is a region called the **asteroid belt** or the **main (asteroid) belt**, consisting of small rocky, metallic, or icy bodies called **asteroids**. It is an example of a **circumstellar disk**, although it should really be called a circumstellar donut because it is shaped like a solid torus around the Sun. (Circumstellar means "star-surrounding".) This "disk" consists of millions of small asteroids separated on average by a distance of one million kilometers. (This means that contrary to sci-fi representations, a spaceship blasting through the belt has almost no chance of reaching an asteroid by accident.) By far the largest object in the asteroid belt is the dwarf planet **Ceres**, with a diameter of 950 km. The only other "large" asteroids in the belt are Vesta, Pallas, and Hygiea with diameters around 500 km.

Beyond Neptune, there is another circumstellar disk called the **Kuiper belt**. It extends from around 30 AU out to a distance of around 50 AU. Whereas most members of the asteroid belt are rocky and/or metallic, most bodies in the Kuiper belt are made of frozen methane, ammonia, and water. Additionally, the dwarf planets **Pluto**, Orcus, Haumea, Quaoar, and Makemake.

The Kuiper belt objects are a subset of the **trans-Neptunian object**s (TNOs). Outside of the Kuiper belt we encounter the scattered disk objects, extreme trans-Neptunian objects, and finally the **Oort cloud object**s. The Oort cloud is a hypothetical spherical shell of icy planetesimals surrounding the Sun between radii of 2,000 to 200,000 AU that was proposed to explain the existence of comets with extremely elliptical and highly inclined orbits (*e.g.* Halley, Hale-Bop, and the recent visitor Tsuchinshan-ATLAS). Such comets are eventually destroyed by their repeated encounters with the Sun, so there must be a large supply of them for us to continue to see them in our mature solar system.

9. One explanation for the asteroid belt is that Jupiter's presence disrupted the formation of what would have been a planet orbiting at the radius of about 2.6 AUs. Let's use this to check my claim about the distance between asteroids in the belt.

The total amount of mass estimated to be in the belt is 3% of the mass of the Moon. Assume an average density of $\rho = 3 \ g/cm^3$ (somewhat less than the Moon and Mars) and model the belt as a solid torus with primary radius $r_a = 2.6 \ AU$, and an unknown inner radius r_b.
a. Find the value of r_b assuming everything in the belt is a fine uniform dust.
b. The material is not a fine unifor dust but a bunch of macroscopic bodies of various sizes ranging from many small pebbles to a handful of objects $\sim 10 \ km$ in diameter (fig. 2 in Bottke *et al.* 2015). The vast majority are small $\sim 1 \ m$, and an analysis of the curve suggests there are just as many asteroids smaller than $\sim 6 \ m$ as there are larger ones. Pretend, then, that the asteroids are all spheres of radius 3 m and calculate the average separation of the asteroids if the inner radius of the belt is measured to be $r_b = 0.6 \ AU$.

18   BEGIN ANSWER

a. The mass of the belt is $M_{belt} \approx \frac{3 \times M_\oplus}{100}$. With a density of $\rho = 3g/cm^3 = 3000kg/m^3$, and the volume occupied by belt is $V_{belt} = \frac{M_{belt}}{\rho}$. The volume of the ring modeled as a solid torus of pirmary radius r_a and inner radius r_b is $V_{belt} = (2\pi r_a) \times (\pi r_b^2) = 2\pi^2 r_a r_b^2$. Therefore,

$$2\pi^2 r_a r_b^2 = \frac{M_{belt}}{\rho} \qquad \Rightarrow \qquad r_b^2 = \frac{M_{belt}}{2\pi^2 \rho r_a} \tag{1}$$

and we find an inner radius for the torus of

$$r_b = \sqrt{\frac{M_{belt}}{2\pi^2 \rho r_a}}$$

```
[5]: from math import pi, sqrt
     M_belt=3/100*M_earth
     rho=3000
     r_a=2.6*AU

     r_b=sqrt(
         M_belt/(
             (2*pi**2)*rho*r_a
         )
     )

     print(f"a. This formula gives an inner radius for the belt of r_b = {round(r_b/
       ↪km)} km.")
```

a. This formula gives an inner radius for the belt of r_b = 3 km.

