

Searching for the extra-tidal stars of globular clusters using high-dimensional analysis and a core particle spray code

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M3 → GC

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purpose of paper: present new way using high dim data analysis to ID extra tidal stars of any galactic GC

ABSTRACT

Three-body interactions can eject stars from the core of a globular cluster, causing them to enter the Galactic halo as extra-tidal stars. While finding extra-tidal stars is imperative for understanding cluster evolution, connecting isolated extra-tidal field stars back to their birth cluster is extremely difficult. In this work, we present a new methodology consisting of high-dimensional data analysis and a particle spray code to identify extra-tidal stars of *any* Galactic globular cluster using M3 as a case study. Using the t-Stochastic Neighbour Embedding (t-SNE) and Uniform Manifold Approximation and Projection (UMAP) machine learning dimensionality reduction algorithms, we first identify a set of 103 extra-tidal candidates in the APOGEE DR17 data catalogue with chemical abundances similar to M3 stars. To confirm each candidate's extra-tidal nature, we introduce **corespray** — a new Python-based three-body particle spray code that simulates extra-tidal stars for any Galactic globular cluster. Using Gaia EDR3 proper motions and APOGEE DR17 radial velocities, we apply multivariate Gaussian modelling and an extreme deconvolution to identify the extra-tidal candidates that are more likely to be associated with a distribution of corespray-simulated M3 extra-tidal stars than the field. Through these methods, we identify 13 new high-probability extra-tidal stars of M3. Future applications of corespray will yield better understandings of core dynamics, star formation histories and binary fractions in globular clusters.

Key words: galaxies: star clusters – globular clusters: individual – stars: kinematics and dynamics – software: simulations

operation

1 INTRODUCTION

Globular clusters (GCs) are some of the most ancient structures in the Milky Way. These collections of stars have been measured to have ages $\gtrsim 12.5$ Gyr, providing evidence that many GCs likely formed around the time of cosmic reionization (Forbes et al. 2018). As such, it is believed that the formation of most GCs occurred before star formation in galaxies (Renzini 2017) and thus likely played a crucial role in forming the Galaxy (Gratton et al. 2019). While GCs are useful for learning about galaxy formation, their dense nature also makes them ideal systems to probe how galaxies evolve over time. Specifically, GC evolution informs star formation histories from initial GC size and mass functions and constrains binary fractions from GC population synthesis studies (Marks & Kroupa 2010; Ivanova et al. 2005). Furthermore, since GCs are densely packed groups of hundreds of thousands to millions of stars with half-light radii ≤ 10 pc, they are ideal environments for learning how dynamical processes influence stars over time (Gratton et al. 2019; van den Bergh 2008). Importantly, while stellar interactions like two-body relaxation and

tidal stripping are primarily responsible for mass loss of a GC (and can thus lead to stellar streams or tidal tails), they do not probe the GC's core or binary systems. Thus, to investigate how stars and binaries escape the *core* of a GC, understanding *three-body interactions* is imperative.

While massive star evolution drives internal GC evolution in the first ~ 1 Gyr after formation, GCs generally evolve due to gravitational forces from the host galaxy (Carlberg 2018) and gravitational encounters between cluster stars (Heggie & Hut 2003). An important example of the latter occurs in three-body systems (Leigh & Geller 2013), usually composed of three stars orbiting a common centre of mass. In a GC's core, the high number density and low volume causes the three-body system to become unstable.¹ Assuming all particles are point-particles (i.e. mergers cannot occur), this instability results in the ejection of a single star from the three-body system while

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¹ Note that these three-body systems are not the same as triple systems, as the interaction involves a single star and a binary (compared to three single stars in a triple system). Leigh et al. (2011) showed that the presence of triple systems in GCs is believed to be rare and as such, they are not considered to be the precursor interactions yielding extra-tidal stars in this study.

leaving the remaining two stars to form a binary (e.g. Stone & Leigh 2019; Manwadkar et al. 2020, 2021). The ejection of the kicked star will also cause the binary to recoil, causing it to get kicked in the opposite direction (Valtonen & Karttunen 2006). In some cases these kicks can result in stars moving beyond a cluster’s tidal radius.

Defined both theoretically and observationally (e.g. King 1962; Innanen et al. 1983; Oh et al. 1992; Webb et al. 2013), the tidal radius of a cluster represents the limit where gravitational forces from the host galaxy balance gravitational forces from the GC. However, a GC’s tidal radius is not constant, rather it changes as a function of time along the cluster’s orbit through the Galaxy (Renaud et al. 2011; Webb et al. 2013). Although it takes time for stars to completely leave the cluster after becoming energetically unbound (Lee & Ostriker 1987), cluster stars that surpass the tidal radius will eventually no longer remain gravitationally bound to the parent GC (Carlberg 2018). Thus, if a star receives enough energy from a kick during a three-body interaction, it will migrate out of the GC’s core and enter the Galactic halo as an *extra-tidal star*.

Dense GCs undergo higher numbers of three-body encounters than their less dense counterparts, which leads to more stars escaping the cluster (e.g. Leigh & Sills 2011; Leigh & Geller 2012, 2015; Leigh et al. 2017, 2018; Barrera et al. 2021; Parischewsky et al. 2021; Reinoso et al. 2022). Generally, three-body encounters between a single star and a binary system will occur approximately once every ~ 10 Myr within GCs (Leigh & Sills 2011). Since these encounters happen while a GC orbits the Galaxy, a given cluster’s extra-tidal stars can be dispersed all throughout the Milky Way. Thus, connecting extra-tidal stars back to their parent cluster is difficult, especially when the stars are located far away. However, all stars in a GC either form from the same giant molecular cloud or GC mergers and as such, the chemical abundances of stars originating in a specific GC should be similar (De Silva et al. 2006, 2007; Bovy 2016; Price-Jones & Bovy 2018). Since certain kinematic quantities are conserved along the GC’s orbit, stars that receive a velocity kick due to a three-body interaction should have similar kinematics as the cluster itself, no matter where along the cluster’s orbit it escaped (Binney & Tremaine 2008). Thus, finding a star beyond the tidal radius of a GC that exhibits similarities in both (i) chemical abundances and (ii) conserved kinematic orbital properties would be strong evidence linking the extra-tidal star back to its original birth cluster.

Clusters that are especially dense and contain many members are ideal environments to search for new extra-tidal stars. GCs with a large number of confirmed cluster members allow for the definition of strong control groups of stars — a requisite tool for conducting important chemical comparisons. As presented in Mészáros et al. (2020), one cluster meeting both of the aforementioned requirements is M3 (NGC 5272). M3 has a right ascension of $\alpha = 205.548^\circ$ and declination of $\delta = 28.377^\circ$ (Baumgardt & Hilker 2018) with an angular size at apogalacticon of $r_t \sim 0.895^\circ$ (computed from M3’s Webb et al. (2013) tidal radius at apogalacticon of $r_t = 159.03$ pc). Although Leon et al. (2000) found extra-tidal structure around M3 through the identification of a tidal tail, later studies by Grillmair & Johnson (2006), Jordi & Grebel (2010) and Carballo-Bello et al. (2014) found no evidence for either extra-tidal stars or structure altogether. However, using the first data release from LAMOST, Navin et al. (2016) identified extra-tidal stars around M3 (and M13). The authors first searched for cluster members of M3 by selecting stars that were within a 5° radius around the GC’s centre and had radial velocities within $\pm 2\sigma$ of M3’s radial velocity. Through establishing V versus $V - K$ and $\log(T_{eff,f})$ versus $\log(g)$ limits and only accepting stars within 10 mas yr^{-1} of the GC’s proper motion, Navin et al. (2016) identified eight new extra-tidal stars of M3.

Since the Navin et al. (2016) study, advances in machine learning have introduced new methods for identifying similar stars in high-dimensional parameter spaces. Recent work by Chun et al. (2020) employed the use of both chemical tagging and kinematic analyses to search for extra-tidal stars around M53 (NGC 5024) and NGC 5053 — two metal-poor GCs in the Galactic halo. Searching a $20^\circ \times 10^\circ$ field around the clusters, Chun et al. (2020) used abundances and radial velocities from the 14th data release (DR14) of the Apache Point Observatory Galactic Evolution Experiment (APOGEE). Using this data, the authors applied a t-Stochastic Neighbour Embedding (t-SNE) machine learning algorithm to identify stars that were both chemically and kinematically similar, allowing them to define members and extra-tidal stars of each cluster. Combined with additional metallicity and radial velocity examinations for stars in which t-SNE was not applicable, Chun et al. (2020) identified three and four extra-tidal stars for M53 and NGC 5053 respectively. Chemical tagging and kinematic constraints have even been used to identify stars that formed in the same birth cluster, despite the host cluster having already dissolved (Webb et al. 2020; Price-Jones et al. 2020).

Increasing the known sample of extra-tidal stars in our Galaxy will allow us to better constrain the underlying Galactic gravitational potential, possible overdensities and binary fractions, ultimately allowing for advancements in our understanding of the evolution of both Galactic GCs and the Milky Way itself. Thus, in this study we build off the work of both Navin et al. (2016) and Chun et al. (2020) to present a new methodology for identifying extra-tidal stars in and around *any* Galactic GC, using M3 as a case study. In Section 3.1, we outline an observational identification scheme of extra-tidal candidates using stars in the 17th data release (DR17) of APOGEE. Specifically, we present methodology to identify stars that are chemically similar to suspected parent GCs using two different machine learning clustering algorithms — t-SNE (as used in Chun et al. (2020)) and Uniform Manifold Approximation and Projection (UMAP). To further confirm the extra-tidal nature of the observationally-identified extra-tidal candidates, Section 3.2 presents *corespray* — a new Python-based code that uses three-body dynamics to simulate the creation of extra-tidal stars and their corresponding recoil binaries. Corespray allows the user to explore where extra-tidal stars of a given GC could end up in a variety of different parameter spaces by taking into consideration the individual GC’s mass, tidal radius, central velocity dispersion, central escape velocity, central potential and core density. Section 4 combines our observational identification and theoretical confirmation methods to identify new high-probability extra-tidal stars of M3. We discuss our results in Section 5 and conclude in Section 6.

2 DATA

To identify extra-tidal candidates, we search the DR17 catalogue from the APOGEE survey (Abdurro’uf et al. 2022). APOGEE is a high-resolution ($R \sim 22,500$), high signal-to-noise ratio ($SNR > 100$), infrared ($1.51 - 1.70 \mu m$) spectroscopic survey (Majewski et al. 2017). As our observational extra-tidal identification scheme centres around chemical abundances, we utilize the high-quality *astroNN* abundances derived by Leung & Bovy (2019a,b) and Bovy et al. (2019). *AstroNN* abundances are determined through a neural network using APOGEE data, providing us with 19 chemical abundances and stellar parameters for an initial sample of 694,932 stars. To examine the kinematics of extra-tidal star candidates throughout our analyses, we use Gaia Early Data Release 3 (EDR3) proper motions (Gaia Collaboration et al. 2021) and APOGEE DR17 barycentric radial velocities

(Abdurro'uf et al. 2022). Of our initial sample, 685,978 stars have Gaia EDR3 proper motions.

