Data Structures and Algorithms

Chapter 3 Linked List

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Outline

Next Section

3.1 Linked List

- A linked list stores a linear sequence of elements.
- Linked lists store elements in non-contiguous memory locations.

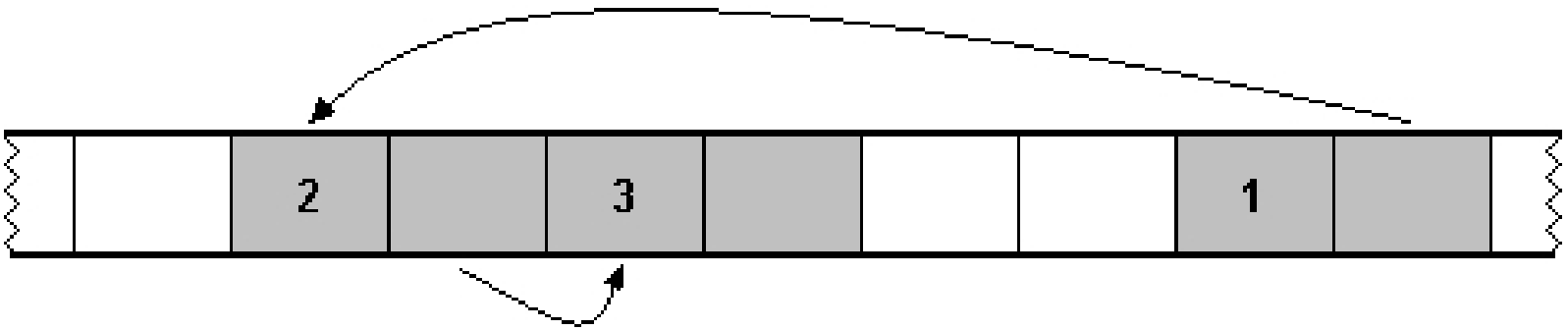
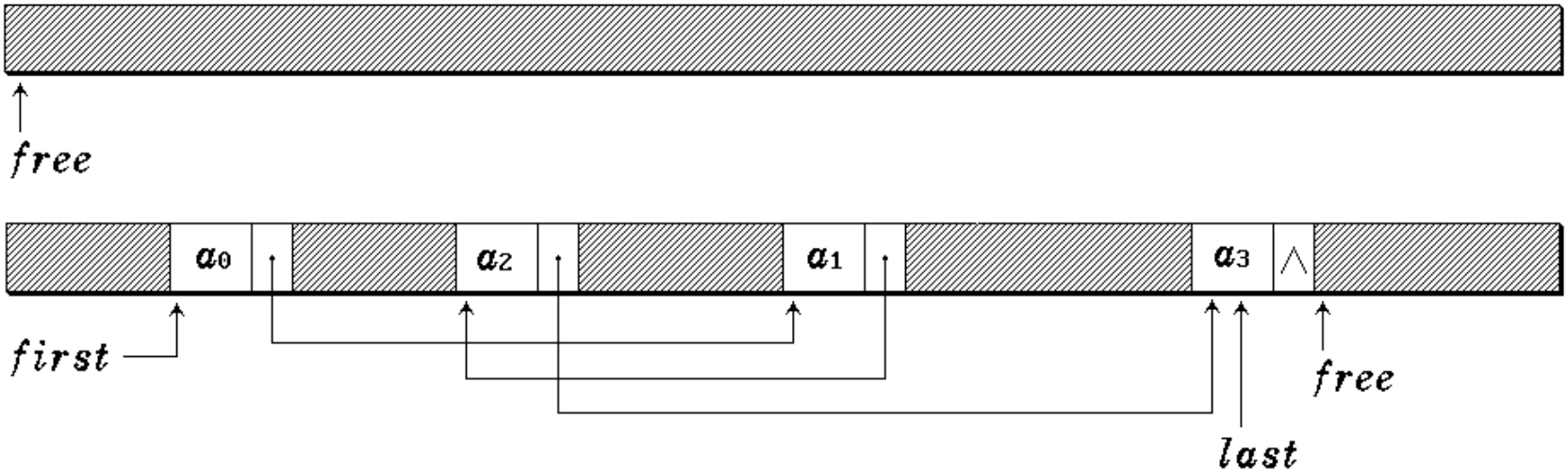


Figure 2 *A linked list in memory*

Next Subsection

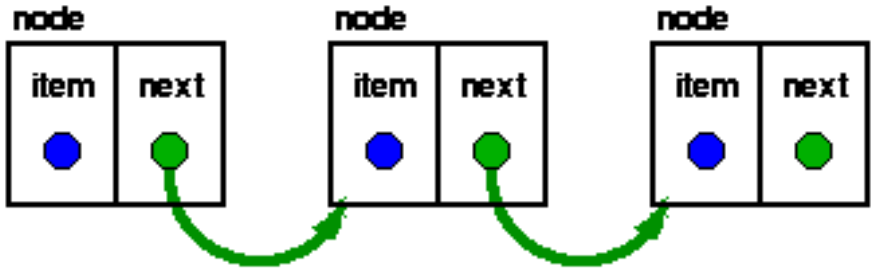
Single Linked list storage image



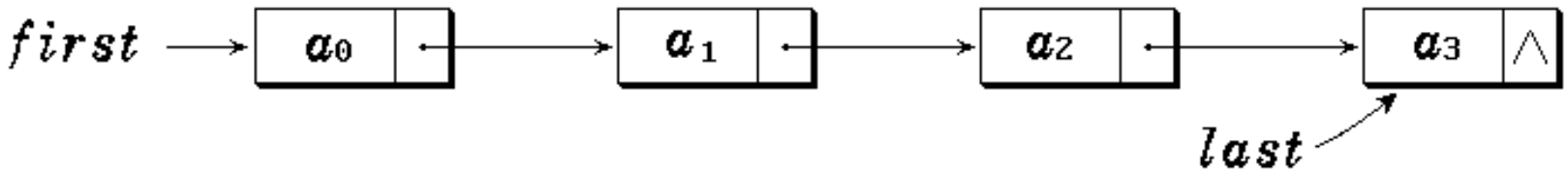
Linked lists store elements in non-contiguous memory locations

3.1.1 Singly Linked List

- Characteristics
  - Node representation



  - Linear structure



  - Extensible

Class definition of Single Linked List

- Class definition of Single Linked List: A linked list consists of:
  - ListNode Class
  - List Class
  - Iterator Class
- Definition
  - Compound definition
  - Nested definition

3.1.1 Singly Linked List

```
1  class List;                    //Compound Class
2
3  class ListNode {          //ListNode Class
4      //List class is its friend
5      friend class List;
6  private:
7      int data; //Node data
8      ListNode *link;      //Pointer
9  };
10
11 class List { //List Class
12 public:
13     ... ...
14 private:
15     //head and tail pointer
16     ListNode *first, *last;
17 };
```

3.1.1 Singly Linked List

```
1  class List { //List Class (Embedded definition)
2  public:
3      //Operations
4      ... ...
5  private:
6      //Embedded ListNode Class
7      class ListNode {
8      public:
9          int data;
10         ListNode *link;
11     };
12     //Head and Tail pointer
13     ListNode *first, *last;
14 };
```

Insert node into Linked List

- Three cases considered, First case:
  - Insert a node at beginning.

```
1 newnode->link = first;
2 first = newnode;
```

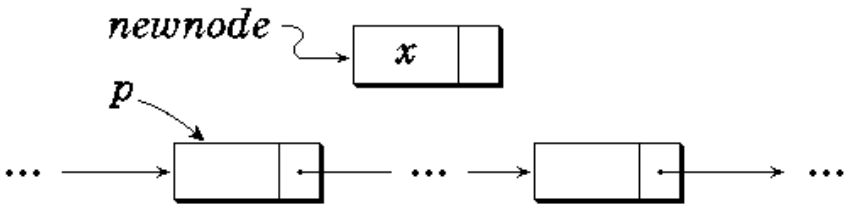
Insert node into Linked List

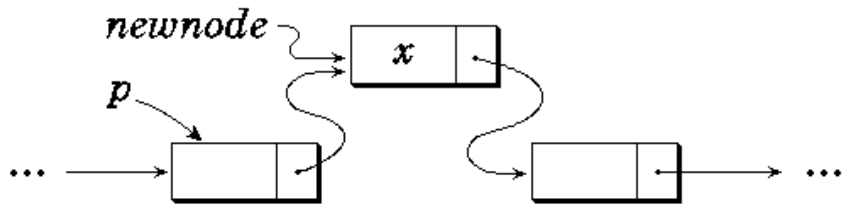
- Three cases considered, First case:
  - Insert a node at beginning.

```
1 newnode->link = first;
2 first = newnode;
```

