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Inter-temporal risk parity

INQUIRE Practitioner Seminar, London

20th January 2015

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Agenda

- Definition of Inter-temporal Risk Parity:
  - Also known as Target risk, Inverse risk weighted, Iso-risk
- Return distributions asset classes and factor premia are not Gaussian
  - Volatility clustering
  - Fat tails
  - Leverage effect
- It is all in the GARCH
  - How GARCH parameters tell you about the success of the strategy
  - Monte Carlo Simulations
- How it works in real life
  - Historical back-tests
  - Improvement of risk-adjusted returns
  - Control of risk in ex-post

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What is inter-temporal risk parity?

- Systematic strategy rebalancing between a risky asset and cash
- Weight of risky asset is set so that ex-ante risk is kept constant
- Return to the strategy is:

$$r^{IRP}{}_t = r_t \frac{\kappa}{\sigma_t} + r_c\left(1 - \frac{\kappa}{\sigma_t}\right)$$

$r^{IRP}{}_t$	return to Inter-temporal Risk Parity strategy
r_t	risky asset return
r_c	cash return

σ_t	ex-ante volatility with all info until t -1
κ	pre-defined target risk budget
κ / σ_t	weight of risky asset

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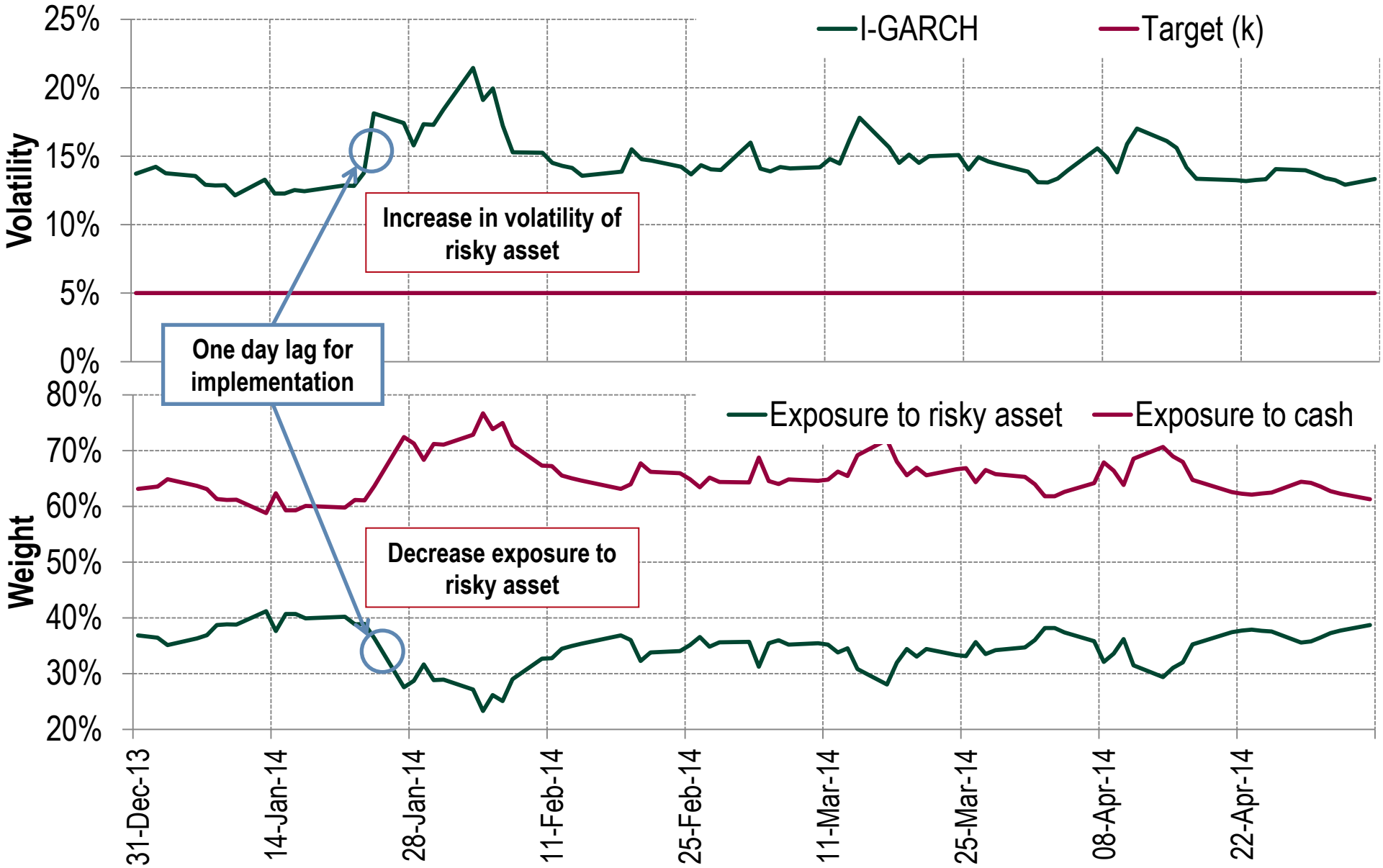
Inter-temporal risk parity strategy for S&P500 at target risk 5%



