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Dynamics of Rotating Triple Systems

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Abstract—We investigate the dynamical evolution of 100 000 rotating triple systems with equal-mass components. The system rotation is specified by the parameter $w = -c^2E$, where c and E are the angular momentum and total energy of the triple system, respectively. We consider $w = 0.1, 1, 2, 4, 6$ and study 20 000 triple systems with randomly specified coordinates and velocities of the bodies for each w. We consider two methods for specifying initial conditions: with and without a hierarchical structure at the beginning of the evolution. The evolution of each system is traced until the escape of one of the bodies or until the critical time equal to 1000 mean system crossing times. For each set of initial conditions, we computed parameters of the final motions: orbital parameters for the final binary and the escaping body. We analyze variations in the statistical characteristics of the distributions of these parameters with w. The mean disruption time of triple systems and the fraction of the systems that have not been disrupted in 1000 mean crossing times increase with w. The final binaries become, on average, wider at larger angular momenta. The distribution of their eccentricities does not depend on w and generally agrees with the theoretical law $f(e) = 2e$. The velocities of the escaping bodies, on average, decrease with increasing angular momentum of the triple system. The fraction of the angles between the escaping-body velocity vector and the triple-system angular momentum close to 90° increases with w. Escapes in the directions opposite to rotation and prograde motions dominate at small and large angular momenta, respectively. For slowly rotating systems, the angular momentum during their disruption is, on average, evenly divided between the escaping body and the final binary, whereas in rapidly rotating systems, about 80% of the angular momentum is carried away by the escaping component. We compare our numerical simulations with the statistical theory of triple-system disruption.

Key words: *celestial mechanics, triple systems, dynamical evolution, simulations.*

INTRODUCTION

Triple systems are widely represented among stars and galaxies. Therefore, the dynamical evolution of such systems has long attracted the attention of researchers [see, e.g., Anosova and Orlov (1985), Valtonen (1988), and Valtonen and Mikkola (1991) for a review].

The dynamical disruption of triple systems can be analyzed both numerically and in terms of the statistical theory of disruption (Monaghan 1976a, 1976b; Nash and Monaghan 1978). This approximate theory is based on the assumption that the phase trajectory of a triple system is quasi-ergodic within the region of strong body interaction (a close triple encounter). The probability of escape with certain parameters of the final state (orbital elements of the final binary and the escaping body) is then proportional to the corresponding volume of phase space in a coordinate system associated with the center of mass of the triple system for the chosen integrals of motion. Another simplifying assumption of this theory is the absence of interaction between the escaping body and the remaining binary.

In several cases, comparison of the theoretical distributions of disruption parameters and numerical simulations shows good agreement, suggesting that the quasi-ergodic hypothesis is suitable for describing the results of close triple encounters that lead to the disruption of triple systems. It is of interest to determine the validity range for the statistical theory of disruption by comparing its predictions with numerical simulations.

The dynamical evolution of rotating triple systems has been numerically simulated for more than 30 years (see, e.g., Anosova 1969; Standish 1972; Saslaw *et al.* 1974; Valtonen 1974; Anosova *et al.* 1984; Mikkola and Valtonen 1986; Anosova and Orlov 1986). These authors showed that an increase

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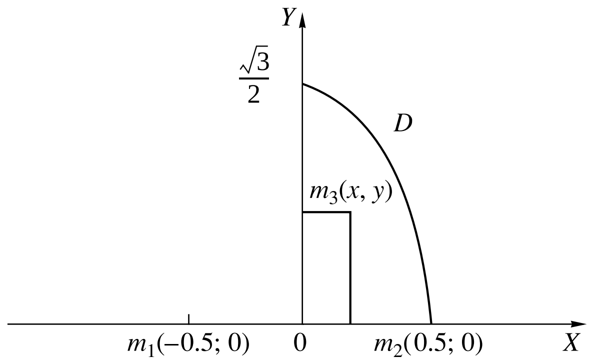

Fig. 1. The domain of all possible configurations of triple systems.

in angular momentum, on average, causes the dynamical evolution to slow down and the fraction of stable hierarchical triple systems to increase.

Comparison of the statistical theory of disruption with numerical simulations (Monaghan 1976b; Nash and Monaghan 1978) shows the qualitative agreement of the theoretical distributions of eccentricity and binding energy for the final binary and the magnitude of the velocity for the escaping body with their distributions derived from the statistical theory of disruption. As the angular momentum of a triple system increases, the escape direction tends to be orthogonal to the angular momentum vector of the triple system (the angle θ between these vectors is close to 90°), which is in qualitative agreement with the numerical simulations of Valtonen (1974).

Here, we consider the dynamical evolution of rotating triple systems in three-dimensional space. We constructed the distributions for several parameters that characterize the final states of the disrupted systems and compare our results with the statistical theory of disruption.