b. First we determine the number of asteroids. A ball of radius R has a volume $\frac{4}{3}\pi R^3$. With a density of $\rho = 3000 \ km/m^3$, this gives a mass of $M_{asteroid} = \frac{4}{3}\pi \rho R^3$. With a total mass of M_{belt} this gives a number
$$N = \frac{M_{belt}}{M_a steroid} = \frac{3M_belt}{4\pi \rho R^3}$$
asteroids in the belt.

These asteroids take up a volume $V_{belt} = 2\pi^2 r_a r_b^2$, as we discussed in part a. Therefore the volume per asteroid is $V_{asteroid} = \frac{V_{belt}}{N}$, or

$$V_{asteroid} = \frac{4\pi \rho R^3 V_{belt}}{3M_{belt}} = \frac{8\pi^3 \rho R^3 r_a r_b^2}{3M_{belt}} = (\pi R)^3 \frac{8\rho r_a r_b^2}{3M_{belt}}$$

The linear distance between asteroids scales like the cube-root of this volume. Therefore, we expect the average distance $\langle r \rangle$ to very roughly be

$$\langle r \rangle \sim \pi R \sqrt[3]{\frac{8\rho r_a r_b^2}{3M_{belt}}}$$

```
[6]: # overload definition of r_b
     r_b=0.6*AU

     # quoted asteroid radius in m
     R=6.0

     # mass of one such spherical asteroid
     M_asteroid = 4/3*pi*R**3

     # number of such asteroids in the belt
     N=M_belt/M_asteroid

     # ugly mess in cube root
     argument=(8*rho*r_a*r_b**2)/(3*M_belt)

     # formula for average separation in terms of ugly combination
     r_ave = pi*R*argument**(1/3)

     print(f"Plugging in the numbers, we find:")
     print(f"There a spherical asteroid with radius R=10 m has a mass of M_asteroid␣
       ↪= {round(M_asteroid)} kg.")
     print(f"With a total mass of M_belt = {M_belt:.1e} kg, this gives N = {N:.1e}␣
       ↪asteroids.")
     print(f"Putting these numbers together, we get a rough estimate for their␣
       ↪average separation of {round(r_ave/km)} km.")
```

```
Plugging in the numbers, we find:
There a spherical asteroid with radius R=10 m has a mass of M_asteroid = 905 kg.
With a total mass of M_belt = 1.8e+23 kg, this gives N = 2.0e+20 asteroids.
Putting these numbers together, we get a rough estimate for their average
separation of 978 km.
```

There are many problems with this very crude analysis, but the most egregious is probably the approximation that all the asteroids are 6 m in diameter. In fact, the distribution of asteroid sizes is not simple. From the reference provided, you can extract a distribution funtion for the number of asteroids of size at least D (in km) of the form $N = 10^{10-\frac{1}{2}D}$. This distribution is not even power-law, so we suspect a naïve use of an average radius of 6 meters for an asteroid is not a very good approximation. In fact, the esimated number of asteroids in the belt is in the millions, not the 10^{20} that we found.

As a concrete objection, let us note that the largest known asteroid in the main belt is Ceres, which we already know a previous packet has a mass of $M_{ceres} \approx 9.4 \times 10^{20}$. Therefore, it alone accounts for 52% of the mass of the belt. Dropping this single asteroid from the pool therefore increases the average separation by almost 20%.

Nevertheless, this analysis goes a long way to giving us some intuition for why asteroid belts are not dangerous: Taking a moon's worth of mass and spreading it out over a torus of radius 2-3 AU makes it *really* sparsely-populated.

19 END ANSWER