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Solving Airlines Disruption by Considering Aircraft and Crew Recovery Simultaneously

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Abstract: When disruptions occur, the airlines have to recover from the disrupted schedule. The recovery usually consists of aircraft recovery, crew recovery and passengers' recovery. This paper focuses on the integrated recovery, which means above-mentioned two or more recoveries are considered as a whole. Taking the minimization of the total cost of assignment, cancellation and delay as an objective, we present a more practical model, in which the maintenance and the union regulations are considered. Then we present a so-called iterative tree growing with node combination method. By aggregating nodes, the possibility of routings is greatly simplified, and the computation time is greatly decreased. By adjusting the consolidating range, the computation time can be controlled in a reasonable time. Finally, we use data from a main Chinese airline to test the algorithm. The experimental results show that this method could be used in the integrated recovery problem.
Key words: aircraft recovery, crew recovery, integrated recovery, airlines optimal recovery
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0 Introduction

Severe weather conditions, air traffic control and mechanical failures resulting in flight schedule disruptions are unavoidable occurrence in the daily operations of an airline. The recovery usually consists of aircraft recovery, crew recovery and passengers' recovery. Now more and more researches pay their attention to the integrated recovery.

Aircraft recovery problem (ARP) is to determine new flight departure time, cancellations and reroute for affected aircrafts including ferrying, diverting, swapping and so on, besides that several decision rules should also be considered: aircrafts balance requirements, maintenance requirements and station departure curfew restrictions. At the end of the recovery period, aircrafts should be positioned to resume operations as planned.

Crew management during irregular operations is usually the bottleneck of the whole system-recovering process due to complicate crew schedules and restrictive crew legalities as well as the size and scope of the hub-and-spoke networks adopted by major carriers.

The integrated recovery considers two or more recoveries simultaneously. The goal of the recovery is to get the optimal solution of the whole system. However, to solve so large-scale problem mathematically in applicable time is almost impossible, so many scientists try to solve a simplified system, in which less factors are considered. In this paper, only aircrafts and crews are considered.

Being different from the aircraft or crew rotation problem in the planning stage, the method to solve the integrated recovery problem should get the optimal solution in a much shorter time, which is very difficult to the most optimization solvers under most reasonable disruption scenarios. Therefore, to find a faster method becomes many scientists' study focus besides formulating the real problem, and this is also the important part of this paper. We use flight string instead of flight and define the recovery scope to reduce the problem size. In this paper, we integrate the aircraft and crew recovery together while satisfying maintenance requirements and meeting the union requirements, allowing crews to deadhead either within the modified pairing or back their crew base.

1 Literature Review

Teodorovic and Guberinic[1] were the first to study the aircraft recovery problem. They considered a situation where one or more aircrafts were taken out of operation and the airlines had to operate the flight schedule with a reduced number of planes. The proposed model

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was solved by a branch and bound (B&B) algorithm, but the research excluded the constraints of station curfews, maintenance requirements and end of recovery period aircrafts balance in the modeling. Afterwards, Yan and Yang[2] successfully combined flight cancellations, delays and ferry flights in a single model. Four systematic strategic models were developed by combining perturbing the basic schedule perturbation model (BSPM) with various scheduling rules. These network models were formulated as pure network flow problems or network flow problems with side constraints. In addition, the constraints of aircraft maintenance and crew scheduling were overlooked.

In response to groundings and delays experienced over the course of the day, Arguello et al.[3] created a greedy randomized adaptive search procedure (GRASP) to reconstruct aircraft routings, which was a fast heuristic based on randomized neighborhood search. Subsequently, Bard et al.[4] developed a time-band optimization model to reconstruct cost-effective aircraft routings in response to groundings and delays. Transforming the routing problem into a time-based network whose time horizon was discretized, the model was solved as an integral minimum cost network flow problem with side constraints. The disadvantage was that both of these researches excluded the maintenance requirements and crew requirements after the aircraft routings altered.

Rosenberger et al.[5] modeled the aircraft recovery problem as a set-packing problem with time window and slots restrictions. Being different from Refs. [3-4], Ref. [5] assumed an aircraft selection heuristic (ASH) for ARO (an optimization model for aircraft recovery), which selected a subset of aircraft for optimization prior to generating new routes. Compared with the network model, this model could check maintenance feasibility using column generation. Then, Eggenberg et al.[6] introduced an extension of the time-space network model to minimize delays, cancellations and plane swappings, and makespan cost. In Ref. [7], the author introduced an airline irregular operation in detail and used the time-band optimization method to solve the aircraft recovery as an example.

In crew recovery aspect, Wei et al.[8] modeled the crew recovery problem as a set of covering problems, and used the primal-dual sub-problem simplex method for solving the linear programming relaxation problem. The proposed framework had been implemented using tree-based data structures for efficient storage and data access. The authors also applied a bucket selection process for choosing a better mix of pairings that price out favorably. A customized branching strategy that could exploit the structure of the problem was developed. Afterwards, the authors in Refs. [9-10] created the same model and used the CALEB technologies to develop the CrewSolver decision-support system to generate crew-recovery solution. They adopted a heuristic-based search algorithm with a generate-and-test approach. In this method, they generated or modified one or a few pairings and tested the status of the problem. The search engine relied on a depth-first-search procedure. Medard et al.[11] integrated both models to solve time critical crew recovery problems arising on the day of operations. They formulated the crew recovery problem as the flight-based crew roster problem. Two solution methodologies were compared within a generated and optimized framework, one based on pure tree search, the other inherited from the pairing approach that was based on reduced cost k-shortest paths.

