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To cite this article: Jingxu Chen, Shuaian Wang, Zhiyuan Liu & Wei Wang (2017) Design of suburban bus route for airport access, Transportmetrica A: Transport Science, 13:6, 568-589, DOI: 10.1080/23249935.2017.1306896

To link to this article: <https://doi.org/10.1080/23249935.2017.1306896>



Accepted author version posted online: 14 Mar 2017.  
Published online: 04 Apr 2017.



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# Design of suburban bus route for airport access

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## ABSTRACT

Suburban bus routes usually serve a city's peripheral areas with sparse density of travel demand. The distribution of many suburban trips is many-to-one, with all the passengers bound for a common destination like an airport. This paper proposes a methodology for the optimal design of a suburban bus route for airport access, with the objective of minimizing the total access time. Its decisions are the selection of pickup locations from candidate stops and the corresponding visiting sequence. We first consider a special model which is polynomially solvable, and then develop a dynamic programming approach to address this special case. Later, we formulate a generic model and prove it to be NP-hard. Apart from the dynamic programming approach, an artificial bee colony (ABC) approach is developed for good-quality solutions of the practical-size problem. Finally, case studies are carried out to validate the applicability of the proposed models and solution methods.

## ARTICLE HISTORY

Received 14 September 2016  
Accepted 12 March 2017

## KEYWORDS

Bus operations; airport access; suburban bus route; dynamic programming approach

## 1. Introduction

As one of the most important components in the transportation system, public transport is considered a pivotal backbone of sustainable urban development. Enhancing public transport service and raising its attractiveness is an effective way to alleviate the problems attributed to the automobile, such as road congestion, energy consumption, and air pollution (Ceder 2007; Currie and Wallis 2008; Yan et al. 2012; Liu et al. 2013; de Oña and de Oña 2015; Orth, Nash, and Weidmann 2015; Chen et al. 2016; Ermagun and Levinson 2017; Fan, Guthrie, and Levinson 2016, among many others). For a bus service, systematic planning is needed before entering into the regular operation, which is composed of five key components: network design, frequency setting, timetabling, vehicle scheduling, and driver scheduling/rostering (Guihaire and Hao 2008; Kepaptsoglou and Karlaftis 2009; Bie, Gong, and Liu 2015; Ibarra-Rojas et al. 2015; Huang et al. 2016).

Bus network design is an elementary planning procedure with the aim of determining the routes, types of vehicles, and stop spacing to meet the demand of travel (Guihaire and Hao 2008). Currently, there are numerous optimization models that have been

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dedicated to the optimal design of bus networks (Lam et al. 1999; Currie and Loader 2010; Roca-Riu, Estrada, and Trapote 2012; Szeto and Jiang 2012, 2014; Yan et al. 2013; Fu and Lam 2014; Amiripour, Mohaymany, and Ceder 2015; Pternea, Kepaptsoglou, and Karlaftis 2015). Most of these previous studies focused on the bus routes located in urban central areas where the majority of bus trips are clustered in these areas. In comparison, the suburban bus routes which are located in the city's outskirts have received relatively less attention.

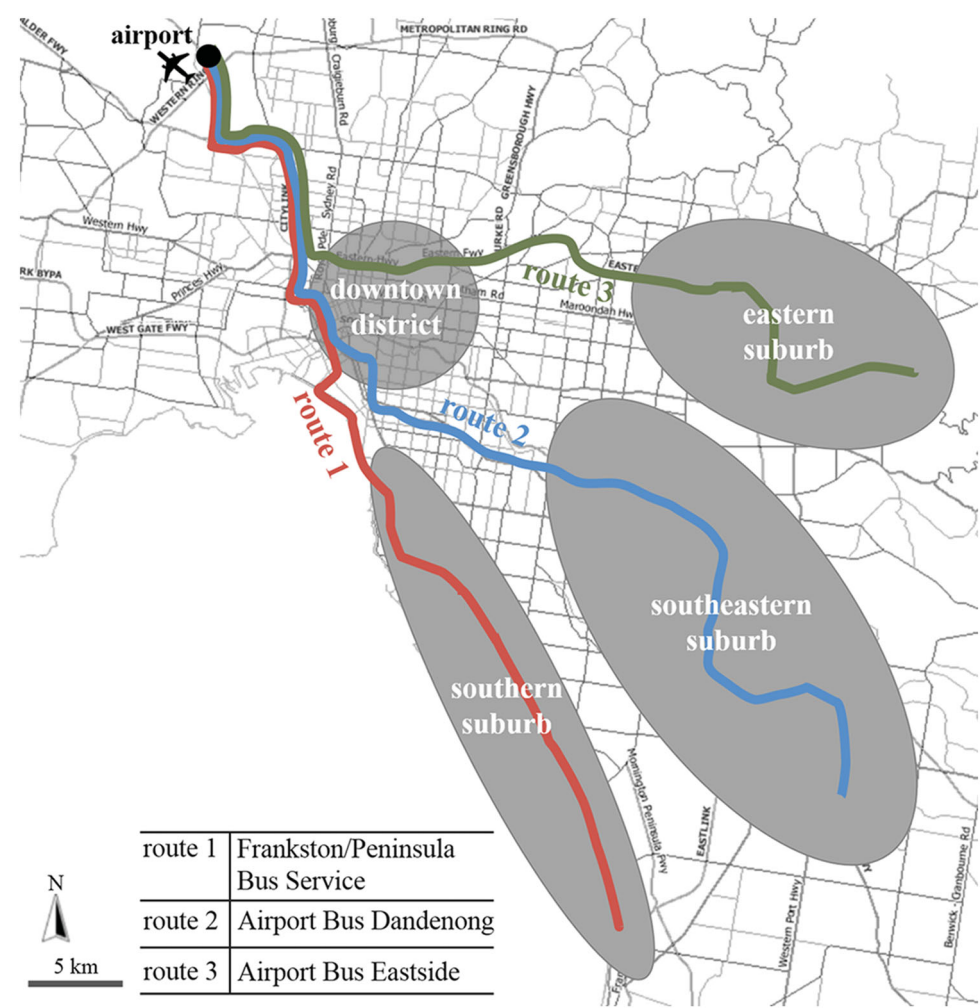
One distinctive feature of suburban bus routes is that they serve peripheral areas with sparse density of travel demand (Bell and Cloke 1991). Therefore, it is not cost-effective to construct a dense bus network in the suburban areas. Instead, suburban bus routes mainly provide a feeder service which connects suburban areas and some demand concentrated places (e.g. airports, transit hubs/stations). Among these places, airports are usually located far away from residential areas due to the aircraft noise. This gives rise to inconvenience for residents living in the suburban residential areas who plan to access the airports. Taking a taxi is quite expensive thus not chosen by many travelers. At the same time, if most suburban residents choose to drive, it will have a detrimental effect on the traffic congestion and also cause a parking problem at the airports. Since these suburban trips present a many-to-one traveling character, the suburban bus route exclusive for airport access and airport shuttle van provide two alternative modes for airline passengers. This study focuses on the former one whose role is to offer an express bus service delivering passengers from multiple origins to a certain airport.

Such suburban bus feeder service is commonly implemented in practice. For instance, Figure 1 presents three typical suburban bus routes access to the airport in Melbourne, Australia. Route 1 mainly serves the Melbourne's eastern suburb, route 2 mainly serves the southeastern suburban area, and route 3 mainly serves the southern suburb of Melbourne. These suburban bus routes have some analogous properties: (i) there is little congestion on suburban roads; (ii) only one bus route is available for airport access in each suburban area, and hence passengers have no other bus route choice; and (iii) the suburban area has a number of candidate pickup locations to collect passengers.

In view of these properties, the design of a suburban bus route for airport access is relatively independent, which differs from the general bus network design in dense urban areas. Regarding the general bus network design, passengers' route choice behaviors have to be considered, due to the frequent transfers and complicated route plans. The general bus network design problem is usually an NP-hard problem, and most of extant studies use heuristic and metaheuristic methods to obtain potentially good solutions (Ibarra-Rojas et al. 2015). In contrast, the many-to-one travel pattern and the three above-mentioned properties make the suburban bus route design problem easier, and hence we expect that exact solution approaches may be proposed to design a practical suburban bus route.

