Guerrieri and Lorenzoni (2017) REMARK

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Abstract

This paper uses the Econ-ARK/HARK toolkit to replicate the results of Guerrieri and Lorenzoni (2017). We create a new AgentType, GLConsumerType, that inherits the IndShockConsumerType and a Solver, GLSolver, that inherits the ConsIndShockSolver. We managed to closely replicate the initial Optimal Consumption and Labor Supply Steady States found Figure 1 of the paper.

Keywords Credit Crises, Precautionary Savings, Liquidity Trap, Replication

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1 Summary

This paper uses a heterogeneous agents model with incomplete markets and endogenous labor supply to analyze the effects of a credit crunch on consumer spending.

Main Findings:

(i) A credit crunch leads to a fall in consumption and real interest rates due to a forced deleveraging and an increase in precautionary savings.

(ii) Adding nominal rigidities to the baseline model may exacerbate the effects on output as the zero lower bound may prevent the real interest rate from falling sufficiently in order to attain the flexible price equilibrium.

(iii) Adding durable goods to the baseline model does not fundamentally alter the baseline result mentioned in (i). In this extension, there is a fall in consumption in both non durables and durables for credit constrained consumers and an increase in precautionary savings through durables and bonds for net lenders.

2 Non-Technical Overview

The authors consider a heterogeneous agents model where there is a continuum of infinitely lived households with idiosyncratic uncertainty to their labor productivity and subject to a borrowing limit. In order to understand the effects of a credit crunch, the authors analyze how a shock to the borrowing limit (a tightening in credit) influences each household's consumption decision and the resulting interest rate dynamics.

2.1 Baseline Model

- Households/Producers

  There is a continuum of infinitely lived households with preferences represented by the utility function:

$$\mathrm{E}\left[\sum_{t=0}^{\infty} \beta^t U\left(c_{it}, n_{it}\right)\right] \tag{1}$$

$$U(c,n) = \frac{c^{1-\gamma}}{1-\gamma} + \psi \frac{(1-n)^{1-\eta}}{1-\eta} \tag{2}$$

  Each household chooses c_{it} and n_{it} to maximize their lifetime expected utility subject to their household budget constraint described below. Production is dependent on the choice of n_{it}

$$y_{it} = \theta_{it} n_{it} \tag{3}$$

  where θ_{it} is an idiosyncratic shock to the labor productivity of household, which follows a Markov chain on the space $\left\{\theta^1, \ldots, \theta^S\right\}$. Let $\theta^1 = 0$.

- Household Budget Constraint

$$q_t b_{it+1} + c_{it} + \tilde{\tau}_{it} \leq b_{it} + y_{it} \tag{4}$$

where q_t is the bond price, $\tilde{\tau}_{it}$ are taxes such that $\tilde{\tau}_{it} = \tau_t$ if $\theta_{it} > 0$ and $\tilde{\tau}_{it} = \tau_t - v_t$ if $\theta_{it} = 0$ where v_t is unemployment insurance. b_{it+1} are bond holdings.

Household debt is bounded below by an exogenous limit $\phi > 0$. That is,

$$b_{it+1} \geq -\phi \tag{5}$$

A credit crunch is equivalent to lowering the value of ϕ.

- Government
  The government chooses the aggregate supply of bonds B_t, the unemployment benefit v_t and the lump sum tax τ_t so as to satisfy the budget constraint

  $$B_t + v_t u = q_t B_{t+1} + \tau_t \tag{6}$$

  where where u = Pr $(\theta_{it} = 0)$ is the fraction of unemployed agents in the population.

  We assume that the supply of government bonds B and unemployment insurance v are kept constant. Taxes τ adjust to ensure the government budget balances.

