QUANTITATIVE PERSPECTIVES

THE RELATIONSHIP BETWEEN THE YIELD CURVE & MORTGAGE CURRENT COUPON

By Eknath Belbase, PhD & Dan Szakallas

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ANDREW DAVIDSON & CO., INC.

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Introduction

A dynamic prepayment model is an integral component of an option-adjusted valuation and risk management framework. Such a model typically requires a forecast of future mortgage rates and possibly index rates such as prime and COFI. Term structure models are used to forecast rates of various maturities on the Libor/Swap yield curve or on the Treasury yield curve. It is therefore necessary to model mortgage rates as functions of these yield curve rates.

In this article we discuss the Andrew Davidson & Co., Inc. approach to forecasting mortgage current coupon as a function of either Treasury or swap rates. Along the way, we compare several statistical approaches to the problem and discuss software implementation issues. In addition, we examine the relative performance of using a single Treasury rate, two Treasury rates or two swap rates to forecast mortgage current coupon rates.

Data

The current coupon is the semi-annual equivalent of the parity-price interpolated coupon, based upon the two bonds whose price brackets the parity price. The mortgage current coupon rates used were based on month-end closing prices. The types used are Fannie Mae 7, 15 and 30 year, Ginnie Mae I 15 and 30 year, and Freddie Mac 5, 7, 15 and 30 year. These rates can be viewed on Bloomberg, e.g. MTGE FNCL <Index> for the FNMA 30 rate.

The Treasury rates used were month-end closing par bond-equivalent yields for the two and ten-year on-the-run Treasuries (GT2 <govt> and GT10 <govt>) or the two and ten-year zero coupon Treasuries (STRP02Y<govt> and STRP10Y<govt>). The swap rates used were either the two and ten-year Libor rates (USSWAP2 <index> and USSWAP10 <index>) or the two and ten-year zero coupon Libor rates, which were derived from the par swap rates using a boot-strapping method with linear interpolation.

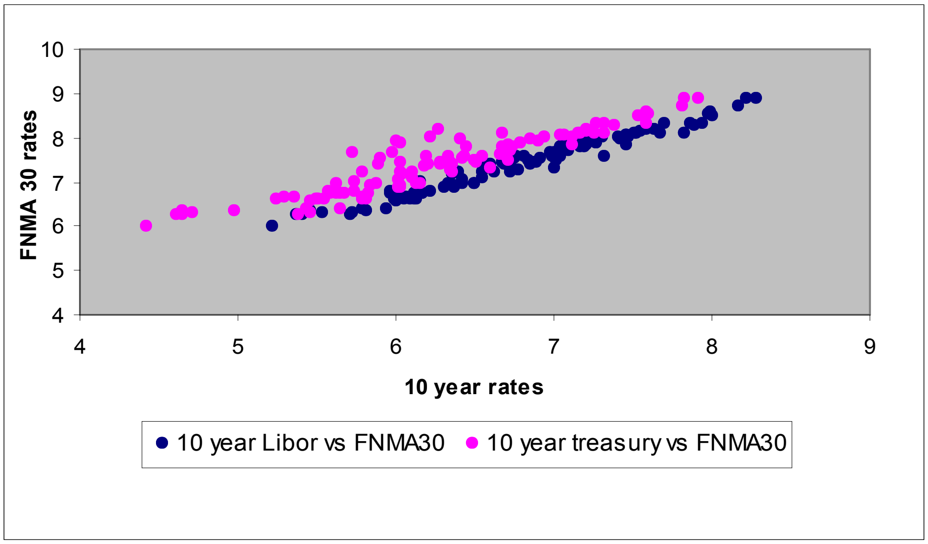
All rates used for this analysis covered November 1991 through August 2000, and there were no missing values.

Preliminary Analysis

Figure 1 is a scatter plot of FNMA 30 year current coupon versus ten-year Treasuries and versus ten-year swap rates on the same scale. This plot suggests that ten-year swap rates may be a better predictor of current coupon than ten-year Treasury rates because the swap versus current coupon points show much less variation about a line through their "center."

Figure 1

FNMA 30 & Libor fit vs FNMA 30 & Treasury fit



Because we also need to predict rates of shorter maturities, such as five and seven-year balloon rates, it is useful to examine the relationship between ten-year rates and a balloon rate. Figure 2 displays scatter plots of a five-year balloon rate versus the ten-year Treasury and swap rates. In contrast to Figure 1, neither Treasury nor swap rates gives a tight fit.

Figure 2

FHG 5 &

Libor fit vs.

