ss_sol

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1 Sol

In the previous section, we explored the ideas leading to the standard model for the formation and evolution of the solar system. In doing so, we were able to account for many of the gross structural features of the solar system such as

- a small number of well-separated planets revolving to within seven degrees of the ecliptic with most rotating in the same direction (in agreement with their revolution)

- differentiation of planetary composition and inner planets of terrestrial type and outer ones of jovian type

- the existence of two circumstellar rings (main and Kuiper belts) and a circumstellar spherical shell (Oort cloud)

- the mini-solar system structure of the jovian planets

We will return to study the planets and their features in more detail, but first we turn our attention to the most glaring omission in this list: the Sun.

When we last left this part of our proto-planetary disk, it was collapsing under the gravitational attraction between the huge number of particles in this part of the cloud fragment. In problem 3 of the previous packet, you verified that the time that fragmentation ceases, the density of the proto-planetary cloud fragment had a density of 10-9 kg/m3 and a central temperature of 10,000 K. Before diving back into the theory of solar formation, let us first take a step back and study those features of the Sun that we can deduce from observation.

2 Gross features

We begin, as we did with our study of the solar system, but familiarizing ourselves with the gross features of the Sun.

2.1 Size, distance, and Mass

1. In class, we discussed how to use angular size and known distance to work out absolute size.

a. We also mentioned that the Sun and the moon both have an angular size of _________ degrees.

b. The Sun is at a distance $r = 1$ au from us which is equivalent to ____________ km.

c. From this and the formula $diameter = \frac{2\pi}{360^\circ}\alpha r$ (with α in degrees), show that the Sun has a radius of approximately $650,000$km.

d. Pull the true radius of the Sun from `astropy.constants`, and calculate how many Suns would fit in 1au.

e. Since the radius of Earth is $R_\oplus = 6,400$ km this means $R_\odot =$ ____________ $R_\oplus$.

[1]:
```python
from astropy.constants import au, R_sun, R_earth
from astropy.units import km
from math import pi, floor

alpha=0.5 #degrees

print(f"1a. alpha = {alpha} degrees.")
print(f"1b. 1 au = {au.to(km):.2e}.")

R=pi/360*alpha*au
N=(au-R_sun)/(2*R_sun)

print(f"1c. R_sun = {int(round(R.to(km).value,-4))} km.")
print(f"1d. At the more accurate radius of {R_sun.to(km):.2e}, {floor(N)} Suns␣
 ↪would fit between us and it.")
print(f"1e. R_sun = {int(round(R_sun/R_earth, 0))} R_earth.")
```

```
1a. alpha = 0.5 degrees.
1b. 1 au = 1.50e+08 km.
1c. R_sun = 650000 km.
1d. At the more accurate radius of 6.96e+05 km, 107 Suns would fit between us
and it.
1e. R_sun = 109 R_earth.
```

2. The mass of the Sun can be determined from Newton's laws. Recall that $GM = 4\pi^2\frac{r^3}{P^2}$ where $G = 6.7\times10^{-11}m^3/kg/s^2$ is the gravitational constant, M is the mass of the principal (in this case the Sun), r is the radius of the orbit of the secondary, and P is the period of the orbit.

a. Use what you know about our orbit around the Sun to confirm that the mass of the Sun is $M_\odot = 2.0 \times 10^{30}$ kg.

b. Using that the mass of the Earth is $M_\oplus = 6.0 \times 10^{24}$ kg, the mass of the Sun is about ________ times the mass of the Earth.

c. Calculate the **average mass density** of the Sun. Check this from table 6.1 and compare this to the density of the planets. Explain in what sense it is large or small by comparing the value to the density of water.

[2]:
```
from astropy.constants import G, au, M_earth, R_sun
from astropy.units import yr, s, g, cm

def M(a,P):
    return 4*pi**2*a**3/P**2/G

mass_sun=M( au , (1*yr).to(s) )

def rho(M,R):
    return 3*M/(4*pi*R**3)

density_sun=rho(mass_sun,R_sun)

print(f"2a. The mass of Sun is {mass_sun:.2e}.")
print(f"2b. The Sun is {int(round(mass_sun/M_earth,-4))} times more massive␣
 ↪than the Earth.")
print(f"    Note that this is a third of a million: If we took a million␣
 ↪Earths, we would have 3 Suns worth of mass.")
print(f"2c. With a radius of {R_sun:.1e}, the density of the Sun is␣
 ↪{round(density_sun.to(g/cm**3) ,1)}.")
print(f"    This is much smaller than the 5.5 g/cm3 of Earth and barely larger␣
 ↪than the density of water.")
```

```
2a. The mass of Sun is 1.99e+30 kg.
2b. The Sun is 330000 times more massive than the Earth.
    Note that this is a third of a million: If we took a million Earths, we
would have 3 Suns worth of mass.
2c. With a radius of 7.0e+08 m, the density of the Sun is 1.4 g / cm3.
    This is much smaller than the 5.5 g/cm3 of Earth and barely larger than the
density of water.
```