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What to expect if returns followed Gaussian distributions

Substantial effort for nothing. But no loss either at least before transaction costs.

Gaussian distributed returns	Buy and Hold	Inter-temporal Risk Parity
Average annualized excess return	7.5%	7.6%
Average annualized volatility	19.0%	19.5%
Sharpe ratio	0.39	0.39
Maximum drawdown (MDD)	-45.5%	-46.5%
Ratio MDD / volatility	-2.4	-2.4
Average exposure	100.0%	101.8%*
Improvement in Sharpe ratio	-	0.00
Std Dev of improvement in Sharpe ratio	-	2.5%

Averages from 5,000 Monte-Carlo simulations of 5,200 daily returns each, i.e. 20 years.
Target 19.0% volatility with 42-day historical ex-ante volatility. Risky asset returns drawn from N(7.5%, 19.0%).

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin, "Inter-temporal risk parity: A constant volatility framework for equities and other asset classes." Forthcoming Journal of Alternative Investments, 2015

* Average of $1 / \sigma_t > 1$ for an uniform function, thus average exposure > 100%

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Evidence

- Managing equities at constant risk seems to add value:
  - Hocquard, Ng and Papageorgiou (2013)
  - Cooper (2010)
  - Kirby and Ostdiek (2012)
  - Ilmanen & Kizer (2012)
  - Giese (2012)

Higher Sharpe ratio and smaller drawdowns with constant volatility strategy..

- No consensus regarding where added value comes from:
  - Hallerbach (2012)

Better volatility forecast and less variability in volatility is sufficient to improve Sharpe ratio.

- Risky assets do not follow Gaussian distributions:
  - Rama Cont (2001)

Volatility clustering, fat tails, leverage effect, etc.

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Return distributions are not Gaussian

Evidence of volatility regimes with different Sharpe ratio => targeting constant risk may add value!

	S&P 500	Russell 1000	MSCI Emerging Markets	S&P GSCI Commodity	US High Yield Bonds	US Inv. Grade Bonds	US 10Y Govern. Bonds
Regime 1							
Volatility	29%	29%	31%	30%	8%	7%	11%
Sharpe Ratio	**-0.5**	**-0.5**	**-1.3**	**-0.2**	**-1.0**	**0.0**	**-0.2**
% observations in regime 2	27%	27%	24%	37%	25%	34%	20%
Probability that return at t+1 is in regime 1	97%	97%	95%	97%	85%	97%	97%
Regime 2							
Volatility	11%	11%	12%	14%	2%	4%	6%
Sharpe Ratio	**1.5**	**1.7**	**2.9**	**0.7**	**6.2**	**1.7**	**0.8**
% observations in regime 2	73%	73%	76%	63%	75%	66%	80%
Probability that return at t+1 is in regime 2	99%	99%	99%	99%	95%	98%	99%

Hidden Markov Model applied to the daily time series of asset classes returns. Maximum likelihood estimation.
1 January 1988 through 31 December 2013.

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin,
"Inter-temporal risk parity: A constant volatility framework for equities and other asset classes." Forthcoming Journal of Alternative Investments, 2015

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What can GARCH models tell us?

Compare buy and hold with inter-temporal risk parity over many simulated scenarios.