To begin our search for extra-tidal candidates, we first follow the work of Chun et al. (2020) and select APOGEE DR17 stars that are within a $10^\circ \times 10^\circ$ field of view (FOV) around a GC's centre. For M3, this spatial cut reduces our sample to 5240 stars, just under 1% of the astroNN APOGEE DR17 catalogue. Furthermore, 187 of these stars do not have any reported chemical abundances, reducing the data set to 5053 stars. Often, APOGEE DR17 contains multiple spectra for the same source. To filter out duplicate sources, we select stars that either (i) only contain a single measurement (1918 stars) or (ii) have the highest signal-to-noise (SNR) spectrum of a source with multiple measurements (1391 stars). Removing duplicate sources reduces our sample size to 3309 stars. Finally, to ensure that we only include stars with high quality spectra in our search, we impose an additional SNR cut, only keeping stars with $\text{SNR} \geq 50$ (which mostly yields red giant stars). Through this filtering, our M3 data set contains 3212 unique stars.

Once this initial sample has been established, we define members of M3 to act as a control group for future analyses and comparisons. Specifically, we wish to identify members that are (i) spatially aligned with the reported cluster centre, (ii) chemically similar and (iii) have similar radial velocities. For members of a given GC, all three of the aforementioned parameters should be similar. While the latter two conditions are applied via machine learning clustering algorithms in Section 3.1, we initially select stars that are within eight times the Baumgardt & Hilker (2018) M3 half-mass radius of $r_{hm} = 6.34\text{pc}$. The spatial constraint of $8 \times r_{hm}$ is chosen as it yields a control group of 133 cluster stars that contain a sufficient amount of stars within the Webb et al. (2013) tidal radius at apogalacticon of $r_t = 159.03\text{pc}$ ². In other words, this control group contains approximately 45% of the stars within the tidal radius of M3, so contamination from field stars is expected to be minimal. Figure 1 highlights the spatial distribution of APOGEE DR17 stars and spatially-identified cluster members of M3 used in this study.

3 METHODOLOGY

We present a new method to identify extra-tidal stars of Galactic GCs that encompasses both observations and theory. By identifying stars that are chemically similar to cluster members through high-order dimensional analysis, we produce a sample of observational extra-tidal candidates. To confirm these candidates, we use corespray — a new Python-based particle spray code that uses three-body dynamics to simulate the creation of extra-tidal stars. We further assess the origins of each extra-tidal candidate by computing the probability that each star belongs to a distribution of (i) simulated Corespray extra-tidal stars of M3 and (ii) field stars around M3. The complete methodology is outlined below and can be applied to search for extra-tidal stars of any GC in the Milky Way.

² For the duration of this study, we use the tidal radius at apogalacticon to only select extra-tidal stars that are located beyond the tidal radius at *all* points along M3's orbit of the Galaxy. The tidal radius at apogalacticon is computed via Equation 8 in Webb et al. (2013).

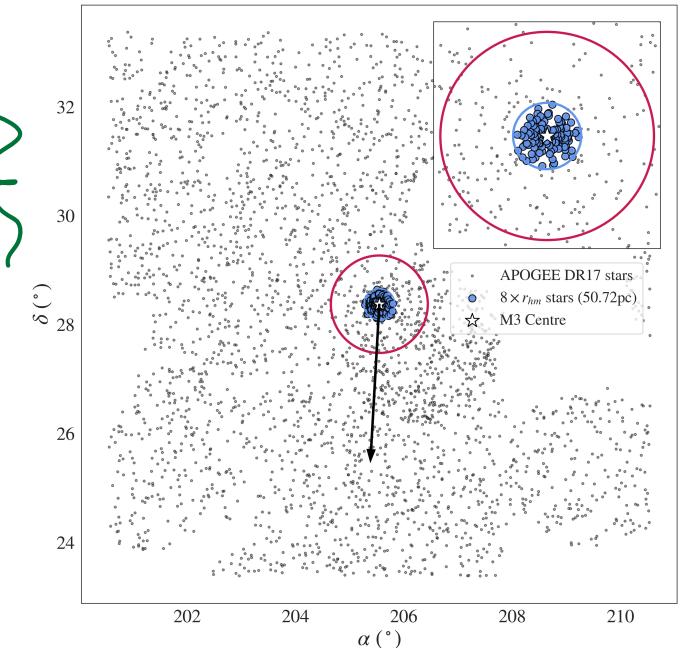


Figure 1. Distribution of 3212 APOGEE DR17 stars in a $10^\circ \times 10^\circ$ FOV around M3. The centre of M3 ($205.548^\circ, 28.377^\circ$) is marked with a white star. Stars that are within eight times the half-mass radius (blue circle) of $8 \times r_{hm} = 50.72\text{pc}$ are indicated as blue points. M3's tidal radius at apogalacticon of $r_t = 159.03\text{pc}$ is shown as a magenta circle and cluster proper motion is indicated with a black arrow. The right ascension, declination, r_{hm} and proper motion of M3 are all obtained from Baumgardt & Hilker (2018) while r_t at apogalacticon is computed using the Webb et al. (2013) r_t at perigalacticon.

3.1 High-Order Dimensional Analysis

3.1.1 Analysis with the t-SNE algorithm

While we have spatially identified a control group of 133 cluster members of M3, we must ensure that these stars are also similar in chemical abundances and radial velocities. Doing so further mitigates the chances of unintentionally including background or foreground field stars in our control group. However, identifying similarities in 19 astroNN abundances plus APOGEE DR17 barycentric radial velocities requires the use of high-dimensional data analysis. We therefore adopt the methodology from Chun et al. (2020) and use the t-Stochastic Neighbour Embedding (t-SNE) algorithm (van der Maaten & Hinton 2008) in the scikit-learn Python package (Pedregosa et al. 2011). T-SNE is a dimensionality-reducing clustering algorithm that finds similarities between data points in high-dimensional parameter spaces and projects them into a two-dimensional plane. Although the t-SNE plane does not correspond to a physical parameter space, it generally groups stars that share similarities in all dimensions of the input data in similar locations in the reduced two-dimensional space.

We run the t-SNE algorithm on both the cluster members and APOGEE DR17 stars in our FOV with 19 chemical abundances and radial velocities to narrow down our cluster member control group. In turn, this step allows us to obtain a new and improved control group of cluster members that are not only spatially similar, but chemically and kinematically similar as well. It is important to note that while Cohen (1978) and Peterson (1980) have shown that GCs can contain multiple stellar populations, we treat cluster members as a single population with similar chemical abundances in this study.

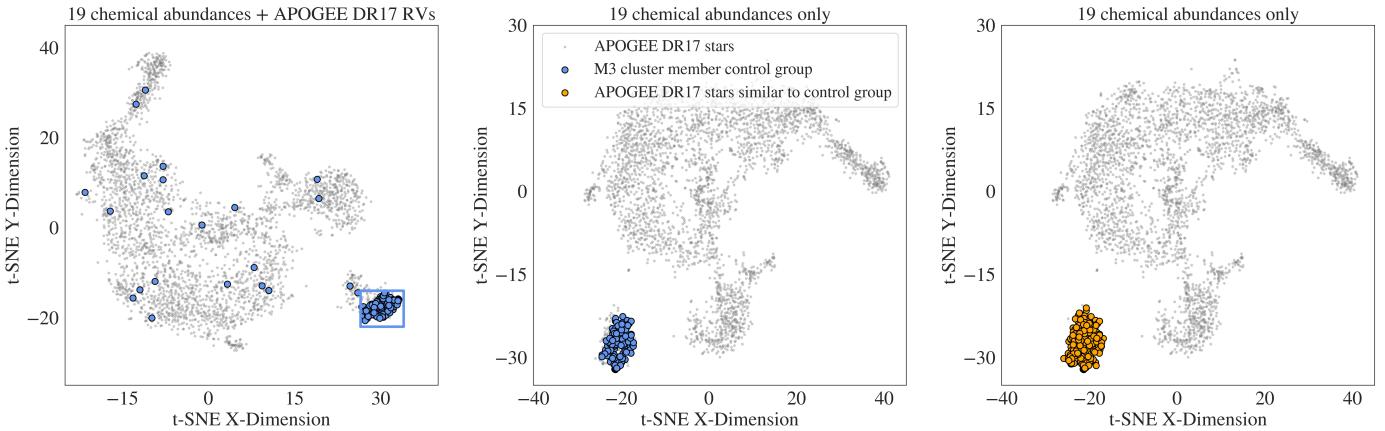


Figure 2. t-SNE projections of 3212 APOGEE DR17 stars (gray points) in a $10^\circ \times 10^\circ$ FOV around M3. With the exception of setting perplexity=100, default t-SNE input parameters are utilized. The left panel highlights the results of a t-SNE reduction of 19 astroNN chemical abundances and APOGEE DR17 radial velocities. A group of 133 stars within $8 \times r_{hm}$ of M3 are highlighted as blue points. To define a stronger control group, we only select cluster members within the blue box, as in addition to spatial proximity, they share similar chemical abundances and radial velocities. As extra-tidal stars may not have similar radial velocities to the suspected parent cluster, we plot the results of a new t-SNE reduction *without* radial velocities (only 19 chemical abundances) in the middle panel. This panel highlights the aforementioned updated control group of 111 stars of M3 along with APOGEE DR17 stars. The right panel is the same as the middle panel, however it highlights 232 APOGEE DR17 stars that have similar chemical abundances to the control group of M3 stars (orange points).

Regardless, it is straightforward to generalize this dimensionality reduction to examine other potential stellar populations.

Although t-SNE is a powerful clustering tool, it is critical to understand that output t-SNE parameter spaces are greatly impacted by (i) changing the input t-SNE parameters (ii) skewed data and (iii) re-running the algorithm (although one can set the initial condition by the random seed used for the data range). For this application, we use default t-SNE input parameters with the exception of perplexity (p). Perplexity is a relative weight between local and global structure in the data (Wattenberg et al. 2016). For large high-dimensional data sets, it is recommended to use higher perplexity values to yield tighter clustering, as data becomes much sparser in higher dimensions. As such, it makes sense that after running t-SNE with four different perplexity values ($p = 2, p = 5, p = 10$ and $p = 100$), we find that $p = 100$ yields the tightest grouping of likely cluster members and is consequently implemented for the duration of this study. We also standardize our data to have $\mu = 0, \sigma = 1$ for all parameters to ensure that one parameter does not dominate or skew the clustering during the dimensionality reduction. Finally, due to a degree of randomness present in the t-SNE algorithm, locations of data (and thus clusters of data) in the two-dimensional output parameter space can vary from run to run. Although the data locations may change, our control group of cluster members always allows us to select APOGEE DR17 stars that are similar to M3, no matter how the data is distributed in a given t-SNE parameter space. The left panel of Figure 2 highlights spatial members within $8 \times r_{hm}$ of M3 relative to the APOGEE DR17 stars in a t-SNE reduction of 19 astroNN chemical abundances and APOGEE DR17 radial velocities. One can observe that almost all stars in our initial M3 control group are clustered tightly in t-SNE space (boxed), so although there is likely some contamination by foreground and background stars in this initial sample, they fall outside of the clustered region and are thus discarded from the M3 control group in subsequent steps.

