2. Basic Quantum Computing with Deutsch Algorithm

Vaughan Sohn

October 13, 2024

Contents

Deutsch Algorithm

Solutions of Deutsch Problem

Key Concept: Discrimination

Discrimination between quantum states
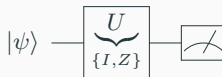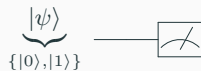
Key Concept: Phase estimation

Universal gate set

Prerequisite: bloch sphere and rotation gate

Example: Controlled Y gate

Circuits

$|0\rangle$ ── ▮ ── H ── ▮ ── U_f ── ▮ ── H ── ▮ ── 📐

$|0\rangle$ ── X ── ▮ ── H ── ▮ ── U_f ── ▮ ── ▮ ── 📐

$\underbrace{|\psi\rangle}_{\{|0\rangle,|1\rangle\}}$ ── 📐

$\underbrace{|\psi\rangle}_{\{|+\rangle,|-\rangle\}}$ ── 📐

$|\psi\rangle$ ── $\underbrace{U}_{\{I,X\}}$ ── 📐

$|\psi\rangle$ ── $\underbrace{U}_{\{I,Z\}}$ ── 📐

$$|0\rangle \quad \boxed{H} \quad \cdots \quad \frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i\varphi 2^{t-1}}|1\rangle)$$

$$|0\rangle \quad \boxed{H} \quad \cdots \quad \frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i\varphi 2^3}|1\rangle)$$

$$|0\rangle \quad \boxed{H} \quad \cdots \quad \frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i\varphi 2^2}|1\rangle)$$

$$|0\rangle \quad \boxed{H} \quad \cdots \quad \frac{1}{\sqrt{2}}(|0\rangle + e^{2\pi i\varphi 2^0}|1\rangle)$$

$$|u\rangle \quad \boxed{U^{2^0}} \quad \boxed{U^{2^1}} \quad \boxed{U^{2^2}} \quad \cdots \quad \boxed{U^{2^{t-1}}} \quad |u\rangle$$

$|0^{\otimes t}\rangle$ —/— $H^{\otimes t}$ —●— $QFT^\dagger$ —📐— φ

$|u\rangle$ —/— U^j — $|u\rangle$

$R_y(\theta)$ = $R_y(\theta/2)$ ⊕ $R_y(-\theta/2)$ ⊕

References
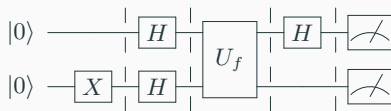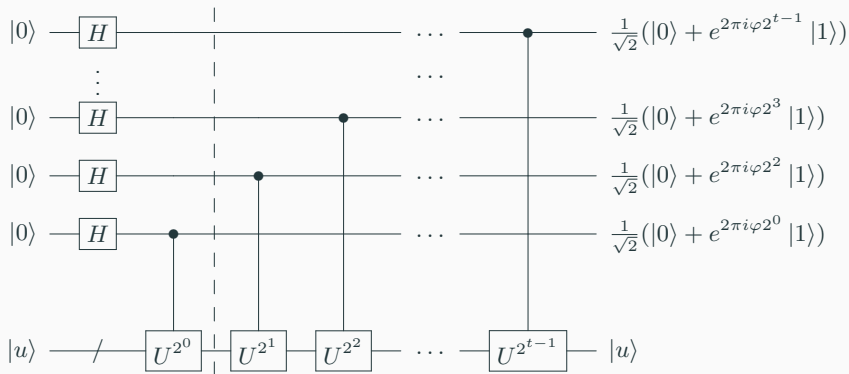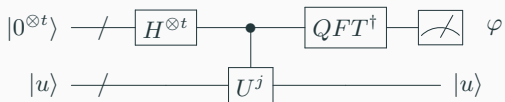
- Lecture notes for EE547: Introduction to Quantum Information Processing (Fall 2024)