THE MODEL

We consider the evolution of triple systems with equal-mass components. The rotation is specified by the parameter

$$w = -c^2E, \tag{1}$$

where c and E are the magnitude of the angular momentum vector and the total energy of the triple system, respectively. This parameter was also used by Mikkola and Valtonen (1986).

We use the following system of units: the gravitational constant $G = 1$, the body masses $m_1 = m_2 = m_3 = 1$, and $E = -1$.

We consider two different methods of specifying the initial conditions.

The first method is similar to that used by Mikkola and Valtonen (1986). The dynamical evolution begins with the encounter of a single body with a close binary along an elliptical orbit with a low binding energy compared to the binding energy of the inner binary; i.e., at the beginning of the evolution, the triple system has an hierarchical structure.

The second method for specifying the initial conditions does not assume any mandatory hierarchical structure. The initial configuration is chosen randomly from the domain D (Fig. 1): the coordinates of the third body are chosen uniformly randomly from the segment bounded by the coordinate axes and by the arc of a circumference of unit radius centered at point $(-0.5, 0)$; the positions of the first and the second bodies are fixed at points $(-0.5, 0)$ and $(+0.5, 0)$, respectively. The domain D includes all possible configurations of triple systems (see, e.g., Agekyan and Anosova 1967). The initial velocities are chosen isotropically, with the virial coefficient k being distributed uniformly randomly over the interval $(0, 1)$.

In both methods of specifying the initial conditions, the value of w (1) is fixed and the initial conditions for which w differs from the assumed value by no more than 0.01 are chosen. We considered $w = 0.1, 1, 2, 4, 6$. For each w and each choice of initial conditions, we traced the evolution of 10 000 triple systems. A total of 100 000 sets of initial conditions were considered. Our computations lasted until one of the bodies escaped from the triple system along a hyperbolic orbit or until the critical time 1000τ was reached, where

$$\tau = \frac{GM^{5/2}}{|2E|^{3/2}} \tag{2}$$

is the mean crossing time of the triple system. Here, M is the total mass of the triple system and G is the gravitational constant. The quantity τ is the characteristic time in which the component moving at the characteristic velocity crosses the system (see, e.g., Valtonen 1988).

Below, the parameters with the dimensions of length are expressed in units of the mean system size

$$d = \frac{G(m_1m_2 + m_1m_3 + m_2m_3)}{|E|}, \tag{3}$$

and the parameters with the dimensions of velocity are given in units of d/τ.

THE RESULTS OF NUMERICAL SIMULATIONS

We determined the following parameters for the disrupted triple systems: the lifetime T of the triple

Mean disruption parameters

w	0.1	1	2	4	6
n_H	9814	9543	8700	4591	883
n_D	9839	9705	9447	7269	4692
$\overline{T}_H$	49.4 ± 1.0	80.0 ± 1.4	120.5 ± 2.0	190.9 ± 3.8	274.2 ± 10.4
$\overline{T}_D$	44.0 ± 0.9	82.7 ± 1.3	112.5 ± 1.7	160.2 ± 2.6	61.9 ± 2.6
$\bar{a}_H$	0.1164 ± 4	0.1349 ± 3	0.1429 ± 2	0.1556 ± 2	0.1636 ± 1
$\bar{a}_D$	0.0976 ± 5	0.1270 ± 3	0.1349 ± 3	0.1404 ± 4	0.1201 ± 6
$\bar{e}_H$	0.710 ± 2	0.689 ± 2	0.705 ± 2	0.722 ± 3	0.634 ± 6
$\bar{e}_D$	0.695 ± 2	0.695 ± 2	0.699 ± 2	0.712 ± 2	0.707 ± 3
$\overline{V}_H$	2.22 ± 2	1.50 ± 1	1.22 ± 1	0.76 ± 1	0.39 ± 1
$\overline{V}_D$	4.04 ± 5	2.02 ± 2	1.69 ± 2	1.51 ± 2	2.34 ± 3
$\overline{\theta}_H$	90.2 ± 3	90.0 ± 1	90.1 ± 1	89.9 ± 1	90.0 ± 1
$\overline{\theta}_D$	90.0 ± 3	89.9 ± 1	90.0 ± 1	90.1 ± 1	90.0 ± 1
$\overline{\lambda}_H$	114.8 ± 4	90.4 ± 4	83.2 ± 4	62.0 ± 4	36.3 ± 5
$\overline{\lambda}_D$	110.6 ± 4	84.3 ± 4	75.0 ± 4	60.1 ± 4	75.9 ± 6
$\bar{\beta}_H$	1.13 ± 2	0.310 ± 1	0.219 ± 1	0.165 ± 1	0.170 ± 1
$\bar{\beta}_D$	0.731 ± 4	0.274 ± 1	0.200 ± 1	0.144 ± 1	0.108 ± 1

system, the semimajor axis a of the final binary, the eccentricity e of the final binary, the asymptotic velocity V of the escaping body, the angle θ between the orbital angular momentum vector of the triple system and the velocity vector of the escaping body, the angle λ between the angular momentum vectors of the final binary and the binary formed by the escaping body and the center of mass of the final binary, and the ratio β of the angular momentum of the final binary to the angular momentum of the outer binary formed by the escaping body and the barycenter of the final binary.

