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Homework #2 CSCE 440

## Homework #2

$$(1) \quad f(x) = x^3 - 15.5x^2 + 44x + 60.5 = 0$$

$$|P - P_n| < 10^{-20} \quad a_1 = -0 \quad b_1 = 10$$

# iterations to solve using bisection method

$$|P_n - P| \leq \frac{b_1 - a_1}{2^n} = \frac{10 - 0}{2^n} = \frac{10}{2^n}$$

$$|P - P_n| \leq \frac{10}{2^n} < 10^{-20}$$

$$n > \frac{21 \ln(10)}{\ln(2)} = 69.76049$$

70 iterations

② - find Lagrange Interpolating Polynomial for 4th Station

- estimate the PM 2.5 at  $T=10$

SN	T	PM	#
4	7	32	0
4	14	34	1
4	21	36	2
4	28	35	3
	x	f(x)	n

$$P_3(x) = \sum_{i=0}^3 L_i(x) f(x_i), \quad f(x_i) = y_i$$

$$L_i(x) = \prod_{j=0, j \neq i}^3 \left( \frac{x - x_j}{x_i - x_j} \right)$$

$$L_0(x) = \frac{3}{\prod_{j=0, j \neq 0}^3 \left( \frac{x - x_j}{x_0 - x_j} \right)} = \left( \frac{x - 14}{7 - 14} \right) \left( \frac{x - 21}{7 - 21} \right) \left( \frac{x - 28}{7 - 28} \right)$$

$$L_1(x) = \frac{3}{\prod_{j=0, j \neq 1}^3 \left( \frac{x - x_j}{x_1 - x_j} \right)} = \left( \frac{x - 7}{14 - 7} \right) \left( \frac{x - 21}{14 - 21} \right) \left( \frac{x - 28}{14 - 28} \right)$$

$$L_2(x) = \frac{3}{\prod_{j=0, j \neq 2}^3 \left( \frac{x - x_j}{x_2 - x_j} \right)} = \left( \frac{x - 7}{21 - 7} \right) \left( \frac{x - 14}{21 - 14} \right) \left( \frac{x - 28}{21 - 28} \right)$$

$$L_3(x) = \frac{3}{\prod_{j=0, j \neq 3}^3 \left( \frac{x - x_j}{x_3 - x_j} \right)} = \left( \frac{x - 7}{28 - 7} \right) \left( \frac{x - 14}{28 - 14} \right) \left( \frac{x - 21}{28 - 21} \right)$$

$$P_3(x) = (32) \left( \frac{x - 14}{7 - 14} \right) \left( \frac{x - 21}{7 - 21} \right) \left( \frac{x - 28}{7 - 28} \right) + (34) \left( \frac{x - 7}{14 - 7} \right) \left( \frac{x - 21}{14 - 21} \right) \left( \frac{x - 28}{14 - 28} \right) \\ + (36) \left( \frac{x - 7}{21 - 7} \right) \left( \frac{x - 14}{21 - 14} \right) \left( \frac{x - 28}{21 - 28} \right) + (35) \left( \frac{x - 7}{28 - 7} \right) \left( \frac{x - 14}{28 - 14} \right) \left( \frac{x - 21}{28 - 21} \right)$$

$$P_3(10) = 11204/343 \approx \boxed{32.664723}$$

$$\frac{(x - 21)}{(28 - 21)}$$

③ Neville's method to estimate PM 2.5 of  $y_{th}$  station

at  $T = 10 = x$

SN	T	PM	#
4	7	32	0
4	14	34	1
4	21	36	2
4	28	35	3
	$x$	$f(x)$	$n$

$$P_{01}(x) = \frac{10-14}{7-14} \cdot 32 + \frac{7-10}{7-14} \cdot 34 = 230/7 \approx 32.85714$$

$$P_{12}(x) = \frac{10-21}{14-21} \cdot 34 + \frac{14-10}{14-21} \cdot 36 = 230/7 \approx 32.85714$$

$$P_{23}(x) = \frac{10-28}{21-28} \cdot 36 + \frac{21-10}{21-28} \cdot 35 = 263/7 \approx 37.57143$$

$$P_{9bc}(x) = \frac{x-c}{9-c} P_{9b}(x) + \frac{9-x}{9-c} P_{bc}(x)$$

i	$x_i$	$x - x_i$				
0	7	3	$P_0(x) = 32$	$P_{01}(x) = 230/7$	$P_{012}(x) = 230/7$	$P_{0123}(x) = 11204/343$
1	14	-4	$P_1(x) = 34$	$P_{12}(x) = 230/7$	$P_{123}(x) = 1544/49$	
2	21	-11	$P_2(x) = 36$	$P_{23}(x) = 263/7$		
3	28	-18	$P_3(x) = 35$			

