Risk Parity Strategies
For Equity Portfolio Management

London, January 2015

Frank Siu
Director, Index Research
Axioma

Agenda

- Methods for constructing risk parity portfolios.
- Risk parity across individual stocks, groupings of stocks, and risk factors.
- How different formulations/variants of risk parity affect portfolio composition and risk profile.
- Risk parity as a “standalone” strategy or an overlay on top of an existing investment process?

Preliminaries

- Increasing popularity of risk-based strategies, “Smart Beta”.
- *Risk parity equalizes sources of risk such that the <u>relative marginal contribution to risk</u> (RMCTR) from each source is equal.*
- Given a n-vector of weights on sources of risk w and their covariance matrix Q, we have:

$$\frac{w_i (Qw)_i}{w^T Q w} = \frac{w_j (Qw)_j}{w^T Q w} \quad \forall i, j = 1 \dots n$$

- In asset allocation setting, n represents asset classes and is small.
- For equity portfolios, n is usually larger and may represent individual stocks, groupings of stocks, or factors.

Solving for Risk Parity Portfolios

- Few technical discussions around exactly how risk parity portfolios are built.
  - Robert M. Anderson, Stephen W. Bianchi, and Lisa R. Goldberg. *Will My Risk Parity Strategy Outperform?* Financial Analysts Journal, 2012.
  - Xi Bai, Katya Scheinberg, and Reha Tutuncu. *Least-Squares Approach to Risk Parity in Portfolio Selection*, 2013.
  - Denis Chaves, Jason Hsu, Feifei Li, and Omid Shakernia. *Efficient Algorithms for Computing Risk Parity Portfolio Weights*. Journal of Investing, 2012.
  - Hakan Kaya and Wai Lee. *Demystifying Risk Parity.* 2012.
  - Hossein Kazemi. *An Introduction to Risk Parity*. 2012.
  - Sebastien Maillard, Thierry Roncalli, and Jerome Teiletche. *The Properties of Equally Weighted Risk Contribution Portfolios*, 2010.

- Analytical solutions with model simplification (e.g. ignoring correlations); iterative numerical algorithms; ad-hoc trial-and-error approaches with rudimentary statistical packages, etc.

Asset Risk Parity

- A commonplace method is to minimize a portfolio's *deviation* from risk parity.
- For example, using a general nonlinear solver, we can solve:

$$w = arg \min_{w} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{w_i (Qw)_i}{w^T Qw} - \frac{w_j (Qw)_j}{w^T Qw} \right)^2 \ \forall i \neq j$$

- Does not guarantee risk parity portfolios. Returns portfolios that have exhausted the particular solver's ability to find a better solution.
- Does not differentiate between large violations from a few stocks vs. many small violations.

Asset Risk Parity

- We propose "enforcing" risk parity through constraints on the RMCTR of each stock instead:

$$\frac{w^T \Lambda_i Q w}{w^T Q w} \geq \frac{1}{n} \quad \forall i = 1 \dots n$$

- This expression can be formulated and solved as a convex optimization constraint. Λ_i is a matrix with $\Lambda_{ii} = 1$ and zero elsewhere.

- It can be shown (e.g. Kaya and Lee 2012) that, for a long-only and fully-invested portfolio, there exists a unique set of weights w that satisfy the RMCTR constraints.

- Uniqueness only applies to asset risk parity case, parity across groupings of stocks or risk factors is more complex.

Asset Risk Parity in Blue-Chip Universes

- Risk parity seems well-suited to these universes containing relatively small numbers of stocks, where there is considerable concentration risk that can be diversified.