FHG 5 &

Treasury fit

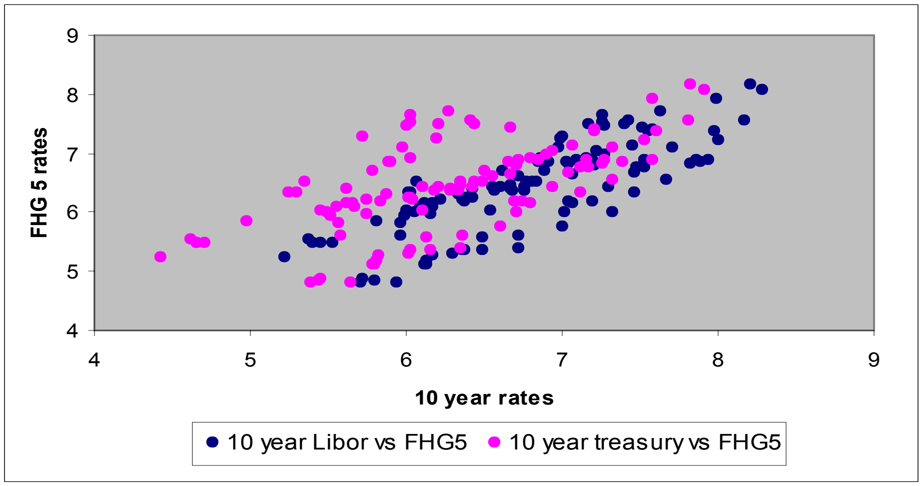


Figure 3 shows FHG five-year balloons plotted against two-year Libor and Treasury. It appears to be a much better fit than that in Figure 2.

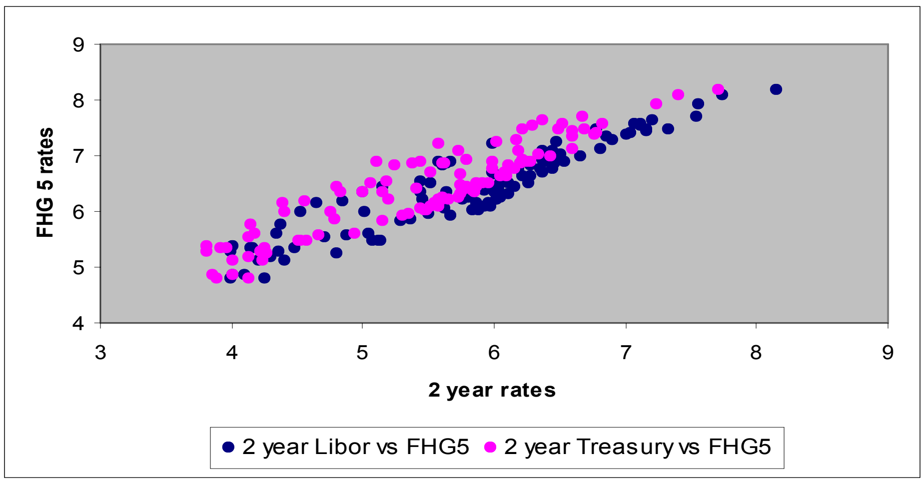
Figure 3

Libor &

FHG 5 fit vs

Treasury &

FHG5 fit



Next, we give an example of a model that simply uses the ten-year Treasury plus a spread to predict the FNMA current coupon. The spread between current coupon and the ten-year Treasury rate in the first month of analysis, November 1991, was used:

FNMA30_CC = 10YR_TREAS + .91

Now, using rates from August 2000, we have:

10YR_TREAS + .91 = 5.72 + .91 = 6.63

The actual value of the FNMA 30 current coupon for August 2000 was 7.7, therefore the prediction using the ten-year Treasury and spread was 107 basis points off.

Next, we attempt to predict the FHG 5 using the ten-year Treasury plus spread, utilizing the same methods as the previous example:

10YR_TREAS + .50 = 5.72 + .50 = 6.22

The actual value of the FHG 5 current coupon for August 2000 was 7.3. Again, the prediction using only the ten-year Treasury ended up 99 basis points off.

Before deciding on a final set of yield curve maturities to use, it is helpful to look at correlations between rates of different maturities and some mortgage current coupon rates. Ideally, the rates chosen to model mortgage current coupon would come from liquid points on the yield curve. In addition, they should have high correlations with mortgage current coupon rates, but low correlations with each other. A high correlation with mortgage current coupon indicates that there is a strong linear relationship between the two rates; a low correlation with each other implies that the additional rate adds to the strength of the relationship. The final set chosen should maximize fit with as simple a set of rates as possible.

The correlation matrix of two and ten-year Libor rates and one, two, five, and ten-year Treasury rates versus the thirty-year FNMA and five-year Freddie balloon mortgage rates is shown below.

