

Exponential Distribution Simulation Exercise

Statistical Inference

Linda Angulo Lopez

01/01/2021

key words: Asymptotic, The Central Limit Theorem, Sample Mean vs Theoretical Mean, Sample Variance vs Theoretical Variance, Distributions, Quantiles, Statistical Inference.

Introduction

The project consists of two parts (i) a simulation exercise and a (ii) basic inferential data analysis, the former is presented here. Asymptotics form the basis for frequency interpretation of probabilities, where the behavior of statistics depends on the sample size or some other relevant quantity of limits to infinity or to zero, the Swiss Army Knives of Statistics.

An exponential distribution is the probability distribution that describes the time between events in a Poisson process, i.e. a process in which events occur continuously and independently at a constant average rate. A thousand simulations, our samples, each of $n = 40$ were run to investigate if the sampling distribution of exponential distribution has a normal distribution with a mean that matches the population mean and a variance that matches the theoretical result. It was found that for the exponential distribution generated the true mean that matches the population mean and a variance that matches the theoretical result in addition the distributions have similar means at the quantiles: 5%, 25%, 50% , 75% and 95%.

```
#R4.0 Environmental Set for:  
library(knitr) # creating a pdf document ;  
library(ggplot2) # making plots  
library(dplyr) #exploring data  
library(DataExplorer) # creating reports
```

Exponential Distribution Simulation:

A discreet case of the CLT, 1000 trials were simulated in with the R function `rexp(n, lambda)` where λ , was set to 0.2, this is the rate parameter. The mean & the standard deviation of an exponential distribution is $1/\lambda$, for $n = 40$. Data was then explored with `glimpse()` and `summary()`.

create, explore & plot data A pseudo-random number generator was used so that the exercise is replicable,data was explored with the dplyr and DataExplorer libraries from CRAN.

Here is a glimpse of the data:Data _SM1, and a basic data summary:

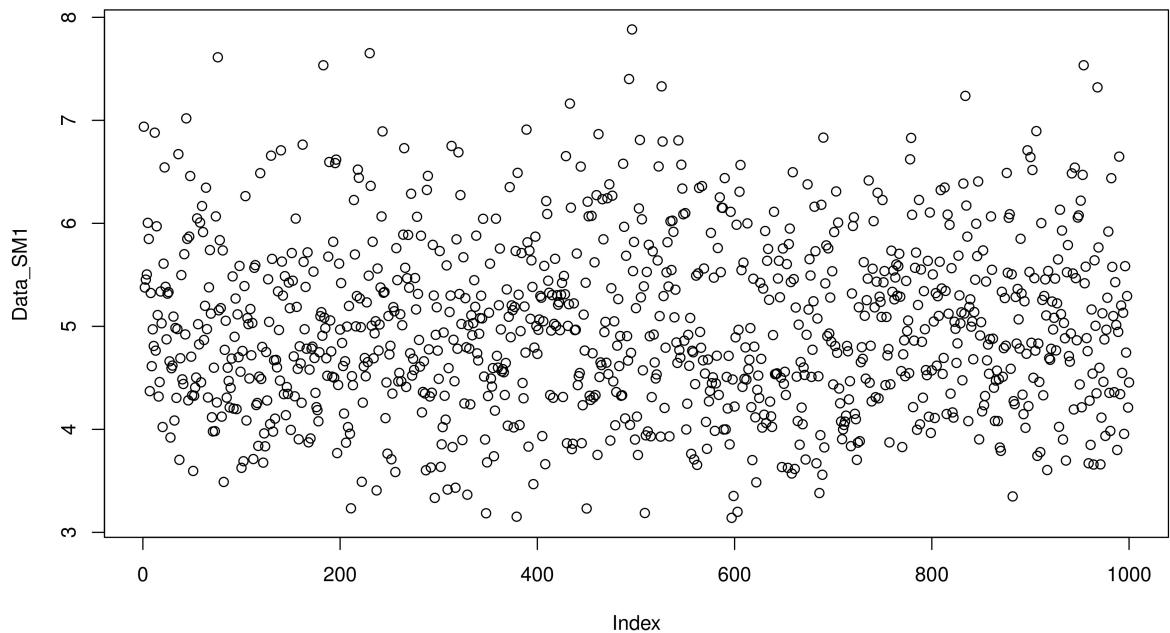
```
set.seed(42); lambda <- 0.2; nosim <- 1000; n <- 40 ;  
SimulationMatrix_1 <- matrix(rexp(nosim * n, rate = lambda), nosim);  
Data_SM1 <- apply(SimulationMatrix_1, 1, mean); glimpse(Data_SM1); summary(Data_SM1)
```

```

##  num [1:1000] 6.94 5.38 5.45 5.5 6 ...
##    Min. 1st Qu. Median     Mean 3rd Qu.    Max.
##  3.141   4.406   4.919   4.987   5.504   7.882

