P for POWER

Statistical Inference

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R4.0 Environment

library(knitr) # creating slides

library(ggplot2) # making plots

library(reshape2) # handling data frames

:: if $\alpha = 5\%$

:: $if\ H_0$ is $\mu_0 = 30$

:: if H_a is $\mu_a > 30$

CASE: Respiratory Distress Index and Sleep Disturbances

Central Theory Limit

Where (X'-30)/($s/\sqrt{(n)}$)measures the number of standard errors the sample mean is from the mean hypothesized by H_0 and the denominator

($s/\sqrt{(n)}$ (is the standard error of the sample mean)

PS: if H_a specified that $\mu_a > \mu_0$

- :: flip the following reasoning
  - :: look at the right tail

POWER

Power is the probability of rejecting the NULL HYPOTHESIS H_0 ,when it is false.

:: Used to determine if your sample size was big enough to yield a meaningful, rather than random result

:: Detect if your ALTERNATIVE HYPOTHESIS, H_a is true, to lower the risk of a Type II errors.

Equation

As beta, β, is the probability of a *Type II error,* for accepting a false null hypothesis, then the complement of this the power is (1-β).

$$P = (1 - \beta)$$

Do you remember this … ?

:: As μ_a gets bigger, the test gets more powerful

:: As n gets bigger, the test gets more powerful

:: Power decreases with increases in variation

:: As α increases, power increases

:: In a one sided test the power is greater as α is bigger than $\alpha/2$

Alpha

Power is the probability that the true mean μ is greater than the (1-α) quantile, in our sleep example:

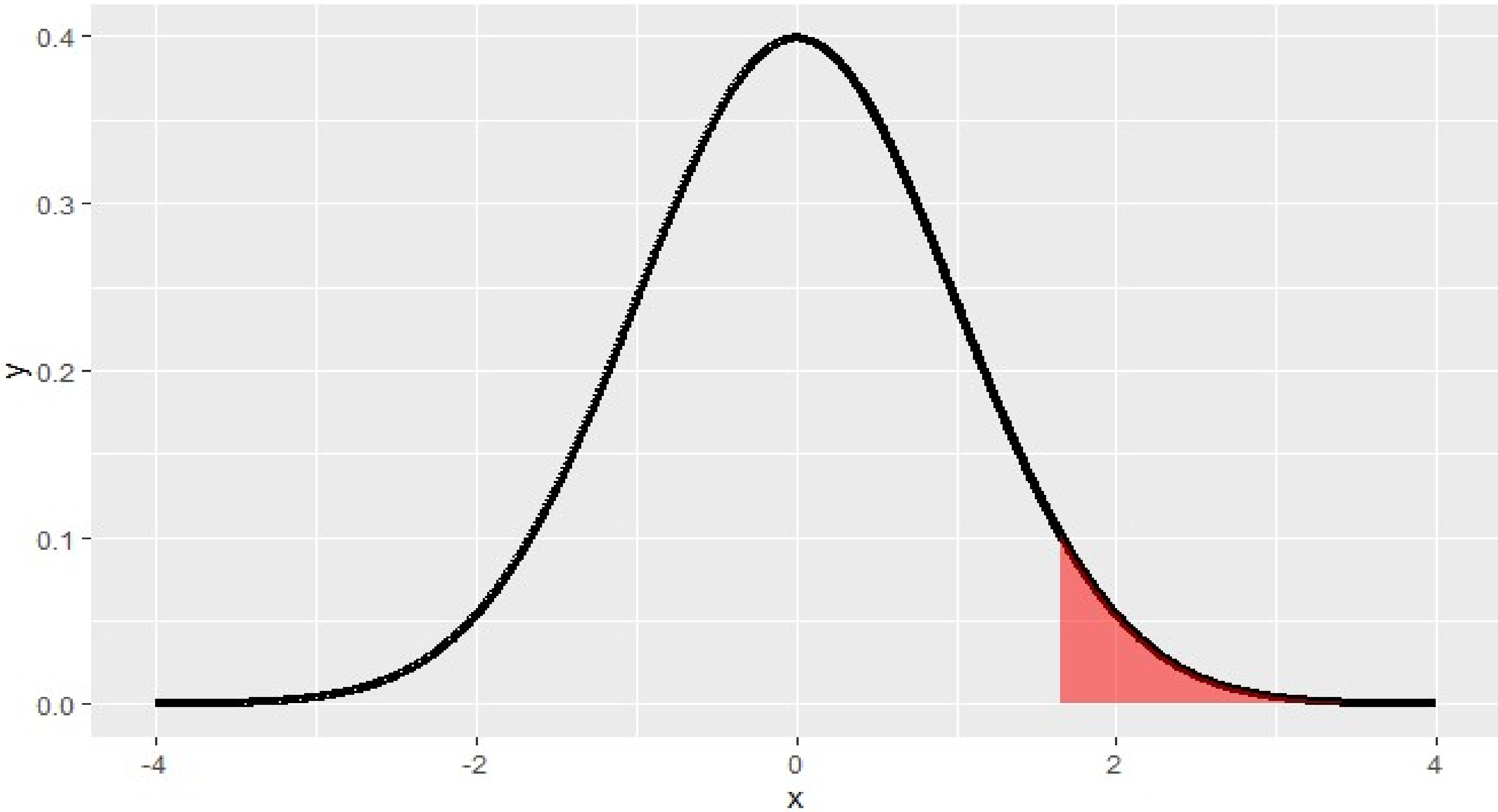
:: if α = .05 and as H_a μ_a30 <

:: for normal distributions of which we know the variances

:: we make p = qnorm(.95) our reference

When to reject H_0 ?