The table lists the mean values of these parameters for the two methods of choosing the initial conditions and the w values considered here. The subscript "H" in the first column corresponds to hierarchical systems (the first method of choosing the initial conditions); the subscript "D" refers to the second method (the initial configuration in the domain D). The first two rows give the number of systems disrupted in a time $T < 1000\tau$. Based on these systems, we computed the mean disruption parameters and constructed their distributions.

As we see from the table, the fraction of the systems that were not disrupted in time 1000τ increases with the angular momentum of the triple system. Note that this increase is much larger for hierarchical systems. For $w = 6$, the fraction of the hierarchical systems disrupted in time $T < 1000\tau$ is less than 10%, whereas this fraction is slightly less than 50% for the second method of specifying the initial conditions. This difference probably results from the emergence of a significant number of nonhierarchical systems disrupted in a short time, whereas among hierarchical systems such initial conditions are much fewer in number. This is probably the reason why the mean lifetime greatly decreases at $w = 6$ for the second method of choosing the initial conditions.

The final binaries become, on average, wider and the velocities of the escaping bodies, accordingly, decrease with increasing angular momentum. In the second method of specifying the initial conditions, the final binaries are, on average, closer and the velocities of the escaping bodies are higher than those for hierarchical systems. This may be because the interaction between the components and the energy redistribution between the escaping body and the final binary in systems with nonhierarchical initial configurations are more intense than those in hierarchical systems.

The mean eccentricities of the final binaries are almost independent of w and of the method for choosing the initial conditions. The mean $\bar{e} \approx 0.7$. Only $\bar{e} = 0.634 \pm 0.006$ for hierarchical triple systems constitutes an exception. Here, the final binaries are, on average, less eccentric.

The mean angles θ are close to 90°, irrespective of the method of choosing the initial conditions.

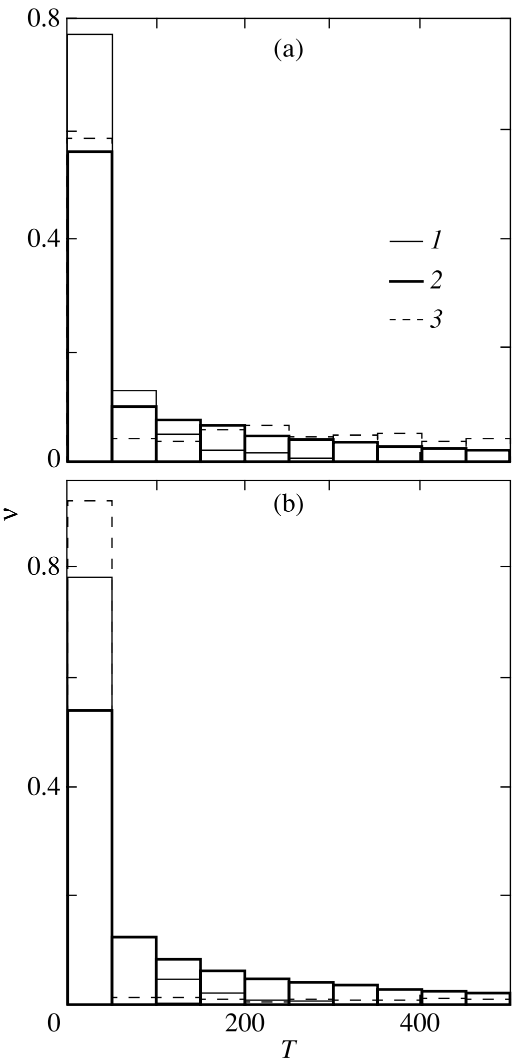

Fig. 2. The distributions of triple-system disruption times for the first (a) and second (b) method of specifying the initial conditions: *1* is $w = 0.1$; *2* is $w = 2$; and *3* is $w = 6$.

