The Distribution of Wealth, Precautionary Saving, and the Business Cycle

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Abstract

In a heterogenous agents New Keynesian model with search and matching frictions and beliefs on job finding, I quantify the role of precautionary saving over the business cycle.

Keywords Precautionary Saving, Heterogeneous Agents, Incomplete Markets, Beliefs, Search and Matching

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1 Introduction

The past decade has seen an eruption of research on the importance of household uncertainty over the business cycle. Increases in household uncertainty, in the form of heightened income or unemployment risk, have shown to generate sizable business cycle fluctuations. Although the effects of uncertainty are transmitted through the precautionary channel[1], few papers study the sensitivity of the precautionary channel to economic conditions. Given precautionary saving is a central mechanism to the effectiveness of unemployment benefits (Kekre (2021)), it is critical to understand the determinants of the magnitude of the precautionary channel in the design of unemployment insurance policy.

In this paper, I quantify the precautionary channel and study its sensitivity to the distribution of liquid wealth in a Heterogeneous Agent New Keynesian(HANK) model with Search and Matching (SAM) frictions and beliefs on job finding probabilities. In particular, I study the determinants of the magnitude of the precautionary channel over the business cycle with a focus on the distribution of liquid wealth. Since precautionary saving is dictated by household expectations, I allow households to form beliefs on job finding expectations to capture the underreaction in job finding expectations found in recent work by Mueller, Spinnewijn, and Topa (2021) and Du, Qiu, and Wang (2022). The model features idiosyncratic income shocks to generate a realistic distribution of wealth in order to both study the relationship between precautionary saving over the cross section of wealth and to match the empirical dynamic consumption response out of unanticipated income shocks documented in Fagereng, Holm, and Natvik (2021). A distribution of liquid wealth that contains a significant share of households who hold little liquidity (but are not constrained) is necessary to generate an empirically consistent dynamic consumption response and, as a consequence, vital to the general equilibrium transmission of shocks with nominal rigidities (Auclert, Rognlie, and Straub (2018)).

In my main result so far, I find the magnitude of the precautionary channel following a monetary policy contraction to rise exponentially with the persistence of the policy shock. This is because the precautionary channel depends on both the persistence of heightened unemployment risk and the interaction of this persistence with the empirically consistent dynamic consumption response of households. When households perceive unemployment risk to rise for a longer duration, they accumulate larger precautionary savings leading to a dramatic decline in aggregate consumption and, as a consequence, a severe rise in the unemployment rate. These fluctuations are amplified as the consumption response of newly unemployed households persist beyond the initial impact of the unemployment spell due to the persistence in the empirical dynamic consumption response. Overall, this result emphasizes the importance of the duration of heightened unemployment risk in the magnitude of the precautionary channel.

[1]The channel through which business cycle fluctutations are amplified from increases in precautionary saving.

With regards to unemployment insurance policy, this result motivates the duration of unemployment benefit extension as opposed to the magnitude of benefits in the efficient stablization of macroeconomic aggregates.

The rest of the paper is as follows. Section 2 details a literature review. Section 3 describes the data. Section 4 presents a preliminary model to be disciplined by expectations data. Section 5 details my next steps in the research process and finally section 6 details possible interesting extensions.

2 Literature Review

The core contribution of this paper is both the inclusion of a realistic distribution of wealth to study the sensitivity of precautionary saving and the evaluation of this sensitivity over the cross section of liquid wealth.

This paper contributes to the literature on the role of unemployment risk over the business cycle. This literature emphasizes the interaction between nominal rigidities, search frictions, and incomplete markets to generate counter-cyclical unemployment risk, and thus countercyclical precautionary savings, that amplify changes in aggregate demand (Den Haan, Rendahl, and Riegler (2018), Leduc and Liu (2016), and Ravn and Sterk (2017)). Within this literature, work by Challe, Matheron, Ragot, and Rubio-Ramirez (2017) do quantify the precautionary channel but do not feature a thorough distribution of wealth to match the dynamic consumption response nor study the sensitivity of precautionary savings. In comparison to these papers, I emphasize the sensitivity of the precautionary mechanism at the center of these papers. Works by Graves (2020) and Broer, Druedahl, Harmenberg, and Öberg (2021) do emphasize the factors that amplify the precautionary channel however both works do not focus on the sensitivity of precautionary saving across the distribution in liquid wealth. In particular, Broer, Druedahl, Harmenberg, and Öberg (2021) quantify the precautionary channel in a tractable HANK and SAM model that assumes a degenerate distribution of wealth and focus on the role job separations and the supply side implications of sluggish vacancies on the strength of the precautionary channel. Graves (2020) constructs a carefully calibrated HANK and SAM model that features realistic a distribution of wealth and finds the flight to liquidity mechanism induced by heightened unemployment risk to amplify the precautionary channel. His analysis captures the indirect importance of the distribution of liquidity on the amplification of the precautionary channel. I instead look to study how households build precautionary savings in direct response to heightened uncertainty across the distribution of liquidity. Heathcote and Perri (2018) emphasize the relationship between liquid wealth and the sensitivity of precautionary saving to heightened uncertainty. Their analysis, however, concerns the overall level of liquid wealth in the economy and studies a highly stylized model of precautionary saving in steady states.

This paper also contributes to the HANK literature that emphasize the distribution of wealth in producing realistic dynamic MPCs Auclert, Rognlie, and Straub (2018) and realistic transmission mechanisms to monetary policy Kaplan, Moll, and Violante (2018); Auclert (2019). Instead of studying the consumption responses across the distribution of wealth, I contribute to this literature by studying the precautionary responses to heightened uncertainty across the distribution of wealth.

3 Empirical Evidence of Underreactive Job Finding Expectations

This section summarizes the findings of Du, Qiu, and Wang (2022) that discipline the job finding expectations of households in the model. In our work, we impute expectations on job finding probabilities from the Survey of Consumer Expectations beginning in 1978 using expectations data from Michigan Survey of Consumer Expectations(MSE) and other real time macroeconomic variables. We find our imputed job finding series is strongly correlated with the realized job finding probability yet is less volatile. Our results suggests households underreact in their job finding expectations to changes in macroeconomic condition in comparison to the realized job finding probability.

3.1 Data

The expectations data on job finding and job separating probabilities are gathered from the Survey of Consumer Expectations and the Michigan Survey of Expectations (MSE). The Survey of Consumer Expectations (SCE) is a nationally representative, Internet-based survey of a rotating panel of approximately 1,300 household heads. The survey tracks a respondent's age, income, education, homeownership status, employment history, and region. The data spans from June 2013 to the present at a monthly frequency. The SCE contains expectations on job finding probabilities at a three month horizon and expectations on job losing probabilities at a one year horizon. The Michigan Survey of Expectations is also a nationally representative survey that conducts 500 interviews by telephone each month however it begins in 1978. The MSE provides expectations of job loss within 5 years at a monthly frequency beginning in December 1997.

3.2 Methodology

We use the MSE data spanning back to 1978 and other real time macro aggregates to impute the job finding expectations in the SCE. In particular, we run a lasso regression on the following equation:

$$\widetilde{\mathrm{JF}}_t = \alpha + \beta \times \mathrm{EXP}_t + \gamma \times \mathrm{REAL}_t + \epsilon_t$$

where $\widetilde{\mathrm{JF}}_t$ is the expectation of job finding probability from the SCE, EXP_t is a vector expectation variables from the MSE, and REAL_t are real time variables. The lasso penalty parameter is chosen from a k-fold[2] cross validation.