Before using t-SNE to identify APOGEE DR17 stars similar to our cluster member control group, we must note that while members of a particular GC have similar radial velocities, extra-tidal stars may not. Recall from Section 1 that extra-tidal stars are created during core three-body interactions, when single stars receive velocity kicks. De-

pending on the configuration of the three-body system, the velocity imparted to the single star can be high. Furthermore, extra-tidal stars escape at various points along a GC's orbit, allowing for escaped stars of the same cluster to experience different phenomena (e.g. gravitational potentials, interactions with other objects, etc.). In fact, the orbital phase of the "escaper" star (i.e. the one that actually escapes the GC's gravitational pull) alone can result in highly different radial velocities. So while extra-tidal stars likely had similar radial velocities as the parent GC when they first originated, they do not necessarily have similar radial velocities at present. Thus, to identify extra-tidal candidates that are similar to the parent cluster, we re-run the t-SNE algorithm on the same control group of cluster members boxed in the left panel of Figure 2 and APOGEE DR17 stars *without* radial velocities. The middle panel in Figure 2 highlights the control group in a t-SNE reduction containing only the 19 astroNN chemical abundances. From here, we select every APOGEE DR17 star that is within a tolerance level of two t-SNE units to a previously confirmed cluster member in the t-SNE two-dimensional plane. The right panel in Figure 2 depicts our final sample of 232 t-SNE selected stars with similar chemical abundances to M3. If we only include stars located beyond M3's tidal radius at apogalacticon, we observationally identify 103 extra-tidal candidates with the t-SNE algorithm.

3.1.2 Analysis with the UMAP algorithm

To further confirm our t-SNE-identified extra-tidal candidates, we perform the same high-dimensional analysis with another machine learning approach — Uniform Manifold Approximation and Projection (UMAP). Developed by McInnes et al. (2018), UMAP is also a dimensionality-reducing clustering algorithm. However, despite its similarities to t-SNE, UMAP better preserves the global structure of the high-dimensional data, permits the addition of new data and allows embedding in arbitrary dimensions (rather than just two-dimensions). As such, it is a useful tool to identify extra-tidal candidates in large data sets like APOGEE DR17.

We follow the same process as outlined in Section 3.1.1; first identifying a control group of cluster members based on chemical abundances and radial velocities, then selecting stars that are similar

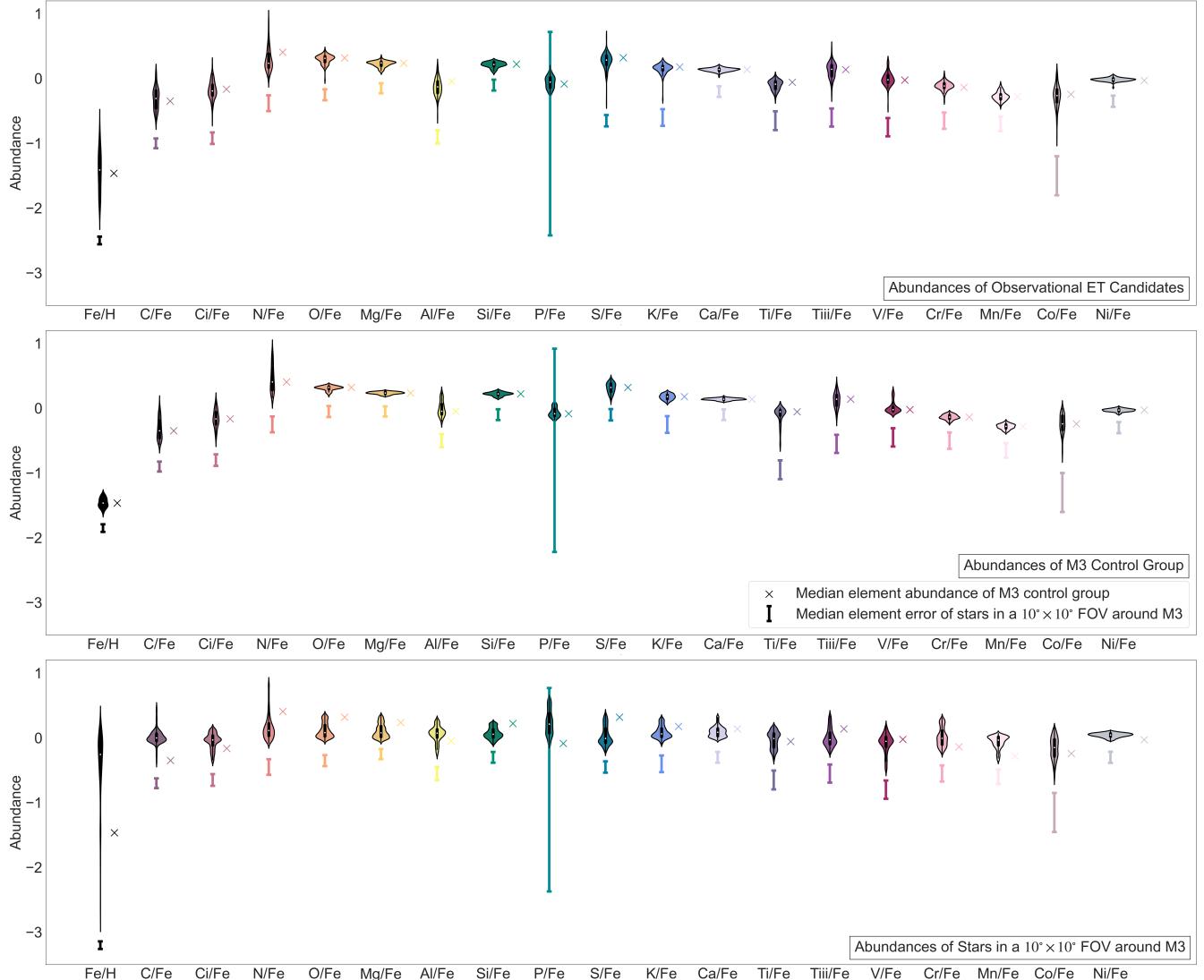


Figure 3. Abundance distributions for 103 observational extra-tidal candidates (top panel), the M3 control group (middle panel) and stars in the entire $10^\circ \times 10^\circ$ FOV around M3 (bottom panel). Outlier stars in the M3 control group and the full $10^\circ \times 10^\circ$ sample are removed by selecting only the central 95% of data to be displayed. The violin plots represent the abundance distributions of all 19 elements in each of the aforementioned samples. The median abundances of each element in the M3 control group are marked with crosses while the median element errors of stars in the full $10^\circ \times 10^\circ$ FOV are indicated with error bars. Each element used in the t-SNE and UMAP reduction is assigned the same colour for the violin, median abundance cross and median error bar. Comparing the violin distributions and median abundance locations in the top and middle plots, it is clear that the 103 observational extra-tidal candidates have similar abundances to those of the M3 control group. The bottom plot highlights M3's unique chemical location relative to the field stars in our $10^\circ \times 10^\circ$ sample.

to the cluster members with just chemical abundances. The initial UMAP analysis that includes radial velocities was run for a range of input parameters, including the size of the local neighbourhood, the minimum distance between points, the number of components of the reduced dimensionality space and choice of the metric parameter. We find that a neighbourhood size of 15, minimum distance of 0.1, and the canberra metric result in M3 candidates being clearly separated from the field stars when reducing the data down to two dimensions (and as such, the selection of chemically similar stars did not require a UMAP tolerance parameter). Repeating the analysis without radial velocities allows us to *observationally identify 119 extra-tidal candidates with the UMAP algorithm*. When we cross-match our t-SNE extra-tidal candidates with those identified by UMAP, we obtain a final observational sample of *103 observationally-identified extra-tidal candidates*. The discrepancy between methods is most likely due to some stars not being as close to cluster members in each of the

t-SNE and UMAP regimes. Regardless, almost all UMAP extra-tidal candidates are also identified with the t-SNE algorithm, providing confidence in this high-dimensionality approach to identifying extra-tidal candidates of M3.

The 19 chemical abundance distributions for the observational extra-tidal candidates, the M3 control group and stars in the entire $10^\circ \times 10^\circ$ FOV around M3 are also presented as violin plots in Figure 3. Outlier stars in the M3 control group and the full $10^\circ \times 10^\circ$ sample are removed by selecting only the central 95% of the data. From the violin shapes, one can observe that the abundance distributions of the extra-tidal candidates are similar to those of M3, which is consistent with those stars originating in that cluster. The median abundance for each element of the M3 control group is also indicated in the three samples, which again highlights both the chemical similarities between the observational extra-tidal candidates and M3 and the unique chemical composition of M3 relative to the surrounding field

stars. Finally, the median abundance error for each element in the $10^\circ \times 10^\circ$ sample is presented in all three plots to show the typical error sizes for each element used in this analysis.

3.2 Corespray Particle Spray Code

While t-SNE and UMAP are useful tools in identifying stars beyond a cluster's tidal radius that are chemically similar to GC members, they neglect to provide information on whether a given star could have escaped a GC. Furthermore, chemical similarities alone do not necessarily indicate that stars were born in the same GC (Ness et al. 2018, 2019). Thus, information on where extra-tidal stars could end up in a variety of parameter spaces is an essential tool that is necessary for confirming the extra-tidal nature of observationally-identified extra-tidal candidates.

To further constrain our extra-tidal candidates and probe three-body dynamics in the cores of GCs, we present `corespray`³ — a Python-based particle spray code that uses three-body dynamics to simulate statistical samples of extra-tidal stars for any Galactic GC. Corespray will simulate N extra-tidal stars, only requiring the user to input a set of conditions unique to their GC of interest (e.g. mass, core density, binary fraction, central velocity dispersion, escape velocity, etc.). These GC parameters can almost always be found in Baumgardt & Hilker (2018).

