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Study on an improved adaptive PSO algorithm for solving multi-objective gate assignment

Wu Deng[a,b,c,d,e], Huimin Zhao[a,b,c,d,*], Xinhua Yang[a,d], Juxia Xiong[b], Meng Sun[a], Bo Li[a]

[a] Software Institute, Dalian Jiaotong University, Dalian, 116028, China
[b] Guangxi Key Laboratory of Hybrid Computation and IC Design Analysis, Guangxi University for Nationalities, Nanning, 530006, China
[c] Sichuan Provincial Key Lab of Process Equipment and Control (Sichuan University of Science and Engineering), Zigong, 64300, China
[d] Dalian Key Laboratory of Welded Structures and Its Intelligent Manufacturing Technology(IMT) of Rail Transportation Equipment, Dalian Jiaotong University, Dalian, 116028, China
[e] Nanjing University of Information Science and Technology, Nanjing, 210044, China

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ABSTRACT

Gate is a key resource in the airport, which can realize rapid and safe docking, ensure the effective connection between flights and improve the capacity and service efficiency of airport. The minimum walking distances of passengers, the minimum idle time variance of each gate, the minimum number of flights at parking apron and the most reasonable utilization of large gates are selected as the optimization objectives, then an efficient multi-objective optimization model of gate assignment problem is proposed in this paper. Then an improved adaptive particle swarm optimization(DOADAPO) algorithm based on making full use of the advantages of Alpha-stable distribution and dynamic fractional calculus is deeply studied. The dynamic fractional calculus with memory characteristic is used to reflect the trajectory information of particle updating in order to improve the convergence speed. The Alpha-stable distribution theory is used to replace the uniform distribution in order to escape from the local minima in a certain probability and improve the global search ability. Next, the DOADAPO algorithm is used to solve the constructed multi-objective optimization model of gate assignment in order to fast and effectively assign the gates to different flights in different time. Finally, the actual flight data in one domestic airport is used to verify the effectiveness of the proposed method. The experiment results show that the DOADAPO algorithm can improve the convergence speed and enhance the local search ability and global search ability, and the multi-objective optimization model of gate assignment can improve the comprehensive service of gate assignment. It can effectively provide a valuable reference for assigning the gates in hub airport.

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1. Introduction

Airport gate assignment problem is an optimization problem [1]. Its task is to assign the flights to the limited gates in order to meet the association between the flight types and gates, the space constraints and the occupancy time, and achieve docking and ensure the effective connection among passengers and goods. So it is a core link for airline operation and management [2,3]. On the one hand, the gate assignment problem takes on the complex properties of internal multi-objective, multi-constraint and multi-resource theory, and usually has hundreds of gates and flights to be assigned for every day in the large international airport. And the gate assignment problem is more complex due to the daily operational changes, such as flight delays, weather and so on. On the other hand, because the airport gates are a scarce, expensive and critical resource, the unreasonable gate assignment will cause the flight delays, reduce resource utilization and increase customers' dissatisfaction [4]. As a result, when the air traffic transportation is increasing, the gate assignment problem is becoming increasingly important.

At present, there are two classes of methods to solve the gate shortage problem. Firstly, the airports are expanded or new airports are established. Although this method can effectively alleviate the problem of airport gate shortage, it has the limitations of more manpower, financial resources and time with hysteresis and so on. Secondly, a reasonable and efficient method is used to optimize and assign the existing resources in order to improve the utilization effi-

* Corresponding author at: Software Institute, Dalian Jiaotong University, Dalian, 116028, China.
E-mail address: hm_zhao1977@126.com (H. Zhao).

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ciency and satisfaction of passengers. This method is simple, easy and quick, and has less investment. Therefore, the reasonable and efficient methods have been more deeply and extensively studied in recent years. But because the hub airport has the layout complexity, more flights, higher frequency of aircraft in and out airport, large turnover of gates, the gate assignment is a very complex optimization problem. So the gate assignment problem belongs to the NP problem. In addition, because the hub airport has more gates and complicated aircrafts, the traditional optimization methods can not effectively solve the gate assignment problem. In recent years, a lot of scholars and experts have proposed a series of the gate assignment methods, such as expert system, mathematical programming, artificial intelligence and so on [5–13]. Expert system establishes the knowledge base system by using configuration rule and considering more non-quantification criteria. Due to the constraints of search scope, this method ignores the key factors that lead to undesirable configuration result. The mathematical programming selects the optimization objective function and uses 0–1 integer planning to explore the feasibility of configuration. The main problem of this method is to select a suitable objective function. There are many factors that affect the airport gate assignment. Artificial intelligence method is used to solve the gate assignment problem. When the number of flights is up to thousands, the complexity will increase with order, and this method is difficult to meet the requirements of real-time gate assignment. Therefore, for these existing shortcomings in solving gate assignment problems, a more reasonable mathematical model of airport gate assignment problem need be established and the effective solving method is proposed to realize the effective assignment of airport gates.

Particle swarm optimization(PSO) algorithm is an intelligent optimization algorithm, which has many advantages, such as less adjustment parameters, fast convergence speed, better optimization effects and so on. The PSO algorithm is widely used in solving NP problems, such as combinatorial optimization problems, function optimization problems and so on. In the study and application of the PSO algorithm, it is found that the PSO algorithm has the shortcomings of premature convergence, slow convergence speed in the later evolution, poor accuracy, and is easy to fall into local extreme point, and so on. Therefore, in order to solve these shortcomings in gate assignment, an improved adaptive PSO(DOADAPO) algorithm based on Alpha-stable distribution theory and dynamic fractional calculus is proposed in this paper. And a multi-objective optimization model of airport gate assignment problem based on the objectives of the minimum walking distances of passengers, the minimum idle time variance of each gate, the minimum number of flights at parking apron and the most reasonable utilization of large gates is established. And the DOADAPO algorithm is used to solve the multi-objective optimization model of gate assignment in order to reasonably and effectively assign the flights to the gates.

The remainder of the paper is organized as follows. The literature review is comprehensively analyzed in Section 2. The optimization model of gate assignment in hub airport is established in Section 3. In Section 4, dynamic fractional calculus and Alpha-stable distribution theory are introduced in detail. In Section 5, an improved adaptive PSO algorithm based on Alpha-stable distribution and dynamic fractional calculus is proposed. The improved adaptive PSO algorithm is used to solve gate assignment optimization model in Section 6. In Section 7, the data simulation and analysis are introduced in detail. Finally, the conclusions are offered and future research direction is discussed in Section 8.

2. Literature review

The airport gate assignment problem has obtained considerable attention and been widely studied in recent years. In this section, a lot of the existing gate assignment models and solution methods for airport gate assignment problems are proposed. These solution methods can be summarized as mathematical programming methods, artificial intelligence methods and computer simulation methods. Next, three solution methods are reviewed respectively.

For the existing mathematical programming methods, Cheng [14] proposed a knowledge-based airport gate assignment system integrated with mathematical programming techniques to provide a solution that satisfies both static and dynamic situations within a reasonable computing time. Yan and Huo [15] proposed a multiple objective model to help airport authorities to efficiently and effectively solve gate assignment problems. The model is formulated as a multiple objective zero-one integer program. Chen et al. [16] proposed a systematic study of the integrated scheduling and runway assignment of both arriving and departing traffic over an airport by using a multiple-point scheduling scheme. This general runway scheduling problem is formulated as mixed-integer linear programming. Jiang et al. [17] proposed an optimization gate assignment model based on minimizing the total walking distance of all passengers and balancing the average walking distance of passengers among different airlines according to the airport passenger service quality. Behrends and Usher [18] proposed a framework that integrates the passenger or freight movement within a terminal with the taxiing of the aircraft to support an integrated approach to solving the gate assignment problem.

For the existing artificial intelligence methods, Bolat [19] proposed a unified framework to specifically deal with the objective functions of the previous models. A genetic algorithm utilizing problem specific knowledge is proposed to provide effective alternative solutions. Ding et al. [20] studied the over-constrained airport gate assignment problem where the objectives are to minimize the number of ungated flights and total walking distances or connection times. Drexl and Nikulin [21] proposed an airport gate assignment problem with multiple objectives of minimizing the number of ungated flights and the total passenger walking distances or connection times and maximizing the total gate assignment preferences. Genç et al. [22] proposed a method that combines the benefits of heuristic approaches with some stochastic approach instead of using a purely probabilistic approach to top-down solution of the problem. Cheng et al. [23] proposed a hybrid approach based on simulated annealing and Tabu search to solve the gate assignment problem. Wang et al. [24] proposed an optimized model based on the characteristics of the flights and the airport gates. Zhao and Cheng [25] proposed a mixed integer model to formulate airport gate assignment problem, and the ant colony optimization algorithm is designed to solve this model. Kim and Feron [26] proposed a queuing model to simulate the airport departure process with the current gate assignment and a robust gate assignment to assess the impact of gate assignment on departure metering. Genc et al. [27] proposed a multi-objective gate assignment problem with the objectives of maximizing gate assignment, minimizing passenger walking distance and maximizing flight to gate preference and a solution strategy based on the evolutionary Single Leap Big Bang-Big Crunch optimization method. Liu et al. [28] proposed to use the genetic algorithm to solve the optimization model based on the main objective of minimizing the dispersion of gate idle time periods for considering operational safety constraints. Benlic et al. [29] proposed a the breakout local search framework based on the iterated local search with a particular focus on the perturbation strategy for the multi-objective gate allocation problem.