Table 1

Correlation Matrix

	Libor 2yr	*Libor 10yr*	*1yr treas*	*2yr treas*	*5yr treas*	*10yr treas*	*fhg5CC*	*fan30CC*
Libor 2yr	1	0.604	0.937	0.979	0.739	0.422	0.932	0.662
Libor 10yr		1	0.386	0.647	0.932	0.932	0.798	0.983
1yr treas			1	0.927	0.580	0.242	0.791	0.464
2yr treas				1	0.818	0.526	0.911	0.708
5yr treas					1	0.914	0.835	0.945
10yr treas						1	0.609	0.915
Freddie 5							1	0.837
Fannie 30								1

The correlation matrix tells us several things. First, using the two-year rate is better than using the one-year rate because the two-year rate has both higher correlations with mortgage current coupon rates and low correlation with the ten-year rate. Second, while the five-year Treasury has a stronger relationship with the thirty-year mortgage rate than does the ten-year Treasury, it has a weaker relationship with the five-year balloon rate than does the two-year Treasury. Because the five-year Treasury has a high correlation with both of these Treasury rates, it is inadvisable to include the five-year in a regression model with either the two-year or ten-year Treasury.

Therefore, based on the scatter plots, the magnitude of the errors that resulted in using only ten-year rates, and the correlation analysis, we found that the two-year and ten-year rates should be used in our regression.

Regression Approaches

A standard simple regression model is of the form $Y_t = ß_0 + ß_1 X_t + \varepsilon_t$. We try to predict readings of the variable Y (e.g. current coupon) at time t in terms of a constant (spread) plus another constant multiplied by the variable X (e.g. 10 year Treasury) at time t. Both constants are the same for all times, and additional variables can be added to the right side to try to improve the fit.

ε_t is the error term, meaning it is the amount a given reading deviates from the value predicted by the line of "best fit." The error terms are assumed to be uncorrelated with each other and are derived from the same random distribution with zero mean and equal variance. However, when modeling time series data, the first assumption is often violated because error terms are auto-correlated, which means that the error in a given period adds explanatory power to the error in the next period.

Fitting a standard regression to a model with auto-correlated error terms can lead to substantial bias in the parameter estimates, resulting in regression lines that do not fit the data as well as a model that corrects for autocorrelation. In addition, ignoring autocorrelation can cause measures of fit that overstate the actual strength of the relationship.

One can test for autocorrelation using the Durbin-Watson statistic, D, which is computed when building a regression model. This test determines whether the autocorrelation parameter ρ is 0. The statistic is computed using methods described in Appendix A. The statistic is then compared to upper and lower bounds that are determined based on sample sizes and the number of independent variables in the model.

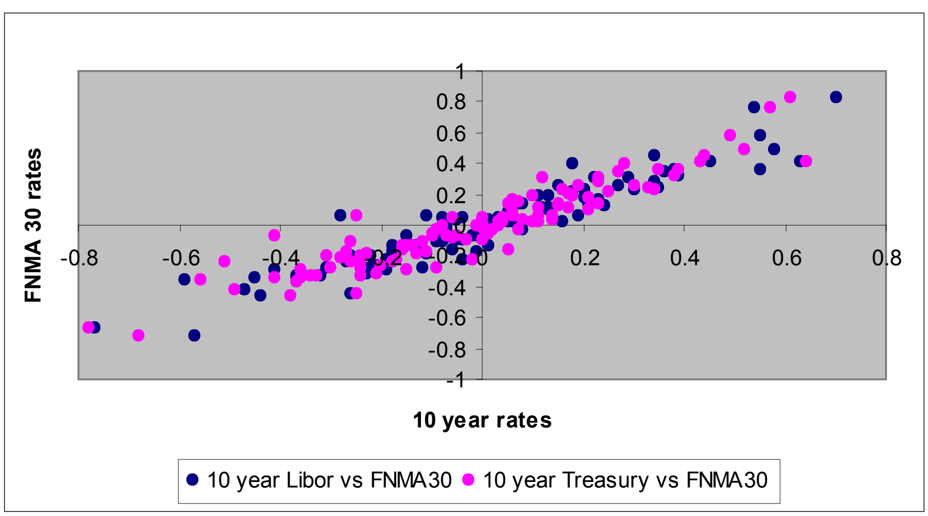
A value of D outside the bounds leads to definite conclusions about autocorrelation. If D is greater than the upper bound, then $\rho=0$ and the error terms are uncorrelated. If D is less than the lower bound, then $\rho>0$ and there is positive autocorrelation. Unfortunately, if the value falls inside the bounds, the test is inconclusive

One way to analyze time-series data with autocorrelation is to perform a regression on differences between observations, rather than on observation values themselves. This technique, commonly called the method of first differences, adjusts for autocorrelation to a significant degree.

When using the method of first differences, the autocorrelation parameter is assumed to be equal to 1, and the model is transformed to $Y'_t = \beta'_1 X'_t$. A) The variables X'_t and Y'_t are the first differences. B) They are represented by equations $X'_t = X_t - X_{t-1}$ and $Y'_t = Y_t - Y_{t-1}$ (it must be noted that the assumption of $\rho=1$ does not typically represent the true value of autocorrelation.) The regression coefficient, β'_1, can be estimated by using the least squares method for regression through the origin. Figure 4 shows a scatter plot of the differences between the points in FNMA 30s and those of ten-year Libor and Treasuries. The correlation coefficients from the first differences method are .942 for ten-year Libor and FNMA 30 and .932 for ten-year Treasury and FNMA 30.