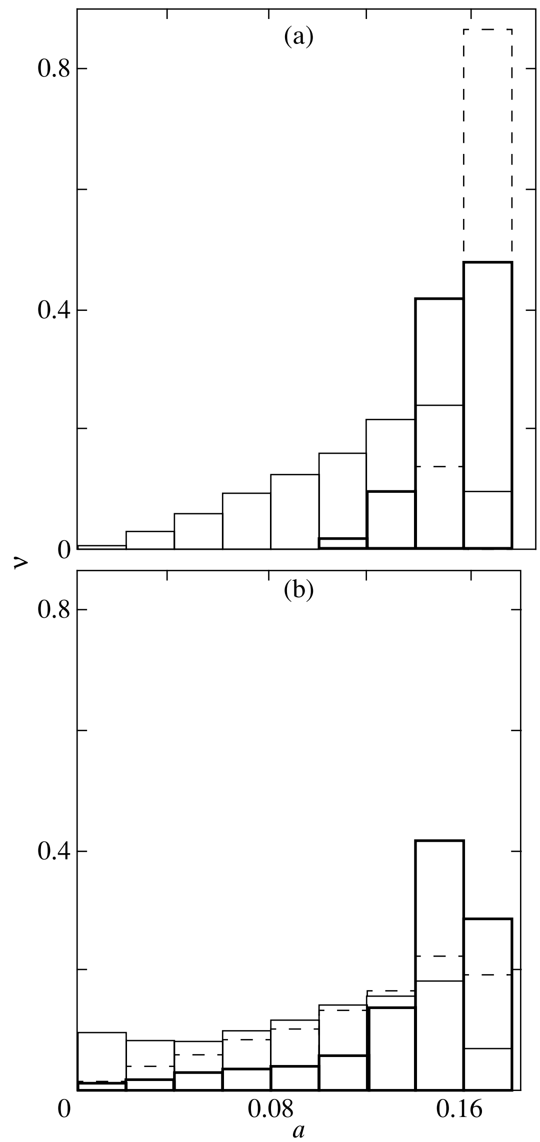

Fig. 3. The distributions of semimajor axes of the final binaries. The notation is the same as that in Fig. 2.

In slowly rotating systems ($w = 0.1$), body escapes in the direction opposite to the rotation of the final binary ($\bar{\lambda} > 90°$) generally dominate, whereas in rapidly rotating systems ($w > 1$), more escapes take place in the direction of the rotation of the final binary ($\bar{\lambda} < 90°$). The values of $\bar{\lambda}_H$ and $\bar{\lambda}_D$ generally agree, although for initially hierarchical systems, $\bar{\lambda}_H$ is slightly larger than $\bar{\lambda}_D$, except for the case $w = 6$, where $\bar{\lambda}_H$ is much smaller than $\bar{\lambda}_D$.

The angular momentum is redistributed almost equally between the final binary and the binary formed by the escaping body and the barycenter of the final binary for both methods of choosing the initial conditions. For hierarchical systems, this ratio is, on average, slightly larger than that for nonhierarchical systems. The mean $\bar{\beta}_H$ and $\bar{\beta}_D$ decrease with increasing angular momentum of the system. This is probably because the escaping body carries away excess angular momentum.

Let us consider the distribution functions for the parameters of the final states of triple systems. These distributions are shown in Figs. 2–8. Figures 2a, 3a, 4a, 5a, 6a, 7a, and 8a and 2b, 3b, 4b, 5b, 6b, 7b, and 8b correspond to the first (hierarchical systems) and second (systems without mandatory initial hierarchy) methods of specifying the initial conditions. The parameter bins and the fractions ν of triple systems in these bins are plotted along the x and y axes, respectively.

In general, the parameter distributions for the first and second methods of choosing the initial conditions

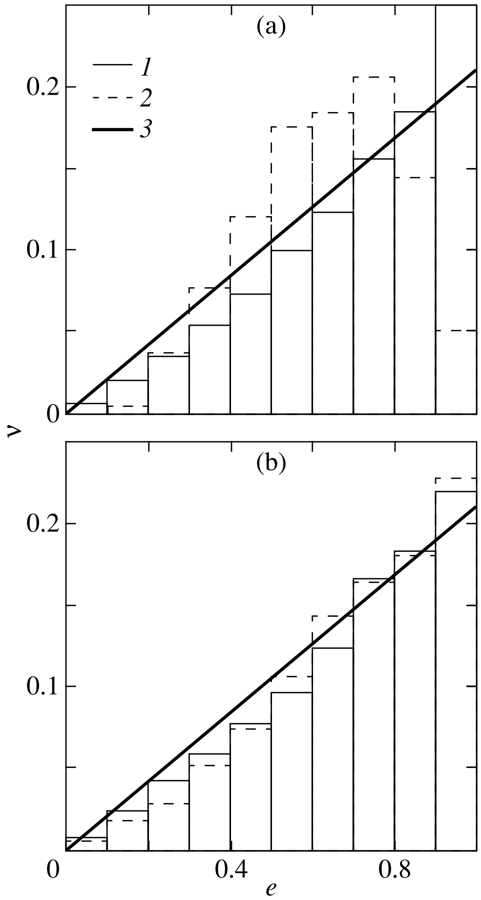

Fig. 4. The distributions of eccentricities of the final binaries for the two methods of specifying the initial conditions (a) and (b) at $w = 0.1$ (*1*) and $w = 6$ (*2*); *3* is the theoretical distribution $f(e) = 2e$.

