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(1) row equivalent: - Swap two rows
- Multiply by non zero Scalar
- Adding a multiple of one row to another row.

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Def: If A and B are $m \times n$ matrices over the field F, we say that B is row equivalent to A if B can be obtained from A by a finite sequence of elementary row operations.

A row swap can be accomplished by a sequence of moves of the other two types...

Sbu: - multiply a row by a non zero Scaler
- add row to another

let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} \to \begin{bmatrix} 0 & -1 \\ -1 & -1 \end{bmatrix} \to \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \to \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \to \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = B.$

We can multiply row by nonzero Scalar and add row to another. But we can't make it row equivalent by our definition unless we perform row swapping

A and B are row equivalent by our def,

(2) Bing: add scaler multiple to any row

let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$ A and B are row equivalent by our def

But A and B will never be row equivalent (Bing def) unless you allow row swaps and multiply by non zero scalar.

③ let $A=\begin{bmatrix}2&0\\0&d\end{bmatrix}$ $B=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

Ⓐ $\xrightarrow{\frac{1}{2}R_1+R_2}\begin{bmatrix}1&1\\0&1\end{bmatrix}=\begin{bmatrix}1&1\\0&1\end{bmatrix}\xrightarrow{R_1-R_2}\begin{bmatrix}1&0\\0&1\end{bmatrix}$

At Albany, they are allowed to add any scaler multiple of any row to any row.

However, this won't be row equivalent at Buffalo because we will need to use scalar multiple of any row to any row.

④ let $A=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, $B=\begin{bmatrix}0&1\\1&0\end{bmatrix}$

Albany: - add any scalar multiple of any row to any row

Buffalo

This would not be row equivalent at Albany because we will need to do a row swap to make A and B row equivalent.

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⑤ For row equivalence to be an equivalence relation, it must satisfy:

① Reflexivity: Every matrix is row-equivalent to itself.

② Symmetry: If A is row-equivalent to B, then B is row equivalent to A

③ Transitivity: A is row equivalent to B and B is row equivalent to C, then A is row-equivalent to C.

Binghamton row equivalence is not symmetric because we cannot go B to A without doing scalar multiplications or row swaps

$\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\begin{bmatrix}0&1\\2&4\end{bmatrix}$

A be an $n \times n$ (square) matrix over the field

An $n \times n$ matrix B such that $BA = I$ is called a left

[inv]erse of A; an $n \times n$ matrix B such that $AB = I$ is

called right inverse of A. If $AB = BA = I$, then B is

called a two sided inverse of A and A is invertible

let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$AB = I$: $\underset{2\times3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}} \underset{3\times2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}} = \underset{2\times2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = I$ ✓ Good.

$\det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 1 - 0 = 1$

This means A is invertible, since $\det \neq 0$

$BA = I$:

$\underset{3\times2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}} \underset{2\times3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}} = \underset{3\times3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$

$\det\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) = (1 \cdot 1 \cdot 0) - (0 \cdot 0 \cdot 0) - (0 \cdot 0 \cdot 1) = 0$

Therefore, A is not invertible, since $\det = 0$

Therefore, A has a left inverse but no right inverse

You've checked that B is not a right inverse of B, but how do you know some other matrix wouldn't work?

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let $R = \mathbb{Z}$ (Ring of integers); $1/2 \notin \mathbb{Z}$...

let $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$...

RRE.

$R_2 - \frac{1}{2}R_2$

$\begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix}$

0/5

A cannot be reduced to row reduced echelon form because the first row is not 1,

Therefore, if we let $R = \mathbb{Z}$ (be an ring of integers), and let $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$, then we can't make row-equivalent to a reduced row-echelon matrix, because we can't row reduced it further.

(8) A matrix over $\mathbb{Z}$ is in integer row reduced echelon form:

- If a row is not 0 and the leading (leftmost) entry is positive.
- The leading entry of the next row is to the right of the leading entry of the previous row above,
- ~~[illegible]~~ All zeros at bottom last row,
- The leading entry of each row is the greatest common divisor (GCD) of all the elements in that row
- All other entries in the column of a pivot must be integer multiples of that pivot
- The leading coefficient in each row is the smallest positive integer that be obtained as a linear integer combination of the row entries.