Figure 4

FNMA 30 & Libor fits vs FNMA 30 & Treasury fits



Another approach to regression with auto-correlated data involves estimation of ρ using the Durbin Watson statistic. Then, in the regression equations, the coefficients are estimated using a variation of first differences, where the estimated ρ is multiplied by the first data point and then subtracted from the second data point, and so on throughout the whole data set in the same manner as the first differences method. This is known as the Hildreth-Liu method[1], and a mathematical illustration of this process is shown in Appendix A.

Analysis

We are interested in comparing par-coupon Treasury rates and swap rates to determine which are more accurate predictors of mortgage current coupon. Because many interest rate term structure models output zero-coupon Treasury or swap rates, we also consider zero-coupon rates as predictors of mortgage current coupon rates.

Due to the way effective duration and convexity are defined, when we perform a parallel yield curve shift, we would like all the mortgage current coupons to shift by the same amount as Treasuries or swap rates. For example, if we make a 50 basis point shift across the entire yield curve, we want all the mortgage current coupons to shift by 50 basis points as well.

We can achieve this by placing a restriction on the model that the sum of the coefficients in the regression must be equal to 1. Another advantage of ensuring that the sum of the coefficients is equal to 1 is that it clearly shows the relative strength of the effect of each covariate on each particular mortgage type. For example, we would expect the two-year rate coefficient to be higher for balloon loans and smaller for 30-year loans.

In all, we had seventy-two models to compare; since there were nine different agency and maturity combinations, four covariate groupings, and two model estimation techniques (finite differences and Hildreth-Liu) per covariate grouping. The analysis was done within the SAS system for

[1] The original Hildreth-Liu method used least-squares to determine ρ; so using the D-W statistic should really be called the modified Hildreth-Liu method.

Windows V8. A sample of the code used to produce the regression models is included in Appendix A.

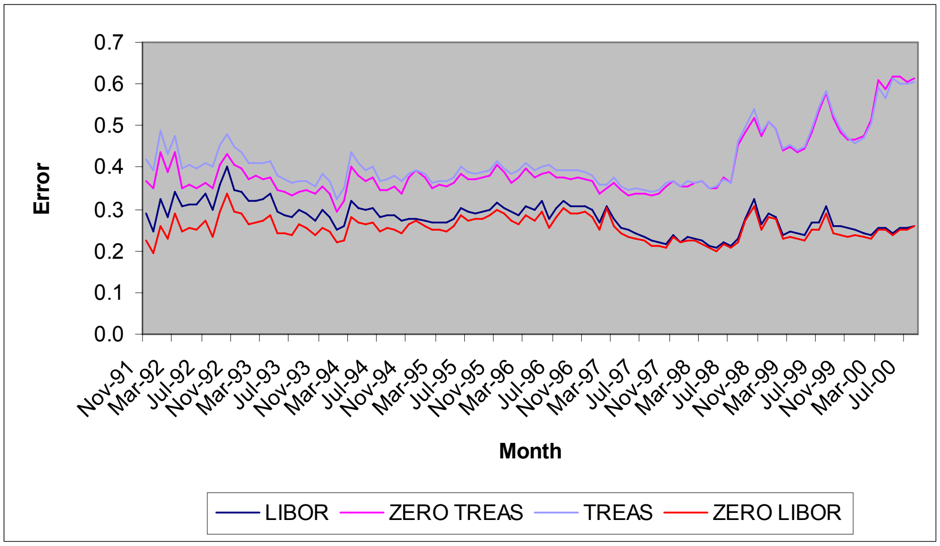
The first measure of error used to compare the four sets of independent variables and the two estimation techniques was the square root of the sum over all nine coupon types of all of the squared errors divided by the number of current coupons.[2] This measure of error utilizes the constant term from the regression, which is equivalent to using an average spread. Increases in this measure of error can be caused either by the non-spread coefficients (and bad explanatory variables) or by changes in this historical spread. To distinguish between the two, we used an additional measure of error based on examining only the changes in the predictors and predicted variable, and not on the spread. This second measure of error was simply the standard deviation of the differences between the predicted and actual values of changes in the underlying variables.

Results

Figure 5 is a graph of the first measure of error as a function of time for the four covariate sets using the first differences estimation approach: Figure 6 displays the results of the modified Hildreth-Liu method.