2.2 Luminosity and Temperature

In `aa:universe`, we will talk about electromagnetic radiation and its interaction with matter. Visible light is an example of electromagnetic radiation, but so are microwaves, radio waves, and X-rays. We are delaying the treatment of electromagnetic radiation, because most of the applications come from studying stars other than the single one in our solar system, and this class has already been physics-heavy. For our current purposes, all we need to know is that any physical object radiates electromagnetic waves in a particular way that is determined by its temperature. The amount of radiation of each type (wavelength) is determined entirely by the object's temperature through something called the Planck radiation curve. Conversely, we can determine the temperature of an object by fitting its radiation curve to the Planck law. In this way, the **effective temperature** of the Sun is determined to be $T = 5800$ K.

3. Study the "More Precisely 3-2" on page 72.

a. State "**Wien's Law**" and use it to determine the peak wavelength of light emitted by the Sun.

b. Explain the "**Stefan-Boltzmann equation**", and use it to calculate the **flux** F of energy radiating from the Sun. Be sure to include the proper units.

c. Multiply your answer from part b by the area of the Sun to show that the total **power** output of the Sun is 4×10^{26} W (Watt = Joules/second). This quantity is called the **luminosity** of the Sun.

d. The most powerful nuclear weapon currently deployed in the US armed forces is the B83 thermonuclear (fission) bomb with a "yield" of 1.2 megatonnes. This is equivalent to a total energy of 5×10^{15} J. Use this to show that to have a power output equivalent to that of the Sun, we would need to detonate around *80 billion B83s every second.*

e. The **solar constant** is the fraction of this total **luminosity** of the Sun that arrives at Earth per square meter. Show that the solar constant is 1400 W/m2 (watts per meter-squared).

3a. The thermal radiation emitted by a body of temperature T peaks at a certain wavelength λ_{max}. Wien's law can be stated as follows: If T is expressed in Kelvin, then

$$\lambda_{max} = \frac{2.9 \text{ mm}}{T}$$

Since the temperature of the photosphere is $T = 5800$ K, we find from Wien's law a peak wavelength of …

[3]:
```
from astropy.units import mm, nm

def lambda_max(T):
    return 2.9*mm/T

print(f"... {lambda_max(5800).to(nm)}. This is right at the cyan-green␣
↪crossover point.")
```

```
… 500.0 nm. This is right at the cyan-green crossover point.
```

3b. The Stefan-Boltzmann law states that an object with temperature T radiates with a flux F in the amount

$$F = \sigma_{SB}\, T^4$$

where σ_{SB} is called the Stefan-Boltzmann constant:

[4]:
```
from astropy.constants import sigma_sb
from astropy.units import K, J, Hz

print(sigma_sb)

def F(T):
    return sigma_sb*(T**4)

print(f"\n The flux of energy eminating from the Sun is {F(5800*K):.1e}.\n")

def A(R):
```

```
    return 4*pi*R**2

L_sun=F(5800*K)*A(R_sun)

print(f"3c. With and area of {A(R_sun):.1e}, this gives a solar luminosity of␣
 ↪{L_sun:.1e}.\n")

yield_b83=5e15*J

N_b83=L_sun/yield_b83

print(f"3d. We divide the luminosity of the Sun by the yield to find␣
 ↪{round((N_b83/1e9).value)} billion B83s per second.\n")

A_au=A(au)

solar_constant=L_sun/A_au

print(f"3e. Dividing by the area of a sphere of radius 1 au, we find that the␣
 ↪flux from the Sun at the position of the Earth is {int(round(solar_constant.
 ↪value,-2))} {solar_constant.unit}.")
```

```
  Name   = Stefan-Boltzmann constant
  Value  = 5.670374419184431 4e-08
  Uncertainty  = 0.0
  Unit  = W / (m2 K4)
  Reference = CODATA 2018

 The flux of energy eminating from the Sun is 6.4e+07 W / m2.

3c. With and area of 6.1e+18 m2, this gives a solar luminosity of 3.9e+26 W.

3d. We divide the luminosity of the Sun by the yield to find 78 billion B83s per
second.

3e. Dividing by the area of a sphere of radius 1 au, we find that the flux from
the Sun at the position of the Earth is 1400 W / m2.
```