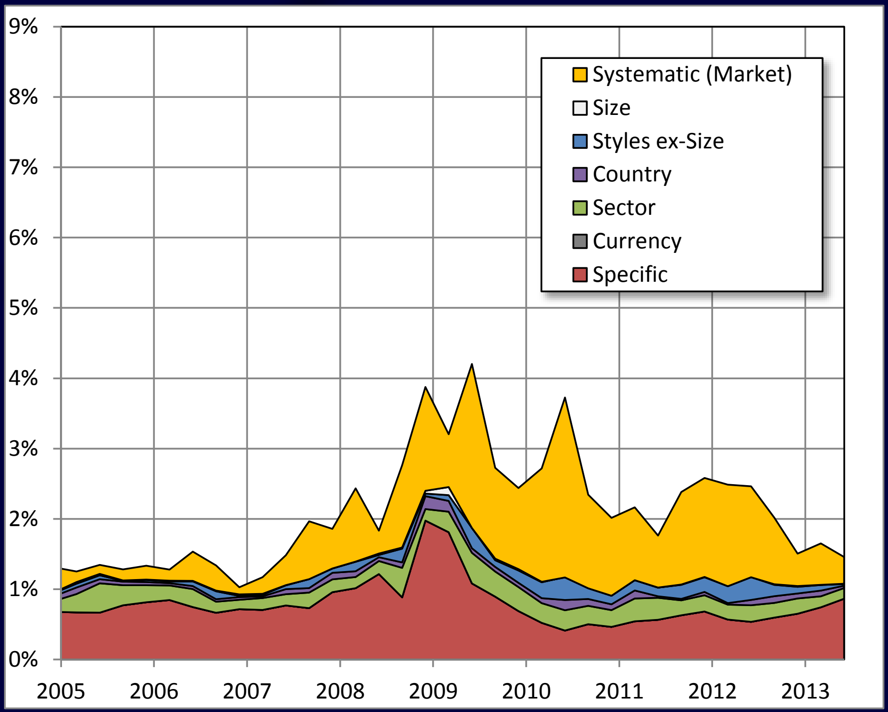
	EURO STOXX 50	Netherlands **AEX**	Austria **ATX**	Belgium **BEL**	France **CAC**	Germany **DAX**	Spain **IBEX**	Italy **MIB**	Switzerland **SMI**
Total Return	6.88%	8.22%	5.82%	5.76%	6.39%	10.38%	6.52%	2.91%	8.56%
Realized Risk	16.29%	18.23%	20.65%	14.06%	18.32%	18.00%	19.11%	19.29%	13.97%
Sharpe Ratio	0.42	0.45	0.28	0.41	0.35	0.58	0.34	0.15	0.61
Active Return	1.77%	3.30%	3.15%	1.83%	1.94%	1.50%	1.07%	3.00%	1.49%
Specific	0.87%	4.51%	2.87%	2.11%	1.51%	0.87%	2.48%	3.70%	1.17%
Factor	0.91%	-1.21%	0.28%	-0.28%	0.43%	0.62%	-1.41%	-0.71%	0.32%
Tracking Error	2.17%	5.48%	7.67%	6.18%	3.39%	4.98%	5.08%	4.94%	5.10%
Information Ratio	0.82	0.60	0.41	0.30	0.57	0.30	0.21	0.61	0.29
Hit Rate	62.96%	63.89%	59.26%	61.11%	57.41%	67.59%	56.48%	58.33%	61.11%
Beta	0.95	0.92	0.86	0.80	0.97	0.91	0.91	0.89	0.95
Avg. Names	50	25	20	20	40	30	35	40	20
Avg. Annual Turnover	36.43%	35.85%	41.00%	35.40%	28.05%	29.98%	35.37%	33.33%	29.50%

Backtest from 2005 through 2013, rebalanced quarterly. Risk and return figures are annualized.
Statistics computed using monthly gross returns in EUR (CHF for SMI).