For the existing computer simulation methods, Yan et al. [30] proposed a simulation framework, that is not only able to analyze the effects of stochastic flight delays on static gate assignments, but can also evaluate flexible buffer times and real-time gate assignment rules. Yan and Tang [31] proposed a heuristic approach embedded in a framework to help the airport authorities to make airport gate assignments that are sensitive to stochastic flight delays. Diepen et al. [32] proposed a completely new integer linear programming formulation that is based on so-called gate plans. The linear programming relaxation is solved through column generation. Prem Kumar and Bierlaire [33] proposed a mathematical model for all complex constraints that are observed at a real airport. Yu et al. [34] proposed a robust gate assignment model based on considering three factors of schedule robustness, facility and personnel cost during tows, and passenger satisfaction level, which is transformed to an equivalent MIP for proving having more efficient than the linearized models. Zhang et al. [35] proposed an efficient gate re-assignment methodology based on the objective functions of minimizing the weighted sum of the total flight delays, the number of gate re-assignment operations and the number of missed passenger connections to deal with the disruptions. Dorndorf et al. [36] proposed a flight gate assignment and a procedure for recovery planning that has proved its practical relevance at numerous airports.

For the gate assignment problems, it is crucial to establish the effective optimization model of gate assignment and obtain solutions with a certain level of robustness in order to maintain effective operations of airport. Some scholars and experts have proposed several the robustness definitions for gate assignments in order to find the optimization solutions, which can tolerate a certain uncertainty degree. Under a particular scenario, it has the difference between the performance and the optimal solution. In this study, the deterministic robust methods are extended by the objective functions of the minimum walking distances of passengers, the minimum idle time variance of each gate, the minimum number of flights at parking apron and the most reasonable utilization of large gates. Then an efficient multi-objective optimization model of gate assignment problem is developed. And an improved adaptive particle swarm optimization algorithm is proposed to solve the established optimization model of gate assignment.

3. Establish an optimization model of gate assignment

3.1. Description of gate assignment problem

Hub airport is to refer the international and domestic airport with intensive flights. It can provide an efficient and convenient service with low cost, so as to let the airline choose it as their destination and allow passengers to select it as a traffic Airport. As the gate in the airport is an extremely scarce and expensive resource, it is the key factor to achieve rapid and secure flight docking, ensure effective connection between flights, and increase airport capacity and efficiency. The characteristics of gates and flights are analyzed in detail.

The characteristics of gates in hub airport mainly include matching characteristics, accessory facilities, available state, instantaneous exclusive, time-sharing, distance difference and adjacency relationship and so on [37].

(1) Matching characteristics

Due to the different geographical location and the size of the area, the parked aircraft types are different. For convenience study, the gates are divided into large, medium and small gates. The large gates can park all aircraft types, and small gates can only park the corresponding small aircraft.

(2) Accessory facilities

It is to refer the boarding bridge, refuelling pipeline and so on. Transfer vehicles are used to meet the passengers to board and get off the aircraft.

(3) Available state

It indicates whether the gate can be used in a certain period. If the gate is used within a certain time, the parked facilities are not available during this period, otherwise they is available.

(4) Instantaneous exclusive

One gate at any time can only be assign to exclusive aircraft. This aircraft occupies the parking space that has exclusive features.

(5) Time-sharing

One gate without overlapping time periods is occupied respectively by aircraft of time-sharing.

(6) Distance difference

In the passengers' transfer, the parked gate of previous flight and the parked gate of later aircraft form a pair of flights. The travelled distance between two gates is called the distance of gate pair. In general, the distance between different pairs of aircraft is not the same.

(7) Adjacency relationship

It means that two adjacent gates can not simultaneously dock two large aircrafts.

A flight is a sum of aircraft and routes that carry out a specific transport task at a particular time according to the pre-established schedule. On the basis of the flight definition, the flight characteristics includes the flight number, airline and aircraft type, planning characteristic, specific characteristic, temporal characteristic, random characteristic and dynamic characteristic.

(1) Flight number

It is a unique identification code of the flight and identifies the flight ascription.

(2) Flight characteristics

The airport determines whether the flight belongs to the departure flight or arrival flight. Round-trip or cross-stop flights include both departure and arrival routes. The departure and arrival are different from the airline's flight number, but the flights are the same. Each route has different properties, including domestic flight or international flight, flight information, departure or arrival times, and so on. Two of the most important characteristics are the departure time or arrival time and ground time, they directly affect the flight schedules and order.

(3) Aircraft type

The aircraft types are divided into three grades. The first level is based on the aircraft's external features, like fuselage width. The width of aircraft is divided into narrow-body and wide-body. The second level is divided according to the aircraft model, such as the Boeing 767, Boeing 737, Airbus 320, etc. They belong to narrow-body. From the display of flight information of airport, most aircrafts belong to the narrow-body, fewer aircrafts are wide-body. The third level is divided according to aircraft characteristics, such as the Boeing 747-400. It is a specific model in Boeing 747 aircraft.

(4) Planning characteristics

Before the flight is executed, its various characteristics are generally determined and formed a planning, which is informed to the relevant agencies.

(5) Temporal characteristic

Each flight has an arrival time and a departure time, and the arrival time must be earlier than the departure time. This time is designed to ensure that the interval time can safely perform the flights.

(6) Random characteristics

Although each flight is planned, there are many uncertainties in flight execution due to unpredictable factors, such as flight cancellations and flight delays and so on.

(7) Specific characteristics

Under normal circumstances, a flight only occupies one gate, which is called flight specificity.

(8) Dynamic characteristics

During the airport operation, the flight plan is adjusted, modified and enriched the relevant information in order to form the dynamic flight plan according to the actual operation of a variety of random factors.

3.2. Determine objective function

Gate assignment result determines the normal operation quality of airport. The reasonable gate assignment can effectively achieve the perfect combination of safety and efficiency. Good optimization objective function will eventually be reflected to the operating costs, benefits and satisfaction of the airlines and airports [38–41]. Usually, the minimum walking distances of passengers are considered as the optimization objective of satisfaction. In real-time operation, the arrival time and departure time of flights are always changed due to various reasons. Thus, the solving algorithm for gate assignment model needs to take on a certain ability to handle dynamic changes and provide space for the real-time operation of gates. That is, the optimization result should have robustness. The specific performance is that the idle time of each gate is as far as possible balanced. In the event of shorter delay, it can provide a certain buffer time so that the flights do not need or only need a small scheduling and adjustment, and will not or as little as possible affect the normal operation of other flights. In addition, the balance of idle time between gates also means the balanced utilization of gates, makes staff and equipment have a relatively balanced working time and intensity in order to ensure the smooth operation and reduce the security risks. Therefore, the minimum idle time variance of each gate is taken as the optimization objective. As the parking apron is away from the terminal, and getting on and off flights by using taxi, it seriously affects the passengers' satisfaction. Thus, the minimum number of flights at parking apron is considered as the optimization objective. The gates have different size, if the large gate is occupied by small and medium-sized aircraft in advance, then the large aircraft behind has fewer options to choose, or even force to park on apron. The assigned result will cause the more inconvenience for passengers. If the large aircraft with large number of passengers is early assigned, it can avoid to strand in the airport and staff congestion. So the most reasonable utilization of large gates is considered as the optimization objective. To summarize, the determined optimization objectives are described in detail as follows:

(1) The minimum walking distances of passengers

On the basis of establishing the walking matrix of passengers, the distances between the different gates and check-in counters can be quantitatively determined. So the walking distances of passengers are determined to distinguish the good and bad gates. The objective function is described.

$$F_1 = \min \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{p} q_{ij} f_k y_{ik} \tag{1}$$

where q_{ij} is the transferred number of passengers for the flight i to gate j, f_k is the walking distances from the check-in counter to gate, y_{ik} is the flight i to be assigned to gate k.

For the objective function, the walking distances include the walking distances of arrival passengers, the walking distances of departure passengers and the walking distances of transferred passengers. The walking distances of arrival passengers are the walking distances from the gate to the airport exit. The walking distances of departure passengers are the walking distances from the check-in counter to the gate. The walking distances of transferred passengers are the walking distances from the gate to the gate of the next flight.

(2) The minimum idle time variance of each gate

For each gate, if the balance of idle time is less worse, then this configuration result often can not cope with the time changes of arrival and departure flights. So the minimum idle time variance of each gate is selected as the objective function. The objective function is described.

$$F_2 = \min \sum_{i=1}^{n}\sum_{k=1}^{p} S_{ij}^2 \tag{2}$$

where S_{ij} represents the idle time between two adjacent flights on the same gate.

(3) The minimum number of flights at parking apron

$$F_3 = \min \sum_{i=1}^{n} G_i \tag{3}$$

where G_i indicates whether the flight parks on the apron. When the flight is only assigned to the apron, the value of G_i is 1. Otherwise the value of G_i is 0.

(4) The most reasonable utilization of large gates

$$F_4 = \min \sum_{i=1}^{n}\sum_{k=1}^{p} w_{ik} \tag{4}$$

where w_{ik} is matching condition between flight size and gate size. When the flight i is only assigned to the gate k and flight i matches the gate k, the value of w_{ik} is 1. Otherwise the value of w_{ik} is 0.

3.3. Establish multi-objective optimization model

Gate assignment optimization model based on operating safety need to comprehensively consider the objectives of the minimum walking distances of passengers, the minimum idle time variance of each gate, the minimum number of flights at parking apron and the most reasonable utilization of large gates in order to achieve the efficiency and safety. A multi-objective optimization model of gate assignment is obtained according to the actual situation. But for the multi-objective optimization of gate assignment problem, a coordination solution for multi-objective optimization problem needs to be found, because it is difficult to directly solve the multi-objective function. The common solving methods include the weighted method, two-level programming, constraint method and so on. Because the weighted method can determine the weight coefficient according to the importance of each objective. It is simple and easy to operate, and takes on higher accuracy. So the weighted method is used to transform multi-objective optimization function into single-objective optimization function. This method is described as follow.

$$F = w_1(\min \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{p} q_{ij} f_k y_{ik}) + w_2(\min \sum_{i=1}^{n}\sum_{k=1}^{p} S_{ij}^2) + w_3(\min \sum_{i=1}^{n} G_i) + w_4(\min \sum_{i=1}^{n}\sum_{k=1}^{p} w_{ik}) \tag{5}$$

where w_1, w_2, w_3 and w_4 are the weight coefficients. According to the importance of each objective, the multiple experiments for different combinations of data are analyzed in order to ultimately determine more appropriate values of weight coefficients.

