

Core dynamics of the MJO; MJ's journey to find true self

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 **NOAA/PMEL, USA

Episode 1. MJ meets little Kelvin

Episode 2. Tune the frequency

Episode 3. Why am I different?

Episode 4. With or without you

ABSTRACT | CONTACT AUTHOR | PRINT | GET iPOSTER

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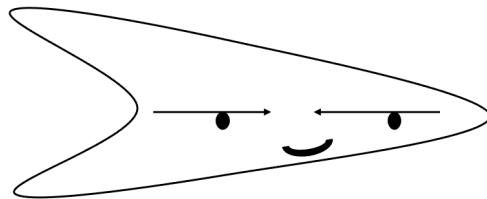


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EPISODE1. MJ MEETS LITTLE KELVIN

Hi, I'm a Madden-Julian oscillation.
Just call me MJ.

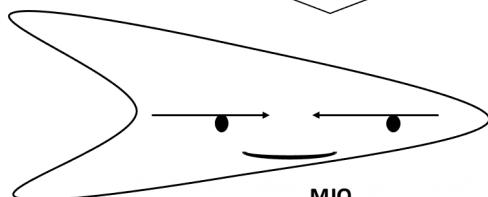


Hi, I'm a moist Kelvin wave.
Just call me Kelvin.



1

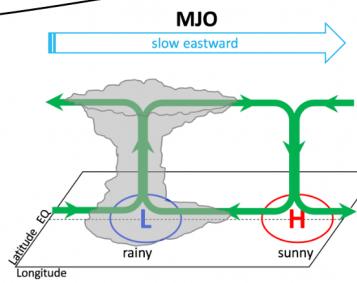
I live in the tropics.
I'm big.
I'm powerful!



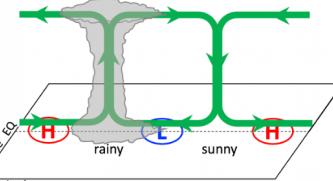
I'm a proud member of convectively coupled equatorial waves.
Shallow water equations are so cool!
They know my family lives in the tropics.



MJO
slow eastward

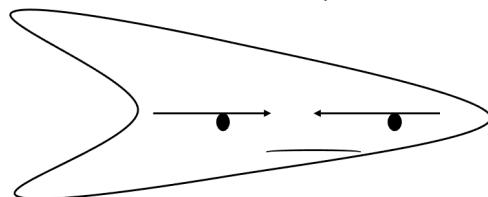


KELVIN
fast eastward



2

You are lucky..
People gave me so many
different nicknames,
but I'm not sure who I am..



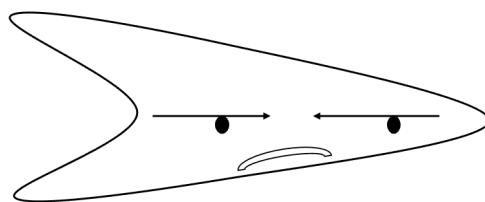
You know what?
Kim & Zhang told me that
you are actually a big
brother of mine!



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What??
 I don't look like you.
 I don't look like any other
 equatorial waves.
 How can I be your brother?
 Are you saying
 I'm also the Kelvin family?

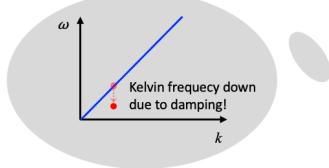
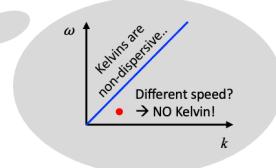
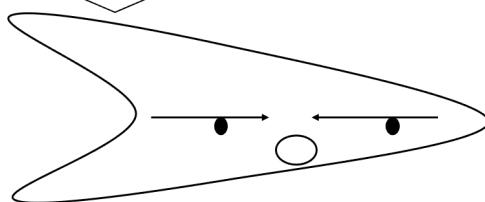
Kinda.
 Because you look so different from
 other Kelvins, you may think you
 are a different type of oscillation.
 But, you are just a big, sluggish
 modified Kelvin wave.



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That doesn't make any sense!
 Kelvin family must have the same
 speed. I'm much slower than you.
 I cannot be a Kelvin member!
 No way!!

