MATH1000 tutorials - Dalhousie University

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Each tutorial is 50 minutes long and will have the following structure:

- **Passive recall:** questions solved on the board. (15-20 minutes)
- **Active recall:** working on select problems in small groups. (15-20 minutes)
- **Break:** we do anything but the course content. (5 minutes)
- **Quiz:** this will be one of the problems of the active learning section. (10 minutes)

I call this method "slightly" spaced repetition. The idea is that after we warm up and let you work on the problems by yourself, we try to take Calculus out of our minds for a split second. Then retention will be much higher when you revisit the question of the quiz.

FAQ

Why do we have tutorials?

Studies suggest that, statistically, attending tutorials reflects in a better grade.

Do I have to interact with my peers?

No, but studies have shown that social ties are a main predictor of success in STEM classes. You still have to write quizzes on your own.

Can I write make-up quizzes?

No. You can submit an SDA form to have a quiz exempted (max twice). Submit your SDAs directly to the MATH 1000 coordinator at rnoble@dal.ca.

Can I use calculators, friends, ChatGPT, or other aids during the quizzes?

No aids of any kind are permitted during the quizzes.

What are we going to do during the break?

You can do whatever you like, but in general I might just play Daily Dose of Pets.

What is your favourite animal?

Lately it's been hedgehogs.

Contents

These notes will be updated as we go. They will always be available at:

https://www.mathstat.dal.ca/~teixeira/25-fall-math1000

Most problems are taken from the textbook, which is freely available online.

1 October 9: limits and continuity

Evaluate the limit $\lim\limits_{x\to -2^-} \dfrac{2x^2+7x-4}{x^2+x-2}$**.**

If we try to substitute $x=-2$, we will have the undefined expression $-10/0$, so the problem is in the denominator. Factoring that we obtain $x^2+x-2=(x+2)(x-1)$, which we can substitute in the expression for the limit

$$\lim_{x\to -2^-} \frac{2x^2+7x-4}{x^2+x-2} = \lim_{x\to -2^-} \frac{2x^2+7x-4}{x-1}\cdot\frac{1}{x+2}.$$

Now $\lim\limits_{x\to 2} \frac{2x^2+7x-4}{x-1} = 14$ and $\lim\limits_{x\to 2} \frac{1}{x+2} = -\infty$, so the original limit is $\boxed{-\infty}$.

Evaluate the limit $\lim\limits_{x\to 0} x^2 \sin\left(\dfrac{1}{x}\right)$

Since $-1 \le \sin(\frac{1}{x}) \le +1$, we can squeeze the function between $-x^2$ and x^2, i.e.

$$\lim_{x\to 0} x^2 \le \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) \le \lim_{x\to 0} x^2.$$

Both surrounding limits evaluate to 0, so the squeezed limit is $\boxed{0}$.

Additional questions

1. Use direct substitution to show that each limit leads to an indeterminate form (such as 0/0). Then, evaluate the limit.
   (a) $\displaystyle\lim_{x\to 4}\frac{x^2-16}{x-4}$
   (b) $\displaystyle\lim_{t\to 9}\frac{t-9}{\sqrt{t}-3}$
   (c) $\displaystyle\lim_{\theta\to \pi}\frac{\sin\theta}{\tan\theta}$
2. Evaluate using the squeeze theorem.
   (a) $\displaystyle\lim_{x\to 1}(x-1)\cos\left(\frac{1}{x^2-1}\right)$
   (b) $\displaystyle\lim_{x\to 0}\sqrt{x^2\cos^2\left(\frac{1}{x}\right)}$
3. Find the value of k that makes the following function continuous.
$$f(x)=\begin{cases}3x+2, & \text{if } x<k\\ 2x-3, & \text{if } x\geq k\end{cases}$$
4. Special Relativity tells that a body with mass m moving at speed v has total energy given by
$$E(v)=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}.$$
Evaluate $\displaystyle\lim_{v\to c^-}E(v)$. What is the physical meaning of this quantity? Why are you evaluating from the left?
5. Look up the basic formula that the CRA uses to calculate the amount of tax due $T(I)$ in terms of the income I.[1] Convince yourself that T is continuous, and that this implies that you always make more money by receiving a higher income.

2 October 23: derivatives

Find $f'(x)$ for $f(x)=5x^4-3\sqrt{x}+\dfrac{2+x}{x^3}$.