3.4. Constraints

On the basis of present circumstance, the constraints are used to control the calculation process, so that the calculation result can meet the actual requirements. In the same airport, the constraints of different gates and flights may be different. The constraint conditions are described in detail.

(1) Each flight is only assigned to one gate.

$$\sum_{j}^{N} x_{ij} = 1, \ \forall i \ \in \ F, j \ \in \ G \tag{6}$$

If the flight i is assigned to gate j, then there is $x_{ij} = 1$. Otherwise there is $x_{ij} = 0$.

(2) Each flight is only assigned to one runway.

$$\sum_{r=1}^{R} p_{ir} = 1 \tag{7}$$

If the flight i is assigned to runway r, then there is $p_{ir} = 1$. Otherwise there is $p_{ir} = 0$.

(3) The gate constraints to aircraft type: when flight i is assigned to gate j, it must comply with: $\varepsilon_i \le \rho_j + (1 - y_{ij})\Omega$, where ε_i is aircraft type of flight i, ρ_j is gate j to allow the largest aircraft type, Ω is arbitrarily positive number.

$$\varepsilon_i \le \rho_j + (1 - y_{ij})\Omega \tag{8}$$

(4) $|A_i - D_i| \ge T$

The interval time between two adjacent flights for the same gate must be greater than the safety interval time T. T is a safety interval time for the same gate.

$$L_{ij} + ST - A_{kj} \le 0 \ \forall i, k \in F, j \in G \tag{9}$$

(5) 0–1 variable constraints: x_{ij}, q_{ij}, w_{ik}, G_i, y_{ik}, $p_{ir} \in \{0, 1\}$.

(6) Positive constraints: $i, j, k \in N^+, f_k, S_{ij}, L_{ij}, A_{kj}, w_1, w_2, w_3, w_4, w_5, \Omega \ge 0$.

4. Dynamic fractional calculus and Alpha-stable distribution theory

4.1. Fractional calculus theory

Fractional calculus [42] is the differential operator and integral operator theory based on the differential order and integral order with arbitrary real number or complex number. It is an extension of integer-order calculus to non-integer-order calculus. The basic operators of fractional calculus is ${}_aD_t^{\lambda}$. Its implication is to solve the λ derivative or integral of function $f(x)$. λ is the fractional derivative and integral order, a and t are the upper and lower bounds of operation.

The general representation of fractional calculus is described.

$${}_aD_t^{\lambda}\mathrm{f(x)} = \begin{cases} \dfrac{d^{\lambda}}{dt^{\lambda}} f(x) & Re(\lambda) > 0 \\ f(x) & Re(\lambda) = 0 \\ \displaystyle\int_a^l f(x)(d\tau)^{-\lambda} & Re(\lambda) < 0 \end{cases} \tag{10}$$

$\mathrm{Re}(\lambda)$ represents the real part of λ. when there is $\mathrm{Re}(\lambda) > 0$, ${}_aD_t^{\lambda} f(x)$ is the differential of fractional order for $f(x)$. When there is $\mathrm{Re}(\lambda) < 0$, ${}_aD_t^{\lambda} f(x)$ the integral of fractional order for $f(x)$. When there is $\lambda = n \in N$, ${}_aD_t^{\lambda} f(x) = f^n(x)$. That is the derivative of n order of $f(x)$.

In the development course of fractional calculus, some mathematicians gave several definitions of different forms of fractional calculus from different perspectives, the definitional rationality and scientificity have been verified in the experiment. At present, the widely used definition of fractional calculus is the definition of Grünwald–Letnikov fractional calculus. It is extended from the higher order derivatives. The representation is described.

$${}_aD_t^{\lambda} f(x) = \lim_{h \to 0} \left[\frac{1}{h^{\lambda}} \sum_{k=0}^{[\frac{t-a}{h}]} \omega_k^{\lambda} f(x - kh) \right] \tag{11}$$

where $\omega_k^{\lambda} = \frac{(-1)^k \Gamma(\lambda+1)}{k!\,\Gamma(\lambda-k+1)}$, h is the time step of the differential, symbols $[\frac{t-a}{h}]$ expresses the integer part of extracting variable $\frac{t-a}{h}$.

4.2. Alpha-stable distribution theory

Alpha-stable distribution $S_a(\sigma, \beta, \mu)$is a kind of important probability distribution, whose concepts are closely related to the large numbers and the central limit theorem of probability theory [43]. The law of large numbers describes the stability of random variable sequences, while the central limit theorem describes the stability of the distribution function. If there are parameters ($0 < a \le 2, \sigma > 0, -1 \le \beta \le 1$ and real number μ), which can make random variables X to meet the following characteristic function.

$$E_{\exp} i\theta X = \begin{cases} \exp\left\{ -\sigma^a |\theta|^a (1 - i\beta sign(\theta) \tan \dfrac{\pi a}{2}) + i\mu\theta \right\}, a \ne 1 \\ \exp\left\{ -\sigma |\theta| (1 + i\beta \dfrac{2}{\pi} sign(\theta) \ln |\theta|) + i\mu\theta \right\}, \ a = 1 \end{cases} \tag{12}$$

Among them, the sign function $sign(\theta) = \begin{cases} 1, & \theta > 0 \\ 0, & \theta = 0 \\ -1, & \theta = 0 \end{cases}$

Four parameters of Alpha-stable distribution have clear meaning. Characteristic factor a (characteristic exponent) is used to control the thickness and length of tail density function. If the value of a is smaller, the corresponding distribution has more heavy tail. When there is $a \le 1$, the distribution have unlimited mean and variance. Scale parameter σ (dispersion coefficient) is used to measure the sample relative mean dispersion, it is similar to the variance of the Gaussian distribution. When there is $a = 2$, the value of σ is the half of variance value. Skew parameter β is used to determine the slope of the stable distribution. Its value is larger, the signal probability density function is more asymmetrical. Position parameter μ is used for a symmetric distribution (S_aS). When there is $0 < a \le 1$, μ indicates the mean value. $1 < a \le 2$, the μ is the mean value.

The probability density function can be expressed as a continuous Fourier transform by using the characteristic function. It is described as follow.

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \varphi(t) e^{-ixt} dt \tag{13}$$

5. An improved adaptive PSO algorithm based on Alpha-stable distribution and dynamic fractional calculus

5.1. Particle swarm optimization(PSO) algorithm

The PSO algorithm is a population-based search algorithm, which simulates the social behavior of birds within a flock [44–46]. In PSO algorithm, the positions of particles within the search space are changed based on the social-psychological tendency of individuals in order to delete the success of other individuals. The changing of one particle within the swarm is influenced by the experience, or knowledge. The consequence of modeling for this social behavior is that the search is processed in order to return toward previously successful regions in the search space. Namely, the velocity(v) and position(x) of each particle will be changed by the particle best value (pB) and global best value (gB). The updating velocity and position of the particle is shown:

$$v_{ij}(t+1) = wv_{ij}(t) + c_1 r_1 \left(pB_{ij}(t) - x_{ij}(t)\right) + c_2 r_2 \left(gB_{ij}(t) - x_{ij}(t)\right) \tag{14}$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \tag{15}$$

where $v_{ij}(t+1)$ is the velocity of particle ith at iteration jth, $x_{ij}(t+1)$ is the position of particle ith at iteration jth. w is inertia weight to be employed to control the impact of the previous history of velocity. t denotes the iteration number, c_1 is the cognition learning factor, c_2 is the social learning factor, r_1 and r_2 are random numbers in [0,1]. Generally, the value of each component in V can be clamped to the range $[-V_{\max}, V_{\max}]$ to control excessive roaming of particle outside the search space.

5.2. The idea of improved adaptive PSO algorithm

The PSO algorithm is a stochastic global optimization computation method based on swarm intelligence. It takes on simple modeling, fast convergence, easy implementation, and so on. However, the PSO algorithm is affected by the individual optimal positions and global optimal position of particles in the optimization process. It has the premature convergence, slow convergence, low search precision, and local optimality, and so on [47–51]. The fractional calculus theory is the differential operator and integral operator theory based on the differential order and integral order. Alpha-stable distribution theory is a kind of important probability distribution [52]. Alpha-stable distribution theory and dynamic fractional calculus theory are introduced into adaptive PSO algorithm, then an improved adaptive PSO(DOADAPO) algorithm based on making full use of the advantages of Alpha-stable distribution and dynamic fractional calculus theory is proposed. The proposed DOADAPO algorithm uses the fractional calculus theory with the memory characteristics to update the particle by adding the trajectory information in the adaptive PSO algorithm in order to improve the convergence speed of the PSO algorithm. And the dynamic inertia weight is favorable to the population search. The order of the dynamic fractional order is also helpful to improve the population search ability. At the same time, Alpha-stable distribution theory is used to replace the uniform distribution in the adaptive PSO algorithm in order to generate random numbers, which can make the particle to take on the ability to escape from the local minimum, and improve the global search ability.

5.3. The strategies of improved adaptive PSO algorithm

5.3.1. Uniformly initialize particles

The initialization method of particles in the PSO algorithm will affect the performance of algorithm. Before one problem is solved, the optimal solution position for this problem is not known. If the initial population is randomly generated, the individual is not representative. If the search space dimension of optimization problem is higher, the initial positions of obtained particles by using random initialization are easily confined to a small space, which makes it is difficult for the PSO algorithm to jump out the local optimum and achieve global optimization. If the particles can be uniformly initialized in the feasible area of search space, the PSO algorithm can search on the whole feasible space, so that the population can search the optimal solution with higher probability. For the initialization problem of particles, one new method is proposed to uniformly initialize the initial positions of particles. This method divides the maximum and minimum of each dimension into three segments according to the optimization problem. The midpoint of each segment is taken as the third initial point.

5.3.2. Generate random function of Alpha-stable distribution

Alpha-table distribution is used to replace the random function in the Eq. (5). Firstly, two independent random variables V and W are generated. V is an uniformly distributed variable on the $[-\pi/2, \pi/2]$. W is an exponentially distributed variable with mean value. According to the relationship of parameters between two parameters, the generating method of random variables for abiding $S(\alpha, \beta, 1, 0)$ distribution is obtained under the standard parameters.

