Statistical Inference Project Part 1 - A Simulation Exercise

Overview:

This project investigates the Exponential distribution in R and compares it with the Central Limit Theorem. The mean of the Exponential distribution is $\frac{1}{\lambda}$ and the standard deviation is also $\frac{1}{\lambda}$. A thousand simulations of the distribution of 40 exponentials would be investigated.

Simulations:

The exponential distribution can be simulated in R with `rexp(n, lambda)`, where `lambda` is the rate parameter and `n` is the number of observations. For the purpose of all the simulations in this project, value of `lambda` is set to `0.2`.
First we load the `ggplot2` plotting library.

```
library(ggplot2)
```

We then initialize the simulation controlling variables.

```
noSim <- 1000
sampSize <- 40
lambda <- 0.2
```

Set the seed of the Random Number Generator, so that the analysis is reproducible.

```
set.seed(3)
```

Create a matrix with thousand rows corresponding to 1000 simulations and forty columns corresponding to each of 40 random simulations.

```
simulationMatrix <- matrix(rexp(n = noSim * sampSize, rate = lambda), noSim, sampSize)
```

Create a vector of thousand rows containing the mean of each row of the `simulationMatrix`.

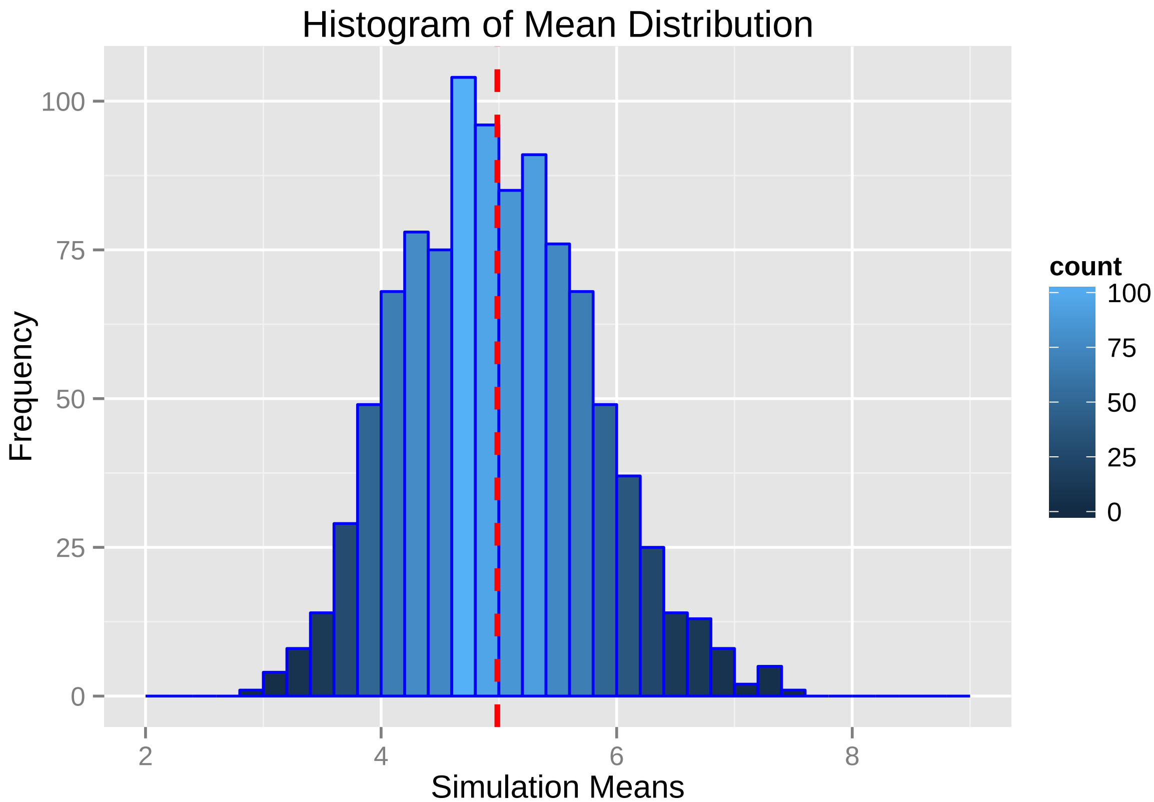
```
simulationMean <- rowMeans(simulationMatrix)
```

Create a data frame containing the whole data.

```
simulationData <- data.frame(cbind(simulationMatrix, simulationMean))
```