3.3 Results

Figure 1 illustrates our imputed job finding expectation series against the realized job finding probability. It is clear that the series are strongly correlated however changes in the realized job finding probability are mirror by attenuated chagnes in the imputed job finding expectation. This suggests that households underreact in comparison to the true job finding probability and that their beliefs are stubborn. When running a regression of our imputed job finding series with the realized job finding probability, we find the coefficient to range from .38 to .65 across different measures of the true job finding probability. A value of .65 would imply that a .01 increase in the actual job finding probability would, on average, be correlated to a .0065 change in the job finding expectation. This potential underreaction is consistent with the work of Mueller, Spinnewijn, and Topa (2021), who study micro data on households expectations from the survey of consumer expectations and find that households do indeed underrreact to changes in the actual job finding probability. The difference between our work and that of Mueller, Spinnewijn, and Topa (2021) is that we compare the imputed macroeconomic job finding expectations with the realized macroeconomic job finding probability as opposed to the microeconomic job finding expectation and a realized job finding probability computed from job transitions within the micro sample.

4 The Model

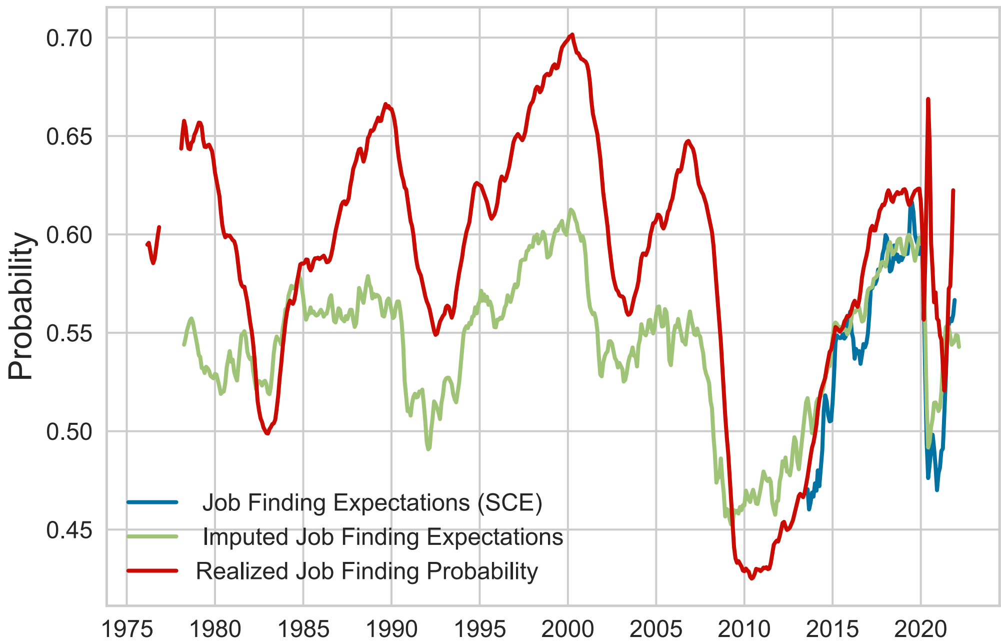
I present an heterogenous agents model with search and matching frictions, nominal rigidities, and beliefs on job finding probabilities.

4.1 Households

There is a continuum of households of mass 1 distributed on the unit interval and indexed by i. Households are ex-ante heterogeneous in their discount factors and subject to idiosyncratic income shocks and stochastic transition in their employment status. Households cannot observe the true job finding rate η_t in period t and instead hold the belief $\hat{\eta}_t$ of the job finding probability in period t. Each household solves the following problem according to their *perceived budget constraint*:

[2] The parameter is .005 for both 5 and 10 fold cross validation

Figure 1 Comparison 3 month job finding imputation with true job finding probability



$$\max_{\{\mathbf{c}_{it+s}\}_{s=0}^{\infty}} \mathrm{E_t}\left[\sum_{s=0}^{\infty} (\not{D}\beta^{t+s} U\left(\mathbf{c}_{it+s}, n_{it+s}\right)\right] \tag{1}$$

subject to

Perceived Budget Constraint

$$\begin{aligned} \mathbf{a}_{it} &= \mathbf{m}_{it} - \mathbf{c}_{it} \\ \mathbf{a}_{it} + \mathbf{c}_{it} &= \mathbf{z}_{it}(\hat{\eta}_t) + (1 + r_t^a)\mathbf{a}_{it-1} \\ \mathbf{a}_{it} &\geq 0 \end{aligned}$$

The perceived budget constraint dictates the consumption policies to be solved while the true budget constraint below dictates the simulations of households in the model.

True Budget Constraint

$$\begin{aligned} \mathbf{a}_{it} &= \mathbf{m}_{it} - \mathbf{c}_{it} \\ \mathbf{a}_{it} + \mathbf{c}_{it} &= \mathbf{z}_{it}(\eta_t) + (1 + r_t^a)\mathbf{a}_{it-1} \\ \mathbf{a}_{it} &\geq 0 \end{aligned}$$

where $U\left(\mathbf{c}_{it}, n_{it}\right) = \frac{\mathbf{c}_{it}^{1-\rho}}{1-\rho} - \varphi\frac{n_{it}^{1+v}}{1+v}$ and β_i is the discount factor of household i. $\mathbf{m}_{it}$ denotes household i's market resources at time t to be expended on consumption or invested at a mutual fund. $\mathbf{c}_{it}$ is the level of consumption and $\mathbf{a}_{it}$ is the value of household i's shares at the mutual fund during period t where the mutual fund's return is r_{t+1}^a. $\mathbf{m}_{it}$ is determined by labor income, $\mathbf{z}_{it}$, and the gross return on assets from the last period, $(1 + r_t^a)\mathbf{a}_{it-1}$. D is the probability of death. Death is included in our model to ensure permanent income, $\mathbf{p}$, and thus wealth, has a limiting distribution. The employment status of household i at time t is denoted by n_{it} and follows a markov chain on the space $\{0, 1\}$. A household is employed when $n_{it} = 1$, otherwise he is unemployed. In particular,

$$P(n_{it} = 1 | n_{it-1}) = \begin{cases} 1 - \omega(1 - \eta_t), & n_{it-1} = 1 \\ \eta_t, & n_{it-1} = 0 \end{cases}$$

$$P(n_{it} = 0 | n_{it-1}) = \begin{cases} \omega(1 - \eta_t), & n_{it-1} = 1 \\ 1 - \eta_t, & n_{it-1} = 0 \end{cases}$$

where ω is the exogenous job separation rate. Households also believe their employment status follows the equation above with the important exception that they believe the job finding probability to be $\hat{\eta}_t$.

Labor income is subject to permanent and transitory idiosyncratic shocks. In particular, household i's labor income is composed of a permanent component, $\mathbf{p}_{it}$ indicating the level of permanent income and a transitory component, ξ_{it}, indicating the transitory income shock received by household i at time t. $\mathbf{p}_{it}$ is subject to permanent income shocks ψ_{it+1} where ψ_{it} is iid mean one lognormal with standard deviation σ_ψ, $\forall t$.

$$\mathbf{z}_{it} = \mathbf{p}_{it}\xi_{it}$$
$$\mathbf{p}_{it+1} = \mathbf{p}_{it}\psi_{it+1}$$

The transitory component follows

$$\xi_{it} = (1 - \tau_t)\theta_{it}w_t n_{it} + u(1 - n_{it})$$

where u are unemployment benefits, w_t is the real wage, and θ_t is an iid mean-one lognormal with standard deviation σ_θ.