If a test statistic fell in the shaded portion, 5% of the area under the curve, we would reject H_0 in favor H_a



Two distributions

The two hypotheses, H_0 and H_a ,represent two distributions

:: since they're talking about means or

:: centers of distributions

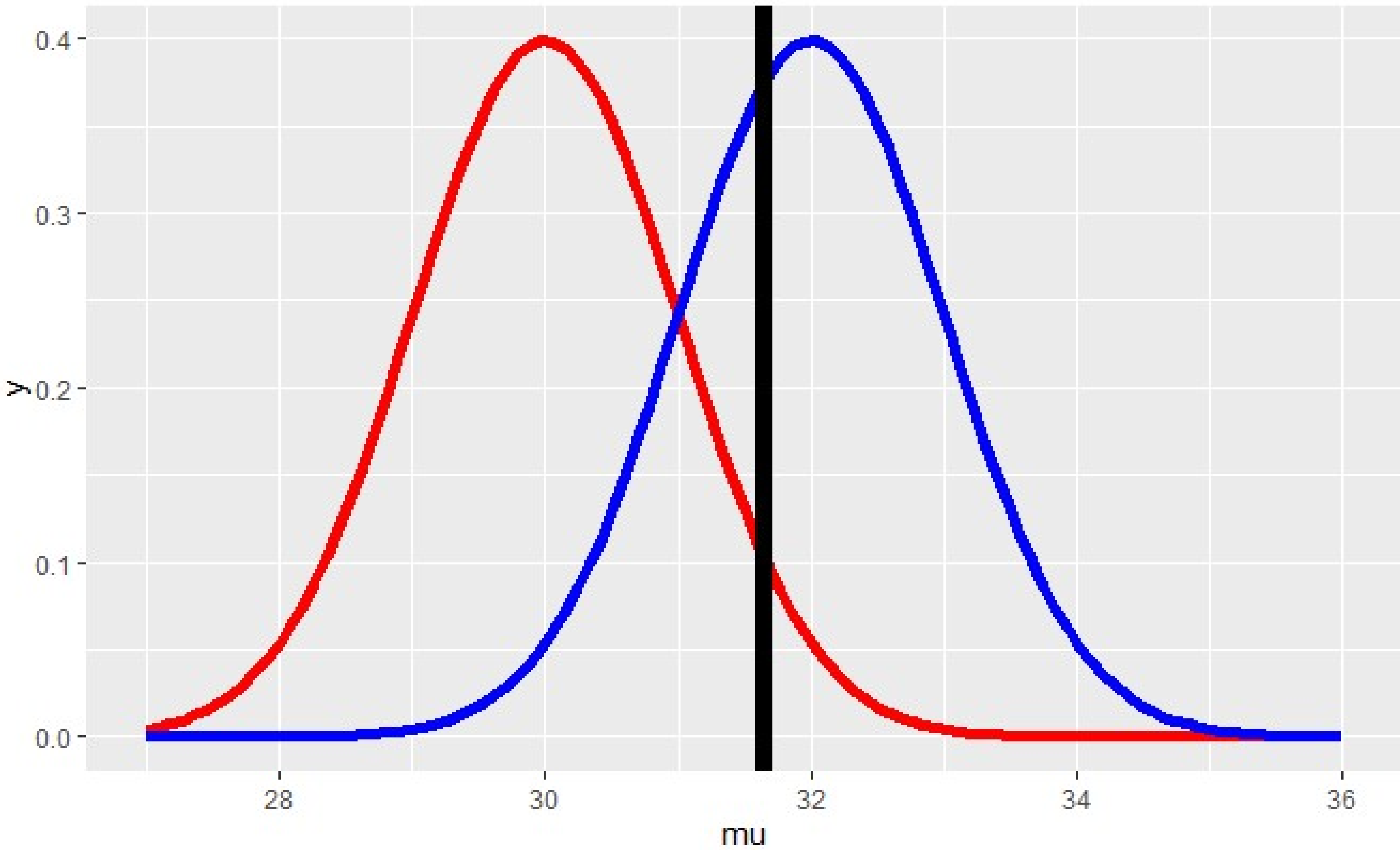
Normal distributed

For a *random variable* X which is distributed as *Normal* with a mean *mu*, μ and variance *sigma squared*, σ^2

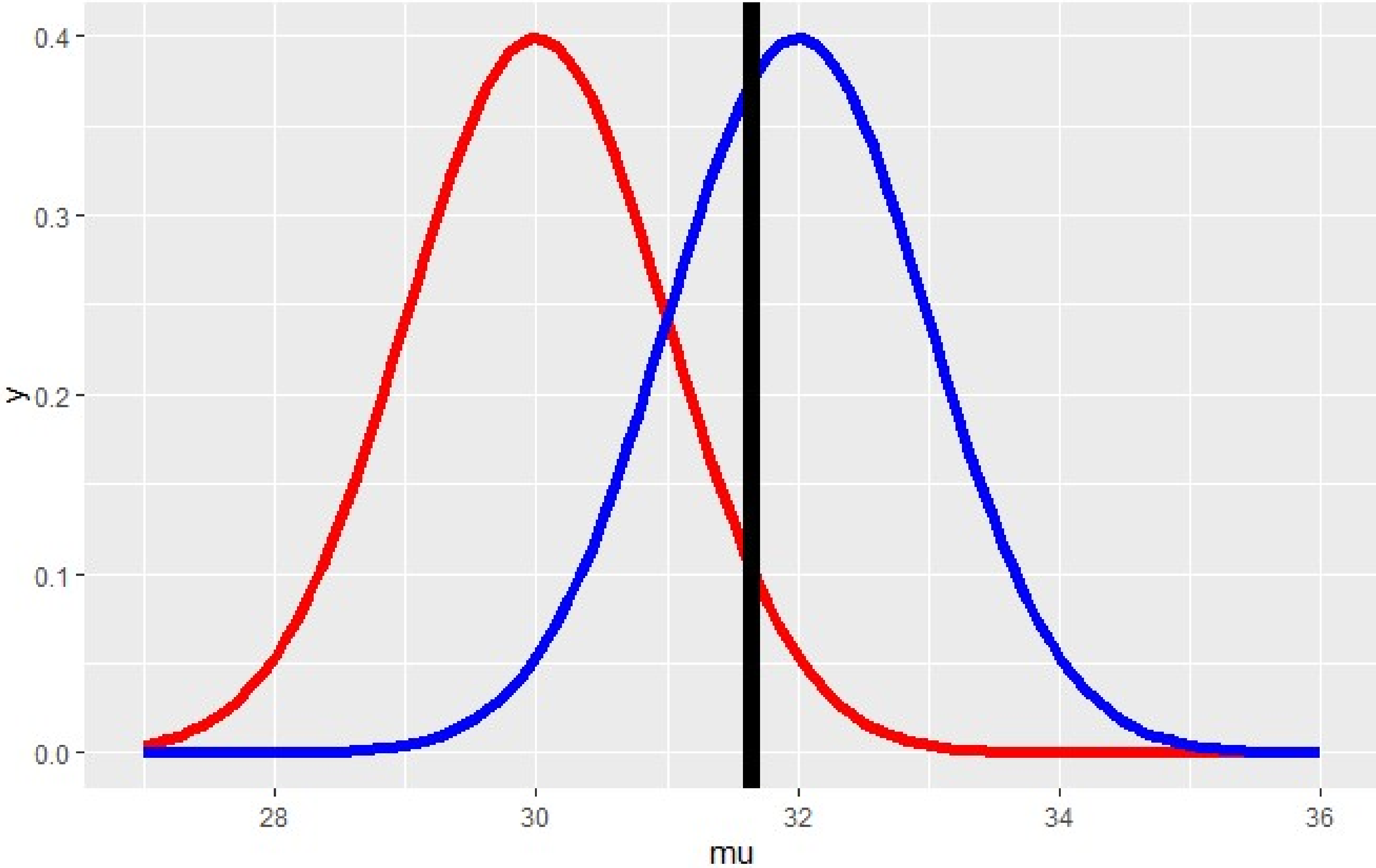
:: under H_0 ,X' is N(μ_0 , σ^2/n)