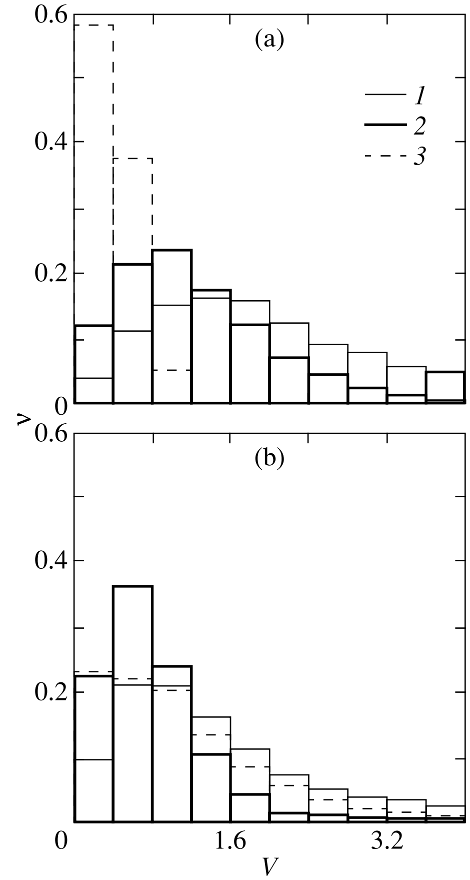

Fig. 5. The velocity distributions for the escaping body for two methods of specifying the initial conditions (a) and (b) at $w = 0.1$ (*1*), $w = 2$ (*2*), and $w = 6$ (*3*).

agree. The general trends of variations in the distributions with w correspond to the trends of variations in the mean parameters (see the table).

Most of the systems are disrupted in time $T < 50\tau$; the fraction of the systems with long lifetimes generally increases with w (the "tail" of the distribution becomes more powerful). Only the distribution $f(T_D)$ for the second method of specifying the initial conditions at $w = 6$ constitutes an exception. Note that $w = 6$ is close to the critical value of $w_{\rm cr} = 6.25$, which corresponds to the instability threshold of a triple system with respect to hierarchy violation (see, e.g., Szebehely and Zare 1977). At $w > w_{\rm cr}$, the triple systems appear to be separated into two categories: (1) stable triple systems and (2) unstable triple systems where one of the bodies escapes in a short time without hierarchy violation. As the stability threshold $w_{\rm cr}$ is approached, the triple systems break up into three classes: (1) nearly stable triple systems with long lifetimes $T > 1000\tau$, (2) unstable systems with fast escape of one of the bodies, and (3) the intermediate class of unstable systems.

For hierarchical triple systems (see Fig. 2a and the table), the fraction of the systems with fast escape of one of the bodies is small, about 5% of the entire sample of initial conditions. More than 90% of the systems were not disrupted in time 1000τ. For the second method of specifying the initial conditions (see Fig. 2b and the table), the fraction of the systems being rapidly disrupted is about 40% of the total number of systems and more than 80% of the systems that were disrupted in time $T < 1000\tau$. This is also the reason why the mean lifetime $\bar{T}_D$ decreases as one passes from $w = 4$ to $w = 6$. The result obtained probably suggests that in nonhierarchical systems with large angular momentum, one of the bodies often rapidly escapes, whereas in hierarchical systems, such situations are much rarer. The results for hierarchical and nonhierarchical systems (the mean lifetimes and their distribution functions) become closer with decreasing angular momentum.

The preference of the $w = 6$ case for nonhierarchical systems also shows up in the distributions of a, V,