4.2 Beliefs

Beliefs over the job finding probabilities $\hat{\eta}_t$ are updated following:

$$\hat{\eta}_t = \beta_\eta \eta + \epsilon_{\eta_t}$$

where β_η is the job finding belief reaction coefficient and captures the reaction of households.

Let $\epsilon_{\eta_{t+1}} = \rho_\eta \epsilon_{\eta_t} + v_\eta$.

4.3 Financial Intermediary

The financial intermediary performs a mutual fund activity where it collects assets from households and invests them into stocks v_{jt}, and nominal reserves M_t at the central bank.

In particular, at the end of period t, the assets collected from households A_t must be invested into shares v_{jt} of firm j at price q^s_{jt}, and nominal reserves M_t.

$$A_t = \int_0^1 q^s_{jt} v_{jt}\, dj + \frac{M_t}{P_t} \tag{2}$$

where A_t is the dollar value of the mutual fund's assets at the end of period t and v_{jt} is the portfolio share of firm j stocks with $\int_0^1 v_{jt}\, dj = 1$.

The mutual fund's return in the next period is then

$$(1 + r^a_{t+1}) = \frac{\int_0^1 (q^s_{jt+1} + D_{jt+1}) v_{jt}\, dj + (1 + i_t)\frac{M_t}{P_{t+1}}}{A_t}$$

where D_{jt+1} are dividends of firm j and i_t is the nominal interest rate on nominal reserves.

The mutual fund is risk neutral and looks to maximize its expected return

$$\max_{M_t, v_{jt}\}} \mathrm{E}_t \left[1 + r^a_{t+1}\right] = \mathrm{E}\left[\frac{\int_0^1 (q^s_{jt+1} + D_{jt+1}) v_{jt}\, dj + (1 + i_t)\frac{M_t}{P_{t+1}}}{\frac{M_t}{P_t} + \int_0^1 q^s_{jt} v_{jt}\, dj}\right]$$

The first order conditions lead to the no arbitrage equations:

$$\mathrm{E}_t \left[1 + r^a_{t+1}\right] = \frac{\mathrm{E}_t \left[q^s_{jt+1} + D_{jt+1}\right]}{q^s_{jt}} = (1 + i_t)\mathrm{E}_t \left[\frac{P_t}{P_{t+1}}\right] \equiv 1 + r_t \tag{3}$$

where r_t is defined to be the real interest rate in period t. In equilibrium ,we will assume $M_t = 0$

4.4 Goods Market

There is a continuum of monopolistically competitive intermediate good producers indexed by $j \in [0, 1]$ who produce intermediate goods Y_{jt} to be sold to a final good producer at price P_{jt}. Using intermediate goods Y_{jt} for $j \in [0,1]$, the final good producer produces a final good Y_t to be sold to households at price P_t.

4.4.1 Final Good Producer

A perfectly competitive final good producer purchases intermediate goods Y_{jt} from intermediate good producers at price P_{jt} and produces a final good Y_t according to a CES production Function.

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\epsilon_p - 1}{\epsilon_p}} \, dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

where ϵ_p is the elasticity of substitution.

Given P_{jt} , the price of intermediate good j , the final good producer maximizes his profit

$$\max_{Y_{jt}} P_t \left(\int_0^1 Y_{jt}^{\frac{\epsilon_p - 1}{\epsilon_p}} \, dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} - \int_0^1 P_{jt} Y_{jt}, \; dj$$

The first order condition leads to demand for good j

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\epsilon_p} Y_t \tag{4}$$

and the price index

$$P_t = \left(\int_0^1 P_{jt}^{1-\epsilon_p} \, dj \right)^{\frac{1}{1-\epsilon_p}} \tag{5}$$

4.4.2 Intermediate Good Producer

Intermediate goods producers employ labor and produce according to a Cobb Douglas Production function.

$$Y_{jt} = Z_t N_{jt}$$

where $log(Z_t) = \rho_Z log(Z_{t-1}) + \epsilon_Z$

Firm j chooses P_{jt} to maximize its dividend D_{jt} and its stock price q^s_{jt} facing price stickiness a la Rotemberg (1982).

$$\max_{\{P_{jt}\}} \overbrace{\frac{P_{jt}Y_{jt}}{P_t} - w_t N_{jt} - \kappa v_{jt} - \frac{\varphi}{2}\left(\frac{P_{jt}-P_{jt-1}}{P_{jt-1}}\right)^2 Y_t}^{\equiv D_{jt}} + q^s_{jt}\left(P_{jt}\right)$$

Given $q^s_{jt}\left(P_{jt}\right) = \frac{\mathrm{E}_t\left[q^s_{jt+1}+D_{jt+1}(P_{jt})\right]}{1+r_t}$, this is equivalent to:

$$\max_{\{P_{jt}\}} \mathrm{E}_t\left[\sum_{s=0}^{\infty} M_{t,t+s} D_{jt+s}\right]$$

subject to

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon_p} Y_t$$

$$N_{jt} = \frac{Y_{jt}}{Z_t}$$

$$v_{jt} = \frac{N_{jt} - (1-\omega)N_{jt-1}}{\phi_t}$$

$$v_{jt} \geq 0$$

where $M_{t,t+s-1} = \prod_{k=t}^{t+s} \frac{1}{1+r_k}$ is the stochastic discount factor.

The problem can be rewritten as

$$\max_{\{P_{jt}\}} \mathrm{E}_t\left[\sum_{s=0}^{\infty} M_{t,t+s}\left(\left(\frac{P_{jt+s}}{P_{t+s}} - MC_{t+s}\right) Y_{jt+s} - \frac{\varphi}{2}\left(\frac{P_{jt+s}}{P_{jt+s-1}} - 1\right)^2 Y_{t+s}\right)\right]$$

where $MC_t = \frac{1}{Z_t}\left(w_t + \frac{\kappa}{\phi_t} - \lambda_t - M_{t,t+1}(1-\omega)\left[\frac{\kappa}{\phi_{t+1}} - \lambda_{t+1}\right]\right)$

Firms can change their price in each period, subject to the payment of the adjustment cost. Hence, all the firms face the same problem, and thus will choose the same price, producing the same quantity. In other words $P_{jt} = P_t$ and $Y_{jt} = Y_t$.

The resulting Phillips Curve is defined by

$$\epsilon_p MC_t = \epsilon_p - 1 + \varphi(\Pi_t - 1)\Pi_t - M_{t,t+1}(\varphi(\Pi_{t+1} - 1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}$$

where $\Pi_t = \frac{P_t}{P_{t+1}}$.

4.5 Labor Market

Every period a proportion ω of workers lose their job, and firm j post vacancies v_{jt}. Each vacancy is filled with probability ϕ_t. Assuming firms are very large so that a large number of vacancies are posted, the level of filled vacancies is $v_{jt}\phi_t$ for firm j.

$$N_{jt} = (1 - \omega)N_{jt-1} + v_{jt}\phi_t$$

Workers search for jobs:

$$m_t = \chi e_t^{\alpha} v_t^{1-\alpha}$$

m_t is the number of matches in period t.

Then define η_t to be the job finding probability and ϕ_t the vacancy filling rate:

$$\eta_t = \frac{m_t}{e_t} = \chi \Theta_t^{1-\alpha}$$

$$\phi_t = \frac{m_t}{v_t} = \chi \Theta_t^{-\alpha} = \eta_t^{\frac{\alpha}{\alpha-1}} \chi^{\frac{-1}{\alpha-1}}$$

Aggregating N_{jt} across j we have the law of motion for labor:

$$N_t = (1 - \omega)N_{t-1} + m_t$$

$$e_t = 1 - (1 - \omega)N_{t-1}$$

4.6 Wage Determination

Following Broer, Druedahl, Harmenberg, and Öberg (2021), I assume the real wage to be fixed.

$$w_t = w_{ss}$$

As Broer, Druedahl, Harmenberg, and Öberg (2021) notes, recent research has documented nominal wage rigidities to be pervasive in the US labor market. Given the shocks I study are contractionary and that the response of inflation in my model to contractionary shocks to be procyclical, this is likely a weak assumption. Further, a fixed real wage reduces the computational complexity of the model as wage bargaining with a distribution of households is not trivial to implement.

