# MATH1000 tutorials - Dalhousie University

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#### Fall 2025

Each tutorial is 50 minutes long and will have the following structure:

- Passive recall: questions solved on the board. (15-20 minutes)
- Active recall: working on select problems in small groups. (15-20 minutes)
- Break: we do anything but the course content. (5 minutes)
- Quiz: this will be one of the problems of the active learning section. (10 minutes)

I call this method "slightly" spaced repetition. The idea is that after we warm up and let you work on the problems by yourself, we try to take Calculus out of our minds for a split second. Then retention will be much higher when you revisit the question of the quiz.

### **FAQ**

#### Why do we have tutorials?

Studies suggest that, statistically, attending tutorials reflects in a better grade.

#### Do I have to interact with my peers?

No, but studies have shown that social ties are a main predictor of success in STEM classes. You still have to write quizzes on your own.

#### Can I write make-up quizzes?

No. You can submit an SDA form to have a quiz exempted (max twice). Submit your SDAs directly to the MATH 1000 coordinator at rnoble@dal.ca.

#### Can I use calculators, friends, ChatGPT, or other aids during the quizzes?

No aids of any kind are permitted during the quizzes.

### What are we going to do during the break?

You can do whatever you like, but in general I might just play Daily Dose of Pets.

#### What is your favourite animal?

Lately it's been hedgehogs.

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These notes will be updated as we go. They will always be available at:

Most problems are taken from the textbook, which is freely available online.

# 1 October 9: limits and continuity

Evaluate the limit 
$$\lim_{x \to -2^-} \frac{2x^2 + 7x - 4}{x^2 + x - 2}$$
.

If we try to substitute x = -2, we will have the undefined expression 14/0, so the problem is in the denominator. Factoring that we obtain  $x^2 + x - 2 = (x + 2)(x - 1)$ , which we can substitute in the expression for the limit

$$\lim_{x \to -2^{-}} \frac{2x^2 + 7x - 4}{x^2 + x - 2} = \lim_{x \to -2^{-}} \frac{2x^2 + 7x - 4}{x - 1} \cdot \frac{1}{x + 2}.$$

Now  $\lim_{x\to 2} \frac{2x^2+7x-4}{x-1} = 14$  and  $\lim_{x\to 2} \frac{1}{x+2} = -\infty$ , so the original limit is  $-\infty$ .

Evaluate the limit  $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$ 

Since  $-1 \le \sin(\frac{1}{x}) \le +1$ , we can squeeze the function between  $-x^2$  and  $x^2$ , i.e.

$$\lim_{x \to 0} x^2 \le \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \le \lim_{x \to 0} x^2.$$

Both surrounding limits evaluate to 0, so the squeezed limit is  $\boxed{0}$ .

## Additional questions

- 1. Use direct substitution to show that each limit leads to an indeterminate form (such as 0/0). Then, evaluate the limit.
- (a)  $\lim_{x\to 4} \frac{x^2 16}{x 4}$ (b)  $\lim_{t\to 9} \frac{t 9}{\sqrt{t} 3}$ (c)  $\lim_{\theta\to \pi} \frac{\sin \theta}{\tan \theta}$ 2. Evaluate using the squeeze theorem.
  - (a)  $\lim_{x \to 3} (x^2 9) \left( 3 \cos \left( \frac{1}{x^2} \right) \right)$ (b)  $\lim_{x \to 0} \sin x \cos^2 x$
- 3. Special Relativity tells that a body with mass m moving at speed v has total energy given by

$$E(v) = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

- Evaluate  $\lim E(v)$ . What is the physical meaning of this quantity? Why are you evaluating from the left?
- 4. Find the value of k that makes the following function continuous.

$$f(x) = \begin{cases} 3x + 2, & \text{if } x < k \\ 2x - 3, & \text{if } x \ge k \end{cases}$$

5. Look up the basic formula that the CRA uses to calculate the amount of due tax T(I) in terms of the income I. Convince yourself that T is continuous, and that this implies that you always make more money by receiving a higher income.

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https://www.canada.ca/en/revenue-agency/services/tax/individuals/frequently-asked-questions-individuals/ canadian-income-tax-rates-individuals-current-previous-ye