Monte Carlo simulations with scenarios generated from parametric models

- Apply stochastic models **[1]** to risky asset returns
  - Keep risk premium μ constant over time
- Apply different volatility models **[2]**
  - GARCH family of models
  - Introduce effects, i.e. leverage effect
- Use different noise **[3]**
  - Gaussian
  - t-student for higher probability of fat tail events
  - skewed for larger extreme events

1 $$r_t = \mu + \sigma_t z$$

2 $$\sigma_t{}^2 = \omega + \alpha(r_{t-1} - \mu)^2 + \beta\sigma_{t-1}{}^2$$

ω long-term volatility level
α volatility clustering
higher alpha => larger clustering effect
β persistency of volatility
~ 1 => few changes in volatility
$\alpha + \beta$ must be < 1 for stationarity
Features like leverage effect, i.e. volatility more impacted by negative returns, can also be added

3 $$Z \sim N(\dots)$$

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Volatility clustering explains better Sharpe ratio

GARCH model Monte Carlo simulations

- Volatility clustering with constant risk premium
  - Higher Sharpe ratio in lower volatility regimes, Lower Sharpe ratio in higher volatility regimes
- Clustering of volatility adds predictability while:
  - Increased exposure in lower volatility regimes, Decreased exposure in higher volatility regimes

GARCH with α = 9% and β = 90%	Buy and Hold	Inter-temporal Risk Parity
Excess return	7.5%	9.1%
Volatility	18.8%	18.9%
Sharpe ratio	0.40	0.48
Average exposure	100%	122%
Sharpe ratio improvement	-	0.08

Averages from 5,000 Monte-Carlo simulations of 5,200 daily returns each, i.e. 20 years.
Target 18.8% volatility with GARCH ex-ante volatility. Risky asset returns drawn from N(7.5%, 18.8%).

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin,
"Inter-temporal risk parity: A constant volatility framework for equities and other asset classes." Forthcoming Journal of Alternative Investments, 2015

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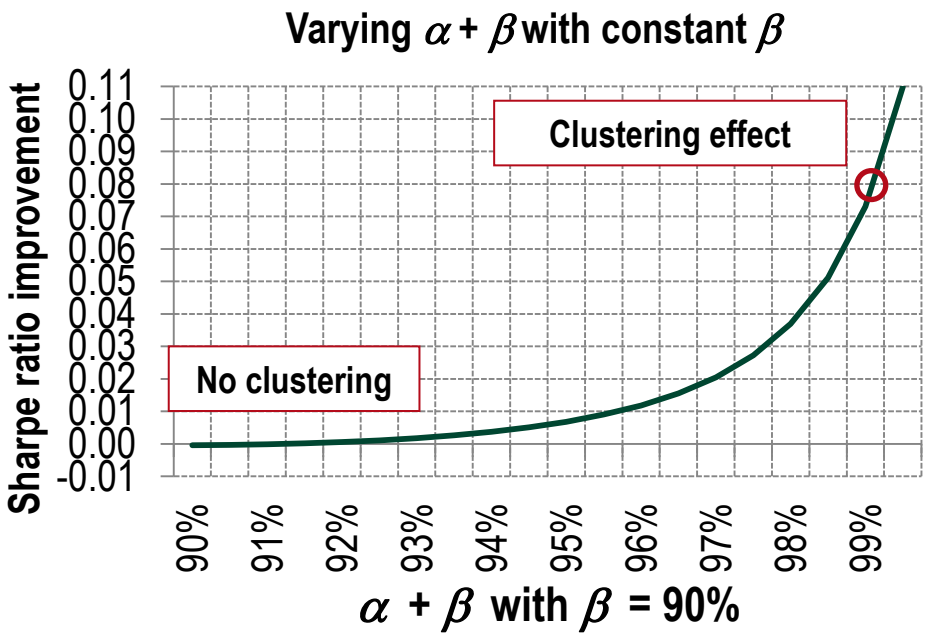
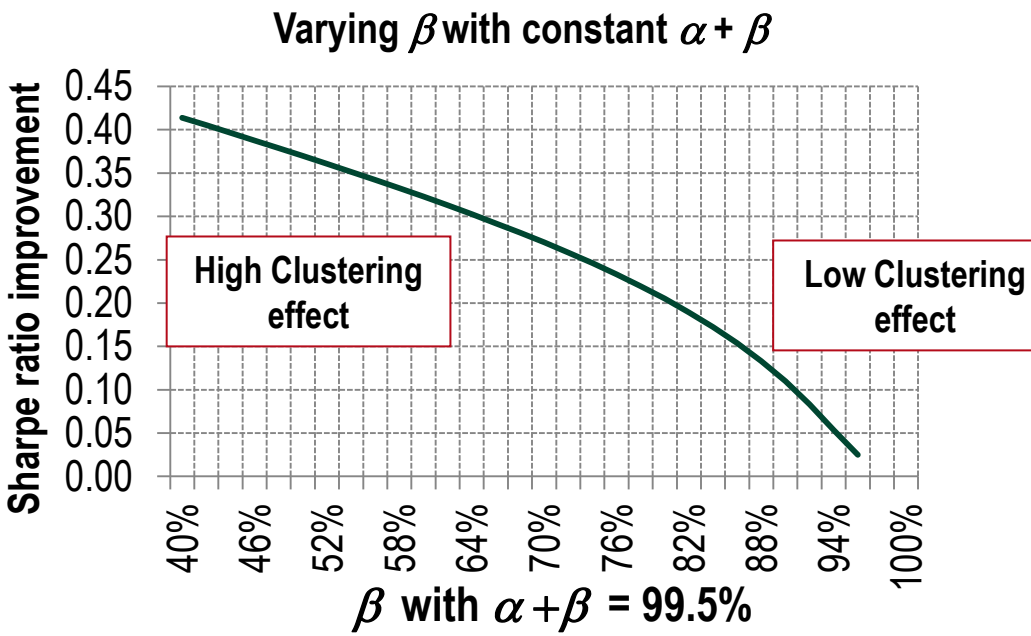
Volatility clustering explains better Sharpe ratio