4.7 Government

4.7.1 Fiscal Policy

I follow Kekre (2021) and Challe, Matheron, Ragot, and Rubio-Ramirez (2017) by having the government tax labor income from employed households to fund unemployment insurance for the unemployed.

$$u(1 - N_t) = \tau_t w_t N_t$$

4.7.2 Monetary Policy

The central bank follows the taylor rule:

$$i_t = r^* + \phi_\pi \pi_t + \epsilon_t^m$$

where ϕ_π is the Taylor rule coefficient for inflation, r^* is the steady state interest rate, Y_{ss} is the steady state level of output, $\epsilon_t^m = \rho_v \epsilon_{t-1}^m + \varepsilon_t$ are innovations to the taylor rule.

4.8 Equilibrium

An equilibrium in this economy is a sequence of:

- Policy Functions $(c_{it}(m))_{t=0}^{\infty}$

- Prices $\left(r_t, r_{t+1}^a, i_t, q_t^s, w_t, \pi_t, \tau_t\right)_{t=0}^{\infty}$

- Aggregates $(C_t, Y_t, N_t, \hat{\eta}_t, \eta_t, \phi_t, v_t, D_t, A_t)_{t=0}^{\infty}$

Such that:

$(c_{it}(m))_{t=0}^{\infty}$ solves the household's maximization problem given $(w_t, \hat{\eta}_t, r_t^a)_{t=0}^{\infty}$.

The Mutual fund, final goods producer, intermediate goods producers maximize their objective function.

The nominal interest rate is set according to the central bank's Taylor rule.

The tax rate is determined by the government budget constraint.

Markets clear:

$$A_t = q_t^s = \int_0^1 \mathbf{p}_{it}\left(m_{it} - c_{it}(m_{it})\right)\, di$$

$$C_t = w_t N_t + D_t$$

where $C_t \equiv \int_0^1 \mathbf{p}_{it} c_{it}(m_{it})\, di$

5 Calibration

Table 1 and 2 summarize the calibration of the model. The model is calibrated to a quarterly frequency.

5.1 Households

I set the CRRA to equal a standard value of 2. I follow Carroll, Slacalek, Tokuoka, and White (2017) and let the standard deviation of permanent shocks to be .06 and the standard deviation of transitory shocks to be .2 . To target a 40 year work life, the probability of death is set to 1 - .99735. The tax rate is set to .18 and the real wage is 2.0. The unemployment replacement rate is set to 35% of the real wage . Although the typical replacement rate across states in the US is around 50%, the majority of the unemployed do not receive unemployment benefits(Graves (2020), Chodorow-Reich and Karabarbounis (2016)). The job separation rate is set to .1 in line with Graves (2020) and Kekre (2021) and the job finding probability is .667 to target a 5% steady state unemployment rate and a 1.5 quarter duration of unemployment. The steady state real interest rate is set to 3% per annum and the resulting discount factor is internally calibrated to reach an employed MPC of .46, consistent with the findings of Kekre (2021). The job finding belief reaction coefficient is calibrated to .6, in the upper range of estimates reported by Du, Qiu, and Wang (2022).

Table 1 Household Calibration

Calibrated Parameters			
Description	Parameter	Value	Source/Target
CRRA	ρ	2	Standard
Belief Reaction Coefficient	β_η	.6	Du, Qiu, and Wang (2022)
Real Interest Rate	r	$1.03^{.25} - 1$	3% annualized real rate
Discount Factor	β	0.961	Employed MPC = 0.47
Probability of Death	$\not{D}$	0.00625	40 Year Work Life
Tax Rate	τ	0.18	
Real Wage	w	2.0	
Unemployment Benefits	u	0.7	35% Replacement Rate
Job Finding Rate	η	0.667	5% Unemployment Rate
Job Separation Rate	ω	0.1	Graves (2020)
Std Dev of Log Permanent Shock	σ_ψ	0.06	Carroll et al. (2017)
Std Dev of Log Transitory Shock	σ_θ	0.2	Carroll et al. (2017)

Table 2 Economy Calibration

Calibrated Parameters			
Description	Parameter	Value	Source/Target
Elasticity of Substitution	ϵ_p	310	$\beta = 0.961$
Price Adjustment Costs	φ	3100	Slope of Phillips Curve = .1
Vacancy Filling Rate	ϕ	0.71	Wouter et al. (2000)
Matching Elasticity	α	0.72	Silva and Toledo (2009)
Vacancy Cost	κ	0.156	7.8% of Real Wage
Government Spending	G	0.30	
Taylor Rule Inflation Coefficient	ϕ_π	1.5	Standard
Liquid Assets to Output Ratio	$\frac{A}{Y}$	.44	
Persistence of Monetary Policy Shock	ρ_v	.85	Graves (2022)

5.2 Rest of the Economy

The quarterly vacancy filling rate is .71 as in Ramey, den Haan, and Watson (2000). The matching elasticity is .72 following Silva and Toledo (2009) and the vacancy cost is set to 7.8% of the real wage, near the 7% target calibrated in Christiano, Eichenbaum, and Trabandt (2016) and within the range of plausible values of this statistic reported by Silva and Toledo (2009)[3]. The elasticity of substitution is set to 310 to target the discount factor value of .961. This assumption is required to clear the asset market clearing condition found at the end of section 4.8. Asset supply in this model is a function of dividends earned by firms and thus a large markup is incompatible with a low level of asset demand generated form a smaller discount factor. In addition, this assumption is almost equivalent to having firms receive zero profits in equilibrium and does not affect the results below. If anything, having zero profits attenuates the amplification mechanism as households do not receive profits. In Kekre (2021) he assumes a elasticity of substitution of 6 although when calibrating his model to the distribution of liquid assets, assumes firms face a tax rate to lead to earn no profits in equilibrium. Instead of adding a tax rate to firms, I simply assume a large elasticity of substitution to ensure near zero proftis to allow my asset market to clear. The price adjusment cost parameter is set to 3100 to target phillips curve slope of .1. With a tax rate of .18, Government Spending is set to .3 to clear the government budget constraint and the discount factor of .961 implies a liquid assets to output ratio of .44. The job finding belief reaction coefficient is calibrated to .6, in the upper range of estimates reported by Du, Qiu, and Wang (2022).

