MTH 309 MIDTERM 1 MARCH 9TH

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45/80

Please show **all** your work! Answers without supporting work will not be given credit. Clearly label your problems on separate paper.

-3

1. (20 points) Let A be a matrix and v a vector given as follows.

$$D = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{u} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

(a) Are the vectors that make up the columns of D linearly independent?

(b) Is $\vec{v}$ in Col(D), the column space of D?

(c) is $\vec{u}$ in Nul(D), the null space of D?

(d) Find an explicit description of Nul(D) by listing vectors that span the null space.

-9

2. (16 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which first scales the vector by a factor of 2 and then reflects the point through the x_2-axis.

(a) Find the standard matrix A of T.

(b) Determine if the transformation is one-to-one, onto, or both.

pivot in col

pivot in row

-17

3. (24 points) Given the following matrices:

3x3

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -3 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 3 & 2 \end{bmatrix}_{3\times 2}, \quad C = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}_{3\times 1}$$

For each expression given below if it is defined, compute it. If it not defined, give a reason why.

a) $C + AC$ b) AB^T c) ABA d)A^TB e)A^{-1}

-6

4. (20 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

(a) If $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are vectors in $\mathbb{R}^3$ and the set $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent, then the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is also linearly independent.

(b) Let B be a matrix and $\vec{u}$ a vector in Nul(B). If A is another matrix, such that the product AB is defined then $\vec{u}$ is also in Nul(AB).

✓ (c) If A is 3×5 matrix with 3 pivot columns then the matrix transformation $T_A : \mathbb{R}^5 \to \mathbb{R}^3$ given by $T_A(\vec{v}) = A\vec{v}$ is onto.

(d) If A is a 2×2 matrix, such that $A\vec{x} = \vec{b}$ has infinitely many solutions for some vector $\vec{b}$ in $\mathbb{R}^2$, then A is invertible.

Ivan Way

a) $D=\begin{bmatrix}1 & -1 & 1 & 2\\ 0 & 0 & 1 & -3\\ 0 & 0 & 0 & 1\end{bmatrix}$

L1 ~ one solution

$$\left[\begin{array}{cccc|c}1 & -1 & 1 & 2 & 0\\ 0 & 0 & 1 & -3 & 0\\ 0 & 0 & 0 & 1 & 0\end{array}\right]$$

No b/c x_2 is a free variable and therefore has infinitly many solutions

$$\cancel{0\begin{bmatrix}1\\0\\0\end{bmatrix}+0\begin{bmatrix}-1\\0\\0\end{bmatrix}+0\begin{bmatrix}1\\1\\0\end{bmatrix}+0\begin{bmatrix}2\\-3\\1\end{bmatrix}=\vec{0}}$$

b) Yes, $\begin{bmatrix}1\\1\\0\end{bmatrix}\in Col(D)$

$Col(D)=\{\text{All vets of } \vec{V}\}$

c) $$\left[\begin{array}{cccc|c}1 & -1 & 1 & 2 & 0\\ 0 & 0 & 1 & -3 & 0\\ 0 & 0 & 0 & 1 & 0\end{array}\right] \quad A\vec{x}=\vec{0}$$

$$\xrightarrow{R_2+3R_3}\left[\begin{array}{cccc|c}1 & -1 & 1 & 2 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\end{array}\right]\xrightarrow{R_1-R_2}\left[\begin{array}{cccc|c}1 & -1 & 0 & 2 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\end{array}\right]$$

$$\xrightarrow{R_1-2R_3}\begin{array}{c}\begin{array}{cccc}x_1 & x_2 & x_3 & x_4\end{array}\\ \left[\begin{array}{cccc|c}1 & -1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\end{array}\right]\\ \text{free}\end{array}$$

$x_1=0$
$x_2=x_2$
$x_3=0$
$x_4=0$

$\vec{u}\notin Nul(D)$

$\begin{bmatrix}1\\-1\\0\end{bmatrix}$

d) $A\vec{x} = \vec{0}$

-3

$$Null(D) = \left\{ \begin{bmatrix} 0 \\ x_2 \\ 0 \\ 0 \end{bmatrix} \right.$$

from where?

$$\begin{bmatrix} 0 \\ x_2 \\ 0 \\ 0 \end{bmatrix} \in span(D) = Null(D)$$

Ivan Wang

) $T_A C$

-9

e_1, e_2 → What does it do!

True!

b) If the trasformation has a pivot in the columns, then it's onto.

+,

If the trasformation has a pivot in every row, then it is one-to-one.

If the trasformation has a pivot in every ~~every~~ row and column, then it's both.

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c) $C + AC$ $\underline{-3}$

$\cancel{\begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}}$ Not defined because A has more columns then C.

A has as many cols as C has rows.

$A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

$C = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$

$= (1 + 0 + 4)$

$6 + (-3)$

b) $\cancel{AB^T}$

$B^T = \begin{bmatrix} 1 & 1 & 3 \\ 0 & -1 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 0 & -3 \\ 0 & 1 & 0 \end{bmatrix}$ 3×3

2×3

$1\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$

$1\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix}$ 3×3

$B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 3 & 2 \end{bmatrix}$ 3×2

d) $A^T B$

$\underline{-3}$

$A^T B = 1\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} - 2\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

$0\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

$1\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

$= ?$ should be 3×2

e) $A^{-1} \stackrel{?}{=} \begin{bmatrix} 1/2 & 0 & 1/2 \\ 2/2 & 1/2 & -3/2 \\ 0 & 1/2 & 0 \end{bmatrix}$

$\underline{-7}$

b) AB^T

-2

$$B^T = \begin{bmatrix} 1 & 1 & 3 \\ 0 & -1 & 2 \end{bmatrix}_{2\times3} \qquad A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -3 \\ 0 & 1 & 0 \end{bmatrix}_{3\times3}$$

not defined!

$$\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$$

$$AB^T = \begin{bmatrix} 2 & 0 \\ -4 & 4 \\ 3 & 2 \end{bmatrix}$$

c) ABA

-2

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -3 \\ 0 & 1 & 0 \end{bmatrix}_{3\times3}, \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 3 & 2 \end{bmatrix}_{3\times2}$$

→ . . . . so?

Ivan Wag

False, because there could be a free variable in $\vec{V}_3$, thus causing infinite solutions. When there's infinite solutions, then it's linear dependent.

b) ~~False~~ True, because $\vec{u}$ is a vector in Nul(B), getting the product AB won't change Nul(B), so ~~Nul(AB)~~ $\vec{u} \in$ Nul(AB)

-3 True! ~~False~~

c) ~~True~~, because all the ~~pivot~~ columns must be pivot ~~columns~~ rows! for T_A to be onto.

d) -3 True, because ~~it~~ having infinitely many solutions doesn't affect A invertibility.

it does!

$$\begin{bmatrix} * & * \\ * & * \end{bmatrix}$$

2x2