HANK with Housing Block

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In [1]:

```
import numpy as np
from numba import njit
import matplotlib.pyplot as plt
from copy import deepcopy
import importlib
import pickle

from sequence_jacobian import grids, interpolate,simple, create_model, solved
from sequence_jacobian.blocks.stage_block import StageBlock
import warnings
warnings.filterwarnings('ignore')
```

Households

- Each period, households choose to be either owners or renters
- Owners can borrow.
- One asset consumption saving problem
- Idiosyncratic income shocks

The Household problem

Stage 1: Enter period t and draw idiosyncratic labor productivity shock

Labor productivity follows a discretized AR(1) process in logs

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(0, \sigma_z^2)$$

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$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(0, \sigma_z^2)$$

$$V_t^{(1)}(h_{t-1}, z_{t-1}, a_{t-1}) = \Pi_z V_t^{(2)}(h_{t-1}, z_t, a_{t-1})$$

Stage 3: Aggregate shocks are realized and households make discrete choice

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- If renters, choose to either buy house $h = 1$ or keep renting $h = 0$

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- If owner $h = 1$, otherwise renter $h = 0$
- Owners choose to either sell house $h = 0$ or continue owning $h = 1$
- If renters, choose to either buy house $h = 1$ or keep renting $h = 0$

$$V_t^{(2)}(h_{t-1}, z_t, a_{t-1}) = \max_{h_t \in \{0,1\}} V_t^{(3)}(h_t, z_t, a_{t-1}) + \underbrace{\varepsilon(h_t)}_{\text{taste shock}}$$

Stage 4: Solve consumption saving problem

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$$V_t^{(3)}(h_t, z_t, a_{t-1}) = \max_{c_t, a_t} u(c_t, h_t) + \beta \mathbb{E}_t V_{t+1}^{(1)}(h_t, z_t, a_t)$$

Owners Budget Constraints

If owner choosing to continue owning,

$$
\begin{aligned}
c_t + a_t &= (1 + r_t) a_{t-1} + y_t z_t \\
a_t &\geq \min\{-\kappa_h P_t, a_{t-1}\}
\end{aligned}
$$

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a_t &\geq \min\{-\kappa_h P_t, a_{t-1}\}
\end{aligned}
$$

If owner choosing to sell, and become a renter

$$
\begin{aligned}
c_t + a_t &= (1 + r_t) a_{t-1} + y_t z_t + P_t - \xi - p_{r,t} \\
a_t &\geq 0
\end{aligned}
$$

Renters Budget Constraints

If renter looking to continue renting

$$
\begin{aligned}
c_t + a_t &= (1 + r_t) a_{t-1} + y_t z_t - p_{r,t} \\
a_t &\geq 0
\end{aligned}
$$

Renters Budget Constraints

If renter looking to continue renting

$$
\begin{aligned}
c_t + a_t &= (1 + r_t)a_{t-1} + y_t z_t - p_{r,t} \\
a_t &\geq 0
\end{aligned}
$$

If renter looking to buy a house

$$
\begin{aligned}
c_t + a_t &= (1 + r_t)a_{t-1} + y_t z_t - P_t - \xi \\
a_t &\geq -\kappa_h P_t
\end{aligned}
$$

General Equilibrium

General Equilibrium

- Standard New Keynesian setup
    - Sticky prices
    - Sticky wages
- Government taxes labor income to pay off interest on government debt.
    - Tax schedule is determined by a standard fiscal rule.
- Inertial Taylor rule

Housing and rental markets

There is a fixed supply of houses $\bar{H}$.

Houses are either owned by homeowners or by the rental sector.

$$\bar{H} = H_o + H_r$$

Rental Sector

- There is a rental sector that borrows from a bank to buy houses H_r and converts them into rental units at rate $\psi > 1$.
    - Rental units are rented out to households at rent price p_r.

$$H_{seg} = \psi H_r$$

where H_{seg} is the supply of rental properties.

The rental sector makes zero profits in expectation.

$$p_{r,t} = \frac{1}{\psi}\mathbb{E}_t\left[r_{t+1}P_t + P_t - P_{t+1}\right]$$

And consumes all unexpected profit

$$C_{rental} = p_{r,t}H_{seg,t} + \frac{1}{\psi}(P_t - P_{t-1})H_{seg,t-1} - \frac{1}{\psi}r_t P_{t-1}H_{seg,t-1}$$

Market Clearing

Asset market

- In aggregate, households deposit A_t at the bank.
- The bank uses these deposits to loan to the rental sector and buy government bonds B_t

$$A_t = P_t H_r + B_t$$

Asset market

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- The bank uses these deposits to loan to the rental sector and buy government bonds B_t

$$A_t = P_t H_r + B_t$$

Goods market

$$Y_t = C_t + C_{rental,t}$$

Bringing the model to code

In [2]:

```
# Import the households
from python.hhproblem_WD import hh, cali
import python.hhproblem_WD as hhprob_WD_st
```

Household calibration

- Key calibration targets:
    - Median house price to median earnings: 32 times quarterly income
    - Share of households that are homeowners : 64%
    - Median rental price as a proportion of household income: 26%
    - Transactional costs of buying a house: 1% of the house price
    - Loan to house price: 75%

Specifying General Equilibrium Blocks into SSJ toolkit

In [3]:

```
# Some general equilibrium equations

@simple
def rentprice(rb,ph,relHR):
    pr = ((rb(+1)*ph+ph-ph(+1))*relHR  ) # arbitrage equation for the rental company, asset pricing equation
    return pr

@simple
def mkt_clearing(AD, NE, C, L, Y, B, mu,hours,Div,ph,HR,Eph,beta,pr,rb,HOO,HOR,HRO,transac,relHR,hbar):

    asset_mkt = AD - B - ph*HR*relHR # bank issues deposits to rental companies who buy rental housing off them

    housing_mkt=hbar-relHR*HR-HOO

    crent=pr*HR+(ph-ph(-1))*HR(-1)*relHR-rb*ph(-1)*HR(-1)*relHR # rental companies consumption

    labor_mkt = hours*NE - L

    goods_mkt = Y - C -crent -transac*(HOR+HRO)

    return asset_mkt, goods_mkt,labor_mkt,crent,housing_mkt
```