Yes, you are slow because you feel drag a lot.
 Friction is everywhere, but I don't feel much
 because I'm small so my footstep frequency is high.
 I can outrun before drag catches me up.
 However, because your body is big so your time
 scale is long enough to feel atmospheric damping.
 The drag pulls you down before your next footstep.

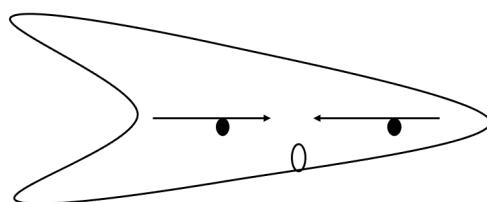


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I still don't understand.
 Show me the equations!

Sure, in my linearized equatorial
 shallow water equations,
 just add one extra term—
 DAMPING!

Then, everything follows.



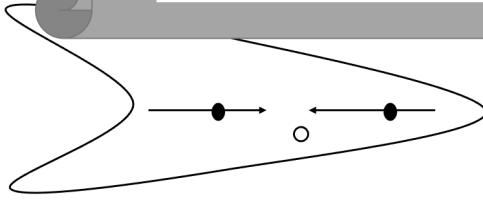
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EPISODE2. TUNE THE FREQUENCY

The Three Conservations

$$(1) \frac{\partial u}{\partial t} + Du + \frac{\partial \phi}{\partial x} = 0$$

$$(2) \beta y u + \frac{\partial \phi}{\partial y} = 0$$

$$(3) \frac{\partial \phi}{\partial t} + gH \frac{\partial u}{\partial x} = M$$


There are all we need.
How simple is that!!

- ✓ Same as Matsuno's Kelvin equations except for **Du** and **M**.
- ✓ **Du** is damping.
- ✓ **M** could be zero, external force, or feedback. All three cases can generate MJOs.



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Combine the zonal momentum (1) and mass conservation (3) equation, then we have a **harmonic oscillator** equation!

$$\frac{\partial^2 u}{\partial t^2} + D \frac{\partial u}{\partial t} + \omega_0^2 u = X$$

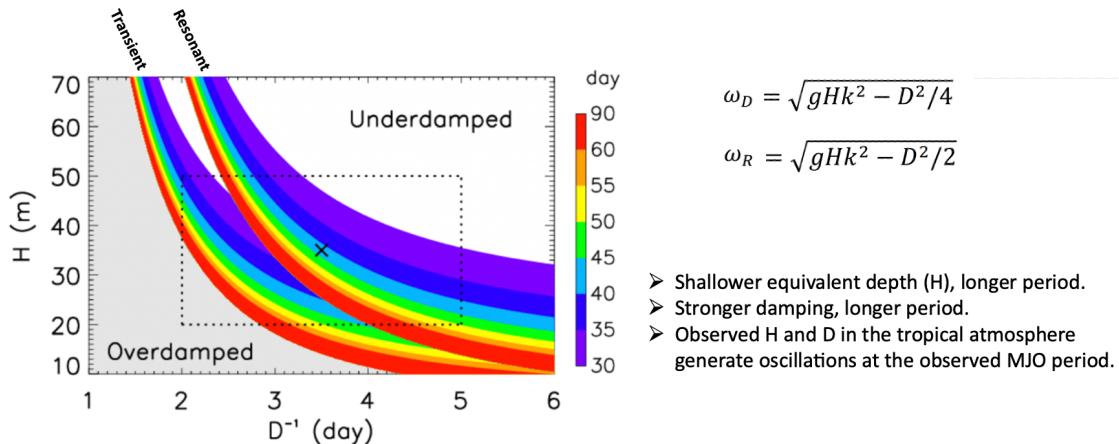
This can be easily solved (and solutions are well known):

- $D=0$ & $X=0$: Simple harmonic \rightarrow Classical Kelvin waves: $u = u_0 e^{i(kx - \omega_0 t)}$, $\omega_0 = k\sqrt{gH}$
- $D \neq 0$ & $X=0$: Damped harmonic \rightarrow Transient solution: $u = u_0 e^{-\frac{D}{2}t} e^{i(kx - \omega_D t)}$, $\omega_D = \sqrt{\omega_0^2 - D^2/4}$
- $D \neq 0$ & $X \neq 0$: Driven harmonic \rightarrow Resonant solution: $u = u_0 e^{i(kx - \omega_R t)}$, $\omega_R = \sqrt{\omega_0^2 - D^2/2}$