In the recent time of the integrated recovery, Abdelghany et al.[12] developed an integrated decision support tool for airlines schedule recovery (DSTAR). DSTAR aimed at integrating aircraft and crew recovery operations and consisted of two modes: schedule simulation mode and optimization mode. In a certain rolling horizon, recovery scheme was divided into several stages by DSTAR. In each stage, the mode simulated the daily schedule operation and projected the list of disrupted flights in the system as function of the severity of anticipated disruptions. The recovery problem at each stage was formulated mathematically in the form of a mixed integer program (MIP). Compared with other models, the study was more integrated. However, the problem was still solved separately and maintenance was not taken into consideration.

Jafari and Zegordi[13] extended the model in Ref. [14] for solving the aircraft recovery problem and reassigning disrupted passengers simultaneously. The modeling employed aircraft rotation and passengers' itineraries instead of flights and reduced the size of the problem. However, the study did not consider the maintenance constraints and the time limitation may be one of the other shortages. Eggenberg et al.[15] provided a constraint-specific recovery network model which could apply in aircraft, crew and passenger recovery problem respectively. As in Ref. [16], they integrated all kinds of recovery simultaneously, and employed the Bender's decomposition to decompose the model into a master problem (schedule recovery problem) and three sub-problems (aircraft recovery problem, crew recovery problem and passenger recovery problem), using an optimization-based approach to solve the situation of hub closure. However, the integrated model was solved separately. Jin[17] and his students also wrote some articles about airlines recovery and obtained some achievement, making some progress.

2 Integrated Recovery Model

The aggregate cost comprised of assigning strings (assignment cost), assigning crew to the flight string and recovering aircrafts (delay cost and cancellation cost) in the recovery scope can be minimized by

$$\min \sum_{k\in K_n(e)} \sum_{s\in S_n} C^k_{e,s} x^k_{e,s} + \sum_{f\in F_n} C^{\mathrm{d}}_f (1-z_f)(t^{\mathrm{d}}_f - T_f) + \sum_{f\in F_n} C^{\mathrm{c}}_f z_f + \sum_{r\in R^e_n} \sum_{p\in P^r_n} C^r_p w^r_p + \sum_{f\in F_n} C^r_f n^{\mathrm{sur}}_f, \tag{1}$$

$$x^k_{e,s} = \begin{cases} 1, & \text{aircraft } k \text{ covers flight string } s \\ 0, & \text{otherwise} \end{cases},$$

$$w^r_p = \begin{cases} 1, & \text{crew } r \text{ is assigned to pairing } p \\ 0, & \text{otherwise} \end{cases},$$

$$z_f = \begin{cases} 1, & \text{flight } f \text{ is canceled} \\ 0, & \text{otherwise} \end{cases},$$

where $C^k_{e,s}$ is the cost of assigning aircraft $k \in K_n(e)$ to flight string s, C^{d}_f is the cost of 1-min delay of flight f, C^{c}_f is the cost of canceling flight f, C^r_p is the cost of assigning crew r to pairing p, C^r_f is the cost of deadheading a crew r on flight f, t^{d}_f is the actual departure time of flight f, T_f is the scheduled departure time of flight f, n^{sur}_f is the number of surplus or deadhead crew on flight f, N is the set of recovery scope, F_n is the set of all flight legs in recovery scope N, $K_n(e)$ is the set of aircraft of fleet type e in recovery scope N, P^r_n is the set of all feasible pairings that can be flown by crew r in recovery scope N, R^e_n is the set of crews available (disrupted, involved, and reserve crews) of a given fleet type e in recovery scope N, and S_n is the set of flight string in recovery scope N.

Either a flight must be contained in exactly one string or it is cancelled:

$$\sum_{k\in K_n(e)} \sum_{s\in S_n} x^k_{e,s} a^s_f + z_f = 1, \quad \forall f \in F_n, \tag{2}$$

where a^s_f is an indicator variable. If flight f is in the flight string s, it is 1; otherwise, it is 0. Ensure that each aircraft is assigned to no more than one string. The cover constraints are split into the following formulae to distinguish between the mandatory and optional legs:

$$\sum_{k\in K_n(e)} \sum_{s\in S_n} x^k_{e,s} a^s_f = 1, \quad \forall f \in F^{\mathrm{man}}_n, \tag{3}$$

$$\sum_{k\in K_n(e)} \sum_{s\in S_n} x^k_{e,s} a^s_f \leqslant 1, \quad \forall f \in F^{\mathrm{opt}}_n, \tag{4}$$

where F^{man}_n is the set of mandatory flight legs in recovery scope N, and F^{opt}_n is the set of optional flight legs that are candidates for deletion.

The maintenance cover constraint simply ensures a maintenance opportunity to be built in, and the specific maintenance planning can be done post-optimization:

$$\sum_{s\in S_n} \sum_{m\in A(e)} I^s_m x^k_{e,s} = 1, \quad \forall k \in K_n(e), \tag{5}$$

$$I^s_m = \begin{cases} 1, & \text{in the case of Condition 1} \\ 0, & \text{otherwise} \end{cases},$$

where Condition 1 is that an eligible maintenance station $m \in A(e)$ is visited by flight string s, and $A(e)$ is the set of stations that are capable of performing schedule maintenance of aircraft k of fleet type e.