GARCH model Monte Carlo simulations

More volatility clustering => larger improvement in Sharpe ratio





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Fat tails, leverage effect, skew explain even higher Sharpe ratio

Inter-temporal risk parity strategy improves returns and filters out fat tails.

Negative correlation between return volatility add to the benefit.

- Fat tails (GARCH with t-student noise)
  - Increase the probability of extremes events
  - ⇒ Improvement of the Sharpe ratio
  - ⇒ Reduces largest drawdown events
- Leverage effect (GJR-GARCH):
  - Volatility increases more with negative returns, negative correlation between vol and returns
  - ⇒ Reduces largest drawdown events
- Larger negative return (Skewed-GARCH)
  - Increase probability of larger negative return
  - ⇒ Reduces largest drawdown events

GARCH with t-Student noise	Buy and Hold	Inter-temporal Risk Parity
Excess return	7.4%	10.3%
Volatility	17.8%	18.8%
Sharpe ratio	0.41	0.55
Maximum drawdown (MDD)	-37%	-35%
Ratio MDD / volatility	-2.1	-1.9
Sharpe ratio improvement	-	0.13

	GJR-GARCH		Skewed-GARCH	
	Buy and Hold	Inter-temporal Risk Parity	Buy and Hold	Inter temporal Risk Parity
Excess return	7.7%	9.4%	7.2%	9.0%
Volatility	19.1%	18.8%	18.4%	18.9%
Sharpe ratio	0.40	0.50	0.39	0.48
Maximum drawdown (MDD)	-43%	-38%	-39%	-37%
Ratio MDD / volatility	-2.2	-2.0	-2.1	-2.0
Sharpe ratio improvement	-	0.10	-	0.08

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin, "Inter-temporal risk parity: A constant volatility framework for equities and other asset classes." Forthcoming Journal of Alternative Investments, 2015

