MTH420 PROBLEM SET 5

(1) We consider $W := \{a_0 + a_1x + a_2x^2 : a_0, a_1, a_2 \in \mathbb{R}\} \leq \mathbb{R}[x]$.
   (a) Prove that W is a subspace of $\mathbb{R}[x]$.
   (b) Prove that $\{x^2, (x-1)^2, (x-2)^2\}$ is a basis of W.
   (c) Let $\mathcal{B} = (x^2, (x-1)^2, (x-2)^2)$, and compute $[(x+1)^2]_{\mathcal{B}}$.
   (d) Let $\mathcal{C} = (1, x, x^2)$, which is usually the most convenient ordered basis for W. Compute the change of basis matrix $P_{\mathcal{B},\mathcal{C}}$.
   (e) Compute $[(x+1)^2]_{\mathcal{C}}$ and verify that the identity which is supposed to relate $P_{\mathcal{B},\mathcal{C}}, [\alpha]_{\mathcal{B}}$ and $[\alpha]_{\mathcal{C}}$ actually checks out in this case.
(2) For $A, X \in F^{n\times n}$ let $B_A(X) = AXA$. Is B_A linear? Justify your answer.
(3) For $A, X \in F^{n\times n}$ let $f_X(A) = AXA$. Is f_X linear? Justify your answer.

Date: February 24, 2025.