Figure 5

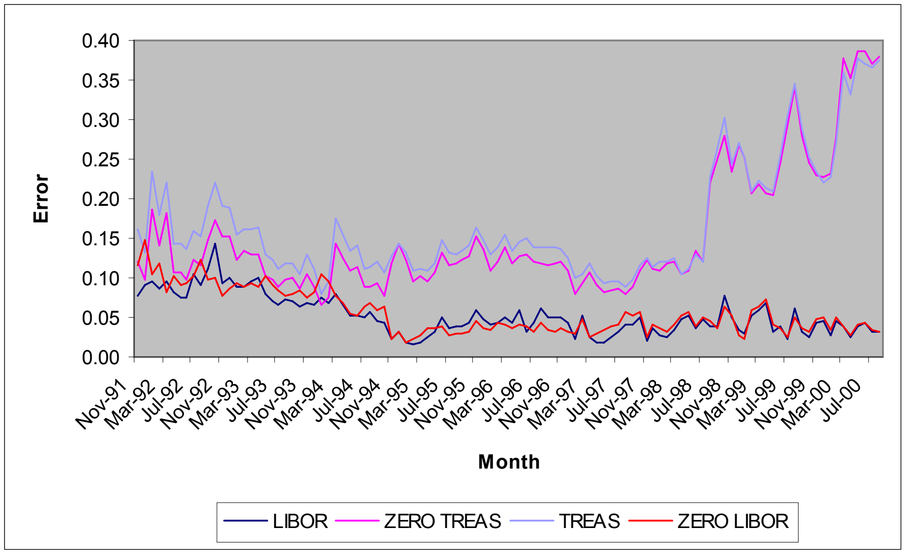
Error Between Estimated and Actual Current Coupon for First Differences Method using Spread Term



[2] This is one measure of average distance over coupons and it does not provide information on whether some coupons were better predicted and others less well predicted.

Figure 6

Error Between Estimated and Actual Current Coupon for Hildreth-Liu Method using Spread Term



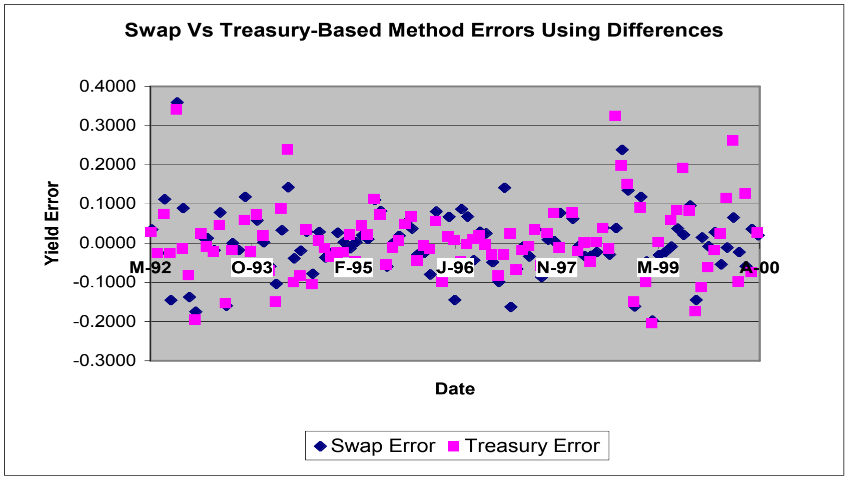
Two features of these graphs are very striking. First, the overall difference in error between the two estimation methods is significant. The range for Figure 5 is 20 to 60 bp, while the range for Figure 6 is only about 2 to 40 bp. This indicates that estimation by the Hildreth-Liu method is much more accurate than with the first differences method.

The second interesting result is that the swap rate errors are consistently lower than Treasury errors after March 1994. Between March 1994 and 1995, the errors for both swap models drop to about 5 bp and hover around that mark for the rest of the time period covered. In contrast, errors using the Treasury model remain more than twice as high until July 1998, when there is a very dramatic increase that continues to August 2000.

Appendix B contains coefficient estimates and R^2 statistics for the four estimations (par/zero, swap/Treasury) performed using the modified Hildreth-Liu method. For a given yield curve type, there appears to be no significant difference between using zero or par rates, as one would expect. In general, as the maturity of the mortgage rate being modeled increases, the coefficients shift from a heavy weight on the two-year rate and a low weight on the ten-year rate, to a low weight on the two-year rate and a higher weight on the ten-year. In addition, there does not appear to be a great amount of variation in the measures of fit within a particular method. These results add to the evidence in Figure 5: not only are the average errors significantly lower using swap rates, but the fit is better for each and every current coupon type.

It remains unclear, however, whether this difference in the fit is due to shifts in the spread level (the constant term in the regression) between Treasuries and the other rates, or simply that none of the coefficients are as good when we use Treasuries. In order to answer this question, we examined the standard deviation of errors using monthly movements in swap rates and Treasury rates to predict monthly movements in mortgage rates using the same coefficients as before, without the constant terms. Figure 7 displays these errors over time for FNMA 30 year current coupon.

