① Give an example of three non zero matrices A, B, C, such that AB = AC but B ≠ C (This is just inoculation against future logic errors...)

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$AB = \underset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}} \cdot \underset{1\times 2}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} = \begin{bmatrix} 1\cdot 1 + 0\cdot 2 \\ 0\cdot 1 + 0\cdot 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\cancel{AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+0 \\ 0+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}}$$

$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 0\cdot -1 \\ 0\cdot 1 + 0\cdot -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ 5/5

AB = AC, but B ≠ C

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Ivan Wang

② Let $R$ be a ring. Prove that $\forall a \in R$ $0 \cdot a = 0$

$(R, +, \cdot)$ is ring.

$0 + 0 = 0$, satifies identity property of zero.

$0 \cdot a = (0+0)a \Rightarrow 0 \cdot a = 0 + 0 = 0 = 0.$

satifies distributive property

why?

You can deduce $0 \cdot a = 0 \cdot a + 0 \cdot a$. But since "$0 \cdot a = 0$" is what you're trying to prove, you can't use it in your proof

$a(b) + a(-b) = a(b + (-b)) = a \cdot 0 = 0$

Cancellation to conclude $0 \cdot a = 0$

Can't be used in proof of itself.

Therefore since $\forall a \in R$ $0 \cdot a = 0$

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Proofs: ① Write complete sentences, with correct grammar, and punctuation.
② Never start a sentence with a symbol.
③ It's hard to see the sentence structure when some of the words are expressed using math notation. Reading aloud helps.

lecture, I forgot to include the condition $1 \neq 0$
the definition of field (or of ring with unity).
w that a ring R with unity such that $1=0$
has only one element.

$1=0$

For any element $a \in R$, $1 \cdot a = a$, $a \cdot 1 = a$

$1 \cdot a = a \cdot 1 = a$

There exists multiplicative Identity in R

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since $1=0$, for any element $a \in R$

$0 \cdot a = a \cdot 0$, Need some explanation of how you are deducing $a=0$, and from what.

$a=0$

Therefore, since $a \in R$ must be 0 and that 0 is the only element in R since $1=0$.

④ How many ways can the following addition and m[ultiplication] tables be filled in so that $\{0,1\}$ is a fie[ld] with additive identity 0 and multiplicative identity 1

| + | 0 | 1 |
|---|---|---|
| 0 | | |
| 1 | | |

| $\cdot$ | 0 | 1 |
|---|---|---|
| 0 | | |
| 1 | | |

$0+0=0$

$0+1=1$

$1+0=1$

$1+1=0$

1

additive inverse

$0 \cdot 0 = 0$

$0 \cdot 1 = 0$

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$

| * | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

| $\cdot$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

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There is 1 way to fill the addition and multiplication tables for the set $\{0,1\}$

"Justify your answer."

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many ways can the following addition and multiplication tables be filled so that $\{0,1,x\}$ is a field with additie identity 0 and multiplicatio identity 1? (You can assume that 0, 1, and x are all distinct)

| + | 0 | 1 | x |
|---|---|---|---|
| 1 | | | |
| 0 | | | |
| x | | | |

| · | 0 | 1 | x |
|---|---|---|---|
| 0 | | | |
| 1 | | | |
| x | | | |

justify your answer.

$1+0=1$
$1+1=x$ (inverse)
$1+x=0$ (inverse)
$0+0=0$
$0+1=1$
$0+x=x$
$x+0=x$
$x+1=0$ (inverse of
$x+x=1$ (inverse of x

$0\cdot 0=0$
$0\cdot 1=0$
$0\cdot x=0$
$1\cdot 0=0$
$1\cdot 1=1$
$1\cdot x=x$
$x\cdot 0=0$
$x\cdot 1=x$
$x\cdot x=1$ (inverse)

| ~~+~~ | ~~0~~ | ~~1~~ | ~~x~~ |
|---|---|---|---|
| ~~1~~ | ~~1~~ | ~~x~~ | ~~0~~ |
| ~~0~~ | ~~0~~ | ~~1~~ | ~~x~~ |
| ~~x~~ | ~~x~~ | ~~0~~ | ~~1~~ |

| * | 0 | 1 | x |
|---|---|---|---|
| 0 | 0 | 1 | x |
| 1 | 1 | x | 0 |
| x | x | 0 | 1 |

| · | 0 | 1 | x |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | x |
| x | 0 | x | 1 |

There is 1 way to fill in addition and multiplication tables for the set $\{0,1,x\}$

Q Suppose that F is a field whose characteristic is positive integer n. Show that n is prime. (Hint: if the characteristic were 6, what would the product of $(1_F + 1_F + 1_F)$ and $(1_F + 1_F)$ be? Why would that be a problem?)

$$n \cdot 1_F = \underbrace{1_F + 1_F + \dots}_{} = 0_F$$

multiplicative identity

additive identity

Assume n is not prime: $a \in n$, $b \in n$

$1 < a < n$, $1 < b < n$

$\Rightarrow n = a \cdot b$

$$(a \cdot 1_F) \cdot (b \cdot 1_F) = (a \cdot b) \cdot 1_F = n \cdot 1_F = 0_F$$

$a \cdot 1_F = 0_F$

or

$b \cdot 1_F = 0_F$

Therefore, a or b is a smaller positive integer such that $a \cdot 1_F = 0_F$ or $b \cdot 1_F = 0_F$

the field has finite characteristic, and n is prime