To simulate extra-tidal stars, `corespray` initially defines a three-dimensional position and velocity within the cluster for single stars and binaries of three-body systems at a random time along the orbit of the GC around the Galaxy. It is important to note that `corespray` does not directly simulate triple systems (i.e. three-body systems composed of three single stars and no binary), as the presence of these systems in GCs is believed to be rare (Leigh et al. 2011). All orbits and potentials in `corespray` are defined and integrated using `galpy`⁴ — a Python-package for galactic dynamics (Bovy 2015). For each three-body simulation, masses of the single star (m_s) and recoil binary (m_a, m_b) are sampled from a power-law distribution with a slope of $\alpha = -1.35$ (Salpeter 1955) and a mass range between $0.08M_\odot < m < 1.4M_\odot$. The system masses determine the probability that a single star escapes the three body system (P_s) and is computed in Equation 1 (Valtonen & Karttunen 2006).

$$P_s = \frac{m_s^{-3}}{m_s^{-3} + m_a^{-3} + m_b^{-3}} \quad (1)$$

By randomly sampling the probability function, `corespray` determines if the star escapes the system and if so, computes the system's total energy. The total energy of the three-body system (E_0) is computed by summing the kinetic and gravitational potential energies of both the single kicked star (E_s) and the recoil binary (E_B). To compute E_B , we first recognize that binaries with circular orbits have constant velocity, so $\dot{r} = 0$. Thus, E_B is totally dependent on the gravitational potential energy of the binary which is sampled between twice the hard-soft boundary and twice the contact boundary between two solar mass stars. To compute both the kinetic and gravitational potential energy of E_s , we require the position vector between (i) the single star and the centre of mass of the binary (\mathbf{r}_s) and (ii) the reduced mass of the motion relative to the single star

($m = \frac{m_B m_s}{M}$). The total mass of the three-body system (M) is represented as $M = m_s + m_B$. Thus, we sum the components of E_s and E_B to compute E_0 in Equation 2, where $\dot{\mathbf{r}}_s$ is with respect to the reference frame of the binary (Valtonen & Karttunen 2006).

$$E_0 = \frac{1}{2} m \dot{\mathbf{r}}_s^2 - G \frac{m_s m_B}{r_s} + E_B \quad (2)$$

With E_0 , `corespray` computes the escape velocity distribution of the star $f(v_s)$ via Equation 3 (Valtonen & Karttunen 2006).

$$f(v_s) dv_s = \frac{(3.5 |E_0|^{7/2} m_s M / m_B) v_s dv_s}{(|E_0| + \frac{1}{2} (m_s M / m_B) v_s^2)^{9/2}} \quad (3)$$

By computing the maximum of Equation 3 via $\frac{df}{dv_s} = 0$, one can solve for the peak escape velocity $v_{s,peak}$ through Equation 4 (Valtonen & Karttunen 2006).

$$v_{s,peak} = \frac{1}{2} \sqrt{\frac{(M - m_s)}{m_s M}} \sqrt{|E_0|} \quad (4)$$

Through randomly sampling the escape velocity distribution function in Equation 3 between $0 < v_{s,test} < 5 \times v_{s,peak}$ (Equation 4), we determine that the kicked star is extra-tidal only if the sampled test velocity is larger than the Baumgardt & Hilker (2018) escape velocity of the GC. Corespray will continue to sample three-body interactions until N escaper stars are produced. Once N extra-tidal stars are simulated, `corespray` projects the escape velocities onto the initial three-dimensional velocity vectors to determine the directions of motion of the escaper stars. Escaper positions and velocities are updated and new orbits are defined for each star. It is important to note that `corespray` assumes that each cluster is isotropic and thus has no rotation. Combined with the fact that `corespray` ejects stars at various times along the GC's orbit throughout the Galaxy, this assumption ensures that the stars kick velocity vector is oriented in a random direction.

After escaping the cluster at some random time during the duration of one azimuthal period of the GC around the Galaxy, the simulated stars are integrated from their escape time to the present day in a combined Milky Way and King potential to encompass influences from both the Galaxy and the GC itself. Along with three-dimensional positions and velocities, escape times and velocities for all the simulated stars are computed in `corespray`. With these quantities, a variety of orbital parameters (i.e. right ascensions, declinations, proper motions, radial velocities and distances) can be computed with `galpy`. Ultimately, `corespray` allows the user to define a variety of parameter spaces to obtain a statistical representation of single extra-tidal stars and binary extra-tidal systems for a given GC. Figure 4 highlights three example parameter spaces generated from a `corespray` simulation of 40,000 extra-tidal stars of M3.

3.3 Extra-Tidal Candidate Probabilities

In Section 3.1, we use high-order dimensional analysis to identify 103 extra-tidal candidates that are both spatially and chemically similar to a control group of M3 stars. While t-SNE and UMAP are successful at identifying candidate stars isolated in the field, these algorithms just confirm that the stars share similar chemistry across 19 different elements. Thus, in Section 3.2 we develop `corespray` to further constrain the origin of our observationally-identified extra-tidal candidates. However, quantifying the probability that each extra-tidal

³ For a complete description of `corespray`'s installation instructions and capabilities, please visit <https://github.com/webbjj/corespray>.

⁴ For a complete description of `galpy`, please visit <http://github.com/jobovy/galpy>.

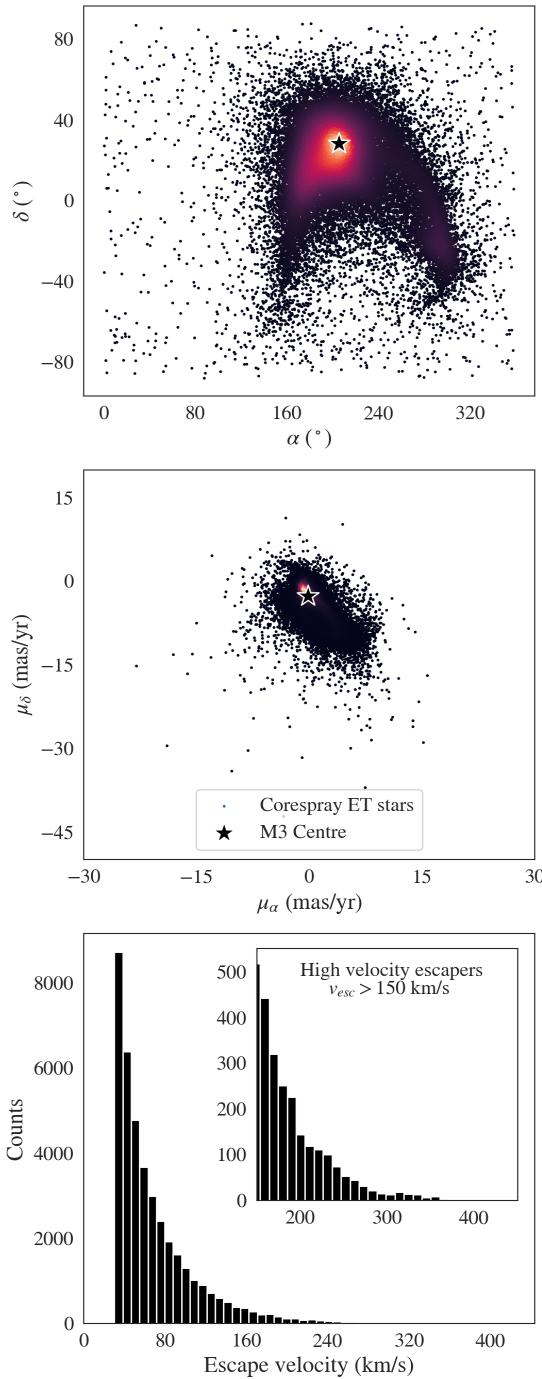


Figure 4. Spatial (top panel), proper motion (middle panel) and escape velocity (bottom panel) distributions of 40,000 extra-tidal stars of M3 simulated with corespray. Each extra-tidal star escaped the core of M3 at a random time during one azimuthal orbital period (~ 387 Myr) of the GC around the Galaxy. Spatial locations and proper motions are shown as scatter density representations with a kernel-density estimate using Gaussian kernels from `scipy.stats.gaussian_kde`. The location and proper motion of M3 is indicated with a black star in both plots respectively. Escape velocities for all simulated stars are depicted in a histogram with a sub-panel that highlights the high velocity escapers (i.e. $v_{esc} > 200$ km/s). One can observe that while the locations and proper motions of extra-tidal stars of M3 are most concentrated near the cluster itself, extra-tidal stars can leave the cluster with a wide variety of escape velocities and proper motions.

candidate is an extra-tidal star of M3 requires us to infer two distributions: one distribution of extra-tidal stars of M3 (corespray) and another distribution of field stars surrounding M3.

To investigate our extra-tidal candidates' origins, we first use corespray to simulate 40,000 extra-tidal stars of M3 over the duration of one azimuthal orbital period of $P_{orb} = 387.82$ Myr (Figure 4). However, since we restrict our observational search to a $10^\circ \times 10^\circ$ FOV, we also constrain our sample of corespray extra-tidal stars to be confined within this same spatial range. Altogether, this yields a sample of 3110 simulated extra-tidal stars that can be used to probe cluster associations for each extra-tidal candidate. It is important to note that corespray extra-tidal stars located within a $10^\circ \times 10^\circ$ FOV around M3 are ones that either received low velocity kicks or escaped recently, as they are still in proximity to the GC. Stars within this FOV represent a small sample of the full 40,000 star simulation, indicating that the majority of core interactions that could have occurred over the past orbital period result in extra-tidal stars being kicked beyond a $10^\circ \times 10^\circ$ FOV of M3.

From our simulated corespray distribution, we have produced a sample that represents where extra-tidal stars of M3 could end up in a variety of parameter spaces. In contrast, we must produce a distribution that represents stars that are clearly not extra-tidal stars of M3 for comparison. Thus, we define a "field star" distribution by removing all sources from our original $10^\circ \times 10^\circ$ FOV sample that are either (i) members of the M3 control group defined in Section 3.1 or (ii) any of the 103 extra-tidal candidates. Our field star distribution contains 2998 stars. With these two samples, we can now compute the probability that each extra-tidal candidate belongs to the corespray extra-tidal star distribution or field star distribution. To do this calculation, we incorporate proper motions and radial velocities into both a multivariate Gaussian distribution model and an extreme deconvolution (XD).

To compute the probabilities of extra-tidal candidates belonging to the corespray distribution, we model the corespray data with a multivariate Gaussian distribution in scikit-learn (Pedregosa et al. 2011). Using Gaia EDR3 proper motions and APOGEE DR17 radial velocities, a mean and covariance matrix can be generated and used to construct the multivariate Gaussian. To ensure accuracy in the covariance, the covariance matrix must be computed for each extra-tidal candidate by summing the covariance from corespray and the covariance from the extra-tidal candidate distribution. From this model, the probability distribution function (PDF) can be computed and individual extra-tidal candidate corespray probabilities, $P(c)$, can be extracted.

Contrary to the simulated corespray distribution, the field star distribution is observed and thus subject to measurement error. Unfortunately, the previous multivariate Gaussian modelling is unable to directly incorporate error when defining the distribution and thus must be modified for noisy data sets. However, Bovy et al. (2011) outline an approach that can analyze noisy, heterogeneous and incomplete data; specifically, an extreme deconvolution (XD).

An XD operates similarly to a Gaussian mixture model, where Bayesian estimation and Gaussian modelling output a corrected distribution (Vanderplas et al. 2012). To estimate a distribution for the M3 field stars, we utilize astroML — a Python module for machine learning and data mining (Vanderplas et al. 2012). Specifically, we use the XDGMM function with one Gaussian component (`n_components=1`) and a default number of iterations (`max_iter=100`) to perform the XD. By inputting the field star proper motions, radial velocities and associated errors into XDGMM, we obtain a corrected field star distribution.