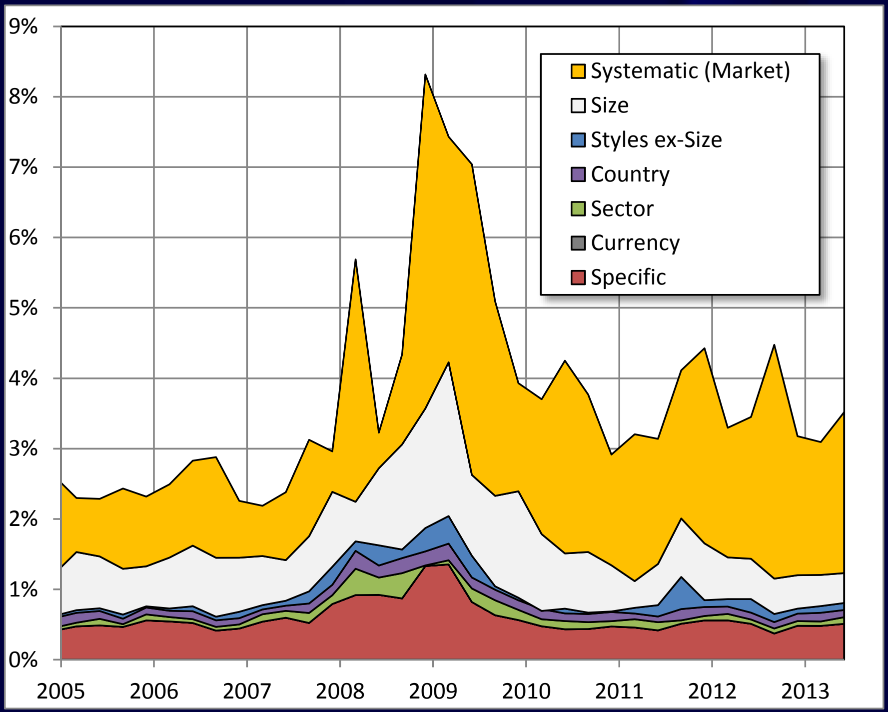
Asset Risk Parity in Broader Universes

- With broader universes, the risk profile of the strategy becomes dependent on *benchmark structure*, as every stock must be held.

- Active risk decomposition of two asset risk parity strategies, on the EURO STOXX 50 and EURO STOXX universes, respectively:



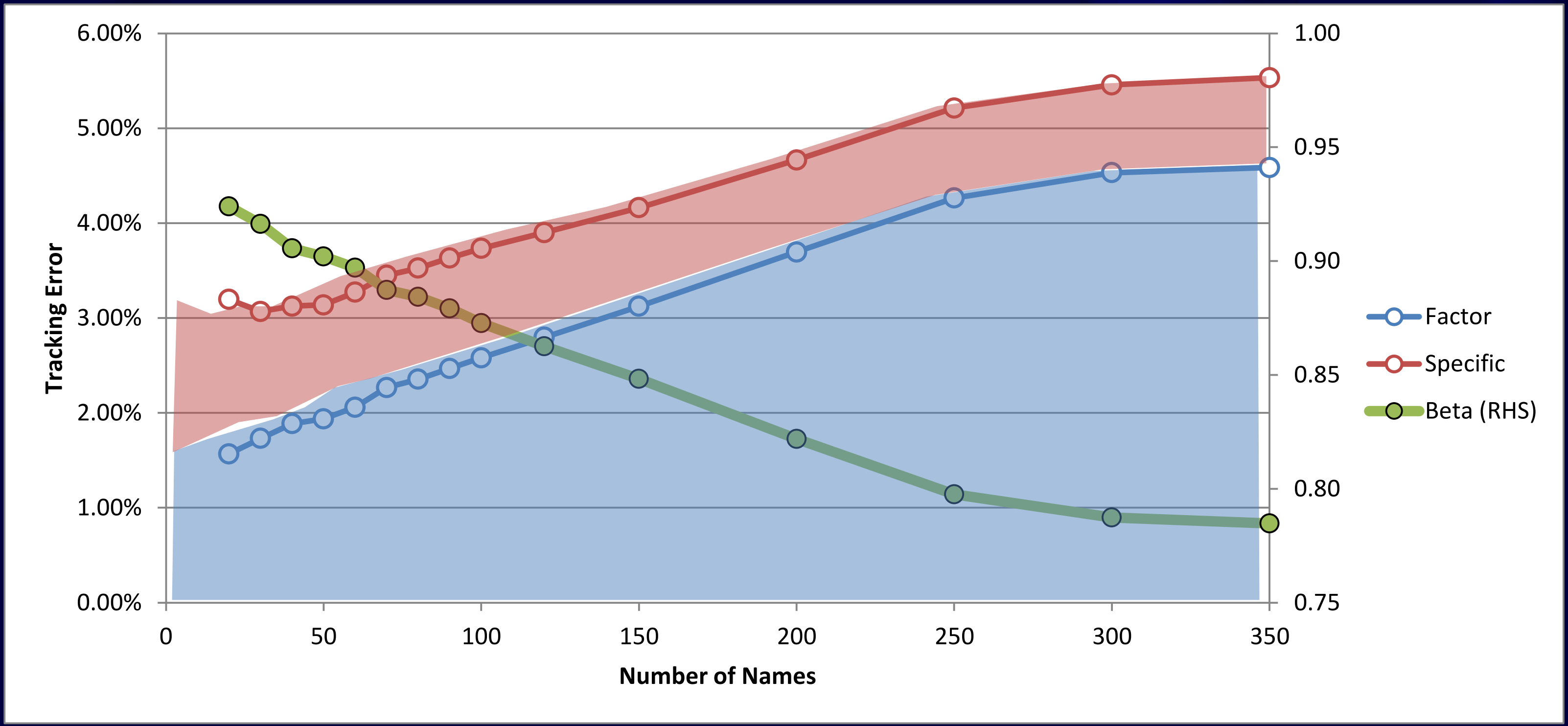
EURO STOXX 50 (50 names)