When there is $\alpha \neq 1$, the definition is obtained.

$$M_{\alpha,\beta} = [1 + \beta^2 \tan^2 \frac{\pi\alpha}{2}]^{\frac{1}{2\alpha}} \tag{16}$$

The expression is used to represent the transformation relationship between $\sigma^2$and σ.

Define:

$$N_{\alpha,\beta} = -\frac{\arctan(\beta \tan \frac{\pi\alpha}{2})}{\alpha} \tag{17}$$

The expression is used to represent the transformation relationship between β^2 and β in order to replace S^2 for generating V_0.

The following expression can be obtained:

$$X = M_{\alpha,\beta} \frac{\sin(\alpha(V - N_{\alpha,\beta}))}{(\cos(V))^{\frac{1}{\alpha}}} \left(\frac{\cos(V - \alpha\left(V - N_{\alpha,\beta}\right))}{W}\right)^{\frac{1-\alpha}{\alpha}} \tag{18}$$

When there is $\alpha = 1$, $M = \pi/2$, $\beta^2 = \beta$

The following expression can be obtained:

$$X = M\left[(\frac{\pi}{2} + \beta V) \tan V - \beta \log \left[\frac{W \cos V}{\frac{\pi}{2} + \beta V} \right] \right] \tag{19}$$

$$= \frac{\pi}{2}\left[(\frac{\pi}{2} + \beta V) \tan V - \beta \log \left[\frac{W \cos V}{\frac{\pi}{2} + \beta V} \right] \right] \tag{20}$$

The generated X is the random variable for abiding $S(\alpha, \beta, 1, 0)$ distribution under standard parameters. The random variable of Alpha-stable distribution has two following characters:

Character one. If there is $X \sim S(\alpha, \beta, \sigma, \mu)$, a_0 is a nonzero real constant, then

$$a_0 X \sim \begin{cases} S(\alpha, sign(\alpha_0)\beta, |a_0|\sigma, \alpha_0\mu) & \alpha \neq 1 \\ S(\alpha, sign(\alpha_0)\beta, |\alpha_0|\sigma, \alpha_0\mu - \dfrac{2}{\pi}\alpha_0(\log|\alpha_0|)\sigma\beta) & \alpha = 1 \end{cases} \tag{21}$$

Character two. If there is $X \sim S(\alpha, \beta, \sigma, \mu)$, a_0 is a real constant, then

$$X + \alpha_0 \sim S(\alpha, \beta, \sigma, \mu + \alpha_0) \tag{22}$$

These ccharacters are used in here. If there is $X \sim S(\alpha, \beta, 1, 0)$, then

$$Y = \begin{cases} \sigma X + \mu & \alpha \neq 1 \\ \sigma X + \dfrac{2}{\pi}\beta\sigma \ \log \ \sigma + \mu & \alpha = 1 \end{cases} \tag{23}$$

Random variable Y meets $Y \sim S(\alpha, \beta, \sigma, \mu)$. It is possible to derive the method of generating a random variable under the standard parameter system that has a combination of arbitrary parameter values within the prescribed range of four parameters. In this paper, Levy distribution with $\alpha = 1.5$ is used, this distribution is between Gaussian distribution $\alpha = 2.0$ and Cauchy distribution $\alpha = 1.0$. This distribution differs to the [0,1] distribution in standard algorithm. The random function based on this distribution can obtain larger value under the certain probability condition, which allows the particles to have opportunity to escape from the local minimum, and expand the search scope.

Table 1
Detailed information of gates in hub airport.

Gates	Type	Distance to exit(m)	Distance to gate(m)	Initial available time	Service end time
Gate 1	Large	560	235	7:00	24:00
Gate 2	Large	400	250	7:00	24:00
Gate 3	Large	500	700	7:00	24:00
Gate 4	Large	476	1042	7:00	24:00
Gate 5	Large	128	894	7:00	24:00
Gate 6	Small	858	567	7:00	22:00
Gate 7	Large	1083	239	7:00	24:00
Gate 8	Medium	990	245	7:00	22:00
Gate 9	Medium	133	285	7:00	23:00
Gate 10	Small	297	266	7:00	22:00
Gate 11	Medium	238	382	7:00	24:00
Gate 12	Medium	641	967	7:00	22:00
Gate 13	Medium	212	247	7:00	22:00
Gate 14	Small	801	299	7:00	22:00
Gate 15	Small	442	999	7:00	22:00
Gate 16	Medium	488	279	7:00	23:00
Gate 17	Large	153	729	7:00	24:00

Table 2
Flight schedules of one hub airport in one typical day.

C/S	TYPE	PN	ATD	ETD
F01	Medium	140	09:00	09:50
F02	Small	138	08:30	09:10
F03	Large	340	09:00	09:50
F04	Large	293	08:30	10:20
F05	Large	252	08:15	09:00
F06	Small	105	07:30	08:50
F07	Small	130	10:15	11:20
F08	Medium	166	10:00	10:50
F09	Medium	191	10:25	11:55
F10	Medium	178	10:40	11:35
F11	Large	293	11:00	12:35
F12	Medium	166	11:00	12:25
F13	Large	255	11:15	12:15
F14	Medium	183	11:30	12:20
F15	Medium	176	11:30	12:40
F16	Large	220	12:00	13:55
F17	Medium	173	11:45	13:40
F18	Medium	145	11:50	12:45
F19	Small	118	11:55	12:55
F20	Medium	153	12:10	14:00
F21	Large	256	12:45	14:30
F22	Small	104	12:05	13:00
F23	Large	269	12:10	13:30
F24	Medium	140	12:20	13:20
F25	Medium	142	12:20	13:40
F26	Small	129	12:20	13:30
F27	Medium	190	12:30	14:10
F28	Medium	142	12:25	14:10
F29	Medium	163	12:40	14:40
F30	Medium	185	12:55	13:40
F31	Large	258	12:50	14:30
F32	Small	133	12:00	12:55
F33	Medium	156	13:00	14:25
F34	Small	120	13:10	14:45
F35	Medium	193	14:25	16:10
F36	Large	300	14:30	15:20
F37	Small	104	14:00	15:20
F38	Large	219	14:55	16:00
F39	Large	239	15:00	15:50
F40	Small	111	19:55	21:25
F41	Medium	166	18:30	19:25
F42	Large	242	19:30	20:50
F43	Medium	167	18:55	19:55
F44	Large	278	21:00	22:55
F45	Medium	195	20:10	22:00
F46	Large	255	20:00	21:15
F47	Large	219	22:30	23:25
F48	Large	254	21:25	23:15
F49	Large	258	21:30	22:45
F50	Large	276	22:15	23:10

Table 3
Schedules of gate assignment.

Gates	Assignment results	Count
Gate 1	F16 F49	2
Gate 2	F13 F21 F39 F47	4
Gate 3	F05 F25	2
Gate 4	F18 F33 F50	3
Gate 5	F03 F11 F29 F38 F42 F44	6
Gate 6	F22	1
Gate 7	F23	1
Gate 8	F15 F30	2
Gate 9	F01 F08 F12 F27 F35 F41 F45	7
Gate 10	F07 F19 F34 F40	4
Gate 11	F09 F24	2
Gate 12	F10 F17	2
Gate 13	F14 F28	2
Gate 14	F32	1
Gate 15	F06 F26 F37	3
Gate 16	F02 F20 F43	3
Gate 17	F04 F23 F36 F46 F48	5

5.3.3. *Speed calculation based on dynamic fractional order*

Fractional differential is used to update the velocity of particle. The inertia weight is set $w = 1.0$. The Eq. (14) can be rewritten as following expression.

$$v_{ij}(t+1) - v_{i,j}(t) = c_1 stbr\left(pB_{ij}(t) - x_{ij}(t)\right) + c_2 stbr\left(gB_{ij}(t) - x_{ij}(t)\right) \tag{24}$$

The $v_{ij}(t+1) - v_{i,j}(t)$ is first-order difference. Because the flight of particles in the PSO algorithm is discrete-time, its minimum interval is $T = 1$. The difference expression is given.

$$\alpha D_\alpha^t = c_1 stbr\left(pB_{ij}(t) - x_{ij}(t)\right) + c_2 stbr\left(gB_{ij}(t) - x_{ij}(t)\right) \tag{25}$$

According to the recurrence formula of gamma function, the approximate expression of front five items in the difference expression is described.

$$\begin{aligned} v_{ij}(t+1) &= \alpha v_{ij}(t) + \frac{1}{2}\alpha v_{ij}(t-1) + \frac{1}{6}\alpha(1-\alpha)v_{ij}(t-2) \\ &+ \frac{1}{24}\alpha(1-\alpha)(2-\alpha)v_{ij}(t-3) \\ &+ \frac{1}{120}\alpha(1-\alpha)(2-\alpha)(3-\alpha)v_{ij}(t-4) \\ &+ c_1 stbr\left(pB_{ij}(t) - x_{ij}(t)\right) + c_2 stbr\left(gB_{ij}(t) - x_{ij}(t)\right) \end{aligned} \tag{26}$$

In order to make the particles to extend the search space, the order α can be dynamically adjusted according to the state of particle and trajectory information of optimal particle. The initial value of order α is set as 0.5, the upper and lower bounds are [0.4,0.8], the order α is adjusted by each interval 20 iterations. The adjustment process is described.

(1) Calculate the sum of distances between each particle and other particle.

$$d_i = \frac{1}{M-1}\sum_{j=1,j+i}^{M}\sqrt{\sum_{k=1}^{N}(x_{i,k}-x_{j,k})^2} \tag{27}$$

(2) Calculate the sum of distances d_{gbest} between the optimal particle and all particles.

(3) Determine β value according to the particle trajectory information. If the optimal position of particle within 20 steps don't change, the β value is added 0.1. Otherwise β value is 0.