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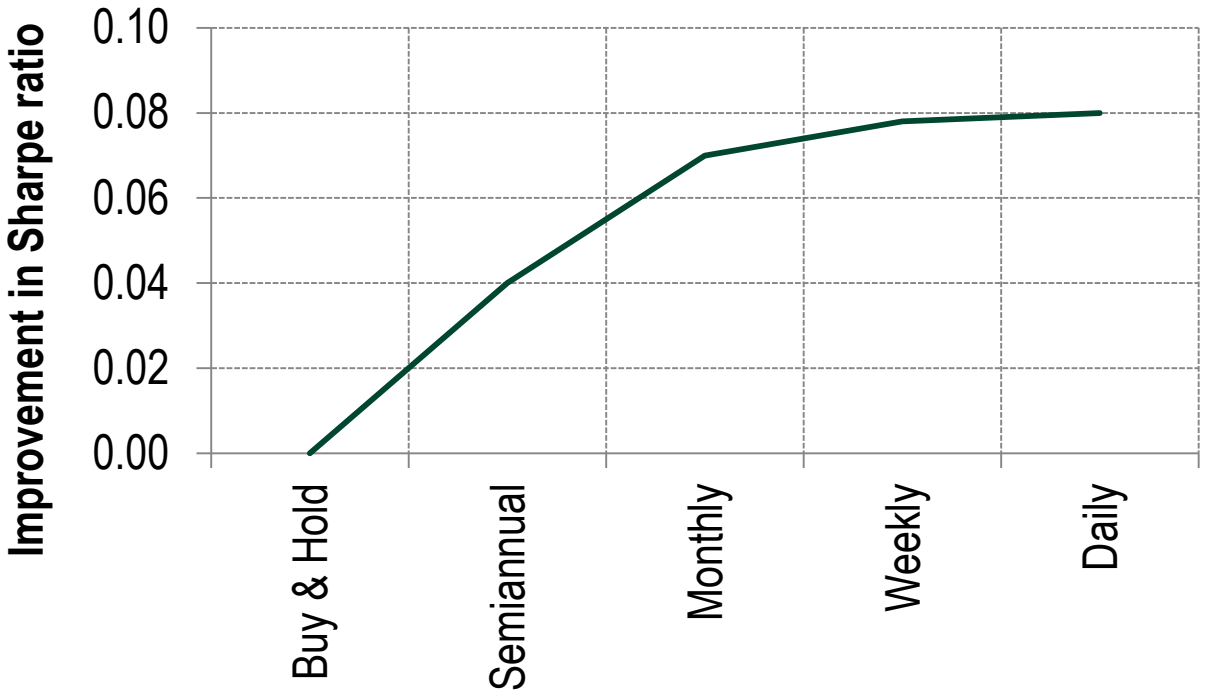
Rebalancing frequency

Lower frequency means substantially lower turnover.
Optimal strategy: daily monitoring and rebalancing only when significant changes are observed.