- Three cases considered, Second case:
  Insert a node in middle of the list.

```
1 newnode->link = p->link;
2 p->link = newnode;
```

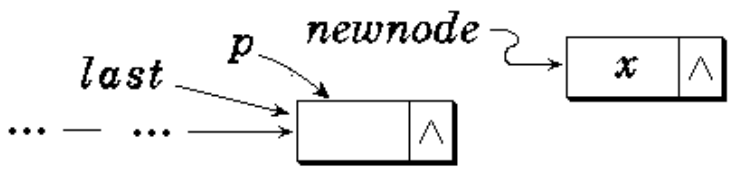


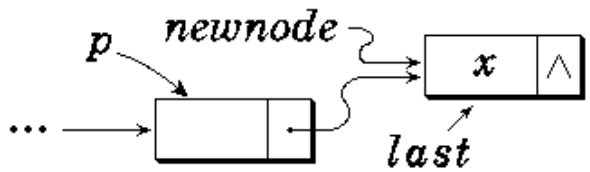


3.1.1 Singly Linked List

- Three cases considered, Third case:
  - Insert a node at end.

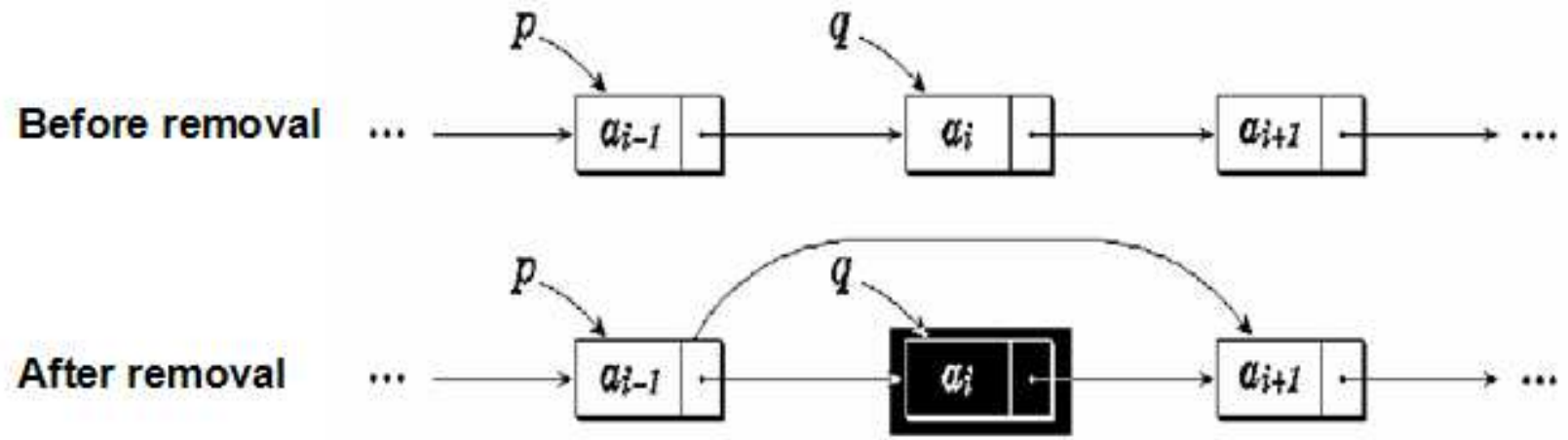
```
1 newnode->link = p->link;
2 p->link = last = newnode;
```





Remove node from Linked List

- Two cases considered
  - First case: Remove the first node of list;
  - Second case: Remove the node within the list or the last node.



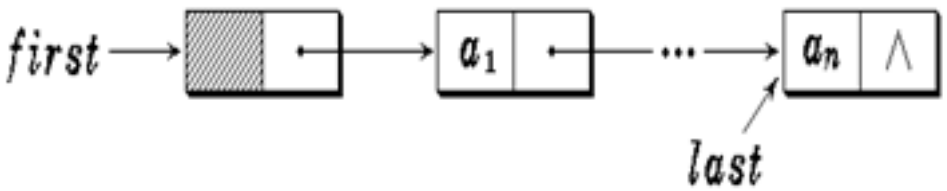
```
1  int List::Remove ( int i ) {
2      //Remove i-th node of the list
3      Node *p = first, *q;
4      int k = 0;
5      // Locate k and p (point to the node before the one
           will be removed)
6      while ( p != NULL && k < i-1 )
7      {
8          p = p->link;
9          k++;
10     }     //Find out i-1-th node
11     if ( p == NULL || p->link == NULL )
12     {
13         cout << "Invalid pos for removal!\n";
14         return 0;
15     }
16
17     if ( i == 0 )
18     {    //First case
19         q = first;
20         //Modify first  pointer
```

```
21          p = first = first->link;
22      }
23      else
24      {   //Second case
25          q = p->link;
26          p->link = q->link;
27      }
28      if ( q == last )
29          last = p;   //Modify last pointer
30      k = q->data;
31      delete q;         //Release q
32      return k;
33  }
```

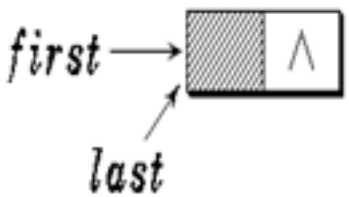
Next Subsection

3.1.2 List with Dummy node

- Dummy node
  - at the beginning of the list
  - No data, just a indicator
- The aim of dummy node
  - Unify the operations of list, whether it is a Empty list or Non-empty list;
  - Simplify the operations of list