$$P_{012}(x) = \frac{10-21}{7-21} \cdot \frac{230}{7} + \frac{7-10}{7-21} \cdot \frac{230}{7} = 230/7 \approx 32.85714$$

$$P_{123}(x) = \frac{10-28}{14-28} \cdot \frac{230}{7} + \frac{14-10}{14-28} \cdot \frac{263}{7} = 1544/49 \approx 31.51020$$

$$P_{0123}(x) = \frac{10-28}{7-28} \cdot \frac{230}{7} + \frac{7-10}{7-28} \cdot \frac{1544}{49} = 11204/343 \approx \boxed{32.664723}$$

④ Newton Divided Differences Method  
 4th Station      Pm 2.5      at T = 10

S N	T	Pm	#
4	7	32	0
4	14	34	1
4	21	36	2
4	28	35	3
	X	f(x)	4

X	f[X]	1st DD	2nd DD	3rd DD
7	32	$\frac{34-32}{14-7} = \frac{2}{7}$	$\frac{2/7 - 2/7}{21-7} = 0$	$\frac{-3/48 - 0}{28-7} = -1/686$
14	34	$\frac{36-34}{21-14} = \frac{2}{7}$	$\frac{-1/7 - 2/7}{28-14} = -\frac{3}{98}$	
21	36	$\frac{35-36}{28-21} = -\frac{1}{7}$		
28	35			

$$P_3(X) = 32 + \frac{2}{7}(X-7) + \frac{-1}{686}(X-7)(X-14)(X-21)$$

$$P_3(10) = 32 + \frac{2}{7}(3) + \frac{-1}{686}(3)(-4)(-11)$$

$$= \frac{11204}{343} \approx 32.664723$$

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>> interpolation

/-----/  
/-----Station 1-----/  
/-----/

/-----Lagrange-----/

outputFunction =

$$29*(x/3 - 4)*(x/10 - 1/2)*(x/7 - 8/7)*(x/4 - 19/4)*(x/14 - 1/14)*(x/7 - 22/7)*(x/11 - 26/11)*(x/14 - 29/14) - 27*(x/4 - 2)*(x/3 - 5)*(x/7 - 5/7)*(x/11 - 1/11)*(x/10 - 11/5)*(x/7 - 19/7)*(x/14 - 13/7)*(x/17 - 29/17) + 25*(x/4 - 13/2)*(x/10 - 6/5)*(x/3 - 19/3)*(x/14 - 4/7)*(x/7 - 15/7)*(x/17 - 5/17)*(x/7 - 29/7)*(x/21 - 1/21) + 35*(x/4 - 3)*(x/3 - 5/3)*(x/7 - 1/7)*(x/7 - 15/7)*(x/14 - 11/7)*(x/18 - 13/9)*(x/11 - 19/11)*(x/21 - 29/21) - 32*(x/4 - 15/4)*(x/7 - 12/7)*(x/3 - 22/3)*(x/11 - 8/11)*(x/14 - 5/14)*(x/18 - 1/18)*(x/7 - 26/7)*(x/10 - 29/10) - 37*(x/4 - 11/2)*(x/14 - 6/7)*(x/18 - 4/9)*(x/7 - 19/7)*(x/3 - 29/3)*(x/11 - 15/11)*(x/21 - 5/21)*(x/25 - 1/25) - 33*(x/4 - 1/4)*(x/3 - 8/3)*(x/10 - 3/2)*(x/7 - 12/7)*(x/14 - 19/14)*(x/17 - 22/17)*(x/21 - 26/21)*(x/24 - 29/24) + 39*(x/3 - 26/3)*(x/7 - 22/7)*(x/10 - 19/10)*(x/14 - 15/14)*(x/17 - 12/17)*(x/21 - 8/21)*(x/24 - 5/24)*(x/28 - 1/28) + 30*(x/4 - 5/4)*(x/7 - 8/7)*(x/11 - 12/11)*(x/14 - 15/14)*(x/18 - 19/18)*(x/21 - 22/21)*(x/25 - 26/25)\\n*(x/28 - 29/28)$$