In [4]:

```
# Remaining Equations

@solved(unknowns={'i':0.005},targets=['i_res'],solver='brentq')
def Taylor(i,pi, rstar, phi,rhor,epsr):
    i_res=(1-rhor)*(rstar+phi*pi)+rhor*(i(-1))+epsr-i # Taylor Rule
    return i_res


@simple
def Fisher(i,pi):
    rb=(1+i(-1))/(1+pi)-1
    r=rb
    return rb,r

@simple
def wageinflation(w,pi):
    piw=w-w(-1)+pi
    return piw

@simple
def firm(Y, Z, pi, mu, kappa,w):
    L = Y/Z
    Div = Y - w * L #- mu/(mu-1)/(2*kappa) * (1+pi).apply(np.log)**2 * Y
    return Div,L

@solved(unknowns={'B': 5.6,'Tax': 0.02 }, targets=['B_res','Tax_res'], solver="brentq")
def fiscal(Tax,B,rb,gammatax,rhotax,Taxss,Bss,w,L,Y,Yss,gammay):
    B_res=B(-1)*(1+rb)-Tax*w*L-B

    Tax_res = Taxss*((Tax(-1)/Taxss)**rhotax)*((B/Bss)**(gammatax*(1-rhotax)))*((Y/Yss)**(gammay*(1-rhotax)))-Tax
    return B_res,Tax_res

@simple
def Agglabsupply(w,wss,frisch,beta,phil,piw,hourss,kappaw):
    hours=(frisch*((w/wss).apply(np.log)+ 1/kappaw*(piw-beta*piw(+1)))+hourss.apply(np.log)).apply(np.exp)
    return hours

@simple
def expectations(ph):
    Eph=ph(+1)
    return Eph

@simple # dynamic philips curve
def nkpc(pi, w, Z, Y, r, mu, kappa):
    nkpc_res = kappa * (w / Z - 1 / mu) + Y(+1) / Y * (1 + pi(+1)).apply(np.log) / (1 + r(+1))\
               - (1 + pi).apply(np.log)
    return nkpc_res

@simple
def TaylorSS(pi, rstar, phi):
    i=rstar+phi*pi # Taylor Rule
    return i

@simple
```

```
def fiscalSS(rb, B,w,L):
    Tax = (rb * B(-1))/(w*L)
    return Tax

@simple
def AgglabsupplySS(w,frisch,L,Ladj):
    hours=L/Ladj
    phil=w/hours**(1/frisch)
    return phil,hours

@solved(unknowns={'q': 0.2/0.005}, targets=['eq_arb'], solver="brentq")
def eqarb(q,rb,Div):
    eq_arb=1+rb-(Div(+1)+q(+1))/q
    return eq_arb

@simple
def fpricesSS(Div,rb,relHR,phss):
    q=Div/rb
    ph=phss
    pr=rb*ph*relHR
    return q, pr,ph

@simple
def nkpc_ss(Z, mu):
    w = Z / mu
    return w
```

Solve for Steady State

Solve for Steady State

In [5]:

```
%%time
blocks_ss = [hh, firm, TaylorSS,Fisher, fiscalSS,AgglabsupplySS,fpricesSS, mkt_clearing,nkpc_ss,expectations]

hank_ss = create_model(blocks_ss, name="One-Asset HANK SS")

unknowns_ss = {'beta': [0.975,0.985],'vh2':[0,0.4]}

targets_ss = {'asset_mkt': 0,'housing_mkt': 0}

ss0 = hank_ss.solve_steady_state(cali, unknowns_ss, targets_ss, solver="broyden_custom")
```

```
CPU times: user 13.9 s, sys: 85.3 ms, total: 14 s
Wall time: 14 s
```