General Non-Empty List

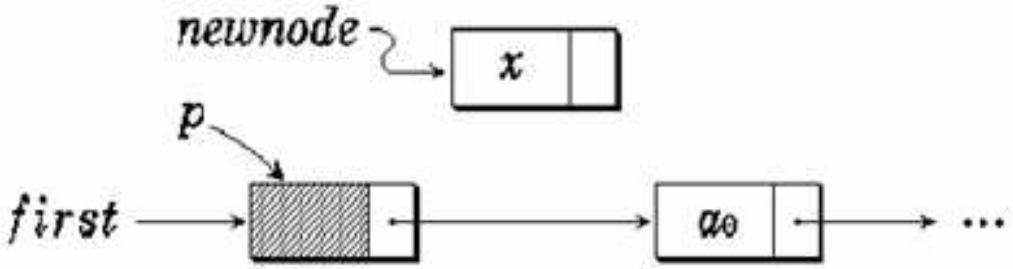


Empty List

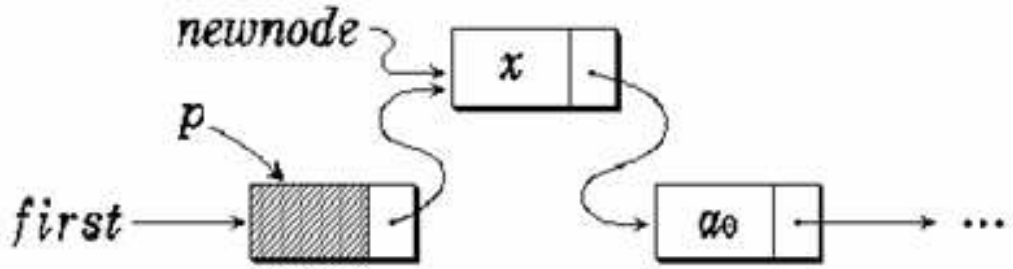
Insertion for List with Dummy Node

Non-Empty list

Before insertion

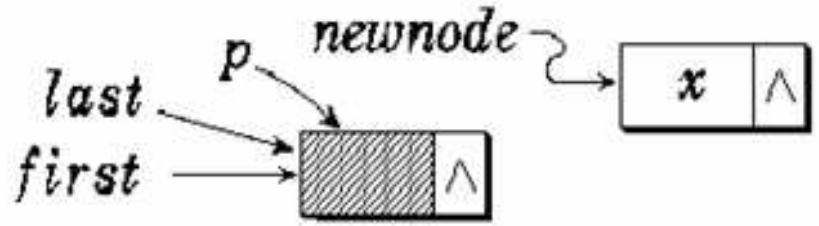


After insertion

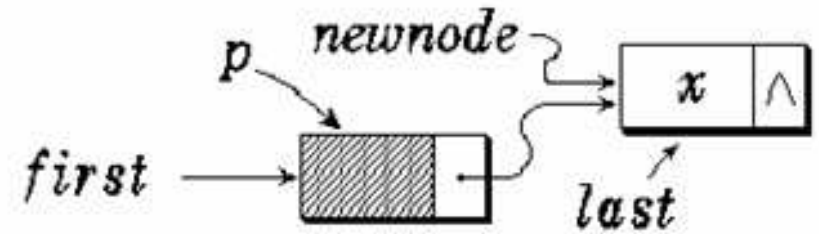


3.1.2 List with Dummy node

Before insertion



After insertion

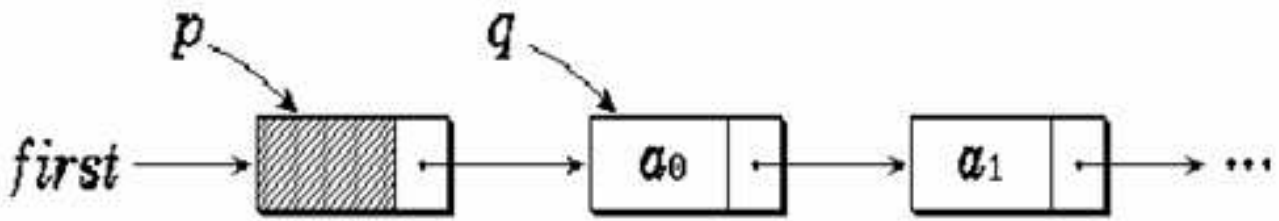



```
1  newnode->link = p->link;
2  if ( p->link == NULL ) last = newnode;
3  p->link = newnode;
```

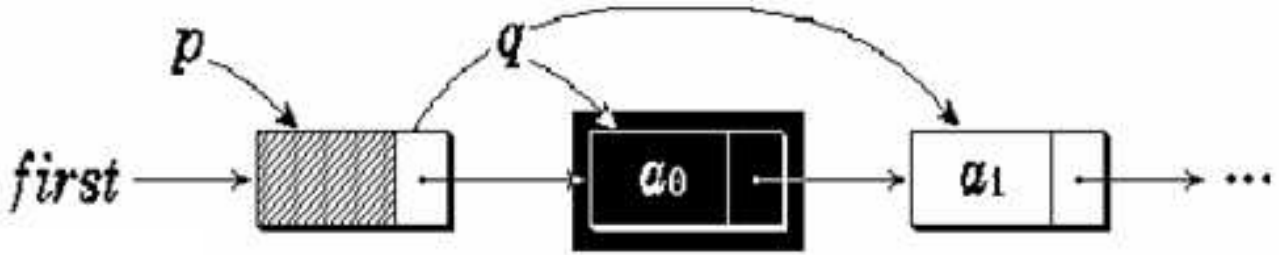
3.1.2 List with Dummy node

Non-Empty list

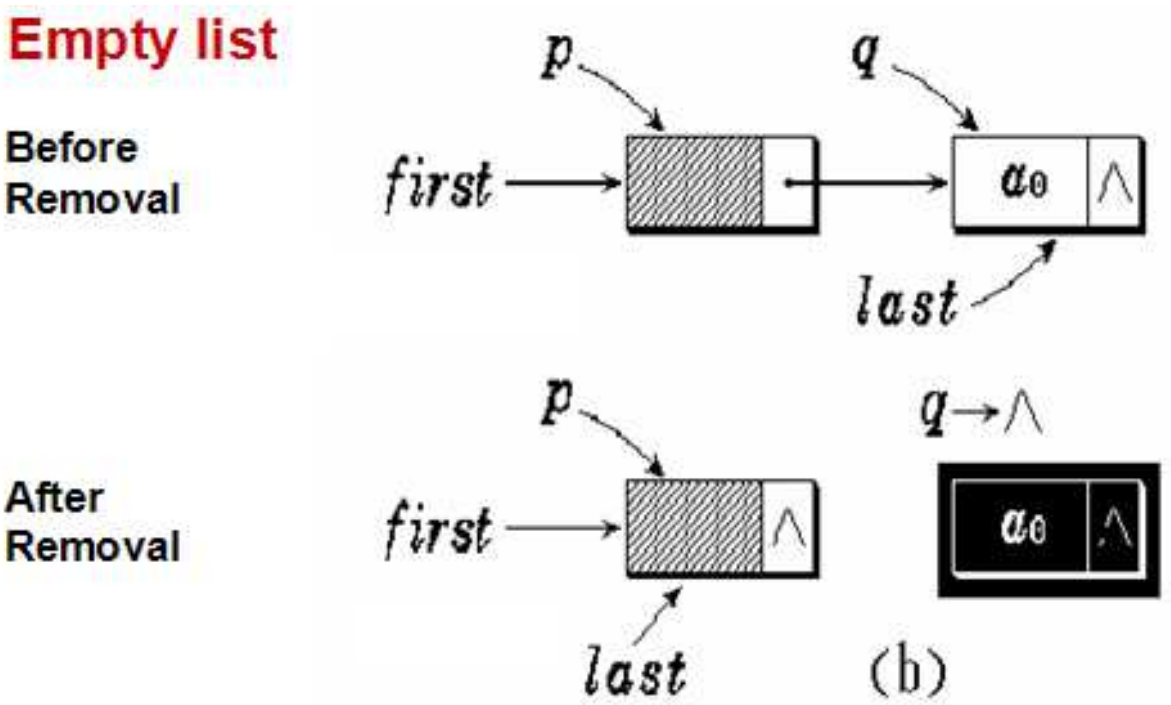
Before Removal



After Removal



3.1.2 List with Dummy node




```
1  q = p->link;
2  p->link = q->link;
3  delete q;
4  if(p->link == NULL)
5      last = p;
```

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```
1  template <class Type> class List;
2
3  template <class Type> class ListNode {
4      friend class List<Type>;
5      Type data;
6      ListNode<Type> *link;
7  public:
8      ListNode ( );
9      ListNode ( const Type& item );
10     //Get the address of next node(successor)
11     ListNode<Type> *NextNode ( ) { return link; }
12     //New a node with (item, next)
13     ListNode<Type> *GetNode ( const Type&
14         item, ListNode<Type> *next );
15     //Insert p-node after the current node
16     void InsertAfter ( ListNode<Type> *p );
17     //Remove the node after current node
18     ListNode<Type> *RemoveAfter ( );
19 };
20
```

```
21 template <class Type> class List {
22     ListNode<Type> *first, *last;
23 public:
24     List ( const Type & value ) {
25         last =first = new ListNode<Type>( value );
26     } //constructor
27     ~List ( );
28     void MakeEmpty ( );
29     int Length ( ) const;
30     ListNode<Type> *Find ( Type value );
31     ListNode<Type> *Find ( int i );
32     int Insert ( Type value, int i );
33     Type *Remove ( int i );
34     Type *Get ( int i );
35 };
36
37 //Constructor 1
38 template <class Type>
39 ListNode<Type> :: ListNode ( ) : link (NULL){ }
40
```

```
41 //Constructor 2
42 template <class Type>
43 ListNode<Type>::
44 ListNode( const Type & item ) :data (item), link (
       NULL){ }
45
46 //New node
47 template <class Type>
48 ListNode<Type>  * ListNode<Type> ::
49 GetNode ( const Type & item,  ListNode<Type> *next
       = NULL )
50 {
51     ListNode<Type> *newnode = new ListNode<Type> (
           item );
52     newnode->link = next;
53     return newnode;
54 }
55
56
57
```

```
58 //Insert a node p after current node
59 template <class Type>
60 void ListNode <Type> :: InsertAfter ( ListNode<
       Type> *p )
61 {
62     p->link = link;
63     link = p;
64 }
65
66
67 //Remove the node after current node
68 template <class Type>
69 ListNode<Type>* ListNode<Type> :: RemoveAfter ( )
70 {
71     ListNode<Type> *tempptr = link;
72     if ( link == NULL ) return NULL;
73     delete link;
74     link = tempptr->link;
75     return tempptr;
76 }
```

