Assume that the air resistance (a force), F_R, on a bike is a function of the density of the air, ρ, the cross-sectional area of the bike, A, the bike's speed, V, and the bike's mass, m

$$F_R = f(\rho, A, v, m)$$

note $[\rho] = ML^{-3}$

(a) Identify the fundamental dimensions of all physical observables in the problem.

$[F_R] = MLT^{-2}$ $\quad [m] = M$

$[\rho] = ML^{-3}$

$[A] = L^2$

$[v] = LT^{-1}$

(b) $[F_R] = (\rho^a A^b v^c m^d)$

$$MLT^{-2} = (ML^{-3})^a (L^2)^b (LT^{-1})^c (m)^d$$
$$(M^a L^{-3a})(L^{2b})(L^c T^{-c})(m^d)$$

$M: 1 = a + d$

$L: 1 = -3a + 2b + c$

$T: -2 = -c$ $\qquad c = 2$

Consider the following system of reactions:

$$X \underset{k_2}{\overset{k_1}{\rightleftarrows}} Y + Z$$

$$X + W \xrightarrow{k_3} 2X + Y$$

$$X \xrightarrow{k_1} Y + Z \qquad r_1 = X k_1$$

$$Y + Z \xrightarrow{k_2} X \qquad r_2 = Y Z k_2$$

$$X + W \xrightarrow{k_3} 2X + Y \qquad r_3 = X W k_3$$

$$\frac{dx}{dt} = -X k_1 + Y Z k_2 - X W k_3 + 2 X W k_3$$

$$\frac{dy}{dt} = X k_1 - Y Z k_2 + X W k_3$$

$$\frac{dz}{dt} = X k_1 - Y Z k_2$$

$$\frac{dw}{dt} = -X W k_3$$

Conservation laws:

$$\frac{d}{dt}(X + Y) = 0 \qquad X + Y = X_0 + Y_0$$

$$\frac{d}{dt}(Y - Z + W) = 0 \qquad Y - Z + W = Y_0 - Z_0 + W_0$$

Consider the following system of equations where $x, y \in \mathbb{R}$

$$\frac{dx}{dt} = xy - 1$$

$$\frac{dy}{dt} = x - y^3$$

Steady state solutions: $0 = xy - 1$ $(1,1)$

$0 = x - y^3$ $(-1,-1)$

For each steady state, determine its stability

$$J = \begin{bmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial y} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial y} \end{bmatrix} = \begin{bmatrix} y & x \\ 1 & -3y^2 \end{bmatrix}$$

Plug in $(1,1)$: $\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1-\lambda & 1 \\ 1 & -3-\lambda \end{bmatrix} \Rightarrow (1-\lambda)(-3-\lambda) - 1 = 0$

$-3 - \lambda + 3\lambda + \lambda^2 - 1 = 0$

$\lambda^2 + 2\lambda - 4 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{4 - 4(1)(-4)}}{2}$

$= \frac{-2 \pm \sqrt{20}}{2} \Rightarrow$

$-1 - \frac{\sqrt{20}}{2}, \ -1 + \frac{\sqrt{20}}{2}$

Not asymmetrically stable b/c one is positive

Plug in $(-1,-1)$: $\begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} -1-\lambda & -1 \\ 1 & -3-\lambda \end{bmatrix}$

$\Rightarrow (-1-\lambda)(-3-\lambda) + 1 = 0$

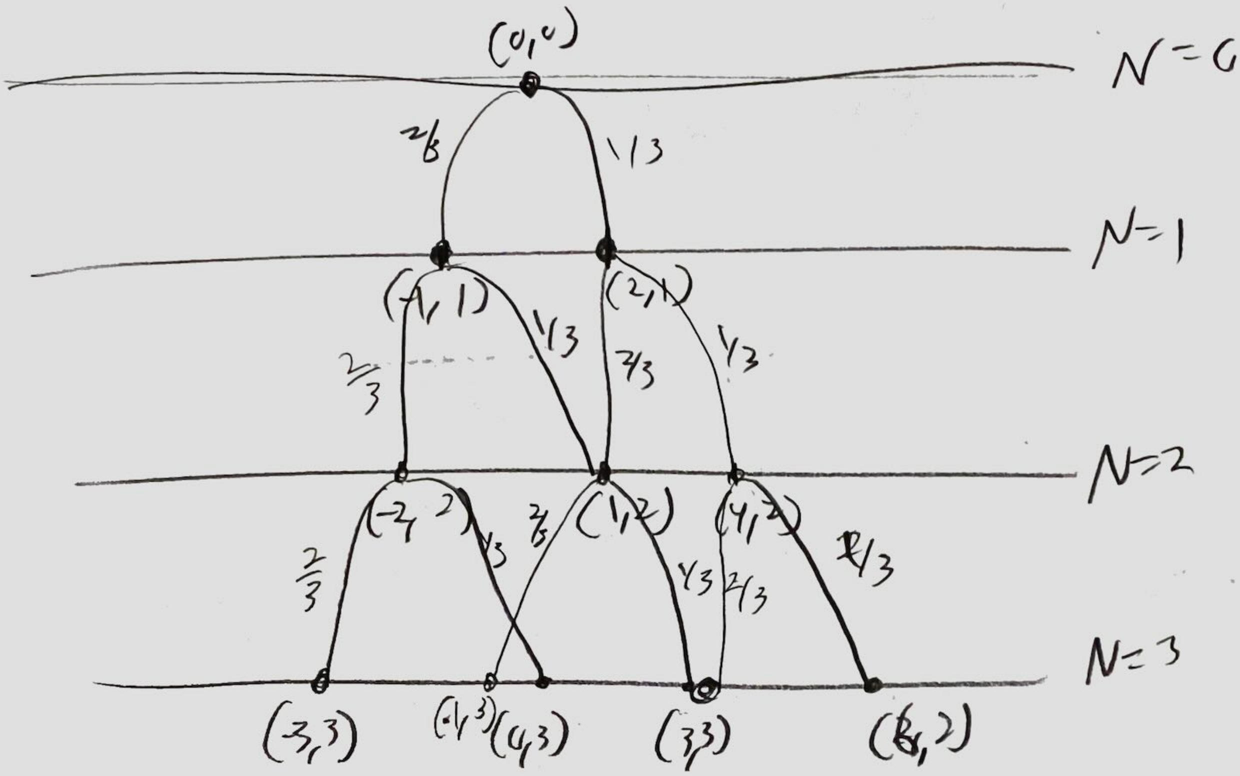
$3 + \lambda + 3\lambda + \lambda^2 + 1 = 0 \Rightarrow \lambda^2 + 4\lambda + 4 = 0$

