# Complex number

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# Contents

# Euler's equation

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*Complex number:*

$$z = x + iy = re^{i\theta}$$

where $x = r\cos\theta, y = r\sin\theta$ and $r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}(y/x)$

**Theorem 1 (Euler's formular)**

$$re^{i\theta} = r\cos\theta + ir\sin\theta$$
$$e^{i\theta} = \cos\theta + i\sin\theta \quad (\textit{ if } \mathrm{r} = 1)$$

**Theorem 2 (Inverse Euler's formular)**

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$$
$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$

# Some special value

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- $e^{i0} = e^{i2\pi} = 1$
- $e^{i\pi/2} = e^{-i3\pi/2} = i$
- $e^{i\pi} = e^{i-\pi} = -1$
- $e^{i3\pi/2} = e^{i-\pi/2} = -i$

# Trigonometric function

## Basic property

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sin\theta = \cos(\theta - \pi/2)$$

$$\cos\theta = \sin(\theta + \pi/2)$$

# Trigonometric function property

$$\begin{aligned}
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
\cos 2\alpha &= 2\cos^2\alpha - 1 \\
&= 1 - 2\sin^2\alpha \\
\tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha}
\end{aligned}$$

$$\begin{aligned}
\sin 3\alpha &= 3\sin\alpha - 4\sin^3\alpha \\
\cos 3\alpha &= 4\cos^3\alpha - 3\cos\alpha \\
\tan 3\alpha &= \frac{3\tan\alpha - \tan^3\alpha}{1-3\tan^2\alpha}
\end{aligned}$$

$$\begin{aligned}
\sin^2\frac{\alpha}{2} &= \frac{1-\cos\alpha}{2} \\
\cos^2\frac{\alpha}{2} &= \frac{1+\cos\alpha}{2} \\
\tan^2\frac{\alpha}{2} &= \frac{1-\cos\alpha}{1+\cos\alpha}
\end{aligned}$$

# Trigonometric function property

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin\alpha\cos\beta = \frac{1}{2}\{\sin(\alpha+\beta) + \sin(\alpha-\beta)\}$$
$$\cos\alpha\sin\beta = \frac{1}{2}\{\sin(\alpha+\beta) - \sin(\alpha-\beta)\}$$
$$\cos\alpha\cos\beta = \frac{1}{2}\{\cos(\alpha+\beta) + \cos(\alpha-\beta)\}$$
$$\sin\alpha\sin\beta = \frac{1}{2}\{\cos(\alpha+\beta) - \cos(\alpha-\beta)\}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$
$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$