:: under H_a ,X' is N(μ_a , σ^2/n)

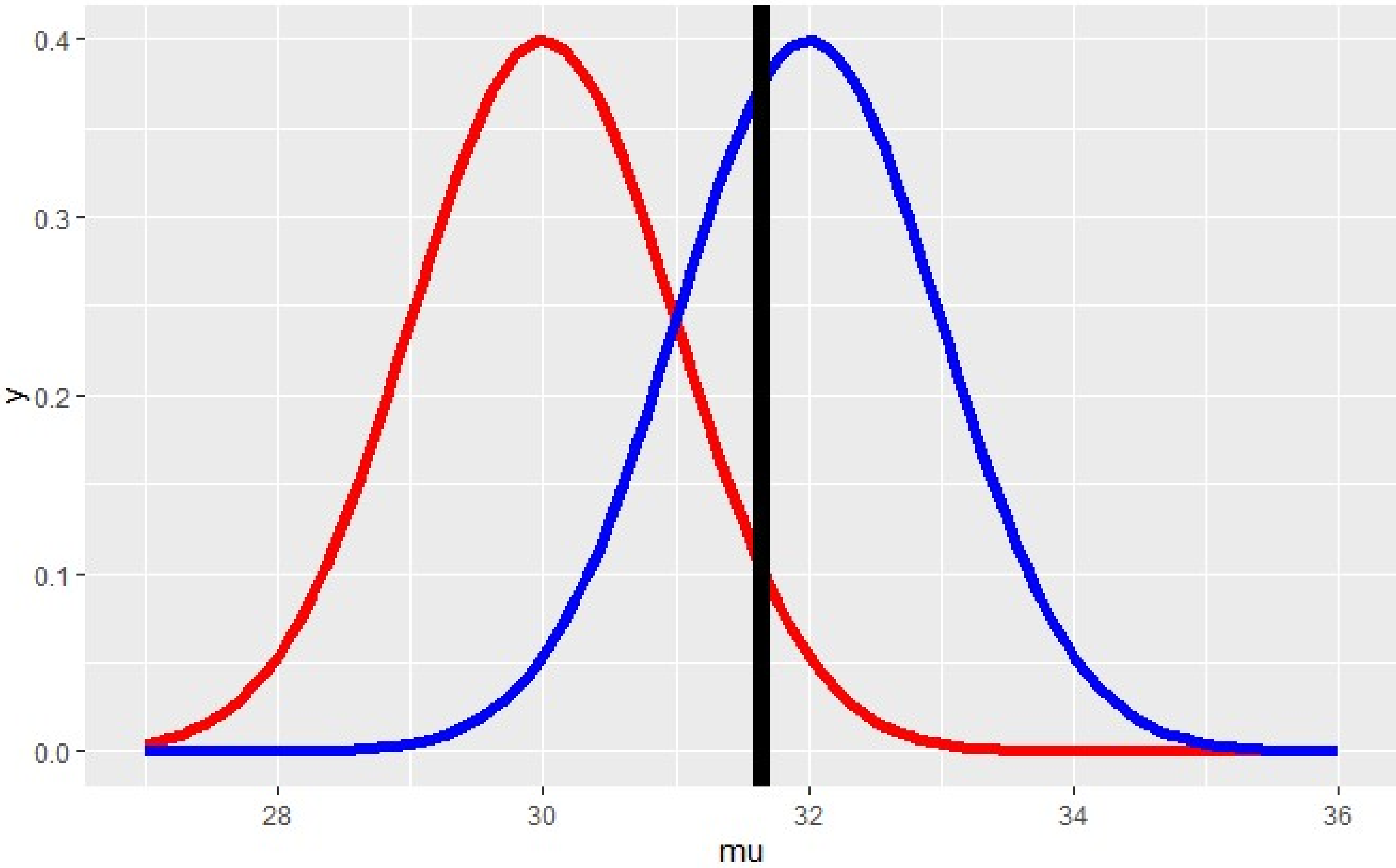
95th percentile of H_0, black vertical on the red plot