Note that $f(x)$ is the sum of three functions, whose derivatives we compute separetely:

- $5x^4 \rightsquigarrow 20x^3$ (power rule)
- $\sqrt{x}=x^{1/2}\rightsquigarrow \frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$ (power rule)
- $\frac{2+x}{x^3}\rightsquigarrow \frac{(1)(x^3)-(2+x)(3x^2)}{x^6}=-\frac{2x+6}{x^4}$ (quotient rule)

Putting these together we obtain $\boxed{f'(x)=20x^3-\frac{3}{2\sqrt{x}}-\frac{2x+6}{.}x^4}$.

The derivative of a^x is $a^x\ln a$. Using this, find $g'(x)$ for $g(x)=6x^2\,a^x$.

Using the product rule we obtain $\boxed{g'(x)=12xa^x+6x^2a^x\ln a}$.

[1] https://www.canada.ca/en/revenue-agency/services/tax/individuals/frequently-asked-questions-individuals/canadian-income-tax-rates-individuals-current-previous-ye

Additional questions

1. Find $f'(x)$ for each function.
   (a) $8x^4 + 9\sqrt{x} - 1$
   (b) $3x^{2/3} - \dfrac{5}{x^{2/3}}$
   (c) $\dfrac{2}{x^2 - 5}$
2. Given that $f(2) = 3$ and $f'(2) = -1$, compute the derivative at $x = 2$ in each case:
   (a) $x\,f(x) - x^3$
   (b) $\dfrac{2x^2}{f(x) + 2}$
3. Using the table, find $f'(x)$ in each case.

$f(x)$	$\sin x$	$\cos x$
$f'(x)$	$\cos x$	$-\sin x$

   (a) $\sin x + \cos x$
   (b) $\sin x \cos x$
   (c) $\tan\ x$
4. Recall that $f'(a)$ is the slope of the tangent to f at $x = a$. Interpret each situation:
   (a) $f'(x) = 0$
   (b) $\lim_{x\to a} f'(x) = +\infty$
5. A ball is dropped from an initial height h_0. Its height after t seconds is given by the following equation

$$h(t) = h_0 - \frac{gt^2}{2},$$

   where g is a constant.
   (a) Find the speed of the ball at time t.
   (b) Find its acceleration at time t.

3 October 30: more derivatives

Find $f'(x)$ for $f(x) = \ln\left(\dfrac{\cos(x^2)}{x^3}\right)$.

Our function is of the form $f(x) = \ln(g(x))$ for $g(x) = \cos(x^2)/x^3$. The chain rule gives

$$f'(x) = g'(x) \cdot \frac{1}{g(x)}.$$

So we need to find $g'(x)$. The quotient rule gives

$$g'(x) = \frac{(2x(-\sin(x^2)))(x^3) - (\cos(x^2))(3x^2)}{(x^3)^2}.$$

(We used the chain rule to differentiate $\cos(x^2)$.) Putting it together:

$$\boxed{f'(x) = \frac{(2x(-\sin(x^2)))(x^3) - (\cos(x^2))(3x^2)}{(x^3)^2} \cdot \frac{x^3}{\cos(x^2)}}$$

Find the equation for the line tangent to $y^2 = x^3 + 1$ at $(2, 3)$.[2]

[2] Equations of the form $y^2 = x^3 + ax + b$ are called elliptic curves and are surprisingly hard to study.

Notice that $(2,3)$ satisfies the equation since $3^2 = 2^3 + 1$. We will find the slope via implicit differentiation. Differentiating:

$$y^2 = x^3 + ax + b \overset{d/dx}{\Longrightarrow} 2\frac{dy}{dx}y = 3x^2.$$

Substituting $(x_0, y_0) = (2,3)$ we obtain $2y' \cdot 3 = 3 \cdot 2^2 \implies y' = 2$. The derivative is the slope of the tangent line, that is,

$$\frac{dy}{dx} = \frac{y - y_0}{x - x_0} \implies 2 = \frac{y-3}{x-2}.$$

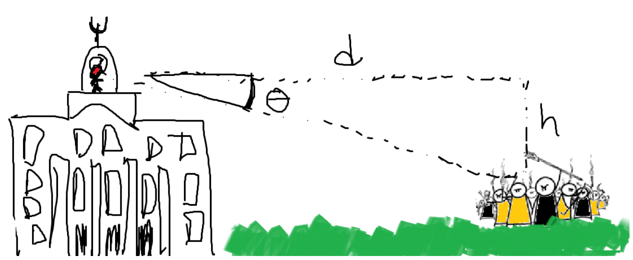
Solving for y gives $\boxed{y = 2x + 7}$.