Figure 7: Errors for FNMA 30 Using Treasury vs Swap Rates Using Rate Changes



In contrast to Figure 5 and 6, we see no obvious difference in the magnitudes of Treasury-based error and swap-based error. There are times when both are roughly the same and others where one is somewhat higher than the other. T-tests on the means of the errors confirmed that the average error is not different from zero in a statistically significant way: F-tests confirmed that the standard deviations of the errors are not different.[3]

Table 2: A Comparison of Means & Standard Deviations of FN30 Errors using Treasuries & Swap Rates Changes over 3 Historical Periods

	Swap	Treasury	P-Value
1992-1994	0.1143	0.1161	0.464
1995-1997	0.0700	0.0478	0.014
1998-2000	0.0836	0.1224	0.019

However, if we break down the error analysis into three distinct periods, we begin to see differences. Table 2 shows that from 1992-1994 both Treasuries and Swaps had higher errors and that the level of error was not statistically distinguishable. In 1995-1997, both had much lower errors and Treasuries had lower errors than Swap rates. Finally, in 1998-2000, Swap

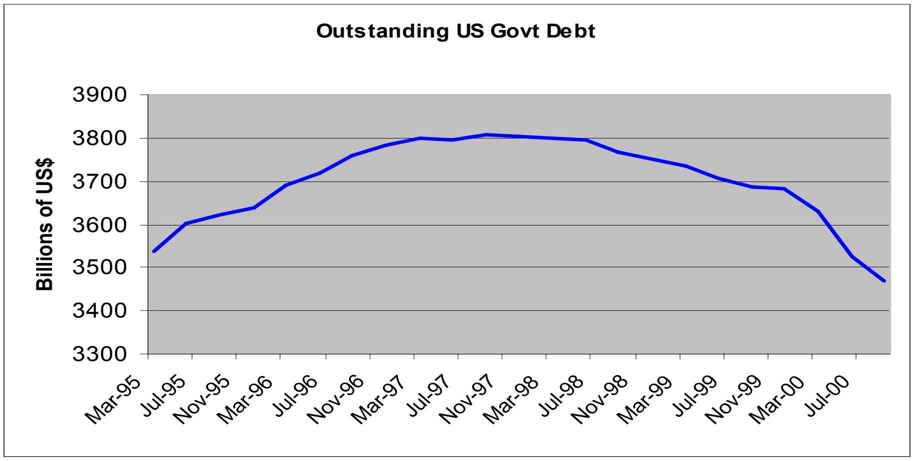
[3] Both at the 90% and 95% significance levels.

errors continue to hold their 1995-1997 levels, whereas Treasury errors are much higher.

Figure 8 displays the total outstanding US government debt as a function of time (DOUT <index> on Bloomberg).[4]

Figure 8

Publically Held Outstanding US Debt



In the summer of 1998, U.S. government debt outstanding peaked at around $3.8 trillion and began to decline as the U.S. Treasury retired debt using rising budget surpluses. Both the three and seven-year Treasury bonds were eliminated, and market expectations of future surpluses and debt pay-down began to grow. In addition, the Russian debt crisis and the collapse of Long Term Capital occurred later that year.

It is precisely in this context that the errors graphed in Figure 7 appear to break and increase from their historical levels. A closer look at the errors shows that around the summer of 1998, the equations begin to consistently under-predict mortgage current coupon, whereas before that period, errors of both signs occur. This is consistent with a premium paid for Treasuries, which would have decreased their yields without affecting yields on swaps or mortgages.[5]

Out of Sample Tests

In addition to the overall error comparisons between the two statistical techniques and four sets of predictors, along with the individual measure of fit comparisons, we performed out-of-sample tests of fit for FNMA 30 and GNMA 30 coupons using par Treasury and swap rates. This was done

[5] The retirement of debt would create a scarcity premium while flight-to-quality creates an increase in the Treasury credit premium. Both would have increased spreads.

[4] This index does not include the Social Security 'IOUs'.

compare historical fit within the sample used to estimate the model with forecasting fit on market data generated after the model estimation. Furthermore, to compare against a "control", we used the original ten-year plus spread method discussed in the preliminary analysis.

In all three methods, the spread was calculated based on August 2000 rates. These are the assumptions that someone running an analysis as of August 2000 would have made if they required mortgage rate forecasts for September 2000 through January 2001, but had access to perfect Treasury forecasts for this period. Figures 9 and 10 display these errors.