To recapitulate: We have found that the **luminosity** of the Sun is

$$L = 4 \times 10^{26}\ W \tag{1}$$

This is a **power**: which is an energy per unit time. For example, an incandescent light bulb bought in the store is labeled by the amount of energy it converts into light and heat per second. As we already know, the SI unit of energy is Joules so the unit of power is Joules/second which is called "Watt": 1 Watt = 1 Joule/second. The higher the wattage, the brighter the bulb (and the higher your parents' electricity bill).

The luminosity of an object emitting light is always spread out over some area, so it is useful to define the **(energy) flux** as the luminosity per unit area:

$$\text{Flux} = \frac{\text{Luminosity}}{\text{Area}}$$

Note that this is an energy per unit area per unit time. The Stefan-Boltzmann law is great because it tells us that the flux is fixed by the temperature: If we can measure the flux, we will know the temperature (and *vice versa*). Since we can measure the amount of energy arriving from the Sun per unit time per unit area (a.k.a. the solar constant), we will know exactly the surface temperature of the Sun!

2.3 Rotation

We will now study the rotational period of the Sun. The Sun spins **differentially**, meaning that it has different rotational periods at different latitudes. This is only possible because the Sun is not a solid ball. Some of these periods are easy to determine by tracking **Sun spots**.

4. Sun spots appear at different latitudes, but they never appear above 60° where it takes them 31 days to go all the way around. At the equator, it only takes 25 days. Through other methods, we find that the rotational period at the poles is 36 days.

a. Using a compass and protractor, draw a cross-sectional view of the Sun and label the periods at these latitudes.

b. Go to helioviewer.org and play around with the times and filters. Track a solar storm as it progresses across the face of the Sun, and use it to estimate the rotational period.

```
[ ]:
```

3 Structure of the Sun

Let's delve a bit into the interior of the Sun. The Sun is roughly a series of concentric shells:

- **core** with radius 200,000 km
- **radiation zone** with thickness 300,000 km
- **convection zone** with thickness 200,000 km
- **photosphere** with thickness 500 km
- **chromosphere** with thickness 1500 km
- **transition zone** with thickness 8500 km
- **corona** 8,000,000 km

5. Using a compass and ruler, draw a *to-scale cross section* of the Sun and label the layers. Do not include the entire corona. Make it large, so that you can label it easily. You will need to refer back to this when we study energy transport in the solar interior.

a. The photosphere is the part of the Sun that we see as the surface. Assuming this, compute the size of the Sun and compare to your data above.

b. Calculate the fractional size of the photosphere's thickness as compared to the radius of the Sun. This should be a tiny number that explains why the "edge" of the Sun looks so "sharp" to us. (See fig. 16.1.)

c. Compare your understanding of your diagram with figure 16.2. Use table 16.1 (not fig. 16.1) to label the temperatures and densities of the regions in your diagram.

d. In a separate diagram, sketch the temperature as a function of radius (distance to the center of the Sun). Label the regions above.

e. In a separate diagram, sketch the density as a function of radius (distance to the center of the Sun). Label the regions above. The density of iron is 7800 kg/m3. Label your regions by roughly how many times the density of iron they have.