```
77 //Constructor (defined in the class declaration)
78 template <class Type>
79 List<Type> :: List ( const Type & value )
80 {
81     last =first = new ListNode<Type>( value );
82 }
83
84 //Destructor
85 template <class Type>
86 List<Type> :: ~List ( )
87 {
88     MakeEmpty ( );  delete first;
89 }
90
91 //Release the linked list
92 template <class Type>
93 void List<Type> :: MakeEmpty ( )
94 {
95     ListNode<Type> *q;
96     while ( first->link != NULL )
```

```
97      {
98          q = first->link;  first->link = q->link;
99          delete q;
100     }
101     last = first;
102 }
103
104 //Get the number of the nodes
105 template <class Type>
106 int List<Type>::Length ( ) const {
107     ListNode<Type> *p = first->link;
108     int count = 0;
109     while ( p != NULL ) {
110         p = p->link;
111         count++;
112     }
113     return count;
114 }
115
116
```

```
117 //Search a value in the list
118 template <class Type>
119 ListNode<Type>*List <Type>:: Find ( Type value ) {
120     ListNode<Type> *p = first->link;
121
122     while ( p != NULL && p->data != value )
123         p = p->link;
124     return p;
125 }
126
127 //Find out the i-th node, return 'its address
128 template <class Type>
129 ListNode<Type> *List<Type> :: Find ( int i ) {
130     if ( i < -1 ) return NULL;
131     if ( i == -1 ) return first;
132     ListNode<Type> *p = first->link;
133     int j = 0;
134     while ( p != NULL && j < i )
135       {
136         p = p->link;
```

```
137             j = j++;
138         }
139         return p;
140     }
141
142     //Insert a new node (value) before the i-th node
        in the list
143     template <class Type>
144     int List<Type> :: Insert ( Type value, int i ) {
145         ListNode<Type> *p = Find ( i-1 );
146         if ( p == NULL ) return 0;
147         ListNode<Type> *newnode =
148             GetNode ( value, p->link );
149         if ( p->link == NULL )
150             last = newnode;
151         p->link = newnode;
152         return 1;
153     }
154
155
```

```
156 //Remove the i-th node in the list
157 template <class Type>
158 Type *List<Type>::Remove ( int i ) {
159     ListNode<Type> *p = Find (i-1), *q;
160     if ( p == NULL || p->link == NULL )
161         return NULL;
162     q = p->link;
163     p->link = q->link;
164     Type value = new Type ( q->data );
165     if ( q == last )
166         last = p;
167     delete q;
168     return &value;
169 }
170
171 //Find out the i-th node, return 'its value
172 template <class Type>
173 Type *List<Type>::Get ( int i ) {
174     ListNode<Type> *p = Find ( i );
175
```

```
176     if ( p == NULL || p == first )
177         return NULL;
178     else
179         return & p->data;
180 }
```

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3.1.4 Iterator Class of Linked List

- The objective of iterator class
  - To search in the linked list
- Principal of iterator class
  - Friend class of ListNode and List classes
  - Iterator object can refer existing CList object
  - Point to the position of the current node in the list
  - Provide several methods for testing and searching

Template description of three Classes of Linked list

```
1  enum Boolean { False, True };
2  template <class Type> class List;
3  template <class Type> class ListIterator;
4
5  template <class Type> class ListNode {
6  friend class List <Type>;
7  friend class ListIterator <Type>;
8  public:
9      ... ...
10 private:
11     Type data;
12     ListNode<Type> *link;
13 };
```

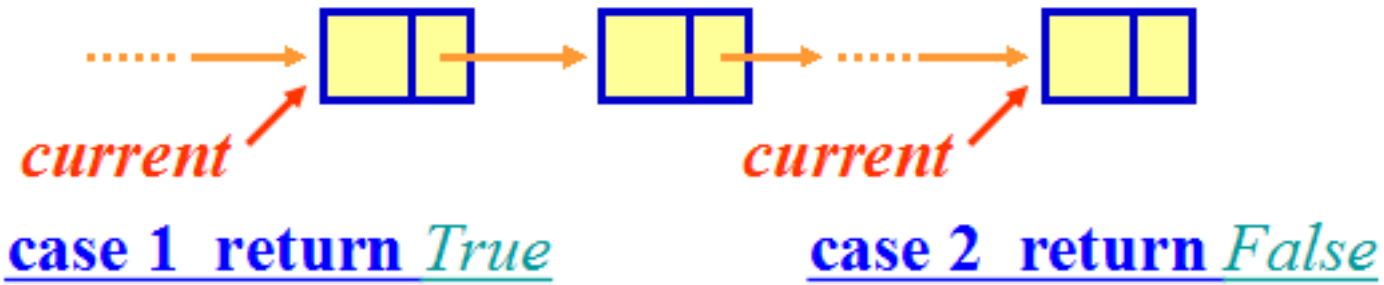
3.1.4 Iterator Class of Linked List

```
1  template <class Type> class  ListIterator {
2  public:
3      ListIterator ( const List<Type> & l )
4  : list ( l ), current ( l.first ) { }
5      Boolean NotNull ( );
6
7      Boolean NextNotNull ( );
8
9      ListNode <Type> *First ( );
10     ListNode <Type> *Next ( );
11 private:
12     const List<Type> & list;
13     ListNode<Type> *current;
14 }
```

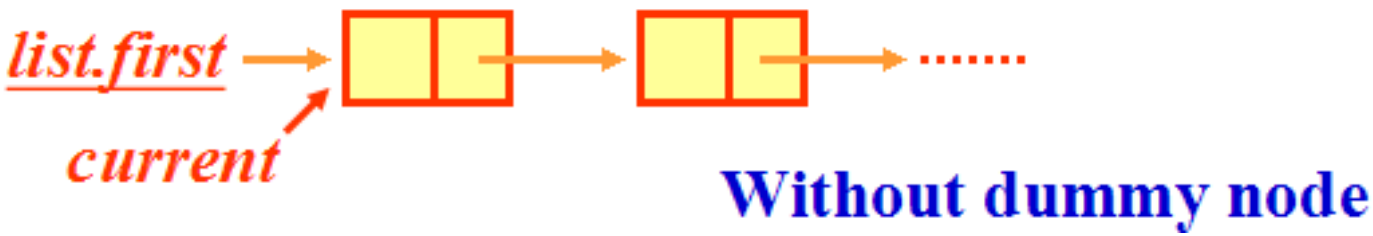
Implementation of methods of Iterator Class

```
1 //Check whether the current node is NULL or not template
      <class Type>
2 Boolean ListIterator<Type> :: NotNull ( ) {
3       if ( current != NULL ) return True;
4       else return False;
5 }
```

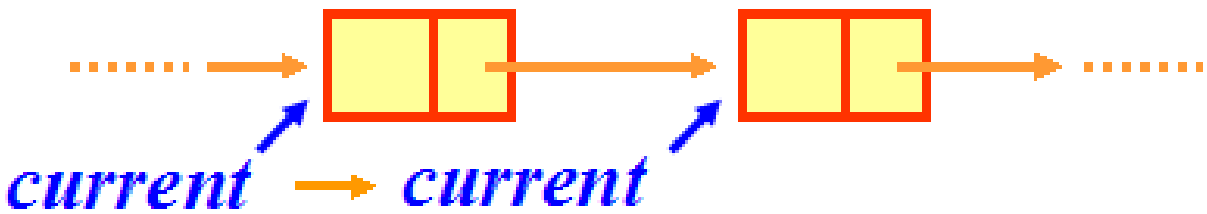
```
1  //Check whether the next node of the current
2  //node is NULL or not
3  template <class Type>
4  Boolean ListIterator<Type>::NextNotNull ( ) {
5      if ( current != NULL &&
6           current->link != NULL ) return True;
7      else return False;
8  }
```



```
1  //return the address of the first node
2  template <class Type>
3  ListNode<Type>* ListIterator<Type> :: First ( ) {
4      if ( list.first != NULL ){
5          current = list.first;
6          return current;
7      }
8      else { current = NULL;  return NULL; }
9  }
```