(4) Calculate the order of fractional differential.

$$\alpha = \frac{0.55}{1+1.5e^{-2.6\times\left|\frac{d_{gbest}-d_{\min}}{d_{\max}-d_{\min}}\right|}} \tag{28}$$

(5) Adjust the order according to the upper and lower bounds.

5.4. The steps of DOADAPO algorithm

The steps of DOADAPO algorithm based on Alpha-stable distribution and dynamic fractional calculus are described as follow.

Step 1. Initialize the DOADAPO algorithm

Set the size of particle(M), the maximum number of iterations, the upper and lower bounds of search space, learning factor c_1 and c_2. Randomly generate M particles and their initial velocity. The best position of individual is regarded as the current position of particle and the global optimum is best individual.

Step 2. Evaluate the particle

Calculate the fitness value of each particle. If this value is better than that of current particle, this value is regarded as new best value of individual. Then the global optimum value is found according to the individual optimum value of each particle.

Step 3. Update speed

Generate a random number by using Alpha-stable distribution theory, and update the velocity of particle by using dynamic fractional calculus.

Step 4. Update particles

The current position of particle is updated according to Formula (15). If the position of particle exceeds the search space, then it is set as the boundary value of search space.

Step 5. Determine the end condition

Determine whether the end conditions are met. If the end condition is met, the global optimum value and position of particle are recorded. Otherwise, turn to Step 2.

6. Solve the optimization model of gate assignment

The gate assignment is a NP problem, which has complex constraints and larger scale, so it is difficult to find the accurate optimal solution by using traditional optimization methods. And the general intelligent optimization algorithms are also difficult to find the optimal solution to meet the actual requirements. Therefore, the proposed DOADAPO algorithm with stronger optimization performance is used to solve the optimization model of gate assignment in hub airport in order to obtain the optimization assignment result of gates. The solving steps are described as follows:

Step 1. Initialize parameters

According to the matching flights and information of gates, the parameters of the DOADAPO algorithm are initialized. These parameters include the particle scale, maximum number of iteration, the current number of iteration, the number of random initial solutions and so on.

Step 2. Select the fitness function

The fitness function is used to evaluate the quality of particles in the DOADAPO algorithm. The fitness value represents the quantification index of particle, and the more suitable individual is closer to the optimal solution. In the gate assignment problem, due to the security time constraint process, the penalty strategy is used to change the fitness value and solve the minimum value problem. Therefore, the constraint condition (9) is regarded as a penalty item to be added to the original objective in order to the fitness function, which is taken as the objective function:

$$\begin{aligned} F = {} & w_1\left(\min\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{p}q_{ij}f_k y_{ik}\right) + w_2\left(\min\sum_{i=1}^{n}\sum_{k=1}^{p}S_{ij}^2\right) \\ & + w_3\left(\min\sum_{i=1}^{n}G_i\right) + w_4\left(\min\sum_{i=1}^{n}\sum_{k=1}^{p}w_{ik}\right) + \\ & w_5\left(\mathrm{M}\sum_{i=1}^{n}\sum_{k=1}^{p}\left[\min\left(0, A_{i,j}-\mathrm{ST}-L_{i,j}\geq 0\right)\right]^2\right) \end{aligned} \tag{29}$$

where M is a sufficiently large positive number, $w_i \geq 0$ is weight coefficient according to the weighted method. When the gate assignment conflicts, the obtained fitness function value will be very large, which makes the probability of conflict particle not to be selected by the next generation in order to achieve the purpose of punishment. In the matching constraint processing between the aircraft types and the gates, the constraint method of search space is used.

Step 3. Normalization processing

In the multi-objective optimization model of gate assignment problem, because each objective function has its own dimension, their units and orders of magnitude will be different to bring great influence on solving this optimization model. Therefore, it is not only necessary to set the weight coefficients by using the weighted method for obtaining the final objective function, but also normalize the obtained objective function. Normalization method is a method, which can simplify the calculation. It will transform the dimensional expression into the dimensionless expression to form scalar quantity. This method can simplify the calculation, reduce the quantity value and is often used in many kinds of computation. The following method is used to normalize the objective function.

$$F_i^{trans} = \frac{F_i - F_i^{\min}}{F_i^{\max} - F_i^{\min}} \tag{30}$$

where F_i^{trans} is the ith normalized objective function. F_i is the ith original objective function value. $F_i^{\min}$ and $F_i^{\max}$are the minimum and maximum values of the ith objective function, respectively.

In here, the obtained objective function is normalized in order to obtain the final normalized objective function.

$$F = w_1F_1^{trans} + w_2F_2^{trans} + w_3F_3^{trans} + w_4F_4^{trans} + w_5F_5^{trans} \tag{31}$$

Step 4. According to the current position of particle, the obtained objective function is used to calculate the current fitness value of particle l_0, which is set as the current individual extreme value *PlBest* and the current particle position *PxBest*. The current global extreme value *GlBest* and the current particle position *GxBest* are found by using the current individual extreme values.

Step 5. Calculate the position $X_0(j)$ of the jth particle, and execute the crossover operation with *GxBest* and *PlBest*(j). Then the mutation operation is executed in order to obtain new particle position $X_1(j)$.

Step 6. Calculate the fitness value $l_1(j)$ of current position of particle. If there is $l_1(j) < PlBest(j)$, then the $l_1(j)$ is set as new optimal value of individual, that is, $PlBest(j) = l_1(j)$, $PxBest(j) = X_1(j)$. Otherwise the individual extreme value does not change.

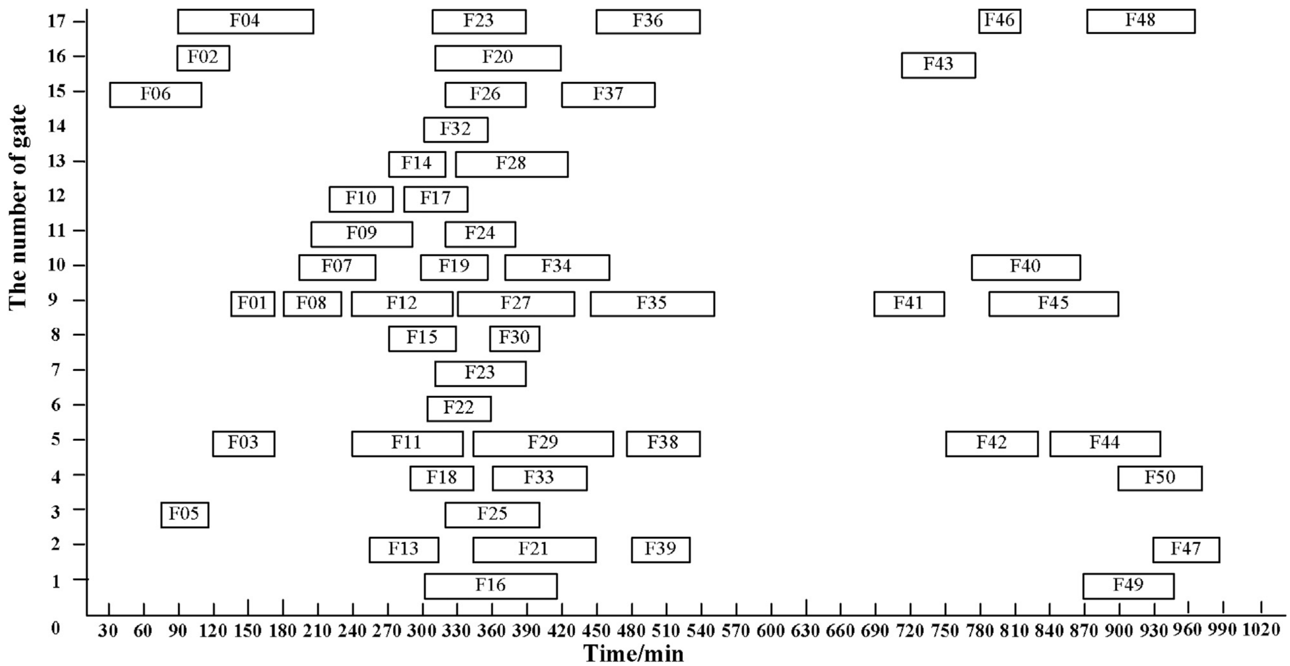

Fig. 1. Gantt chart of airport gate assignment.

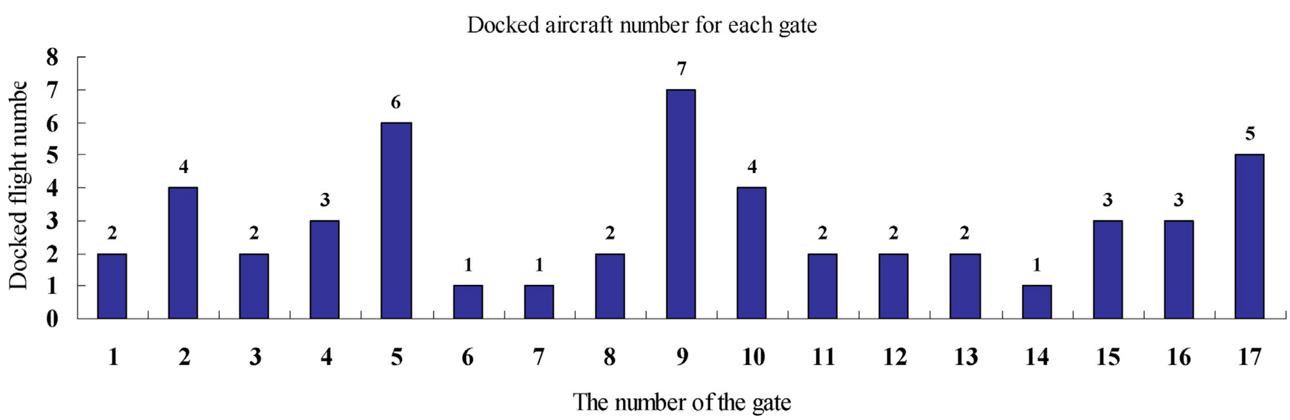

Fig. 2. The number of assigned flights for each gate.

Step 7. According to the individual extreme *PlBest* of particle, the global extreme value *GlBest* and global extreme position *GxBest* are obtained.

Step 8. Alpha-stable distribution theory is used to generate random number, and the speed calculation method based on dynamic fractional calculus is used to update the velocity of the particle.

Step 9. According to the Formula (15), the current position of the particle is updated. If the position of the particle exceeds the search space, then it is set as the boundary value of search space.

Step 10. Determine whether the end conditions are met. If the end condition is not met, turn to Step 2. Otherwise output the global extreme value *GlBest* and global extreme position *GxBest*, which is the optimal gate assignment result.

7. Data simulation and analysis

7.1. Test environment and experiment data

The proposed DOADAPO algorithm is applied to solve the gate assignment problem in an airport. According to the typical flight schedule for one day in the airport, 17 available gates for 50 flights between 7:00 and 24:00 are used to test and simulate in this paper. 17 gates are numbered Gate 1–Gate17. 7 large gates are Gate1–Gate 5, Gate 7 and Gate 17, 6 medium gates are Gate 8, Gate 9, Gate 11–Gate 13 and Gate 16, 4 small gates are Gate 6, Gate 10, Gate 14 and Gate 15. The large gates can spark all flights, the medium gates can spark medium flights and small flights, and the small gates can only spark small flights. The detailed information of these gates is shown in Table 1. 50 flights need to be assigned to 17 gates in the limited time. The information of flights is described in detail in Table 2. The described information includes flight number(C/S), the type of aircraft(TYPE), the number of passengers(PN), the estimated departure time(ETD) and the estimated arrival time(ATD). 50 flights include 19 large flights, 21 medium flights and 10 small flights. The safety time is 10 min between two adjacent flights on the same gate.

The parameters of the proposed DOADAPO algorithm are set as follow. The number of particles is $M = 100$, the inertia weight is $w = 1$, the order of the fractional differential dynamically changes between 0.4 and 0.9, the learning factor $c_1 = c_2 = 2.0$, the maximum number of iterations is $T_{\max} = 800$. If the global optimal value changes less than 1e-20 at each interval 100 iterations in the search,

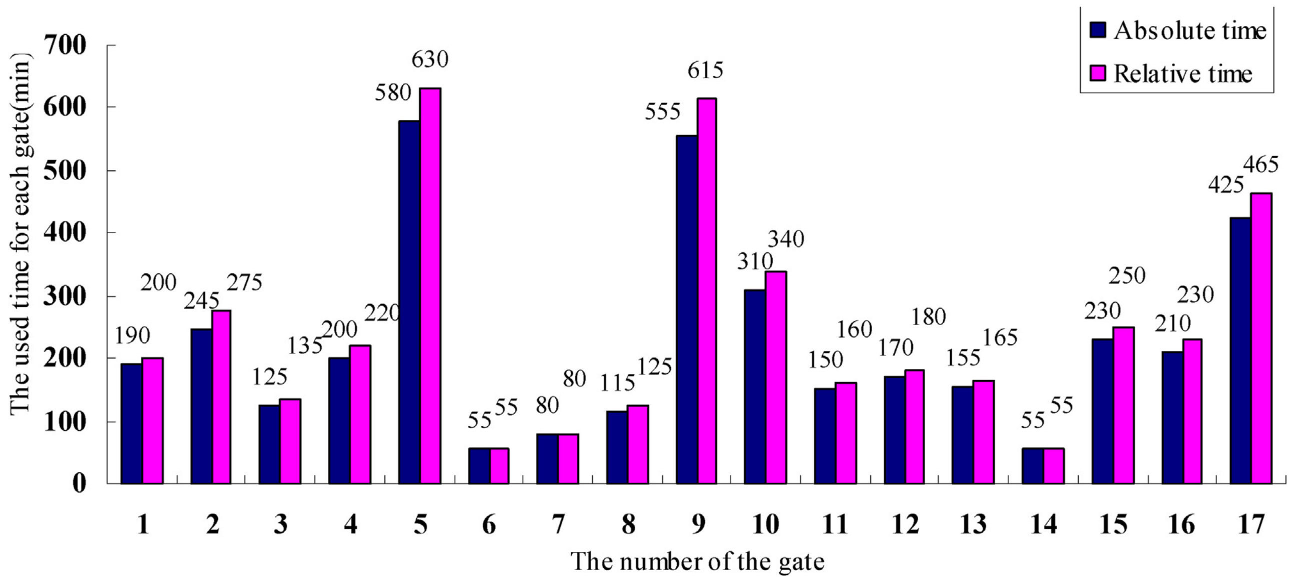

Fig. 3. The absolute utilization time and relative utilization time of each gate.

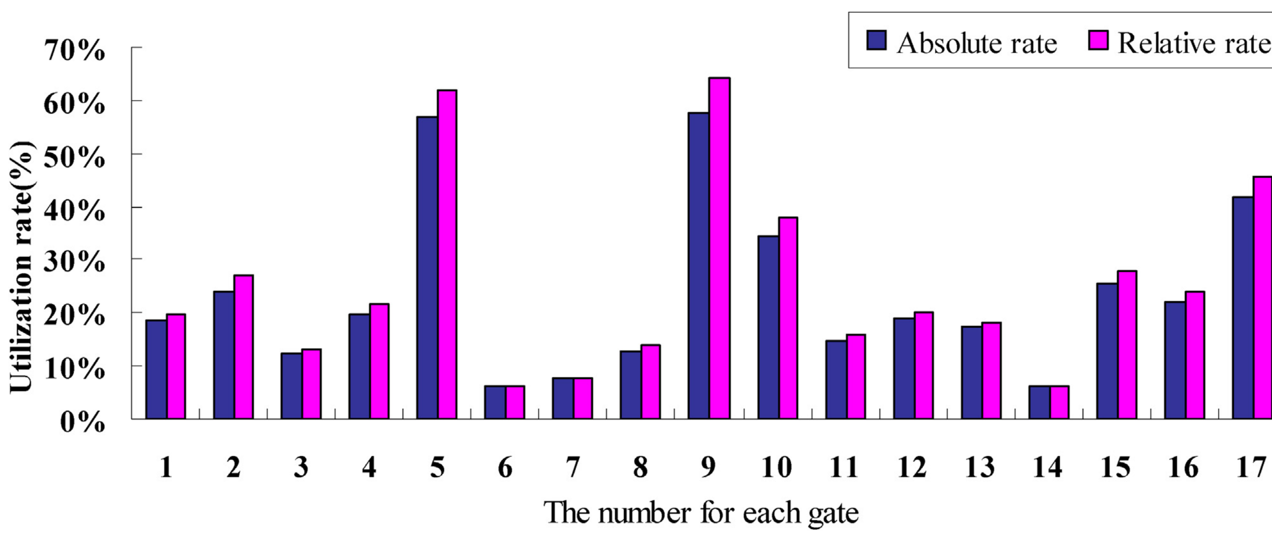

Fig. 4. The absolute utilization rate and relative utilization rate of each gate.

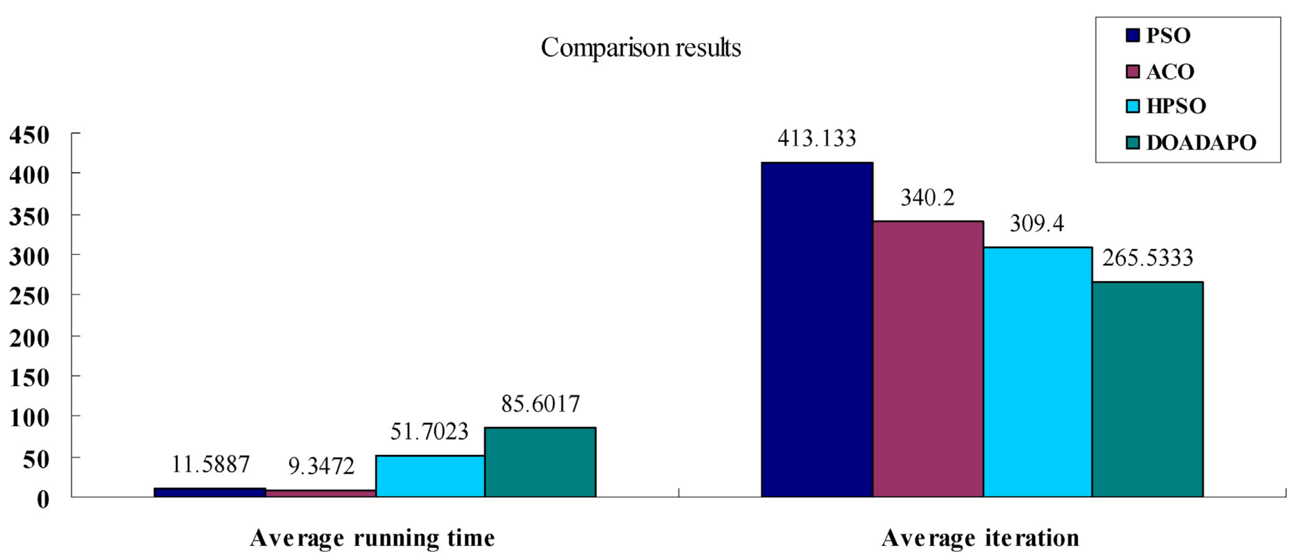

Fig. 5. The comparison results of average running time and average iteration by using four algorithms.

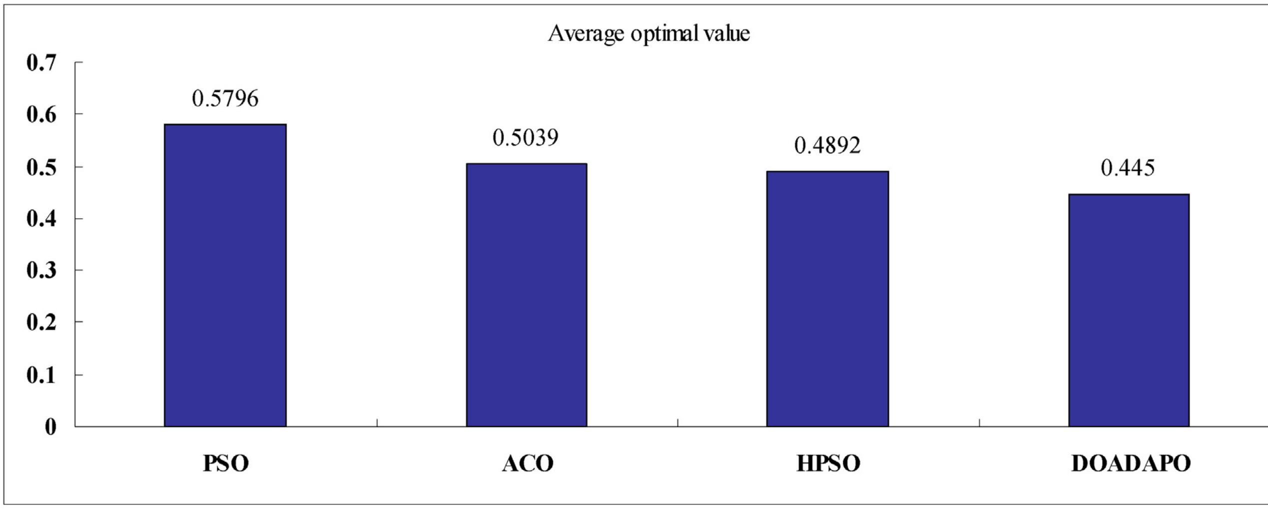

Fig. 6. The comparison results of the optimal solution value by using four algorithms.

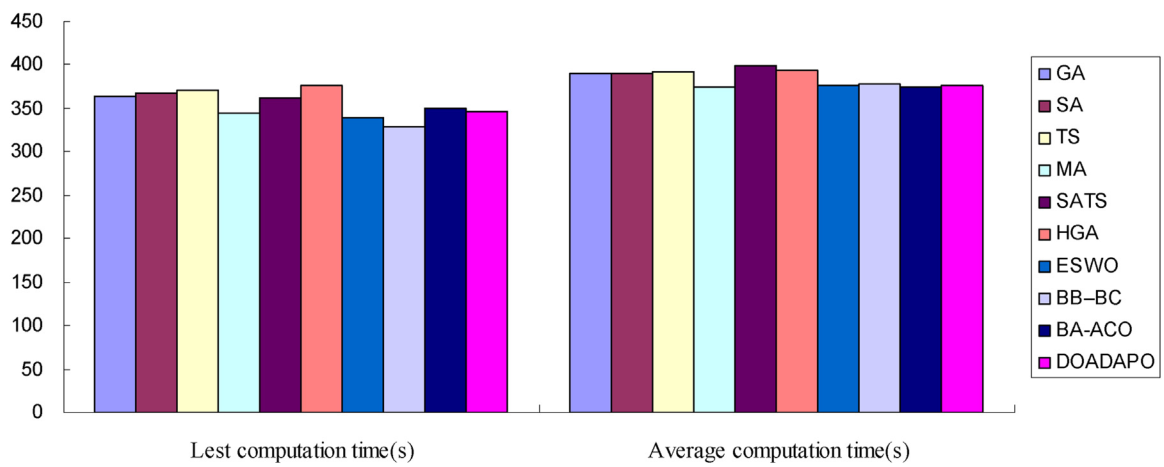

Fig. 7. The comparison results of computation time by using ten algorithms.

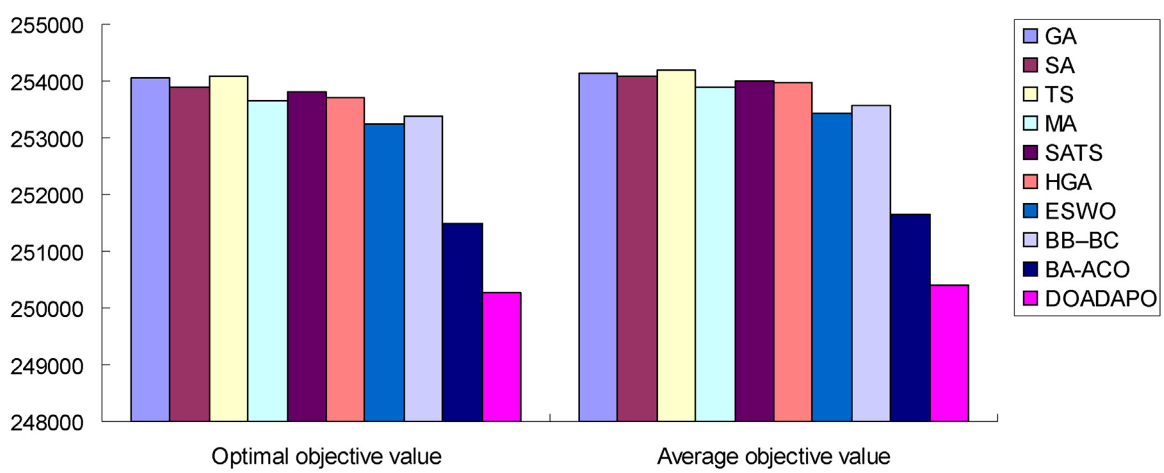

Fig. 8. The comparison results of objective values by using ten algorithms.

this search is stopped. According to the measurement of the actual operation condition and the importance degree of each objective, a set of appropriate weight values are determined based on analyzing the experiments with different data combination. Finally, the determined weight values are $w_1 = 0.40$, $w_2 = 0.25$, $w_3 = 0.15$, $w_4 = 0.15$ and $w_5 = 0.05$.

7.2. Experimental results and analysis of the gate assignment

The proposed DOADAPO algorithm is independently run 15 times in solving the gate assignment problem. An optimal gate assignment scheme is selected to analyze the effectiveness of the DOADAPO algorithm and optimization model in here. The opti-

Table 4
The calculation and comparison results.

Time	PSO algorithm			ACO algorithm			HPSO algorithm			DOADAPO algorithm		
	Run time(s)	Iterations	Optimal value	Run time(s)	Iterations	Optimal value	Run time(s)	Iterations	Optimal value	Run time(s)	Iterations	Optimal value
1	11.6572	462	0.5918	8.3054	375	0.4903	50.1354	344	0.4861	79.9515	318	0.4432
2	10.3481	389	0.5744	9.1328	352	0.5132	51.3645	316	0.4902	78.4416	431	0.4441
3	11.5733	417	0.5805	8.9535	386	0.5104	52.7541	289	0.4845	86.8880	347	0.4449
4	12.7691	426	0.5697	8.1402	297	0.5035	53.0516	234	0.4802	77.8503	387	0.4419
5	11.8402	386	0.5781	10.0453	362	0.5167	52.0578	189	0.4931	95.4725	175	0.4449
6	11.6429	467	0.5638	9.9451	341	0.4935	51.7602	451	0.4873	78.1423	115	0.4434
7	11.4379	385	0.5904	8.3023	305	0.4817	51.9649	238	0.4906	89.0283	160	0.4452
8	10.8095	421	0.5883	8.9426	407	0.4979	50.8570	346	0.4894	87.9840	324	0.4429
9	10.6843	379	0.5896	9.4504	279	0.5032	52.8405	331	0.4910	87.7115	82	0.4391
10	11.4854	427	0.5963	10.5832	349	0.5205	51.0561	365	0.4807	89.5230	423	0.4478
11	12.0460	419	0.5768	9.5370	356	0.4947	52.1642	278	0.4892	87.4770	53	0.4485
12	12.4631	463	0.5872	9.4932	318	0.5016	52.1971	253	0.4918	89.1883	178	0.4469
13	11.6489	417	0.5769	10.9417	295	0.4983	50.8468	341	0.4975	88.0790	217	0.4451
14	11.3683	343	0.5386	9.3041	345	0.5346	51.7345	302	0.4964	79.3742	410	0.4512
15	12.0565	396	0.5916	9.1305	336	0.4980	50.7492	364	0.4895	88.9139	363	0.4461
Avg. value	11.5887	413.133	0.5796	9.3472	340.2	0.5039	51.7023	309.4	0.4892	85.6017	265.5333	0.4450

Table 5
The calculation and comparison results.

Algorithm	Computation time(s)		Objective value	
	Lest computation time(s)	Average computation time(s)	Optimal objective value	Average objective value
GA	363.045	390.242	254,046	254,127
SA	366.790	389.851	253,901	254,083
TS	370.941	391.492	254,083	254,179
MA	345.304	374.908	253,657	253,904
SATS	362.591	398.174	253,804	254,003
HGA	376.735	393.750	253,690	253,964
ESWO	338.932	375.433	253,235	253,438
BB-BC	329.507	378.627	253,368	253,581
BA-ACO	350.271	375.106	251,492	251,641
DOADAPO	346.957	376.539	250,273	250,396

mal objective value is 0.4391. The assignment result of 50 flights is shown in Table 3. The Gantt chart of the optimal gate assignment result is shown in Fig. 1. Each section of the strip is parking gate time in Fig. 1.

As can be seen from Table 3, Figs. 1 and 2, for 17 gates and 50 flights at 7:00–24:00, the gate assignment result does not appear an idle gate. A minimum interval time T = 10 min between two adjacent flights on the same gate is set in order to avoid the conflict. For the same type of gates, if the gates are closer to the check-in, the more flights are assigned to this type of gates, such as Gate 2, Gate 5, Gate 9,Gate 10 and Gate 17. Gate 2 sparks 4 flights, Gate 5 sparks 6 flights, Gate 9 sparks 7 flights, Gate 10 sparks 4 flights and Gate 17 sparks 5 flights. Gate 6, Gate 7 and Gate 14 are assigned the least flights, these gates only spark 1 flight. The reason is that Gate 6, Gate 7 and Gate 14 are farther to the check-in in all gates and the passengers need to walk more distances to arrive at these gates. In general, the near gates from the check-in have higher utilization rate. But the excessive use of these gates will damage equipment and easily cause equipment fault. Therefore, the balanced utilization gate problem need to be considered in the optimization model of gate assignment. As is known to all from Figs. 1 and 2, the established multi-objective optimization model of gate assignment based on the minimum walking distances of passengers, the minimum idle time variance of each gate, the minimum number of flights at parking apron and the most reasonable utilization of large gates can improve the utilization efficiency and balance rate of gates and satisfactory degree of passengers. The multi-objective optimization model of gate assignment takes on a certain ability to deal with the dynamic changes of flights due to the delay of flights. And the proposed DOADAPO algorithm can fast and effectively solve the multi-objective optimization model of gate assignment problem. It takes on better optimization performance in solving complex optimization problem.