Household transitions

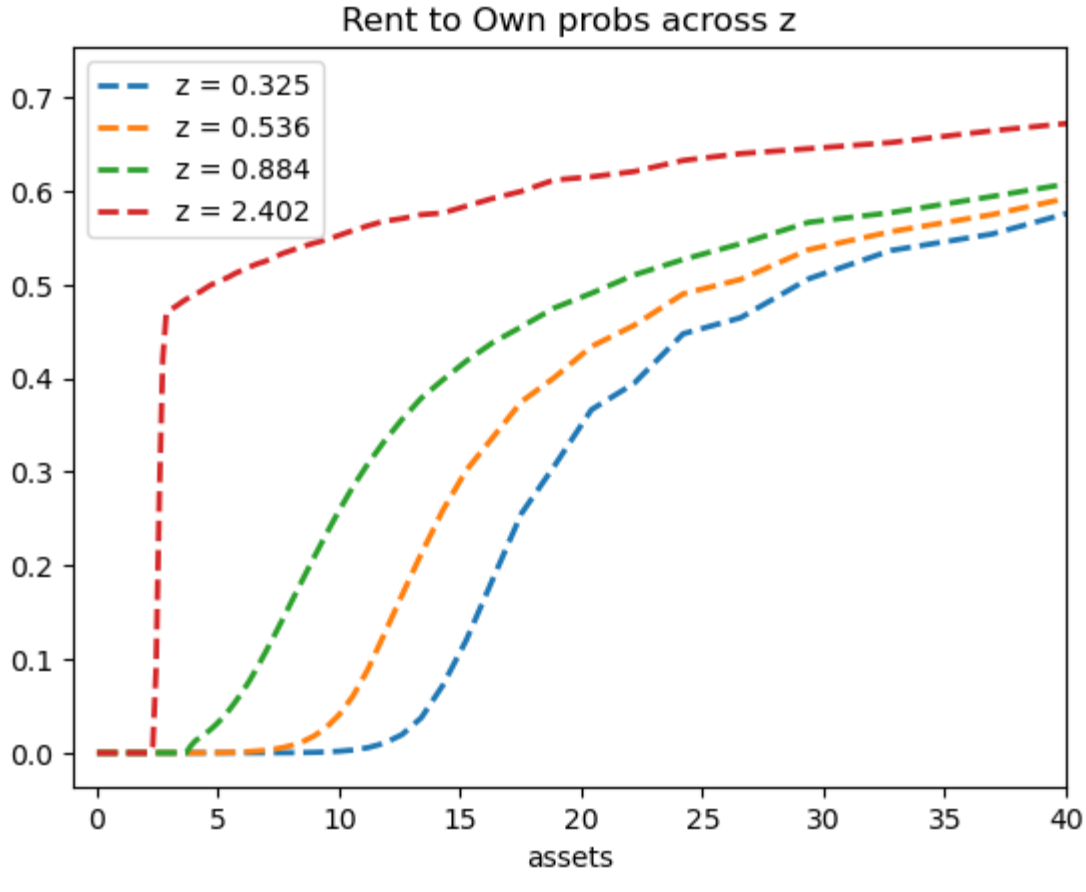
Household transitions

In [6]:

```
ss0['hourss']=ss0['hours']
ss0['HRSS']=ss0['HR']

P_transition = ss0.internals['hh']['house_choice']['law_of_motion'].P # transition matrix between states across assets and
a_grid_r = ss0.internals['hh']['a_grid_r'] # renters asset grid
a_grid_o = ss0.internals['hh']['a_grid_o'] # owners asset grid
z_grid = ss0.internals['hh']['z_grid'] # idiosyncratic prod. grid
```

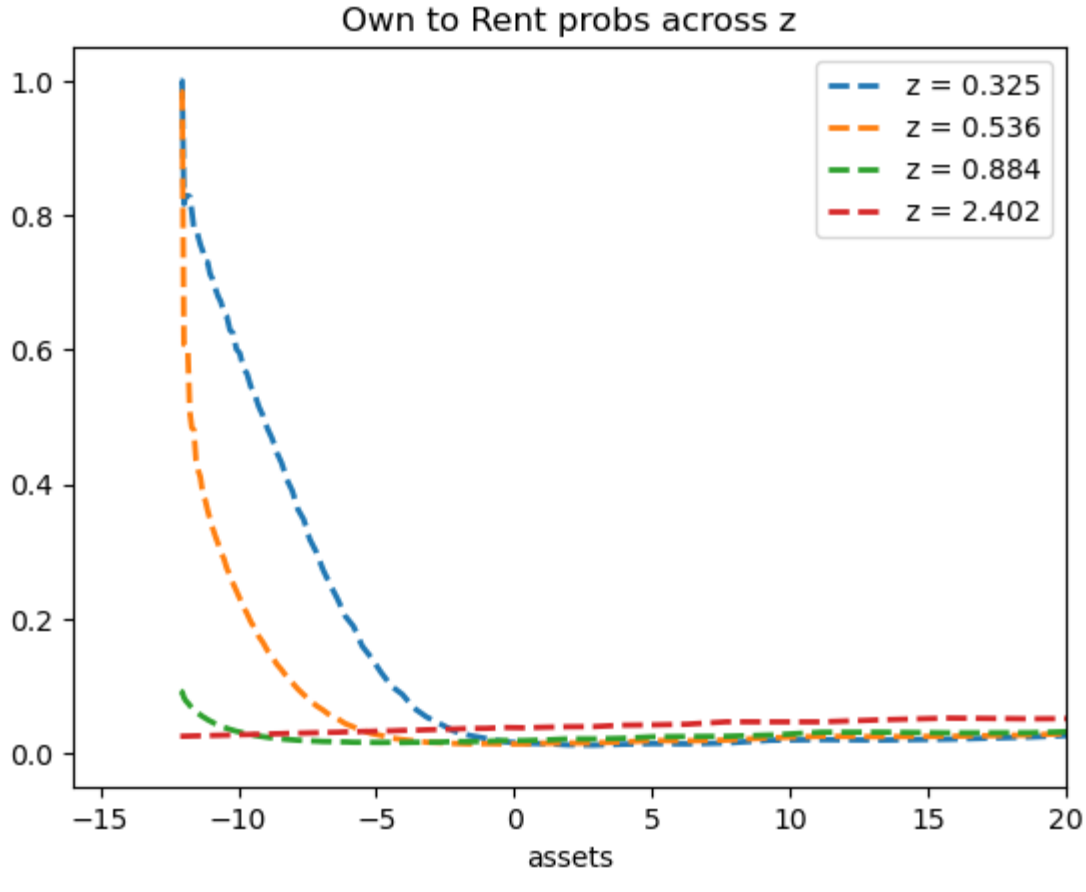
In [7]:

```
plt.plot(a_grid_r, P_transition[2, 3, 0], linewidth=2, linestyle = '--' , label = 'z = ' + str(round(z_grid[0],3) )) #
plt.plot(a_grid_r, P_transition[2, 3, 1], linewidth=2, linestyle = '--' , label = 'z = ' +str(round(z_grid[1],3))) #
plt.plot(a_grid_r, P_transition[2, 3, 2], linewidth=2, linestyle = '--' , label ='z = ' +str(round(z_grid[2],3))) #
plt.plot(a_grid_r, P_transition[2, 3, 4], linewidth=2, linestyle = '--' , label ='z = ' +str(round(z_grid[4],3))) #
plt.title('Rent to Own probs across z')
plt.legend()
plt.xlim(- 1,40)
plt.xlabel('assets')
plt.show()
```



In [8]:

```
plt.plot(a_grid_o, P_transition[1, 0, 0], linewidth=2, linestyle = '--' , label = 'z = ' + str(round(z_grid[0],3) )) #
plt.plot(a_grid_o, P_transition[1, 0, 1], linewidth=2, linestyle = '--' , label = 'z = ' +str(round(z_grid[1],3))) #
plt.plot(a_grid_o, P_transition[1, 0, 2], linewidth=2, linestyle = '--' , label ='z = ' +str(round(z_grid[2],3))) #
plt.plot(a_grid_o, P_transition[1, 0, 4], linewidth=2, linestyle = '--' , label ='z = ' +str(round(z_grid[4],3))) #
plt.title('Own to Rent probs across z')
plt.legend()
plt.xlim(-ss0['ph']*ss0['kappah'] - 4,20)
plt.xlabel('assets')
plt.show()
```



Computing the het agent Jacobians in the code

Computing the het agent Jacobians in the code

In [9]:

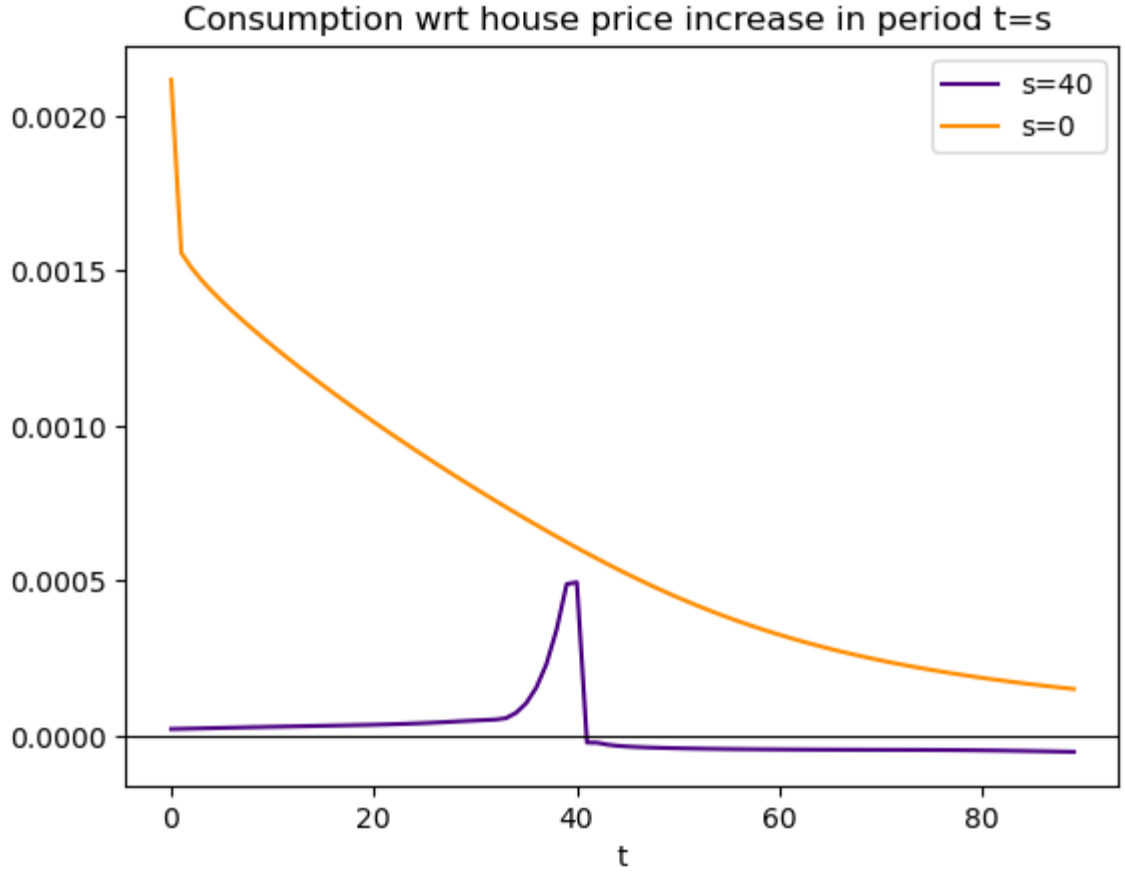
```
%%time
T = 300
J_hh = hh.jacobian(ss0, inputs=['Tax','Div', 'r','w','hours','pr','ph', 'MPC'], T=T)
J_hh_base = deepcopy(J_hh)
```

```
CPU times: user 5.8 s, sys: 3.33 s, total: 9.14 s
Wall time: 3.45 s
```

Jacobians with respect to increase in house price

In [10]:

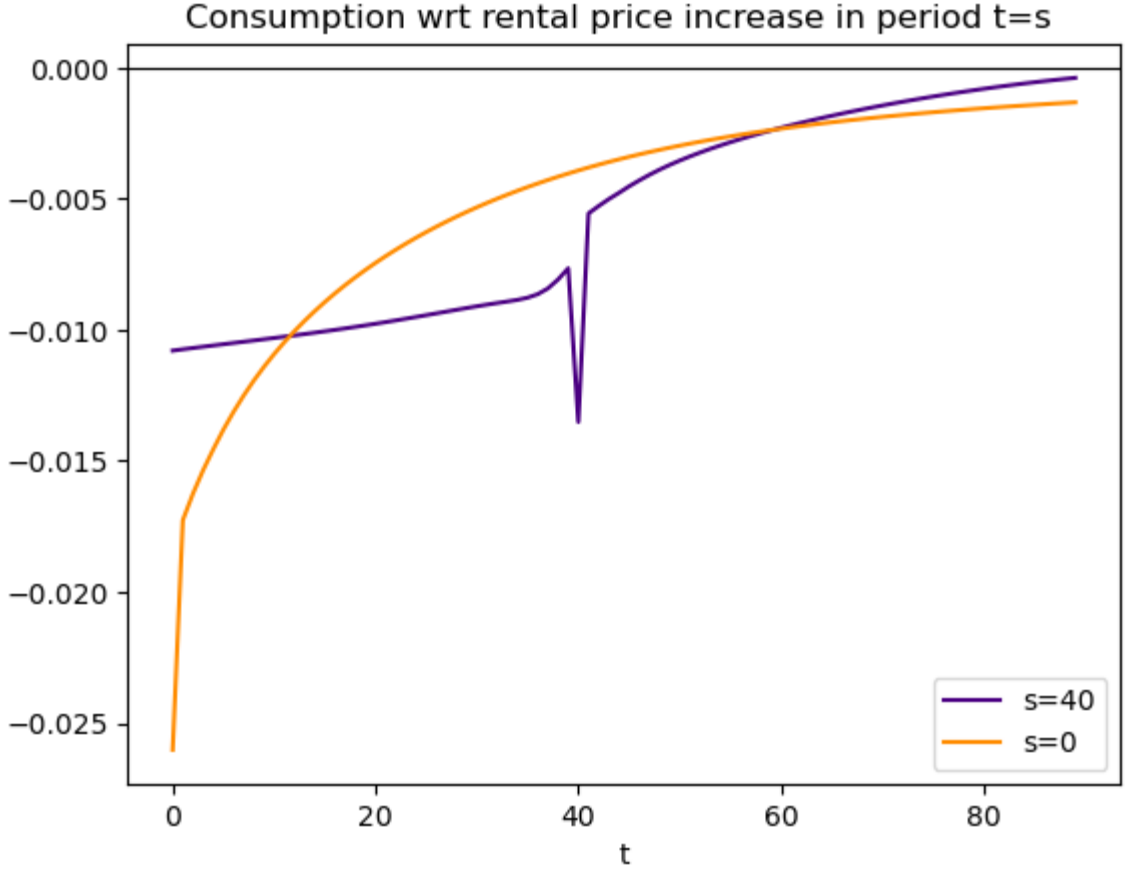
```
plt.plot( J_hh['C']['ph'][:90, 40], color = 'indigo', label = 's=40')
plt.plot( J_hh['C']['ph'][:90, 0], color = 'darkorange', label = 's=0')
plt.xlabel('t')
plt.axhline(0, color='k', linestyle='-', linewidth = .8)
plt.title('Consumption wrt house price increase in period t=s ')
plt.legend()
plt.show()
```