Figure 9: GNMA 30 Forecasting Errors Sept 2000-January 2001

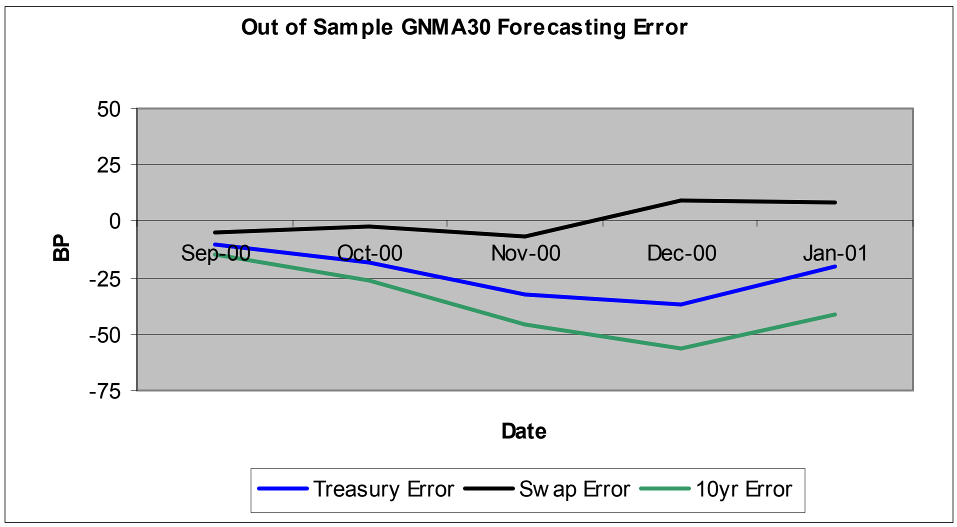
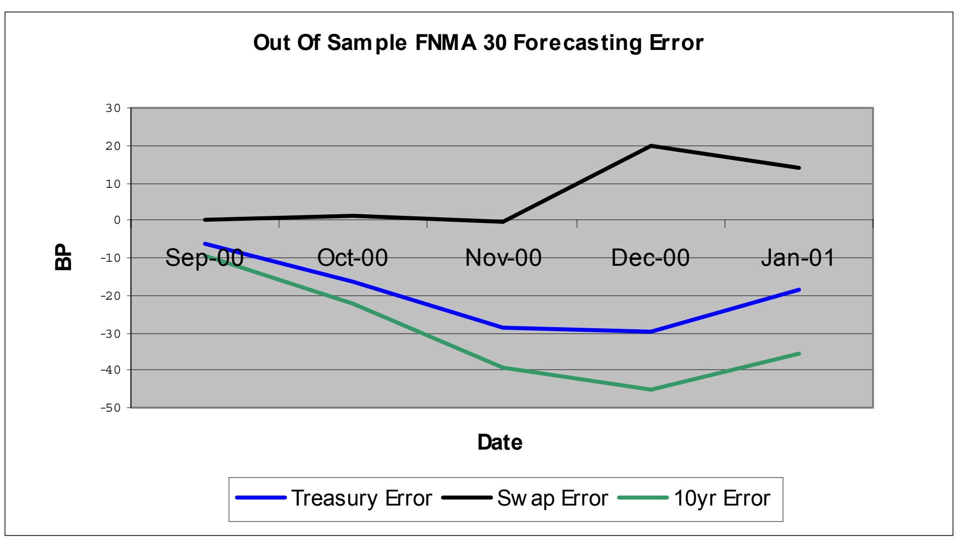


Figure 10: FNMA 30 Forecasting Errors Sept 2000-January 2001



In both graphs, it is clear that the ten-year plus spread method performs the most poorly, with errors reaching 40-50 bp in just the first few months. Using two Treasury rates is somewhat better, with the errors in the period covered not exceeding 35 bp. However, both of these methods have systematically negative errors over the period, while the errors using swaps are both negative and positive. Moreover, the magnitude of the errors is considerably lower, with the majority under 10 bp.

Software Implementation

Our prepayment model libraries provide a number of utility functions in addition to prepayment model setup and forecasting functions. Two of these utility functions incorporated the results of this study as of the fourth quarter of 2000.

The first, *adco_spread_calc*, calculates spreads based on the most recent available yield curve and mortgage rates. Most recent can mean rates based on the most recent Bloomberg feed, the prior day's close, or the close on the last day of the previous month, depending on the application and the local user configuration. This spread is used in the second routine, *adco_cc_calc*, which asks for two and ten-year interest rate forecasts and outputs the mortgage rate forecasts.

Both functions now allow the user to specify which of the four combinations (par vs. zero, Treasury vs. Libor) are being used to specify the yield curve rates. This enhancement to the spread calculation and current coupon forecasting utility functions is available in v4.2 and all subsequent versions of our MBS prepayment models. It is also available in our ABS prepayment library for use in the Home Equity Loan and Manufactured Housing models. For more information on these models, see the following Quantitative Perspectives:

ARM Home Equity Loan Prepayment Model, July 2000
Home Equity Loan Prepayment Model, May 1999
Fixed Rate Mortgage Prepayment Model, March 1999
The ARM Prepayment Model, February 1999
New Prepayment Model: Mobile Homes, March 1998

Conclusion

Based on a comparison of two related regression methodologies and four sets of yield curve variables, we found that zero coupon or par Libor swap rates are better predictors of mortgage current coupon than the corresponding Treasury rates, with significant changes in the predictive power of Treasuries beginning around July 1998. Both the in-sample fits and out-of-sample forecasting supported this conclusion. Given the ongoing uncertainty about the impact of supply-related technical factors on the Treasury market, we believe that option-adjusted valuation based on swap rates is preferable at this time.