The following count ensures that the total number of aircraft k in the air and on the ground does not exceed the size of fleet type e:

$$\sum_{k\in K_n(e)} \sum_{s\in S_n} l^k_s x^k_{e,s} + \sum_{k\in K_n(e)} \sum_{j\in G^k} m^k_j y^k_j \leqslant N_e, \quad \forall e \in E_n, \tag{6}$$

$$m^k_j = \begin{cases} 1, & \text{in the case of Condition 2} \\ 0, & \text{otherwise} \end{cases},$$

where l^k_s is the number of flight string s being executed by aircraft k cross the count time, N_{e} is the number of aircrafts in fleet type e, y^k_j is a ground variable used to count the number of aircraft k on the ground j, E_n is the set of fleet type in recovery scope N , Condition 2 is that ground arc $j \in G^k$ for aircraft k crosses the count time, and G^k is the set of ground arcs of aircraft k which cross the count time.

The balance of crew is guaranteed by

$$\sum_{r\in R^e_n} \beta^p_f w^r_p + z_f - n^{\mathrm{sur}}_f = 1, \quad \forall f \in F_n, \tag{7}$$

$$\beta^p_f = \begin{cases} 1, & \text{flight } f \text{ is included in pairing } p \\ 0, & \text{otherwise} \end{cases}.$$

The following formula forces us to assign crew r to a pairing p or to deadhead to its crew base:

$$\sum_{p\in P^r_n} w^r_p + v_r = 1, \quad \forall r \in R^e_n, \tag{8}$$

$$v_r = \begin{cases} 1, & \text{crew } r \text{ has no pairing assigned} \\ 0, & \text{otherwise} \end{cases}.$$

If the crew chooses pairing p, and pairing p includes flight f, it means that crew r covers flight f:

$$\sum_{p\in P^r_n} \beta^p_f w^r_p = u^r_f, \quad \forall r \in R^e_n, \tag{9}$$

$$u^r_f = \begin{cases} 1, & \text{flight } f \text{ is assigned to crew } r \\ 0, & \text{otherwise} \end{cases}.$$

The following formula ensures that crew r is assigned to at most one pairing:

$$\sum_{p \in P_n^r} w_p^r \leqslant 1, \quad \forall r \in R_n^e. \tag{10}$$

The following formula ensures that each available aircraft cannot be assigned to two different strings at the same time:

$$x_{e,s}^k a_f^s + \sum_{f'} x_{e,s'}^k a_{f'}^{s'} \leqslant 1, \quad \forall k \in K_n(e), \tag{11}$$

$$\begin{aligned} & f' \in \{\text{first flight of string } s' | T_{f'} > T_f, \\ & \qquad T_{f_{\text{last}}} + T_{f_{\text{last}}}^{\text{e}} \geqslant T_{f'} + U_{\text{delay}}\}, \\ & f \in \{\text{first flight of string } s\}, \\ & f_{\text{last}} \in \{\text{last flight of string } s\}, \\ & s, s' \in S_n, \end{aligned}$$

where U_{delay} is the allowed maximum delay time, and $T_{f_{\text{last}}}^{\text{e}}$ is the expected trip time of flight f_{last}.

Rotation aircraft use is defined as

$$\begin{aligned} & x_{e,s}^k a_{f_{i+1}}^s - x_{e,s}^k a_{f_i}^s = 0, \\ & \forall k \in K_n(e), \\ & f_i \in s, \quad s \in S_n. \end{aligned} \tag{12}$$

All flights in a rotation use one aircraft not different ones. By using the concept of rotation and by defining rotations in the model, aircraft balance at each airport is satisfied.

The departure time of each flight is determined by

$$t_f^{\text{d}} \geqslant A_{e,f}^k x_{e,s}^k a_{f_{\text{first}}}^s, \tag{13}$$

$$t_{f'}^{\text{d}} \geqslant t_f^{\text{a}} x_{e,s}^k a_{f_{\text{last}}}^s x_{e,s'}^k a_{f'}^{s'} + U_{\min}, \tag{14}$$

$$t_{f_{\text{first}}(i+1)}^{\text{d}} \geqslant t_{f_{\text{first}}(i)}^{\text{a}} + U_{\min}, \tag{15}$$

$$\begin{aligned} & k \in K_n(e), \\ & f' \in \big\{\text{first flight of string } s' | T_{f_{\text{last}}} + \\ & \qquad T_{f_{\text{last}}}^{\text{e}} \geqslant T_{f'} + U_{\text{delay}}\big\}, \\ & s, s' \in S_n \end{aligned}$$

where t_f^{a} is actual arrival time of flight f, t_f^{d} and $t_{f'}^{\text{d}}$ are actual departure time of flight f and f' respectively, f_{first} is first flight of flight string s, $f_{\text{first}(i)}$ and $f_{\text{first}(i+1)}$ are connecting flight, $f_{\text{first}(i+1)}$ just follows $f_{\text{first}(i)}$, and $U_{\min}$ is the aircraft minimum connecting time. Equation (13) states that a flight cannot depart earlier than the ready time of its assigned aircraft. Equation (14) ensures that when two flight strings are flown by the same aircraft, the second string cannot depart earlier than real arrival time of first string (because of the minimum connecting time). In a flight string, the departure time of a flight cannot be earlier than the arrival time of its previous flight, stated as Eqs. (13)—(15).