Additional questions

1. Find $f'(x)$ for each function.
   (a) $e^{-2\pi x^2}$
   (b) $\ln(\sec x)$
   (c) $3^{\sin 3x}$
   (d) $(\sin 3x)^3$

2. Use implicit differentiation to find $\frac{dy}{dx}$.
   (a) $(xy)^2 + 3x = y^2$
   (b) $xy - \cos(xy) = 1$
   (c) $\arctan(x+y) = x^2 + \dfrac{\pi}{4}$

4. A Dark Overlord stands atop the Henry Hicks building, staring down at an angry mob of students that storms through the Studley Quad. The crowd moves following the equation $d = 200 - 6t$, and the height of the HH is around $h = 20$. What is the rate of change of the angle θ of the neck of the Dark Overlord?



3. Find the equation of the line tangent to

$$2x^3 + 2y^3 - 9xy = 0$$

at $(2,1)$. Use some app to graph the function and the tangent line.

5. Try and fail to find the tangent to the circle

$$x^2 + y^2 = 1$$

at the point $(1,0)$ via implicit differentiation. Interpret your failure geometrically.

4 November 6: graphing

Let $f(x) = f(x) = (x^2 - 2x - 2)e^x$.

Find the maxima and minima of f, and the intervals where it is increasing or decreasing.

First we find $f'(x)$:

$$f'(x) = (2x - 2)e^x + (x^2 - 2x - 2)e^x \implies f'(x) = (x^2 - 4)e^x,$$

then we solve $f'(x) = 4$. Since $e^x > 0$, this only happens at the roots of $(x^2 - 4)$, i.e. $\boxed{x = \pm 2}$.

To determine which one are maximum or minimum, we look at the sign of $f'(x)$:

		-2 $\qquad$ 2	
$(x^2 - 4)$	$+$ •	$-$ •	$+$
e^x	$+$	$+$	$+$
$f'(x)$	$+$	$-$	$+$
$\nearrow$?	$\nearrow$	$\searrow$	$\nearrow$

We see that $x = -2$ is a minimum and $x = +2$ is a maximum.

Find the inflections points of f, and the intervals where it is concave up or down.

First we find $f''(x)$,

$$f''(x) = (2x)e^x + (x^2 - 4)e^2 \implies f''(x) = (x^2 + 2x - 4)e^x,$$

then we solve $f''(x) = 0$. The roots $\boxed{x = -1 \pm \sqrt{5}}$ are the inflection points. For concavity, we use the second derivative test:

		$-1 + \sqrt{5}$ $\qquad$ $1 + \sqrt{5}$	
$x^2 + 2x - 4$	$+$ •	$-$ •	$+$
e^x	$+$	$+$	$+$
$f''(x)$	$+$	$-$	$+$
concave?	up	down	up

Additional questions

Consider the following functions for 1 - 4:

(a) $e^{-\pi x^2}$ (b) $1+\ln x$ (c) x^3-6x^2

1. Find the maxima and minima of $f(x)$.
2. Find the intervals where f is increasing or decreasing.
3. Find the inflection points of f.
4. Find the intervals where f is concave up and concave down.
5. In high school, you might have learned the formula $h=-\frac{b}{2a}$ for the position of the maximum or minimum of a parabola $y=ax^2+bx+c$. Prove this formula using calculus.
6. The equation for the angle of the sun above the horizon in Halifax (at noon) can be approximated by the function

$$\alpha(d)=45.4^\circ+23.4^\circ\sin\left(\frac{2\pi}{365}(d-80)\right)$$

where d is the day of the year. Solve $\alpha'(d)=0$ and interpret your findings in your calendar.

5 November 20: misc derivatives

Let $f(x)=f(x)=(x^2-2x-2)e^x$.

Sketch the graph of the function in your last quiz.

We work out the example $f(x)=e^{-x^2}$.