[16]:
```
from astropy.units import km

R_core=200_000*km
t_rad=300_000*km
t_conv=200_000*km
t_photo=500*km
t_chromo=1_500*km
t_trans=8_500*km
t_corona=8_000_000*km

R_photo=R_core+t_rad+t_conv

print(f"5a. The radius of the Sun is about {int(R_photo.value)} {R_photo.unit}.
 ↪")

print(f" b. The fractional thickness of the photosphere compared to the radius␣
 ↪of the Sun is {t_photo/R_photo:.1e}, or 0.07%.\n")
```

```
5a. The radius of the Sun is about 700000 km.
 b. The fractional thickness of the photosphere compared to the radius of the
Sun is 7.1e-04, or 0.07%.
```

3.1 Solar wind

6. From the previous problem, you know the inner radius of the corona to be around 7×10^8 m.

a. Recall that the escape speed $v_e = \sqrt{\frac{2GM}{r}}$ is the speed an object needs to escape the gravitational pull of a body of mass M starting from a distance r from its center. Confirm that this formula has the correct dimensions, that is, that the combination on the RHS has units of speed (*i.e.* meters per second).

b. Use this formula and the mass of the Sun to show that the escape speed of particles from the inner corona is $v_e = 618$ km/s.

c. Is the answer you found in part b high? Be quantitative in your answer. If you don't know how to start to answer this question, talk to your table mates and/or ask me.

d. The plasma making up the corona has as an insanely high temperature of $T \sim 3$ MK (mega-Kelvin). Recall from formula (2) of packet 3.1 Solar System II that the average speed of a particle in a gas is related to the temperature of that gas. Use this to show that average speed of a proton in the coronal plasma is only around 270 km/s.

Although the thermal speed of the coronal protons is lower than the escape speed, measurements show that there is a **solar wind** of 1×10^{36} particles escaping the Sun every second. These are mostly photons, protons (i.e. ionized hydrogen), and electrons, 8% helium, and trace amounts of other stuff.

e. Show that the Sun is carrying away around 2×10^9 kg of matter every second. (For perspective, this is the equivalent of 20 Ford-class aircraft carriers per second.)

f. This sounds like a huge number, but write the rate of mass loss as a fraction of solar mass.

g. At this rate, show that it would take 4×10^{13} years to evaporate. (This is a *very* rough estimate, but for comparison, the universe is only 14 billion years old.)

h. Assuming the Sun formed 4.6 billion years ago, make a rough estimate of the percentage of the solar mass carried away by the solar wind over the Sun's current lifetime.

[17]:
```
from astropy.constants import M_sun, R_sun
from astropy.units import m, kg



def v_esc(M,R):
    return (2*G*M/R)**(1/2)

print(f"6a.The units of G are {G.unit} so the units of GM are {(G*kg).unit}.")
print(f"Then the units of 2GM/R are {(G*kg/m).unit}, so the square root has the␣
 ↪units of speed.")

v_esc_sun=v_esc(M_sun,R_sun)

print(f" b. Using the formula, we find an escape speed of {round(v_esc_sun.
 ↪to(km/s))}.")

v_esc_earth=v_esc(M_earth,R_earth)
```

```
print(f" c. The escape speed on Earth is {round(v_esc_earth.to(km/s))}, which␣
↪is {int(v_esc_sun/v_esc_earth)} times smaller. However, the relevant thing␣
↪to compare to is the speet of particles in the atmosphere. The temperature␣
↪of Earth is 20 times smaller than the temperature of the Sun and the␣
↪molecules of our atmosphere are 30 times more massive than the protons in␣
↪the Sun's. The square root of the product of these numbers ~{int(␣
↪(20*30)**(1/2) )} so perhaps this escape speed is not too high.")
```

```
6a.The units of G are m3 / (kg s2) so the units of GM are m3 / s2.
Then the units of 2GM/R are m2 / s2, so the square root has the units of speed.
 b. Using the formula, we find an escape speed of 618.0 km / s.
 c. The escape speed on Earth is 11.0 km / s, which is 55 times smaller.
However, the relevant thing to compare to is the speet of particles in the
atmosphere. The temperature of Earth is 20 times smaller than the temperature of
the Sun and the molecules of our atmosphere are 30 times more massive than the
protons in the Sun's. The square root of the product of these numbers ~24 so
perhaps this escape speed is not too high.
```

6d. Recall that the kinetic energy of a particle of mass m is $\frac{1}{2}mv^2$, and that the energy of a gas of N particles with temperature T is $\frac{3}{2}Nk_BT$. Then, per particle, we find $mv^2 = 3k_BT$. Solving for v, we get the average speed of the particles in the gas

$$v_{th} = \sqrt{\frac{3k_BT}{m}}$$

This is called the thermal speed. Recalling that the Sun's atmosphere is almost entirely made up of protons (and electrons), we take m to be the mass of the proton. Then, the thermal speed for the coronal protons at $T = 3 \times 10^6$ K is...