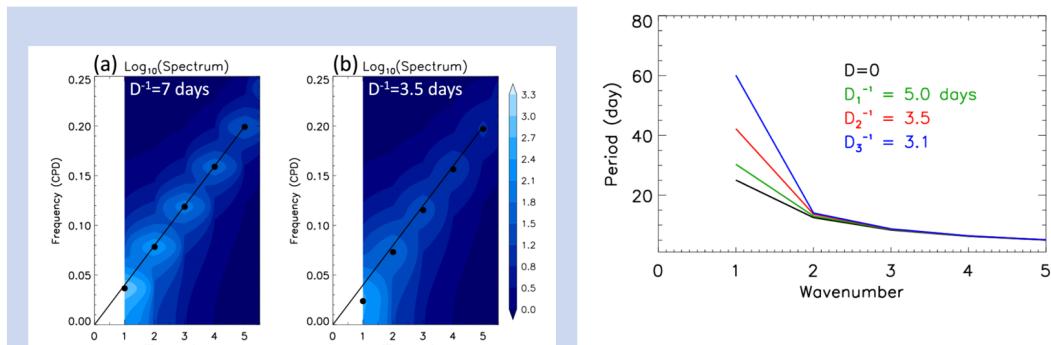
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Double (upside-down) Rainbow of the longest ($k=1$) Earth wave's frequency response



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Spectral response under different damping magnitudes for $H=35m$



- Spectrum is like Kelvin under weak damping
- Spectrum maxima shifts to lower frequencies
- Biggest shift occurs at $k=1$ (like MJO!)

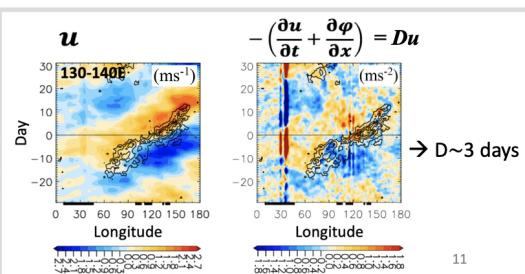
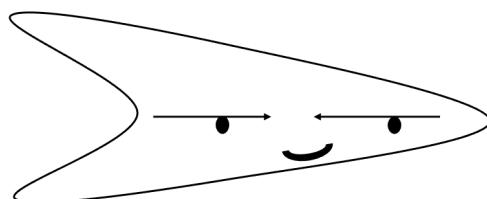
$$\omega_R = \sqrt{gHk^2 - D^2/2}$$

Strong response at $k=1$, negligible at $k \geq 2$
→ Scale selection!

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The used damping seems too strong.
Can the atmosphere have the
damping time scale of ~ 3 days?

Such strong damping is needed to explain
other large-scale convection-circulation
coupled phenomena such as monsoon and
Walker circulation. Many studies have already
used similar scales, and I can prove it with
observational data!