EURO STOXX (~300 names)

Asset Risk Parity in Broader Universes

- As universe breadth increases, drivers of risk/return shift disproportionately towards systematic factors, such as Size.
- Investment thesis changes considerably, portfolio characteristics approach that of a low-beta, small-cap strategy. Returns become dominated by factors rather than the effects of risk diversification.



Asset Risk Parity in Broader Universes

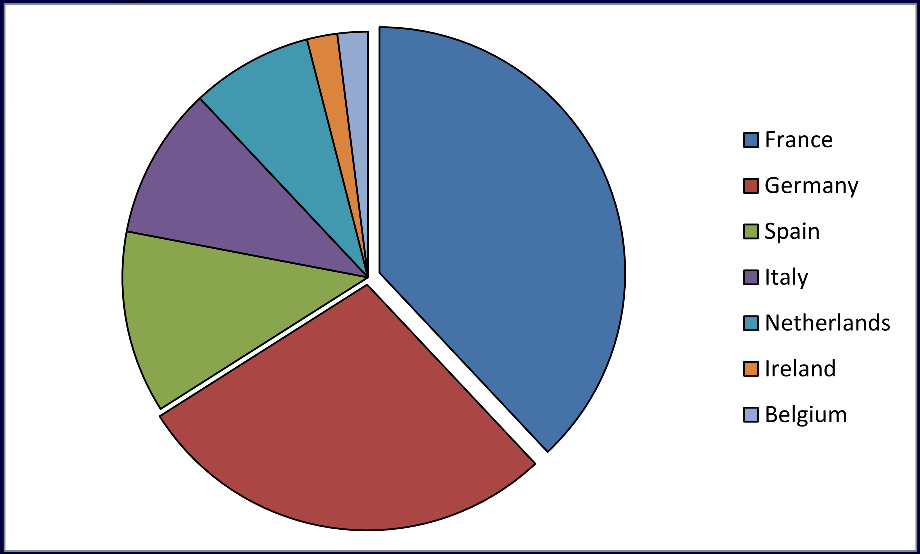
- Increasing universe breadth sometimes improves returns, but changes the drivers of risk and return substantially.
- Increased tracking error worsens risk-adjusted active returns (IR).