```
1  template <class Type>
2  ListNode<Type>* ListIterator<Type> :: Next ( ) {
3       if ( current != NULL
4                 && current->link != NULL ) {
5            current = current->link;
6            return current;
7       }
8       else { current = NULL;  return NULL; }
9  }
```



Example: compute the sum of elements of the list using iterator class

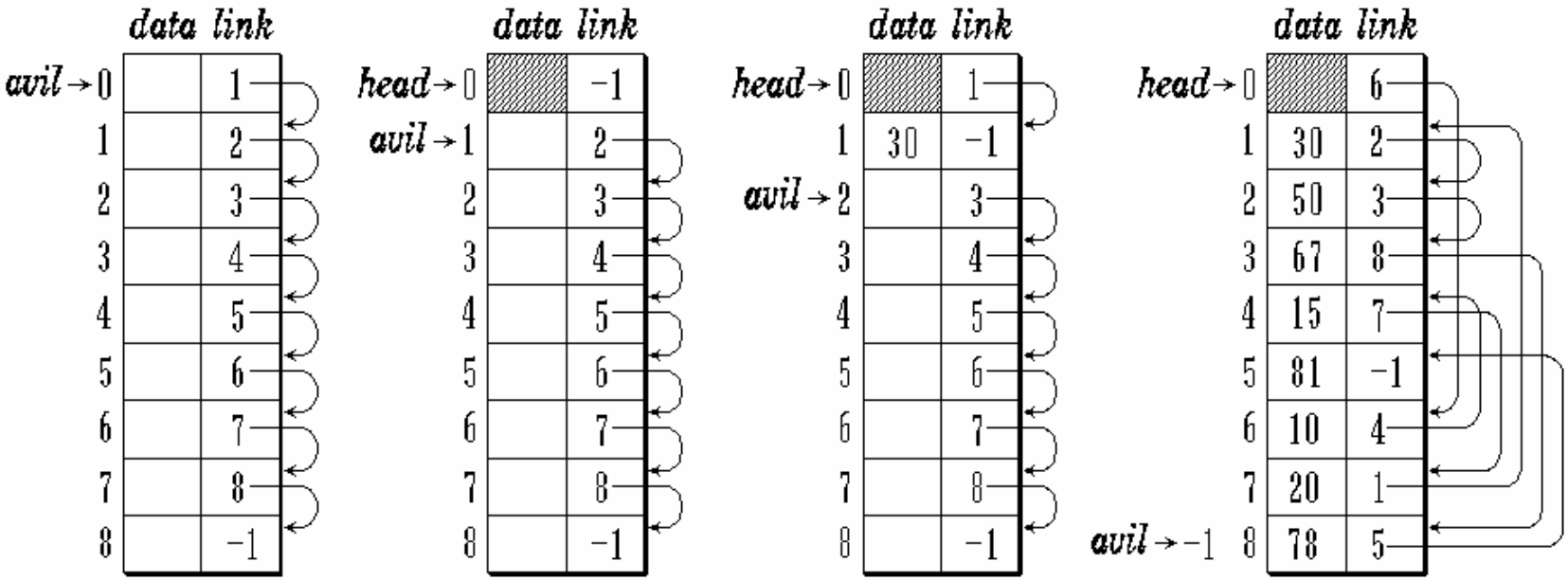
```
1  int sum ( const List<int> &l )
2  {
3      ListIterator<int> li ( l );
4      if ( ! li.NotNull () )
5          return 0;
6      int retval =  li.First()->getData();
7      while ( li.nextNotNull () )
8          retval +=  li.Next()->getData();
9
10     return retval;
11 }
```

Next Subsection

3.1.5 Static Linked List

Defined by array

Store space is invariable



New node j = avil; avil = A[avil].link;

Release A[i].link = avil; avil = i;

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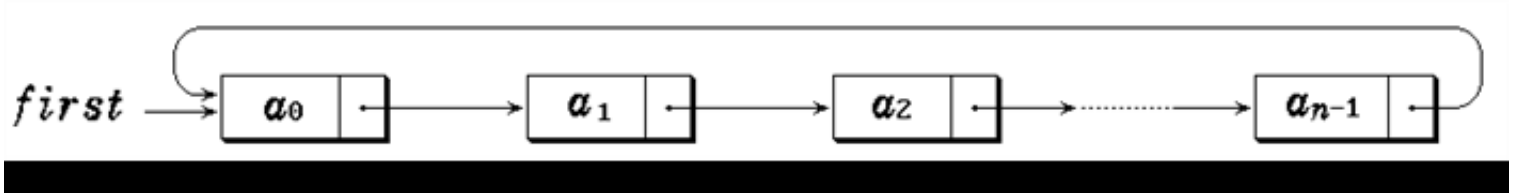
3.1.6 Circular list

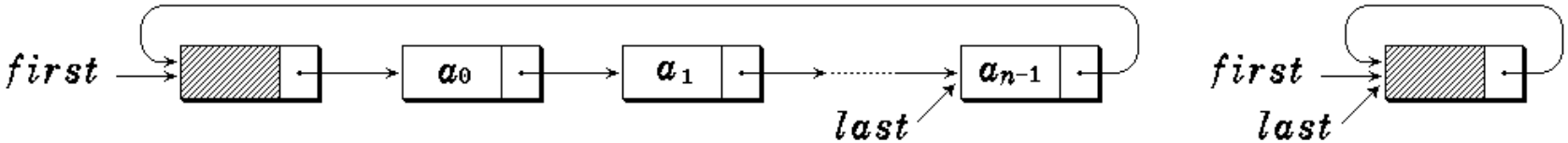
- Circular list is an advanced singly linked list
  - The link field of the last node of the list is not 0 yet, pointing to the head of list;
  - If we know the position of arbitrary node of the circular list, we could access all the nodes one by one.
- Circular list has two kinds of representation
  - Without Dummy node
  - With dummy node

Example

- An example of circular list



- Circular list with DN

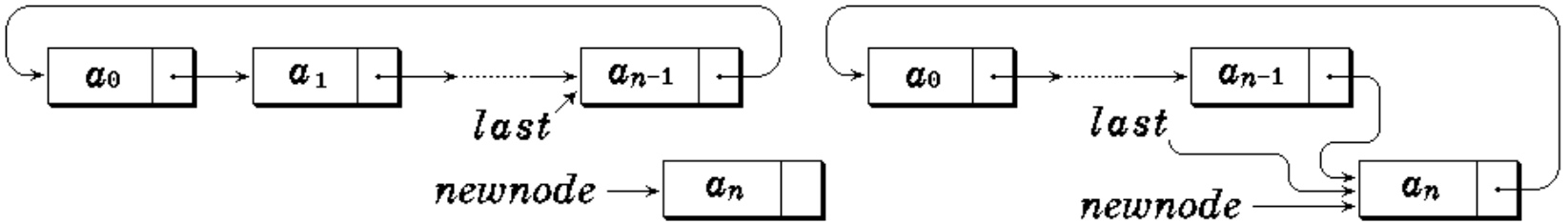


Class of circular list

```
1  template <class Type> class CircList;
2
3  template <class Type> class CircListNode {
4      friend class CircList;
5  public:
6      CircListNode ( Type d = 0,
7          CircListNode<Type> *next = NULL ) :
8      data ( d ), link ( next ) { }
9  private:
10     Type data;
11     CircListNode<Type> *link;
12 }
13
14 template <class Type> class CircList {
15 public:
16     CircList ( Type value );
17     ~CircList ( );
18     int Length ( ) const;
19     Boolean IsEmpty ( )
20     { return first->link == first; }
```