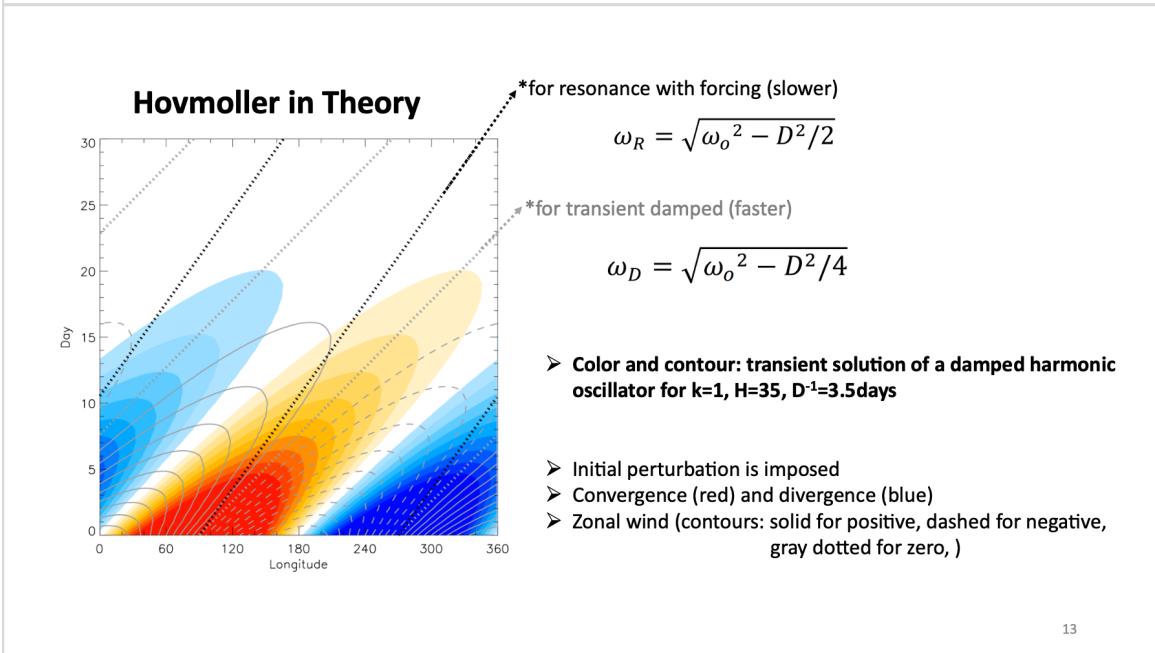


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**It's nonsense.
If damping is ~3-day scales,
then I would be damped
down very quickly,
and I wouldn't have
repeating oscillations.**

**True. Sometimes you disappear after a single event.
In this case, you are weak and fast.
BUT, with lots of forcing sources within the tropics
and from higher latitudes, **resonance** can occur.
Then you have strong, slow successive events.
Interestingly, observations show weak events tend
to move fast with quick decay, while strong events
are slower and have multiple oscillations.**

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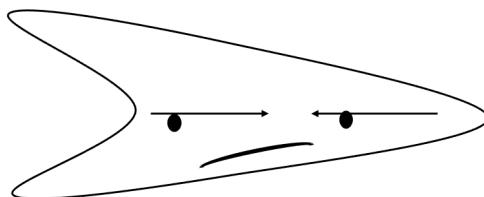
Resonance is commonly observed in other oscillatory systems. Did I say our zonal momentum and mass conservation equations combine into the **universal harmonic oscillator equation? Kelvins and MJOs, thus their impacts on weather on Earth, are after all governed by the same physics for spring motion, swing of a pendulum, RLC circuits, etc. How cool is that?**

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EPISODE3. WHY AM I DIFFERENT?

What about my shape?
It's not a round shape like you.
I look more like a swallowtail.

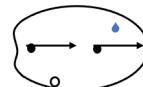
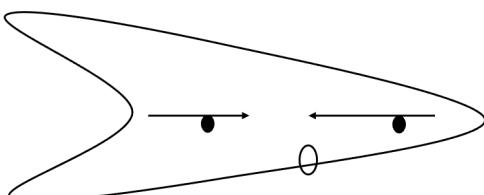
That is also because of damping.
If you look at me closely,
I also have a teeny tiny dent
on my back.



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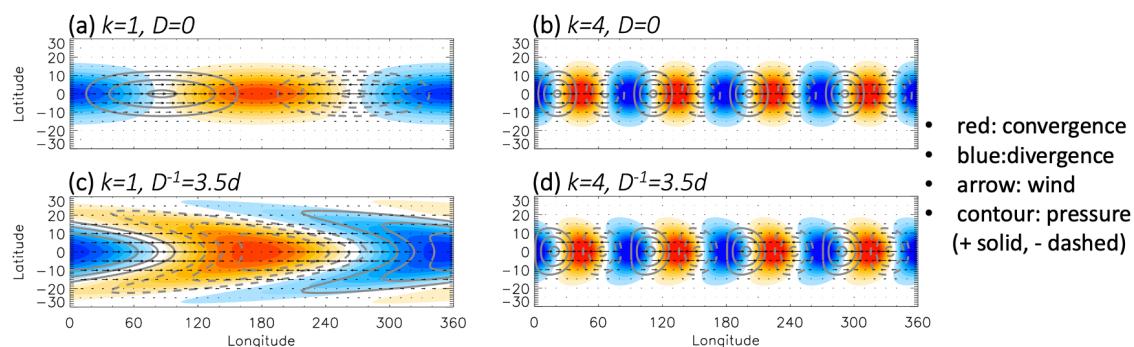
You are right!

