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CSCE 440

Homework #3

① Piece wise linear interpolation for 5th station

SN	T	AM	i
5	5	28	0
5	10	30	1
5	15	33	2
5	20	31	3

n=4

$$S_i(x) = y_i + \frac{y_{i+1} - y_i}{t_{i+1} - t_i}(x - t_i), \quad i = 0, 1, \ldots, n-1$$

$$S_0(x) = 28 + \frac{30-28}{10-5}(x-5)$$

$$S_1(x) = 30 + \frac{33-30}{15-10}(x-10)$$

$$S_2(x) = 33 + \frac{31-33}{20-15}(x-15)$$

$$S(x) = \begin{cases} 28 + \frac{2}{5}(x-5), & 5 \le x \le 10 \\ 30 + \frac{3}{5}(x-10), & 10 \le x \le 15 \\ 33 - \frac{2}{5}(x-15), & 15 \le x \le 20 \end{cases}$$

(2) Piece wise Quadratic interpolation for 5th Station

SN	T	PM	n
5	5	28	0
5	10	30	1
5	15	33	2
5	20	31	3

$z_0 = 0$

$z_{0+1} = z_1 = -0 + 2\left(\frac{30-28}{10-5}\right) = \frac{4}{5}$

$z_{1+1} = z_2 = -\frac{4}{5} + 2\left(\frac{33-30}{15-10}\right) = \frac{2}{5}$

$z_{2+1} = z_3 = -\frac{2}{5} + 2\left(\frac{31-33}{20-15}\right) = \frac{-6}{5}$

$$Q_i(x) = \frac{z_{i+1} - z_i}{2(t_{i+1} - t_i)}(x - t_i)^2 + z_i(x - t_i) + y_i$$

$$Q(x) = \begin{cases} \left(\frac{\frac{4}{5} - 0}{2(10-5)}\right)(x-5)^2 + 0(x-5) + 28, & 5 \le x \le 10 \\ \left(\frac{\frac{2}{5} - \frac{4}{5}}{2(15-10)}\right)(x-10)^2 + \frac{4}{5}(x-10) + 30, & 10 \le x \le 15 \\ \left(\frac{\frac{-6}{5} - \frac{2}{5}}{2(20-15)}\right)(x-15)^2 + \frac{2}{5}(x-15) + 33, & 15 \le x \le 20 \end{cases}$$

(3) Cubic Spline interpolation for 5th Station, tridiagonal with Natural Boundary

SN	T	PM	i
5	5	28	0
5	10	30	1
5	15	33	2
5	20	31	3

$h_0 = h_1 = h_2 = \dots = h = 10 - 5 = 5$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 20 & 5 & 0 \\ 0 & 5 & 20 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} 0 \\ \frac{3}{5}(33-30) - \frac{3}{5}(30-28) \\ \frac{3}{5}(31-33) - \frac{3}{5}(33-30) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3/5 \\ -3 \\ 0 \end{bmatrix} \qquad \vec{X} = \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

$$\vec{X} = A^{-1}\vec{b} = \begin{bmatrix} 0 \\ 9/125 \\ -21/125 \\ 0 \end{bmatrix}$$

$$b_j = \frac{1}{h_j}(a_{j+1} - a_j) - \frac{h_j}{3}(2C_j + C_{j+1})$$

$$d_j = \frac{1}{3h_j}(C_{j+1} - C_j)$$

$$S(x) = \begin{cases} S_0(x) = 28 + \frac{7}{25}(x-5) + 0(x-5)^2 + \frac{3}{625}(x-5)^3, & 5 \le x \le 10 \\ S_1(x) = 30 + \frac{16}{25}(x-10) + \frac{9}{125}(x-10)^2 + \frac{-2}{125}(x-10)^3, & 10 \le x \le 15 \\ S_2(x) = 33 + \frac{4}{25}(x-15) + \frac{-21}{125}(x-15)^2 + \frac{7}{625}(x-15)^3, & 15 \le x \le 20 \end{cases}$$

