

Data Structures and Algorithms

Chapter 1 Introduction

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Outline

- 1 Preliminary understanding
- 2 Introduction of the course
 - The Contents of the course
 - Data Structure Philosophy
 - Purpose of Data Structure and Algorithms
- 3 Related Concepts
 - Data Structure
 - Abstract Data Type (ADT)
 - Algorithm
- 4 Algorithm Efficiency and Analysis
 - Algorithm Criterion
 - Algorithm Efficiency
 - Big-O Examples
 - Space and Time Tradeoff Principle
- 5 Brief Summary

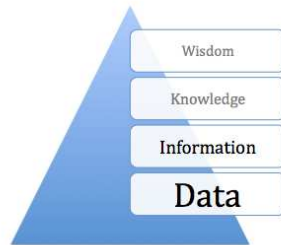
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1.1 Preliminary understanding



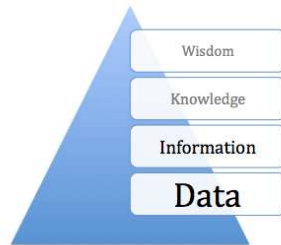
1.1 Preliminary understanding



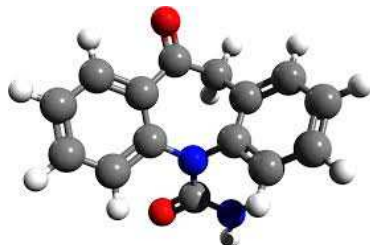
1.1 Preliminary understanding



Symbol



Relation



1.1 Preliminary understanding

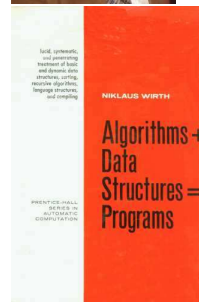
- Program: Find out the max one from n numbers

1.1 Preliminary understanding

- Program: Find out the max one from n numbers
 - ▶ How to organize and store n numbers and other temp numbers?
 - ▶ What are operations of those numbers for processing the problem?
 - ▶ How to use those operations?
 - ▶ What are steps to deal with the problem by those operations?

1.1 Preliminary understanding

- Prof. Niklaus Wirth
- <http://www.inf.ethz.ch/personal/wirth/>
 - ▶ Received Turing Award in 1984
 - ▶ Father of Pascal
 - ▶ Algorithms+Data Structures = Programs
Prentice-Hall, 1976



Where are Algorithms and Data Structures in follow program

```
1  /* Original program segment of Tiny HTTPd */
2  void accept_request(int client){
3      char buf[1024];
4      int numchars;
5      char method[255];
6      char url[255];
7      size_t i, j;
8      struct stat st;
9      int cgi = 0;
10
11     numchars = get_line(client, buf, sizeof(buf));
12     i = 0; j = 0;
13     while (!ISspace(buf[j]) && (i < sizeof(method) - 1))
14     {
15         method[i] = buf[j]; i++; j++;
16     }
17
18     i = 0;
19     while (ISspace(buf[j]) && (j < sizeof(buf)))
20         j++;
21     ... ..
```

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1.2.1 The Contents of the course

- We have 12 weeks to learn fundamental data structures and algorithms for organizing and processing information
- Deeply understand the basic structures used in all software

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 - ▶ Understand the data structures and their trade-offs
 - ▶ Rigorously analyze the algorithms that use them (math!)

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- Deeply understand the basic structures used in all software
 - ▶ Understand the data structures and their trade-offs
 - ▶ Rigorously analyze the algorithms that use them (math!)
- You must learn this course, if you would like to find the answer of following questions:
 - ▶ How does Google quickly find web pages that contain a search term?
 - ▶ What's the fastest way to broadcast a message to a network of computers?
 - ▶ How can a subsequence of DNA be quickly found within the genome?
 - ▶ How does your operating system track which memory (disk or RAM) is free?
 - ▶ In the game Half-Life, how can the computer determine which parts of the scene are visible?
 - ▶

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- A data structure requires:
 - ▶ space for each data item it stores,
 - ▶ time to perform each basic operation,
 - ▶ programming effort.
- Each problem has constraints on available space and time.
- Only after a careful analysis of problem characteristics can we know the best data structure for the task.