In order to further analyze the effectiveness of the DOADAPO algorithm in solving multi-objective optimization model of gate assignment, the indexes of the absolute utilization time and relative utilization time of each gate are used to evaluate the outcomes of the DOADAPO algorithm and multi-objective optimization model of gate assignment. So the obtained results of the absolute utilization time and rate, relative utilization time and rate of each gate are shown in Figs. 3 and 4.

As can be seen from Figs. 3 and 4, the relative utilization times are between 55 min and 630 min for all gates, and the absolute utilization times are between 55 min and 580 min for all gates. The relative utilization rates are between 6.11% and 64.06% for all gates, and the absolute utilization rates are between 6.11% and 56.86% for all gates. The absolute utilization time and the relative utilization time of Gate 5 are 580 min and 630 min, respectively. This gate has the longest utilization time in 17 gates. The absolute utilization rate and the relative utilization rate of Gate 9 are 57.81% and 64.06%, respectively. This gate has the highest utilization rate in 17 gates. The reason is that Gate 9 is nearest to the check-in and the passengers need less time to arrive this gate in all gates. The absolute utilization time and relative utilization time of Gate 6 and Gate 14 are 55 min. This gate is the shortest utilization time in 17 gates. The absolute utilization rate and the relative utilization rate of Gate 6 and Gate 14 are 6.11%. This two gates have the lowest utilization rate in 17 gates. The reason is that Gate 6 and Gate 14 are farther from the check-in and the passengers need to walk longer distances to arrive at this two gates. In general, the gates nearby the check-in are given more priority to assigning the flights and have longer utilization time and higher utilization rate. And each gate obtains the balanced utilization in order to ensure the smooth operation and reduce the security risks caused by human factors. The large gates can be reasonably utilized by the constructed multi-objective optimization model of gate assignment.

7.3. Comparison and analysis of experimental results

In order to further demonstrate the optimization performance of the proposed DOADAPO algorithm, the basic PSO algorithm, ant colony optimization(ACO) algorithm, hybrid particle swarm optimization algorithm with a new particle diversity controller policies and dissipation operation(HPSO) [53], and the proposed DOADAPO algorithm are selected to solve the constructed multi-objective optimization model of gate assignment. The inertia weight of the PSO and HPSO algorithm linearly decreases between 0.95 and 0.40. The parameters of ACO algorithm are set as follow: population size is 50, the control parameters is $\alpha = \beta = 2.0$, pheromone amount is 60 and the evaporation rate is 0.05. The other parameters are set the same with the DOADAPO algorithm. The experiments were independently carried out for 15 consecutive simulation to solve the constructed multi-objective optimization model of gate assignment. The calculation and comparison results are shown Table 4, Figs. 5 and 6.

As can be seen from Table 4, Figs. 5 and 6, the basic PSO algorithm is used to solve the constructed multi-objective optimization model of gate assignment, the optimal objective value is 0.5386, which is found at the 343th iteration, the average number of iterations for finding the optimal value is 413.133 and the average optimal objective value is 0.5796. The ACO algorithm is used to solve the constructed multi-objective optimization model of gate assignment, the optimal objective value is 0.4903, which is found at the 375th iteration, the average number of iterations for finding the optimal value is 340.2 and the average optimal objective value is 0.5039. The HPSO algorithm is used to solve the constructed multi-objective optimization model of gate assignment, the optimal objective value is 0.4802, which is found at the 234th iteration, the average number of iterations for finding the optimal value is 309.4 and the average optimal objective value is 0.4892. The DOADAPO algorithm is used to solve the constructed multi-objective optimization model of gate assignment, the optimal objective value is 0.4391, which is found at the 82th iteration, the average number of iterations for finding the optimal value is 265.5333 and the average optimal objective value is 0.4450. Therefore, for the basic PSO algorithm, ACO algorithm, HPSO algorithm and the DOADAPO algorithm in solving the constructed multi-objective optimization model of gate assignment, the optimal objective value, the average number of iterations and the average optimal objective value of the DOADAPO algorithm is better than those of the basic PSO algorithm, ACO algorithm and HPSO algorithm. That's to say, the solution quality is the best by using the DOADAPO algorithm. But from the experiment results, we can see that time complexity of the DOADAPO algorithm is worse than time complexity of the basic PSO algorithm, ACO algorithm and HPSO algorithm.

In general, although the DOADAPO algorithm uses more time to solve the constructed multi-objective optimization model of gate assignment, and the solution quality of the DOADAPO algorithm has been improved by comparing the solution quality of the basic PSO algorithm, ACO algorithm and HPSO algorithm. The DOADAPO algorithm can effectively improve the comprehensive service performance of gate assignment in hub airport. So the constructed optimization model of gate assignment can significantly balance the utilization rate of gates, reduce the walking distances and improve the service level of airport and satisfactory degree of passengers. The proposed DOADAPO algorithm takes on the ability to escape the local minimum value and improve the global search ability of algorithm. The proposed method can effectively improve the flexibility of gate assignment and avoid the occurrence of a large number of flight delays. It can effectively provide a valuable reference for assigning the gates in hub airport.

In order to further validate the effectiveness of the DOADAPO algorithm, the genetic algorithm(GA) [54], simulated annealing(SA) [23], Tabu search(TS) [23], hybrid approach(SATS) [23], memetic algorithm(MA)[55], evolutionary squeaky wheel optimization(ESWO) algorithm and the hill-climbing GA(HGA) [56], Big Bang-Big Crunch (BB-BC) algorithm [22] and improved ACO(BA-ACO) algorithm [57] are select to solve gate assignment problem. The experiment data came from PEK airport in the literature [58]. These data include 40 flights, 10 gates (7 large gates and 3 medium gates) and an extra ungated apron stand in one day operation between 8:00 am and 8:00 pm. The parameters of GA, SA, TS, MA, SATS, HGA, ESWO, BB-BC and BA-ACO algorithms are same values with the corresponding literatures. The comparison results for average value of ten times are shown in Table 5.

As it can be observed from Table 5, Figs. 7 and 8, the proposed DOADAPO algorithm is used to solve the gate assignment problem in the literature[58], the optimal objective value and average objective value are 250273 and 250396 respectively. For the GA, SA, TS, MA, SATS, HGA, ESWO, BB-BC, BA-ACO and DOADAPO algorithms in solving the gate assignment problem, the optimal objective value and average objective value of the DOADAPO algorithm is best than those of the GA, SA, TS, MA, SATS, HGA, ESWO, BB-BC and BA-ACO algorithms. That's to say, the solution quality is the best by using the DOADAPO algorithm. The lest computation time and the average computation time of the DOADAPO algorithm are 346.957 s and 376.539 s respectively. The lest computation time is 329.507 s by using the BB-BC algorithm, and the average computation time is 374.908 s by using MA algorithm. The lest computation time of the DOADAPO algorithm is better than the lest computation time of GA, SA, TS, SATS, HGA and BA-ACO algorithms, and the average computation time of the DOADAPO algorithm is better than the average computation time of MA, ESWO and BA-ACO algorithms. But the lest computation time of the DOADAPO algorithm is worse than the lest computation time of MA, ESWO, BB-BC algorithms. In general, although the DOADAPO algorithm uses more time to solve the gate assignment problem, the solution quality of the DOADAPO algorithm has been improved by comparing with the solution quality of the GA, SA, TS, MA, SATS, HGA, ESWO, BB-BC, BA-ACO algorithms. So the proposed DOADAPO algorithm takes on the better global optimization ability. It can provide an effective solving method for assigning the gates.

8. Conclusion and future work

In this paper, the minimum walking distances of passengers, the minimum idle time variance of each gate, the minimum number of flights at parking apron and the most reasonable utilization of large gates are used to construct a new multi-objective optimization model of gate assignment problem. And Alpha-stable distribution and dynamic fractional calculus are used to improve the adaptive PSO algorithm and then propose an improved adaptive PSO(DOADAPO) algorithm for improving the slow convergence, search precision and local optimality, which is used to solve the optimization model of gate assignment for effective assigning the gates to different flights in different time. The typical flight schedule for one day in the airport is studied in detail. And the basic PSO, ACO and HPSO algorithms are selected to solve the optimization model of gate assignment in order to demonstrate the optimization performance of the proposed DOADAPO algorithm. The GA, SA, TS, MA, SATS, HGA, ESWO, BB-BC and BA-ACO algorithms are used to further validate the effectiveness of the DOADAPO algorithm. The experiment results show that the optimization performance of the

DOADAPO algorithm is better than the optimization performance of the basic PSO, ACO, GA, SA, TS, MA, HPSO, SATS, HGA, ESWO, BB-BC and BA-ACO algorithms. The constructed optimization model of gate assignment can significantly balance the utilization rate of gates, reduce the walking distance and improve the service level of airport and satisfaction degree of passengers. The proposed method can effectively provide a valuable reference for assigning the gates in hub airport.

Due to the uncertainty of flight schedule and airport condition, a lot of delay flights will occur. And the proposed DOADAPO algorithm exists higher time complexity, it needs more time to solve the gate assignment problem. So the time complexity of the proposed DOADAPO algorithm need to further be studied, and the more effective method is proposed to solve the gate re-assigning problem. The time complexity and gate reassignment problem will be deeply studied in the next work to us.

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