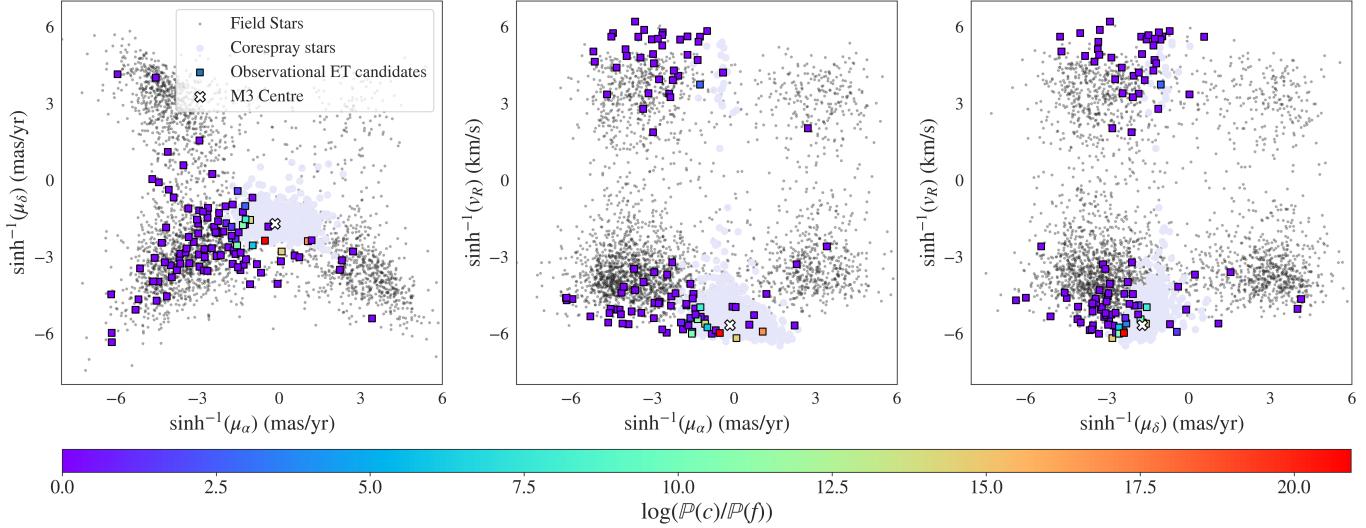


Figure 5. Proper motion and radial velocity spaces for all 103 observationally identified extra-tidal candidates (coloured squares) relative to 2998 field stars (black points) and 3110 corespray stars (lavender circles) in a $10^\circ \times 10^\circ$ FOV around M3. Extra-tidal candidates are coloured by their $\log(\mathbb{P}(c)/\mathbb{P}(f))$ value, where stars with $\log(\mathbb{P}(c)/\mathbb{P}(f)) < 0$ are treated equally and assigned the same colour (dark purple). This is done to highlight stars with positive $\log(\mathbb{P}(c)/\mathbb{P}(f))$ values, as they have higher probabilities of belonging to the corespray distribution than the field. Each representation depicts the \sinh^{-1} of the data, as it best separates low and high values to allow for optimal data visualization. M3's proper motion of $(\mu_\alpha, \mu_\delta) = (-0.152, -2.670)$ mas/yr and radial velocity of $v_R = -147.20$ km/s are obtained from Baumgardt & Hilker (2018) and used to indicate the centre of M3 (white cross) in each panel.

Using this distribution's mean and covariance matrix, we repeat the aforementioned multivariate Gaussian modelling process to compute the PDF and individual extra-tidal candidate field probabilities, $\mathbb{P}(f)$. Ultimately, once $\mathbb{P}(c)$ and $\mathbb{P}(f)$ are computed, the odds that an extra-tidal candidate is more similar to the corespray sample than the field star sample can be quantified by computing the logarithm of the odds ratio: $\log(\mathbb{P}(c)/\mathbb{P}(f))$. The odds ratio — also referred to as the Bayes factor (Kass & Raftery 1995) — represents the ratio of the likelihoods between two distributions and is useful to quantitatively assign a strength of association of the data to the distributions (Jeffreys 1935). Ultimately, if an extra-tidal candidate has an odds ratio of $\log(\mathbb{P}(c)/\mathbb{P}(f)) > 0$, it would indicate that the star has a higher probability of being associated with the corespray distribution than the field and should thus be deemed a high-probability extra-tidal star of M3.

4 RESULTS

For each of the 103 extra-tidal candidates we identify in Section 3.1, we use the methodology presented in 3.3 to compute $\log(\mathbb{P}(c)/\mathbb{P}(f))$. To compute $\mathbb{P}(c)$ via a multivariate Gaussian model, we use a mean and covariance matrix generated from the corespray sample. From the simulated proper motions and radial velocities, we obtain a corespray mean of $\mu_c = (-0.152 \text{ mas/yr}, -2.590 \text{ mas/yr}, -123.084 \text{ km/s})$. The corespray covariance matrix is computed by summing the covariance from the corespray sample (Equation 5) and the covariance of each extra-tidal candidate (Equation 6). After modelling a multivariate Gaussian distribution with these parameters, we extract $\log \mathbb{P}(c)$ for each extra-tidal candidate.

$$\Sigma_c = \begin{bmatrix} 4.599 \times 10^{-1} & -1.139 \times 10^{-1} & -1.200 \times 10^1 \\ -1.139 \times 10^{-1} & 8.110 \times 10^{-1} & 2.615 \times 10^1 \\ -1.200 \times 10^1 & 2.615 \times 10^1 & 1.843 \times 10^3 \end{bmatrix} \quad (5)$$

$$\Sigma_{ET} = \begin{bmatrix} \sigma_{\mu_{\alpha,i}}^2 & 0 & 0 \\ 0 & \sigma_{\mu_{\delta,i}}^2 & 0 \\ 0 & 0 & \sigma_{v_{R,i}}^2 \end{bmatrix} \quad (6)$$

To compute $\mathbb{P}(f)$, we use a mean and covariance generated from the XD. As outlined in Section 3.3, we run the XD by inputting field star proper motions, radial velocities and associated errors into XDGMM. From this analysis, we obtain a field star mean of $\mu_f = (-19.804 \text{ mas/yr}, -11.385 \text{ mas/yr}, -13.570 \text{ km/s})$ and a field star covariance outlined in Equation 7.

$$\Sigma_f = \begin{bmatrix} 3.225 \times 10^3 & -1.720 \times 10^1 & 1.376 \times 10^2 \\ -1.720 \times 10^1 & 1.672 \times 10^3 & -1.982 \times 10^1 \\ 1.376 \times 10^2 & -1.982 \times 10^1 & 9.808 \times 10^2 \end{bmatrix} \quad (7)$$

With μ_f and Σ_f , we replicate our previous probability computation, but this time for $\mathbb{P}(f)$. Like before, the final covariance matrix for the field star distribution is the sum of the covariance from the field star sample (Equation 7) and the covariance of each extra-tidal candidate (Equation 6). With these parameters, we construct a new multivariate Gaussian distribution, ultimately allowing us to extract $\log \mathbb{P}(f)$ for each extra-tidal candidate. Proper motion and radial velocity distributions with $\log(\mathbb{P}(c)/\mathbb{P}(f))$ values are presented for each of the 103 extra-tidal candidates in Figure 5. Although $\log(\mathbb{P}(c)/\mathbb{P}(f))$ ranges from -143719.903 to 20.924 , extra-tidal candidates with $\log(\mathbb{P}(c)/\mathbb{P}(f)) < 0$ are treated equally and coloured the same, as these stars have higher probabilities of being associated with the field than with corespray extra-tidal stars. The kinematic parameter spaces are plotted by computing the \sinh^{-1} of the data, as it best separates low and high values to allow for optimal data visualization.

As described in Section 3.3, our criteria for selecting high-probability extra-tidal candidates of M3 are those with $\log(\mathbb{P}(c)/\mathbb{P}(f)) > 0$. This simply means that based on its kine-