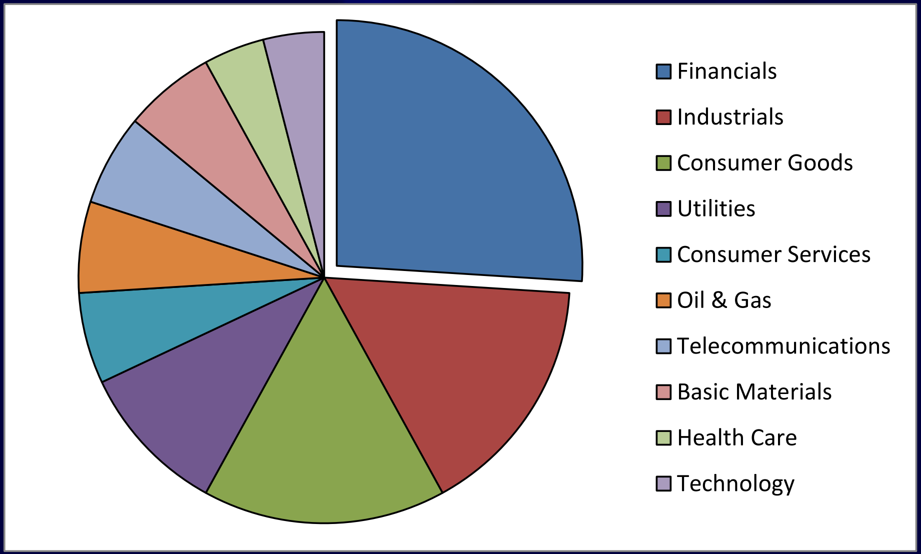
	Eurozone		France		Germany		Spain		Switzerland	
	EURO STOXX 50	**EURO STOXX**	**CAC**	**CAC-All***	**DAX**	**HDAX + SDAX**	**IBEX**	**IGBM**	**SMI**	**SMI Expanded**
Total Return	6.88%	6.82%	6.39%	7.52%	10.38%	11.04%	6.52%	1.82%	8.56%	8.79%
Realized Risk	16.29%	16.61%	18.32%	16.83%	18.00%	18.36%	19.11%	17.63%	13.97%	15.47%
Sharpe Ratio	0.42	0.41	0.35	0.45	0.58	0.60	0.34	0.10	0.61	0.57
Active Return	1.77%	1.21%	1.94%	1.86%	1.50%	1.76%	1.07%	-2.89%	1.49%	1.61%
Specific	0.87%	2.34%	1.51%	2.93%	0.87%	4.59%	2.48%	-0.21%	1.17%	2.83%
Factor	0.91%	-1.13%	0.43%	-1.06%	0.62%	-2.82%	-1.41%	-2.68%	0.32%	-1.22%
Tracking Error	2.17%	3.96%	3.39%	7.25%	4.98%	7.91%	5.08%	9.80%	5.10%	6.89%
Information Ratio	0.82	0.31	0.57	0.26	0.30	0.22	0.21	-0.29	0.29	0.23
Hit Rate	62.96%	62.04%	57.41%	64.81%	67.59%	66.67%	56.48%	53.70%	61.11%	65.74%
Beta	0.95	0.87	0.97	0.70	0.91	0.81	0.91	0.74	0.95	0.90
Avg. Names	50	311	40	260	30	160	35	118	20	50
Avg. Annual Turnover	36.43%	39.83%	28.05%	52.84%	29.98%	48.26%	35.37%	50.90%	29.50%	35.89%

Backtest from 2005 through 2013, rebalanced quarterly. Risk and return figures are annualized.
Statistics computed using monthly gross returns in EUR (CHF for SMI).
* The CAC All-Tradable replaced the SBF 250 in early 2011.

Equalizing Sources of Risk

- With asset risk parity over n stocks, risk is not really being diversified across n independent sources.
- Equities in same sector/country are typically very correlated.
- Consider asset risk parity on the EURO STOXX 50 universe; let us examine risk contributions by grouping stocks by country/sector:





Risk Parity Over Groupings of Stocks

- Consider equalizing risk across groupings of stocks, such as country/sector buckets:

$$\sum_{i \in S} \frac{w^T \Lambda_i Q w}{w^T Q w} \geq \frac{1}{m} \quad \forall S$$

- Here we have m groupings S. Again, Λ_i is a matrix with $\Lambda_{ii} = 1$ and zero elsewhere.

- Unlike the earlier RMCTR constraint, this constraint is continuous non-convex, therefore significantly more difficult to solve.

