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MTH 420

① A is $n \times m$ matrix and B is $m \times n$ matrix.

Therefore, AB must have $n \times n$ dimensions because

$AB = n \times n$

This means that the maximum number of linearly independent columns in AB is at most m. Not clear how this is being deduced

Since AB is $n \times n$, but $m < n$, this means that AB cannot span all F^n. Therefore, AB has to be linearly dependent and AB is singular (not invertible).

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$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix} \in \mathbb{R}^{4 \times 4}$$

Matrix has upper triangular. Therefore, det is product of it's diagonal entries:

$\det(m) = 1 \times 2 \times 3 \times 4 = 24$

Since $\det(m) = 24 \neq 0$, it means that the matrix is invertible

...s we subspaces of $\mathbb{R}^{2\times 2}$ needs:

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contains zero vector: $\begin{pmatrix}0&0\\0&0\end{pmatrix} \in$ set

closed under addition: If A and B are in set, A+B are also in set.

- closed under scaler multiplication: If A is set and c is scalar, then

$cA \in S$

let $A = \begin{pmatrix}1&2\\3&4\end{pmatrix}$ $\quad \begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}1&2\\3&4\end{pmatrix} = \begin{pmatrix}7&10\\15&22\end{pmatrix}$

(a) $\{A \in \mathbb{R}^{n\times n} : A^2 = A\}$

- zero matrix: $0(A) = 0$ ✓

let $0 = c$, satisfies since $0 \in \mathbb{R}$ and $\begin{pmatrix}0&0\\0&0\end{pmatrix} \in$ set

- closed under addition: X

let $A, B \in S$:

$A^2 = A$, $B^2 = B$

check $(A+B)^2 = A+B$:

$(A+B)^2 = A^2 + AB + BA + B^2 = A + AB + BA + B \neq A+B$

Therefore, set is not closed under addition

- Closed under scaler multiplication is also satisfied because $1,2,3,4 \in \mathbb{R}$, $c = 0 \in \mathbb{R}$ ✓

$0\begin{pmatrix}1&2\\3&4\end{pmatrix} = \begin{pmatrix}0&0\\0&0\end{pmatrix} \in \mathbb{R}$

Therefore, $\{A \in \mathbb{R}^{n\times n} : A^2 = A\}$ is not subspace of $\mathbb{R}^{2\times 2}$ because although it satisfies zero matrix and closed under scaler multiplication. It does not satisfy closed under addition.

(b) $\{A \in \mathbb{R}^{2\times 2} : A\begin{pmatrix}1&2\\2&4\end{pmatrix} = \begin{pmatrix}1&2\\2&4\end{pmatrix}A\}$

write a clear organized argument

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zero matrix: let $A = 0$, satisfies since $0 \in \mathbb{R}$, therefore, $0\begin{pmatrix}1&2\\3&4\end{pmatrix} = \begin{pmatrix}1&2\\3&4\end{pmatrix}0 \Rightarrow \begin{pmatrix}0&0\\0&0\end{pmatrix} = \begin{pmatrix}0&0\\0&0\end{pmatrix}$ ✓

closed under addition: let $A, B \in S$

$A\begin{pmatrix}1&2\\2&4\end{pmatrix} = \begin{pmatrix}1&2\\2&4\end{pmatrix}A$ $\quad$ check: $(A+B)\begin{pmatrix}1&2\\2&4\end{pmatrix} = \begin{pmatrix}1&2\\2&4\end{pmatrix}(A+B)$

$B\begin{pmatrix}1&2\\2&4\end{pmatrix} = \begin{pmatrix}1&2\\2&4\end{pmatrix}B$ $\quad (A+B)\begin{pmatrix}1&2\\2&4\end{pmatrix} = A\begin{pmatrix}1&2\\2&4\end{pmatrix} + B\begin{pmatrix}1&2\\2&4\end{pmatrix} \Rightarrow \begin{pmatrix}1&2\\2&4\end{pmatrix}A + \begin{pmatrix}1&2\\2&4\end{pmatrix}B$

Therefore, it satisfies closed under addition

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- closed under scalar ~~multiplication~~: is also satisfied ? because $1,2,3,4 \in \mathbb{R}$, $A = 0 \in \mathbb{R}$, $0\begin{pmatrix}1&2\\3&4\end{pmatrix} = \begin{pmatrix}1&2\\3&4\end{pmatrix}0 = \begin{pmatrix}0&0\\0&0\end{pmatrix} \in \mathbb{R}$ ✓

Closure under scalar mult is not checked correctly

Therefore, $\{A \in \mathbb{R}^{2\times 2} : A\begin{pmatrix}1&2\\2&4\end{pmatrix} = \begin{pmatrix}1&2\\2&4\end{pmatrix}A\}$ is subspace of $\mathbb{R}^{2\times 2}$ because it satisfies zero matrix, closed under addition, and closed under scaler multiplication.

① $\{A \in \mathbb{R}^{2\times 2} : A\begin{pmatrix}1&2\\2&4\end{pmatrix} = 0\}$

✓ zero matrix: let $A = 0$, $0 \in \mathbb{R} \Rightarrow A = 0$.