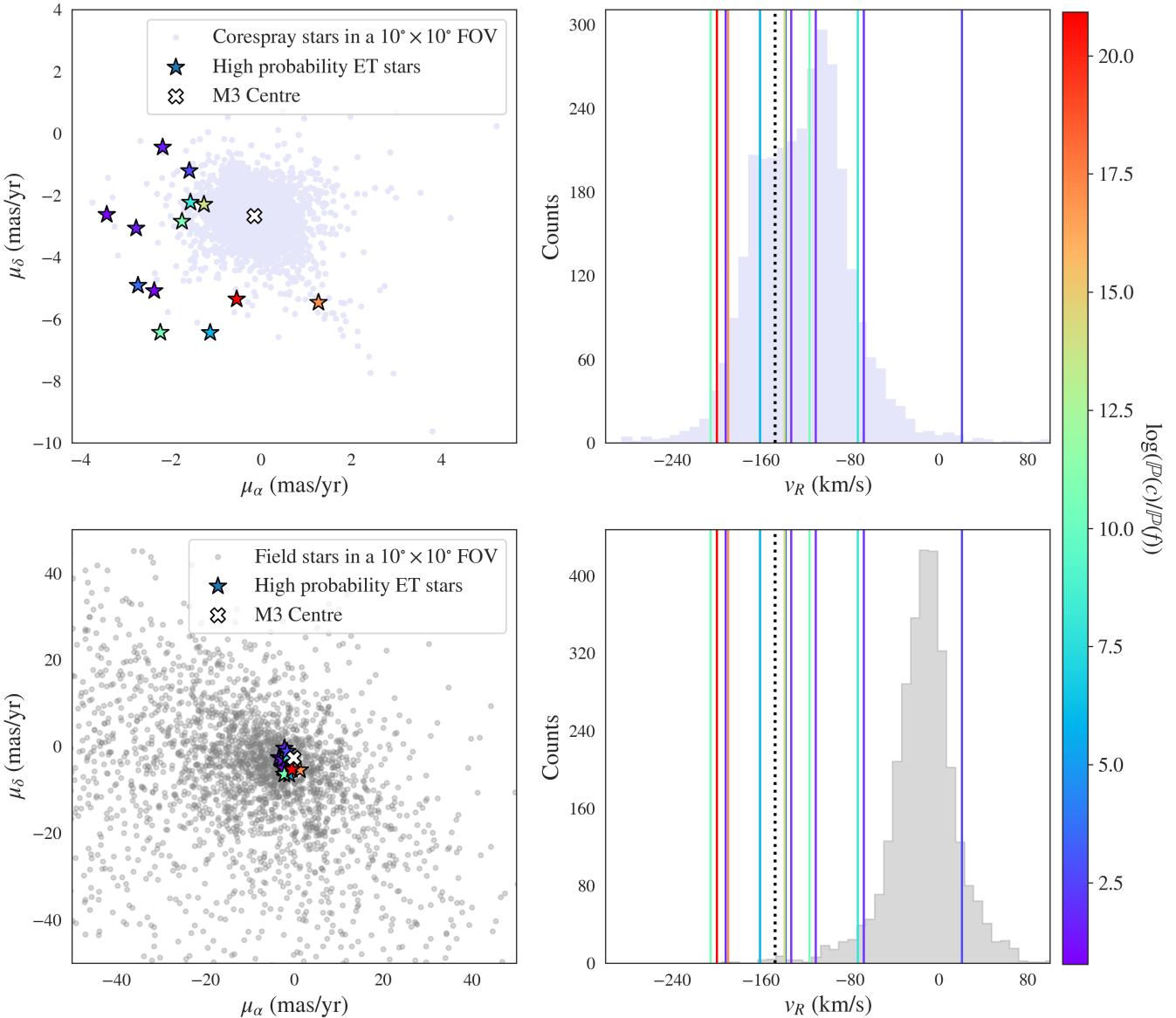


Figure 6. Proper motion and radial velocity distributions of 13 high-probability extra-tidal stars relative to a sample of 3110 corespray extra-tidal stars in a $10^\circ \times 10^\circ$ FOV around M3 (lavender points and distribution) and a sample of 2998 field stars in a $10^\circ \times 10^\circ$ FOV around M3 (gray points and distribution). Left panels: proper motions of the 13 new extra-tidal stars are indicated as stars coloured by their $\log(\mathbb{P}(c)/\mathbb{P}(f))$ values. M3's Baumgardt & Hilker (2018) mean proper motion of $(\mu_\alpha, \mu_\delta) = (-0.152, -2.670)$ mas/yr is indicated with a white cross. The proper motion field star distribution is zoomed in for clarity. Right panels: radial velocities of the 13 new extra-tidal stars are indicated as coloured lines. The colouring for each line again corresponds to the $\log(\mathbb{P}(c)/\mathbb{P}(f))$ of each high-probability extra-tidal star. The Baumgardt & Hilker (2018) mean M3 radial velocity of $v_R = -147.20$ km/s is indicated as a dotted black line.

matics, an extra-tidal candidate is more likely to belong to the corespray distribution of extra-tidal stars of M3 than a sample of field stars. Upon computation of $\log(\mathbb{P}(c)/\mathbb{P}(f))$ for each of our 103 chemically similar extra-tidal candidates, we find that 14 stars have $\log(\mathbb{P}(c)/\mathbb{P}(f)) > 0$.

To remove outlier sources, we only keep stars that are within 5σ of the corespray median proper motion and radial velocity. One extra-tidal candidate (2M13413296+3255410) lies outside this range and is thus discarded from our final *high-probability* extra-tidal star sample. Of these 13 high-probability extra-tidal stars, $\log(\mathbb{P}(c)/\mathbb{P}(f))$ ranges from 0.775 to 20.924. Proper motion and radial velocity distributions for all 13 high-probability stars of M3 are presented in Figure 6. The $\log(\mathbb{P}(c)/\mathbb{P}(f))$ values for the 13 high-probability

extra-tidal stars are presented in Table 1. Complete spatial, kinematic and $\log(\mathbb{P}(c)/\mathbb{P}(f))$ information for the 103 extra-tidal candidates (including the high-probability stars) is presented in Appendix A1.

It is interesting to note that several of the extra-tidal stars are located in the outskirts of the corespray proper motion and radial velocity distributions in Figure 6. The high relative proper motions and radial velocities of these stars are consistent with high-velocity ejections from three-body systems in M3. As seen in Figure 4, three-body interactions producing high velocity ejections are rare, but not completely uncommon. Given the number of high-velocity extra-tidal stars relative to the simulated corespray stars, it may be the case that the core of M3 has a higher density, a flatter stellar mass function, or a sub-population of black holes such that three-body

interactions are primarily between higher-mass stars and remnants than considered here. These findings further indicate that imposing proper motion or radial velocity constraints when initially searching for extra-tidal stars could result in extra-tidal candidates from high-velocity ejections being missed.

5 DISCUSSION

5.1 Extra-Tidal Star Probabilities

After spatial, chemical and kinematic analyses, we have identified 13 high-probability extra-tidal stars of the Galactic GC M3. Our metric for determining the highest-probability extra-tidal stars is simple: we only select stars that have $\log(\mathbb{P}(c)/\mathbb{P}(f)) > 0$. However, Kass & Raftery (1995) famously provide metrics to interpret the logarithm of the odds ratio values and assign a strength of evidence that data is associated with a given distribution. Based on our computed $\log(\mathbb{P}(c)/\mathbb{P}(f))$ and the Kass & Raftery (1995) interpretation, we quantify the strength of evidence for each extra-tidal star in Table 1. From this, we observe that 9 stars are decisive, 2 stars are strong and 2 stars are significant with being associated with the corespray extra-tidal star distribution. It is important to mention that the above interpretation is equivocal and is meant to act more as a general guideline than a definite statement. Regardless, the large $\log(\mathbb{P}(c)/\mathbb{P}(f))$ for almost all 13 stars is evidence that these stars are more likely to belong to the corespray extra-tidal star distribution than the field.

When examining the locations of the highest-probability extra-tidal stars in Figure 6, we see that the majority of stars with large $\log(\mathbb{P}(c)/\mathbb{P}(f))$ are the ones with highly negative radial velocities. This observation occurs despite the fact that these stars are not necessarily the ones that are located closest to M3's proper motion median. It should again be noted that $\log(\mathbb{P}(c)/\mathbb{P}(f))$ is computed from both proper motion and radial velocity information. Thus, it is the combination of these three parameters that determine the probabilities that an extra-tidal candidate is associated with each distribution.

In Figure 6, we see that the corespray radial velocity distribution is skewed to larger negative radial velocities than the field star distribution. Thus, stars that are located far from the radial velocity field star distribution (i.e. stars that have highly negative radial velocities) will be assigned larger probabilities of being associated with the corespray distribution, even though they are not necessarily "close" to M3's proper motion median. Hence, we observe the high $\log(\mathbb{P}(c)/\mathbb{P}(f))$ of these high-velocity stars. The opposite of this effect is observed with extra-tidal star 2M13470503+2834136, where despite its highly positive radial velocity, its consistent proper motions cause it to have a positive $\log(\mathbb{P}(c)/\mathbb{P}(f))$.

The spatial locations of our 13 new high-probability extra-tidal stars relative to APOGEE DR17 field stars in a $10^\circ \times 10^\circ$ FOV around M3 are shown in Figure 8.

5.2 Alternative Probability Tests

While odds ratio computations allow us to statistically infer the highest-probability extra-tidal candidates of M3, other kinematic and photometric tests could also be employed. Specifically, utilizing conserved kinematic quantities like actions would be the ideal way to probe extra-tidal association. Actions are useful because the actions of an extra-tidal star will remain similar to those of the parent cluster itself, no matter where along the cluster's orbit the star escaped (Binney & Tremaine 2008).

Computing the actions of a star requires an assumed Galactic

Extra-tidal Star	$\log(\mathbb{P}(c)/\mathbb{P}(f))$	Evidence Strength
2M13500350+2431542	20.924	Decisive
2M13240682+3020316	16.993	Decisive
2M13553890+3241208	13.982	Decisive
2M13234701+3111279	10.677	Decisive
2M13353852+2939287	10.497	Decisive
2M13271850+2841521	7.990	Decisive
2M13251237+3018535	5.934	Decisive
2M13320566+2614003	3.550	Decisive
2M13470503+2834136	2.651	Decisive
2M13382215+3233031	1.450	Strong
2M13563485+2517471	1.418	Strong
2M13264876+3300388	0.835	Significant
2M13524016+2601592	0.775	Significant

Table 1. The $\log(\mathbb{P}(c)/\mathbb{P}(f))$ values for the 13 new high-probability extra-tidal stars of M3. Stars are organized in descending order of $\log(\mathbb{P}(c)/\mathbb{P}(f))$. The Kass & Raftery strength of evidence metric to quantify each star's association to the corespray distribution is also presented (Kass & Raftery 1995). Values of $0 < \log(\mathbb{P}(c)/\mathbb{P}(f)) < 0.5$ are not worth more than a bare mention, $0.5 < \log(\mathbb{P}(c)/\mathbb{P}(f)) < 1$ are significant, $1 < \log(\mathbb{P}(c)/\mathbb{P}(f)) < 2$ are strong and $\log(\mathbb{P}(c)/\mathbb{P}(f)) > 2$ are decisive of being associated with the corespray extra-tidal stars.

potential and the knowledge of six parameters: α , δ , μ_α , μ_δ , v_R and distance (d). While the APOGEE DR17 actions of the high-probability extra-tidal candidates are consistent with the corespray distribution within error, uncertainties in all six components result in the errors of the actions being large (Figure 7). The most uncertain input parameter is the astroNN distance estimate, with most extra-tidal candidates having $\delta d/d > 0.20$. Alternative methods for measuring distances to Gaia EDR3 stars have improved fractional distance errors, with the extra-tidal candidates having a mean $\delta d/d \sim 0.09$ (Bailer-Jones et al. 2021). While using this catalogue would indeed allow many candidates to have smaller distance uncertainties, more than one-third of our extra-tidal candidates would still have $\delta d/d > 0.10$. Similarly, action computation of the field star distribution would also be challenging, as field stars span a wide range of distances. Since Bailer-Jones et al. (2021) state that their distance estimates are only reliable out to several kpc, and M3 is located 10.2 kpc away, we discard this method of action analysis until more accurate distances to faraway stars are obtained. Nevertheless, computation of the odds ratios using proper motions and radial velocities remains a robust statistical method to probe the kinematic associations of extra-tidal stars to M3 (and other Galactic GCs).