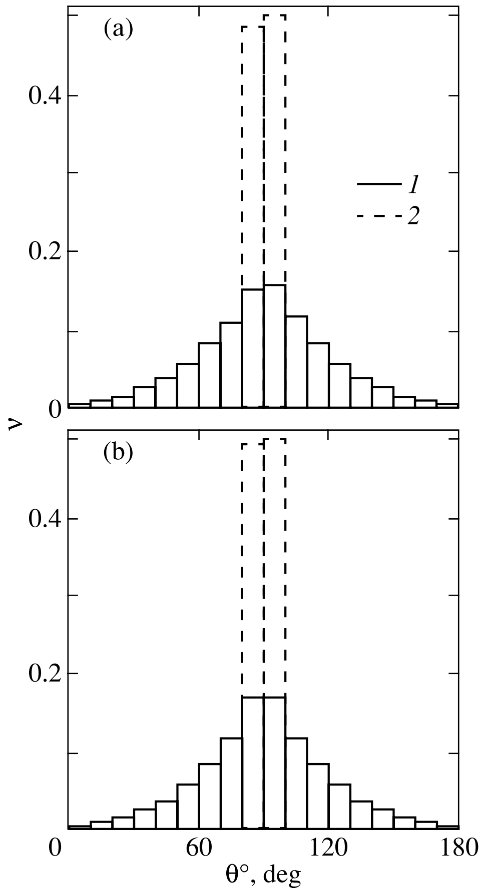

Fig. 6. The distributions of the angle between the orbital angular momentum vector of the triple system and the velocity vector of the escaping body for the two methods of specifying the initial conditions (a) and (b) at $w = 0.1$ (*1*) and $w = 6$ (*2*).

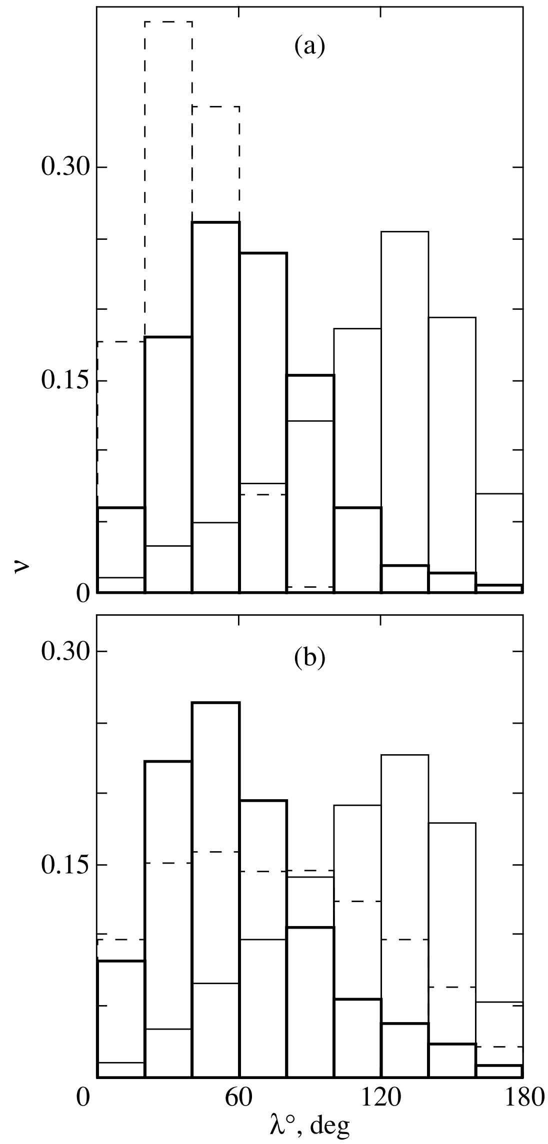

Fig. 7. The distributions of the angle between the orbital planes of the final binary and the escaping body. The notation is the same as that in Fig. 5.

and λ (see Figs. 3, 5, and 7 and the table). At $w = 6$, the final binaries formed during the disruption of nonhierarchical systems become, on average, closer and the escaping bodies carry away more kinetic energy than in the $w = 4$ case (see Figs. 3b and 5b), whereas for hierarchical systems the tendency for the final binaries to widen and for the escape velocity to decrease with increasing w is preserved (see Figs. 3a and 5a). For nonhierarchical systems with $w = 6$, the tendency for the angles λ to decrease (i.e., for prograde motions of the escaping body and the remaining pair to dominate) gives way to their increase. This may be because for systems being rapidly disrupted, the numbers of prograde and retrograde escapes are approximately equal (see Fig. 9).

The distributions of eccentricities for the final binaries are virtually independent of w (Fig. 4). They agree with the theoretical distribution for the probability density

$$f(e)de = 2ede, \tag{4}$$

which corresponds to the dissociative equilibrium of binaries in a stellar field (see Ambartsumyan 1937; Heggie 1975). The straight lines in Fig. 4 correspond to this distribution. The distribution of eccentricities for the final binaries formed during the disruption of hierarchical systems with large angular momentum at $w = 6$ (Fig. 4a) constitutes an exception. In this case, no highly eccentric binaries with $e > 0.9$ are formed and the distribution peak is located at $e \approx 0.6$.