### **1.1. Literature review**

Airports are among the largest activity centers in a city. For decades, transportation planners have endeavored to handle the airport access problem. For airline passengers, public transport is an effective way to provide airport access service that has attracted considerable interest from researchers, especially in recent years.



**Figure 1.** Three typical suburban bus routes access to the Melbourne Airport.

Mandle, Mansel, and Coogan (2000) reviewed the use of public transport modes (e.g. rail, bus, and van service) at US airports. They found that many US cities provide rail transit service between the airport and downtown district (or major activity centers). Nevertheless, most suburban areas do not have direct rail transit service. In this case, bus and shared-ride van services were recommended as a proxy to serve these suburban areas. Coogan (2008) reported a similar result that most major airports in Europe and Asia are served by heavy-rail connections to the city's downtown district, whereas airport access in suburban areas is a bottleneck that greatly affects travel reliability of suburban airline passengers. Orth and Weidmann (2014) conducted a case study of Zurich Airport, which is a pivotal hub in Zurich, Switzerland. They indicated that many offices and shopping centers are located at the airport and these places generate high nonaviation activities (travel to airport to work or shop rather than take a flight). Such nonaviation travel demand is a main reason why Zurich Airport has a high public transport mode share. However, most trips in suburban

areas are aviation-related travel, and hence the Zurich case is not suited to suburban airline passengers.

Apart from the qualitative analysis, a large number of authors have worked on the modeling of mode choice decisions for airport access. Many studies have developed models to identify the dominant factors that influence the airport ground access mode choices, including multinomial logit model (Harvey 1986; Tam, Lam, and Lo 2008; Tsamboulas, Evmorfopoulos, and Moraiti 2012), nested logit model (Hess and Polak 2006; Wen, Wang, and Fu 2012; Hess et al. 2013), mixed logit model (Jou, Hensher, and Hsu 2011), and logistic regression models (Chang 2013; Choo, You, and Lee 2013). All these studies indicated that access time and access cost are most significant explanatory variables in an access mode decision. Therefore, public transport is attractive to more airline passengers only in cases when public transport modes can provide direct and reliable access service with less access time and access cost.

Previous studies have reported many useful findings on how to raise mode shares of public transport users. These findings are mainly applicable to urban central areas where bus, rail, and other access mode choices are available. As multiple access modes are closely related, the design of bus routes in central areas has to consider many external factors, which poses a very complex problem. It is quite different in suburban areas that bus is the primary public transport mode for airport access, and the design of a suburban bus route is relatively independent. In addition, the distribution of suburban trips for airport access is many-to-one, with all airline passengers bound for the airport. Currently, a number of studies have been conducted on many-to-one travel pattern (Kuah and Perl 1988; Spasovic, Boile, and Bladikas 1994; Lam and Huang 1995; Chien and Yang 2000; Daganzo 2005; Sivakumaran et al. 2012; Wang and Qu 2015). These studies can be further divided into two categories: (i) a continuous form, parameters, and variables are approximated as continuous functions, such that stops are expressed per unit length; and (ii) a discrete form, parameters and variables take discrete values, for example, the locations available for stops are expressed by site-specific details.

The many-to-one bus travel access to the airport is a discrete location-routing problem. Before proceeding to the designated airport, a bus vehicle first serves a certain suburban area which is composed of several zones. Each zone has a number of candidate stops, which can be chosen by expert judgment or based on vehicle GPS trace data to identify places with high travel demand (Chen et al. 2014). These candidate stops may be selected as the real pickup locations. For a specific suburban bus route, the design task is to find the optimal bus routing with the least travel time that delivers passengers from selected pickup locations to the airport. The suburban bus feeder service provides benefits for suburban airline passengers, with real implementations in Melbourne (see Figure 1). However, none of previous studies have investigated the optimization of bus routing pertinent to suburban routes for airport access, which is addressed in this paper.

## **1.2. Objectives and contributions**

The main objective and contribution of this study is to propose a methodology for the optimal bus routing of a suburban bus route exclusively for airline passengers. Considering the practical circumstances, the bus routing is not necessary to cover all the candidate stops.

Meanwhile, at least one candidate stop per zone should be selected as a pickup location in ways that walking access distance is reachable for passengers in each zone.

The suburban bus routing problem (SBRP) is a practical research topic which is of particular significance for the airport access. The SBRP modeling framework is proposed for the optimal bus routing through minimizing the total travel time. Its decisions are the selection of pickup locations from the candidate stops and the corresponding visiting sequence. A special model of SBRP is first considered that all the zones are sequentially distributed along the bus route from the original terminal to the airport station. We demonstrate that the special model is polynomially solvable, and then develop a dynamic programming approach to address this special case. Afterwards, a generic model of SBRP is discussed that all the zones of a suburban area are randomly distributed. We prove that the generic model belongs to the NP-hard class. A dynamic programming approach and a metaheuristic approach are proposed to solve the generic model.

The remainder of this paper is organized as follows. Section 2 describes the problem and develops an optimization model. In Section 3, the proposed model is further categorized into a special model and a generic model. Then, the computational complexity of two models and associated solution methods are discussed. Numerical examples are presented in Section 4. Finally, conclusions are provided in the last section.

## 2. Problem statement and model development

In this section, we present an integer programming formulation for the SBRP. The variables used in the model and their notation are summarized in Table 1. Consider a suburban bus route that originates from a bus terminal (e.g. one depot), denoted by node 0, and terminates at the airport station, denoted by node  $N + 1$ . The bus starts from node 0, then travels through the suburban area which contains  $N$  candidate stop pairs, and finally ends at node  $N + 1$ . It should be noted that there are usually two bus stops at a location, one bus stop on each side of a road segment. Then, we use  $1, 1', 2, 2', \dots, N, N'$  to denote the

**Table 1.** List of notation.

<i>Indices</i>	
$i, j$	The index of stops/nodes, $i, j \in \bar{H} = H \cup H' \cup \{0, N + 1\}$
$h, h'$	The index of a stop pair, $h \in H$ and $h' \in H'$ correspond to a stop pair at the same geographical location but on the opposite sides of a street
$c$	The index of zones, $c \in C$
<i>Sets</i>	
$C$	A set of zones within the suburban area
$\bar{H}$	A set of stops, including two terminals and all the intermediate candidate stops
$H, H'$	A set of candidate stops, $H = \{1, 2, \dots, N\}$ and $H' = \{1', 2', \dots, N'\}$
$\underline{H}$	An arbitrary subset of $H$
$G_c$	A set of candidate stops located in zone $c$
<i>Parameters</i>	
$d_{ij}$	Distance between nodes $i$ and $j$
$N$	The number of candidate stops in set $H$ and set $H'$
$v$	Average bus cruising speed
$T$	Average dwell time at each stop
<i>Variables</i>	
$\delta_i$	A binary variable which equals 1 if candidate stop $i \in \bar{H}$ is visited by the suburban bus route, and 0 otherwise
$x_{ij}$	A binary variable which equals 1 if the bus visits stop $j \in \bar{H}$ immediately after stop $i \in H$ , and 0 otherwise

sequence of these candidate stop pairs, where nodes  $h$  and  $h'$  correspond to a stop pair at the identical geographical location but on both sides of a street,  $h = 1, 2, \dots, N$ . Here,  $h$  is denoted as the opposite bus stop of  $h'$ , and vice versa. We define set  $H = \{1, 2, \dots, N\}$ , set  $H' = \{1', 2', \dots, N'\}$ , and set  $\bar{H} = H \cup H' \cup \{0, N+1\}$ .

The travel demand of a bus vehicle is given and the suburban area includes a total of  $|C|$  zones that needs to be visited. Note that zones can be determined based upon the administrative division and some other division criteria. Each zone  $c$  ( $\forall c \in C$ ) subsumes a set of candidate stops, denoted by  $G_c$ . Candidate stops of all these zones constitute the set  $H \cup H'$ . To ensure that any resident is reachable from his/her house to the nearest bus stop, at least one of the candidate stops in each zone must be served by the bus route.