(4) Cubic Spline interpolation for 5th Station

SN	T	PM	i
5	5	28	0
5	10	30	1
5	15	33	2
5	20	31	3

$r = 2+\sqrt{3} \qquad h = 5$

$$\vec{b} = \begin{bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \frac{3(2+\sqrt{3})}{10}\left(\frac{30-28}{5}\right) \\ \frac{3}{25}(28-60+33) \\ \frac{3}{25}(30-66+31) \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{3(\sqrt{3}+2)}{25} \\ \frac{3}{25} \\ \frac{-3}{5} \\ 0 \end{bmatrix}$$

$\alpha_0 = \frac{e_0}{r} = \frac{3}{25}$

$\alpha_1 = \frac{e_1-\alpha_0}{r} = 0 \qquad \alpha_2 = \frac{e_2-\alpha_1}{r} = \frac{3(\sqrt{3}-2)}{5} \qquad \alpha_3 = 0$

$c_3 = 0 \qquad c_2 = \alpha_2 - \frac{c_3}{r} = \frac{3(\sqrt{3}+2)}{5} \qquad c_1 = \alpha_1 - \frac{c_2}{r} = \frac{-3(4\sqrt{3}-7)}{5}$

$c_0 = \alpha_0 - \frac{c_1}{r} = 9\sqrt{3} - \frac{387}{25}$

$b_0 = (q_1-q_0)\frac{1}{h} - \frac{2c_0+c_1}{3}h = 45-26\sqrt{3}$

$d_0 = \frac{1}{3h}(c_1-c_0) = \frac{-(95\sqrt{3}-164)}{125}$

$b_1 = (q_2-q_1)\frac{1}{h} - \frac{2c_1+c_2}{3}h = 7\sqrt{3} - \frac{57}{5}$

$d_1 = \frac{1}{3h}(c_2-c_1) = \frac{5\sqrt{3}-9}{25}$

$b_2 = (q_3-q_2)\frac{1}{h} - \frac{2c_2+c_3}{3}h = \frac{21}{5} - 2\sqrt{3}$

$d_2 = \frac{1}{3h}(c_3-c_2) = \frac{-(\sqrt{3}-2)}{25}$

$$S(t) = \begin{cases} S_0(t) = 28 + \left(9\sqrt{3}-\frac{387}{25}\right)(t-5) + \left(9\sqrt{3}-\frac{387}{25}\right)(t-5)^2 + \left[\frac{-(95\sqrt{3}-164)}{125}\right](t-5)^3, & 5 \le t \le 10 \\ S_1(t) = 30 + \left(7\sqrt{3}-\frac{57}{5}\right)(t-10) + \left(\frac{-3(4\sqrt{3}-7)}{5}\right)(t-10)^2 + \left(\frac{5\sqrt{3}-9}{25}\right)(t-10)^3, & 10 \le t \le 15 \\ S_2(t) = 33 + \left(\frac{21}{5}-2\sqrt{3}\right)(t-15) + \left(\frac{3\sqrt{3}+2}{5}\right)(t-15)^2 + \left(\frac{-(\sqrt{3}-2)}{25}\right)(t-15)^3, & 15 \le t \le 20 \end{cases}$$

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>> homework03

/----------------w1(x,y):------------------/

w =

(7*x)/2 - (595*y)/2 + 17500


/----------------w2(x,y):------------------/

w =

6*x - 78*y + 13080


/----------------w3(x,y):------------------/

w =

567*x + 105*y + 9450

              .

/----------------w4(x,y):------------------/

w =

15400 - 35*y - 259*x


/----------------w5(x,y):------------------/

w =

14*x - 74*y + 12904


/----------------w6(x,y):------------------/

w =

54*x - 102*y + 12600


/----------------w7(x,y):------------------/

w =

378*x + 420*y + 6300


/----------------w8(x,y):------------------/

w =

45*x - 105*y + 12780
```

5

```
/----------------w9(x,y):-----------------/

w =

(117*y)/2 - 132*x + 9915


/----------------w10(x,y):----------------/

w =

390*x + 450*y + 6000


/----------------w11(x,y):----------------/

w =

600*x - (615*y)/2 + 9915


/----------------w12(x,y):----------------/

w =

390*x - (755*y)/2 + 10965


/----------------w13(x,y):----------------/

w =

51*x - 95*y + 12660
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(6)