Questions

Answer the following questions.

1. This is a question about binary min-heaps.

1. Given binary min-heap A = [6, 9, 17, 63, 10, 19, 79, 87, 86, 84, 53, 31, 92] do the following
   (a) Draw the tree representation. [3 marks]
   (b) Show the steps involved when inserting the element 8. [6 marks]
   (c) Show the steps involved when deleting the smallest element from A (note that we are deleting from the *original* heap A, not from the outcome of question 1b). [6 marks]
2. Given the array [37, 97, 60, 27, 92, 4, 95, 14, 44, 62, 38, 13, 59], create a binary min-heap using the procedure MakeHeap from the lectures. Give a sequence of arrays highlighting the two elements that get swapped in the array and draw a tree representation of the final heap. [10 marks]

2. You are given a map datastructure that contains a mapping from student names (or ids) to the sets of books the students need to buy. That is, a map `books_to_buy` has keys that are student ids, and values that are *sets* of books, with one (possibly empty) set of books for each student.

The university administration orders the books, and must distribute the books to the students. For this, you need a datastructure called `book_for_students` that is a map from each book to the set of students who need a copy of it.

Give pseudocode for converting the Map `books_to_buy` to the map `book_for_students`. [10 marks]

3. This is a question about hashing and string-matching.

1. Describe the naive algorithm for string-matching. State its worst-case running time when looking for a pattern of length m in a string of length n. Outline how the Rabin-Karp (rolling hash) algorithm achieves an improvement on this. [10 marks]
2. How would you extend the Rabin-Karp algorithm to compute the set of different strings of length m that occur in a string of length n? Give pseudocode. For example, the string `gggggggttttt` contains only the following substrings of length 3: `ggg`, `ggt`, `gtt`, and `ttt`. [5 marks]

4. Consider the graph Γ in Figure 1.

1. Illustrate the Kruskal's algorithm on the graph Γ. In addition show how Union-Find data structure changes throughout the algorithm. [15 marks]

2. Using Kruskal's algoritm, determine how many minimum spanning trees (MSTs) the graph G. [10 marks]

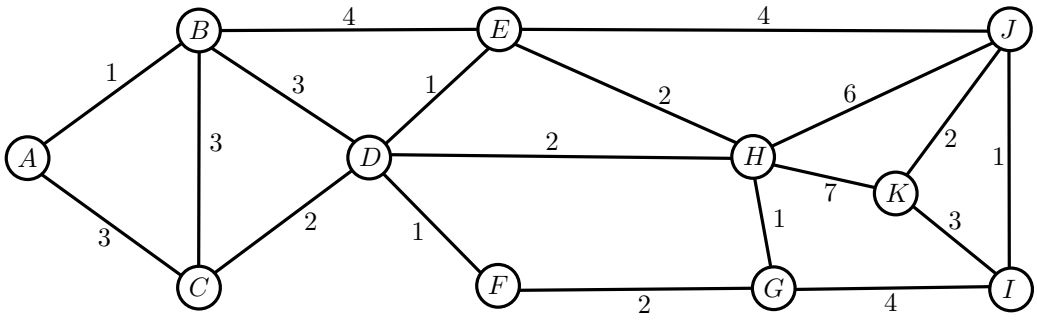

Figure 1: The graph Γ from question 3.

5. Suggest an algorithm for a search engine that lets you plan travel connections between cities. Its input is a list of possible connections, where each connection is given as a combination of a starting time, an ending time, a starting location and a destination location. (For example, one such connection could be "a train departing from London at 08:48, arriving in Manchester 10:55".)

1. Describe an algorithm for finding a fastest journey from one city to another, with a given starting time. Note that a journey can consist of many connections. [15 marks]

2. Explain how to adjust your algorithm to accommodate minimum *changeover times*, e.g., when switching from one train to another at least a 20-minute margin should be given, and when switching from one plane to another a margin of at least one hour is required. [10 marks]