MTH 309 FINAL EXAM MAY 16TH

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Please show **all** your work! Answers without supporting work will not be given credit. Clearly label your problems and work. Have fun!

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1. (24 points) Given the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

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For each expression given below if it is defined, compute it. If it not defined, give a reason why.

a) ABA b) AB^T c) $AC+C$ d)A^TB e)A^{-1}

a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}_{3\times3} \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 1 & 0 \end{bmatrix}_{3\times2} = AB = \begin{bmatrix} 1 & 5 \\ 7 & 8 \\ 11 & 11 \end{bmatrix}_{3\times2}$

$3+8$

$ABA = \begin{bmatrix} 1 & 5 \\ 7 & 8 \\ 11 & 11 \end{bmatrix}_{3\times2} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}_{3\times3}$

Not defined b/c AB column $=2 \neq$ A row 3

b) $B^T = \begin{bmatrix} 3 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}_{2\times3}$ $\quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}_{3\times3} \begin{bmatrix} 3 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}_{2\times3}$

Not defined b/c A column $=3 \neq B^T$ row $=2$

d) $A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$ $\quad A^TB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}_{3\times3} \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 1 & 0 \end{bmatrix}_{3\times2} = \begin{bmatrix} 4 & 5 \\ 7 & 8 \\ 11 & 11 \end{bmatrix}_{3\times2}$

c) $AC = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}_{3\times3} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{3\times1}$

continue on back →

$AC = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $AC + C$

$C = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ $AC + C = \underset{3\times1}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} + \underset{3\times1}{\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

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e) A^{-1} $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 3 & 4 & 6 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & -2 & -3 & -3 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{-1R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - 2R_3}$$

$-2 - 2(-1)$
$-2 + 2$

$-3 - 2(-2)$
$-3 + 4 = 1$

$-3 - 2(-2)$
$-3 + 4$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \xrightarrow[\cancel{R_1 - 2R_2}]{R_1 - 3R_3}$$

$A^{-1} = \begin{bmatrix} -2 & 2 & 1 \\ 0 & -1 & -2 \\ 1 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -2 & 0 & -3 \\ 0 & 1 & 0 & 0 & -1 & -2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$-3 - 2(-2)$
$-3 + 4$

$$\xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 1 \\ 0 & 1 & 0 & 0 & -1 & -2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

2. (20 points) Let D be a matrix given as follows.

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$$D = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

3x3

−3

(a) Find a basis for the Column space of D.

+ (b) Find a basis for the Null Space of D.

+ (c) What is the rank of D?

(a) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 4 & 6 & 0 \\ 3 & 6 & 9 & 0 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 + 2x_2 + 3x_3 = 0$

$x_2 = x_2$

$x_3 = x_3$

$x_1 = -2x_2 - 3x_3$

$\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

$Col(D) = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \right\}$

(b) $Nul(D) = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

c) $rank(D) = 3$

Cont.

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3. (18 points) Consider the following vectors in $\mathbb{R}^3$:

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{b}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{b}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \vec{u} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\vec{u} = c_1\vec{b}_1 + c_2\vec{b}_2 + c_3\vec{b}_3$

The set $\beta = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ is a basis for $\mathbb{R}^3$.

$[\vec{u}]_\beta = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

(a) Compute $[\vec{u}]_\beta$, the coordinate vector of $\vec{u}$ relative to the basis β.

(b) Let $\vec{w} \in \mathbb{R}^3$ be a vector such that $[\vec{w}]_\beta = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$. Compute the vector $\vec{w}$.

a) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 0 & -1 & 1 \\ 1 & 1 & 0 & 0 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & -3 & 1 \\ 0 & 0 & -1 & 0 \end{array}\right] \xrightarrow{-1R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & -3 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$-1-2(1)$

$\xrightarrow{R_2 + 3R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad [\vec{u}]_\beta = \begin{bmatrix} 1/2 \\ -1/2 \\ 0 \end{bmatrix}$

b) $3\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + -1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3+0-1 \\ 6+0+1 \\ 3+0+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}$

$\vec{w} = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}$

4. (20 points) Find the equation $f(x) = ax + b$ of the least square line for the points $(-1, 0)$, $(0, 1)$, $(1, 1)$.