1. *Extrema and intervals of increasing and decreasing.*
   First find $f'(x)=-2\,xe^{-x^2}$ by the chain rule; the only solution to $f'(x)=0$ is $x=0$.
   To determine whether this is a maximum or a minimum, we look at the sign of $f'(x)$:

	0	
$-2\,x$	$+$	$-$
e^{-x^2}	$+$	$+$
$f'(x)$	$+$	$-$
$\nearrow$?	$\nearrow$	$\searrow$

   So $x=0$ is a (global) maximum.
2. *Inflection points and concavity.*
   First we find $f''(x)$:

$$f''(x)=-2e^{x^2}-2x(-2xe^{x^2})\implies f''(x)=(4x^2-2)e^{-x^2}.$$

   The inflection points occur where $4x^2-2=0\implies x=\pm\dfrac{1}{\sqrt{2}}$. We determine the concavity via the second derivative test:

		$-1/\sqrt{2}$ $\quad$ $1/\sqrt{2}$	
$4x^2-2$	$+$	$-$	$+$
e^{-x^2}	$+$	$+$	$+$
$f'(x)$	$+$	$-$	$+$
concave?	up	down	up

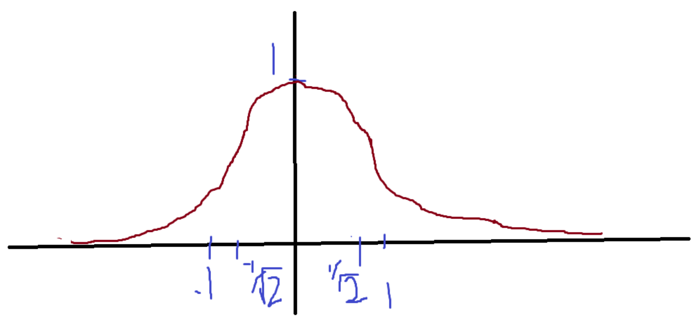
3. *Asymptotes.*

   The limits $\lim_{x\to\pm\infty} e^{-x^2} = 0$ give the asymptotes $x = 0$.

4. *Roots.*

   The function has no roots - it is always positive.

5. *Sketch.* I made this sketch in Paint with my mousepad to emphasize how, despite the terrible drawing skills, you can still see the features that we calculated in this section.



Compute the limit $\lim\limits_{x\to 0}(e^x - x)^{1/x^2}$**.**

Note that direct substitution gives the inderterminate of the form 1^∞, so we need to somehow apply L'Hôpital. Writing L for the limit and applying ln:

$$\ln L = \lim_{x\to 0} \frac{\ln(e^x - x)}{x^2}.$$

Now direct substitution gives an indeterminate of the form $0/0$, so we may L'Hôpital to obtain

$$\ln L \overset{H}{=} \lim_{x\to 0} \frac{\frac{e^x - 1}{e^x - x}}{2x} \implies \ln L = \lim_{x\to 0} \frac{e^x - 1}{2x(e^x - x)}.$$

This is still an indeterminate of the form $0/0$, so we L'Hôpital again:

$$\ln L \overset{H}{=} \lim_{x\to 0} \frac{e^x}{2(e^x - x) + 2x(e^x - 1)}.$$

Direct substitution *now* gives $\ln L = \frac{1}{2}$, so $\boxed{L = \sqrt{e}}$.

Additional questions

1. Calculate by L'Hôpital.

   (a) $\lim_{x\to\infty} \frac{\ln x}{x^2}$ (b) $\lim_{x\to\infty} \frac{x^2}{2^x}$ (c) $\lim_{x\to\infty} \frac{x^{100}}{2^x}$

   (d) $\lim_{x\to 0^+} x^{1/x}$ (e) $\lim_{x\to 1} \frac{x^2-1}{\sin x}$

   (f) $\lim_{x\to\infty} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

2. Find the ratio h/r for an aluminum can of height h and radius r (pretends it's a cylinder) holding 16oz of soda using the least amount of material possible. (The answer doesn't look like a can of coke.)

3. You are choosing between different algorithms for running a piece of code. The distinct strategies give the following options for the running time $O(n)$:

$$\ln n, \quad 2^n, \quad n^2, \quad n!, \quad n^{100}.$$

   Which is the fastest option? Which is the slowest? *Hint: problems 1a-c.*

6 November 27: definite integrals

7 December 4: indefinite integrals and substitution