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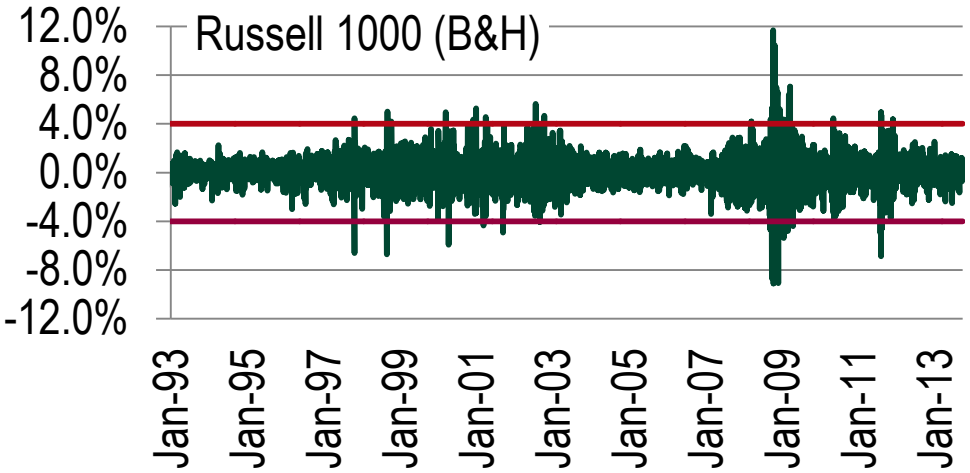
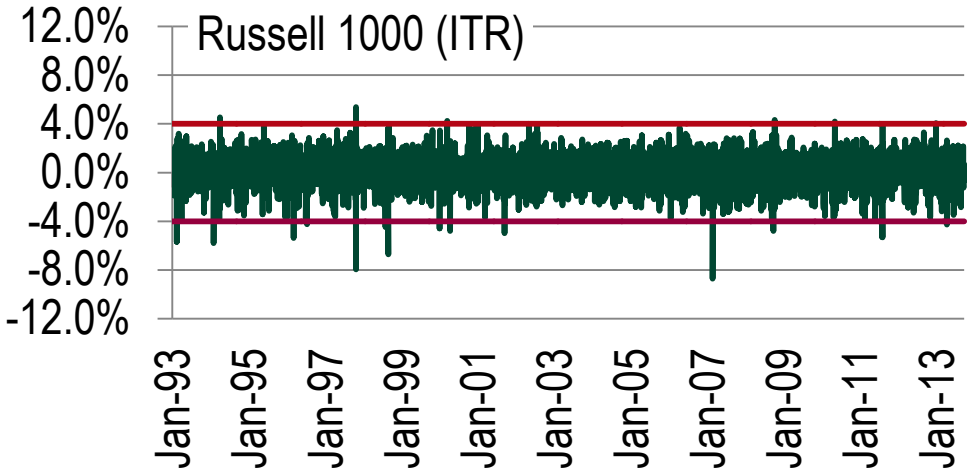
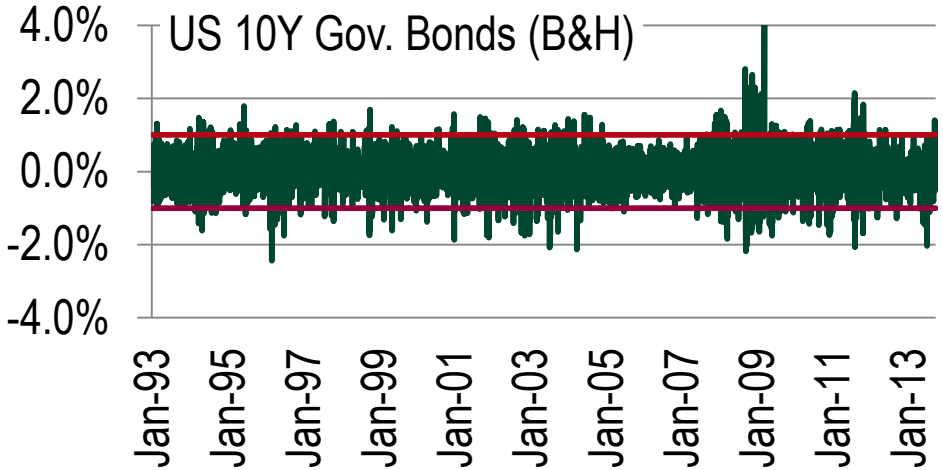
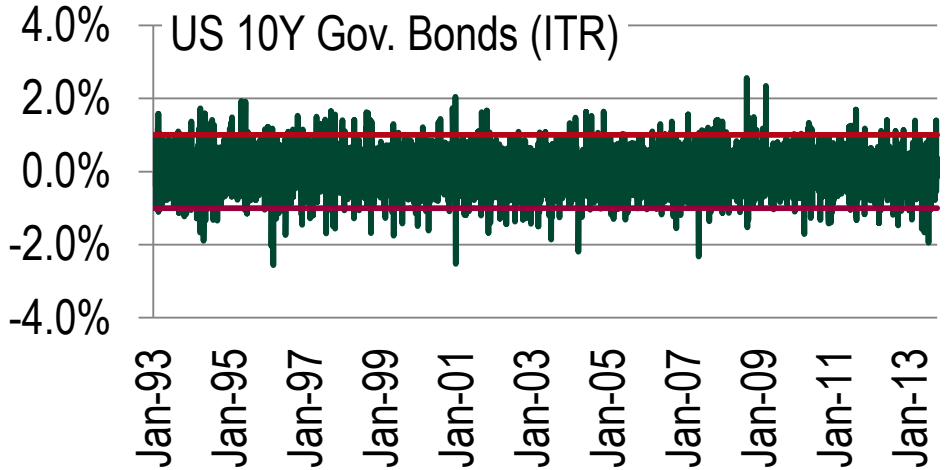
Chunnel returns with Inter-temporal risk parity strategy









Source: BNP Paribas Investment Partners

Using I-GARCH as ex-ante volatility. 1 January 1988 through 31 December 2013.