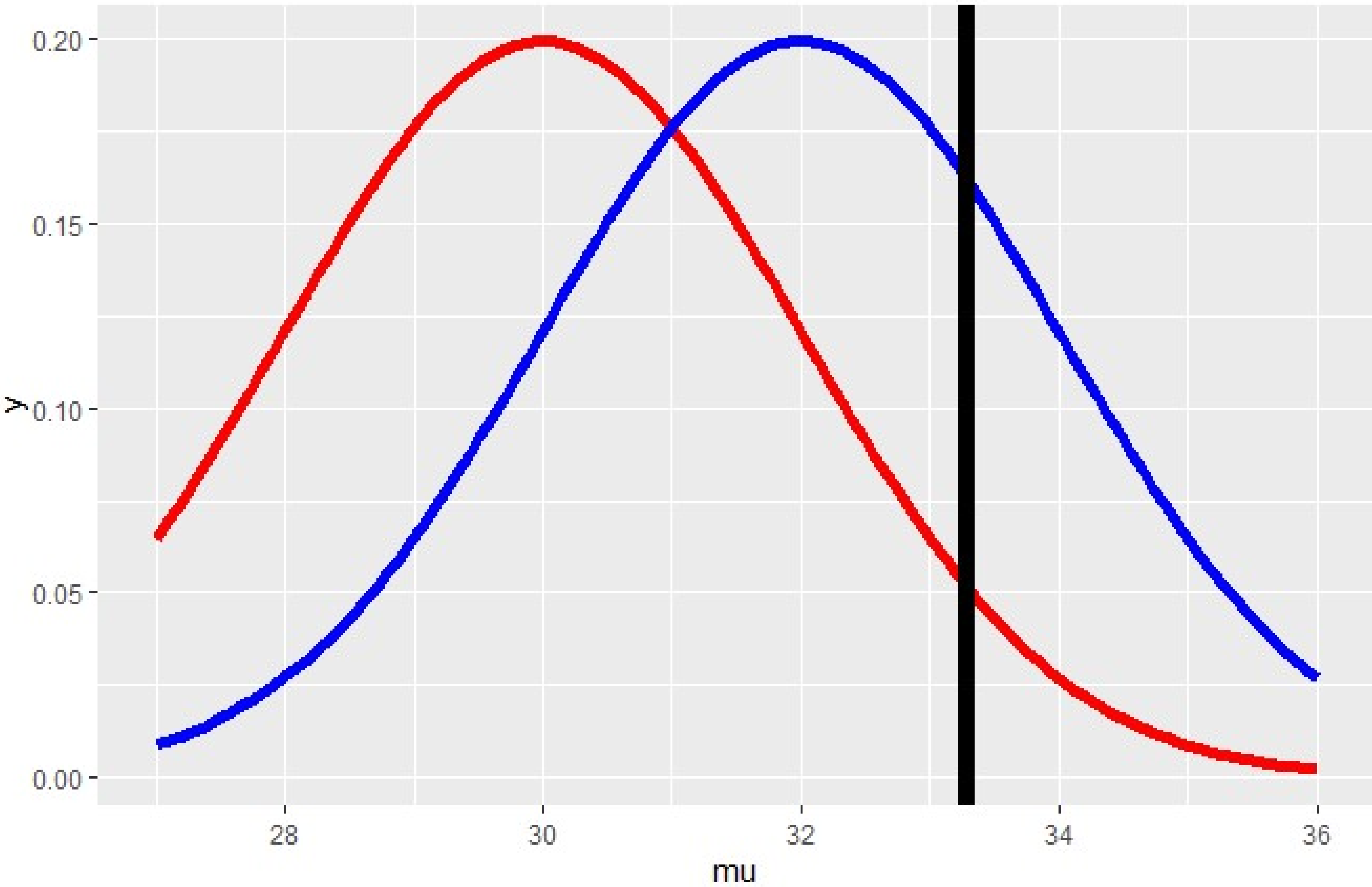
Mean proposed by H_a, peak of blue plot



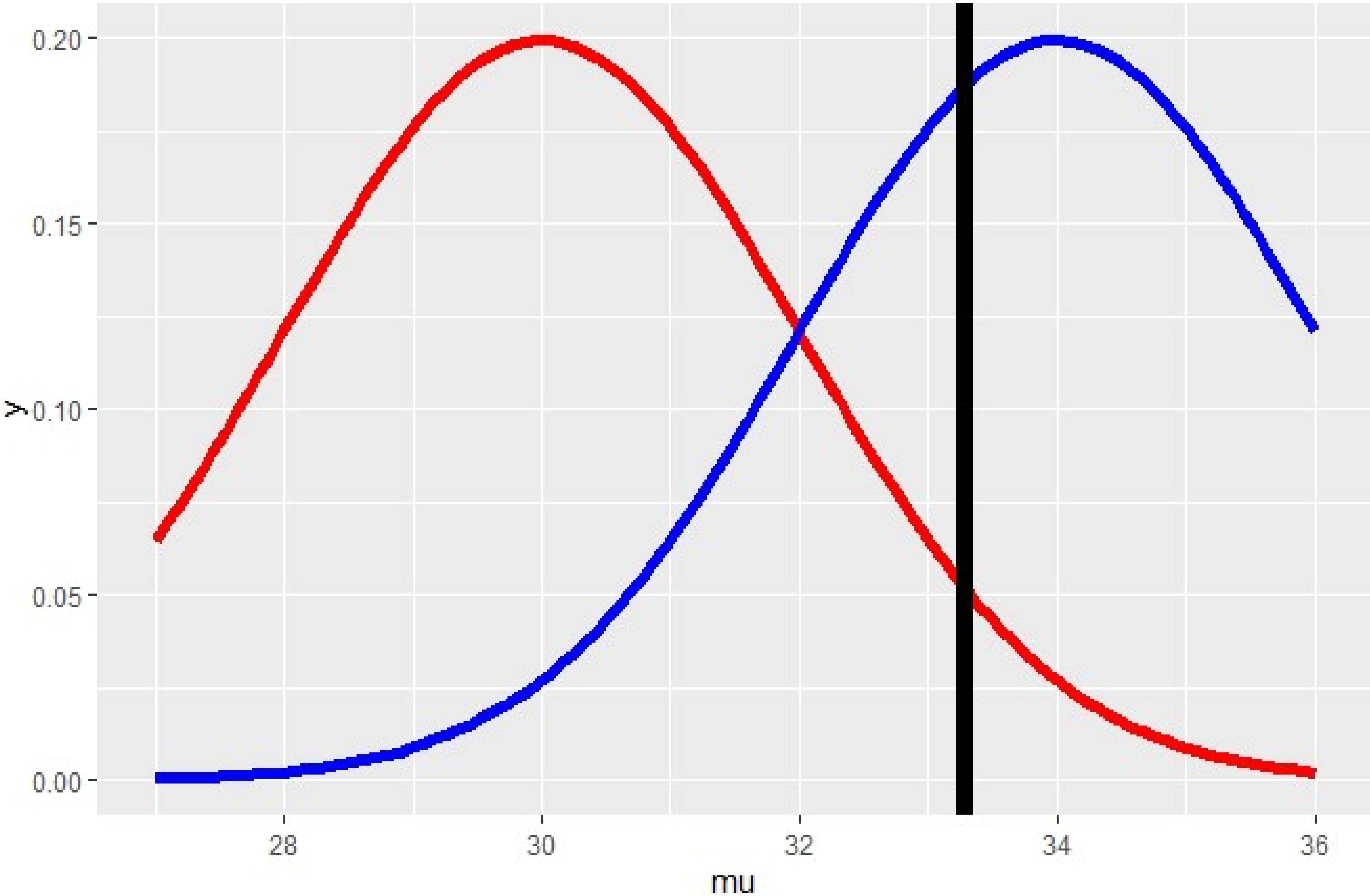
Power is how far H_a is from the right of 95th percentile of H_0