2.2 Dynamic Program

For household i, the Bellman equation is

$$V_{it}(b_{it}, \theta_{it}) = \max_{c_{it}, n_{it}, b_{it+1}} U(c_{it}, n_{it}) + \beta E\left[V\left(b_{it+1}, \theta_{it+1}\right) \mid \theta_{it}\right] \tag{7}$$

$$\text{s.t. } b_{it} + \theta_{it} n_{it} - \tau(\theta_{it}) \geq q_t b_{it+1} + c_{it}, \tag{8}$$

$$b_{it+1} + \phi \geq 0 \tag{9}$$

2.3 Results

- Baseline Model
  Consumers who's borrowing constraint was slack are forced to deleverage when the borrowing limit falls. These consumers both increase their labor supply and reduce their consumption. Furthermore, this deleveraging requires in an increase in the demand for bonds to a fall in the real interest rate.

  There is an increase in precautionary savings for non-constrained consumers (who are not at the right tail of the initial bond distribution) in order to buffer themselves against future shocks. Increasing their precautionary motive requires an increase in the demand for bonds leading the real interest rate to fall.

  Only highly productive consumers (those who did not experience a significant negative productivity shock) will decumulate bonds and increase consumption from lowered interest rates.

3 Replication

Table 1 Model Calibration

Symbol	MATLAB	HARK	Value	Description
γ	'gam'	'CRRA'	4	Coefficient of relative risk aversion
r	'r'	'1-Rfree'	0.00625	Quarterly net interest rate
η	'eta'	'eta'	1.5	Curvature of utility from leisure
β	'bet'	'DiscFac'	0.9457	Discount factor
v	'nu'	'nu'	0.16	UI benefits
B	'B'	'B'	2.56	Net supply of bonds
ψ	'pssi'	'pssi'	18.154609	Disutility from labor
ϕ	'phi'	'-BoroCnstArt'	1.60	Borrowing limit
$\pi_{e,u}$	'sep'	'sep'	0.0573	Separation probability
$\pi_{u,e}$	'fin'	'fin'	0.8820	Job-finding probability
ρ			0.967	Persistence of wage process
σ_ε^2			0.017	Variance of wage process

The wage process is approximated by a 12-state Markov chain, following the approach in Tauchen (1986). Since the GLConsumerType is a subclass of the IndShockConsumer-Type , we must specify an income distribution. However, because there are no transitory nor permanent shocks to income in the model, the Income distribution is degenerate.

3.1 Initial Steady States

See Figure 3.1 and Figure 1 for consumption and labor supply functions for $\theta_{it} = \theta^2, \theta^8$ from out replication codes.

It is apparent that consumption varies with bond holdings. At high levels of bond holdings, the consumer's behavior is similar to that of the Permanent Income Hypothesis. The consumption function is concave due to the precautionary motive as there is positive probability that the agent will be unemployed. The labor supply functions are convex as higher levels of bond holdings capture an income effect that in turn lowers the amount of labor supplied.