Larger clustering effect in riskier asset classes

- Clustering effect and fat tails
  - Equities and US High Yield

  Large α => more volatility clustering
  - US Gov Bonds and US Inv Grade Bonds

  Smaller α => less volatility clustering
- Fat tails
  - US High Yield and Russell 1000

  Smaller t-Student => more extreme events
- Explanatory power
  - For all asset classes

  $\alpha + \beta \sim 1$ => volatility well explained by past volatility and returns
- Small impact of long-term volatility

	Russell 1000	MSCI Emerging Markets	S&P GSCI Commo	US High Yield Bonds	US Inv. Grade Bonds	US 10Y Gov. Bonds
ω	7.0E-7	1.6E-6	8.0E-7	2.0E-7	1.0E-7	3.0E-7
α	6%	10%	5%	22%	4%	4%
β	93%	89%	94%	76%	95%	95%
$\alpha + \beta$	99%	99%	100%	97%	99%	99%
t-Student	5.6	7.1	7.4	3.7	6.7	7.6

GARCH with t-Student noise. 1 January 1988 through 31 December 2013.

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin,
"Inter-temporal risk parity: A constant volatility framework for equities and other asset classes." Forthcoming Journal of Alternative Investments, 2015

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Historical simulations for different asset classes

Higher Sharpe ratio for asset classes with stronger volatility clustering and fat tails.

	Russell 1000	MSCI EM Markets	S&P GSCI Commo	US High Yield Bonds	US Inv. Grade bonds	US 10Y Gov. Bonds
			Buy and hold strategy			
Sharpe ratio	0.42	0.35	0.11	1.09	0.73	0.4
Ratio MDD / volatility	-2.90%	-3.4	-3.4	-6.6	-3.3	-1.8
I-GARCH			Inter-temporal risk parity strategy			
Sharpe ratio	0.56	0.56	0.15	1.55	0.76	0.4
Ratio MDD / volatility	-2	-3.5	-3.2	-5.2	-2.2	-2
Sharpe ratio improvement	0.14	0.21	0.05	0.45	0.04	0.00

Using I-GARCH as ex-ante volatility. 1 January 1988 through 31 December 2013.

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin,
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Application to Value and Momentum premia

Inter-temporal risk parity strategy applied to Value and Momentum factors:

- Equities: daily data from Ken French's web-site:
  - Value premium: HML (High-Minus-Low factor)
  - Momentum premium: Mom (Momentum)
- Foreign exchange based on 10 countries*:
  - Value premium: carry strategy using inter-bank rates
  - Momentum premium: past twelve month cumulative returns of forward returns
- Government Government bonds based on 10 countries**:
  - Value premium: slope of the yield curve (10-year bond yields minus cash rates)
  - Momentum premium: past twelve month cumulative returns of total return indices

* Australia, Canada, Germany or Euro zone after 1999, Japan, New Zealand, Norway, Sweden, Switzerland, UK and US

** Australia, Canada, Germany, Japan, Denmark, Norway, Sweden, Switzerland, UK and US

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Improvement of information ratios in factor investing

Comparable results for value and momentum in equities, foreign exchange and government bonds.

	Equity		Foreign Exchange		Government Bonds	
	Momentum	Value	Momentum	Value	Momentum	Value
			Constant leverage			
Information ratio	0.59	0.34	0.19	0.44	-0.06	0.52
Ratio MDD / volatility	-4.5	-4.7	-3	-3.9	-5.1	-1.9
I-GARCH			Inter-temporal risk parity strategy			
Information ratio	1.43	0.42	0.46	0.63	0.16	0.57
Ratio MDD / volatility	-2.6	-4.2	-2.8	-3.2	-3.5	-1.7
Information ratio improvement	0.83	0.08	0.27	0.19	0.22	0.05

1 January 1985 to 31 December 2013

Source: R Perchet, R Leote de Carvalho and P Moulin,
"Inter-temporal risk parity: A constant volatility framework for factor investing." Journal of Investments Strategies, Vol. 4, No 1 (2015)