Jacobians with respect to increase in rental price

In [11]:

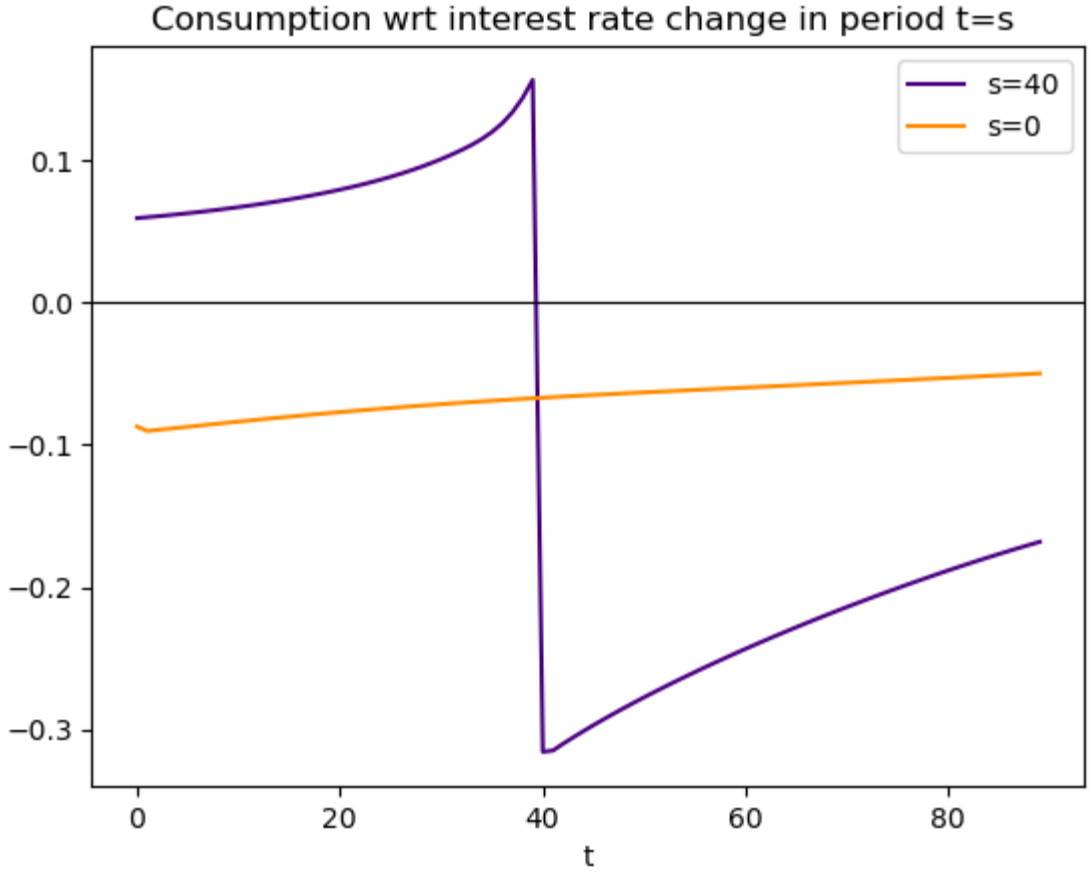
```
plt.plot( J_hh['C']['pr'][:90, 40], color = 'indigo', label = 's=40')
plt.plot( J_hh['C']['pr'][:90, 0], color = 'darkorange', label = 's=0')
plt.xlabel('t')
plt.axhline(0, color='k', linestyle='-', linewidth = .8)
plt.title('Consumption wrt rental price increase in period t=s ')
plt.legend()
plt.show()
```



Jacobians with respect to fall in real interest rate

In [12]:

```
plt.plot( -J_hh['C']['r'][:90, 40], color = 'indigo', label = 's=40')
plt.plot( -J_hh['C']['r'][:90, 0], color = 'darkorange', label = 's=0')
plt.xlabel('t')
plt.axhline(0, color='k', linestyle='-', linewidth = .8)
plt.title('Consumption wrt interest rate change in period t=s ')
plt.legend()
plt.show()
```