[18]:
```
from astropy.constants import k_B, m_p

def v_th(m,T):
    return (3*k_B*T/m)**(1/2)

print(f"... {round(v_th(m_p,3e6*K).to(km/s))}.")

mass_loss=1e36/s*m_p

print(f" e. 1e36 protons per second is a mass rate of {mass_loss:.1e}.")

mass_loss_fraction=mass_loss=1e36/s*m_p/M_sun

print(f" f. In terms of solar masses, this is a rate of {mass_loss_fraction:.
↪1e}.")

time_evaporation=1/mass_loss_fraction

print(f" g. The reciprocal of this is {time_evaporation:.1e}, or␣
↪{time_evaporation.to(yr):.1e}.")
```

```
lifetime=4.6e9*yr
fraction_evaporated=mass_loss_fraction*lifetime.to(s)

print(f" h. Assuming this constant rate for 4.6 billion years, we find that the␣
 ↪of mass lost is {round(fraction_evaporated.value*100,2)}%.")
```

```
… 273.0 km / s.
 e. 1e36 protons per second is a mass rate of 1.7e+09 kg / s.
 f. In terms of solar masses, this is a rate of 8.4e-22 1 / s.
 g. The reciprocal of this is 1.2e+21 s, or 3.8e+13 yr.
 h. Assuming this constant rate for 4.6 billion years, we find that the of mass
lost is 0.01%.
```

The rate of mass carried away by the solar wind is larger than what we would get from the thermal speed, that is, from standard evaporation. The mechanisms by which the particles are boosted to higher velocities for escape are still under study, but it is believed that the large magnetic fields of the Sun are responsible for both this phenomenon and the related high temperature of the corona. In 2018, the Parker Solar Probe was sent to investigate this phenomenon.

4 Energy generation

In problem 3c, we derived the luminosity of the Sun to be that given in equation (1). We will now investigate the mechanisms that could potentially give rise to such immense powers.

7. At various points in previous packets, you estimated the size of the gravitational potential energy of an object of mass M and radius R to be $U \sim GM^2/R$ by dimensional analysis. Continuing with dimensional analysis, combine this result with the rate of energy conversion of the Sun from problem 3 (cf. eq. 1) to give a rough estimate of how long the Sun could continue to shine at this rate if it could use all its gravitational energy to do so.

7. To shine away all of its gravitational energy U is in Joules and the luminosity $L = 4 \times 10^{26}$ W, is in Joules per second. Therefore, we need to compute $t \sim U/L$. Plugging in …

[8]:
```
from astropy.constants import L_sun
from astropy.units import W

def U(M,R):
    return G*M**2/R

U_grav=U(M_sun,R_sun)

t_shine=U_grav.to(J)/L_sun

print(f"... the mass {M_sun:.1e} and radius {R_sun:.1e}, we find an energy␣
 ↪{U_grav.to(J):.1e}.")
print(f"At the stated luminosity, it would take a time {t_shine.to(yr):.1e}.")
```

```
… the mass 2.0e+30 kg and radius 7.0e+08 m, we find an energy 3.8e+41 J.
At the stated luminosity, it would take a time 3.1e+07 yr.
```

What you have just derived in problem 7 is called the **Kelvin-Helmholtz time scale**. It is the time it would take for the Sun to collapse completely by gravitational attraction alone if it were to do so by radiating energy at a constant rate equal to the current rate. More realistically, the rate would not be constant, and we would not be able to use up all of the gravitational energy stored in the current configuration of the Sun. These effects would act to reduce the time scale, but even under the simplifying assumptions, we are still a factor of 10 or more short of the 4.6 billion year age of the Sun. From this, we conclude that gravitational energy alone is several orders of magnitude too small to explain the luminosity of the Sun.

4.1 Nuclear fusion

In the previous section, we concluded that the gravitational potential energy of the Sun was insufficient to explain its persistent luminosity. In this section, we will develop evidence that the mechanism responsible for the energy output of the Sun is nuclear fusion.

The basic idea is **Einstein's mass-energy relation**

$$E = mc^2 \tag{3}$$

where

$$c = 299,792,458\ m/s \tag{4}$$

is the speed of light in vacuum: a universal constant like Newton's and Planck's, meaning that any inertial observer would measure the same number for its value.

8. Use dimensional analysis to confirm that–the speed of light in vacuum being a fundamental constant–it makes sense to assign an energy to a mass according to the rule (3).

