Analysis I: Exercises
Rachel Shu
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Chapter 4

Integers and Rationals

4.1 The integers

Exercise 4.1.1

Verify that the definition of equality on the integers is both reflexive and symmetric.

Proof. We first prove that the definition of equality is reflexive. Let $a, b, c, d \in \mathbb{N}$. Let $(a—b) \in \mathbb{Z}$. We wish to prove $a—b = a—b$. By reflexivity for the natural numbers, $a + b = a + b$. The definition of equality for integers states $a—b = c—d \iff a + d = b + c$. Thus, $a—b = a—b$, as desired.

Next we prove the definition of equality is symmetric. Suppose $a, b, c, d \in \mathbb{N}$. We wish to prove $a—b = c—d \iff c—d = a—b$. The definition of equality for integers states $a—b = c—d \iff a + d = b + c$. We know by symmetry for the natural numbers that $a + d = b + c \iff b + c = a + d$. Thus, $a—b = c—d \iff c—d = a—b$, as desired. □

Exercise 4.1.3

Show that $(-1) \times a = -a$ for every integer a.

Proof. Let a in $\mathbb{Z}$. □