We let  $d_{ij}$  denote the distance between stops  $i$  and  $j$  ( $i, j \in \bar{H}$ ). Distances are assumed to be nonnegative and satisfy the triangle inequality ( $d_{ij} \leq d_{ik} + d_{kj}$ ,  $\forall i, j, k \in \bar{H}$ ). The bus is assumed to have an average cruising speed  $v$  and a common dwell time  $\tau$  at each stop. Parameters  $d_{ij}$ ,  $v$ , and  $\tau$  are already known. We define  $\delta_i$  as the decision variable which equals 1 if node  $i \in \bar{H}$  is visited by the bus, and 0 otherwise, and define  $x_{ij}$  as the decision variable which equals 1 if and only if the bus visits node  $j \in \bar{H}$  immediately after node  $i \in \bar{H}$ , and 0 otherwise.

The SBRP is formulated as follows:

$$\min z = \frac{1}{v} \sum_{i \in \bar{H}} \sum_{j \in \bar{H}} d_{ij} x_{ij} + \tau \sum_{i \in \bar{H} \setminus \{0, N+1\}} \delta_i \quad (1)$$

subject to

$$\sum_{j \in \bar{H} \setminus \{0\}} x_{0j} = 1, \quad (2)$$

$$\sum_{i \in \bar{H} \setminus \{N+1\}} x_{i, N+1} = 1, \quad (3)$$

$$\sum_{j \in \bar{H}} x_{ij} - \sum_{j \in \bar{H}} x_{ji} = 0 \quad \forall i \in H \cup H', \quad (4)$$

$$\sum_{i \in G_c} \delta_i \geq 1 \quad \forall c \in C, \quad (5)$$

$$\delta_0 = \delta_{N+1} = 1, \quad (6)$$

$$\delta_h + \delta_{h'} \leq 1 \quad \forall h \in H, \quad (7)$$

$$\sum_{j \in \bar{H} \setminus \{0\}} x_{ij} = \delta_i \quad \forall i \in \bar{H} \setminus \{N+1\}, \quad (8)$$

$$\sum_{i \in \bar{H}} \sum_{j \in \bar{H}} (x_{ij} + x_{ji}) \leq |\bar{H}| - 1 \quad \forall \bar{H} \subseteq \bar{H}, \quad (9)$$

$$x_{ii} = 0 \quad \forall i \in \bar{H}, \quad (10)$$



$$x_{hh'} = 0, \quad x_{h'h} = 0 \quad \forall h \in H, \quad (11)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in \bar{H}, \quad (12)$$

$$\delta_i \in \{0, 1\} \quad \forall i \in \bar{H}. \quad (13)$$

The objective function is to minimize the total vehicular travel time within a suburb area for a suburban bus. It contains two terms: the first term is the total cruise time and the second term is the total dwell time. Constraints (2) and (3) ensure that only one bus stop connects with the terminal node 0 and node  $N + 1$ , respectively. Constraint (4) ensures that with the exception of two terminal nodes, any visiting node on a service route has one preceding node and one following node. If one candidate stop  $i \in \bar{H}$  is not selected as the visiting node, constraint (4) is also satisfied since all the decision variables with respect to node  $i$  (i.e.  $x_{ij}$  and  $x_{ji}$ ) equal zero. Constraint (5) is the zone coverage constraint, and ensures that at least one of the candidate stops in each zone is served by the bus. Constraint (6) underscores that two bus terminal stops (node 0 and  $N + 1$ ) must be visited. Constraints (7) and (8) ensure that exactly one of node  $h$  and its corresponding opposite node  $h'$  is visited if one of them is selected as a visiting stop. Constraint (9) is the sub-tour elimination constraints, which should be satisfied by any subset  $H$  of  $\bar{H}$ . Constraints (10) and (11) underscore that the bus does not visit a bus stop twice including its opposite stop. Constraints (12) and (13) define  $x_{ij}$  and  $\delta_i$  as binary variables.

**Remark 2.1:** Once the optimal bus routing is designed, bus frequencies need to be determined for temporal variations in travel demand to the airport during the practical uses. We can partition one day into many time periods. As the travel demand in the suburban area is far less than that in the city's central district, suburban buses can be dispatched at relatively low frequencies that vary with multiple time periods. At the same time, bus frequencies should satisfy the minimal service level which ensures that the provided bus capacity is greater than or equal to the total passenger demand in any given time period.

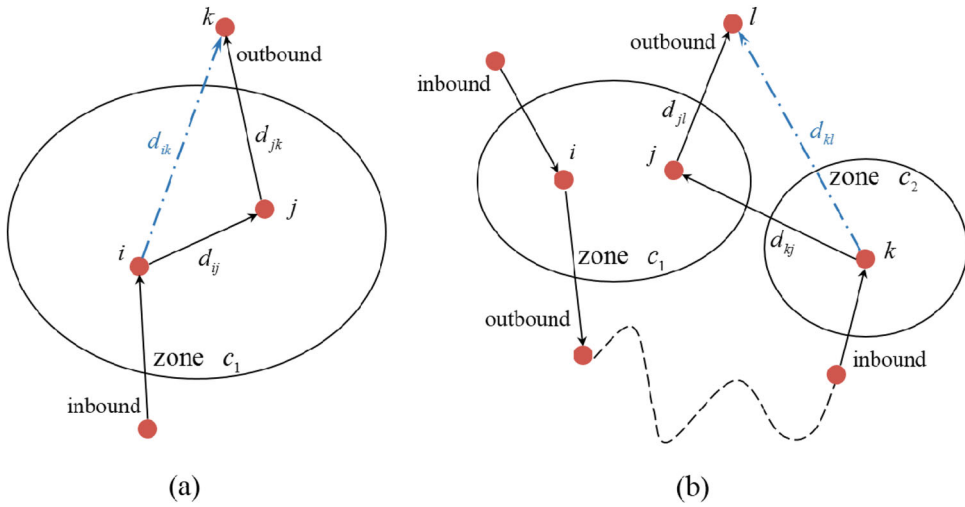
**Remark 2.2:** When the served suburban area is sufficiently large, the wandering time for the first passenger needs to be considered. We define an upper bound limit of wandering time. Then, the wandering time for the first passenger is required to be not larger than the upper bound limit value. Such feature can be seen either as a constraint or as a new term of the objective function in Equation (1) which represents an additional time penalty when the maximum wandering time exceeds the upper bound limit value. It can be easily incorporated into the proposed model and algorithm.

In this formulation, the zone coverage constraint (5) requires that at least one candidate stop of each zone is selected by the bus route. In reality, to achieve the minimization of Equation (1), we find that the bus should visit each zone exactly once and only one stop per zone during the bus tour. Then, we give the proof of this property in Proposition 2.1.

**Proposition 2.1:** *The zone coverage constraint (5) can be replaced by*

$$\sum_{i \in G_c} \delta_i = 1 \quad \forall c \in C. \quad (14)$$





**Figure 2.** Zone coverage constraint in two cases.

**Proof:** This proposition is proven to consider two cases in Figure 2 by contradiction. Consider Case I, suppose that the optimal solution of SBRP contains one zone in which the bus visits two consecutive stops as shown in Figure 2(a). In this case, the bus travels from node  $i$  to node  $j$ , and then leaves zone  $c_1$  to arrive at node  $k$ . Then, the travel distance equals  $d_{ij} + d_{jk}$ . Note that the arcs in Figure 2 are fictitious which correspond to the traveling path in the real streets and roads. We now consider the scenario that bus only visits one stop of zone  $c_1$  (i.e. node  $i$ ) and directly goes to node  $k$ . The travel distance equals  $d_{ik}$ . As distances are assumed to satisfy the triangle inequality, we get  $d_{ij} + d_{jk} \geq d_{ik}$ . Adding the dwell time at node  $j$ , the bus travel time of the optimal solution is larger than the scenario that bus goes directly from node  $i$  to node  $k$ . This is a contradiction, and hence bus visits only one stop of an arbitrary zone each time in the optimal solution.