Power depends on the null distribution's variance



If μ_a = 34 > μ_0 the test **,H_a is more powerful than H_0**

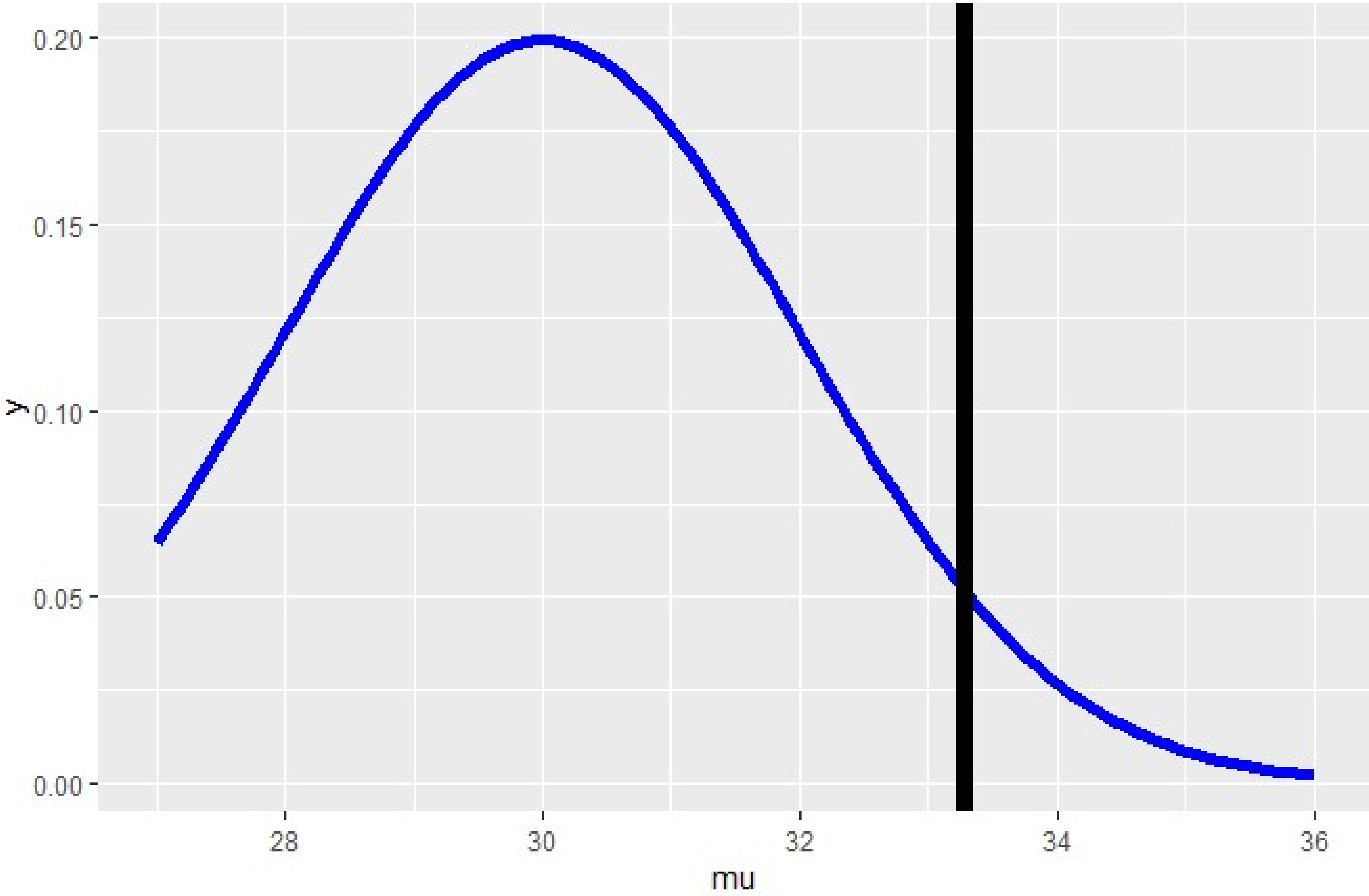


If power is large, we can reject the null hypothesis

The distribution represented by H_a moved to the right, most of the blue curve is to the right of the vertical line, indicating that with μ_a ,34 = the test is more powerful, so it is

:: correct to reject the H_0 as it appears to be false.