```
21      Boolean Find ( const Type & value );
22      Type getData ( ) const;
23      void Firster ( ) { current = first; }
24      Boolean First ( );
25      Boolean Next ( );
26      Boolean Prior ( );
27      void Insert ( const Type & value );
28      void Remove ( );
29  private:
30      CircListNode<Type> *first, *current, *last;
31  };
```

3.1.6 Circular list

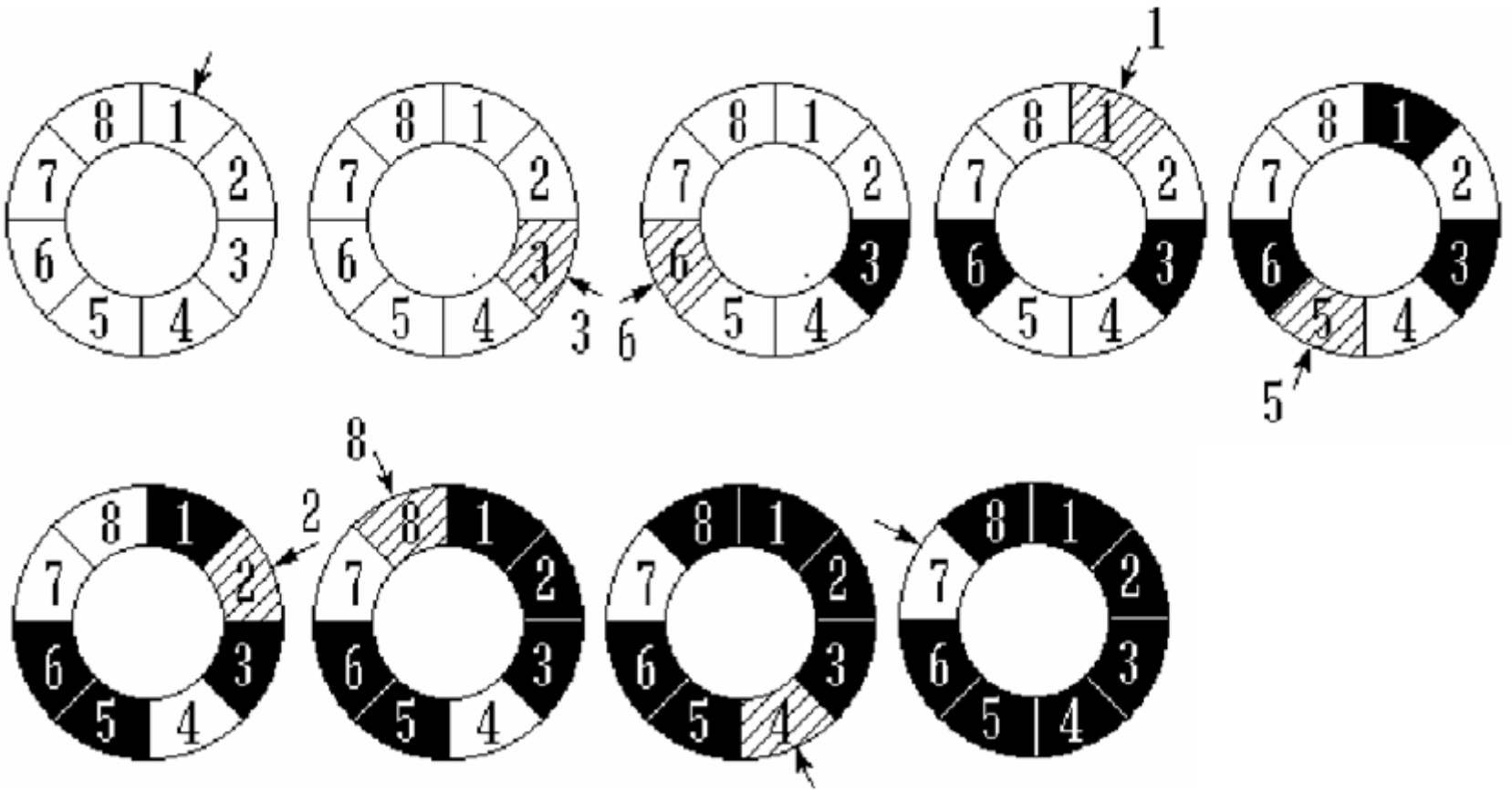


Next Section

Next Subsection

3.2.1 Joseph Problem

- Joseph Problem: m=3, n=8



```
#include <iostream.h>
#include “CircList.”h
Template<Type> void CircList<Type>
:: Josephus ( int n, int m ) {
    Firster ( );
    for ( int i = 0; i < n-1; i++ ) {
        for ( int j = 0; j < m-1; j++ )
            Next ( );
        cout << "The out one is"
            << getData ( )  << endl;
        Remove ( );
    }
}

void main ( ) {
    CircList<int> clist;
    int n, m;
    cout << "Enter the Number of Contestants?";
    cin >> n >> m;
    //Construct Joseph circle
    for ( int i=1; i<=n; i++ )
```

```
22          clist.insert (i);
23      //Call Joseph function
24      clist.Josephus (n, m);
25  }
```

Next Subsection

3.2.2 Polynomial

- Polynomial

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n = \sum_{i=0}^{n} a_i x^i$$

- Node:

$data \equiv Term$

coef	exp	link

Class description of polynomial using Linked List

```
1  struct Term {
2      int coef;
3      int exp;
4      Term ( int c, int e ) { coef = c;  exp = e; }
5  };
6
7  class Polynomial    {
8      List<Term> poly;
9      friend Polynomial & operator +
10           ( Polynomial &, Polynomial &);
11 };
```

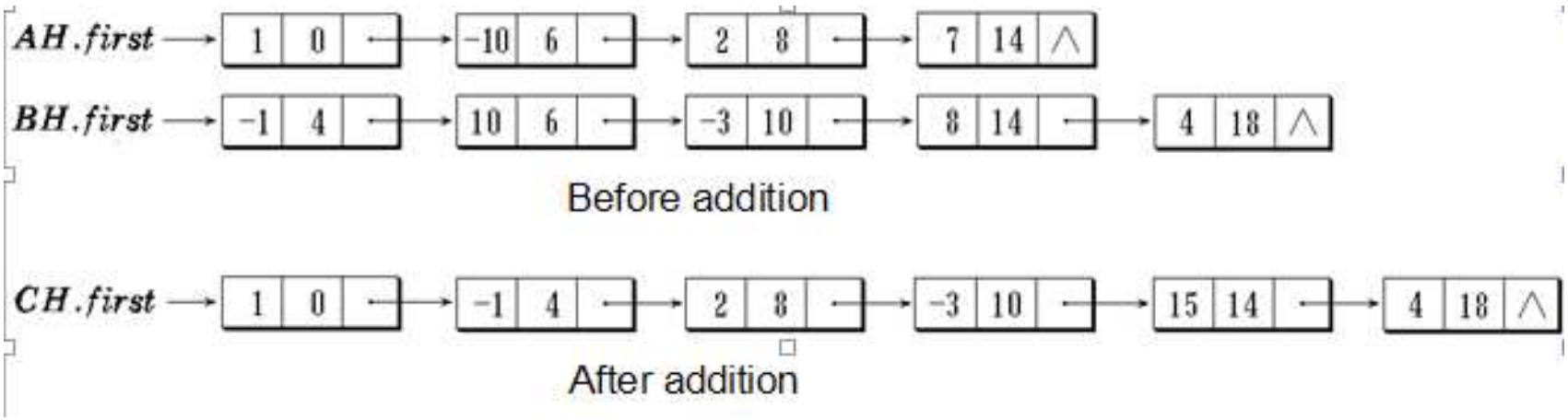
Polynomial Addition

$$AH = 1 - 10x^6 + 2x^8 + 7x^{14}$$

$$BH = -x^4 + 10x^6 - 3x^{10} + 8x^{14} + 4x^{18}$$

$$CH = AH + BH$$

$$= 1 - x^4 + 2x^6 - 3x^{10} + 15x^{14} + 4x^{18}$$



```
1  Polynomial & operator + ( Polynomial & ah,
2                  Polynomial & bh )
3  {
4      ListNode<Term> *pa, *pb, *pc, *p;
5      // Aiter, Biter
6      ListIterator<Term> Aiter ( ah.poly );
7      ListIterator<Term> Biter ( bh.poly );
8      // pa, pb
9      pa = pc = Aiter.First ( );   // ah
10     pb = p = Biter.First ( );     // bh
11     pa = Aiter.Next ( );
12     pb = Biter.Next ( );
13     delete p;
```