Risk Parity Over Groupings of Stocks

- Cannot guarantee optimality or solution uniqueness.
- The number of names is now free to vary, no longer dictated by the breadth of the universe. We don't have to hold every stock.
- Solution quality depends heavily on the numerical techniques employed by the particular solver used.
- Small universes with sparsely populated groupings (e.g. small countries) are especially problematic, giving the optimizer insufficient choices for diversification.
- Unlike asset risk parity, risk parity with groupings might be more suitable for broader universes.

Risk Parity Over Groupings of Stocks

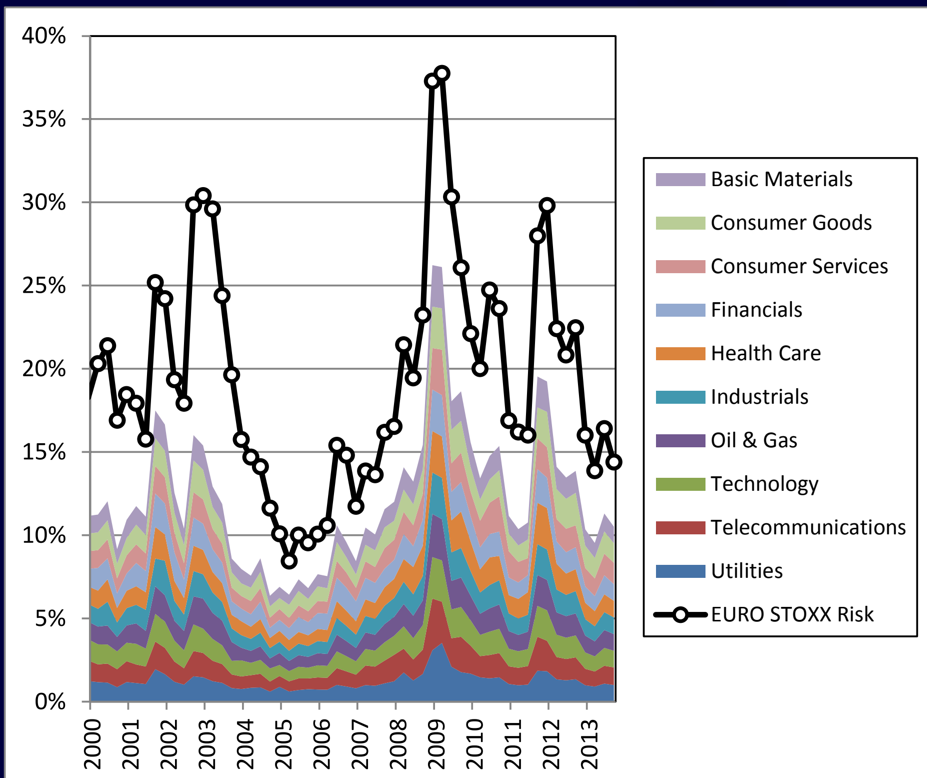
- By enforcing risk parity via constraints, we “free up” the objective function to pursue other investment goals.
- Multiple solutions may exist; objective function matters!

	EURO STOXX	Sector Risk Parity Minimize Risk	Sector Risk Parity Minimize Risk Target 30% Turnover	Sector Risk Parity Minimize Tracking Error Target 30% Turnover
Total Return	3.57%	7.77%	8.39%	3.74%
Realized Risk	18.69%	10.83%	10.93%	18.70%
Sharpe Ratio	0.19	0.72	0.77	0.20
Active Return		4.20%	4.82%	0.17%
Specific		1.04%	2.11%	-0.22%
Factor		3.17%	2.71%	0.39%
Tracking Error		12.43%	11.13%	2.05%
Information Ratio		0.34	0.43	0.08
Hit Rate		69.44%	68.06%	59.72%
Beta		0.51	0.56	1.00
Avg. Names		41	52	172
Avg. Annual Turnover		222.08%	42.98%	32.82%