```

Here is a plot of the data: Data_SM1



Mean vs True Mean We can get the mean of the data with `mean()` and the theoretical mean is $1/\lambda$. We see that The mean of the data, the true mean is close to the theoretical mean:

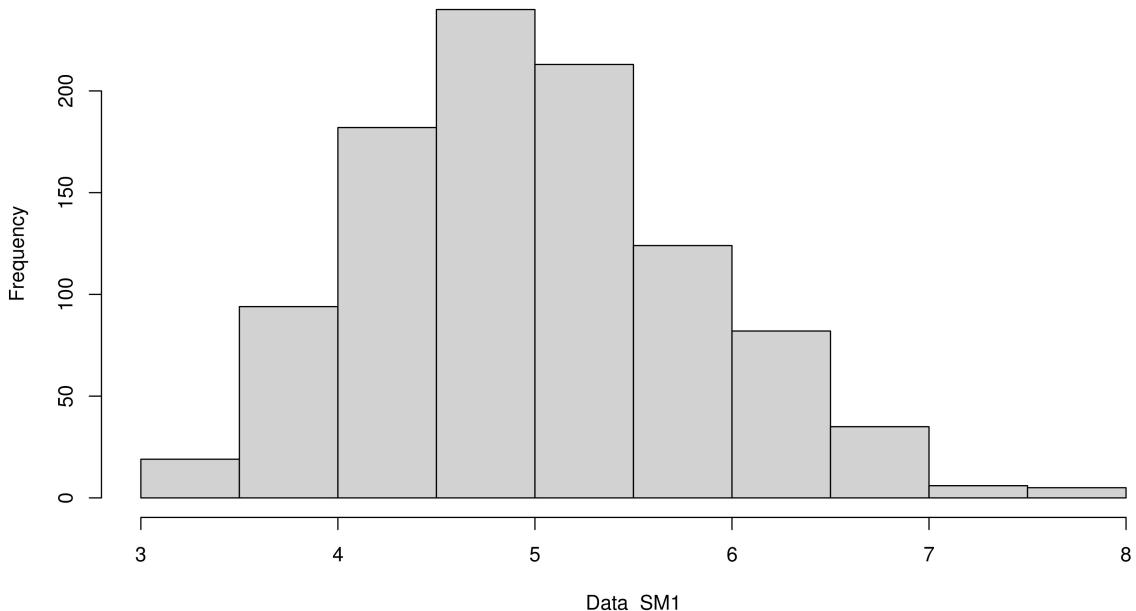
```

## [1] 4.986508
## [1] 5

```

After taking a peak at the report generated with `create_report()`, we see that the histogram has a bell shaped curve it looks like a normal distribution.

Histogram of Data_SM1



True variance vs Theoretical variance We can get the variance of the data by using the result of the `sd()` function squared and the theoretical value is $\sigma^2 n = 1/\lambda^2 n$. We see that The variance of the data is close to the theoretical variance:

```
Variance_Data <- sd(Data_SM1)^2 ; Variance_Data; TheoVar <- 1/((lambda^2)*n); TheoVar
## [1] 0.6793521
## [1] 0.625
```

Comparing Distributions The easiest way to compare the data vs theoretical distributions would be to use the `qnorm()` function to get the normal distributions and `quantile()` to get the quantiles of the data then compare the true vs theoretical values. We see that the distribution of quantiles, for the data is close to the theoretical quantiles:

```
Quantiles_DatavsTheo <-c(0.05, 0.25, 0.5, 0.75, 0.95); quantile(Data_SM1, Quantiles_DatavsTheo);
##      5%      25%      50%      75%      95%
## 3.750774 4.406170 4.919282 5.504413 6.485176
qnorm(Quantiles_DatavsTheo, mean = mean(Data_SM1), sd = sd(Data_SM1))
## [1] 3.630774 4.430575 4.986508 5.542442 6.342243
```

After-note:

- This is an R Markdown document, created as a submission for the Statistical Inference Course by Johns Hopkins University on Coursera.
- Hidden code is available here,