In addition, we found that explicitly taking into account the autocorrelation structure leads to better parameter estimates and significantly lower overall error than using a pure first differences approach. Our latest release of version 4.2 of our MBS prepayment model and all future versions of our software incorporate the results of this study.

References

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Theil, H and Nagar, AL. "Testing the Independence of Regression Disturbances," Journal of the American Statistical Association 56 (1961), pp793-806.

Pindyck, RS and Rubinfeld, DL. Econometric Models and Economic Forecasts. 3rd ed McGraw Hill, 1991.

To obtain D, the Durbin Watson statistic:

$$\mathbf{D} = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=2}^{n} e_t^2}$$

where $e_t = Y_t - \hat{Y}_t$

The Hildreth-Liu Estimation method, using SAS code:

1 - (DW/2) = ρ,

/ **then**: /

```
z  = lag(rate2yr);
w = lag(rate10yr);
q  = lag(fan30CC);
Diff2yr = (rate2yr - ρ *z);
Diff10yr = (rate10yr - ρ *w);
Diff_fan30CC = (fan30CC - ρ *q);
```

/ **where lag(X) is the value of the previous data point** /

/ **then:** /

```
proc reg;
model Diff_fan30CC = Diff2yr + Diff10yr;
run;
```

Appendix B: Regression Coefficients and R2 using Modified Hildreth-Liu

	PAR SWAP				ZERO SWAP		
	AGENCY	2 YEAR	10 YEAR	R - SQUARED	2 YEAR	10 YEAR	R - SQUARED
5	FHG	0.56651	0.43349	95.04%	0.61794	0.38206	94.85%
7	FNMA	0.34450	0.65550	97.70%	0.41912	0.58088	97.27%
	FHG	0.36634	0.63366	93.61%	0.43865	0.56135	92.99%
15	FNMA	0.17570	0.82430	93.80%	0.27112	0.72888	93.13%
	FHG	0.19701	0.80299	91.31%	0.28708	0.71292	90.81%
	GNMA	0.18884	0.81116	89.18%	0.28787	0.71213	89.07%
30	FNMA	0.10213	0.89787	95.40%	0.20684	0.79316	95.18%
	FHG	0.10206	0.89794	95.17%	0.19977	0.80023	94.73%
	GNMA	0.09880	0.90120	92.59%	0.20272	0.79728	92.43%
	PAR TREAS				ZERO TREAS		
	AGENCY	2 YEAR	10 YEAR	R - SQUARED	2 YEAR	10 YEAR	R - SQUARED
5	FHG	0.518357	0.481643	82.45%	0.55953	0.44047	82.45%
7	FNMA	0.504595	0.495405	88.58%	0.54015	0.45985	88.55%
	FHG	0.510901	0.489099	86.20%	0.55953	0.44047	86.02%
15	FNMA	0.357817	0.642183	87.63%	0.42110	0.57890	87.35%
	FHG	0.345772	0.654228	89.72%	0.40540	0.59460	89.70%
	GNMA	0.335591	0.664409	79.99%	0.39987	0.60013	79.85%
30	FNMA	0.236148	0.763852	80.24%	0.30429	0.69571	80.14%
	FHG	0.18466	0.81534	87.04%	0.26141	0.73859	87.32%
	GNMA	0.18445	0.81555	86.39%	0.25495	0.74505	86.62%

About the Authors

***Eknath Belbase** is responsible for the research, development and testing of valuation/risk management tools and quantitative methodology for Andrew Davidson & Co., Inc. He has worked on the MBS fixed-rate and ARM prepayment models, designed a lattice implementation of the Black-Karasinki interest rate process tailored to be extremely efficient on mortgage instruments, and researched LIBOR-Treasury and mortgage current coupon dynamics. He has worked on consulting projects involving structured products, credit risk and regulatory testing.*

Eknath holds a PhD in mathematics with specialization in probability, and an MS in Statistics, both from Cornell University. He received his BA in mathematics and computer science from Ohio Wesleyan University.

***Dan Szakallas** provides statistical support, data collection and analysis to Andrew Davidson & Co., Inc. Dan assists in the development of the MBS fixed-rate and ARM prepayment models. He has also produced many of the new performance reports available on the Andrew Davidson & Co., Inc. website at www.ad-co.com. He graduated from Carnegie Mellon University with a dual major in Statistics and Psychology.*

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