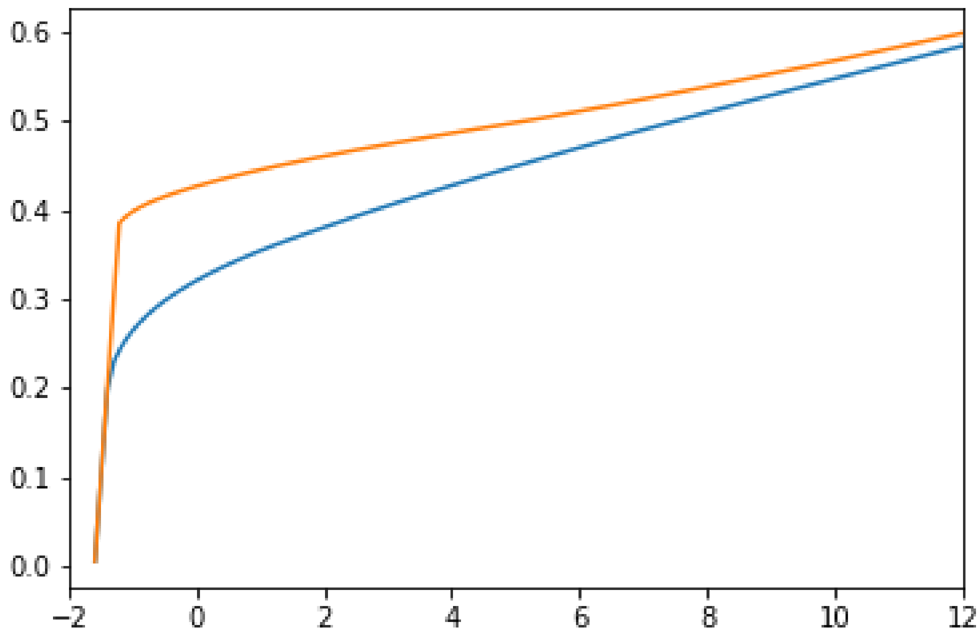

Figure 1 Consumption Functions for $\theta_{it} = \theta^2, \theta^8$

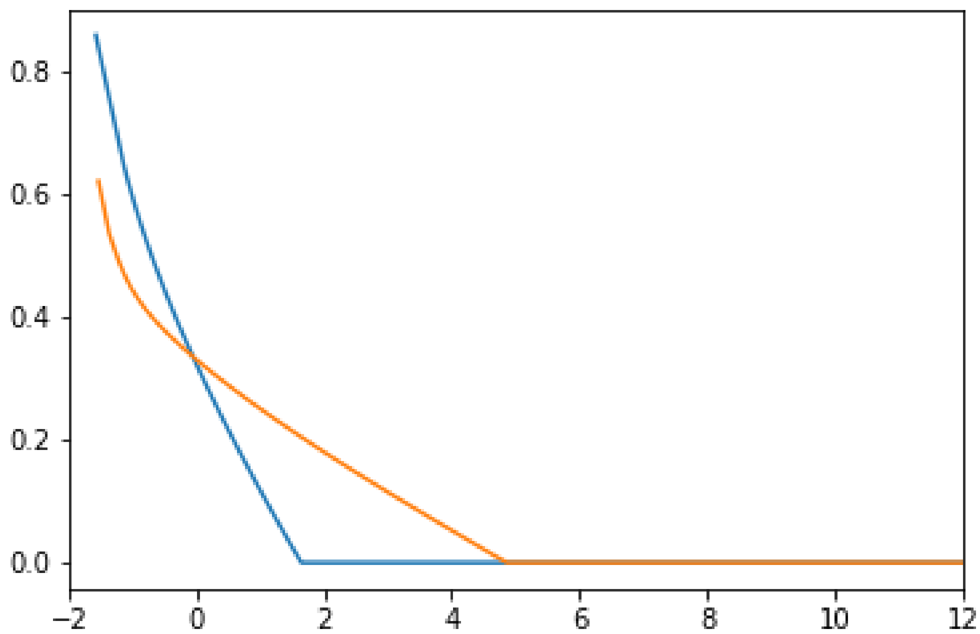

Figure 2 Labor Supply Functions for $\theta_{it} = \theta^2, \theta^8$

3.2 Figures in Paper

See Figure 3.2 for labor supply functions for $\theta_{it} = \theta^2, \theta^8$