We plot the simulation data to visualize it.

```
ggplot(data = simulationData, aes(simulationData$simulationMean)) +
  geom_histogram(breaks = seq(2, 9, by = 0.2), col = "blue", aes(fill = ..count..)) +
  labs(title = "Histogram of Mean Distribution", x = "Simulation Means", y = "Frequency") +
  geom_vline(aes(xintercept=mean(simulationData$simulationMean)), color="red",
             linetype="dashed", size=1)
```



Sample Mean Versus Theoretical Mean:

The actual mean of the simulated mean sample data is 4.9866197, calculated by:

```
actualMean <- mean(simulationMean)
```

And the theoretical mean is 5, calculated by:

```
theoreticalMean <- (1 / lambda)
```

Thus, we can see that the actual mean of the simulated mean sample data is very close to the theoretical mean of original data distribution.

Sample Variance Versus Theoretical Variance:

The actual variance of the simulated mean sample data is 0.6257575, calculated by:

```
actualVariance <- var(simulationMean)
```

And the theoretical variance is 0.625, calculated by:

```
theoreticalVariance <- ((1 / lambda) ^ 2) / sampSize
```

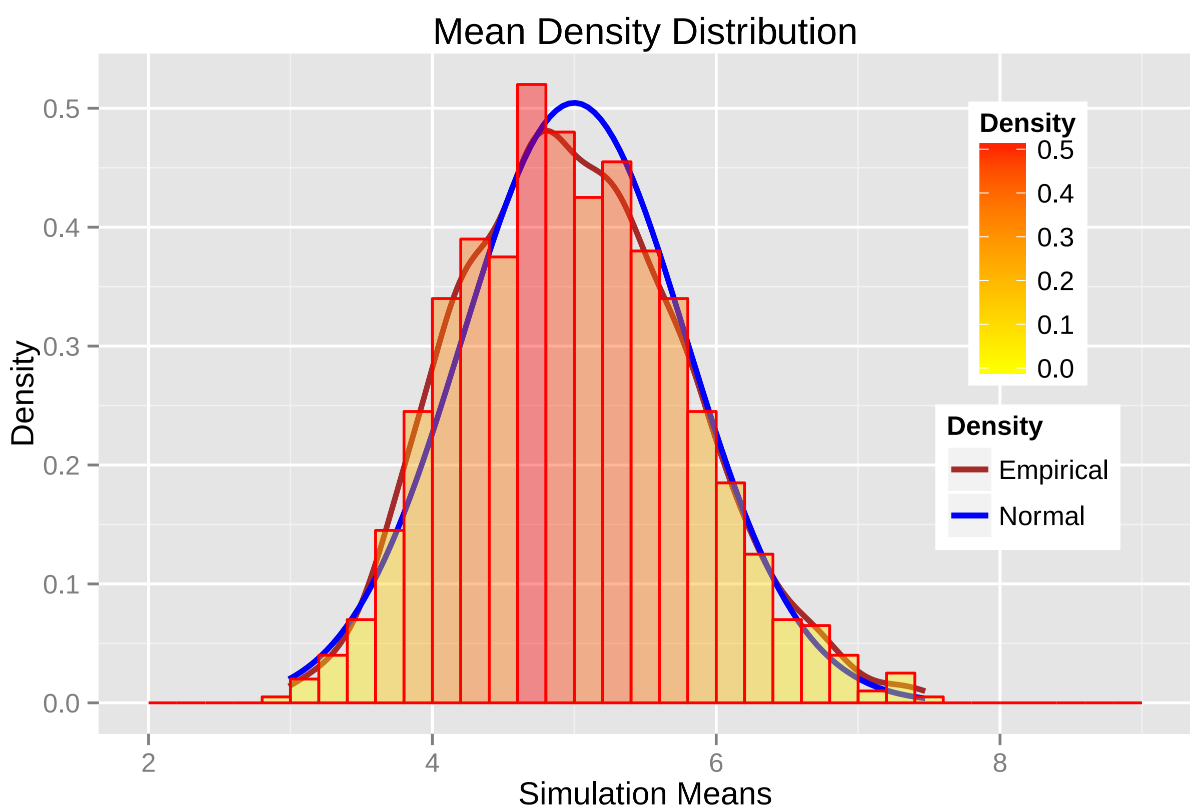
Thus, we can see that the actual variance of the simulated mean sample data is very close to the theoretical variance of original data distribution.