```
1  while ( Aiter.NotNull ( ) && Biter.NotNull ( ) )
2      switch ( compare ( pa$\to$exp, pb$\to$exp ) ) {
3      case ' = ' :
4          pa->coef = pa->coef + pb->coef;
5          p = pb;   pb = Biter.Next ( );  delete p;
6          if ( !pa->coef ) {
7              p = pa;  pa = Aiter.Next ( );
8              delete p;
9          }
10         else {
11             pc->link = pa;   pc = pa;
12             pa = Aiter.Next ( );
13         }
14          break;
15     case ' > ' :       // pa->exp> pb->exp
16          pc->link = pb;   pc = pb;
17          pb = Biter.Next ( );   break;
18    case ' < ' :        // pa->exp< pb->exp
19       pc->link = pa;   pc = pa;
20       pa = Aiter.Next ( );
21    }
```

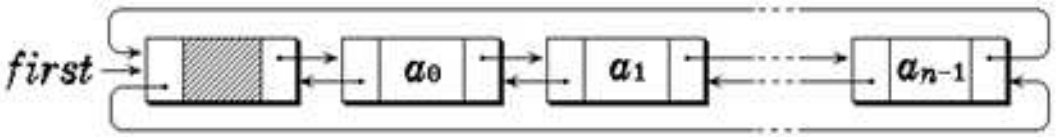
```
22      if ( Aiter.NotNull ( ) )
23              pc->link = pa;
24      else
25              pc->link = pb;
26  }
```

Next Section

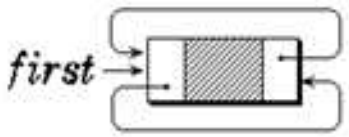
3.3 Doubly Linked List (DblList)

- Drawback of singly linked list
  - Can't visit the predecessor of the current node
- Improved criteria
  - Besides the existed successor linked pointer, add another pointer which used to point to the predecessor node

3.3 Doubly Linked List (DblList)



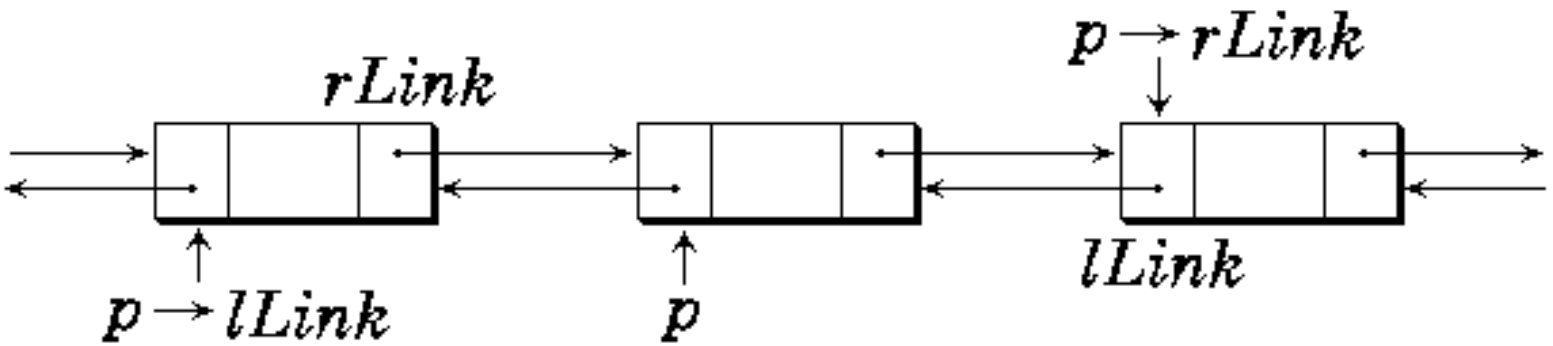
Non-empty list



Empty list

- Relations between the predecessor, current node and successor

```
1   p==p->lLink->rLink==p->rLink->lLink
```

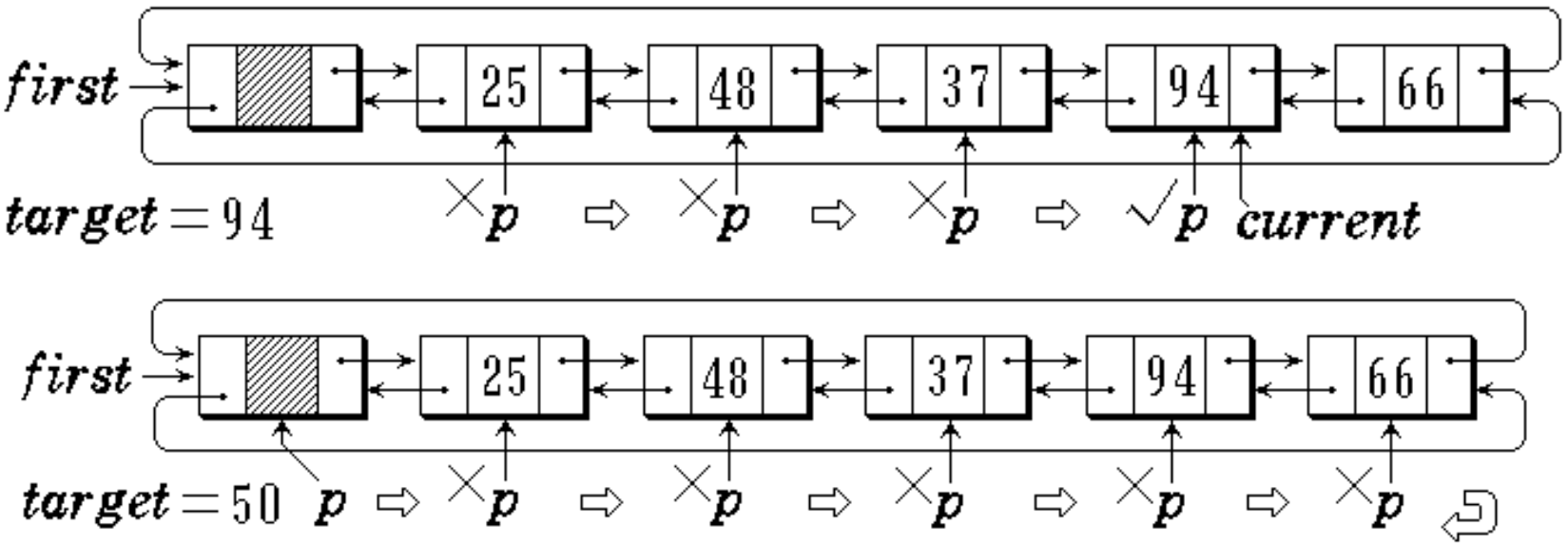


Class of DblList

```
1  template <class Type> class DblList;
2  template <class Type> class DblNode {
3  friend class DblList<Type>;
4  private:
5      Type data;                          //data
6      DblNode<Type> *lLink, *rLink;       //pointer
7
8  DblNode(Type value, DblNode<Type> *left,
9      DblNode<Type> *right):data (value), lLink (left),
           rLink (right)
10    { }
11 DblNode ( Type value ) : data (value),
12           lLink (NULL), rLink (NULL){ }
13 };
```