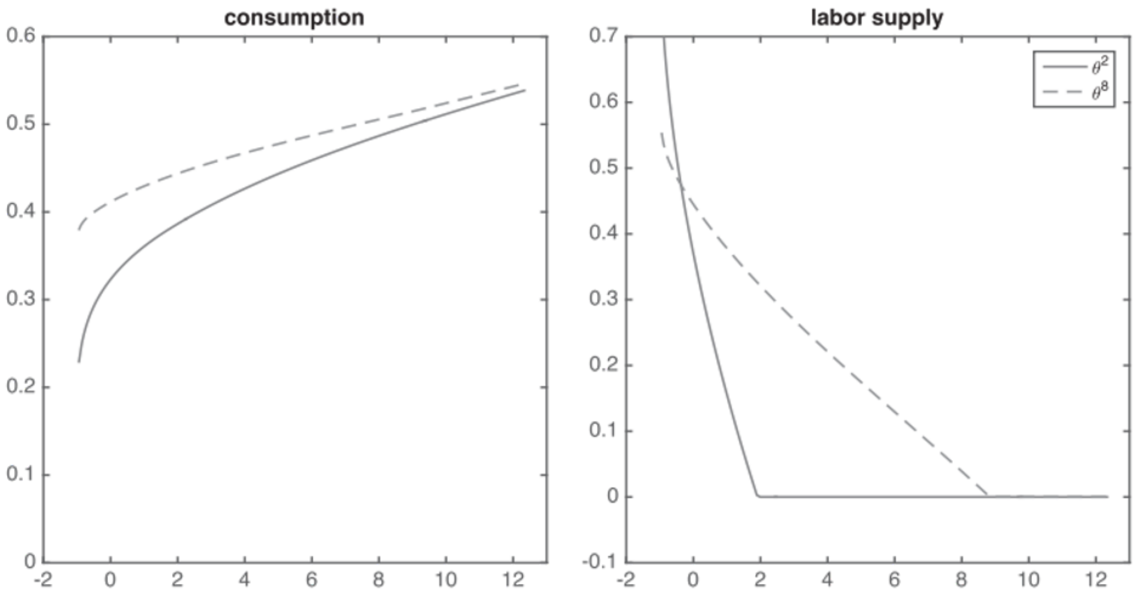

FIGURE I

Optimal Consumption and Labor Supply in Steady State

Figure 3 Optimal Consumption and Labor Supply in Steady State

The figures produced in our code closely resemble those found in the paper. The only notable difference is the labor supply function when $\theta_{it} = \theta^8$.