Distribution:

To prove that the simulated mean sample data approximately follows the Normal distribution, we perform the following three steps:

Step 1: Create an approximate normal distribution and see how the sample data alligns with it.

```
qplot(simulationMean, geom = 'blank') +
  geom_line(aes(y=..density.., colour='Empirical'), stat='density', size=1) +
  stat_function(fun=dnorm, args=list(mean=(1/lambda), sd=((1/lambda)/sqrt(sampSize))),
              aes(colour='Normal'), size=1) +
  geom_histogram(aes(y=..density.., fill=..density..), alpha=0.4,
               breaks = seq(2, 9, by = 0.2), col='red') +
  scale_fill_gradient("Density", low = "yellow", high = "red") +
  scale_color_manual(name='Density', values=c('brown', 'blue')) +
  theme(legend.position = c(0.85, 0.60)) +
  labs(title = "Mean Density Distribution", x = "Simulation Means", y = "Density")
```



From above histogram, the simulated mean sample data can be adequately approximated with the normal distribution.

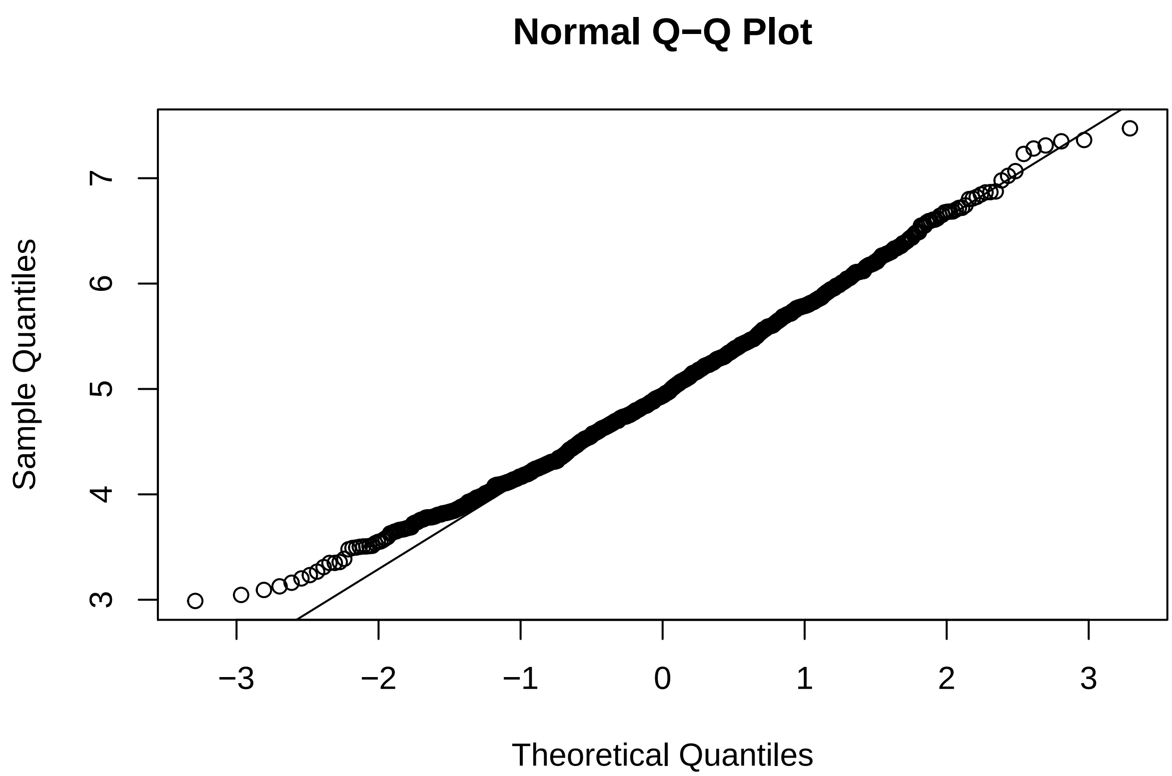
Step 2: Compare the 95% confidence intervals of the simulated mean sample data and the theoretical normally distributed data.

```
actualConfInterval <- actualMean+c(-1,1)*1.96*sqrt(actualVariance)/sqrt(sampSize)
theoreticalConfInterval <- theoreticalMean+c(-1,1)*1.96*
  sqrt(theoreticalVariance)/sqrt(sampSize)
```

Actual 95% confidence interval is [4.7414712, 5.2317681] and Theoretical 95% confidence interval is [4.755, 5.245] and we see that both of them are approximately same.

Step 3: q-q Plot for Qunatiles.

```
qqnorm(simulationMean)
qqline(simulationMean)
```



The actual quantiles also closely match the theoretical quantiles, hence the above three steps prove that the distribution is approximately normal.