

Please show all your work! Answers without supporting work will not be given credit.

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20
20

1. (5 points) Find the determinant of the matrix $A = \begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & -4 \\ 3 & -5 & 1 \end{bmatrix}$.

$$\det(A) = 1 \begin{vmatrix} 2 & -4 \\ -5 & 1 \end{vmatrix} - 0 \begin{vmatrix} 3 & -4 \\ 3 & 1 \end{vmatrix} - 3 \begin{vmatrix} 3 & 2 \\ 3 & -5 \end{vmatrix}$$

$$= 2 - 20 + 45 + 18 = 45$$

$$-3(-15 - 6) = 45 + 18$$

2. (7 points) Find $\det(B)$, where $B = \begin{bmatrix} -4 & 4 & 0 & 8 \\ 2 & 7 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 8 & 5 & -9 & 4 \end{bmatrix}$

$$\det(B) = 0 C_{31} + 1 C_{32} + 0 C_{33} + 0 C_{34}$$

$$C_{32} = (-1)^5 \det \begin{bmatrix} -4 & 0 & 8 \\ 2 & 0 & 0 \\ 8 & -9 & 4 \end{bmatrix}$$

$$= -1 \cdot -144 = 144$$

$$\det(B) = 144$$

$$2 C_{21} = (-1)^3 \det \begin{bmatrix} 0 & 8 \\ -9 & 4 \end{bmatrix} = 0 - (-72) = 72$$

$$(-1) \cdot 72 = -72$$

$$2 \cdot -72 = -144$$

3. (8 points) Given the matrix C of size 3×3 , with $\det(C) = 3$ determine the following determinants.

a.) The determinant of the matrix G obtained by subtracting row 1 from row 2 of C .

$$\det(G) = 3$$

b.) The determinant of the matrix H obtained by swapping rows 2 and 3 of C , followed by multiplying row 3 by 4.

$$\det(H) = -12$$

c.) The determinant of C^T

$$\det(C^T) = \det(C) \Rightarrow 3$$

d.) The determinant of C^{-1} .

$$\det(C^{-1}) = \det(C)^{-1}$$

$$3^{-1} = \frac{1}{3}$$

$$\det(C^{-1}) = \frac{1}{3}$$

$$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \xrightarrow{\begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}} \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \xrightarrow{R_3 \cdot 4} \begin{bmatrix} * & * & * \\ * & * & * \\ 4* & 4* & 4* \end{bmatrix}$$

$$-3 \cdot 4 = -12$$

$$\begin{bmatrix} * & * & * \\ * & * & * \\ 4* & 4* & 4* \end{bmatrix}$$

- Swapping

$$\det(A) = -\det(B)$$

- Scalar multiplication

$$\det(A) = k \cdot \det(B)$$

- Adding/ Subtracting row by row

$$\det(A) = \det(A)$$