station1Lagrange =

30.4768

/-----Neville-----/

station1Neville =

30.4768

/-----Newton-----/

station1Newton =

30.4768

/-----/  
/-----Station 2-----/  
/-----/

/-----Lagrange-----/

outputFunction =

$$\begin{aligned} & 34*(x/2 - 9)*(x/12 - 1/3)*(x/5 - 11/5)*(x/14 - 1/7)*(x/7 - 9/7)*(x/14 - 15/7)*(x/7 - 23/7)*(x/9 - 25/9) - \\ & 32*(x/2 - 8)*(x/9 - 1)*(x/12 - 5/2)*(x/14 - 2/7)*(x/7 - 11/7)*(x/16 - 1/8)*(x/5 - 23/5)*(x/7 - 25/7) + \\ & 30*(x/9 - 2)*(x/5 - 4/5)*(x/2 - 11/2)*(x/7 - 2/7)*(x/7 - 16/7)*(x/21 - 10/7)*(x/14 - 23/14)*(x/16 - \\ & 25/16) + 40*(x/5 - 5)*(x/12 - 3/2)*(x/14 - 8/7)*(x/21 - 3/7)*(x/7 - 23/7)*(x/26 - 2/13)*(x/28 - \\ & 1/14)*(x/19 - 11/19) - 28*(x/2 - 9/2)*(x/7 - 4/7)*(x/9 - 2/9)*(x/5 - 16/5)*(x/7 - 18/7)*(x/12 - \\ & 23/12)*(x/14 - 25/14)*(x/19 - 30/19) - 35*(x/2 - 1)*(x/5 - 9/5)*(x/12 - 4/3)*(x/7 - 11/7)*(x/14 - \\ & 9/7)*(x/26 - 15/13)*(x/19 - 23/19)*(x/21 - 25/21) - 37*(x/5 - 6)*(x/2 - 23/2)*(x/7 - 18/7)*(x/9 - \\ & 16/9)*(x/14 - 11/14)*(x/16 - 9/16)*(x/21 - 4/21)*(x/23 - 2/23) + 36*(x/5 - 18/5)*(x/2 - 25/2)*(x/7 - \\ & 16/7)*(x/12 - 11/12)*(x/14 - 9/14)*(x/19 - 4/19)*(x/7 - 30/7)*(x/21 - 2/21) + 36*(x/2 - 2)*(x/7 - \\ & 9/7)*(x/9 - 11/9)*(x/14 - 8/7)*(x/16 - 9/8)*(x/28 - 15/14)*(x/21 - 23/21)*(x/23 - 25/23) \end{aligned}$$

station2Lagrange =

28.2242

/-----Neville-----/

station2Neville =

28.2242

/-----Newton-----/

station2Newton =

28.2242

/-----/

/-----Station 3-----/

/-----/

/-----Lagrange-----/

outputFunction =

$$\begin{aligned} & 40*(x/21 - 2/7)*(x/7 - 20/7)*(x/14 - 13/14) - 42*(x/7 - 13/7)*(x/14 - 10/7)*(x/21 - 9/7) - 38*(x/14 - \\ & 3/7)*(x/7 - 13/7)*(x/7 - 27/7) + 36*(x/7 - 6/7)*(x/7 - 20/7)*(x/14 - 27/14) \end{aligned}$$

station3Lagrange =

37.1254

/-----Neville-----/

station3Neville =

37.1254

/-----Newton-----/

station3Newton =

37.1254

/-----/  
/-----Station 4-----/  
/-----/

/-----Lagrange-----/

outputFunction =

$34*(x/7 - 1)*(x/7 - 3)*(x/14 - 2) - 36*(x/7 - 2)*(x/7 - 4)*(x/14 - 1/2) + 35*(x/7 - 3)*(x/14 - 1)*(x/21 - 1/3) - 32*(x/7 - 2)*(x/14 - 3/2)*(x/21 - 4/3)$

station4Lagrange =

32.6647

/-----Neville-----/

station4Neville =

32.6647



/-----Newton-----/

station4Newton =

32.6647

>>

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$$f(x) = x^3 - 5x, \quad p_0 = 1, \quad \text{Newton method}$$

$$f'(x) = 3x^2 - 5$$

$$p_0 = 1$$

$$p_1 = 1 - \frac{-4}{-2} = -1$$

$$p_2 = -1 - \frac{4}{-2} = 1$$

$$p_3 = 1 - \frac{-4}{-2} = -1$$

Newton's method leads to a cycle,

so the method does not converge, and

no solution is obtained