1.2.2 Data Structure Philosophy

How to select a Data structures?

- Analyze the problem to determine the resource constraints a solution must meet.
- Determine the basic operations that must be supported.
- Quantify the resource constraints for each operation.
- Select the data structure that best meets these requirements.

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- Be able to make good design choices as a developer, project manager, etc.
- Be able to justify and communicate your design decisions

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- Be able to make good design choices as a developer, project manager, etc.
- Be able to justify and communicate your design decisions
- Must be included in post-graduate entrance examination (computer related)

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What is Data Structure?

Preliminary Concepts

Data

- The difference between Data (Raw) and Information (Processed)
- (In computer) Data is carrier of Information, is symbols inputted, stored, processed and outputted by computers

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024020	Data Structure	64
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Data Object

A SET containing the same type of data elements

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1.3.1 Data Structure

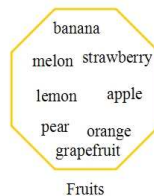
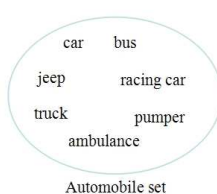
Data Structure

- A data object + specific existing relations between elements
- Set + Relations (Constraint), Ordered pair
- Four basic relations in Data Structure

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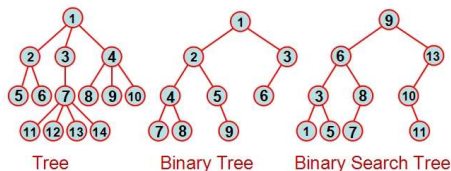
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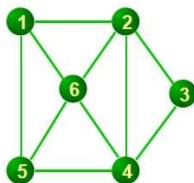
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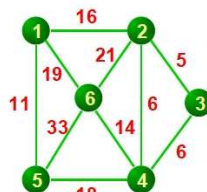
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- Four basic relations in Data Structure
 - ▶ Sets: Uniform type, No specific relations between elements
 - ▶ Linear structure: One to one
 - ▶ Hierarchical Structure: One to many
 - ▶ Graph Structure: Many to many



Graph



Net

Logical Form and Physical Form

Data structure have

- Logical Form

- ▶ Definition of the data item within an Abstract Data Type (ADT)
- ▶ Problem orient: for example, Integers in mathematical sense: +, -

- Physical Form

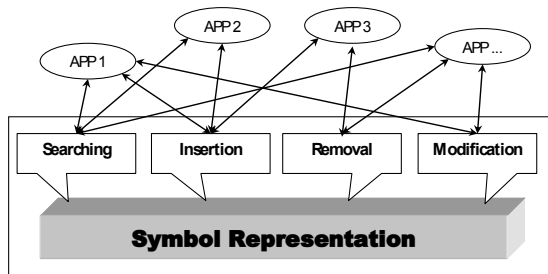
- ▶ Implementation of the data item within a data structure.
- ▶ Computer orient: for example, 16/32 bit integers, overflow, Storage Structure in computer
 $(123)_{10} = (1111011)_2$

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1.3.2 Abstract Data Type (ADT)

- Abstract Data Type (ADT): An abstract data type is defined as "class of objects whose logical behavior is defined by a set of values and a set of operations"
 - ▶ Encapsulation: Each ADT operation is defined by its inputs and outputs (Interface), Hide implementation details.
 - ▶ Abstract: Extract problem essential.
 - ▶ Data objects, Relations, Functions (Special relations)