The distributions of the angle θ are symmetric about $\theta = 90°$ (Fig. 6). The concentration of the distribution toward $\theta = 90°$ increases with w: body escapes generally occur near the stationary Laplace

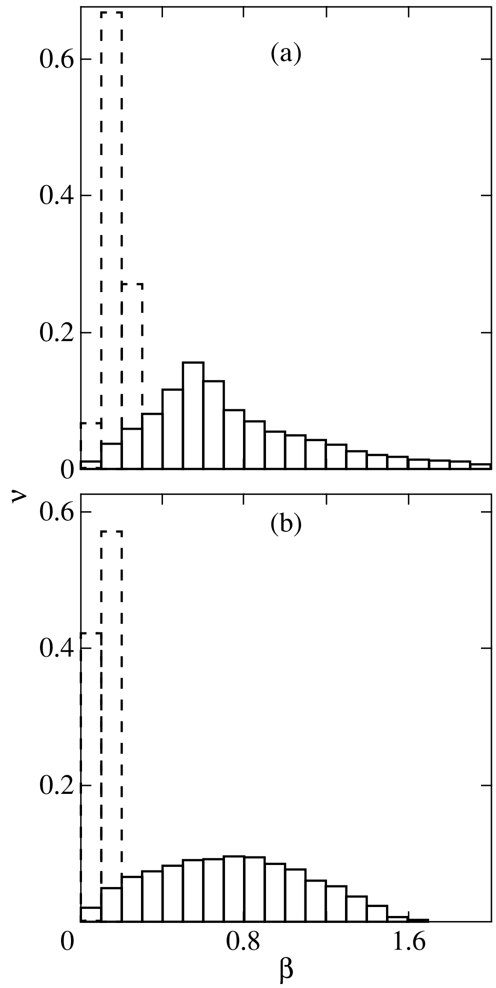

Fig. 8. The distributions of the ratio of angular momenta for the final binary and the binary formed by the escaping body and the barycenter of the final binary. The notation is the same as that in Fig. 6.

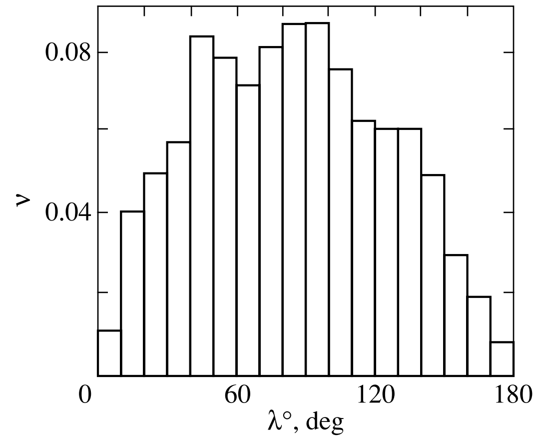

Fig. 9. The distributions of the angle λ between the angular momentum vectors of the final binary and the escaping body at $w = 6$ for systems with lifetimes $T < 5$ for the second method of choosing the initial conditions.

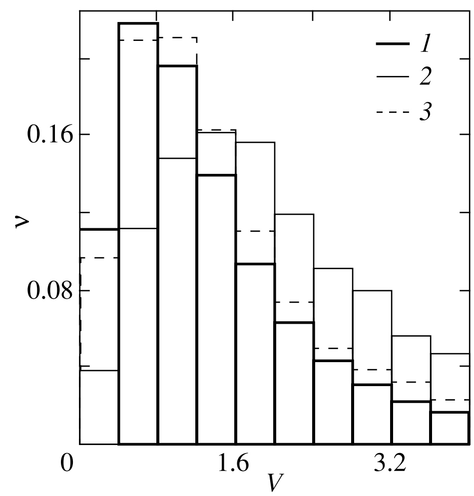

Fig. 10. The velocity distributions for the escaping body: theoretical distribution (*1*), at $w = 0.1$ for the first (*2*) and second (*3*) methods of specifying the initial conditions.

plane. Fast rotation hinders escapes in other directions.

The distributions of λ (Fig. 7) show that in slowly rotating triple systems ($w = 0.1$), escapes in the directions opposite to the rotation of the triple system ($\lambda > 90°$) dominate, because the rotations of the inner and outer binaries are mutually compensated. In rapidly rotating systems ($w \geq 1$), escapes more commonly occur in the direction of rotation of the triple system; rotation supplies additional energy to the escaping body.

The angular momentum is redistributed between the outer and inner binaries differently for slowly and rapidly rotating systems (see Fig. 8 and the table). Whereas in slowly rotating systems ($w = 0.1$) the angular momentum is, on average, evenly distributed, in rapidly rotating systems ($w \geq 2$) the lion's share (on average, from 80% to 90%) is carried away by the escaping body and only from 10% to 20% of the angular momentum remains in the final binary.