See?
This is a sign of damping
dragging me just a little bit.



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Theoretical horizontal structures



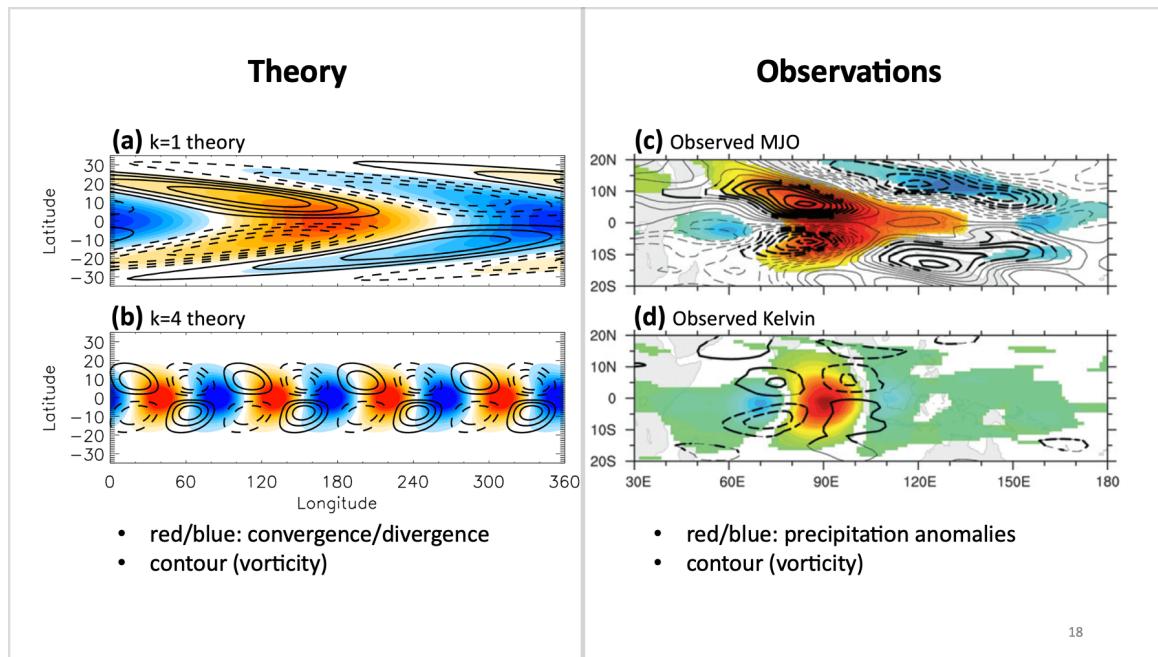
Combine equations (1) and (2): $\frac{\partial^2 u}{\partial y \partial t} + D \frac{\partial u}{\partial y} - \beta y \frac{\partial u}{\partial x} = 0$