Consider the Case II, suppose that the optimal solution of SBRP subsumes one zone that is visited twice. As shown in Figure 2(b), node  $j$  is the second stop that bus visits in zone  $c_1$ . In this case, the bus travels from node  $k$  in zone  $c_2$  to node  $j$  in zone  $c_1$ , and later arrives at node  $l$ . The total travel distance of this route segment is  $d_{kj} + d_{jl}$ . Here, consider the other scenario that bus travels directly from node  $k$  to node  $l$ , the travel distance equals  $d_{kl}$ . Since the triangle inequality holds (i.e.  $d_{kj} + d_{jl} \geq d_{kl}$ ) and the optimal solution includes an extra dwell time at node  $j$ , its total travel time is no longer optimal, and we have a contradiction.

Thus, in the optimal solution, the bus visits only one stop of each zone during the bus tour, and Equation (14) is acquired. ■

**Proposition 2.2:** The objective function (1) is equivalent to

$$\min z = \sum_{i \in \bar{H}} \sum_{j \in \bar{H}} d_{ij} x_{ij}. \quad (15)$$

**Proof:** According to Proposition 2.1, each zone only has one stop to be visited by the bus vehicle. Since the number of zones within the suburban area is known, the second term in the objective function (1) equals a constant value, namely  $|\bar{C}| \tau$ . The bus cruising

speed  $v$  is also a known parameter, and thus the objective function (1) is equivalent to  $\min \sum_{i \in \bar{H}} \sum_{j \in \bar{H}} d_{ij} x_{ij}$  in Equation (15). ■

### 3. Quantitative analysis and solution method

#### 3.1. A special model of SBRP

We first discuss a special model that the bus starts from node 0 and proceeds to visit  $|C|$  zones within a suburban area. Fortunately, all the zones are sequentially distributed along the bus route from the original terminal to the airport station, and they are ordered by their distance from node 0 (see Figure 3). In this simple case, it is easy to verify that to obtain the optimal solution, the bus should take turns to visit zone 1, zone 2, etc. and finally zone  $|C|$  within the suburban area.

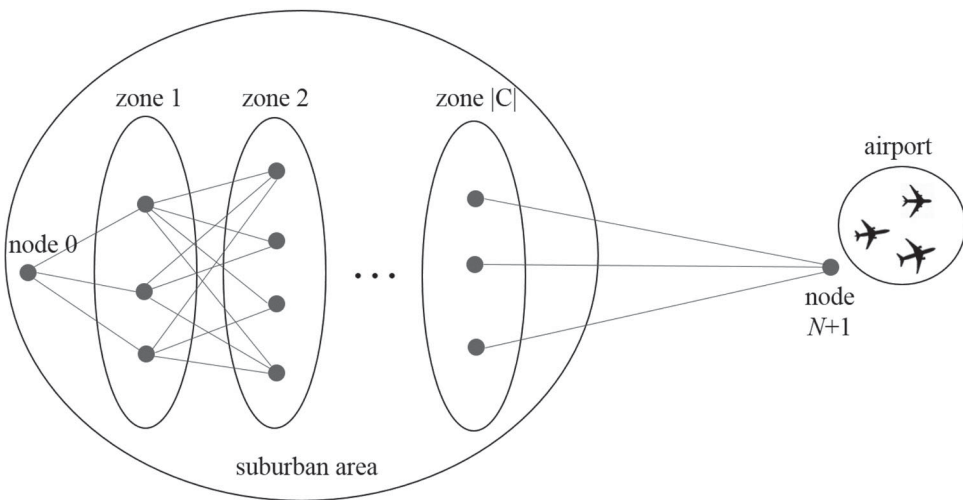
According to Proposition 2.2, the special model is equivalent to the shortest path problem. Hence, it can be polynomially solved by several methods (e.g. simplex method, dynamic programming approach). Here, a dynamic programming approach is proposed to address this special model of SBRP.

##### 3.1.1. Dynamic programming approach

Consider the problem as a multistage decision problem. Computations are carried out for each stage separately. The crucial element of the dynamic programming approach is the definition of recursive equation at stage  $c$ :

$$f_c(i, S) := \text{The shortest sub-tour from node } i \text{ located in zone } c \text{ (i.e. } i \in G_c) \text{ to node } N+1, \text{ and set } S \text{ denotes the candidate stops that are potentially visited along this sub-tour, } S \subseteq H \cup H'. \quad (16)$$

It is noteworthy that in the above function, set  $S$  is a collection of intermediate candidate stops that may be visited at the succeeding stages (i.e. stage  $c+1, c+2, \dots, |C|$ ). For an



**Figure 3.** Schematic diagram of the special model.

arbitrary stop pair,  $h \in H$  and  $h' \in H'$  belong to the identical zone, and hence this candidate stop pair should satisfy either (i)  $h \in S$  and  $h' \in S$  or (ii)  $h \notin S$  and  $h' \notin S$ .

The special model of SBRP has a salient feature: the sequence of stages is the same as the visiting sequence of zones. It means that the state at stage  $c$  corresponds to the candidate stops in set  $G_c$ .

In this study, we use backward recursion in which the computations proceed from stage  $|C|$  to stage 1. Clearly, we have

$$f_{|C|}(i, S) = d_{i, N+1} \text{ if } i \in G_{|C|} \text{ and } S = \emptyset. \quad (17)$$

Then, these distances in stage  $|C|$  are used as input data to compute the immediately preceding stage. Without the loss of generality, we discuss the recursive equation of an arbitrary stage  $c$ :

$$f_c(i, S) = \min_{j \in G_{c+1}} \{d_{ij} + f_{c+1}(j, S/G_{c+1})\} \quad i \in G_c, \quad c < |C| \text{ if } S \neq \emptyset. \quad (18)$$

The dynamic programming approach is executed in an iterative form. In each iteration, several candidate stops of one zone in  $S$  will be eliminated. When  $S = \emptyset$ , we can use Equation (17) to compute the final stage. Therefore, at any stage, the corresponding function  $f_c(i, S)$  can be computed.

The recursive equation of node 0, denoted by stage 0, can also be obtained:

$$f_0(0, H \cup H') = \min_{i \in G_1} \{d_{0i} + f_1(i, S/G_1)\} \quad i \in G_1. \quad (19)$$

Then, we can readily get the optimal bus tour from node 0 to node  $N + 1$ , which just is the optimal solution of Equation (1). It should be mentioned that the proposed approach in this section can be utilized to solve the special model of SBRP in polynomial time with regard to the size of the suburban bus route design.

### 3.2. A generic model of SBRP

We now turn our attention to a generic model that zones are no longer sequentially distributed along the bus route from the original terminal 0 to the airport station  $N + 1$ . Instead, they are randomly distributed within the suburban area. In other words, the generic model of SBRP has no information about the visiting sequence of all the zones located in the suburban area. We first examine the tractability of this generic model and later give the algorithm design.

#### 3.2.1. Computational complexity

Computational complexity theory presents that if an optimization problem belongs to the NP-hard class, an efficient algorithm for the exact solution of this problem does not exist (e.g. Garey and Johnson 1979; Papadimitriou 2003). In this section, we prove that the generic model of SBRP is NP-hard by proving that its decision version is NP-complete. The decision version of SBRP (called dSBRP) is that are there any values for variables  $x_{ij}$  and  $\delta_i$  of SBRP  $\forall i, j \in \bar{H}$  that satisfy constraints (2)–(13) and such that the total vehicular travel time in Equation (1) is less than a given constant  $T$ ?

We propose a polynomial reduction from the Traveling Salesman Problem (TSP), whose NP-completeness is assured by Papadimitriou (1977), to dSBRP. The decision version of TSP is described as: given a set of cities  $\{1, 2, \dots, N\}$  and the distance  $d_{ij}$  from city  $i$  to  $j$ , is there a tour that starts and ends at city 0, and visits the remaining city exactly once such that the total travel distance is of the tour is shorter than  $T$ ?