$\boxed{A^TAx = A^TB}$

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$-x + b = 0$
$0x + b = 1$
$x + b = 1$

$$\underbrace{\begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}}_{A} \begin{bmatrix} a \\ b \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}}_{B}$$

$$A^T = \underbrace{\begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{2\times 3}$$

$$A^TA = \underbrace{\begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{2\times 3} \underbrace{\begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}}_{3\times 2} = \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}}_{2\times 2}$$

$$A^TB = \underbrace{\begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{2\times 3} \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}}_{3\times 1} = \underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}_{2\times 1}$$

$$\left[\begin{array}{cc|c} 2 & 0 & 1 \\ 0 & 3 & 2 \end{array}\right] \xrightarrow{\frac{1}{3}R_3} \left[\begin{array}{cc|c} 2 & 0 & 1 \\ 0 & 1 & 2/3 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & 0 & 1/2 \\ 0 & 1 & 2/3 \end{array}\right]$$

$a = 1/2$
$b = 2/3$

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5. (18 points) Given the following vectors in $\mathbb{R}^4$,

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 12 \\ 2 \\ 0 \end{bmatrix}$$

The set $\beta = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis for some subspace of $\mathbb{R}^4$. Use the Gram-Schmidt process to create an orthogonal basis that spans the same subspace.

$$W_1 = \vec{V}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$\vec{V}_2 \cdot W_1 = 2+2+0-1 = 3$

$W_1 \cdot W_1 = 1+1+0+1 = 3$

$\vec{V}_3 \cdot W_1 = 0+12+0+0 = 12$

$\vec{V}_3 \cdot W_2 = 0+12+0+0 = 12$

$W_2 \cdot W_2 = 1+1+0+4 = 6$

$$W_2 = \vec{V}_2 - \left(\frac{\vec{V}_2 \cdot W_1}{W_1 \cdot W_1}\right) W_1$$

$$\begin{bmatrix} 2 \\ 2 \\ 0 \\ -1 \end{bmatrix} - \left(\frac{3}{3}\right) \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix}$$

$$W_3 = \vec{V}_3 - \left(\frac{\vec{V}_3 \cdot W_1}{W_1 \cdot W_1}\right) W_1 - \left(\frac{\vec{V}_3 \cdot W_2}{W_2 \cdot W_2}\right) W_2$$

$$= \begin{bmatrix} 0 \\ 12 \\ 2 \\ 0 \end{bmatrix} - \left(\frac{12}{3}\right) \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} - \left(\frac{12}{6}\right) \begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix}$$

$-4+2$

$$= \begin{bmatrix} 0 \\ 12 \\ 2 \\ 0 \end{bmatrix} - 4 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 12 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -4 \\ -4 \\ 0 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ -2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \\ 2 \\ 0 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -6 \\ 6 \\ 2 \\ 0 \end{bmatrix} \right\}$$

6. (20 points) The matrix A, given below, has eigenvalues $\lambda = 1$ and $\lambda = 4$. Find a basis for each eigenspace.

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$$A = \begin{bmatrix} 4 & 0 & 3 \\ -3 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$p(x) = \det(A - \lambda I)$

$$\begin{bmatrix} 4-\lambda & 0 & 3 \\ -3 & 1-\lambda & -3 \\ 0 & 0 & 1-\lambda \end{bmatrix} = 0 \begin{bmatrix} 0 & 3 \\ 1-\lambda & -3 \end{bmatrix} - 0 \begin{bmatrix} 4-\lambda & 3 \\ -3 & -3 \end{bmatrix} + 1-\lambda \begin{bmatrix} 4-\lambda & 0 \\ -3 & 1-\lambda \end{bmatrix}$$

$p(x) = 1-\lambda(\lambda^2 - 5\lambda + 4) = 0$

$(4-\lambda)(1-\lambda)$

$4 - 4\lambda - \lambda + \lambda^2$

$\lambda^2 - 5\lambda + 4 = 0$

$(-\lambda+1)(\lambda-4)(\lambda-1) = 0$

$\lambda = 1, \lambda = 4, \lambda = 1$

$A = PDP^{-1}$

$\lambda = 1 \quad (A - I)x = 0$

$$\left[\begin{array}{ccc|c} 3 & 0 & 3 & 0 \\ -3 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = x_1$
$x_2 = x_2$
$x_3 = x_3$

$x_3 = 0$

$\lambda = 4, (A - 4I)x = 0$

$$\left[\begin{array}{ccc|c} 0 & 0 & 3 & 0 \\ -3 & -3 & -3 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right] \xrightarrow[R_2+R_1]{R_3+R_1} \left[\begin{array}{ccc|c} 0 & 0 & 3 & 0 \\ -3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[\frac{1}{3}R_1]{\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$-x_1 - x_2 = 0$

$-x_1 = x_2$

$$= \underbrace{\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}}_{D} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

Cont.

110 + 23 = 133

7. (40 points total, 5 each) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false explain why or give a counterexample.

5 (a) If a square matrix A has an eigenvalue, $\lambda = 0$, then the matrix A is not invertible.

4 (b) If a 2×2 matrix A has the property that $A^2 = A$, then the only possibility is $\det(A)=1$.

