MTH420 PROBLEM SET 1

(1) Give an example of three nonzero matrices A, B, C such that $AB = AC$ but $B \neq C$. (This is just innoculation against future logic errors....)

(2) Let R be a ring. Prove that
$$\forall_{a \in R} 0 \cdot a = 0.$$

(3) In lecture, I forgot to include the condition $1 \neq 0$ in the definition of field (or of ring with unity). Show that a ring R with unity such that $1 = 0$ has only one element.

(4) How many ways can the following addition and multiplication tables be filled in so that $\{0, 1\}$ is a field with additive identity 0 and multiplicative identity 1?

+	0	1
0		
1		

$\cdot$	0	1
0		
1		

Justify your answer.

(5) How many ways can the following addition and multiplication tables be filled in so that $\{0, 1, x\}$ is a field with additive identity 0 and multiplicative identity 1? (You can assume that $0, 1$ and x are all distinct.)

+	0	1	x
0			
1			
x			

$\cdot$	0	1	x
0			
1			
x			

Justify your answer.

(6) Suppose that F is a field whose characteristic is a positive integer n. Show that n is prime. (Hint: if the characteristic were 6, what would the product of $(1_F + 1_F + 1_F)$ and $(1_F + 1_F)$ be? And why would that be a problem?)

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