ADT of Natural Number

```
1 ADT NaturalNumber is
2 {
3   Objects: An ordered subset of integers, beginning at 0, ending
         at the maximum (MaxInt).
4
5   Relation:  $C = \{ \langle i, i+1 \rangle \mid i = 0, 1, \dots, \text{MaxInt} \}$ 
6
7   Function: For any x, y belong NaturalNumber;
         False, True belong Boolean, +, -, <, ==, = are all available
         properties and methods
9   Zero( ) :NaturalNumber      return 0
10  IsZero(x) :Boolean  if (x==0) return True else return False
11  Add (x, y) :NaturalNumber
12      if (x+y<=MaxInt) return x+y  else return MaxInt
13  Subtract (x, y) :NaturalNumber
14      if (x < y) return 0 else return x - y
15  Equal (x, y) :Boolean
16      if (x==y) return True else return False
17  Successor (x) :NaturalNumber
18      if (x==MaxInt) return x else return x+1
19 } ADT NaturalNumber
```

ADT of Queue

```
1 ADT Queue is
2 {
3   Objects:
4
5   Relation:
6
7   Function:
8
9
10  } ADT NaturalNumber
```

Data Type vs ADT

- A data type consists of the values it represents and the operations defined upon it.
- A data type is the physical implementation of an ADT.
 - ▶ Each operation associated with the ADT is implemented by one or more subroutines in the implementation
- Data type usually refers to an organization for data in main memory.

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What is Algorithm?

Algorithm

- Is $n(n > 2)$ prime number?

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Algorithm

- Is $n(n > 2)$ prime number?
- An algorithm is a finite set of instructions that, if followed, accomplishes a particular task.
- In addition, all algorithms must satisfy the following characteristic:
 - ▶ Input: zero, one, or more
 - ▶ Output: at least one output
 - ▶ Definiteness: each instruction is clear and unambiguous
 - ▶ Effectiveness: every instruction must be basic enough to be carried out, in principle, by a person using only pencil and paper
 - ▶ Finiteness: terminates after a finite number of steps
- In computational theory
 - ▶ Main distinguish between an algorithm and a program is the characteristic Finiteness.

Algorithm Description

- Using a natural language
- Using flow diagram
- Using a programming language
- Combine with the above

Is $n(n > 2)$ prime number?

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1.4.1 Algorithm Criterion

- There are often many algorithms to solve a problem. How do we choose between them?

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- Algorithm Criterion
 - ▶ Validity (正确性)
 - ▶ Usability (可用性)
 - ▶ Readability (可读性)
 - ▶ Robustness (健壮性)
 - ▶ Efficiency (效率)

1.4.1 Algorithm Criterion

- There are often many approaches (algorithms) to solve a problem. How do we choose between them?
- At the heart of computer, program design are two (sometimes conflicting) goals.
 - ▶ (1) To design an algorithm that is easy to understand, code, and debug.
 - ▶ (2) To design an algorithm that makes efficient use of the computer's resources.

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- Goal (1) is the concern of Software Engineering
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- Goal (1) is the concern of Software Engineering
- Goal (2) is the concern of data structures and algorithm analysis.
- When goal (2) is important, how do we measure an algorithm's cost?

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Algorithm Efficiency: How to Measure the Efficiency?

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 - ▶ Critical resources;
 - ▶ Factors affecting running time;
 - ▶ Strategy of algorithm
 - ▶ Scale of problem
 - ▶ Program language
 - ▶ Compiler
 - ▶ Computer speed

Algorithm Efficiency: How to Measure the Efficiency?

- (1) Empirical comparison (run programs)
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 - ▶ Factors affecting running time;
 - ▶ Strategy of algorithm
 - ▶ Scale of problem
 - ▶ Program language
 - ▶ Compiler
 - ▶ Computer speed
 - ▶ For most algorithms, running time depends on "size" of the input.
 - ▶ Running time is expressed as $T(n)$ for some function T on input size n .