5 (c) If a system of linear equations has 3 equations with 3 unknowns then the system has a unique solution.

4 (d) If $\vec{b}$ is not in Col(A), then the least squares solution $\hat{x}$ to $A\vec{x} = \vec{b}$, is the vector such that $\hat{x}$ is the closest to $\vec{b}$.

1 (e) If $\vec{u}$, $\vec{v}$ are two distinct vectors in Nul(A), where A is an $n \times n$ matrix, then $\vec{u}+\vec{v}$ is also in Nul(A).

2 (f) If $A\vec{x} = \vec{0}$ has a unique solution, then for any $\vec{b} \in \mathbb{R}$, $A\vec{x} = \vec{b}$ has a unique solution.

1 (g) If A is an $n \times n$ matrix, then the rank of A has to be at most n.

1 (h) If A is an $n \times n$ matrix, then the rank of A has to be at least n.

(a) Ex) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\underset{3\times3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \\ 4 & 5 & 6 \end{bmatrix}}$ & $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

True, the $\lambda = 0$ will leave a free variable in that column, (No pivot) Matrix A is not invertible because all cols needs to be invertible.

(b) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\det(A) = 1$

False, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\det(A) = -1 \neq 1$

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\det(A) = 0 - 1 = -1$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\frac{13}{7} - \frac{14}{7}$

(c) $\begin{cases} x + 2y + 3z = 5 \\ 2x + 3y + 4z = 8 \\ 4x + y + 5z = 7 \end{cases} \Rightarrow$ row echelon form $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & 3 & 4 & 8 \\ 4 & 1 & 5 & 7 \end{array}\right]$

$\xrightarrow{R_3 - 4R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & 3 & 4 & 8 \\ 0 & -7 & -7 & -13 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & -7 & -7 & -13 \end{array}\right] \xrightarrow{-\frac{1}{7}R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 1 & 1 & \frac{13}{7} \end{array}\right]$

$1 - 4(2)$
$1 - 8$
$5 - 4(3)$

$\xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & -1 & -\frac{1}{7} \end{array}\right] \xrightarrow{-1R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right]$

Cont.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 1 & 1/7 \end{array}\right] \xrightarrow{R_2+2R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -1 & 0 & -12/7 \\ 0 & 0 & 1 & 1/7 \end{array}\right]$$

$\frac{35}{7} - \frac{3}{7} = \frac{32}{7}$

$-2+2(1/7)$

$-2+2/7$

$-\frac{14}{7}+\frac{2}{7}$

$5-3(1/7)$

$5-3/7$

$$\xrightarrow{-1R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 0 & 12/7 \\ 0 & 0 & 1 & 1/7 \end{array}\right]$$

$$\xrightarrow{R_1-3R_3} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 32/7 \\ 0 & 1 & 0 & 12/7 \\ 0 & 0 & 1 & 1/7 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8/7 \\ 0 & 1 & 0 & 12/7 \\ 0 & 0 & 1 & 1/7 \end{array}\right]$$

$\frac{32}{7}-2\left(\frac{12}{7}\right)$

$\frac{32}{7}-\frac{24}{7}$

Depends, after row reducing and if there's a row with all 0's, then it's gonna have infinite solutions.
If we're able to fully row reduce it and theres a pivot in every column the the system has an unique solution.

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d) True, the least square solution $\vec{x}$ to $A\vec{x}=\vec{b}$, tries to get a vector to get as close to $\vec{b}$.

e) $\vec{u} = \cancel{\begin{bmatrix} 2 & 1 \\ 3 & 7 \end{bmatrix}}$ $\quad$ $\vec{u}, \vec{v} \in Nul(A)$ vectors

$\vec{v} = \cancel{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}$

$\vec{u}+\vec{v} = \begin{bmatrix} 3 & 1 \\ 3 & 9 \end{bmatrix} \in Nul(A)$

False!

f) $A\vec{x}=\vec{0}$, $\vec{b}\in\mathbb{R}$, $A\vec{x}=\vec{b}$

True because $\vec{b}$ is vector of all real values, $A\vec{x}=\vec{b}$ will have unique solutions.

why?

let $\vec{b}=\begin{bmatrix}1\\7\\12\end{bmatrix}$

$$\left[\begin{array}{ccc|c}5&3&7&0\\7&5&1&0\\1&7&11&0\end{array}\right] \qquad \left[\begin{array}{ccc|c}5&3&7&1\\7&5&1&7\\1&7&11&12\end{array}\right]$$

g) False, rank(A) has to be ~~atleast~~ n for A(nxn matrix.

At most

h) True, rank(A) is atleast n because it's the amount of columns in A.

Can be zero pivot cols.

The End.