Next, we show the proposition of the complexity for dSBRP.

**Proposition 3.1:** *dSBRP is NP-complete.*

**Proof:** The description of two steps for the complexity proof are the following.

(1) dSBRP  $\in$  NP

Assume that there exists an algorithm that generates a solution for dSBRP. Determining if the solution is feasible for dSBRP needs several steps. First, check if constraints (2)–(13) are satisfied for each decision variable of the solution (i.e.  $x_{ij}$  and  $\delta_i$ ). Then, verify if the sum of the travel time resulted from the solution is less than or equal to  $T$  (one constraint). Considering all the constraints, we need a polynomial number of steps to verify the feasibility of a dSBRP solution, that is, dSBRP  $\in$  NP.

(2) TSP  $\leq_p$  dSBRP

Consider an arbitrary instance of TSP. We build a particular instance of dSBRP, called dSBRP\*, as follows. Suppose that node 0 and node  $N + 1$  coincide. Hence,  $d_{0i} = d_{N+1,i}$ ,  $d_{i0} = d_{i,N+1}$  for all  $i \in H \cup H'$ . Then, suppose that node  $h$  and  $h'$  coincide for all the candidate stop pairs  $h \in H$ , that is  $d_{ih} = d_{ih'}$ ,  $d_{hi} = d_{h'i}$  for all  $i \in \bar{H}$  and all  $h \in H$ . Suppose further that each zone only has one candidate stop. Therefore, the number of zones equals the number of intermediate stops, namely  $|C| = |N|$ , and based on constraint (5), each candidate stop is visited exactly once, that is,  $\delta_i = 1 \forall i \in \bar{H}$ . Set the average bus cruising speed to  $v = 1$ . The dwell time at each stop is set to be  $\tau = 0$ , and then the second term of objective (1) vanishes. As per dSBRP\* definition, the solution of dSBRP\* is equivalent to TSP. Thus, it is easily to verify that TSP can be solved by solving a dSBRP. ■

### 3.2.2. Dynamic programming approach

The generic model of SBRP can also be solved by the dynamic programming approach. Computations are implemented in a fashion analogous to the proposed method in Section 3.1.1. The main difference between these two models of SBRP is that in the special model, the sequence of stages is identical to the visiting sequence of zones, while in the generic model, there are still a total of  $|C|$  stages but the visiting sequence of zones is not predetermined.

For the generic model, the state at stage  $c$  corresponds to the candidate stops in set  $H \cup H'$ . It is because there is no information about the visiting zone sequence, and the bus can dwell at almost any candidate stop in set  $H \cup H'$  at stage  $c$ .

Thus, we redefine the recursive equation of the generic model. The backward recursion is utilized to compute from stage  $|C|$  to stage 1. The recursive equation of stage  $|C|$  is

$$f_{|C|}(i, S) = d_{i,N+1}, \quad i \in H \cup H' \text{ if } S = \emptyset. \quad (20)$$

With a little abuse of the notation, all the zones are also labeled from 1 to  $|C|$  but have no relations with the sequence of stages. Given that node  $j \in G_c$ , we first let  $\varphi(j)$  be the zone number  $c$  in which node  $j$  is located, that is,  $\varphi(j) = c$ .

Then, we discuss the recursive equation of an arbitrary stage  $c$ ,  $f_c(i, S)$ :

$$f_c(i, S) = \min_{j \in S} \{d_{ij} + f_{c+1}(j, S/G_{\varphi(j)})\}, \quad i \in H \cup H'c < |C| \text{ if } S \neq \emptyset. \quad (21)$$

Equation (21) means that the bus travels from node  $i$  to node  $j \in S$  with the travel distance  $d_{ij}$ . When the bus is at node  $j$ , according to Proposition 2.1, none of candidate stops in  $G_{\varphi(j)}$  need to be visited.  $f_c(i, S)$  is acquired when the best node  $j \in S$  is selected.

Later, the recursive equation of node 0 at stage 0 is calculated as follows:

$$f_0(0, H \cup H') = \min_{i \in H \cup H'} \{d_{0i} + f_1(i, S/G_{\varphi(i)})\}. \quad (22)$$

Finally, we procure the optimal bus tour from node 0 to node  $N + 1$ , which is the optimal solution of the objective function (1). Since the generic model of SBRP is proved to be NP-hard, the above dynamic programming approach cannot be applied to solve the practical-size problem. We can use some heuristic and metaheuristic methods (e.g. genetic algorithm, artificial bee colony [ABC] algorithm) to search good-quality solutions, which can be seen as an approximation of the optimal suburban bus route design.

### 3.2.3. Metaheuristic approach

An ABC approach is proposed in this section to solve the practical-size problem in regard to the generic model of SBRP. The ABC algorithm is a population-based metaheuristic approach, which is inspired by the foraging behaviors of honey bees searching nectar sources around the hive (Karaboga 2005). More recently, the ABC algorithm has been applied to solve several transit optimization problems (Szeto, Wu, and Ho 2011; Szeto and Jiang 2012, 2014; Chen et al. 2015; Jiang and Szeto 2015).

**3.2.3.1. ABC algorithm.** In the ABC algorithm, a food source represents a feasible solution and nectar amount of a food source reflects the solution quality. The bee colony is classified into three categories: employed bees, onlookers, and scouts. Employed bees are mainly responsible for exploiting available food sources and getting their nectar amount. Then, they share such information with onlookers, and each onlooker tends to select a food source found by employed bees according to the probability proportional to the quality of that food source. In this study, the traditional roulette wheel selection method is used to calculate the probability (Szeto and Jiang 2014). Onlookers continue to exploit the solution space in the hope of finding better quality solutions. When the quality of one food source cannot improve for a predetermined number of iterations, this source will be abandoned, and the employed bee will become a scout and start to look for a new source in the vicinity of the hive. Any interested readers could refer to Chen et al. (2015) for a detailed description of the ABC algorithm.

We describe the basic structure of the proposed algorithm below.

*Step 1:* Initialize the parameters: set the colony size  $N_c$ , the number of employed bees  $N_e$ , and the number of onlookers  $N_o$ ; set the counter of iterations,  $l$  and its initial value as one; set the maximum number of iterations,  $l_{\max}$ ; let  $l_m$  be the count of previous successive iterations that food source  $m$  does not improve; let *limit* be the maximal trial count of iterations; let the fitness of food source  $m$  be the reciprocal of  $z(m)$  in Equation (1).

*Step 2:* Randomly generate a set of solutions as initial food sources. Assign each employed bee to an arbitrary food source  $m$ , and set the limit counter  $l_m$  to be zero.

*Step 3:* Perform the employed bee phase. Conduct a neighborhood search for food source  $m$  on  $m \rightarrow \tilde{m}$ . If  $z(\tilde{m}) < z(m)$ , then replace  $m$  with  $\tilde{m}$  and  $l_m = 0$ , else  $l_m = l_m + 1$ .

*Step 4:* Perform the onlooker phase. Execute the fitness-based roulette wheel selection method to select a food source  $m$  for an onlooker from all the food sources obtained by employed bees. Continue to conduct a neighborhood search for food source  $m$  on  $m \rightarrow \tilde{m}$ . If  $z(\tilde{m}) < z(m)$ , then replace  $m$  with  $\tilde{m}$  and  $l_m = 0$ , else  $l_m = l_m + 1$ .

*Step 5:* Perform the scout phase. Compare the fitness of all the renewed food sources originated from  $N_e$  employed bees, and find the best food source with the highest fitness. If the limit counter  $l_m$  of food source  $m$  reaches the maximal trial number *limit*, and meanwhile it is not the best food source, replace it with a new solution generated randomly, and set its limit counter to zero.

*Step 6:* Termination test. Increase the number of iterations by 1, that is,  $l = l + 1$ . Check the stopping criterion: If  $l < l_{\max}$ , return to Step 3; otherwise, terminate and output the best solution.

