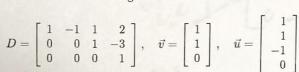
Ivan Wang

Please show all your work! Answers without supporting work will not be given credit. Clearly label your problems on separate paper.

1. (20 points) Let A be a matrix and v a vector given as follows.



- (a) Are the vectors that make up the columns of D linearly independent?
- (b) Is  $\vec{v}$  in Col(D), the column space of D?
- (c) is  $\vec{u}$  in Nul(D), the null space of D?
- (d) Find an explicit description of Nul(D) by listing vectors that span the null space.
- (16 points) Let  $T: \mathbb{R}^2 \to \mathbb{R}^2$  be the linear transformation which first scales the vector by a factor of 2 and then reflects the point through the  $x_2$ -axis.
  - (a) Find the standard matrix A of T.
  - (b) Determine if the transformation is one-to-one, onto, or both
- 3. (24 points) Given the following matrices:  $A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -3 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 3 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$

For each expression given below if it is defined, compute it. If it not defined, give a reason why.

- a) C + AC b)  $AB^T$  c) ABA d) $A^TB$  e) $A^{-1}$
- 4. (20 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
  - (a) If  $\vec{v}_1, \vec{v}_2, \vec{v}_3$  are vectors in  $\mathbb{R}^3$  and the set  $\{\vec{v}_1, \vec{v}_2\}$  is linearly independent, then the set  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is also linearly independent.
  - (b) Let B be a matrix and  $\vec{u}$  a vector in Nul(B). If A is another matrix, such that the product AB is defined then  $\vec{u}$  is also in Nul(AB).
- $\sqrt{\phantom{a}}$  (c) If A is  $3 \times 5$  matrix with 3 pivot columns then the matrix transformation  $T_A : \mathbb{R}^5 \to \mathbb{R}^3$  given by
  - (d) If A is a 2 ×2 matrix, such that  $A\vec{x} = \vec{b}$  has infinitely many solutions for some vector  $\vec{b}$  in  $\mathbb{R}^2$ . then A is invertible.

$$D = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1$$

- 0 ->2 -0 - TAC

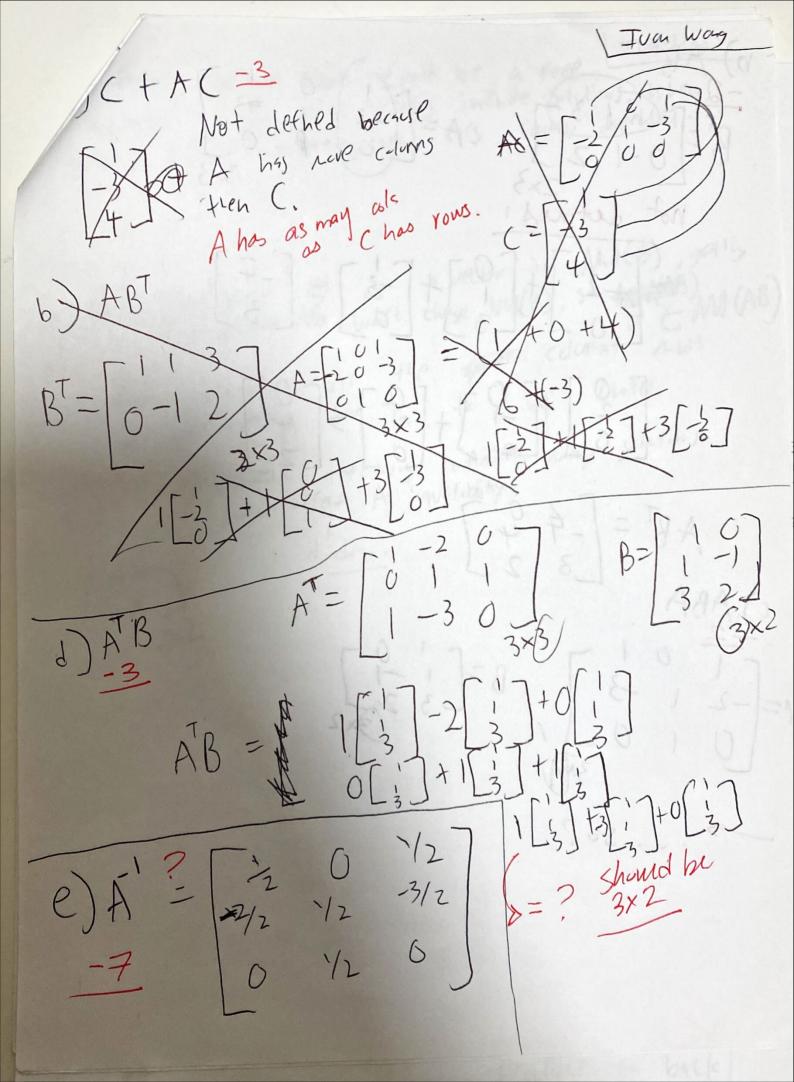
-9

e, e, e, but dos it do!

The transformation has a pivot in the transformation has a pivot in every now, then it is one to cone.

If the transformation has a to cone.

If the transformation has a pivot in every now, then it is one to both.



b) 
$$AB^{T}$$
 $=\frac{2}{13}$ 
 $B^{T} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ 
 $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ 
 $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ 
 $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ 
 $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ 
 $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ 
 $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ 
 $ABA$ 
 $ABA$ 

> . . . 50!

Faise, becase there could be a free Ivanhage variable in V3, this causing infinite solutions. When there's infinite solutions, then it's linear dependant,

b) the because V is a vector in NU(B), getting

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B), so the M(AB)

1 the product AB work charge NU(B)

1 the product AB work charge NU(B)

1 the product AB work char