```
1  template <class Type> class DblList {
2  public:
3    DblLIst ( Type uniqueVal );
4    ~DblList ( );
5    int Length ( ) const;
6    int IsEmpty ( ) { return first->rlink == first; }
7    int Find ( const Type & target );
8    Type getData ( ) const;
9    void Firster ( ) { current = first; }
10   int First ( );
11   int Next ( );
12   int Prior ( );
13   int operator!(){ return current != NULL; }
14   void Insert ( const Type & value );
15   void Remove ( );
16 private:
17   DblNode<Type> *first, *current;
18 };
```

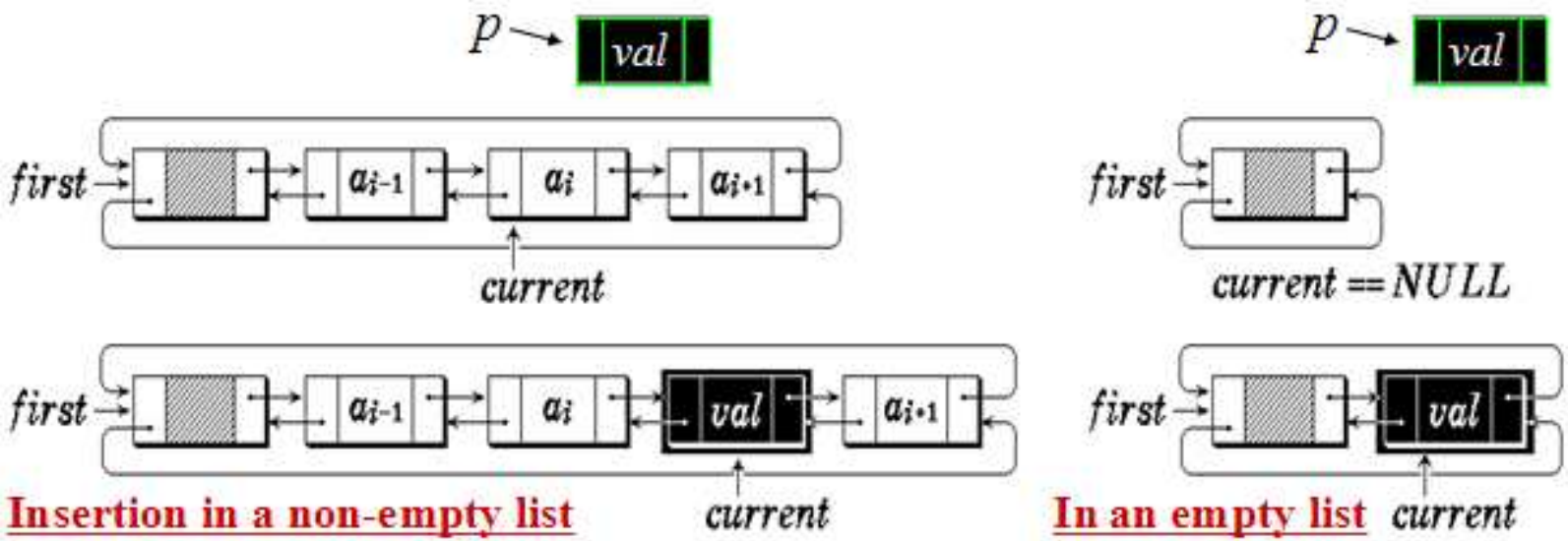
Searching in DblList



```
1  //Find out the target in the DbL-List
2
3  template <class Type>
4  int DblList<Type>::Find ( const Type & target ) {
5      //Return 1 if success, otherwise return 0
6      DblNode<Type> *p = first->rLink;
7      while ( p != first && p->data != target )
8        p = p->rLink;
9      if ( p != first ) { current = p;  return 1; }
10     return 0;
11 }
```

Insertion after current node in DblList

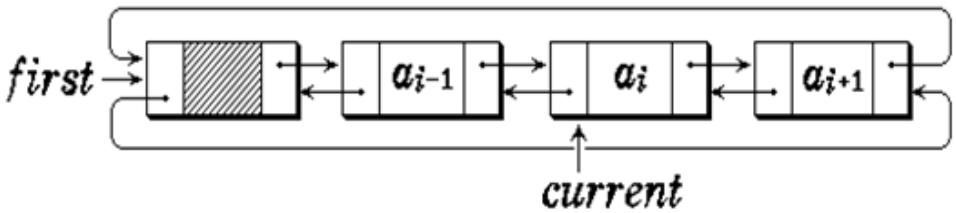
```
1 p->rLink =current->rLink;
2 current->rLink = p;
3 current->rLink->Link = current;
4 current = current->rLink;
5 current->rLink->Link = current;
```

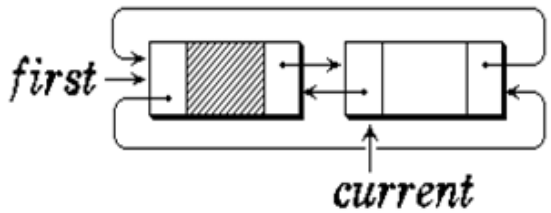


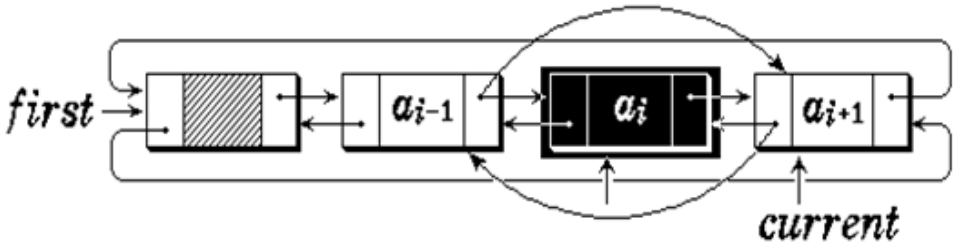
```
1  //Insert a node (value) after the current node
2  template <class Type>
3  void DblList<Type>::Insert ( const Type & value ) {
4       if ( current == NULL )
5            current = first->rLink =
6            new DblNode ( value, first, first );
7       else {
8            current->rLink =new DblNode
9          ( value, current, current->rLink );
10      current = current->rLink;
11      }
12      current->rLink->lLink = current;
13 }
```

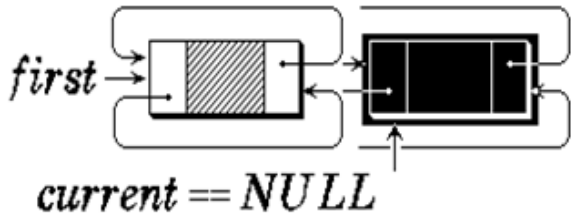
Node removal in DblList

```
1 current->rLink->lLink=current->lLink;
2 current->lLink->rLink=current->rLink;
3 current=current->rLink;
```









```
1  //Remove the current node
2  template <class Type>
3  void DblList<Type>::Remove ( ) {
4      if ( current != NULL ) {
5          DblNode *temp = current;
6          current = current->rLink;
7          current->lLink = temp->lLink;
8          temp->lLink->rLink = current;
9          delete temp;
10         if ( current == first )
11            if ( IsEmpty ( ) )
12                current = NULL;
13            else
14                current = current->rLink;
15     }
16 }
```

```
1 //Constructor
2
3 template <class Type>
4 DblList<Type>::DblLIst ( Type uniqueVal ) {
5      first = new DblNode<Type> ( uniqueVal );
6      first->rLink = first->lLink = first;
7      current = NULL;
8 }
```

```
1  //Get the number of the nodes in the DbL List
2
3  template <class Type>
4  int DblList<Type>::Length ( ) const {
5      DblNode<Type> * p = first->rLink;
6      int count = 0;
7      while ( p != first )
8          { p = p->rLink;  count++; }
9      return count;
10 }
```

```
1 //Move the current pointer to the first node
2
3 template <class Type>
4 int DblList<Type>::First ( ) {
5     if ( !IsEmpty ( ) )
6           { current = first->rLink;  return 1; }
7     current = NULL;
8     return 0;
9 }
```

```
1 //Move the current pointer to the next node
2
3 template <class Type>
4 int DblList<Type>::Next ( ) {
5     if ( current->rLink == first )
6      { current = NULL;  return 0; }
7     current = current->rLink;
8     return 1;
9 }
```

```
1 //Move the current pointer to the prior node
2
3 template <class Type>
4 int DblList<Type>::Prior ( ) {
5      if ( current->lLink == first )
6      { current = NULL;  return 0; }
7      current = current->lLink;
8      return 1;
9 }
```

Next Section

3.4 Summary

1. Linked List
   - Singly Linked List
   - List with Dummy node
   - Class definition of Linked List Using Template
   - Iterator Class of Linked List
   - Static Linked List
   - Circular list
2. Applications
   - Joseph Problem
   - Polynomial
3. Doubly Linked List (DblList)
4. Summary
5. QUIZ

Next Section

3.5 QUIZ and homework

1、设计一个算法，从顺序表中删除其值在s和t（包含相等）之间的所有元素，若顺序表为空，则显示出错信息并退出运行。

```
1 template <class DataType> int deleteNo_stot(
      SeqList &L, DataType s, DataType t)
2 {}
```

2、设有一个表头指针为h的单链表。设计一个算法，通过遍历一趟链表，将连表中所有节点的链接方向逆转。

```
1 void Reverse(LinkNode *h)
2 {}
```

3.5 QUIZ and homework

3、试设计一个算法，改造一个带表头结点的双向循环链表，所有结点的原有次序保持在各个结点的右链域rLink中，并利用左链域lLink把所有结点按照其值从小到大的顺序连接起来。

```
1 typedef struct DblNode{
2     int data;  struct DblNode *lLink, rLink;
3 } DblNode;
4 typdef DblNode* DblList;
5 void SortedList(DblList dblist){}
```

- 4、 For a large number, how to represent it and implement the $+$,$-$, $*$, $/$ operations? For example: