

Please show all your work! Answers without supporting work will not be given credit.

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- 3 1. (8 pts) Determine which of the following augmented matrices are in echelon form, reduced echelon form, or neither. If the matrix is in either echelon form, is the associated linear system consistent? Make sure to explain your reasoning.

(a)  $\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$

echelon form

consistent

because there is a solution to the linear system

(b)  $\left[ \begin{array}{cc|c} 1 & 0 & 0 \\ -1 & 1 & 2 \end{array} \right]$

neither

(c)  $\left[ \begin{array}{cccc|c} 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right]$

neither

(d)  $\left[ \begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right]$

echelon form

reduced echelon form

inconsistent

because  $0x + 0y + 0z = 1$

$0x + 0y + 0z = 1$   
 $0 = 1$

- 2 2. (12 points) Use Gauss-Jordan elimination (row reduction) in order to solve the linear system given below. Make sure to clearly identify which elementary row operations you are using.

$-10 + 6 + 8 = 4$   
 $-10 + 14 = 4$

augmented matrices

$x = -5$   
 $y = 3$   
 $z = 4$

$2x + 2y + 2z = 4$   
 $x + 4y + z = 8$   
 $-x - y = 2$

$2(-5) + 2(3) + 2(4) = 4$

$1 - \frac{1}{2}(2) = 1 - 1 = 0$

$4 - \frac{1}{2}(2) = 4 - 1 = 3$

$1 - \frac{1}{2}(2) = 0$

$\left[ \begin{array}{ccc|c} 2 & 2 & 2 & 4 \\ 1 & 4 & 1 & 8 \\ -1 & -1 & 0 & 2 \end{array} \right] \xrightarrow{R_2 - \frac{1}{2}R_1} \left[ \begin{array}{ccc|c} 2 & 2 & 2 & 4 \\ 0 & 2 & 0 & 6 \\ 0 & 0 & 1 & 4 \end{array} \right]$

$\xrightarrow{R_1 - R_2} \left[ \begin{array}{ccc|c} 2 & 0 & 2 & -2 \\ 0 & 2 & 0 & 6 \\ 0 & 0 & 1 & 4 \end{array} \right] \xrightarrow{\frac{1}{2}R_1} \left[ \begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 2 & 0 & 6 \\ 0 & 0 & 1 & 4 \end{array} \right] \xrightarrow{\frac{1}{2}R_2} \left[ \begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array} \right]$

$x + z = -1$

$y = 3$   
 $z = 4$

$x + 4 = -1$

$x = -5$

$-1 + 4 = -5$

$-1 + \frac{1}{2}(2) = 0$

$-1 + \frac{1}{2}(2) = 0$

$0 + \frac{1}{2}(2) = 1$

$2 + \frac{1}{2}(4) = 4$