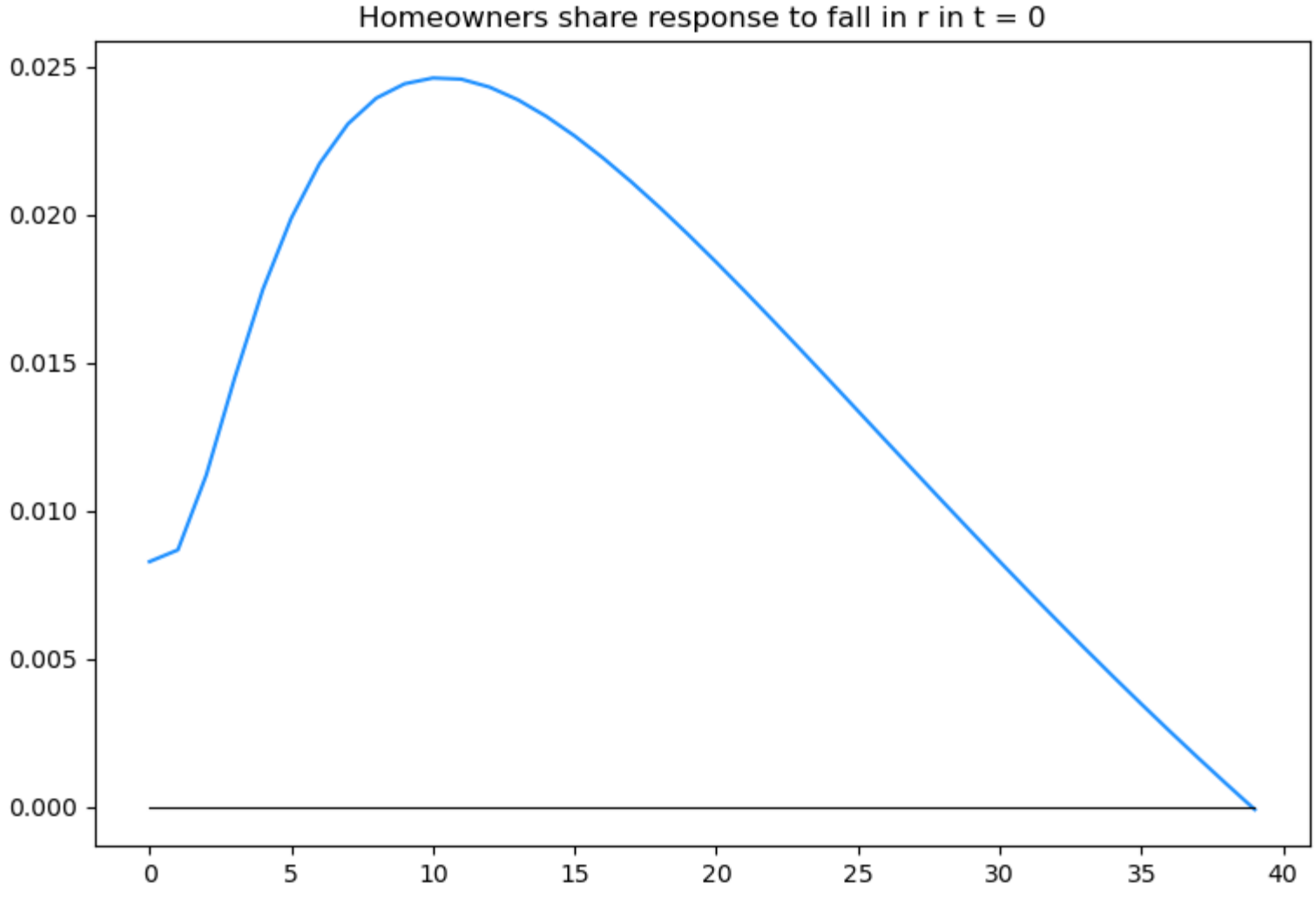


In [13]:

```
fig, axs = plt.subplots(1, figsize=(9, 6))
axs.plot(-J_hh['HOO']['r'][:40, 0] , color = 'dodgerblue')
axs.plot(np.zeros(40), color = 'k', linewidth = .8)
axs.set_title('Homeowners share response to fall in r in t = 0')
```

Out[13]:

```
Text(0.5, 1.0, 'Homeowners share response to fall in r
in t = 0')
```


Homeowners share response to fall in r in t = 0
0.025
0.020
0.015
0.010
0.005
0.000
0
5
10
15
20
25
30
35
40

General equilibrium impulse responses

General equilibrium impulse responses

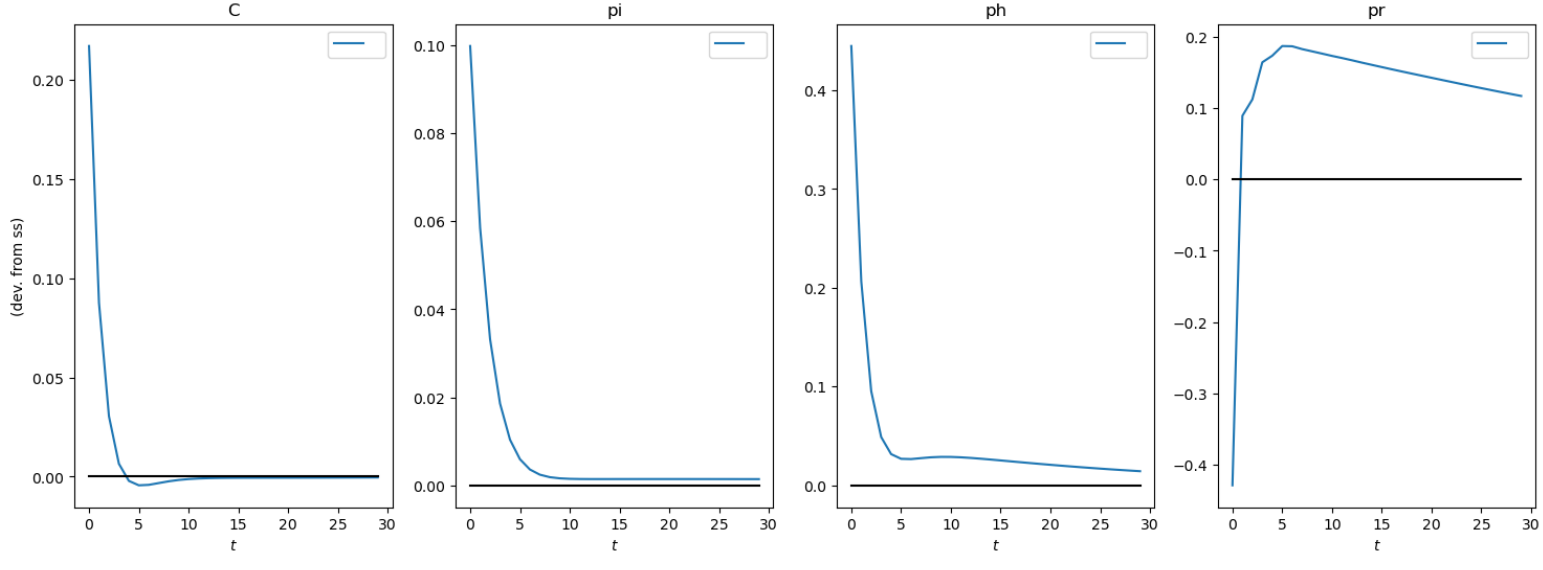
In [14]:

```
%%time
show_irfs = hhprob_WD_st.show_irfs # function to plot Irfs

exogenous = ['epsr', 'Z']
unknowns = [ 'pi','Y','w','ph']
targets = ['nkpc_res','labor_mkt', 'asset_mkt','housing_mkt']

drstar=np.zeros(T)
drstar[0]= -0.0025

rstar_shock_path = {"epsr": drstar}
blocks = [hh, firm, Taylor,Fisher, fiscal,Agglabsupply, mkt_clearing,rentprice,eqarb, nkpc,wageinflation,expectations]
hank = create_model(blocks, name="One-Asset HANK")

IPRs = hank.solve_impulse_linear(ss0, unknowns, targets, rstar_shock_path)
```

```
CPU times: user 17.5 s, sys: 8.97 s, total: 26.5 s
Wall time: 8.55 s
```