If μ_a = 30 = μ_0 the power is at α



If power is similar or equal to the 95th percentile of H_0, we cannot reject the null hypothesis

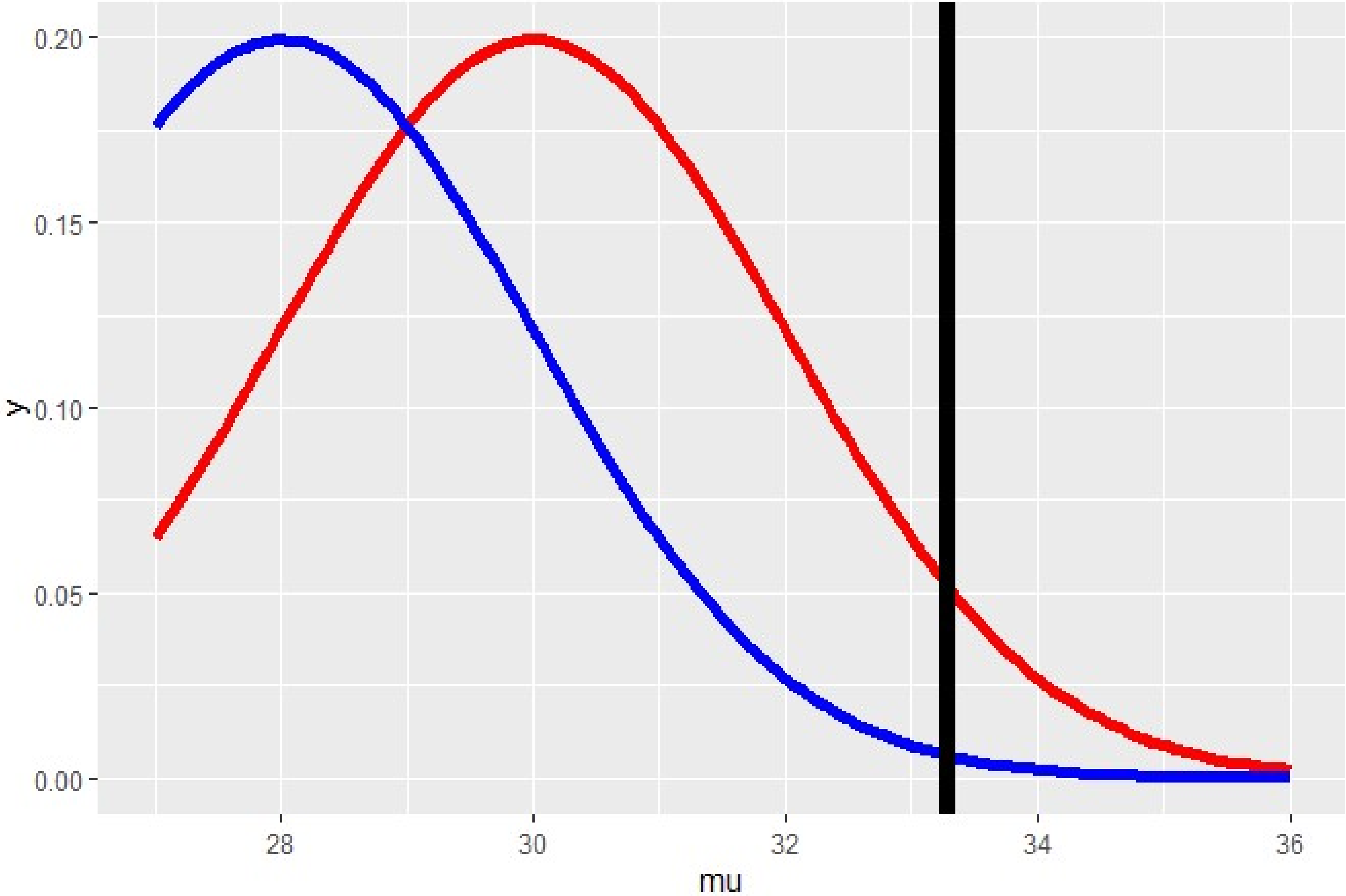
The distribution represented by H_a ,in the above graph moved under the blue curve, indicating that with

:: μ_a = 30 = μ_0 , the test,

:: H_a is almost as powerful as H_0 ,so it is

:: incorrect to reject the null hypothesis since it does not appear to be false.

If μ_a > 28 = μ_0 the test's power is weaker



Not worth investigating

The distribution represented by H_a will move to the left of μ_0 ,30 = the area under the blue curve is less than the 5% our α, so the test is not only less powerful,
it even contradicts H_a

∷ it is therefore not worth looking into.

RECAP

Power is a function that depends on a specific value of an alternative mean, μ_a ,which is any value greater than μ_0, the mean hypothesized by H_0

