Complex number

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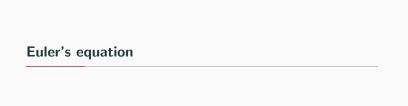
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Euler's equation

Complex number:

$$z = x + iy = re^{i\theta}$$

where $x=r\cos\theta,y=r\sin\theta$ and $r=\sqrt{x^2+y^2},\theta=\tan^{-1}(y/x)$

Theorem 1 (Euler's formular)

$$re^{i\theta} = r\cos\theta + ir\sin\theta$$
$$e^{i\theta} = \cos\theta + i\sin\theta \quad (\text{ if } r = 1)$$

Theorem 2 (Inverse Euler's formular)

$$\cos \theta = \frac{1}{2} \left(e^{i\theta} + e^{-i\theta} \right)$$
$$\sin \theta = \frac{1}{2i} \left(e^{i\theta} - e^{-i\theta} \right)$$



Some Special value

•
$$e^{i0} = e^{i2\pi} = 1$$

•
$$e^{i\pi/2} = e^{-i3\pi/2} = i$$

•
$$e^{i\pi} = e^{i-\pi} = -1$$

$$\bullet \ e^{i3\pi/2} = e^{i-\pi/2} = -i$$



Basic property

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
$$\sin(-\theta) = -\sin(\theta)$$
$$\cos(-\theta) = \cos(\theta)$$
$$\sin \theta = \cos(\theta - \pi/2)$$
$$\cos \theta = \sin(\theta + \pi/2)$$

Trigonometric function property

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$= 1 - 2 \sin^2 \alpha$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$\tan 3\alpha = \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}$$

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

$$\tan^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

Trigonometric function property

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\sin \alpha \cos \beta = \frac{1}{2} \{\sin(\alpha + \beta) + \sin(\alpha - \beta)\}$$

$$\cos \alpha \sin \beta = \frac{1}{2} \{\sin(\alpha + \beta) - \sin(\alpha - \beta)\}$$

$$\cos \alpha \cos \beta = \frac{1}{2} \{\cos(\alpha + \beta) + \cos(\alpha - \beta)\}$$

$$\sin \alpha \sin \beta = \frac{1}{2} \{\cos(\alpha + \beta) - \cos(\alpha - \beta)\}$$

$$\sin \alpha \sin \beta = \frac{1}{2} \{\cos(\alpha + \beta) - \cos(\alpha - \beta)\}$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}$$