5.3 Extra-Tidal Versus Tidal Tail Stars

As previously mentioned, we have only considered three-body core encounters as the dynamical interaction that produces extra-tidal stars. However, other dynamical processes like tidal stripping can also cause stars to migrate beyond the tidal radius of a GC, albeit resulting in stars populating tidal tails rather than being isolated field stars. To determine if any of our high-probability extra-tidal stars are more likely to be associated with M3's tidal tails than with extra-tidal stars produced via three-body encounters (i.e. a corespray distribution) we first generate mock tidal tails for M3. The mock tidal tails are simulated using the particle spray method of Fardal et al. (2015), which has recently been implemented in galpy Bovy (2015); Banik & Bovy (2019). For the simulation we generate 10,000 stars over a disruption timescale of 1 Gyr, assuming M3 has a mass of $59,800 M_\odot$ (Baumgardt & Hilker 2018). M3's current orbital parameters are taken from Vasiliev (2019) and the clus-

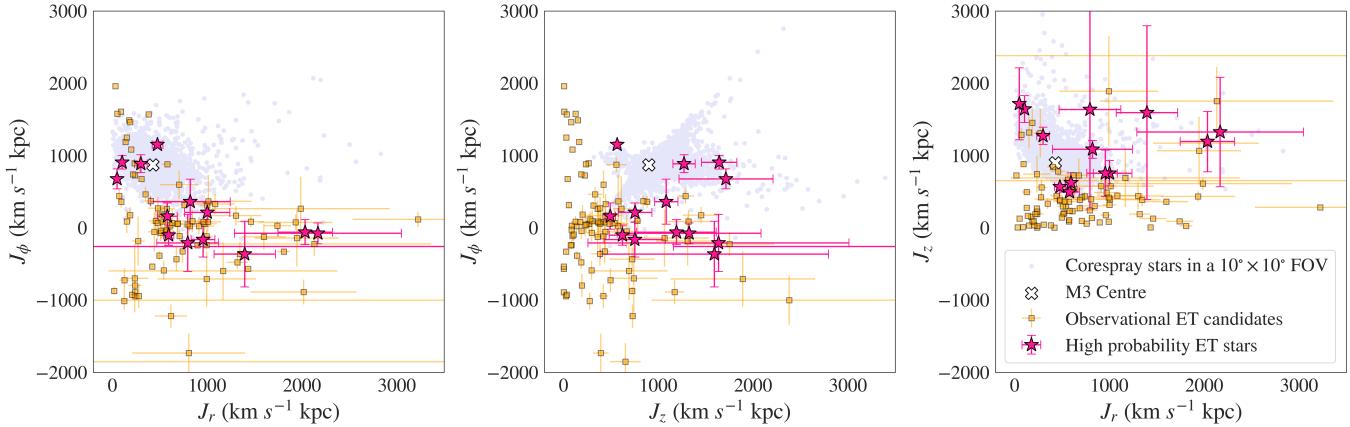


Figure 7. Radial (J_r), azimuthal (J_ϕ) and vertical (J_z) action variable distributions for stars within a $10^\circ \times 10^\circ$ FOV around M3 (white star). 3110 corespray simulated extra-tidal stars are indicated as lavender points, 103 observationally-identified extra-tidal candidates are indicated as orange squares and our final sample of 13 high-probability extra-tidal stars with $\log(p_c/p_f) > 0$ are indicated as pink stars. Actions and action errors for the observed extra-tidal candidates and stars are obtained from APOGEE DR17 (Abdurro'uf et al. 2022).

ter orbit is integrated in the MWPotential2014 Milky Way galaxy model from Bovy (2015). Once the mock tail stars are generated (we repeat the analysis presented in Section 3.3, but with the mock tail stars replacing the corespray stars to compute the individual tidal tail probabilities, $P(t)$). We find the range of $\log(P(c)/P(t))$ is $39.208 < \log(P(c)/P(t)) < 88278113.255$, which indicates that all 103 candidates are far more likely to be associated with the corespray distribution than M3's tidal tails.

6 CONCLUSIONS

In this study, we present a new methodology that combines observational and theoretical techniques to identify 13 new extra-tidal stars of the Galactic GC M3. Using two different unsupervised machine learning algorithms (t-SNE and UMAP), we identify stars beyond the tidal radius of M3 that are chemically similar to a control group of cluster members of M3. While machine learning clustering algorithms like t-SNE and UMAP are excellent tools in identifying stars with similar chemical abundances, particle-spray simulations like corespray are necessary for tracing the extra-tidal candidates back to their suspected birth cluster and assigning probabilities of association. Ultimately, each of our identified extra-tidal stars has passed rigorous tests to confirm its extra-tidal nature. The results and implications can be summarized as follows:

(i) An application of the t-SNE and UMAP dimensionality-reduction algorithms to stars within a $10^\circ \times 10^\circ$ FOV around M3 identifies 103 extra-tidal candidates that are chemically similar to a control group of stars within M3 (Figure 2). Chemical abundance distributions further confirm that the 103 observationally-identified extra-tidal candidates have similar abundances to M3; both samples occupying a unique chemical location relative to stars in the full FOV (Figure 3).

(ii) A corespray simulation of 40,000 extra-tidal stars of M3 (Figure 4) finds that only 3110 stars are located within a $10^\circ \times 10^\circ$ FOV around M3 (our observational FOV). This result indicates that the majority of core three-body interactions that could have occurred over M3's past orbital period result in extra-tidal stars being kicked beyond this FOV. Hence, any extra-tidal stars found within the FOV will have either escaped M3 recently or with a low kick-velocity.

(iii) A multivariate Gaussian model and an XD compute the probabilities that each extra-tidal candidate either belongs to a corespray distribution of extra-tidal stars of M3 or the field stars around M3 (Figure 5). By computing $\log(P(c)/P(s))$, we find that 14 stars have $\log(P(c)/P(s)) > 0$. One star is likely an outlier, having proper motions $> 5\sigma$ of the corespray mean, leaving 13 high-probability extra-tidal stars of M3 (Table 1). This result indicates that although stars beyond the tidal radius of a GC can be chemically similar, only a fraction have kinematics that are consistent with being extra-tidal.

(iv) A proper motion and radial velocity analysis highlights that while all 13 extra-tidal candidates have properties consistent with M3, some were likely produced in rare three-body interactions (Figure 6). This finding suggests that imposing proper motion and radial velocity constraints when using high-dimensional analysis to initially search for extra-tidal stars could result in high-velocity extra-tidal stars being missed.

(v) A meaningful action variable analysis is unable to be performed due to large uncertainties in distances and pre-computed APOGEE DR17 actions (Figure 7). Once more accurate distances to stars at ~ 10 kpc are obtained, conserved kinematic quantities like actions can provide additional constraints in defining high-probability extra-tidal stars of M3 and other Galactic GCs at similar distances.

Ultimately, all 13 extra-tidal stars identified in this study have passed spatial, chemical and kinematic analyses, providing strong evidence that each candidate is indeed an extra-tidal star of M3. None of the eight extra-tidal stars presented in Navin et al. (2016) are recovered in this study, as (i) APOGEE and LAMOST observe different regions of the sky (eliminating seven out of Navin et al. (2016) eight stars) and (ii) the proper motion of the remaining extra-tidal star is far different than the stars in our corespray sample. Thus, each extra-tidal star presented in this work is a new extra-tidal star of M3. As new extra-tidal stars are discovered, one will be able to better understand core dynamics and star formation histories in GCs. Furthermore, as corespray also computes the orbital parameters of the recoiled binaries of the simulated three-body systems, binary fractions and their locations in the Galaxy can also be inferred. Thus, future applications of corespray to other Galactic GCs have the potential to inform us not just about GC evolution, but formation and evolution of our Galaxy itself.

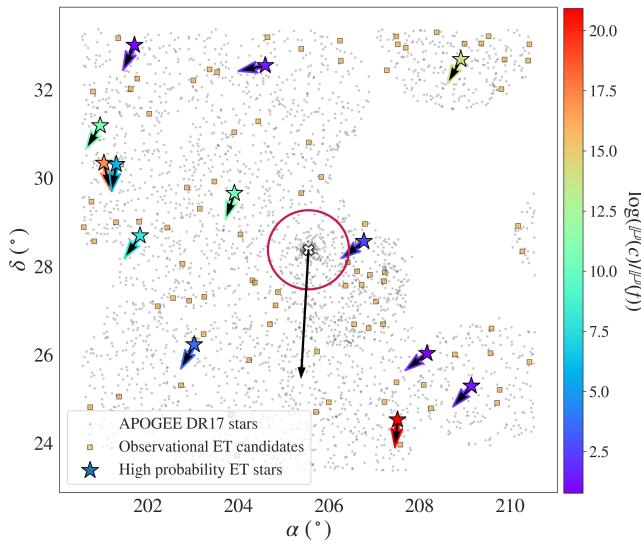


Figure 8. Spatial distribution of the 13 new high-probability extra-tidal stars of M3 identified in this study relative to APOGEE DR17 stars in a $10^\circ \times 10^\circ$ FOV around M3. Each extra-tidal star is coloured by its $\log(P(c)/P(f))$ value. Proper motion directions of each star are marked with coloured arrows. M3's tidal radius at apogalacticon of $r_t = 159.03\text{pc}$ is shown as a magenta circle, its proper motion with a black arrow and the cluster centre with a white cross.

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DATA AVAILABILITY

All data and parameters in this study are obtained using apogee tools (Bovy 2016), which can be downloaded at <https://github.com/jobovy/apogee>. Chemical abundances are acquired from the astroNN catalogues (Leung & Bovy 2019a,b), which can be downloaded at <https://astronn.readthedocs.io/en/latest/>. The corespray simulation software is available for download at <https://github.com/webbjj/corespray>. Important individual GC parameters of M3 are obtained from Baumgardt & Hilker (2018), where the online database is accessible at <https://people.smp.uq.edu.au/HolgerBaumgardt/globular/>. All high-dimensional analysis and multivariate Gaussian modelling are performed using scikit-learn (Pedregosa et al. 2011), which can be accessed at <https://scikit-learn.org/stable/>. Finally, the XD technique is implemented via the astroML software and can be installed from <https://www.astroml.org/>.

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APPENDIX A: EXTRA-TIDAL CANDIDATE PARAMETERS

As mentioned in Section 3.1, we identify 103 stars located beyond M3’s tidal radius that have similar chemical abundances to a control group of M3 members. In Section 4, we observe that 14 of these observationally-identified extra-tidal candidates have higher probabilities of belonging to a simulated corespray distribution of M3 extra-tidal stars than the surrounding field stars (however one of these stars has a proper motion $> 5\sigma$ of the corespray mean and is deemed an outlier). Although the other 89 extra-tidal candidates have higher probabilities of being associated with the field rather than with corespray, these stars still exhibit chemical similarities to M3 members (Figure 2). Consequently, these candidates could still have a connection to the GC M3. In Table A1, we list the APOGEE DR17 identifiers, spatial locations, proper motions, radial velocities and probabilities of belonging to the corespray extra-tidal distribution for all 103 observationally-identified extra-tidal candidates. The 13 high-probability extra-tidal stars are indicated with a \star .

Table A1: Spatial, kinematic and probability parameters of the full 103 observationally-identified extra-tidal star sample around M3. While these stars had lower probabilities of belonging to the corespray extra-tidal star distribution, they are chemically similar to a control group of cluster members of M3. Parameters obtained from the APOGEE DR17 catalogue (Abdurro'uf et al. 2022) are marked with a $*$, parameters obtained from astroNN are marked with a \dagger (Leung & Bovy 2019a,b; Bovy et al. 2019) and parameters obtained from Gaia EDR3 (Gaia Collaboration et al. 2021) are marked with a \ddagger . The 13 high-probability extra-tidal stars are indicated with a \star .