In [15]:

```
show_irfs([IPRs],ss0,[ 'C' , 'pi' , 'ph', 'pr'  ])
```

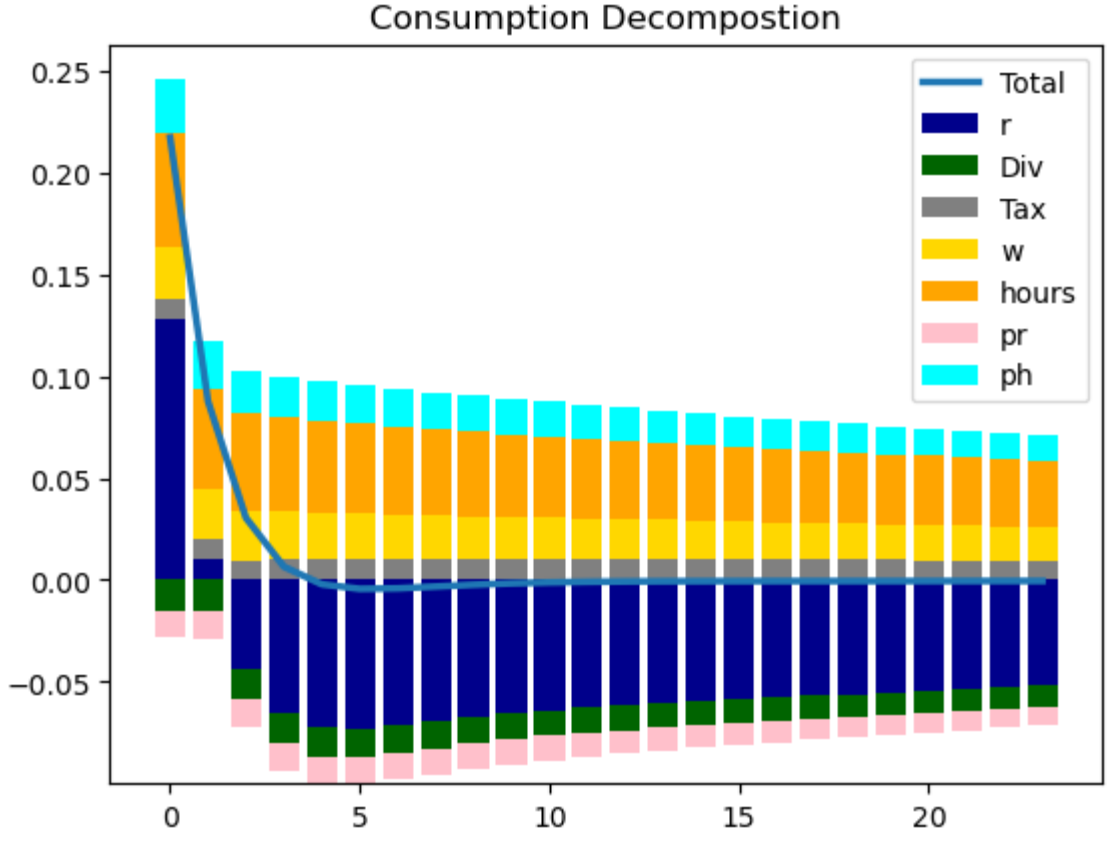


Decomposing Consumption

Decomposing Consumption

In [16]:

```
plot_C_decomp = hhprob_WD_st.plot_C_decomp # function to plot Irfs
plot_C_decomp(IPRs,J_hh,ss0,T) # base
```



Including sticky expectations

- Suppose when an aggregate shock occurs,
    - In any given period,
    - 25% of hh do not update their forecast of the path of aggregate variables.