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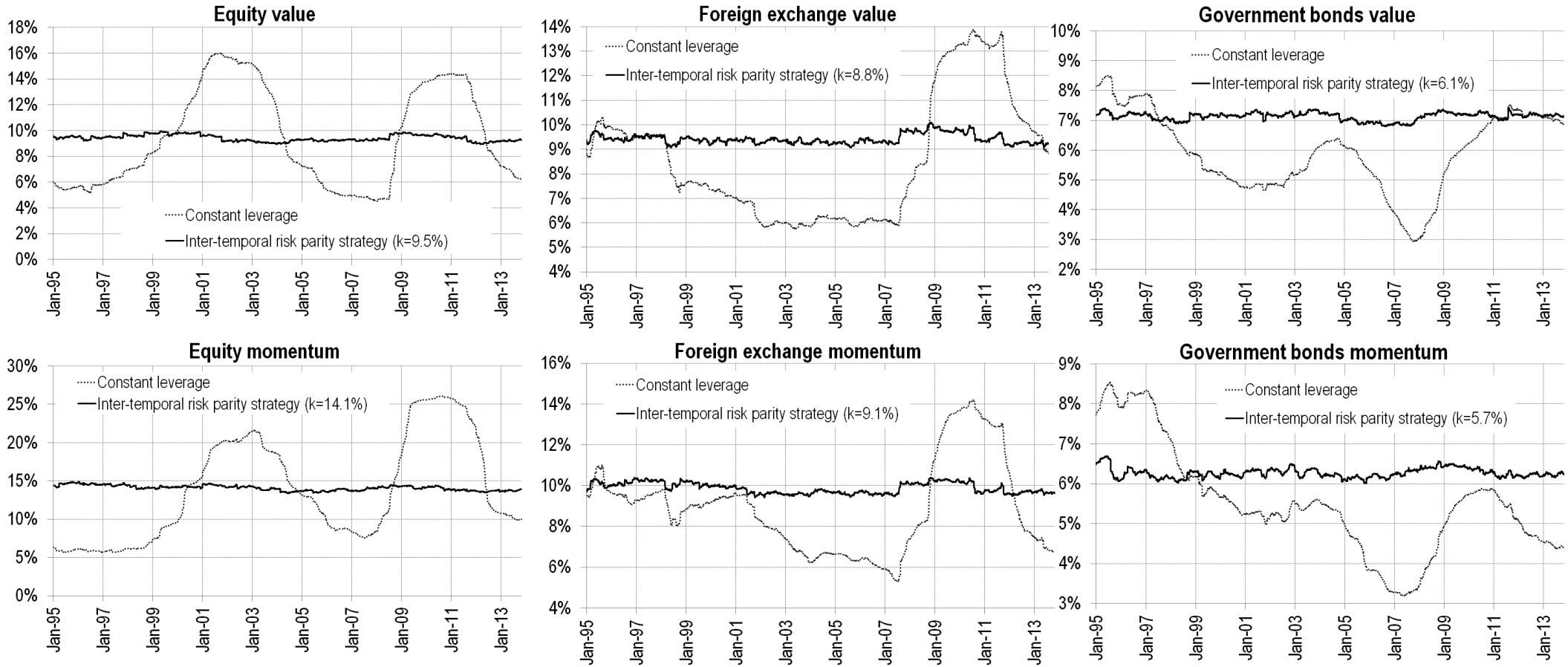
Success in controlling volatility

Risk can be successfully tamed and targeted

3-year rolling ex-post volatility of returns



USD daily returns, 1 Jan 1990 to 31 Dec 2013.

Source: R Perchet, R Leote de Carvalho and P Moulin,
"Inter-temporal risk parity: A constant volatility framework for factor investing." Journal of Investments Strategies, Vol. 4, No 1 (2015)

Conclusions

- Constant volatility strategy add value because return distributions are not Gaussian
- Improvement of Sharpe ratio and information ratio explained by volatility clustering
- Presence of fat tails events increase volatility clustering effect
- Benefit of risk management is larger if return and volatility are negatively correlated
- Clear benefit for risky asset classes: equities, high yield and foreign exchange rates
- For less risky asset classes, the added value is smaller but risk exposures are better control
- Extension to factor premia in different asset classes
- Investors should think in terms of risk budget allocation rather than fixed weights

Risk management can improve risk-adjusted returns!

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