APOGEE ID*	α^* [deg]	δ^* [deg]	d^l [pc]	μ_α^\dagger [mas/yr]	μ_δ^\dagger [mas/yr]	v_R^* [km/s]	$\log(\mathbb{P}(c)/\mathbb{P}(f))$
2M13221962+2853203	200.581	28.888	418.45	-233.614	-286.569	-55.512	-143719.903
2M13225389+2448532	200.724	24.814	9289.86	-2.094	-3.826	-43.63	-0.852
2M13230059+2927561	200.752	29.465	2200.571	-26.797	-14.122	83.181	-969.316
2M13230915+3157296	200.788	31.958	3687.256	-9.291	-14.912	-61.228	-255.638
2M13231525+2834137	200.813	28.57	464.202	-239.481	-44.355	-50.988	-74693.77
\star 2M13234701+3111279	200.945	31.191	13846.063	-1.756	-2.846	-116.314	10.677
\star 2M13240682+3020316	201.028	30.342	4421.892	1.272	-5.464	-189.679	16.993
2M13251080+2900003	201.295	29.0	5748.236	-9.713	-4.014	3.161	-100.028
\star 2M13251237+3018535	201.301	30.314	7344.123	-1.13	-6.438	-160.858	5.934
2M13252308+3310108	201.346	33.169	22747.718	-1.757	-1.612	54.494	-0.587
2M13252729+2503126	201.363	25.053	3046.792	-9.326	2.223	-18.561	-113.301
2M13253774+3018220	201.407	30.306	4835.265	-4.207	-9.506	-42.262	-75.083
2M13262865+3200458	201.619	32.012	7762.88	-5.355	-3.065	14.557	-24.358
\star 2M13264876+3300388	201.703	33.01	7246.469	-2.371	-5.087	-110.683	0.835
2M13270393+3215104	201.766	32.252	2112.221	4.622	-16.842	-149.049	-255.575
2M13271442+2900409	201.81	29.011	1428.097	-31.06	-47.74	-41.122	-3226.424
\star 2M13271850+2841521	201.827	28.697	18603.378	-1.568	-2.231	-73.028	7.99
2M13280718+3127198	202.029	31.455	5923.512	0.816	-10.302	-99.347	-67.723
2M13285221+2705512	202.217	27.097	5293.979	-3.165	-6.959	-173.036	-5.337
2M13294218+2852493	202.425	28.88	3018.643	-42.454	-26.872	156.198	-2712.866
2M13301333+2717046	202.555	27.284	4156.3	-0.712	-18.958	-178.933	-255.425
2M13305572+2518385	202.732	25.31	3974.775	-7.637	-10.19	-58.625	-121.665
2M13310270+2803397	202.761	28.061	6704.045	-2.78	-4.369	-64.231	-3.826
2M13312657+2947187	202.86	29.788	739.22	15.237	-113.417	-6.728	-15612.418
2M13313781+2713289	202.907	27.224	3109.405	4.994	-11.641	-13.592	-192.129
2M13320138+3212483	203.005	32.213	529.207	-97.777	-80.343	-104.566	-17112.749
\star 2M13320566+2614003	203.023	26.233	9871.155	-2.731	-4.913	-137.845	3.55
2M13320644+2620099	203.026	26.336	4620.158	-9.442	-9.242	-112.258	-132.231
2M13323523+2918295	203.146	29.308	3175.738	-1.949	-17.01	-76.786	-248.667
2M13325603+2627593	203.233	26.466	7557.041	-2.558	-7.847	-149.407	-14.421
2M13325885+2718504	203.245	27.314	6231.899	-10.174	-2.254	122.425	-95.567
2M1332728+3227326	203.363	32.459	2759.147	-13.509	-13.077	176.94	-365.166
2M13335179+2955261	203.465	29.923	14820.654	-4.484	-1.695	36.146	-12.335
2M13340645+3223497	203.526	32.397	2684.888	-2.661	-13.385	66.794	-205.137
2M13351884+2439111	203.828	24.653	163.775	-189.121	31.3	-52.838	-50777.16
2M13353738+3102263	203.905	31.04	541.548	-87.566	-53.618	75.996	-11666.421
\star 2M13353852+2939287	203.91	29.657	6552.756	-2.236	-6.434	-205.337	10.497
2M13355149+2343075	203.964	23.718	1262.867	-9.426	-1.502	47.752	-84.833
2M13360924+2358096	204.038	23.969	5923.698	-16.814	0.602	135.273	-295.605
2M13361042+2843314	204.043	28.725	540.263	-82.954	-15.896	51.014	-8742.756
2M13365674+2553016	204.236	25.883	2184.551	-25.688	-22.565	63.794	-1191.561
2M13371399+2741118	204.308	27.686	2691.399	1.504	-5.265	-43.34	-18.767
2M13372695+2738074	204.362	27.635	2910.639	-28.813	-0.395	-32.718	-1054.831
2M13374538+3117169	204.439	31.288	13732.833	-3.529	-1.224	121.074	-6.62
\star 2M13382215+3233031	204.592	32.55	7675.512	-2.184	-0.45	-191.682	1.45
2M13383329+3311212	204.638	33.189	2328.157	-1.281	-29.694	-116.427	-834.228
2M13384718+2641264	204.696	26.69	8351.948	-1.926	-8.614	-97.11	-35.406
2M13391725+2707122	204.821	27.12	1083.342	-33.237	-12.553	-80.604	-1436.225
2M13392970+2725086	204.873	27.419	1347.444	-18.862	-13.228	-37.422	-522.071
2M13393858+2324093	204.91	23.402	2803.387	-18.843	-9.022	244.367	-440.696
2M13413310+2939316	205.387	29.658	1253.692	-48.445	-10.534	-91.72	-3017.713
2M13413296+3255410	205.387	32.928	5910.173	0.096	-8.231	-244.763	14.581

Continued on next page.

Table A1 – continued from previous page.

APOGEE ID*	α^* [deg]	δ^* [deg]	d^l [pc]	μ_α^\dagger [mas/yr]	μ_δ^\dagger [mas/yr]	v_R^* [km/s]	$\log(\mathbb{P}(c)/\mathbb{P}(f))$
2M13421226+3048148	205.551	30.804	1940.452	-31.733	-3.141	-96.47	-1295.412
2M13425125+2442197	205.713	24.705	2196.549	-15.501	-8.553	60.006	-308.838
2M13425643+2604283	205.735	26.074	10111.364	-4.662	-2.412	24.875	-15.531
2M13431999+3001209	205.833	30.022	11764.436	-1.595	-2.779	136.178	-16.16
2M13435960+2455333	205.998	24.925	570.664	7.577	-8.218	3.71	-179.063
2M13445801+3306304	206.241	33.108	8645.228	-3.811	-3.697	29.321	-15.875
2M13450302+3239166	206.262	32.654	2987.031	-53.007	0.03	14.196	-3578.081
2M13452795+2734001	206.366	27.566	47296.464	-1.144	-0.754	169.044	-5.727
2M13453968+2805362	206.415	28.093	4972.64	-6.902	-2.083	144.143	-38.526
2M13461283+2641252	206.553	26.69	4600.086	-1.702	-13.81	111.228	-243.49
2M13462942+2753266	206.622	27.89	3088.608	-2.155	-15.265	-51.806	-204.782
2M13463425+2833348	206.642	28.559	4212.972	-0.906	-13.501	-205.726	-82.103
2M13465357+2735089	206.723	27.585	2471.714	-50.672	-26.497	-132.811	-3638.089
★ 2M13470503+2834136	206.77	28.57	16598.224	-1.593	-1.209	20.833	2.651
2M13471172+2857471	206.798	28.963	4743.041	-14.962	-7.277	-90.816	-278.262
2M13473000+2636130	206.875	26.603	2263.907	-7.89	-17.271	-62.865	-303.73
2M13473772+2748479	206.907	27.813	8267.7	-5.094	-3.829	12.701	-25.443
2M13480854+2712278	207.035	27.207	6036.497	0.599	-9.622	-144.539	-37.134
2M13484600+2641250	207.191	26.69	3942.425	-1.542	-11.163	-106.196	-75.493
2M13485213+2740093	207.217	27.669	224.053	-234.975	-198.961	-50.308	-102548.672
2M13485492+2455467	207.228	24.929	4621.769	-6.915	-17.607	-174.538	-247.869
2M13485603+2751541	207.233	27.865	7080.795	-8.147	-1.59	125.37	-54.637
2M13490745+3226207	207.281	32.439	1501.892	-0.053	-29.16	-71.615	-857.832
2M13495713+3312327	207.488	33.209	4057.047	-2.376	-9.061	-29.203	-63.283
2M13495806+2522400	207.491	25.377	5158.221	-5.777	0.234	-20.387	-31.678
★ 2M13500350+2431542	207.514	24.531	9663.527	-0.541	-5.356	-199.733	20.924
2M13500435+3301440	207.518	33.028	5588.955	-4.943	-9.793	-48.061	-84.222
2M13500482+2427061	207.52	24.451	4572.004	-9.749	-2.686	68.862	-92.252
2M13501865+2357507	207.577	23.964	19619.564	-2.658	-1.124	136.996	-4.392
2M13504647+3256579	207.693	32.949	1659.718	-24.038	-0.745	-141.824	-762.212
2M13522181+2519046	208.09	25.317	608.27	-45.196	-56.555	136.265	-5497.98
★ 2M13524016+2601592	208.167	26.033	8461.93	-3.425	-2.624	-132.903	0.775
2M13533112+3317545	208.379	33.298	5718.64	-7.795	-7.435	25.513	-96.297
2M13530057+2447273	208.252	24.79	2455.072	-37.238	-5.82	40.441	-1695.783
2M13550799+2600242	208.783	26.006	10707.711	-0.407	-3.037	33.309	-6.829
★ 2M13553890+3241208	208.912	32.689	16030.677	-1.268	-2.299	-138.898	13.982
2M13555858+3302315	208.994	33.042	5813.009	-6.936	-1.693	96.118	-38.423
★ 2M13563485+2517471	209.145	25.296	12684.391	-2.773	-3.067	-67.637	1.418
2M13572764+3302442	209.365	33.045	4588.824	-11.484	-5.632	14.797	-152.778
2M13580623+3312372	209.525	33.21	10350.6	-6.794	-1.329	161.259	-32.892
2M13581572+2602122	209.565	26.036	1769.465	-21.404	-17.072	-85.888	-730.967
2M13582241+2635398	209.593	26.594	5644.837	-4.761	-3.952	-12.575	-21.141
2M13585105+2454427	209.712	24.911	2140.677	-29.907	1.325	-138.452	-1235.104
2M13590531+2512219	209.772	25.206	5616.199	0.067	-12.109	-72.584	-114.074
2M13591443+3200423	209.81	32.011	6563.059	-5.017	-3.585	-68.815	-18.829
2M13593064+3241036	209.877	32.684	4682.448	-19.575	-10.683	-16.317	-508.173
2M14004220+2855119	210.175	28.919	3637.32	-5.869	-7.203	-198.696	-33.928
2M14010561+2820306	210.273	28.341	8799.474	-13.822	-4.638	-27.22	-219.346
2M14013546+3239245	210.397	32.656	1108.471	-41.541	-0.091	-80.28	-2275.449
2M14014321+2550085	210.43	25.835	5852.216	-14.037	-1.366	8.016	-220.284
2M14014338+3301395	210.43	33.027	1647.218	-46.407	27.245	-77.696	-4250.183

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