For the proposed ABC algorithm, solution generation and neighborhood search procedures should be specifically designed in order to search all of the possible suburban bus route structures.

**3.2.3.2. Solution generation procedure.** In the ABC algorithm, new solutions are generated in the initialization and scout phases, both of which use the identical steps to produce a random solution.

For a new solution, a set of decision variables  $x_{ij}$  and  $\delta_i$  need to be initialized. According to Proposition 2.1, each zone only has one stop to be visited by the bus. The suburban area has a total of  $|C|$  zones, and hence only  $|C|$  intermediate candidate stops will be visited along the bus tour. It means that the number of binary variables  $x_{ij}$  and  $\delta_i$  whose values equal 1 is  $|C|$  and  $|C| + 1$ , respectively. All the remaining decision variables are equal to 0.

To create a random solution, the following steps are carried out. First, generate a random permutation of the integers from 1 to  $|C|$  inclusive. As above-mentioned, all the zones already have a set of serial numbers (i.e. 1, 2, ...,  $|C|$ ). The generated permutation is seen as the visiting sequence of  $|C|$  zones. Then, for an arbitrary zone  $c$ , randomly select one of the candidate stops in set  $G_c$  as the visiting node. Finally, variables  $\delta_i$  corresponding to the visiting nodes and variables  $x_{ij}$  corresponding to the bus tour equal 1. Set the rest of decision variables, if any, to zero.

**3.2.3.3. Neighborhood search procedure.** Each employed bee or onlooker conducts a neighborhood search around a food source. For each food source, since the visiting sequence of zones is given, the dynamic programming approach of the special model in Section 3.1.1 can be incorporated in the neighborhood search procedure. It is available to find the optimal bus tour under the current zone visiting sequence in polynomial time. Furthermore, a greedy search procedure is also considered. Specifically, for a food source, swap the visiting sequence of two zones each time to form a new permutation, and then execute the dynamic programming approach in Section 3.1.1. As the number of zones is  $|C|$ , the dynamic programming approach is conducted  $\frac{1}{2}|C|(|C| - 1)$  times.

Then, evaluate the fitness of all neighbor solutions and find the best neighbor solution. Replace the solution with the best neighbor solution, if the fitness of this neighbor solution is better. Otherwise, hold the current solution. The pseudo-code of the neighborhood search procedure is as follows:

```

For each ABC solution  $m$ 
  Set  $c_1 = 1$ 
  While  $c_1 \leq \text{the number of zones } |C| - 1$ 
    Set  $c_2 = c_1 + 1$ 
    While  $c_2 \leq |C|$ 
      Swap the visiting sequence of zone  $c_1$  and  $c_2$ 
      Conduct the dynamic programming approach of the special model
      Record the temporarily optimal neighbor solution  $\tilde{m}$ 
      Restore the original solution  $m$ 
       $c_2 = c_2 + 1$ 
    endwhile
     $c_1 = c_1 + 1$ 
  endwhile
  Search the best neighbor solution  $\hat{m}, z(\hat{m}) = \min\{z(\tilde{m})\}$ 
  If the objective value  $z(\hat{m}) < z(m)$ ,
    then replace  $m$  with  $\hat{m}$ 
  retain  $m$ 
endif
Next ABC solution

```

## 4. Numerical examples

In this section, the proposed models are verified based on a square-block example and two real-case examples in Melbourne, Australia. The solution methods are coded in *Matlab* R2013b and implemented on a personal computer with Inter Core i7-4700 CPU @ 2.40GHz, 2.40 GHz and 8.00 GB RAM. Both models can be efficiently solved.

### 4.1. A square-block example

A square-block example, set out in Figure 4, is first used to demonstrate the calculation of the special model of SBRP. In this example, two bus terminal stops 0 and  $N + 1$  are located at the bottom-left and upper-right corner of the square-block suburban area, respectively. Furthermore, three zones are sequentially distributed along the suburban bus route. Each zone has two candidate stop pairs and each stop pair has two stops on both sides of a road segment.

To be coherent with the real-case examples in the next section, it is required that drivers should drive on the left side in Figure 4. We further assume that (i) vehicles are only allowed to make left-turn, right-turn, and U-turn at intersections, rather than in the middle of route segments; (ii) an average cruising speed is considered to adequately account for the effects of turning at intersections and other such factors which are not explicitly modeled, because there is little congestion in suburban area. Then, the values of  $v$  and  $\tau$  in the objective



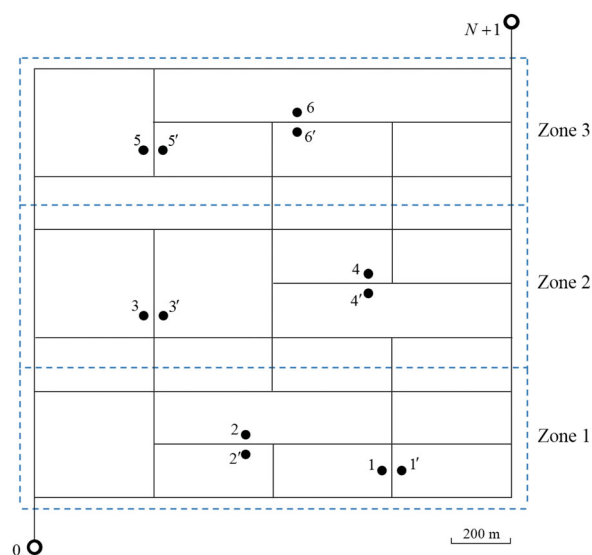


Figure 4. A square-block example of special model.

Table 2. Pairwise distances of candidate stops (unit: km).

<i>i</i>	<i>J</i>													
	0	1	1'	2	2'	3	3'	4	4'	5	5'	6	6'	7
0	0	1.5	1.7	1.1	1.3	1.3	1.9	2.1	2.3	1.9	2.5	2.5	3.1	3.6
1	1.7	0	0.2	1.2	0.6	1.4	2.0	1.4	1.6	2.0	2.2	1.8	2.4	2.1
1'	1.5	0.2	0	1.4	0.8	1.6	2.2	1.6	1.8	2.2	2.4	2.0	2.6	2.3
2	1.3	0.8	0.6	0	0.2	1.0	1.6	1.8	2.0	2.4	2.6	2.2	2.8	2.5
2'	1.1	1.4	1.2	0.6	0	0.8	1.4	1.6	1.8	2.2	2.4	2.0	2.6	3.1
3	1.9	2.2	2.0	1.4	1.6	0	0.6	1.2	1.6	1.4	1.6	1.2	1.8	2.3
3'	1.3	1.6	1.4	0.8	1.0	0.2	0	1.0	1.2	1.6	1.8	1.4	2.0	2.5
4	2.3	1.8	1.6	1.8	2.0	1.2	1.6	0	0.2	1.4	1.6	1.2	1.0	1.5
4'	2.1	1.6	1.4	1.6	1.8	1.0	1.2	0.6	0	1.2	1.4	1.0	1.6	2.1
5	2.5	2.4	2.2	2.4	2.6	1.8	1.6	1.4	1.6	0	0.2	0.6	1.2	1.7
5'	1.9	2.2	2.0	2.2	2.4	1.6	1.4	1.2	1.4	0.2	0	0.8	1.4	1.9
6	3.1	2.6	2.4	2.6	2.8	2.0	1.8	1.6	1.0	1.4	1.2	0	0.6	1.1
6'	2.5	2.0	1.8	2.0	2.2	1.4	1.2	1.0	1.2	0.8	0.6	0.2	0	1.3
7	3.6	2.3	2.1	3.1	2.5	2.5	2.3	2.1	1.5	1.9	1.7	1.3	1.1	0

Note:  $d_{ij}$  is the distance from the row node  $i$  to the column node  $j$ .

function (1) are taken as 20 km/h and 2 min, respectively. Table 2 presents the pairwise distances of all the candidate stops.