5.3 Model Validation

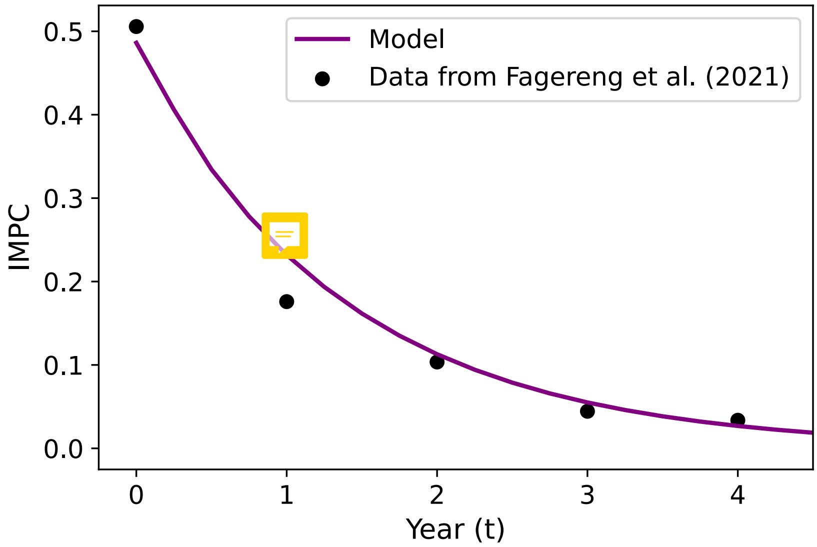
In this section, I give confidence to the calibration of households by verifying that my model is consistent with some key empirical moments documented in the microdata on household consumption. In particular, I verify that the model is consistent with the empirical dynamic consumption response to income shocks and the MPCs of the employed and unemployed documented in the microdata.

Figure 2 demonstrates the intertemporal MPCs[4] of the model are consistent with the estimates from Fagereng, Holm, and Natvik (2021). This implies the model nearly captures the empirical dynamic response of consumption to income shocks. In year 0, the empirical estimate for the MPC is .5 while the model produces an MPC of .485 while in year 0 the empirical estimate is .176, .056 less than its model counterpart. Still, the overall timepath of the consumption response is consistent with Fagereng, Holm, and Natvik (2021) and vital to the general equilibrium transmission of shocks in models with heterogeneous agents and nominal rigidities (Auclert, Rognlie, and Straub

[3] The range of plausible values lie between 4% and 14%

[4] The intertemporal MPCs, or IMPCs, is defined as the consumption response in t + j to a transitory income shock in period t, where $j \geq 0$. Intuitively, it is the consumption response tomorrow (or today) to a transitory income shock today. Fagereng, Holm, and Natvik (2021) use Norwegian administrative data to estimate the dynamic consumption responses to income shocks.

Figure 2 Comparison of Model IMPCs vs Data from Fagereng et al. (2021)



This figure plots the IMPCs produced by the model against the IMPCs found from the data in Fagereng et al.(2021). The IMPCs of the model are computed by simulating an unanticipated one time transitory shock to income in period 0. Then the IMPC is computed as $IMPC_t = \frac{dC_t}{dZ_0}$ where dC_t is the change in consumption relative to the steady state in period t, and dZ_0 is the size of the income shock.

(2018)).

Table 3 demonstrates the model matches the annual MPC out of unexpected, transitory income shocks of both the employed and unemployed documented by Kekre (2021) [5] . These are important moments to match as employed households are taxed to fund unemployment benefits. Thus, a shock that leads to changes in the unemployment rate will produce an empirically consistent consumption response to both changes in taxes for the employed and increases in UI receipts for the unemployed.

[5] Kekre (2021) uses the 2010 Survey of Household Income and Wealth to estimate the sample mean of the self reported MPCs.

Table 3 Moments

Moment	Estimate	Source	Model
Employed MPC - Unemployed MPC (Annual)	.25	Kekre (2021)	.24
Employed MPC (Annual)	.47	Kekre (2021)	.47
Unemployed MPC (Annual)	.72	Kekre (2021)	.71

6 Impulse Response to a Monetary Policy Contraction

In this section, I study how underreaction in job finding expectations influence fluctuations due to a monetary policy shock. Figure 3 illustrates the impulse responses to a 10bp (annualized) shock to the nominal interest rate with persistence .85 under two different model calibrations. I compare a model with rational households ($\beta_\eta = 1$) against a model with underreactive households ($\beta_\eta = .6$) to illustrate the effects of underreactive beliefs.

The impulse responses to the model with underreactive beliefs are attenuated compared to the model with rational households (model with $\beta_\eta = 1$). In both calibrations of the model, a rise in the nominal rate raises the real interest rate causing downward pressure on aggregate consumption due to the intertemporal substitution channel. Given that output is demand determined, output and labor must fall decreasing labor tightness and the job finding probability. The fall in the job finding probability raises the probability of being unemployed and induces households to increase their precautionary savings. This rise in precautionary savings leads consumption to decline further and as a consequence amplifies the business cycle. Because the households in the model with underreactive model do not perceive their unemployment risk to rise as substantially in comparison to the baseline HANK and SAM, the rise in precautionary saving is dampened and therefore the amplification of the business cycle is attenuated.

This attenutation exhibits the effects of underreactive beliefs on aggregate demand. The supply side of effects of underreactive job finding expectations may reverse the attenuation of the impulse responses. Notably, the inclusion of wage bargaining may amplify the impulse responses of the underreactive model as stubborn job finding beliefs will induce wage stickiness (Menzio (2022)).

7 Quantifying The Precautionary Channel

In this section, I quantify the precautionary channel to a monetary policy contraction. I find the precautionary channel to explain a significant share of the volatility in fluctuations following a monetary policy shock.

7.1 Magnitude of The Precautionary Channel

To quantify the precautionary channel, I compute the share of the variance of the responses of consumption and the unemployment due to precautionary saving.

Figure 4 plots the impulse responses to a monetary policy contraction of the baseline model, and a model with no cyclical unemployment risk. In particular, I compare a model with underreactive beliefs and a model where households do not change their