$$u(y) = e^{-l^2 y^2} e^{im^2 y^2}$$

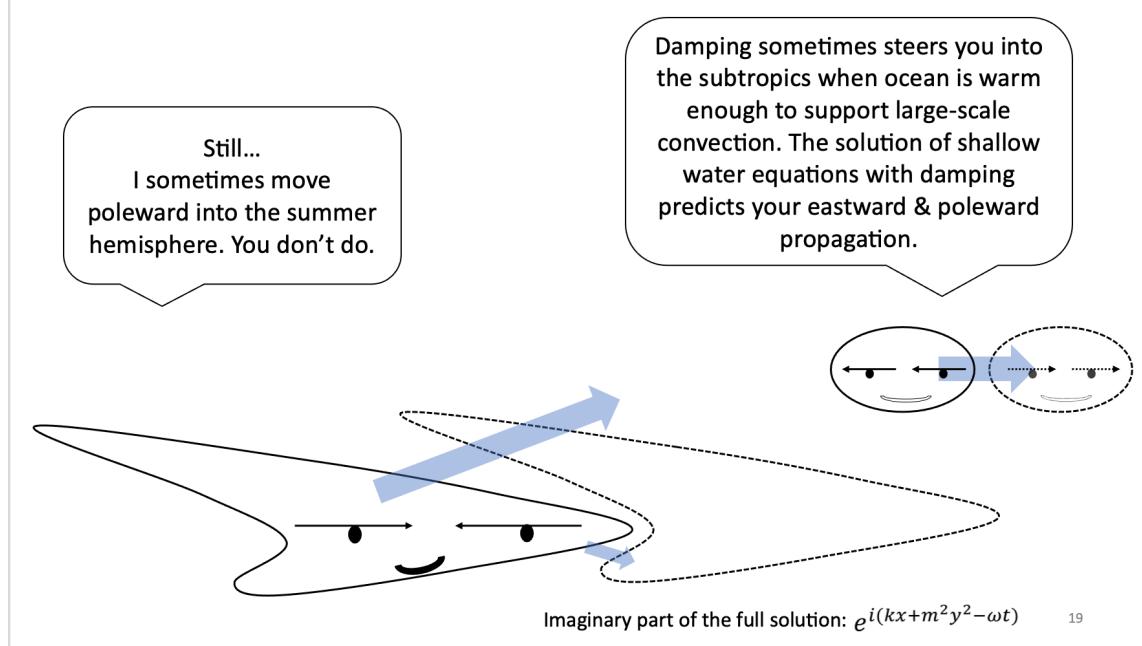
$$l^2 = \frac{\beta c_r}{2(c_r^2 + c_i^2)} \text{ and } m^2 = \frac{\beta c_i}{2(c_r^2 + c_i^2)} \text{ with } c_r = \omega/k, c_i = D/k$$

* If $D=0$; $u(y) = e^{-\beta y^2/2c_0}$ where $c_0 = \omega_0/k$ (classical Kelvin waves!)

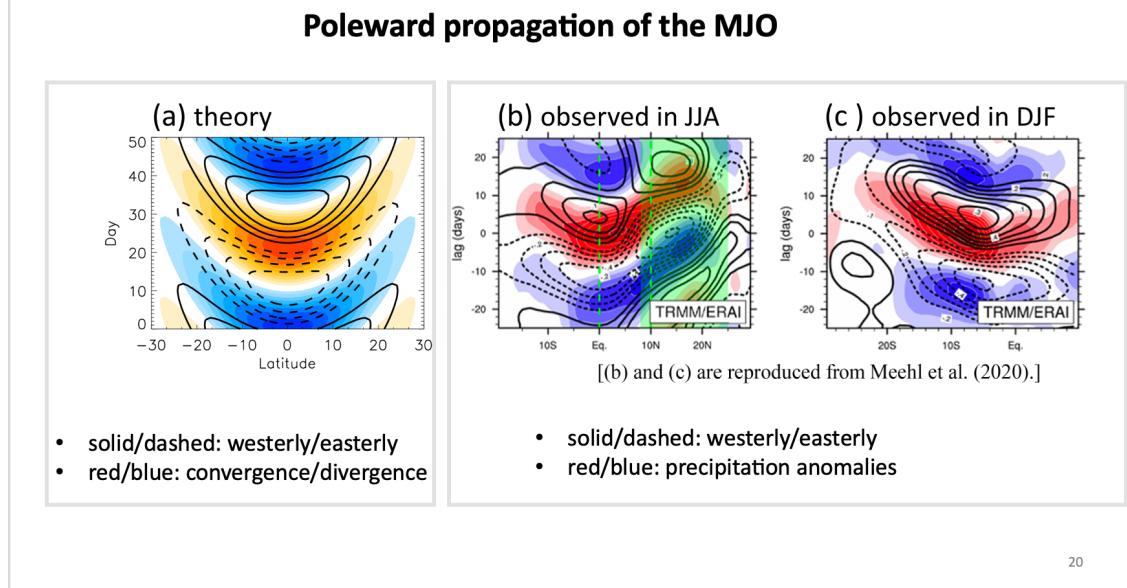
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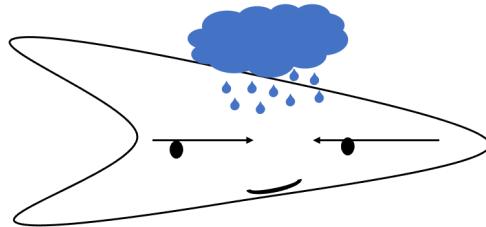
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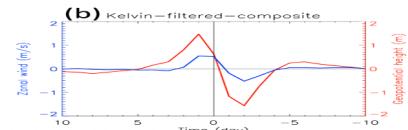
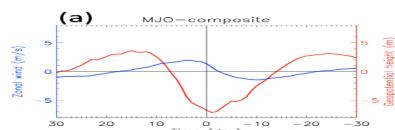
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EPISODE4. WITH OR WITHOUT YOU