1.4.2 Algorithm Efficiency

- Algorithm = control structure + meta operation

```
1 int largest(int array[],int n)
2 {
3     int currlarge=0; //Largest value seen
4     for (int i=1; i<n; i++) //For each val
5     {
6         if(array[currlarge]<array[i])
7             currlarge=i;//Remember pos
8     }
9     return currlarge;//Return largest
10 }
```

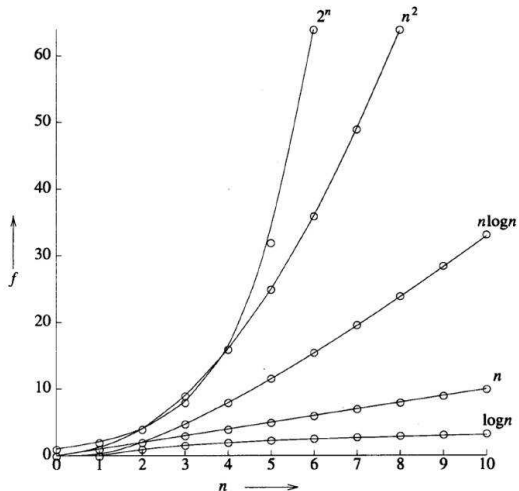
$$T(n) = C_1n + C_2 \text{ steps}$$

compute n squared

```
1 //compute n squared
2 int n2(int n)
3 {
4     sum=0;
5     for(i=1;i<=n;i++)
6         for(j=1;j<n;j++)
7             sum++;
8     return sum;
9 }
```

$$T(n) = C_1 n^2 + C_2$$

1.4.2 Algorithm Efficiency



Grow Rate of Functions

1.4.2 Algorithm Efficiency

Growth Rates Compared

	n=1	n=2	n=4	n=8	n=16	n=32
1	1	1	1	1	1	1
$\log n$	0	1	2	3	4	5
n	1	2	4	8	16	32
$n \log n$	0	2	8	24	64	160
n^2	1	4	16	64	256	1024
n^3	1	8	64	512	4096	32768
2^n	2	4	16	256	65536	4294967296

$$f(n) = n^2 + 100n + \log_2 n + 1000$$

Asymptotic Analysis: Big-O

- Running time of the algorithm takes to execute depends on the value of N
asymptotic complexity

Time Complexity $T(n) = O(f(n))$

Space Complexity $S(n) = O(f(n))$

Asymptotic Analysis: Big-O

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asymptotic complexity

Time Complexity $T(n) = O(f(n))$

Space Complexity $S(n) = O(f(n))$

- Definition:

$f(n)$ is $O(g(n))$ if there exist positive numbers c and N such that $f(n) \leq cg(n)$ for all $n > N$

- Usage:

The algorithm is in $O(n^2)$ in **[best, average, worst]** case.

Best, Worst, Average Cases

Not all inputs of a given size take the same time to run.

- Sequential search for K in an array of n integers:
- Begin at first element in array and look at each element in turn until K is found
 - ▶ Best case: 1 time of comparison
 - ▶ Worst case: n times of comparison
 - ▶ Average case: $(n+1)/2$

```
1  int lookup(int k, int array[], int n)
2  {
3      int i;
4      for (i = 0; i < n; i++)
5      {
6          if (k == array[i])
7              { return k; }
8      }
9      return -1;
10 }
```

1.4.2 Algorithm Efficiency

Which is fairest? Which is most important? it may be difficult to determine!

A Common Misunderstanding

- "The best case for my algorithm is $n = 1$ because that is the fastest."
WRONG!
- Big-O refers to a growth rate as n grows to ∞ .
- Best case is defined as which input of size n is cheapest among all inputs of size n .

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1.4.3 Big-O Examples

Example 1:

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1 a = b;
```

This assignment takes constant time, so it is $T(n) = c$. We say this is in $O(1)$.

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Example 2: $T(n) = c_1n^2 + c_2n$ in average case.

$$c_1n^2 + c_2n \leq c_1n^2 + c_2n^2 \leq (c_1 + c_2)n^2 \text{ for all } n > 1.$$

$$T(n) \leq cn^2 \text{ for } c = c_1 + c_2 \text{ and } n_0 = 1.$$

Therefore, $T(n)$ is in $O(n^2)$ by the definition.

Simplifying Rules

- If $f(n)$ is in $O(g(n))$ and $g(n)$ is in $O(h(n))$, then $f(n)$ is in $O(h(n))$.
- If $f(n)$ is in $O(kg(n))$ for any constant $k > 0$, then $f(n)$ is in $O(g(n))$.
- If $f_1(n)$ is in $O(g_1(n))$ and $f_2(n)$ is in $O(g_2(n))$, then $(f_1 + f_2)(n)$ is in $O(\max(g_1(n), g_2(n)))$.
- If $f_1(n)$ is in $O(g_1(n))$ and $f_2(n)$ is in $O(g_2(n))$ then $f_1(n)f_2(n)$ is in $O(g_1(n)g_2(n))$.

1.4.3 Big-O Examples

Example 3:

```
1 sum = 0;  
2 for (i=1; i<=n; i++)  
3     sum += n;
```

1.4.3 Big-O Examples

Example 3:

```
1 sum = 0;  
2 for (i=1; i<=n; i++)  
3     sum += n;
```

$O(n)$.

1.4.3 Big-O Examples

Example 4:

```
1 sum = 0;
2 for (j=1; j<=n; j++)
3     for (i=1; i<=j; i++)
4         sum++;
5 for (k=0; k<n; k++)
6     A[k] = k;
```

1.4.3 Big-O Examples

Example 4:

```
1 sum = 0;
2 for (j=1; j<=n; j++)
3     for (i=1; i<=j; i++)
4         sum++;
5 for (k=0; k<n; k++)
6     A[k] = k;
```

$O(n^2)$.

1.4.3 Big-O Examples

Example 5:

```
1 sum1 = 0;
2 for (i=1; i<=n; i++)
3     for (j=1; j<=n; j++)
4         sum1++;
5
6 sum2 = 0;
7 for (i=1; i<=n; i++)
8     for (j=1; j<=i; j++)
9         sum2++;
```

1.4.3 Big-O Examples

Example 5:

```
1 sum1 = 0;
2 for (i=1; i<=n; i++)
3     for (j=1; j<=n; j++)
4         sum1++;
5
6 sum2 = 0;
7 for (i=1; i<=n; i++)
8     for (j=1; j<=i; j++)
9         sum2++;
```

$O(n^2)$.

1.4.3 Big-O Examples

Example 5:

```
1 sum1 = 0;
2 for (k=1; k<=n; k*=2)
3     for (j=1; j<=n; j++)
4         sum1++;
5
6 sum2 = 0;
7 for (k=1; k<=n; k*=2)
8     for (j=1; j<=k; j++)
9         sum2++;
```


1.4.3 Big-O Examples

Example 5:

```
1 sum1 = 0;
2 for (k=1; k<=n; k*=2)
3     for (j=1; j<=n; j++)
4         sum1++;
5
6 sum2 = 0;
7 for (k=1; k<=n; k*=2)
8     for (j=1; j<=k; j++)
9         sum2++;
```

First loop is $\sum n$ for $k = 1$ to $\log n$, $O(n \log n)$.

Second loop is $\sum 2^k$ for $k = 0$ to $\log(n-1)$, $O(n)$.

1.4.3 Big-O Examples

Example 5:

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1  sum1 = 0;
2  for (k=1; k<=n; k*=2)
3      for (j=1; j<=n; j++)
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7  for (k=1; k<=n; k*=2)
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First loop is $\sum n$ for $k = 1$ to $\log n$, $O(n \log n)$.

Second loop is $\sum 2^k$ for $k = 0$ to $\log(n-1)$, $O(n)$.

$O(n \log_2 n)$.

Space Bounds

Space bounds can also be analyzed with asymptotic complexity analysis.

Time: Algorithm

Space: Data Structure

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1.4.4 Space and Time Tradeoff Principle

- One can often reduce time if one is willing to sacrifice space, or vice versa
 - ▶ Table lookup
 - Factorials
 - Cryptography

1.4.4 Space and Time Tradeoff Principle

- One can often reduce time if one is willing to sacrifice space, or vice versa
 - ▶ Table lookup
 - Factorials
 - Cryptography
- Disk-based Space/Time Tradeoff Principle:
The smaller you make the disk storage requirements, the faster your program will run.

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1.5 Brief Summary

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