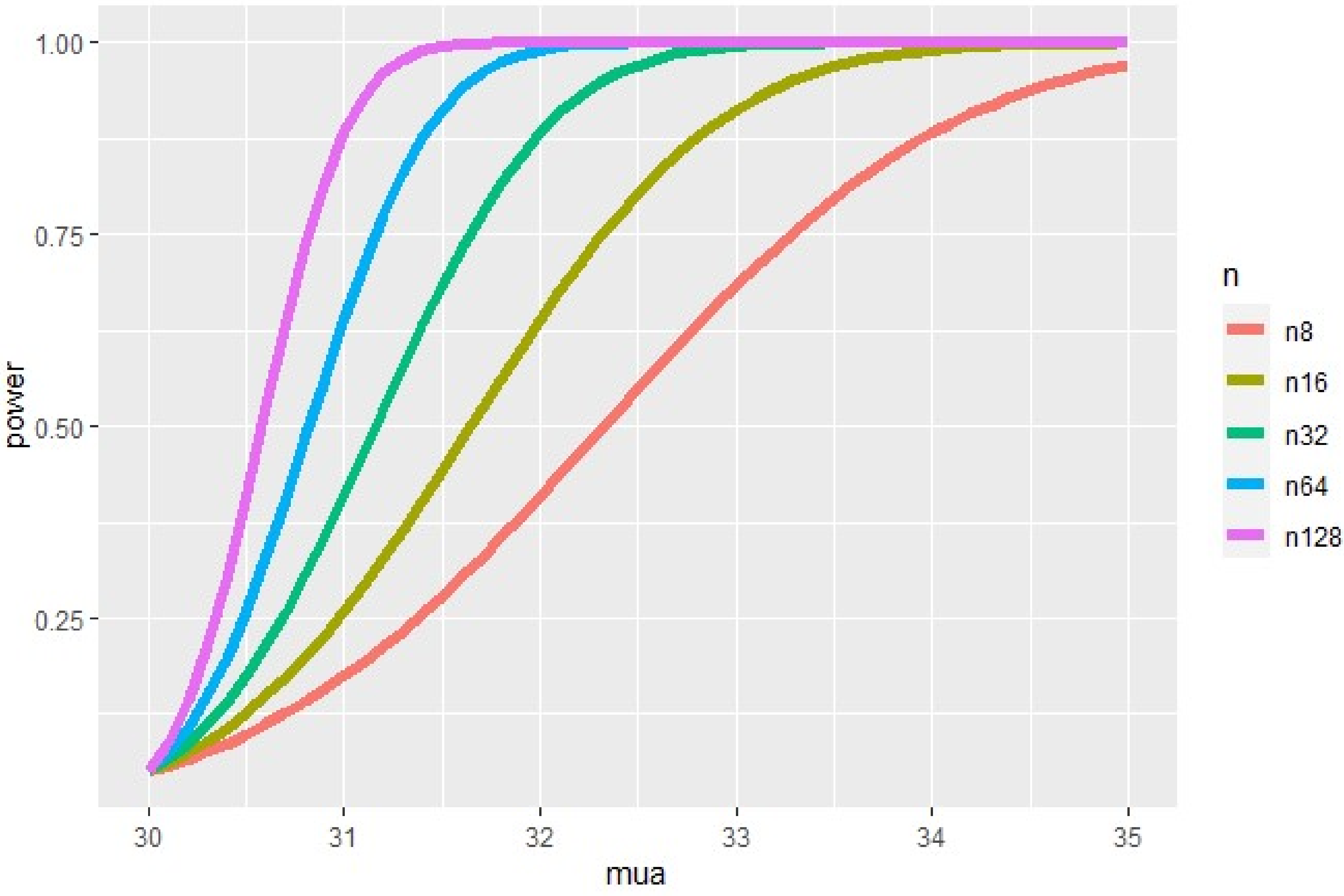
∷ Recall that H_a specified μ > 30, in the sleep case.

RECAP

If μ_a is much bigger than μ_0 30 = then the power, a probability, is bigger than if μ_a is close to 30.

:: As μ_a approaches 30, the mean under H_0 ,the power approaches α.

Power Curves n



z <- qnorm(.95)

```r
# mean=30

z <- qnorm(.95)

pnorm(30+z,mean=30,lower.tail=FALSE)
## [1] 0.05
```

With the mean set to μ_0 the two distributions, null and alternative, are the same and power = α = 5%.

$\mu_a < \mu_0$

```r
#mean=32

z <- qnorm(.95)

pnorm(30+z, mean=32,lower.tail=FALSE)
## [1] 0.63876
```

With $\mu_a < \mu_0$ the power is greater than α, at 64%.

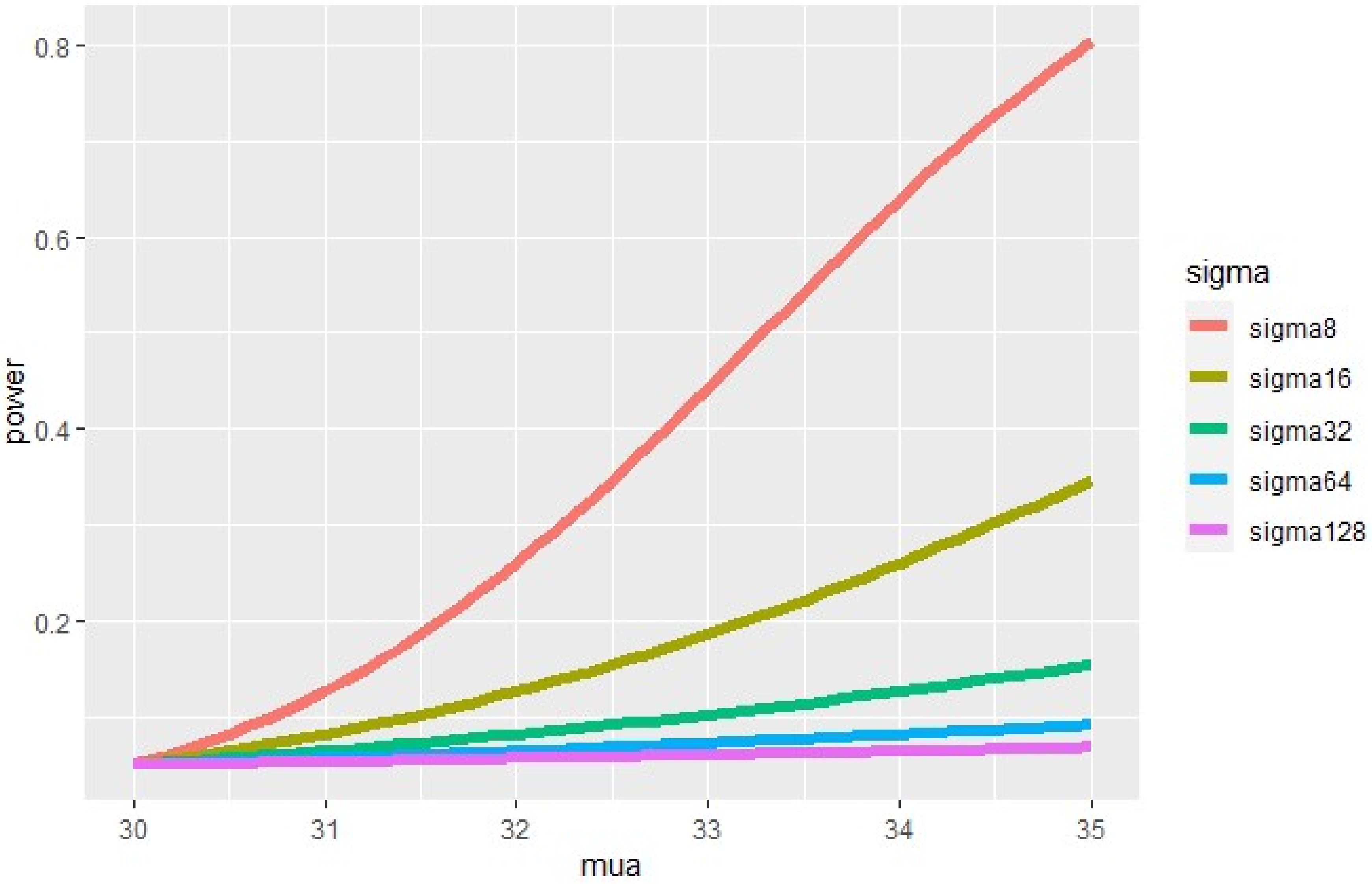
:: When the sample mean is many standard errors greater than the mean hypothesized by the null hypothesis,
:: the probability of rejecting H_0 is false is much higher.