8. The dimensions of energy $[E] = \frac{M \cdot L^2}{T^2}$, whereas $[c] = \frac{L}{T}$. Therefore, if we take a mass and multiply by c^2, we get and energy.

In analogy to the kinetic energy, it may be tempting to think the formula should be $E = \frac{1}{2}mc^2$, but recall that according to the virial theorem the average potential energy is twice the kinetic energy. Together with the sign, we can interpret equation (3) as the statement that the "binding energy" holding a particle of mass m together is $U = -mc^2$, so "unbinding" it should release an energy in the amount given by equation (3).

9. Use the NIST CODATA website to find the masses of a(n)

a. electron

b. proton

c. neutron

d. deuteron (2H^+ nucleus)

e. helion (3He++ nucleus)

f. alpha particle (4He++ nucleus)

Go nuts with significant figures; we're gonna need 'em!

[19]:
```
from astropy.constants import m_e, m_n

particles=["electron","proton  ","neutron ","deuteron","helion  ","alpha   "]

m_d=3.343_583_7768e-27*kg
m_h=5.006_412_7862e-27*kg
m_a=6.644_657_3450e-27*kg

masses=[m_e.to(kg),m_p.to(kg),m_n.to(kg),m_d,m_h,m_a]

alphabet="abcdefg"

for i in range(6):
    print(f"9{alphabet[i]}.The mass of a(n) {particles[i]} is {masses[i]:.5}.")
```

```
9a.The mass of a(n) electron is 9.1094e-31 kg.
9b.The mass of a(n) proton   is 1.6726e-27 kg.
9c.The mass of a(n) neutron  is 1.6749e-27 kg.
9d.The mass of a(n) deuteron is 3.3436e-27 kg.
9e.The mass of a(n) helion   is 5.0064e-27 kg.
9f.The mass of a(n) alpha    is 6.6447e-27 kg.
```

Einstein's mass formula (3) allows us to convert between mass and energy: It makes sense to say that the neutron has a mass of 1.5×10^{-10} J. What we mean by this is that, if we take the mass of the neutron you found above and multiply by the square of the speed of light, we get this much energy.[^1]

It is common in the physics of fundamental particles to refer to masses by an energy, but the unit of choice is not the Joule. Strangely, the convention is to refer to the **electron-volt** which is defined to be the kinetic energy of an electron that has been accelerated in an electric field of intensity 1 Volt. Its value is

$$1\ eV = 1.602176634 \times 10^{-19}\ J$$

Most of the fundamental particles have masses in the $10^6\ eV$ range, or mega-electron-volt MeV.

$$1\ J = 6.241509074 \times 10^{12}\ MeV$$

Combining this with Einstein's relation, the energy equivalent of one kilogram is

$$\begin{aligned} 1\ kg &= 6.241\ 509\ 074 \times 10^{12} \times (299792458)^2 \\ &= 5.609\ 588\ 603 \times 10^{29}\ MeV/c^2 \end{aligned} \tag{1}$$

10. Convert the masses of the fundamental particles you found in problem 7 into MeV.

[10]:
```
from astropy.constants import c
from astropy.units import eV, MeV

def kg_to_MeV(m):
    return (m*c**2).to(MeV)

for i in range(6):
    print(f"10{alphabet[i]}. The mass of a(n) {particles[i]} is␣
 ↪{kg_to_MeV(masses[i]):.3f}.")
```

```
10a. The mass of a(n) electron is 0.511 MeV.
10b. The mass of a(n) proton   is 938.272 MeV.
10c. The mass of a(n) neutron  is 939.565 MeV.
10d. The mass of a(n) deuteron is 1875.613 MeV.
10e. The mass of a(n) helion   is 2808.392 MeV.
10f. The mass of a(n) alpha    is 3727.379 MeV.
```

By definition, a proto-stellar object does not become a star until it fuses hydrogen nuclei (*i.e.* protons) into helium.