Figure 3 Impulse Responses to Monetary Contraction Shock

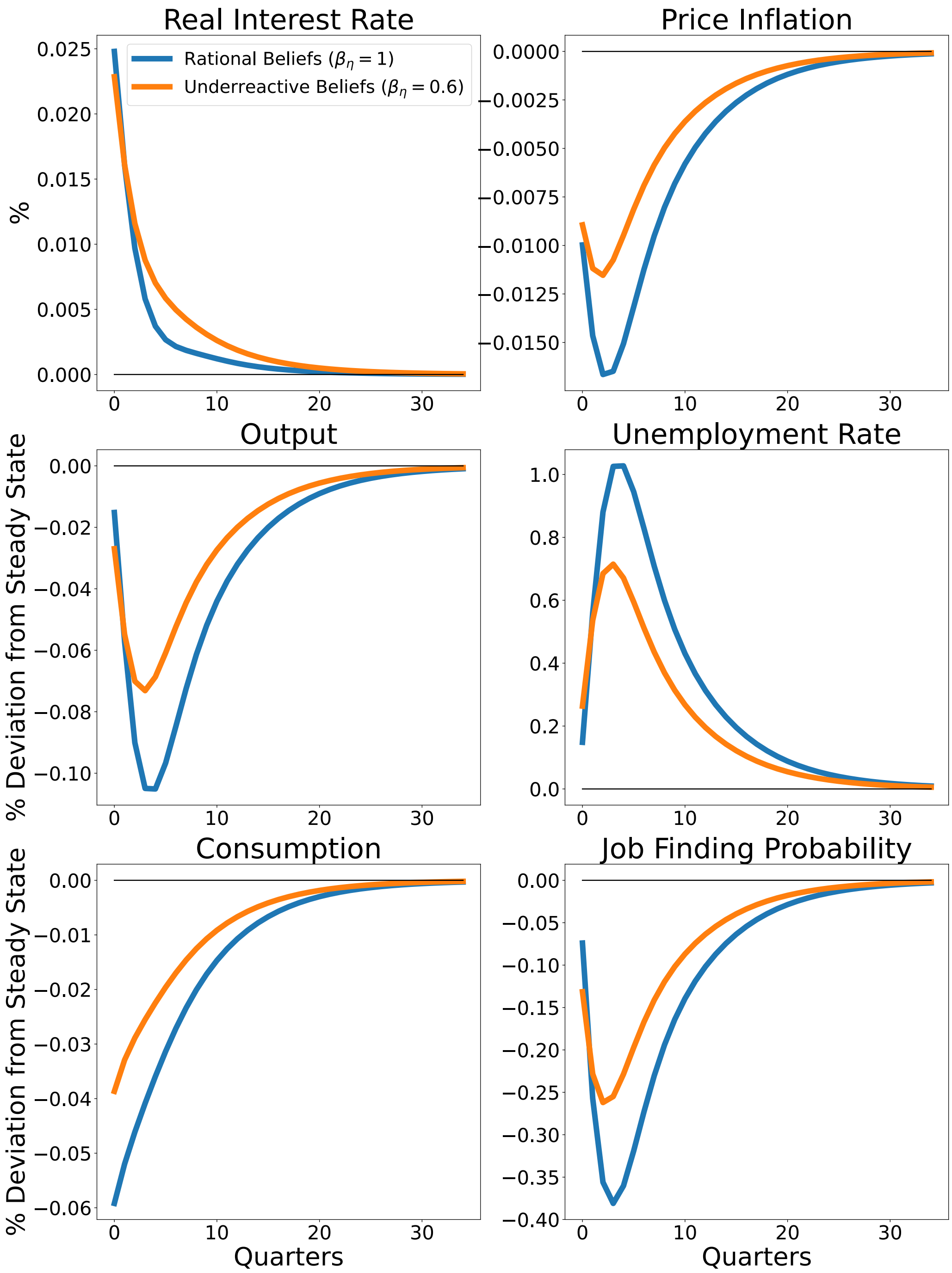


Figure 4 Impulse Responses to Monetary Contraction Shock (Precautionary Channel)

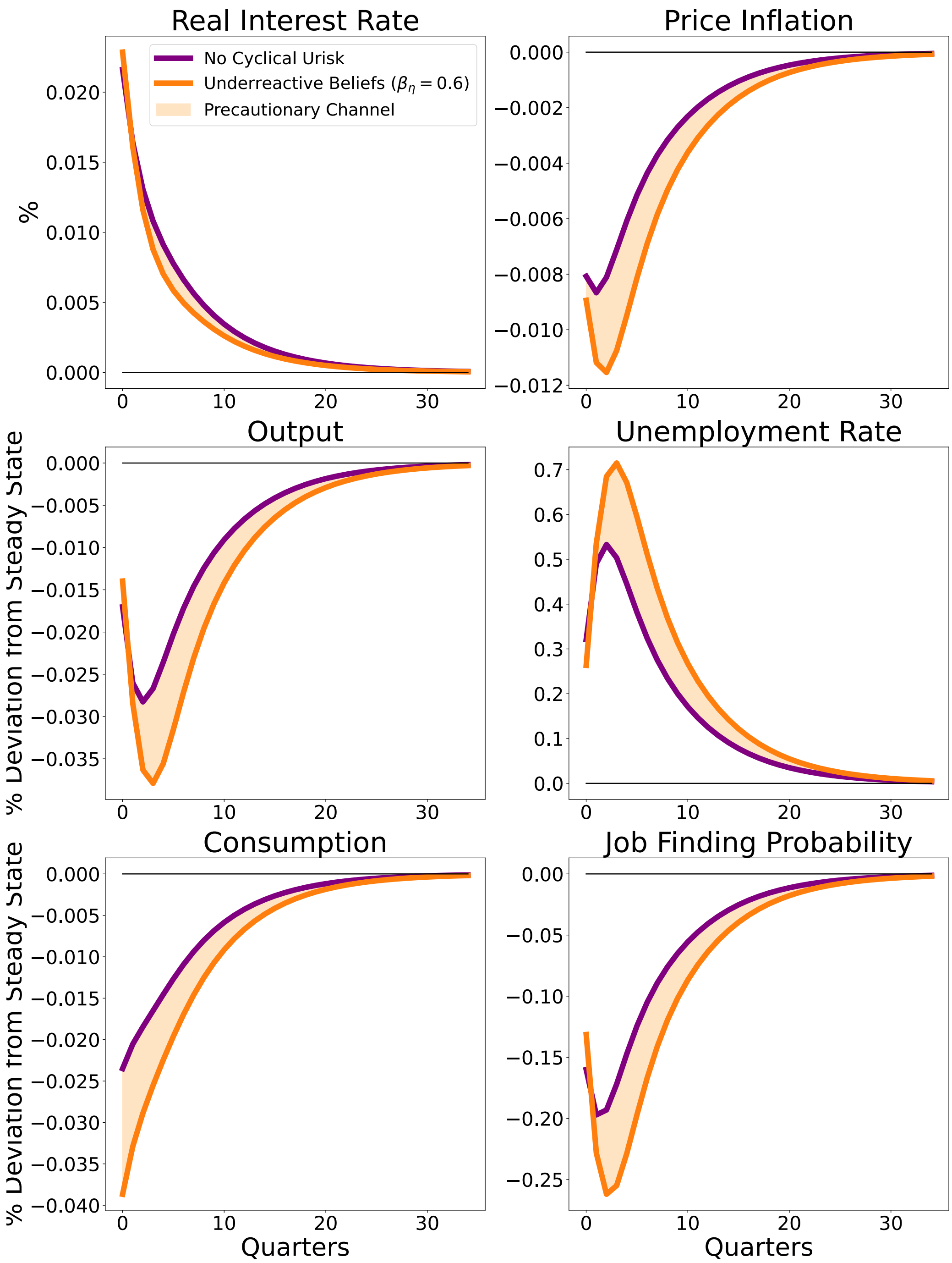
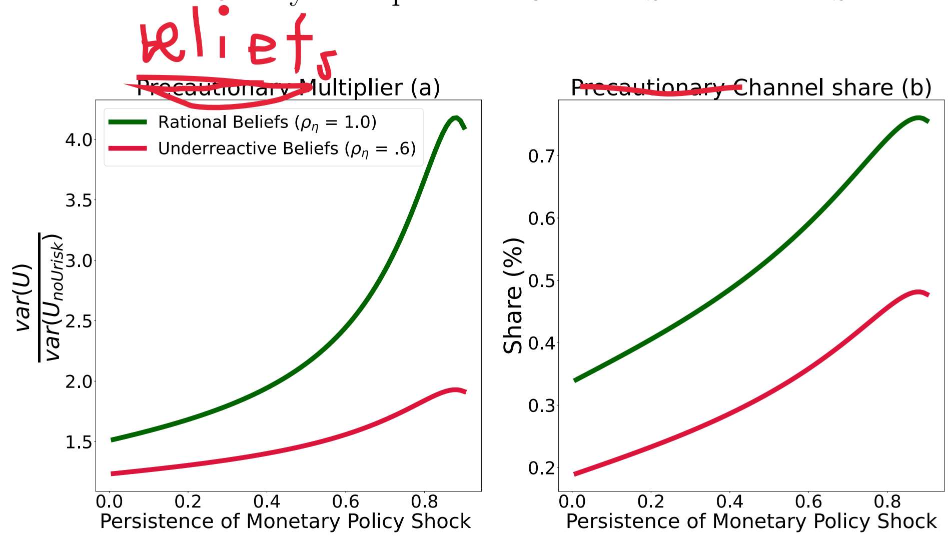