Thus, $0\begin{pmatrix}1&2\\2&4\end{pmatrix} = 0$. Therefore since, $0\begin{pmatrix}1&2\\2&4\end{pmatrix} = 0$, the zero matrix is satisfied.

Closed under addition:

let $A, B \in \{A \in \mathbb{R}^{2\times 2} : A\begin{pmatrix}1&2\\2&4\end{pmatrix} = 0\}$, then $AB = 0$ and $CB = 0$:

$(A+C)B = AB + CB = 0 + 0 = 0$

$\Rightarrow A+C \in \{A \in \mathbb{R}^{2\times 2} : A\begin{pmatrix}1&2\\2&4\end{pmatrix} = 0\}$

✓ Closed under Scalar multiplication: For $A \in \mathbb{R}$ and $A = 0 \in \mathbb{R}$. It also satisfies because $1, 2, 2, 4 \in \mathbb{R}$ and $A = 0 \in \mathbb{R}$ and $\begin{pmatrix}0&0\\0&0\end{pmatrix} \in \mathbb{R}$?

therefore, since all 3 conditions are satisfied. $\{A \in \mathbb{R}^{2\times 2} ; A\begin{pmatrix}1&2\\2&4\end{pmatrix} = 0\}$ is a subspace of $\mathbb{R}^{2\times 2}$

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④ $\{A \in \mathbb{R}^{2\times 2} : A\begin{pmatrix}1&2\\2&4\end{pmatrix} = \begin{pmatrix}1&2\\2&4\end{pmatrix}A\}$

let $B = \begin{pmatrix}1&2\\2&4\end{pmatrix}$:

$= \{A \in \mathbb{R}^{2\times 2} : AB = BA\}$

$\{A \in \mathbb{R}^{2\times 2} : A\begin{pmatrix}1&2\\2&4\end{pmatrix} = 0\}$

$\{A \in \mathbb{R}^{2\times 2} : AB = 0\}$

since $AB = BA$ and $AB = 0$, it means that $0 = BA$

Therefore, $\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}1&2\\2&4\end{bmatrix} = 0$, $\begin{bmatrix}a+2b & 2a+4b\\ c+2d & 2c+4d\end{bmatrix} = \begin{bmatrix}0&0\\0&0\end{bmatrix}$

$a+2b=0$, $2a+4b=0$, $c+2d=0$, $2c+4d=0$

$2a+4b=0$
$-2a-4b=0$

$a=-2b$
$c=-2d$

$A = \begin{bmatrix}-2b & b\\ -2d & d\end{bmatrix} = b\begin{bmatrix}-2&1\\0&0\end{bmatrix} + d\begin{bmatrix}0&0\\-2&1\end{bmatrix}$

The solution set of all matrices $= \text{span}\left\{\begin{bmatrix}-2&1\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\-2&1\end{bmatrix}\right\}$

✗

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? ... ... a subspace

(5) let $Q = \{A \in \mathbb{R}^{2\times2} \mid AB = BA\}$

let $V = \{A \in \mathbb{R}^{2\times2} \mid AB = C\}$ $\qquad$ $M = q + v$, where $q \in Q$, $v \in V$

$Q + V = \{q + v \mid q \in Q, v \in V\}$

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(6)

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(7) subspace properties: Zero function; closed under addition; closed under scalar multiplication

(a) The set of all continuous elements

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zero function: $f(x) = 0$ is continuous because it has no breaks in its domain, Therefore, $0 \in S$

closed under addition: If f and g are continuous, then $f+g$ is also continuous as the sum of continuous functions is continuous.

closed under scalar multiplication: let $c \in \mathbb{R}$ and f is continuous, then cf is still continuous as scalar multiples of continuous functions remain continuous

Therefore, the set of all continuous elements is a subspace of $\mathbb{R}^{\mathbb{R}}$.

(b) The set of all odd elements

~~Zero function~~ is satisfied because $f(x)=0$ and $f(x)=-0$, being in the set.

Closed under addition: $\cancel{X}(f+g) = f(-x)+g(-x) = -f(x)-g(x) = -(f+g)(x)$

therefore, $f+g$ is also odd.

Closed under scalar multiplication: If $c \in \mathbb{R}$ and f is odd function,

$(cf)(-x) = cf(-x) = \cancel{*}\ c(-f(x)) = -cf(x)$

Therefore, cf is odd

Therefore since all conditions are meet, the set of odd functions is a subspace of $\mathbb{R}^{\mathbb{R}}$

(c) The set of discontinuous elements

Fails to meet zero function condition because $f(x)=0$ is continuous (it has no breaks in domain). Discontinuous function has breaks or gaps.

Therefore not all conditions were meet, the set of discontinuous elements is not a subspace of $\mathbb{R}^{\mathbb{R}}$.

(d) The set of all solutions to the ODE $y''-2y'+y=x^2$

Zero function: $0''-2(0)'+0=x^2$

therefore the zero function is not a solution. Thus it fails to be a subspace.