11. a. Carefully calculate the **binding energy** of the helium nucleus by comparing the energy associated to its mass with that associated to 2 free protons and 2 free neutrons. To do this, use your results from problem 9. Hint: This energy should come out to 28 MeV.

b . Show that this is equivalent to about 7 MeV or 1.1×10^{-12} `J` per nucleon.

c. Using the mass of a proton, show that fusing 1 kg worth of protons results in an energy of 7×10^{14} `J`.

d. Use this energy per unit mass together with the luminosity of the Sun from equation (1) to show that the Sun burns hydrogen at an eyebrow-raising rate of 600 billion kilograms per second.

e. The mass you found in part d is the size of a small mountain. Demonstrate this by modeling a mountain by a cone and using the density of 2700 `kg/m3` of the upper crust of the Earth to estimate the height of the cone. (Mountains and their size classification are discussed in this Wikipedia page.)

f. To put yourself at ease, calculate the time in years that it will take to convert one solar mass of hydrogen completely to helium.

[11]:
```
mass_diff=2*m_p+2*m_n-m_a

N_kg_of_protons=(1/m_p).value

print(f"11a. The mass difference of {mass_diff:.1e} is equivalent to␣
 ↪{round(kg_to_MeV(mass_diff))}.")
```

```
print(f"  b. The mass difference of {mass_diff/4:.1e} is equivalent to␣
↪{round(kg_to_MeV(mass_diff/4))}\n     or {kg_to_MeV(mass_diff/4).to(J):.1e}␣
↪per nucleon.")

E_kg=kg_to_MeV(mass_diff/4*N_kg_of_protons)

print(f"  c. 1 kg of protons is {N_kg_of_protons:.1e} protons. The mass deficit␣
↪for this many protons is of {mass_diff/4*N_kg_of_protons:.1e} or {E_kg:.1e}.
↪\n     This is equivalent to {E_kg.to(J):.1e}.")

rate=( L_sun/(E_kg.to(J)) ).to(Hz)

print(f"  d. Dividing the luminosity of the Sun by the energy obtained from a␣
↪kilogram gives a rate of {round(rate/1e9).value} billion kg/s.")
#print(f"  d. {(rate2/1e9).to(kg/s)}")
```

```
11a. The mass difference of 5.0e-29 kg is equivalent to 28.0 MeV.
  b. The mass difference of 1.3e-29 kg is equivalent to 7.0 MeV
     or 1.1e-12 J per nucleon.
  c. 1 kg of protons is 6.0e+26 protons. The mass deficit for this many protons
is of 7.5e-03 kg or 4.2e+27 MeV.
     This is equivalent to 6.8e+14 J.
  d. Dividing the luminosity of the Sun by the energy obtained from a kilogram
gives a rate of 565.0 billion kg/s.
```

The volume of a cone of height h and radius r is $V = \frac{\pi}{3}hr^2$. If we take $r = h$ then $V \sim h^3$. Since mass density $\rho = \frac{M}{V}$, the volume is also the mass divided by the density $V = \frac{M}{\rho}$. Thus $h \sim \sqrt[3]{\frac{M}{\rho}}$. Plugging in the numbers gives a height of …

[12]:
```
rho=2700*kg/m**3
h=(rate*kg/rho/Hz)**(1/3)
print(f"  e. ... {round(h)}.")
```

```
  e. … 594.0 m.
```

According to the Wikipedia link, this is the height of a medium class 6 mountain.

[20]:
```
from astropy.units import Gyr

time_sun=M_sun/kg/rate

print(f"   f. If the Sun could burn though all of its mass (which is not␣
↪realistic) at the current luminosity (which is also not realistic, it would␣
↪take {round(time_sun.to(Gyr))}. (One Gyr= 1 billion years.)")
```

```
   f. If the Sun could burn though all of its mass (which is not realistic) at
the current luminosity (which is also not realistic, it would take 112.0 Gyr.
(One Gyr= 1 billion years.)
```

This is an overestimate, because it assumes that the Sun can use all its hydrogen, whereas only the hydrogen in the core can be used for fusion. Unfortunately, you need to use calculus to estimate this fraction, but the answer is that roughly 20% of the mass of the Sun is available for fusion.

12. Use this estimate to adjust your calculation in 11f.

[14]:
```
print(f"12. The adjusted lifetime would be {round(0.2*time_sun.to(Gyr))}.")
```

```
12. The adjusted lifetime would be 22.0 Gyr.
```

The answer is still off by a factor of roughly 2 because as the hydrogen gets used up, it becomes rarer, so the fusion process slows down well before the hydrogen runs out.
Taking all this into account, we find the Sun's lifetime to be roughly 10 billion years. Since it is already 4.6 billion years old, we can expect the Sun to continue happily on the main sequence of another 5 billion years.