Figure 5 Precautionary Multiplier and Channel Share Across Shock Persistence



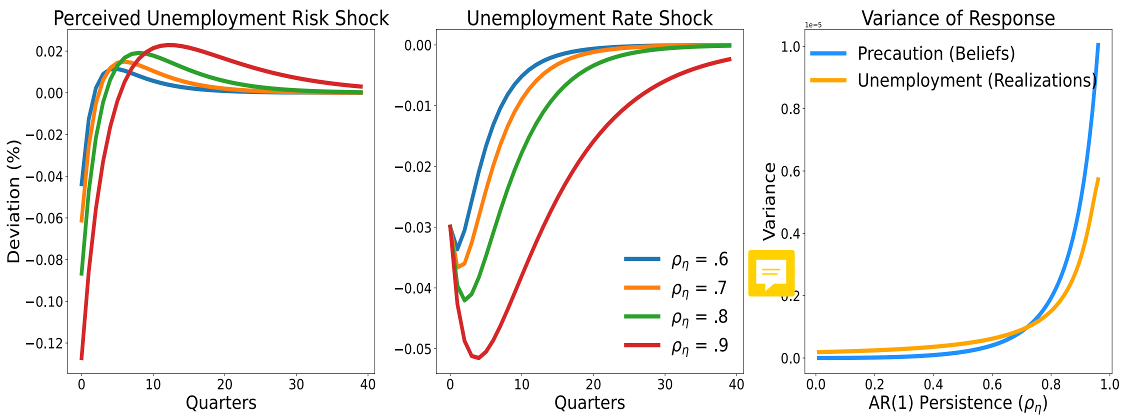
precautionary savings in response to changes in unemployment risk. The difference in volatility between the impulse responses of these models is the precautionary channel. While the volatility of the impulse responses of the economy with underreactive beliefs are attenuated compared to the model with rational beliefs, the model still demonstrates sizeable amplification through the precautionary channel. In particular, the share of fluctuations in aggregate consumption and the unemployment rate due to the precautionary channel. In particular, the precautionary channel is large and accounts for 46% of the variance in the unemployment rate response and 60.5% of the variance in the aggregate consumption response.

8 The Role of Shock Persistence

In this section I clarify the role of the persistence of a monetary policy shock in explaining the size of the precautionary channel.

Figure 5 illustrates the role of shock persistence on the amplification of the precautionary channel. Graph (a) plots the the ratio of the variance of the unemployment rate response in the baseline model to the variance of the unemployment rate response in the model with no cyclical unemployment risk across monetary policy shock persistences. I define this ratio to be the precautionary multiplier. Graph (b) plots the share of variance in the unemployment rate response due to the precautionary channel across monetary policy shock persistences. The plots demonstrate that the increased volatility due to the precautionary channel increases exponentially with the persistence of the shock while a larger share of the increased volatility is attributed to the precautionary channel. Overall, the amplification of the volatility of the business cycle through precautionary saving rises with the persistence of the monetary policy shock.

Figure 6 Partial Equilibrium Consumption Response to AR(1) Shocks



Note: The first two panels plot the partial equilibrium impulse responses of consumption to AR(1) shocks at differing shock persistences ρ_η. The first panel plots the impulse response to a shock to job finding beliefs while the second panel plots the response to a shock to the realized job finding probability assuming households never perceive the job finding probability to change. The last panel plots the variances of the responses of consumption across different shock persistences.

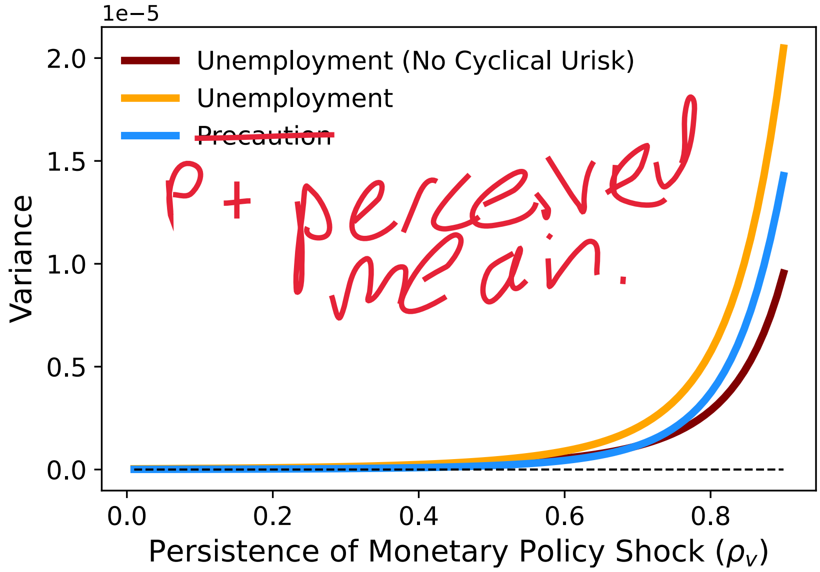
The exponential rise of the precautionary multiplier stems from both the sensitivity of the precautionary motive to the persistence of heightened unemployment risk and the interaction of this sensitivity with the persistence in the IMPCs. The first two panels in figure 6 illustrate the partial equilibrium impulse responses of consumption to AR(1) shocks of varying persistences on job finding beliefs (perceived unemployment risk)[6] and job finding realizations (unemployment rate)[7]. I study the response of aggregate consumption as it amplifies the business cycle through nominal rigidities (Demand determined output). The shocks to perceived unemployment risk (first panel) illustrate that the precautionary motive is sensitive to the duration of heightened unemployment risk with the impact of the shock increasing exponentially with the persistence of the shock. The consumption response to shocks to the unemployment rate (second panel) are also sensitive to the persistence of the shock however the initial impact of all shock persistences are the same because the size of shocks are all the same. Under shocks to the unemployment rate, the persistence of the consumption responses differ with persistence due to the slow decay of the IMPCs. The longer the duration of heightened unemployment causes persistence in losses to aggregate income which leads

[6] AR shocks to job finding beliefs are effectively shocks to perceived unemployment risk and therefore the consumption response captures the precautionary response to heightened unemployment risk.

[7] AR shocks to job finding realizations are shocks to the unemployment rate as the job finding probability dictates the mass of individuals who will transition into unemployment.

the consumption response to persist. To quantify and compare the exponential rise of both shocks across persistence, the third panel plots the variance of consumption to both perceived unemployment risk shocks and unemployment rate shocks. The panel illustrates the exponential rise in the variance of consumption for both shocks with the perceived unemployment risk shocks (Blue Line) increasing at a greater rate than the unemployment rate shock.

Figure 7 Variance of Consumption Responses in General Equilibrium



Note: This figure plots the variances of the general equilibrium consumption response to the endogenous changes in perceived unemployment risk and the unemployment rate. These plots are the general equilibrium counterparts of the third panel in figure 6

Figure 7 illustrates the variances of consumption to both heightened unemployment risk and increased unemployment in general equilibrium. In general equiilbrium, the endogenous path of the job finding probability does not follow an AR(1) and therefore the graph is plotted across different monetary policy shock persistences to capture the resulting general equilibrium paths of unemployment risk and unemployment. The rise in the variances are amplified due to their interaction in general equilibrium. Comparing the rise in the variance due to the unemployment effect between the baseline model (Orange) and model with no cyclical unemployment risk (Maroon), the dramatic rise in the variance attributed to the unemployment effect stems from its interaction with the precautionary effect. The addition of precautionary saving magnifies the initial drop in aggregate consumption as households immediately build a larger precautionary buffer stock upon learning of the shock. This decline in consumption transmits through output and raises the unemployment rate which proliferates the fall in consumption because of the persistence in the IMPCs. With the precautionary motive rising sharply with the persistence of heightened unemployment risk, the intensity of the proliferation in consumption from the interaction between persistent IMPCs and precautionary saving rises further with the persistence of the monetary policy shock.

