MTH420 PROBLEM SET 2

(1) At Stony Brook, students are only allowed to use two types of elementary row operations: they are allowed to multiply a row by a nonzero scalar, and they are allowed to add one row to another, but the operation of switching two rows with each other is not considered an elementary row operation there. Are there any pairs of matrices which would be considered "row equivalent" by our definition, but not by theirs? Justify your answer.

(2) At Binghamton, students are only allowed to use one type of elementary row operation. They are allowed to add any scalar multiple of any row to any other row. But multiplying a row by a scalar is not considered elementary, and neither is switching two rows. Are there any pairs of matrices which would be considered "row equivalent" by our definition, but not by theirs? Justify your answer.

(3) At Albany, students are only allowed to use one type of elementary row operation. They are allowed to add any scalar multiple of any row to any row. (So, it's similar to Binghamton, but students at Binghamton are required to use two different rows, and students at Albany are not.) Are there any pairs of matrices which would be considered row equivalent at Albany, but not at Buffalo? Justify your answer.

(4) Are there any pairs of matrices which would be considered row equivalent at Buffalo, but not at Albany? Justify your answer.

(5) All four schools use the same definition of "row equivalent," namely: "if R is a ring, m, n are positive integers, and $A, B \in R^{m\times n}$, then A is row equivalent to B if there is a finite sequence of elementary row operations which transforms A into B." At one of the schools, "is row equivalent to " is not an equivalence relation. Which one and why?

(6) Give an example of a matrix A which has a left inverse but no right inverse.

(7) We have seen that, for every field F, any matrix with entries in F is row-equivalent to a row-reduced echelon matrix. Give an example of a ring R and a matrix with entries in R which is not row-equivalent to a row-reduced echelon matrix (with entries in R).*

(8) How would you adapt the definition of "Row reduced echelon" to work for the ring $\mathbb{Z}$? Your adaptation shoiuld have two properties:
   - it should be true that every matrix with entries in $\mathbb{Z}$ is row equivalent to a matrix, with entries in $\mathbb{Z}$, which is "RRE" according to the adapted definition,
   - it should be easy to read off the solution set of a system whose augmented matrix is "RRE".

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* I recommend $R = \mathbb{Z}$. Of course, a matrix with entries in $\mathbb{Z}$ has entries in $\mathbb{Q}$ and is thus row-equivalent to a row-reduced echelon matrix with entries in $\mathbb{Q}$. But it might not be row-equivalent to an RRE matrix with entries in $\mathbb{Z}$.

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