The dynamic programming approach in Section 3.1.1 is applied to solve this square-block example. Table 3 presents the computation results: the set  $G_c$  of three zones, pairwise distances of candidate stops that are really used during the calculation, and optimal solutions of backward recursion. After calculation, we get the optimal suburban bus routing:  $0 \rightarrow 2' \rightarrow 3 \rightarrow 4 \rightarrow 7$  as described in Figure 5. It is noteworthy that this example has alternative optima apart from the already obtained optimal solution (see Figure 5). In practice, alternative optima are beneficial because we can select from several bus routing plans without experiencing deterioration in the objective value.

Table 3. Results of the square-block example.

SBRP														
Node 0			Zone 1			Zone 2			Zone 3			Node 7		
Set $G_c$			$G_1 = \{1, 1', 2, 2'\}$			$G_2 = \{3, 3', 4, 4'\}$			$G_3 = \{5, 5', 6, 6'\}$					
Pairwise distances $d_{ij}$ (unit: km)														
$i$	$j$	$d_{ij}$	$i$	$j$	$d_{ij}$	$i$	$j$	$d_{ij}$	$i$	$j$	$d_{ij}$	$i$	$j$	$d_{ij}$
0	1	1.5	1	3	1.4	1'	3	1.6	3	5	1.4	3'	5	1.6
	1'	1.7		3'	2.0		5'	1.6		5'	1.8			
	2	1.1		4	1.6		6	1.2		6	1.4			
	2'	1.3		4'	1.8		6'	1.8		6'	2.0			
			2	3	1.0	2'	3	0.8	4	5	1.4	4'	5	1.2
				3'	1.6		5'	1.6		5'	1.4			
				4	1.8		6	1.2		6	1.0			
				4'	2.0		6'	1.0		6'	1.6			

Dynamic programming (backward recursion)

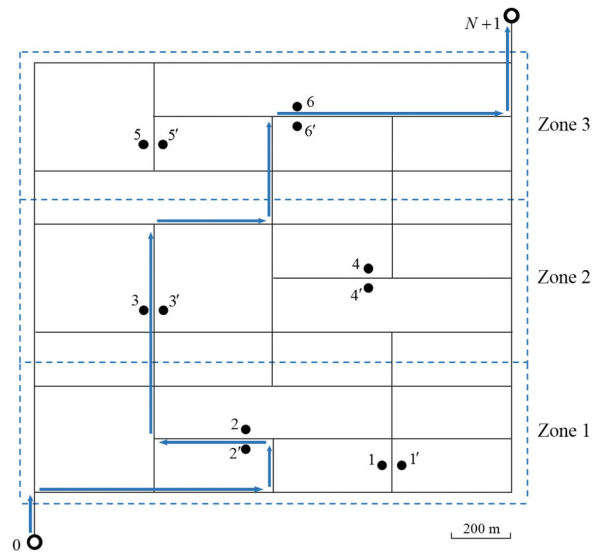
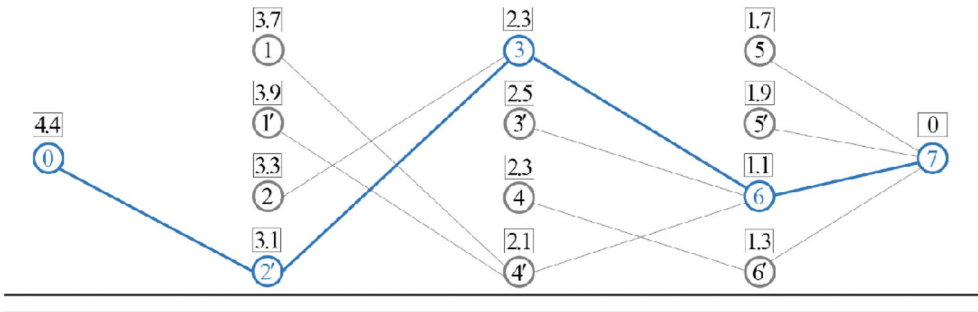


Figure 5. Optimal bus route design via the dynamic programming approach.

4.2. Real-case examples

4.2.1. Example I

The suburban bus route 1 in Figure 1 is taken as the first real-case example. It mainly provides airport access service for the southern suburb of Melbourne. The suburban area in this real case contains 15 zones. In each zone, the candidate stops are selected at the locations with aggregate demand nearby, such as residential neighborhoods, hotels, universities, and some public places like local shopping centers. A total of 36 places are chosen to be the candidate stop pairs which are located at both sides of associated road segments.

In addition, route 1 has a special property that the bus route is parallel to the coastline of Port Phillip Bay and all the suburban zones are sequentially distributed along the bus route.