Standard deviation, sd

```
# sd=1

z <- qnorm(.95)

pnorm(30+z, mean=32, sd= 1, lower.tail=FALSE)
## [1] 0.63876
# sd=2

z <- qnorm(.95)

pnorm(30+z, mean=32, sd= 2, lower.tail=FALSE)
## [1] 0.5704709
```

:: Power decreases with increases in variation

Power Curves, σ

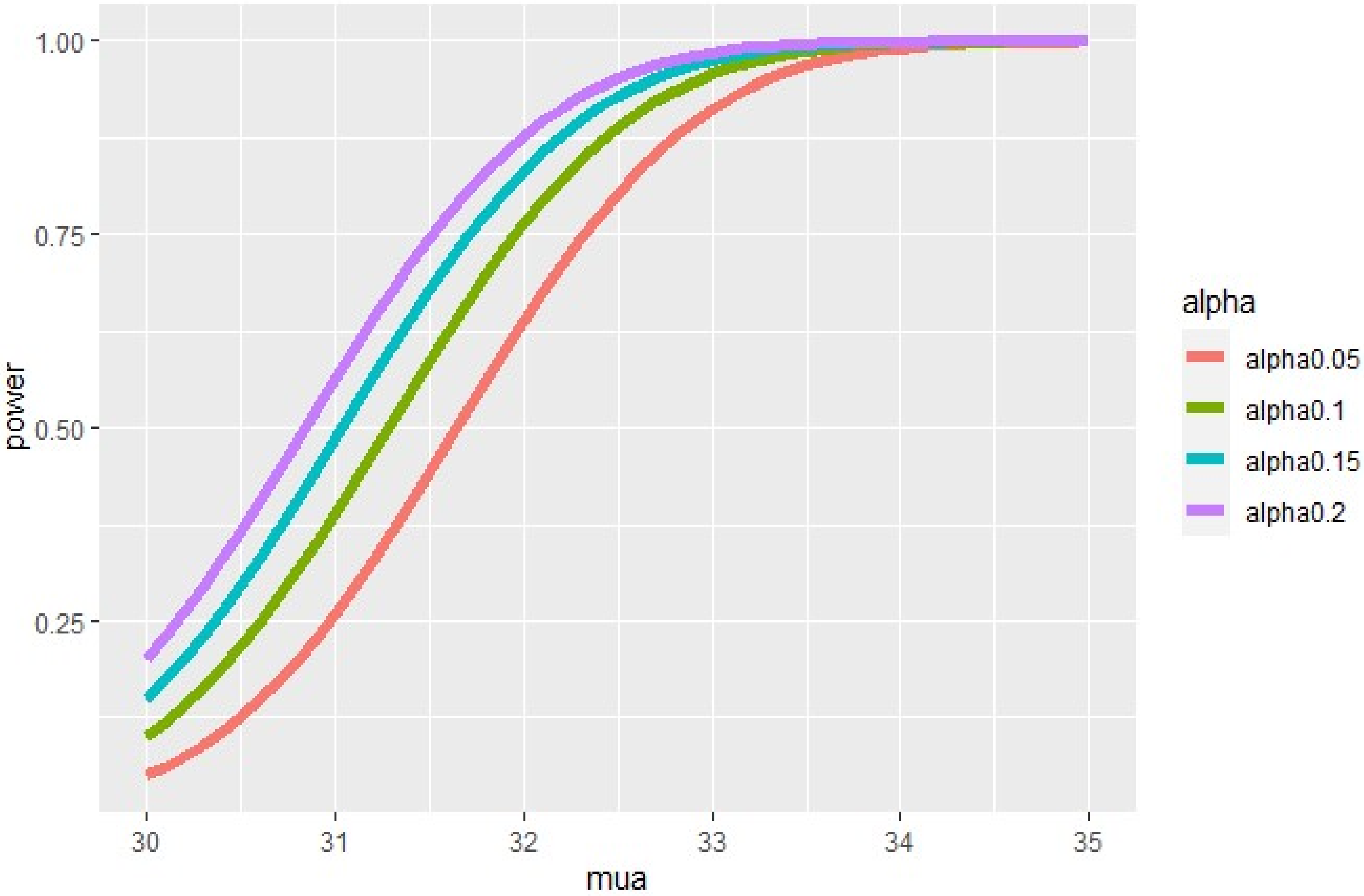


RECAP

:: As α increases, power increases

:: In a one sided test the power is greater as α is bigger than $\alpha/2$

Power Curves, α



POWERful RECAP

:: As μ_a gets bigger, the test gets more **powerful**

:: As n gets bigger, the test gets more powerful

:: Power decreases with increases in variation

:: As α increases, power increases, so

:: A one sided test has greater power

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Statistical Inference

myCOURSERA notes:

plots from swirl