3.3 Optimal Consumption and Labor Supply for All θ_{it}

See Figure 3.3 and Figure 4

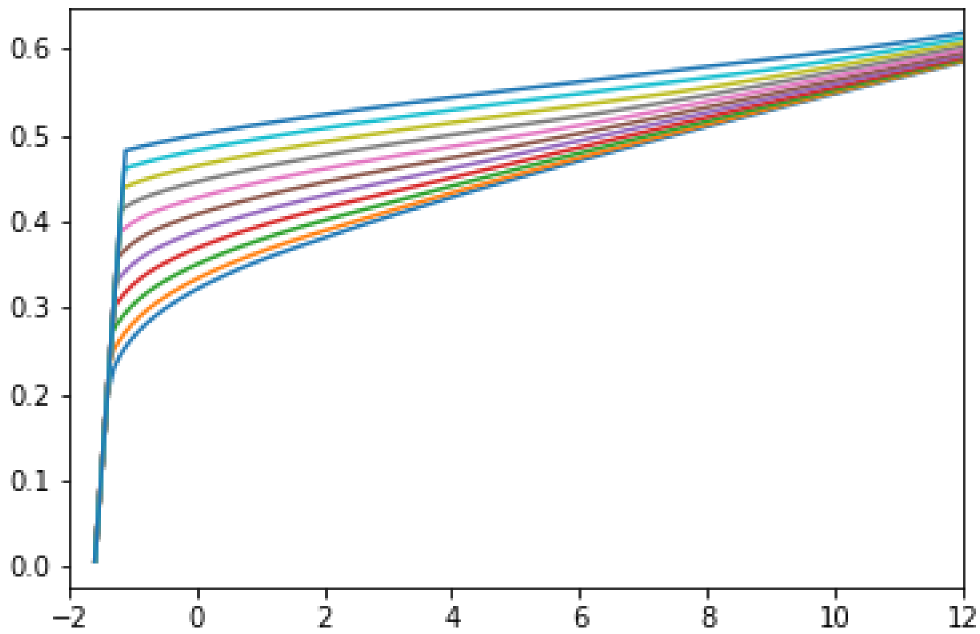

Figure 4 Optimal Consumption for All θ_{it}

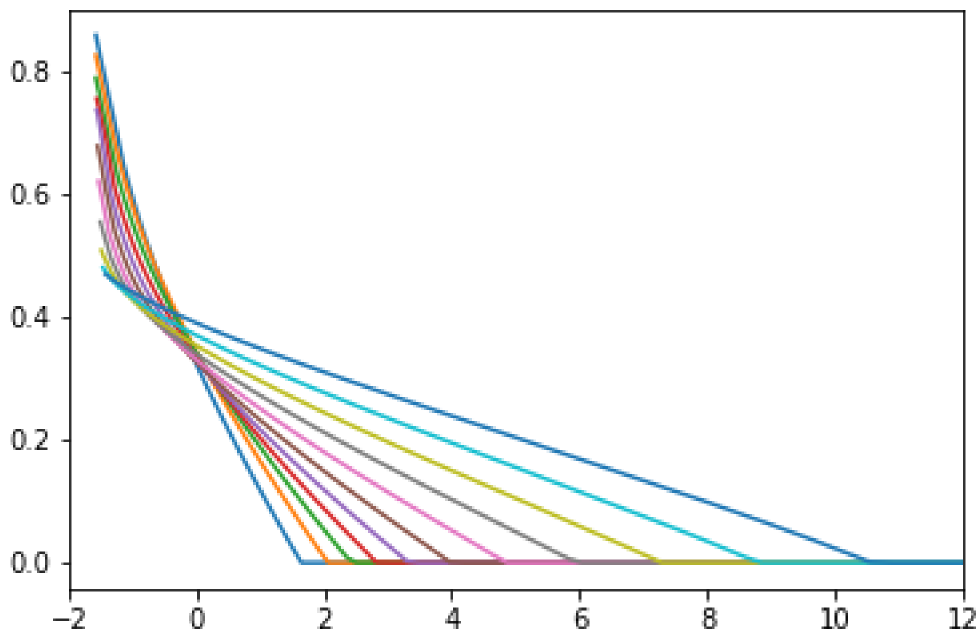

Figure 5 Optimal Labor Supply for All θ_{it}

Appendices

This Appendix derives the first order conditions that characterizes the solution to the agent's problem.

Dropping the subscript i, the problem in bellman form is

$$V_t(b_t,\theta_t) = \max_{c_t,n_t} \&U(c_t,n_t) + \beta E[V_{t+1}(b_{t+1},\theta_{t+1})|\theta_t].$$

The first order conditions for consumption and labor are

$$\begin{aligned} U_c(c_t,n_t) &= \beta R_t E[V^b_{t+1}(b_{t+1},\theta_{t+1})|\theta_t] \\ U_n(c_t,n_t) &= -\beta\theta_t R_t E[V^b_{t+1}(b_{t+1},\theta_{t+1})|\theta_t]. \end{aligned}$$

Combining these two FOCs, we obtain

$$U_n(c_t,n_t) = -\theta_t U_c(c_t,n_t).$$

Let $\underline{V}_t\left(b_t,\theta_t,c_t,n_t\right) = U\left(c_t,n_t\right) + \beta E\left[V_{t+1}\left(b_{t+1},\theta_{t+1}\right) \mid \theta_t\right]$. It follows that

$$\begin{aligned} \underline{V}^b_t\left(b_t,\theta_t,c_t,n_t\right) &= \beta E\left[V^b_{t+1}\left(b_{t+1},\theta_{t+1}\right) \mid \theta_t\right] \\ \underline{V}_t\left(b_t,\theta_t,c_t\left(b_t\right),n_t\left(b_t\right)\right) &= V_t\left(b_t,\theta_t\right). \end{aligned}$$

Since $c'_t(b_t) = 0$ and $n'_t(b_t) = 0$, then

$$\begin{aligned} &\underline{V^b}_t(b_t,\theta_t,c_t(b_t),n_t(b_t)) = V^b_t(b_t,\theta_t), \text{ so} \\ &V^b_t(b_t,\theta_t) = \beta E\left[V^b_{t+1}\left(b_{t+1},\theta_{t+1}\right) \mid \theta_t\right]. \end{aligned}$$

Combining with the Consumption FOC,

$$\begin{aligned} &V^b_t(b_t,\theta_t) = U_c(c_t,n_t) \\ &U_c(c_t,n_t) = \beta R_t \mathbb{E}\left(U_c(c_{t+1},n_{t+1})\right). \end{aligned}$$