9 Looking Forward

Looking forward, I plan to compute the consumption and savings jacobians across distribution of wealth to determine the effect of uncertainty across the cross section of wealth. In particular, I may compute the consumption jacobian to a unemployment risk shock for the bottom third of the wealth distribution and compare it to the jacobian of the whole economy to determine whether a particular part of the distribution is driving the results. I am also thinking of expanding the modeling of unemployment insurance to be more realistic. In particular, at the moment, unemployed households receive unemployment benefits no matter the duration of their unemployment spell. This is clearly not realistic and therefore I plan to have unemployment benefits only lasting two periods into the unemployment spell. With a more realistic model of unemployment insurance, I may study shocks to unemployment benefit duration since the precautionary channel is sensitive to the persistence of unemployment risk. Furthermore, I may solve an analytical HANK and SAM commonly used in the literature to demonstrate the importance of full fledge heterogeneity on the results of the model. Finally, this dissertation proposal may split into two separate avenues. This paper concerns the role of precautionary saving over the business cycle and the determinants of this this channel in a full fledged HANK model. The addition of beliefs lends a more realistic calibration of beliefs however I believe the inclusion of job separation expectations could lead to an interesting result in its own right, away from the analysis of the sensitivity the precautionary channel. Thus, I may pursue a separate paper that includes both job finding expectation and job separations data and analyze how the model discpline by expectations data differs than its rational counterpart. In such a framework, as a first basis, I would estimate a shock series from the job separation expectations and simulate the model with those shocks to determine any differences.

References

Auclert, Adrien (2019): "Monetary policy and the redistribution channel," *American Economic Review*, 109(6), 2333–67.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub (2018): "The intertemporal keynesian cross," Discussion paper, National Bureau of Economic Research.

Broer, Tobias, Jeppe Druedahl, Karl Harmenberg, and Erik Öberg (2021): "The Unemployment-Risk Channel in Business-Cycle Fluctuations," .

Carroll, Christopher, Jiri Slacalek, Kiichi Tokuoka, and Matthew N White (2017): "The distribution of wealth and the marginal propensity to consume," *Quantitative Economics*, 8(3), 977–1020.

Challe, Edouard, Julien Matheron, Xavier Ragot, and Juan F Rubio-Ramirez (2017): "Precautionary saving and aggregate demand," *Quantitative Economics*, 8(2), 435–478.

Chodorow-Reich, Gabriel, and Loukas Karabarbounis (2016): "The Cyclicality of the Opportunity Cost of Employment," *Journal of Political Economy*, 124(6), 1563–1618.

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt (2016): "Unemployment and Business Cycles," *Econometrica*, 84, 1523–1569.

Den Haan, Wouter J, Pontus Rendahl, and Markus Riegler (2018): "Unemployment (fears) and deflationary spirals," *Journal of the European Economic Association*, 16(5), 1281–1349.

Fagereng, Andreas, Martin B. Holm, and Gisle J. Natvik (2021): "MPC Heterogeneity and Household Balance Sheets," *American Economic Journal: Macroeconomics*, 13(4), 1–54.

Graves, Sebastian (2020): "Does Unemployment Risk Affect Business Cycle Dynamics?," International Finance Discussion Papers 1298, Board of Governors of the Federal Reserve System (U.S.).

Heathcote, Jonathan, and Fabrizio Perri (2018): "Wealth and volatility," *The Review of Economic Studies*, 85(4), 2173–2213.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante (2018): "Monetary policy according to HANK," *American Economic Review*, 108(3), 697–743.

Kekre, Rohan (2021): "Unemployment Insurance in Macroeconomic Stabilization," NBER Working Papers 29505, National Bureau of Economic Research, Inc.

Leduc, Sylvain, and Zheng Liu (2016): "Uncertainty shocks are aggregate demand shocks," *Journal of Monetary Economics*, 82, 20–35.

Menzio, Guido (2022): "Stubborn Beliefs in Search Equilibrium," *NBER Working Paper*, (w29937).

Mueller, Andreas I, Johannes Spinnewijn, and Giorgio Topa (2021): "Job seekers' perceptions and employment prospects: Heterogeneity, duration dependence, and bias," *American Economic Review*, 111(1), 324–63.

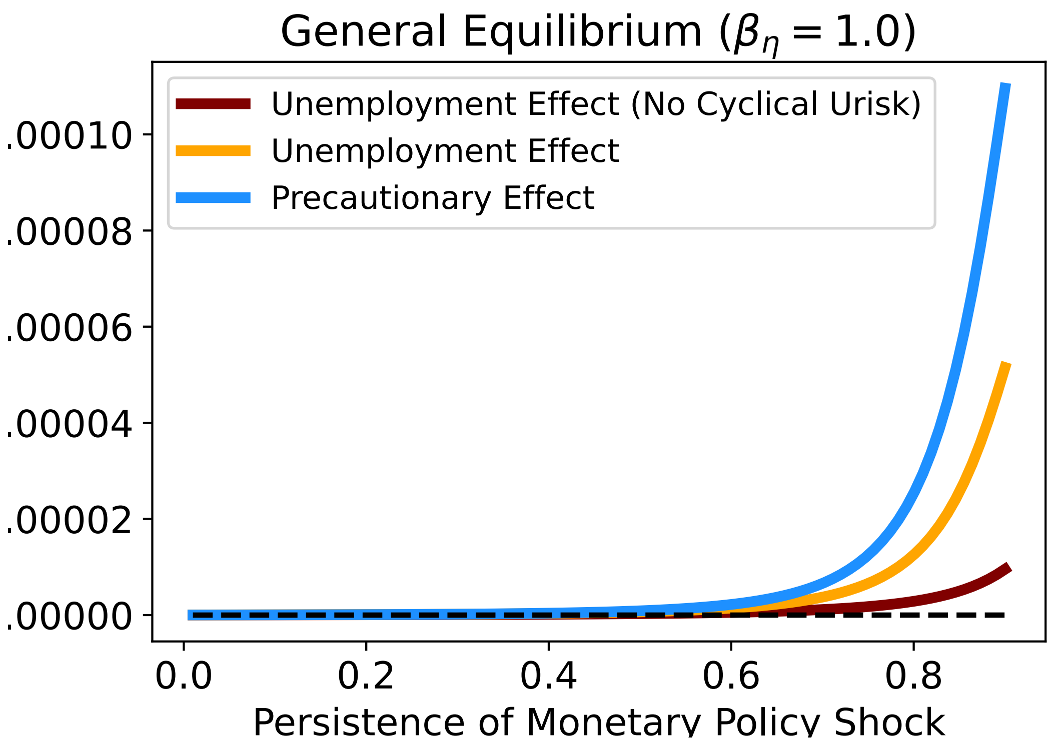
Ramey, Garey, Wouter J. den Haan, and Joel Watson (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90(3), 482–498.

Ravn, Morten O, and Vincent Sterk (2017): "Job uncertainty and deep recessions," *Journal of Monetary Economics*, 90, 125–141.

Rotemberg, Julio J (1982): "Sticky prices in the United States," *Journal of political economy*, 90(6), 1187–1211.

Silva, Josh(=|e(', and Manuel Toledo (2009): "LABOR TURNOVER COSTS AND THE CYCLICAL BEHAVIOR OF VACANCIES AND UNEMPLOYMENT," *Macroeconomic Dynamics*, 13(S1), 76–96.

Figure 8 Consumption Variance Decomposition



Appendices

Figure 9 Consumption Variance Decomposition (Model with Government Spending)

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