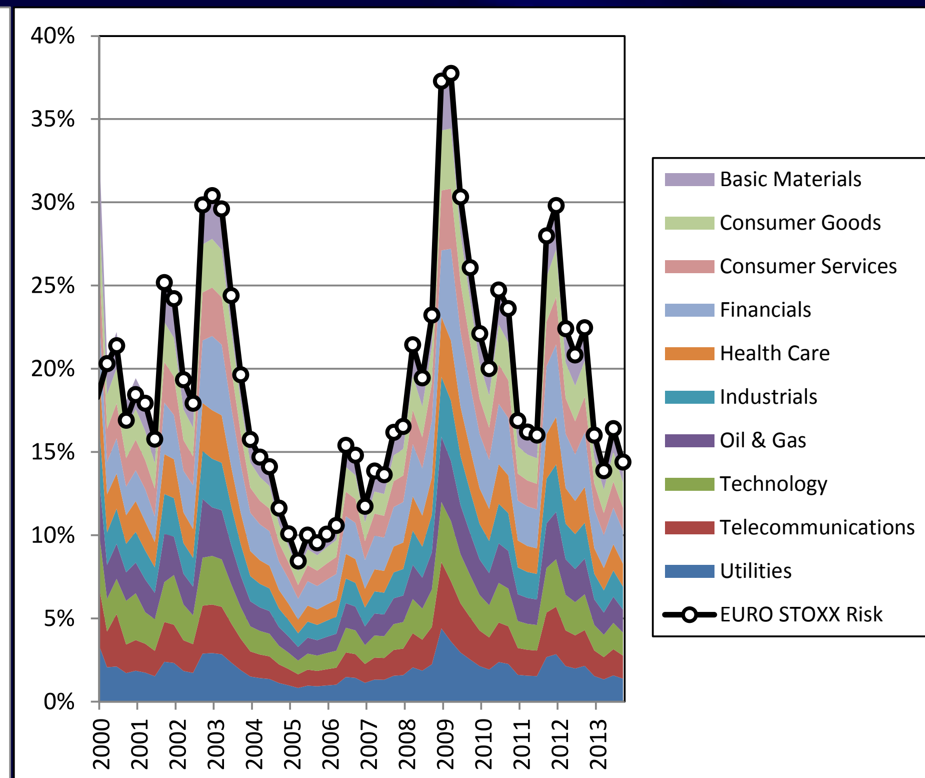
Backtest from 2002 through 2013, rebalanced quarterly. Risk and return figures are annualized.
Statistics computed using monthly gross returns in EUR.

Risk Parity Over Groupings of Stocks

- Compare risk decomposition of two sector risk parity strategies, one that minimizes total risk and another tracking error.



Sector Risk Parity
Minimize Total Risk



Sector Risk Parity
Minimize Tracking Error (vs. EURO STOXX)

Factor Risk Parity

- Use a factor risk model to decompose the asset covariance matrix Q into common factor and specific components:

$$Q = X\Omega X^T + \Delta$$

- X is a matrix of factor exposures, Ω the factor covariance matrix, Δ a diagonal matrix of stock specific risks.

- Break down into country, industry, and style factors:

$$X = \lfloor X_C \quad X_I \quad X_S \rfloor$$

$$\Omega = \begin{bmatrix} \Omega_{CC} & \Omega_{CI} & \Omega_{CS} \\ \Omega_{IC} & \Omega_{II} & \Omega_{IS} \\ \Omega_{SC} & \Omega_{SI} & \Omega_{SS} \end{bmatrix}$$

- Risk parity across country risk factors:

$$\frac{f_C^T \ \Lambda_i \Omega_{CC} f_c}{f_C^T \ \Omega_{CC} f_c} = \frac{f_C^T \ \Lambda_j \Omega_{CC} f_c}{f_C^T \ \Omega_{CC} f_c} \quad \forall i, j \ \epsilon C \quad f_c \equiv X_C^T w$$

Summary

- We proposed an optimization-based approach to solving for risk parity portfolios using constraints on RMCTR.
- Asset risk parity is a standalone strategy, well-suited to concentrated asset universes with few names.
- Other forms of risk parity do fully define a portfolio strategy. The choice of objective function and other constraints matter greatly.
- Risk parity over groupings of stocks is better suited to broader universes, as a risk-control overlay to an existing investment strategy.
- Risk parity with risk factors is complex and results are highly sensitive to the factor modeling methodologies used.