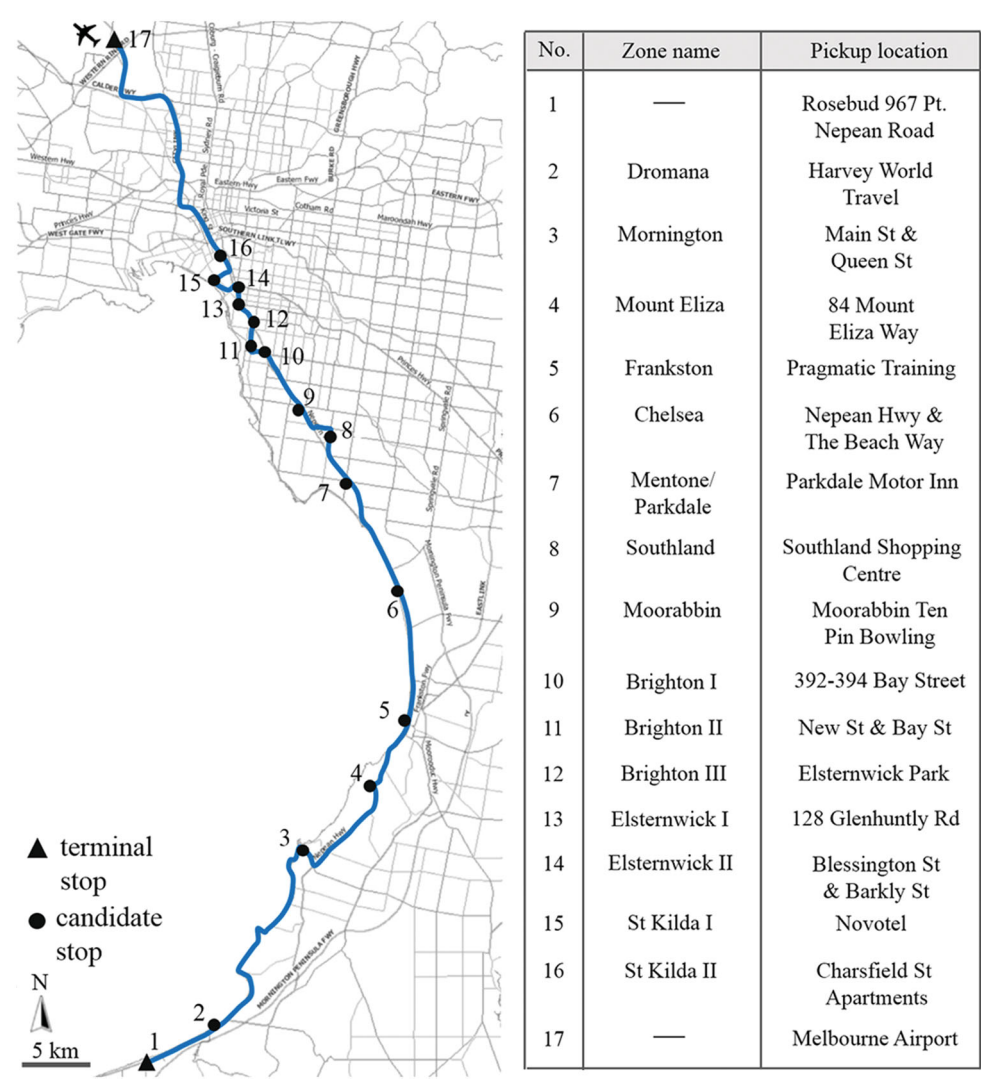


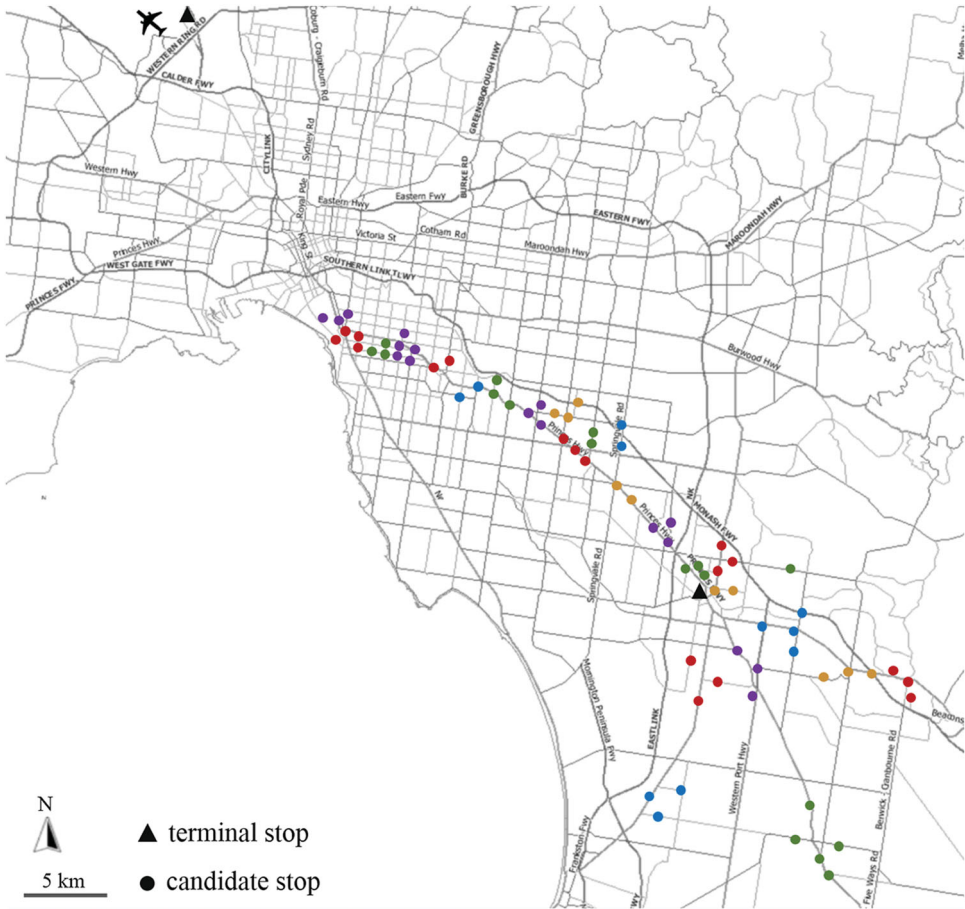
Figure 6. Optimal bus routing of route 1 serving the southern suburban area.

Therefore, route 1 can be seen as a special case of SBRP. We then use the dynamic programming approach in Section 3.1.1 to compute the optimal bus routing of the suburban bus route 1. Figure 6 presents the results of the final selected pickup locations, their attached zones and the corresponding visiting sequence.

#### 4.2.2. Example II

The second real-case study is conducted on the suburban bus route 2, Airport Bus Dandenong. It mainly serves the southeastern suburb of Melbourne. The suburban area in this case contains 25 zones and a total of 73 candidate stops. Figure 7 shows all the candidate stops of these suburban zones. Candidate stops in various zones are distinguished in different colors.

Different from route 1, the suburban zones of route 2 are randomly distributed which correspond to the generic model of SBRP in Section 3.2. In this case, we use the proposed ABC algorithm to seek the optimal bus routing of the suburban bus route 2. Figure 8 intuitively illustrates the results of the final selected stops, their attached zones and the corresponding visiting sequence in the suburban bus route 2.



**Figure 7.** Zones and associated candidate stops along the suburban bus route 2.

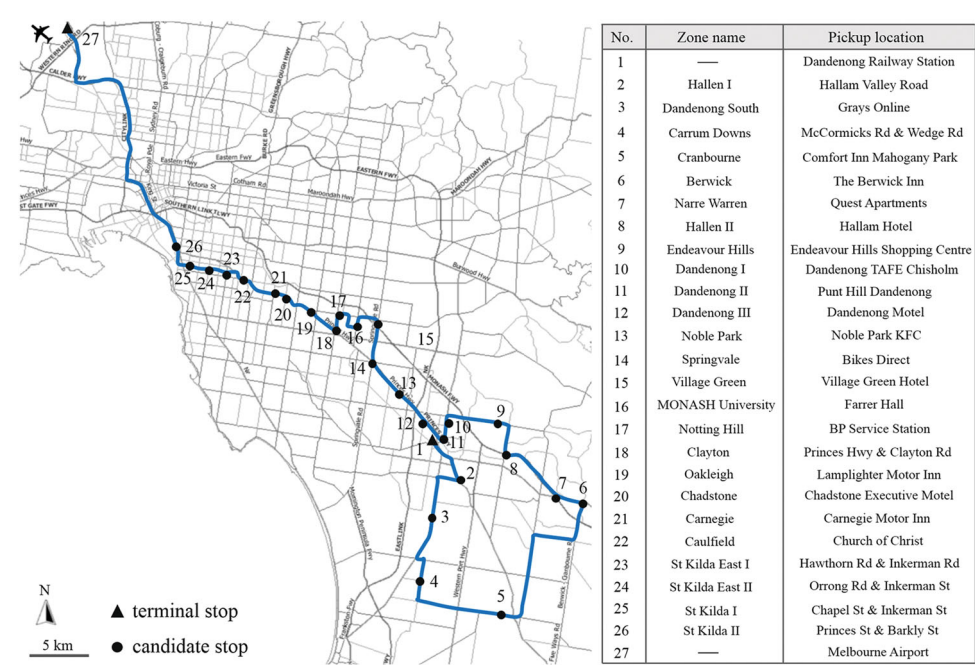


Figure 8. Optimal bus routing of route 2 serving the southeastern suburban area.

5. Conclusions

This paper focused on the bus routing problem of a suburban bus route for airport access. The bus route serves a certain suburban area including several zones. Each zone has a number of candidate stops. The bus routing does not necessarily cover all the candidate stops while at least one candidate stop per zone must be visited in ways that walking access distance is reachable for airline passengers in each zone. The SBRP in this circumstance was formulated as an optimization model, with the objective of minimizing the total travel time.

The proposed model of SBRP was further categorized into a special model and a generic model. The special model was first considered in which suburban zones are sequentially distributed along the bus route from the original terminal to the airport station. It can be solved in polynomial time. A dynamic programming approach was developed to address this special model. Later, the generic model, in which all the suburban zones are randomly distributed, was discussed. We proved that the generic model is NP-hard. A dynamic programming approach and a metaheuristic method were developed. Case studies based on a square-block example and two suburban bus routes access to the Melbourne Airport were conducted to demonstrate the applicability of the proposed models and solution methods. Our findings could serve as guidelines for designing the bus routing of suburban bus routes for airport access.

Admittedly, our proposed models come with some limitations, and the following improvements are suggested: (i) It could be considered to extend the model to apply into a broader service area for airport access rather than the suburban area only; (ii) Research is needed to propose a more generalized model which is capable to consider congestion effect and multiple bus routes; (ii) Different population-based algorithms could be

systematically compared to solve the proposed problem in this study. The authors recommend that future studies could focus on these issues.

## Acknowledgements

The authors would like to thank the Editor and four anonymous reviewers for their valuable comments and suggestions to improve the quality of the article.

## Disclosure statement

No potential conflict of interest was reported by the authors.

## Funding

This study was supported by the Projects of International Cooperation and Exchange of the National Natural Science Foundation of China (No. 51561135003), Key Projects (No. 51338003), Surface Projects (No. 51378120) and the Youth Projects (No. 71501038) of the National Natural Science Foundation of China and Housing and Rural Construction Science and Technology Plan of Shaanxi Province (2015-K54).

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