

Please show all your work! Answers without supporting work will not be given credit.

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1. Given the vectors $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$, determine if the following vectors are in $\text{Span}\{\vec{u}, \vec{v}\}$.

If the vector is in the span, express it as a linear combination of \vec{u} and \vec{v} .

a.) $\begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$ $0\vec{u} + \vec{v} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$ In $\text{Span}\{\vec{u}, \vec{v}\}$, $\begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} \in \text{Span}\{\vec{u}, \vec{v}\}$

b.) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $0\vec{u} + 0\vec{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ In $\text{Span}\{\vec{u}, \vec{v}\}$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in \text{Span}\{\vec{u}, \vec{v}\}$

c.) $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ $1\vec{u} + 1\vec{v} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \in \text{Span}\{\vec{u}, \vec{v}\}$

d.) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ -2 \end{bmatrix}$ $\notin \text{Span}\{\vec{u}, \vec{v}\}$
Not in $\text{Span}\{\vec{u}, \vec{v}\}$

2. (12 points) Determine if the following vectors are linearly independent. Make sure to explain your reasoning.

$\vec{v}_1 - \vec{v}_3 = 0$
 $\vec{v}_2 = 0$
 $\vec{v}_3 = 0$
Linearly independent b/c there's only one solution (no inf solutions)

$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$

$1-1=0$

$3-1=2$

$2-0=2$

$[\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3 \ | \ \vec{0}]$

$\begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 1 & 3 & 2 & | & 0 \\ 0 & 1 & 2 & | & 0 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 0 & 2 & 2 & | & 0 \\ 0 & 1 & 2 & | & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 0 & 2 & 2 & | & 0 \end{bmatrix}$

$0\vec{v}_1 + 0\vec{v}_2 + 0\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$2 - \frac{1}{2}(2) = 2 - 1 = 1$

$R_3 - \frac{1}{2}R_2$
 $0 - \frac{1}{2}(0) = 0$
 $1 - \frac{1}{2}(2) = 0$

$\begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 0 & 2 & 2 & | & 0 \\ 0 & 0 & 1 & | & 0 \end{bmatrix} \xrightarrow{R_2 - 2R_3} \begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 0 & 2 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \end{bmatrix} \xrightarrow{R_1 - \frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 2 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \end{bmatrix}$
 $0 - \frac{1}{2}(2) = -1$
 $2 - 2(1) = 0$
 $0 - 2(0) = 0$
 $2 - 2(0) = 2$