# MATH1000 tutorials - Dalhousie University

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Each tutorial is 50 minutes long and will have the following structure:

- **Passive recall:** questions solved on the board. (15-20 minutes)
- **Active recall:** working on select problems in small groups. (15-20 minutes)
- **Break:** we do anything but the course content. (5 minutes)
- **Quiz:** this will be one of the problems of the active learning section. (10 minutes)

I call this method "slightly" spaced repetition. The idea is that after we warm up and let you work on the problems by yourself, we try to take Calculus out of our minds for a split second. Then retention will be much higher when you revisit the question of the quiz.

## FAQ

**Why do we have tutorials?**

Studies suggest that, statistically, attending tutorials reflects in a better grade.

**Do I have to interact with my peers?**

No, but studies have shown that social ties are a main predictor of success in STEM classes. You still have to write quizzes on your own.

**Can I write make-up quizzes?**

No. You can submit an SDA form to have a quiz exempted (max twice). Submit your SDAs directly to the MATH 1000 coordinator at rnoble@dal.ca.

**Can I use calculators, friends, ChatGPT, or other aids during the quizzes?**

No aids of any kind are permitted during the quizzes.

**What are we going to do during the break?**

You can do whatever you like, but in general I might just play Daily Dose of Pets.

**What is your favourite animal?**

Lately it's been hedgehogs.

# Contents



These notes will be updated as we go. They will always be available at:

`https://www.mathstat.dal.ca/~teixeira/25-fall-math1000`

Most problems are taken from the textbook, which is freely available online.

# 1 October 9: limits and continuity

**Evaluate the limit** $\lim_{x\to -2^-} \dfrac{2x^2+7x-4}{x^2+x-2}$**.**

If we try to substitute $x=-2$, we will have the undefined expression $-10/0$, so the problem is in the denominator. Factoring that we obtain $x^2+x-2=(x+2)(x-1)$, which we can substitute in the expression for the limit

$$\lim_{x\to -2^-} \frac{2x^2+7x-4}{x^2+x-2} = \lim_{x\to -2^-} \frac{2x^2+7x-4}{x-1}\cdot\frac{1}{x+2}.$$

Now $\lim_{x\to 2} \frac{2x^2+7x-4}{x-1} = 14$ and $\lim_{x\to 2}\frac{1}{x+2} = -\infty$, so the original limit is $\boxed{-\infty}$.

**Evaluate the limit** $\lim_{x\to 0} x^2 \sin\left(\dfrac{1}{x}\right)$

Since $-1 \le \sin(\frac{1}{x}) \le +1$, we can squeeze the function between $-x^2$ and $x^2$, i.e.

$$\lim_{x\to 0} x^2 \le \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) \le \lim_{x\to 0} x^2.$$

Both surrounding limits evaluate to 0, so the squeezed limit is $\boxed{0}$.

**Additional questions**

1. Use direct substitution to show that each limit leads to an indeterminate form (such as 0/0). Then, evaluate the limit.
   (a) $\lim_{x\to 4} \dfrac{x^2-16}{x-4}$
   (b) $\lim_{t\to 9} \dfrac{t-9}{\sqrt{t}-3}$
   (c) $\lim_{\theta\to \pi} \dfrac{\sin\theta}{\tan\theta}$
2. Evaluate using the squeeze theorem.
   (a) $\lim_{x\to 1} (x-1)\cos\left(\dfrac{1}{x^2-1}\right)$
   (b) $\lim_{x\to 0} \sqrt{x^2\cos^2\left(\dfrac{1}{x}\right)}$
3. Find the value of $k$ that makes the following function continuous.

$$f(x) = \begin{cases} 3x+2, & \text{if } x < k \\ 2x-3, & \text{if } x \geq k \end{cases}$$

4. Special Relativity tells that a body with mass $m$ moving at speed $v$ has total energy given by

$$E(v) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}.$$

   Evaluate $\lim_{v\to c^-} E(v)$. What is the physical meaning of this quantity? Why are you evaluating from the left?
5. Look up the basic formula that the CRA uses to calculate the amount of tax due $T(I)$ in terms of the income $I$.[1] Convince yourself that $T$ is continuous, and that this implies that you always make more money by receiving a higher income.

## 2 October 23: derivatives

**Find $f'(x)$ for $f(x) = 5x^4 - 3\sqrt{x} + \dfrac{2+x}{x^3}$.**

Note that $f(x)$ is the sum of three functions, whose derivatives we compute separetely:

- $5x^4 \rightsquigarrow 20x^3$ (power rule)
- $\sqrt{x} = x^{1/2} \rightsquigarrow \frac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$ (power rule)
- $\dfrac{2+x}{x^3} \rightsquigarrow \dfrac{(1)(x^3)-(2+x)(3x^2)}{x^6} = -\dfrac{2x+6}{x^4}$ (quotient rule)

Putting these together we obtain $\boxed{f'(x) = 20x^3 - \dfrac{3}{2\sqrt{x}} - \dfrac{2x+6}{.}x^4}$.

**The derivative of $a^x$ is $a^x\ln a$. Using this, find $g'(x)$ for $g(x) = 6x^2\,a^x$.**

Using the product rule we obtain $\boxed{g'(x) = 12xa^x + 6x^2a^x\ln a}$.

[1] https://www.canada.ca/en/revenue-agency/services/tax/individuals/frequently-asked-questions-individuals/canadian-income-tax-rates-individuals-current-previous-ye

**Additional questions**

1. Find $f'(x)$ for each function.
   (a) $8x^4 + 9\sqrt{x} - 1$
   (b) $3x^{2/3} - \dfrac{5}{x^{2/3}}$
   (c) $\dfrac{2}{x^2 - 5}$
2. Given that $f(2) = 3$ and $f'(2) = -1$, compute the derivative at $x = 2$ in each case:
   (a) $x\,f(x) - x^3$
   (b) $\dfrac{2x^2}{f(x) + 2}$
3. Using the table, find $f'(x)$ in each case.

| | | |
|---|---|---|
| $f(x)$ | $\sin(x)$ | $\cos(x)$ |
| $f'(x)$ | $\cos(x)$ | $-\sin(x)$ |

   (a) $\sin(x) + \cos(x)$
   (b) $\sin(x)\cos(x)$
   (c) $\tan(x)$
4. Recall that $f'(a)$ is the slope of the tangent to $f$ at $x = a$. Interpret each situation:
   (a) $f'(x) = 0$
   (b) $\lim_{x \to a} f'(x) = +\infty$
5. A ball is dropped from an initial height $h_0$. Its height after $t$ seconds is given by the following equation

$$h(t) = h_0 - \frac{gt^2}{2},$$

   where $g$ is a constant.
   (a) Find the speed of the ball at time $t$.
   (b) Find its acceleration at time $t$.

## 3 October 30: implicit differentiation & min/max problems

**First question.**

**Second question.**

**Additional questions**

1.

2.

3. Ommitting the constants, the daylight duration at a given day of the year $d$ in Halifax is given by

$$L(d) = \arccos(\tan(0.409\frac{2\pi}{365}\sin(d-80))).$$

Calculate the derivative $L'(d)$ and find the minima and maxima of $L(d)$ by setting $L'(d) = 0$ in a symbolic calculator.

# 4 November 6: graphing

# 5 November 20: optimization problems

# 6 November 27: definite integrals

# 7 December 4: indefinite integrals and substitution