ANALYTIC APPROXIMATIONS OF THE DISTRIBUTIONS

The Eccentricities of Final Binaries

In the statistical theory of disruption of triple systems (Monaghan 1976a, 1976b; Nash and Monaghan 1978), the eccentricity distributions for the final binaries were obtained as a function of the triple-system angular momentum. The distribution function of eccentricities for three-dimensional motions with small angular momentum is given by formula (4). Comparison with numerical simulations (see Fig. 4

and the table) shows that this law agrees with the simulations. The fraction of the final binaries with highly eccentric orbits decreases with increasing angular momentum (Nash and Monaghan 1978). This result also qualitatively agrees with the numerical simulations of hierarchical triple systems at $w = 6$ (Fig. 4a). At the same time, for nonhierarchical systems, the function $f(e) = 2e$ satisfactorily describes the simulation results for all of the w values considered.

The Velocities of Escaping Bodies

In the statistical theory of disruption, the velocity distributions for the escaping bodies are unimodal; as the angular momentum increases, the peak shifts toward higher velocities (Nash and Monaghan 1978). In the numerical models, the distribution is also unimodal (Fig. 5), however, the peak shifts toward lower velocities as the angular momentum increases. This discrepancy probably stems from the fact that at small angular momentums, the triple encounters that result in escapes are closer than those in the cases of fast rotation [see, e.g., Anosova and Orlov (1985) for a review]. For a small angular momentum, the theoretical distribution $f(V)$ peaks at $V = 2/3$ (in our system of units). This value does not differ greatly from the corresponding values for the model distributions at $w = 0.1$ (see Fig. 5).

The theoretical distribution itself is (Monaghan 1976a)

$$f(V) = \frac{3}{2}\frac{V}{(1 + 3/4V^2)^2}. \quad (5)$$

Figure 10 shows a plot of this distribution (heavy solid line) and, for comparison, histograms of the model distributions at $w = 0.1$ for the first (solid line) and second (dashed line) methods of specifying the initial conditions. The theoretical distribution agrees with the numerical simulations of nonhierarchical systems (dashed line) but shifts to the left of the model distribution for hierarchical systems. The mean for the theoretical distribution (5) is equal to $\bar{V} = \pi/\sqrt{3} \approx 1.81$. This value is closer to $V_H = 2.22$, whereas, as was pointed out above, the form of the theoretical distribution is closer to the model function for nonhierarchical systems.

The Escape Angle

Nash and Monahgan (1978) obtained theoretical distributions of the angle θ between the velocity vector of the escaping body and the angular momentum vector of the triple system. These distributions are symmetric about $\theta = 90°$, as are the model distributions (see Fig. 6). The peaks of the theoretical distributions become sharper with increasing angular momentum, which is also in agreement with the numerical simulations.

CONCLUSIONS

We numerically simulated the dynamical evolution of 100 000 rotating triple systems. The following two methods of specifying the initial conditions were considered:

(1) Hierarchical systems in which the encounter of a single body with a binary system takes place at the beginning of the evolution; and

(2) Triple systems with randomly chosen configurations in the domain D (Fig. 1) with an isotropic velocity distribution of the bodies.

Our analysis of the final states for disrupted triple systems showed that the results for the two methods of specifying the initial conditions generally agree.

The following evolutionary trends are observed as the angular momentum of a triple system increases:

(1) The fraction of the systems that were not disrupted in time 1000τ increases.

(2) The mean lifetime of the systems being disrupted increases.

(3) The final binaries generally become wider.

(4) The velocities of the escaping bodies during disruption, on average, decrease.

(5) Body escapes occur, on average, closer to the Laplace plane and in the direction of rotation of the triple system.

(6) The fraction of the angular momentum carried away by the escaping body increases and the ratio of the final-binary and triple-system angular momenta decreases.

We compared the numerical simulations with the statistical theory of disruption for triple systems.

The form of the eccentricity distribution for the final binaries is almost independent of the angular momentum and agrees with the theoretical distribution $f(e) = 2e$, which is valid for small angular momenta (Monaghan 1976a). Only hierarchical systems with $w = 6$, where no highly eccentric binaries with $e > 0.9$ are formed, constitute an exception.

The general forms of the theoretical and model distributions of escape velocities are in agreement. These distributions have one peak. However, the tendencies for the forms of the distributions to change with increasing w are different. The theoretical distributions shift toward higher velocities, whereas in model systems, the escape velocities, on average, decrease. This discrepancy may be due to the different degree of closeness of triple encounters for slowly and rapidly rotating triple systems. Closer triple encounters in slowly rotating systems increase the probability of escapes with high velocities. The statistical theory of disruption, which assumes all encounters to be equally close, appears to ignore this effect.

The distributions of angles θ qualitatively agree and have a similar dependence on angular momentum: the larger the triple-system angular momentum, the stronger the concentration of the escape angles to 90°.

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