$(\lambda + 2)(\lambda + 2) = 0$

$\lambda = -2$

Asymmetrically stable b/c it's negative

Assume that a particle undergoes a random walk in one dimension such that at time, $t = N\Delta t$, its position is given by $x = m\Delta x$. At each point in time, the particle moves to $x = (m+2)\Delta x$ with probability $1/3$ or to $x = (m-1)\Delta x$ with probability $2/3$. Assume that particle starts at $(m,N) = (0,0)$



$\underline{N=0:}$ $w(0,0) = 1$

$\underline{N=1:}$ $w(-1,1) = \frac{2}{3}$, $w(2,1) = 1/3$

$\underline{N=2:}$ $w(-2,2) = \frac{2}{3}\cdot\frac{2}{3} = \frac{4}{9}$, $w(1,2) = \frac{1}{3}\cdot\frac{2}{3} + \frac{2}{3}\cdot\frac{1}{3} = \frac{4}{9}$

$w(4,2) = \frac{1}{3}\cdot\frac{1}{3} = \frac{1}{9}$

$\underline{N=3:}$ $w(-3,3) = \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \frac{8}{27}$, $w(-1,3) = \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{2}{3} =$

$w(0,3) = \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}$, $w(3,3) = \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}$ + $\frac{1}{}$

Write down a general master equation for $w(m, N)$ in terms of $w(m-2, N-1)$ and $w(m+1, N-1)$

$A = \frac{1}{3}$

$\beta = \frac{2}{3}$

$$W(m, N) = \frac{1}{3} w(m-2, N-1) + \frac{2}{3} w(m+1, N-1)$$

Show that the Fourier transform of $f(x)e^{iax}$ is $F(k-a)$

$$F(k) = \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx$$

$$\mathcal{F}(f(x)) = \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx$$

table 1, row 4

$= F(k-a)$

$U_t + U_{xx} = U_{xxx}$

$\cancel{A}\ U_t + (ik)^2 U = (ik)^3 U$

5.2) Consider the situation of when two lanes of traffic merge down to one lane, as shown in Fig 5.37. Assume a steady flow, so the density and velocity do not depend on time, and assume all variables are non negative.

(a) Using the result of Exercise 5.1a, find an equation that relates the values on the right with those on the left

$$J(\rho_a) - J(\rho_0) = 0$$

$$\rho_1 V_1 + \rho_2 V_2 = \rho_3 V_3$$

5.3) There are various recommendation concerning safe following.

(a) The National Security Council recommends, the three-second rule. This means that you are allowed at least 3 s between,

uniform density: $\rho = \frac{1}{l+d}$ $\qquad t = 3s$, $d = vt = 3v$

$$\rho = \frac{1}{l+3v}$$

$$\rho(l+3v) = 1$$

$$\rho l + 3\rho v = 1$$

$$\frac{3\rho v}{3\rho} = \frac{1-\rho l}{3\rho}$$

$$v = \frac{1-\rho l}{3\rho}$$

$\boxed{\text{Solve for } V}$

8) In the early days of motoring, it was recommended that you keep one car length back (about 20 ft) for each 10 mph of speed.

$$f \quad \frac{1}{\ell + td}$$

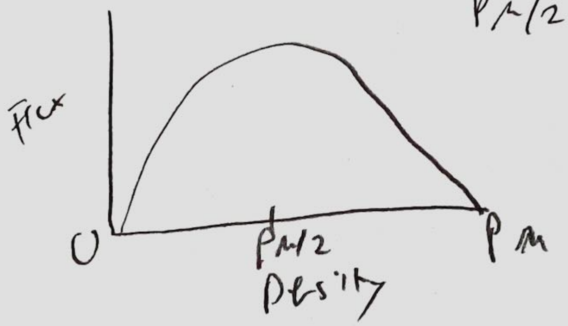
5.5) This problem explores some of the consequences of the Greenshields model as identified in a typical traffic engineering manual.

(a) Sketch the flux as a function of density. A what density is the flux a maximum?

$$V = V_m\left(1 - \frac{\rho}{\rho_m}\right)$$

$$J = \rho V = \rho V_m\left(1 - \frac{\rho}{\rho_m}\right)$$

Flux max at $\rho_m/2$



(b) Jam density $(\rho_m) \Rightarrow V = 0$

Jam density means that traffic is stopped, here traffic jam.

Free-flow velocity (V_m) - Velocity occurs $\Rightarrow$ traffic can flow