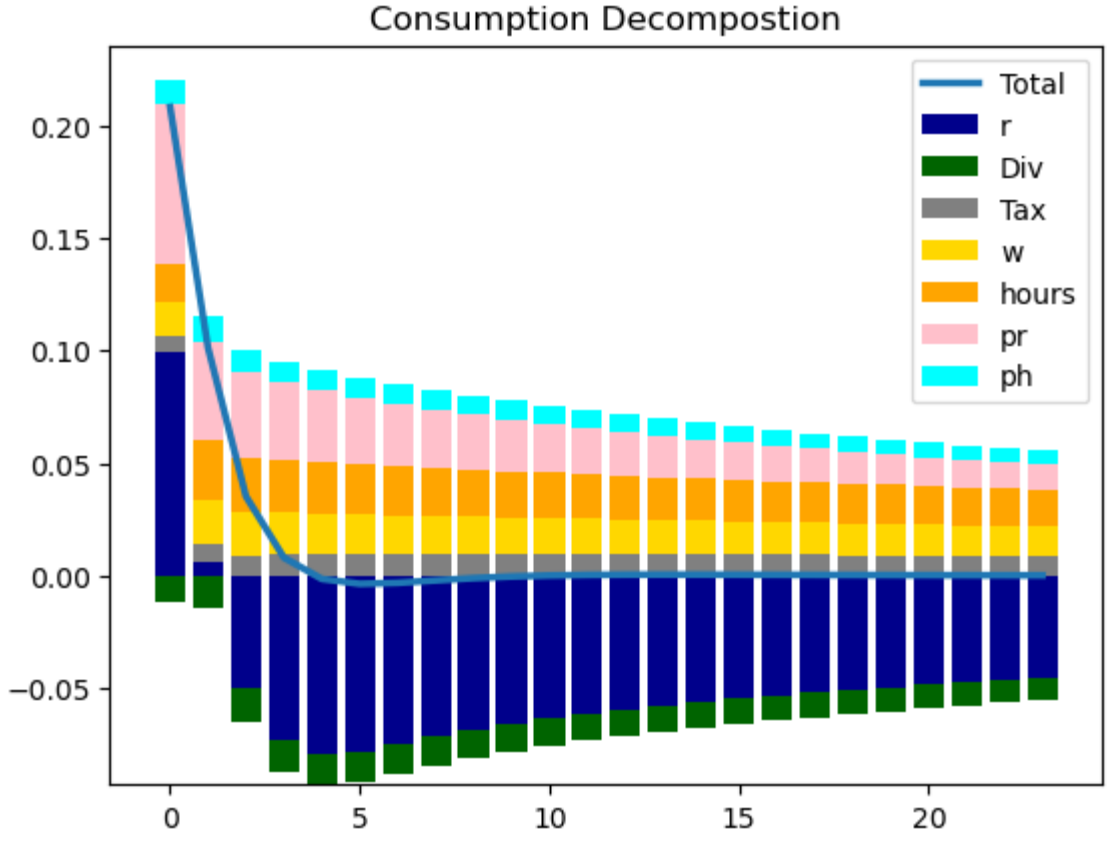
In [17]:

```
%%time
theta = .25 # Sticky expectations parameter

stick_jacob = hhprob_WD_st.stick_jacob

J_hh_sticky = stick_jacob(J_hh,theta)

G_sticky = hank.solve_jacobian(ss0, unknowns, targets, exogenous, T=T,Js={'hh': J_hh_sticky})
# convert G jacobian dictionary into impulse response dictionary
IPRs_sticky = {}
for output in G_sticky:
    IPRs_sticky[output] = G_sticky[output]['epsr'] @drstar
```

```
CPU times: user 9.52 s, sys: 3.14 s, total: 12.7 s
Wall time: 4.65 s
```

In [18]:

```
plot_C_decomp(IPRs_sticky,J_hh_sticky,ss0,T) # base
```

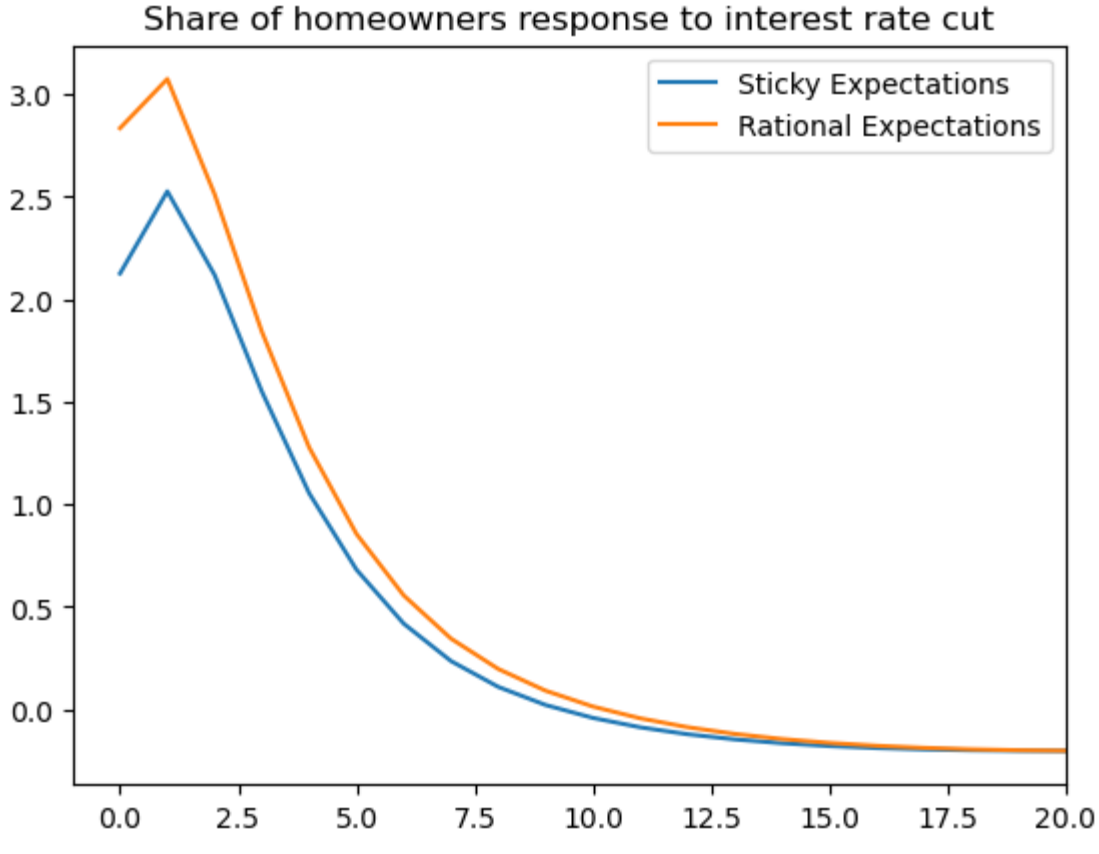


In [19]:

```
plt.plot(100 * J_hh_sticky['HOO']['r']@IPRs_sticky['r']/ss0['HOO'], label =  'Sticky Expectations')
plt.plot(100 * J_hh_base['HOO']['r']@IPRs_sticky['r']/ss0['HOO'], label = 'Rational Expectations')
plt.xlim(-1,20)
plt.title('Share of homeowners response to interest rate cut')
plt.legend()
```

Out[19]:

```
<matplotlib.legend.Legend at 0x7feac8f498e0>
```



Future directions

- Add mortgages as second asset
- Add quality of housing to utility function
- Add landlords
- Decompose consumption responses between wealth effects and collateral effects