Well..
My convergence-convection center is in the middle of my low pressure.
Yours is totally different!



The phase between zonal wind and pressure depends on a wave scale relative to the damping time scale! Look at the zonal momentum equation.



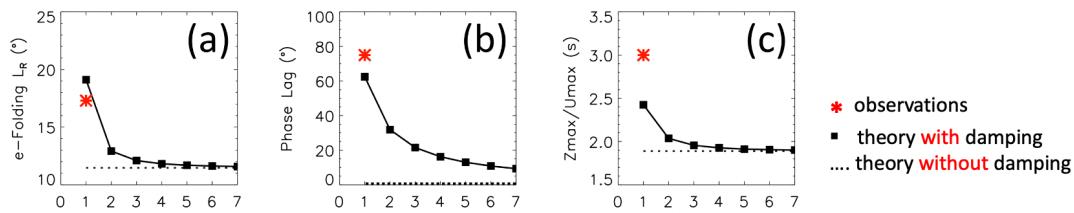
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From equation (1), the lag between wind and geopotential is given:

$$Re(u) = u_0 \cos \theta$$

$$Re(\phi) = u_0 A \cos(\theta + \theta_{lag})$$

$$\theta_{lag} = \tan^{-1}(D/\omega)$$



* observations
■ theory with damping
.... theory without damping

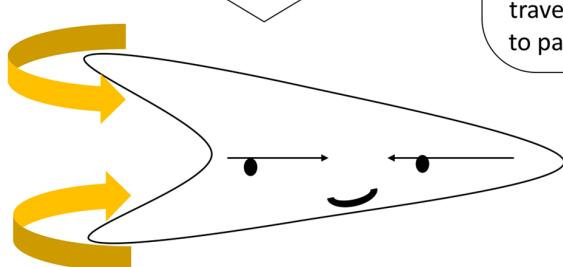
➤ Much wider meridionally

➤ Almost quarter angle lag

➤ Stronger pressure response

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I am always with my best buddy Rossby, but you don't have anyone. I'm a Kelvin-Rossby couplet, but you are single.



Rossby only follows you because your convective heating is large and slow enough to wake him up. You are who you are with or without Rossby. Nonetheless, it's nice to have such a friend! He travels around the world to pass on tropical news.

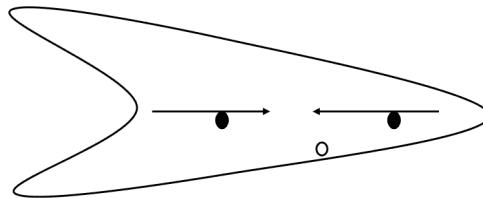
I'm too small and too fast to shake big and slow Rossby. Yanai is antisymmetric and I am symmetric to the equator, so he doesn't respond too. That's why I play solo.



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If there is feedback,
what happens to me?
Can I be made from
instability due to feedback?

Good question!
You can maintain your amplitude longer
with a right combination of feedback
effects. But, mathematics and observations
say that you are not an unstable mode. Your
growth rate under feedback is still negative.
Keep in mind you can show up
with or without feedback!