The following formula states that no flight is allowed to depart before its scheduled departure time:

$$t_f^{\text{d}} \geqslant T_f, \quad \forall f \in F_n. \tag{16}$$

The following formula relates the departure and arrival times for each flight:

$$t_f^{\text{a}} = t_f^{\text{d}} + E[T_f](1 - z_f), \quad \forall f \in F_n, \tag{17}$$

where $E[T_f]$ is the expected trip time of flight f.

If a selected flight f is delayed, no duty violation is allowed for crew r:

$$t_f^{\text{a}} \leqslant u_f^r d_r + (1 - u_f^r) U_{\max}, \quad \forall f, r, p, \tag{18}$$

where $U_{\max}$ is the maximum sit-connection time between flights, and d_r is the duty limit for crew r.

The following formula indicates that the number of deadhead crews cannot exceed the reserved seats for deadhead crews in flight:

$$0 \leqslant n_f^{\text{sur}} \leqslant \max f, \quad \forall f \in F_n, \tag{19}$$

The following formula ensures that $x_{e,s}^k$ and z_f are binary variables, and t_f^{d} and t_f^{a} are nonnegative:

$$x_{e,s}^k \in \{0, 1\}, \quad z_f \in \{0, 1\}, \quad t_f^{\text{d}} \geqslant 0, \quad t_f^{\text{a}} \geqslant 0. \tag{20}$$

3 Solution Methodology

Even by limiting the scope of the problem to make computational tractable, to most airlines, the problem is likely too large and complex to return a global optimal solution with optimization solver for the most reasonable disruption scenarios. Thus, we seek the hybrid method, which is optimization method with heuristic approach. The heuristic we used is so-called iterative tree growing with node-combination. Before using this heuristic method, we first use two preprocessing methods, namely, node aggregation and island isolation.

3.1 Node Aggregation and Island Isolation

Node aggregation allows consecutive arrival nodes and subsequent consecutive departure nodes to be combined. It also eliminates unnecessary ground arcs. In Fig. 1, there are 8 flights and 16 nodes. But in the Fig. 2(a) there are only seven nodes (six aggregated nodes and one original node).

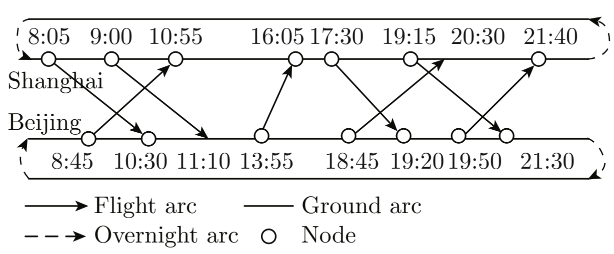

Fig. 1 A time-space network of two cities and eight flights

Island isolation can eliminate a ground arc if it is not necessary to have aircrafts on the ground arc during the specific period. In Fig. 2(b), after island isolation, two ground arcs are reduced while compared with Fig. 2(a).

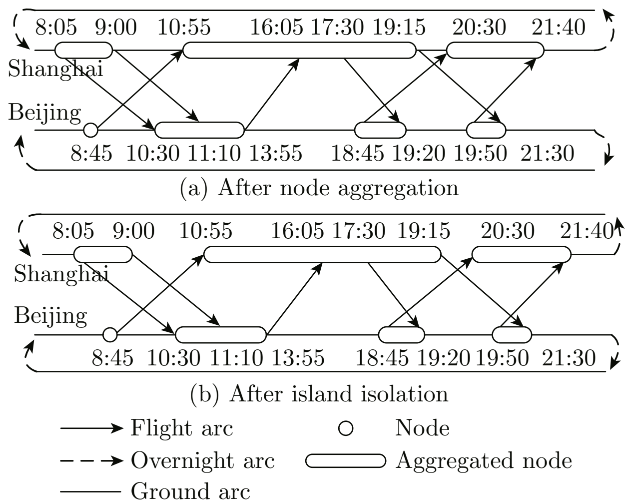

Fig. 2 The simplified time-space network

3.2 Iterative Tree Growing with Node-Combination

The time-space network is used to describe our heuristic method. In the graph, the cities and times are represented horizontally and vertically respectively. Each node represents an airport-departure or airport-arrival event. All the arcs denote flights. Except first node (time-earliest node) and last node (time-termination node, usually night curfew time for departure), we draw all parallel arcs (copy arcs) if the arcs lie above the node and originate from the same node (airport). As the flight arcs are placed in the graph iteratively, the tree grows downward. There are three kinds of arcs in the graph. One is the original flight arc; the other is the copy arc, which is actually the opportunity flight arc due to flight cancellation. The third is the overfly arc, which is actually the delay flight arc. Most probably, each copy arc generates a new node. From this new node, copy arcs and overfly arcs can be originated or connected again. It is an iterative procedure. By so stretching, the tree grows downward. Obviously, there will be more and more nodes and arcs as the graph stretches downward. Every route from top to down represents a routing. In order to simplify such a combinatorial problem, we use circle to replace a dot to represent a node. In other words, a node does not represent a single airport-departure or airport-arrival event, but a cluster of airport-departure or airport-arrival events. All the airport-departure or airport-arrival dots within the time circle are aggregated to this node. The delay time is counted from departure circle node to arrival circle node, not the difference between real departure and arrival time. Under the extreme condition, such aggregating method may calculate delay time the circle diameter difference.

The network transformation procedure is listed as follows.

Step 1 Create the set of airport (A) and flight $f \in F_n$ in recovery scope N, containing the departure airport (DepAirport), arrival airport (ArrAirport), ready time, departure time (DepTime), arrival time (ArrTime) and turnaround time between flights.

Step 2 Create the set of aircraft $k_n(e)$ in recovery scope N, containing aircraft tail number, and fleet type.

Step 3 Create the origin node in the recovery scope N, containing the number of nodes, time-band, time-up and time-down.

Step 4 Transform the flight schedule into the time-space network in recovery scope N. The origin node is equal to the current node (CurNode). Create the nodelist (Nodelist).

Step 5 While CurNode is true, if CurNode $\notin$ Nodelist, turn to Step 6; otherwise turn to Step 10.

Step 6 For flight $f \in F_n$, if flight.DepAirport = CurNode.Airport, create a NewNode. The arrival airport of the new node is equal to the arrival airport of the flight, calculating the actual time of the NewNode.

Step 7 If NewNode.TimeDown $<$ CurfewStart or NewNode.TimeUp $\geqslant$ CurfewEnd, next to Step 8; otherwise cancel the NewNode, and then turn to Step 6.

Step 8 If Node $\in$ State.Time and
Node == NewNode,
NewNode.ReadyTime =
min {NewNode.ReadyTime, Node.ReadyTime},
Node.ReadyTime =
min {NewNode.ReadyTime, Node.ReadyTime},
else, State.Time = State.Time $\cup$ NewNode. Add the copy arc from CurNode to NewNode, containing DepTime, ArrTime, DepAirport and ArrAirport, and calculate the delay Cost. Create the copy of NewNode (NewNodeCopy). Let NewNodeCopy = NewNode; add NewNodeCopy to the NodeList; circularly calculate until no more new node.

Step 9 Create the flight of the CurNode.

Step 10 Order the Newlist according to the sequence; let CurNode = Node; delete the node from the NewList.

Step 11 Delete the surplus arc from the time-space network.

Step 12 End.

4 Case Study

The test instances used as benchmark problems in this study are acquired from real flight schedule of a medium-size airline in China. The schedule consists of 170 flights served by 4 fleets, 35 aircrafts over a network of 51 airports all over the country.

We choose test instances from the flight schedule. The relative data listed in Table 1 consists of 8 flight strings, 4 fleet types and 43 flight legs. Each station requires a minimum of 40 min turnaround time. Execute midnight arrival/departure curfew (no arrival or departure after midnight is allowed). Each minute of delay on any flight costs the airline 20 USD. Each duty should not exceed 8 hours in 24 hours. The minimum and maximum sit-connection times are 10 minutes and 3 hours respectively. Flight legs should start and end at the same crew base. In Table 1, DStat denotes the departure station, AStat denotes the arrival station, STD1 denotes the scheduled departure time, STA1 denotes the scheduled arrival time, CAN denotes Guangzhou, NKG denotes Nanjing, XMN denotes Xiamen, SYX denotes Sanya, SZX denotes Shenzhen, SHA denotes Shanghai, PEK denotes Beijing, and TSN denotes Tianjin.

We transform Table 1 to a time-space networks as shown in Fig. 3. The figure on the arc is flight number. The figure besides the node is departure or arrival time. The node is marked according to the vertical time and horizontal airport coordinates. In order to reflect whether two flight legs can be connected, the arrival time has been added turnaround time. For example, FM9308 arrives in Shanghai at 10:15 and connects to FM9303, which is available for departure at 10:55. We use 30 min as the diameter of the circle. So, FM9303 is ready for departure not at 10:55 but at 11:30.

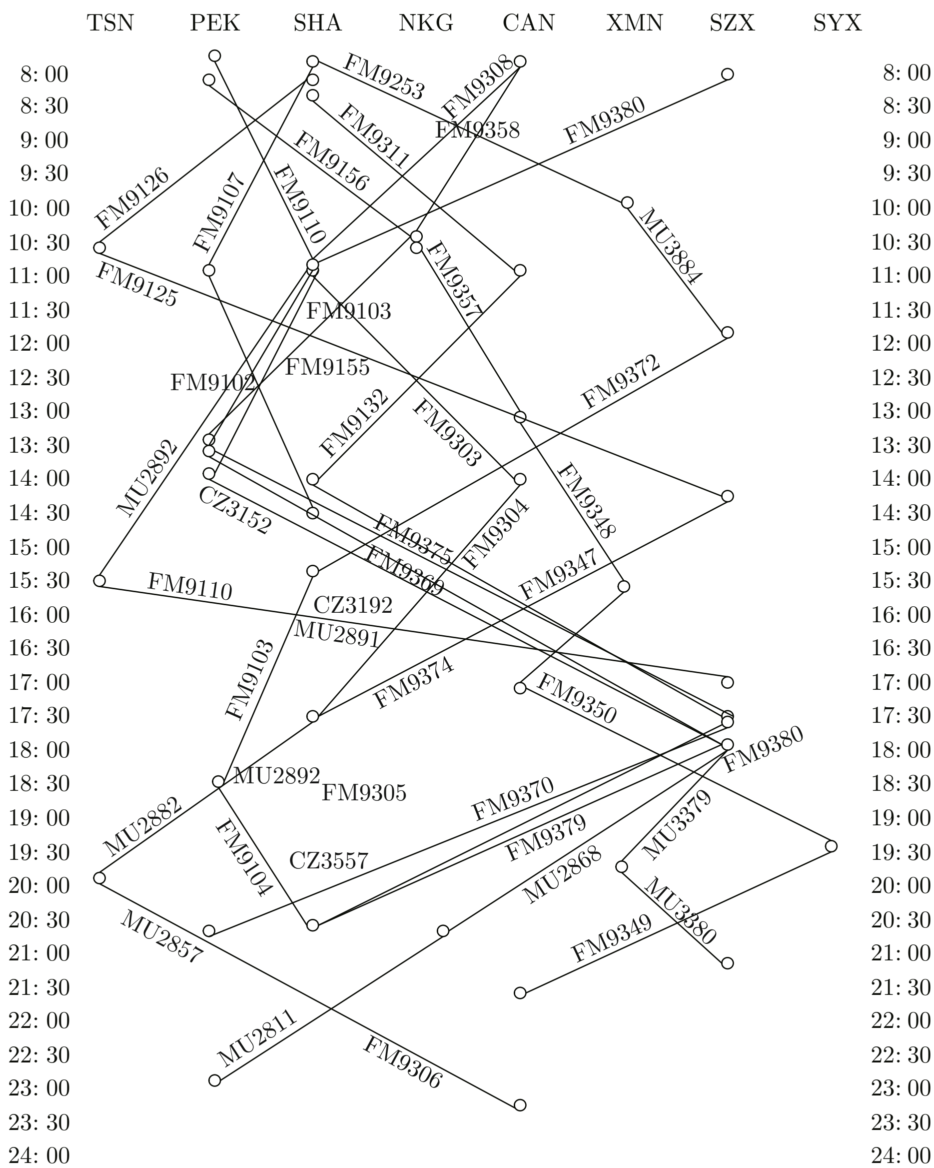

Fig. 3 The time-space network for the case study

Table 1 The flight schedule and cancellation cost

Flight string	Tail no.	Fleet type	Crew pairing	Flight	DStat	STD1	AStat	STA1	Duration/min	Cancellation cost/USD
S_1	AC1	757-200	#1	FM9358	CAN	8:00	NKG	10:00	120	16 500
				FM9357	NKG	10:45	CAN	12:50	125	17 800
			#2	FM9348	CAN	13:35	XMN	14:45	70	9 500
				FM9347	XMN	15:40	CAN	16:45	65	8 500
				FM9350	CAN	17:40	SYX	18:55	75	8 900
				FM9349	SYX	19:50	CAN	21:10	80	7 900
S_2	AC2	757-200	#3	FM9311	SHA	8:30	CAN	10:35	125	17 500
				FM9312	CAN	11:30	SHA	13:35	125	17 600
			#4	FM9375	SHA	14:35	SZX	16:45	130	18 750
				FM9376	SZX	17:50	SHA	20:15	145	19 100
S_3	AC3	737-800	#5	FM9107	SHA	8:05	PEK	10:30	145	17 870
				FM9102	PEK	11:30	SHA	13:40	130	18 850
			#6	FM9369	SHA	14:30	SZX	16:45	135	17 100
				FM9370	SZX	17:40	SHA	19:50	130	17 850
S_4	AC4	737-800	#7	FM9126	SZX	8:15	TSN	10:05	110	16 870
				FM9125	TSN	11:00	SZX	13:50	170	20 500
				FM9374	SZX	14:30	SHA	16:40	130	17 650
			#8	FM9305	SHA	17:40	CAN	20:05	145	18 150
				FM9306	CAN	21:00	SHA	23:20	140	17 950
S_5	AC5	767-300	#9	FM9380	SZX	8:10	SHA	10:25	135	17 850
				MU2892	SHA	11:20	TSN	13:05	165	15 650
				MU2891	TSN	14:00	SZX	17:00	180	22 500
			#10	MU3379	SZX	18:00	XMN	19:00	60	11 500
				MU3380	XMN	20:00	SZX	21:05	65	12 050
S_6	AC6	CRJ-200	#11	FM9156	PEK	8:25	NKG	10:15	110	17 550
				FM9155	NKG	11:05	PEK	12:50	105	16 950
			#12	CZ3152	PEK	13:30	SZX	16:40	190	23 450
				MU2868	SZX	17:40	NKG	20:00	140	16 550
				MU2811	NKG	21:00	PEK	22:50	110	13 750
S_7	AC7	737-800	#13	FM9110	PEK	8:00	SHA	10:10	130	18 350
				FM9103	SHA	11:00	PEK	13:15	135	18 750
			#14	CZ3192	PEK	14:00	SZX	17:10	190	21 750
				CZ3557	SZX	18:00	PEK	21:15	195	24 550
S_8	AC8	737-800	#15	FM9253	SHA	8:05	XMN	9:30	85	13 500
				MU3384	XMN	10:20	SZX	11:30	70	9 500
				FM9372	SZX	12:30	SHA	14:40	130	17 500
			#16	FM9105	SHA	15:30	PEK	17:50	140	18 100
				FM9104	PEK	18:50	SHA	21:00	130	18 900
S_9	AC9	767-300	#17	FM9308	CAN	8:10	SHA	10:15	125	17 500
				FM9303	SHA	11:30	CAN	13:30	120	16 950
			#18	FM9304	CAN	14:30	SHA	16:35	125	17 150
				MU2882	SHA	17:30	TSN	19:15	105	15 950
				MU2857	TSN	20:10	CAN	23:15	185	21 150

We use two scenarios to test the method.

Scenario 1 Aircraft recovery.

Suppose aircraft in airport CAN is out of service from 8:00 to 12:00. The trivial solution 1 is to cancel flights FM9358 and FM9357, and the cancellation cost is 35 300 USD. The trivial solution 2 is to delay S_1 (flight FM9358, FM9357, FM9348, FM9347, FM9350, FM9349), the ready time of flight FM9349 is 22:55, the arrival time would be 24:15 against the curfew constraint, so the flight FM9349 cannot be performed. The total delay and cancellation cost is 30 300 USD. Considering crew recovery, we divide the S_1 and S_2 into the same part.

Figure 4 is deduced form Fig. 3 and Table 2 based on the method mentioned above. In the Fig. 4, the Node 1 means the time-band from 8:30 to 8:59, as the same as Node 2, 3, 4, etc, which means the 30 min diameter of the circle.

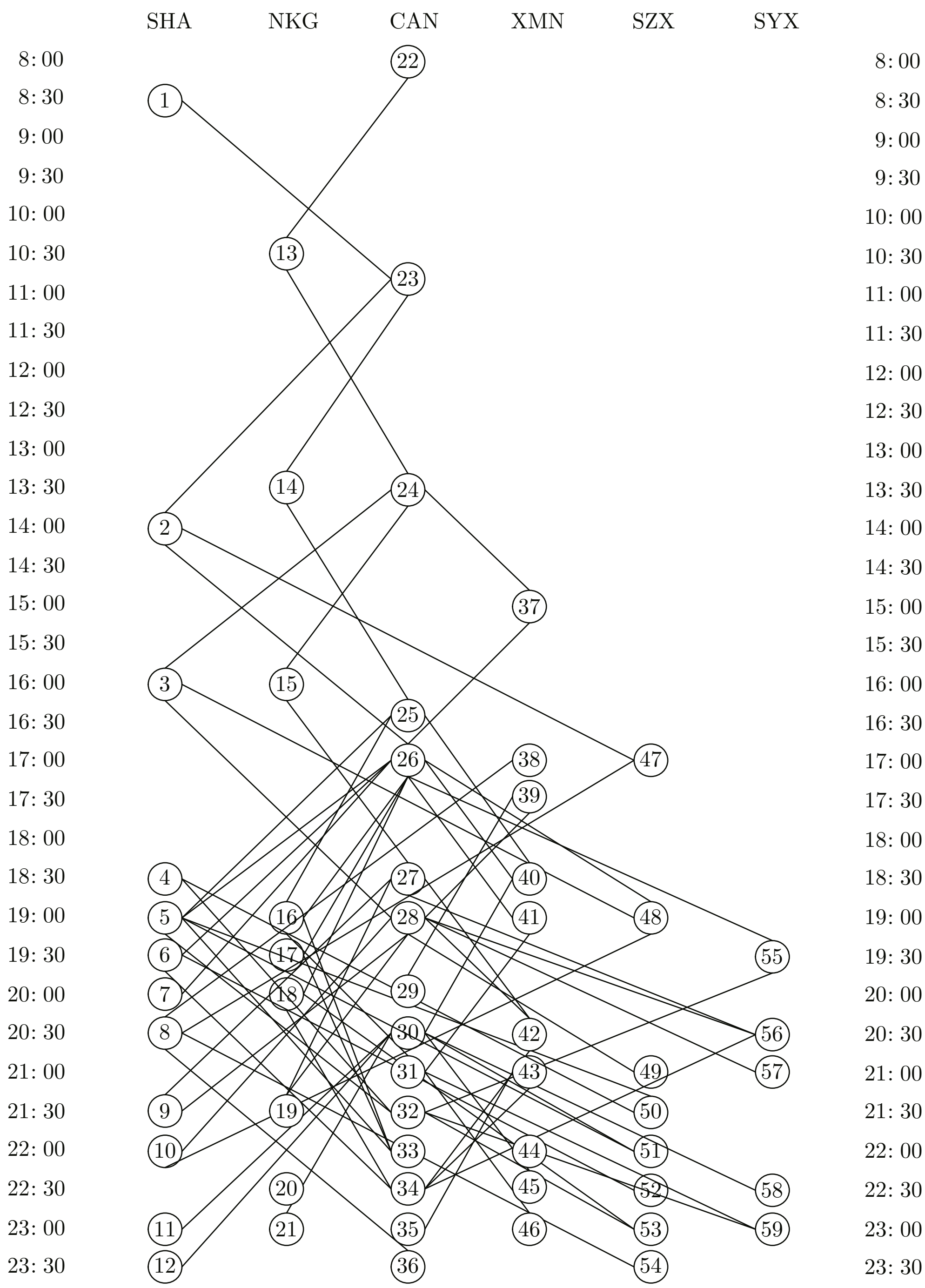

Fig. 4 Iterative tree growing with node-combination of the flight string S_1 and S_2

Table 2 Solution for Scenario 1

Flight string	Crew pairing	Tail no.	Flight	DStat	STD1	AStat	STA1	Option	Delay cost/USD	Total cost/USD
S_1	#1	AC2	FM9311	SHA	8:30	CAN	10:35	—	—	—
	#2	AC2	FM9358	CAN	11:15	TSN	13:15	Delay	3 600	3 600
			FM9357	TSN	13:55	CAN	16:00	Delay	3 000	3 000
	#3	AC2	FM9348	CAN	16:40	XMN	17:50	Delay	3 000	3 000
			FM9347	XMN	18:30	CAN	19:15	Delay	2 400	2 400
			FM9350	CAN	19:15	SYX	20:30	Delay	1 800	1 800
			FM9349	SYX	21:10	CAN	22:20	Delay	1 800	1 800
S_2	#1	AC1	FM9312	CAN	12:00	SHA	14:05	Delay	600	600
	#4	AC1	FM9375	SHA	14:45	SZX	16:55	Delay	200	200
			FM9376	SZX	17:50	SHA	20:15	—	—	—
Total	—	—	—	—	—	—	—	—	16 400	16 400

Table 3 The integrated recovery solution for Scenario 2

Flight string	Crew pairing	Tail no.	Flight	DStat	STD1	AStat	STA1	Option	Delay cost/USD	Total cost/USD
S_7	#13	AC7	FM9110	PEK	8:00	SHA	10:10	—	—	—
	#13	AC3	FM9107	SHA	10:20	PEK	12:45	Delay	2 700	2 700
			FM9102	PEK	13:25	SHA	15:35	Delay	2 300	2 300
	#6		FM9369	SHA	16:15	PEK	18:35	Delay	900	900
			FM9370	PEK	19:15	SHA	21:35	Delay	500	500
S_3	#16	AC7	FM9103	SHA	11:00	PEK	13:15	—	—	—
	#14		CZ3192	PEK	14:00	SZX	17:10	—	—	—
			CZ3557	SZX	18:00	PEK	21:15	—	—	—
S_8	#5	AC8	FM9253	SHA	8:05	XMN	9:30	—	—	—
			MU3884	XMN	10:20	SZX	11:30	—	—	—
			FM9372	SZX	12:30	SHA	14:40	—	—	—
	#16		FM9103	SHA	15:20	SZX	17:35	Delay	1 000	1 000
			FM9104	SZX	18:15	SHA	20:25	Delay	700	700
S_4	#7	AC4	FM9126	SHA	8:15	TSN	10:05	—	—	—
			FM9125	TSN	11:00	SZX	13:50	—	—	—
			FM9374	SZX	14:30	SHA	16:40	—	—	—
	#8		FM9305	SHA	17:40	CAN	20:05	—	—	—
			FM9306	CAN	21:00	SHA	23:20	—	—	—
Total	—	—	—	—	—	—	—	—	—	8 100

In Fig. 4, one arc is drawn from Node 2 to Node 26, and it represents flight FM9311. Actually it is a copy arc of FM9311; the delay time is 360 min, not the actual time minus the schedule time. This is because FM9311 is scheduled to arrive at CAN at 11:15. If this flight occurs in Node 2, the arrival time is calculated as 17:00. Considering the nodes within 30 min circle, this delay spans from 11:00 to 17:00, a total of 360 min. Each minute of delay costs the airline 20 USD, so flight FM9311 has a delay cost of 7 200 USD, if it departs from Node 2.

We use ILOG CPLEX to calculate the scenario, through a series of aircraft rerouting and cancellations in an effort to minimize the total cost to the airlines. The total cost for the recovery solution is 16 400 USD, smaller than the trivial solution of 34 300 USD, resulting from canceling FM9358 and FM9357, and smaller than cost of 30 300 USD by delaying the flight string S_1.

Scenario 2 Aircraft integrated with crew recovery.

In this case, we assume that crew pairing of #15 gets ill and flights FM9253, MU3384 and FM9372 in flight string S_8 should be canceled. The cancellation cost should be 40 500 USD. In our schedule, only the crew pairing of fleet type 737-800 can be used to represent crew pairing #15. We use the same method in the above to solve Scenario 2.

According to Table 3, we get the integrated recovery solution for Scenario 2. The total cost for the solution given by our method is 8 100 USD, smaller than the trivial solution of 40 500 USD resulting from canceling all flights operated by aircraft 8. The crew pairing #7 and pairing #8 remain unchanged as original schedule. The crew pairing #13 takes the flight FM9369 as deadhead back to their base — Beijing, and the crew pairing #5 takes the flight FM9104 as deadhead back to Shanghai.

5 Conclusion

The paper presents a more practical formulation for airline optimal recovery. In order to get the solution in a reasonable time, a new approach to solve the problem is studied. The computational results state the method which could be used in the airline recovery.

Airlines recovery is a more complex and large-scale problem. Not only should aircrafts and crew be considered, but passengers should be considered, too. In the future, a more comprehensive recovery model would be studied. The model can be extended through considering passenger constraints or slot times as other resources in airline scheduling. Furthermore, developing solution techniques for large scale problems is another extension which authors are now working on.

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