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- For the mass equation (3), take a simplest form to represent feedback effects that are relevant to wind and pressure.

$$\frac{\partial \phi}{\partial t} + gH \frac{\partial u}{\partial x} = au + b\phi$$

- Combine with Equation (1).

$$\frac{\partial^2 u}{\partial t^2} + D \frac{\partial u}{\partial t} + \omega_0^2 u = -a \frac{\partial u}{\partial x} + b \frac{\partial u}{\partial t} + bDu$$

- Solution would be like this.

$$u(x, t) = e^{\alpha t} e^{i(kx - \omega t)}$$

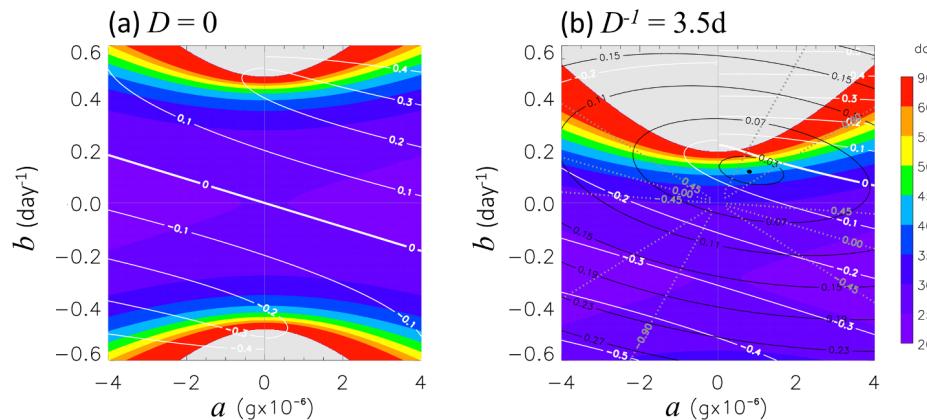
- Solve!

Then, we get the growth/decay rate and dispersion relation.

$$\alpha = \frac{1}{2} \left(a \frac{k}{\omega} + b - D \right)$$

$$\omega^2 = \frac{1}{8} \left\{ 4gHk^2 - (b + D)^2 + \sqrt{[4gHk^2 - (b + D)^2]^2 + 16a^2k^2} \right\}$$

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Periods (color) and the growth/decay rate (white curves). Negative values of white curves represent decay. (b) Black lines show RMSD between the observation-based budget and theoretical estimates, and gray dotted lines are correlation coefficients between them. The minimum RMSD and maximum correlation between observations and theory is located at the black point.

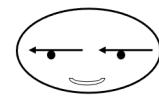
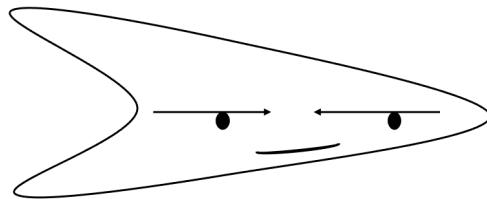
The black point in (b) suggests -- the most likely values of two feedback coefficients (a, b)
-- negative growth rate (so, no instability)
-- period matching the observed one

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Hmm..
Everything seems to make sense,
but I still have lots of questions.

Find more information
in Kim & Zhang (2020)!

<https://doi.org/10.1175/JAS-D-20-0193.1>



ABSTRACT

Substantial progress has been made in understanding the dynamics of the MJO during the past decades. However, there is no consensus on the underlying mechanism for the MJO. Most current ideas of MJO dynamics involves atmospheric deep convection and its interaction with moisture. These theories explain the eastward propagation of the MJO through moisture enhancement to the east of large-scale convection. Here, using a simple analytical approach with equatorial shallow water equations, we revisit the view on the MJO as slowed-down Kelvin waves due to damping. We show that the dynamic core of the MJO can be described in terms of a harmonic oscillator that can be excited by stochastic forcing. The mechanism for selecting MJO scales comes from momentum damping, and the mode propagates eastward because of exactly the same reason for Kelvin waves to propagate eastward due to the Coriolis effect. Our unified theory of fast Kelvin waves at synoptic scales and the slow MJO at planetary scales supports the notion of a distinction and continuous transition between equatorial Kelvin waves and the MJO. We also discuss the role of feedback mechanisms that help sustain an MJO event longer.