Follow-up Exam 7

Graded

Student

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Total Points

0 / 0 pts

Question 1

Question Sheet

0 / 0 pts

✔ **− 0 pts** See handwritten grades and comments on paper.

Name: Ivan Wang

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I1	I2
I3	I4

I5

I6

AI1

AI2

$$\int \frac{x^3}{\sqrt{4-x^2}}\,dx \qquad \sqrt{a^2-x^2} \Rightarrow \sqrt{2^2-x^2}$$

$a=2$

$x = 2\sin\theta$

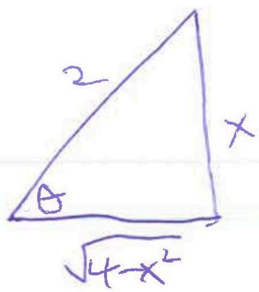
$dx = 2\cos\theta\, d\theta$

$\sqrt{2^2-x^2} = 2\cos\theta$

$$\int \frac{(2\sin\theta)^3}{2\cos\theta} \cdot 2\cos\theta\, d\theta$$

$$\int (2\sin\theta)^3\, d\theta$$

$$\Rightarrow 8\int \sin^3\theta\, d\theta$$



$\sin\theta = \frac{x}{2}$

$\cos\theta = \frac{\sqrt{4-x^2}}{2}$

$$8\int \sin\theta \cdot \sin\theta \cdot \sin\theta\, d\theta$$

$$\Rightarrow 8(-\cos\theta \cdot -\cos\theta \cdot -\cos\theta) + C$$

$$= 8\left(-\frac{\sqrt{4-x^2}}{2} \cdot -\frac{\sqrt{4-x^2}}{2} \cdot -\frac{\sqrt{4-x^2}}{2}\right) + C$$

P

AI3

AI4

$\frac{1}{5}\ln|5x+2| + \frac{1}{5}\left(\frac{1}{-1}(5x+2)^{-1}\right) + \frac{2}{2}\ln|x^2+36| + \frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + C$

m

AI5

AI6

S1

S2

S3	S4
S5	ST1

ST2 $\sum_{n=1}^{\infty} \frac{1}{(3n+4)^{7/2}}$ $f(x) = \frac{1}{(3x+4)^{7/2}}$ P

✓ Continuous? Yes radical in denominator

✓ Positive? Yes

✓ decreasing?

$1 \le x_1 < x_2$

$(3x_1+4)^{7/2} < (3x_2+4)^{7/2}$

$\frac{1}{(3x_1+4)^{7/2}} > \frac{1}{(3x_2+4)^{7/2}}$

Improper integral

$\lim_{b\to\infty} \int_1^b \frac{1}{(3x+4)^{7/2}} \Big|_{x=1}^{x=b}$

Need antiderivative

$\lim_{b\to\infty} \frac{1}{(3b+4)^{7/2}} - \frac{1}{(3+4)^{7/2}}$

$\Rightarrow -\frac{1}{(7)^{7/2}}$

Convergent b/c the limit exist & the improper integral.

Since the improper integral is convergent and $f(x_1) > f(x_2)$, we can conclude that the series is convergent by the integral test.

ST3

p-Series

ST4 $\sum_{n=1}^{\infty} \frac{6n^2+3n+1}{5n^3+7n+6}$ $\sum b_n = \sum \frac{6n^2}{5n^3}$ $\frac{6}{5n}$ ~~geometric series~~ $r=1$ $|r| \geq 1$ divergent series with $p=1$

M

positive? Yes, not negatives

limit? $\lim_{n\to\infty} \frac{a_n}{b_n} \Rightarrow \lim_{n\to\infty} a_n \cdot \frac{1}{b_n} = \lim_{n\to\infty} \frac{6n^2+3n+1}{5n^3+7n+6} \cdot \frac{5n^3}{6n^2}$

$\lim_{n\to\infty} \frac{30n^5+15n^4+5n^3}{30n^5+42n^3+36n^2} \cdot \frac{\left(\frac{1}{n^5}\right)}{\left(\frac{1}{n^5}\right)} \Rightarrow \lim_{n\to\infty} \frac{30+\frac{15}{n}+\frac{5}{n^2}}{30+\frac{42}{n}+\frac{36}{n^3}} \Rightarrow 1$

since $\lim_{n\to\infty} \frac{30n^5+15n^4+5n^3}{30n^5+42n^3+36n^2} = 1$ and $\sum b_n$ is divergent, we can conclude that this series is also divergent by the limit comparison test.

ST5 $\sum_{n=1}^{\infty} (-1)^n \frac{5n}{4n^2+2}$ $\sum_{n=1}^{\infty} \frac{5n}{4n^2+2}$

P

positive? ✓

inequality: $\frac{5n}{4n^2+2}$, $\frac{5(n+1)}{4(n+1)^2+2}$

ST6 $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2 \cdot (4^n)}{n!}$

M

$\lim_{n\to\infty} \left| a_{n+1} \cdot \frac{1}{a_n} \right| \Rightarrow \lim_{n\to\infty} \left| \frac{(-1)^n (n+1)^2 \cdot 4^{(n+1)}}{(n+1)!} \cdot \frac{n!}{(-1)^{n-1} n^2 \cdot 4^n} \right| \Rightarrow \lim_{n\to\infty} \left| \frac{(-1)^n}{(-1)^{n-1}} \cdot \frac{(n+1)^2}{(n+1)!} \cdot \frac{4^{(n+1)}}{4^n} \cdot \frac{n!}{n^2} \right|$

If $L < 1$ series convergent
If $L > 1$ series divergent
If $L = 1$ series inconclusive

$\lim_{n\to\infty} \left| \frac{(n+1)^2}{(n+1)n!} \cdot 4 \cdot \frac{n!}{n^2} \right| \Rightarrow 4 \lim_{n\to\infty} \left| \frac{(n+1)}{n^2} \right|$

$4 \lim_{n\to\infty} \frac{n+1}{n^2} \cdot \frac{\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)} = \frac{\frac{1}{n}+\frac{1}{n^2}}{1} = 0$

since $\lim_{n\to\infty} \left| a_{n+1} \frac{1}{a_n} \right| = 0 < 1$, the series is convergent from the ratio test

ST8 $\sum_{n=1}^{\infty} (-1)^n \frac{4n^2+3n+2}{n^4+6n^3+n}$

$\sum_{n=1}^{\infty} \left| (-1)^n \frac{4n^2+3n+2}{n^4+6n^3+n} \right| = \sum_{n=1}^{\infty} \frac{4n^2+3n+2}{n^4+6n^3+n}$

$\lim_{n\to\infty} \frac{4n^2+3n+2}{n^4+6n^3+n} \cdot \frac{\left(\frac{1}{n^4}\right)}{\left(\frac{1}{n^4}\right)} = \lim_{n\to\infty} \frac{\frac{4}{n^2}+\frac{3}{n^3}+\frac{2}{n^4}}{1+\frac{6}{n}+\frac{1}{n^3}} = 0$

P

PS1 $\sum_{n=1}^{\infty} (-1)^n \frac{2^n (x-3)^n}{n^{1/3}}$

ratio test

$\left| a_{n+1} \cdot \frac{1}{a_n} \right| \Rightarrow \left| \frac{(-1)^{n+1} \cdot 2^{n+1} (x-3)^{n+1}}{(n+1)^{1/3}} \cdot \frac{n^{1/3}}{(-1)^n 2^n (x-3)^n} \right|$

$\Rightarrow \left| \cancel{\frac{(-1)^{n+1}}{(-1)^n}} \cdot \frac{2^{n+1}}{2^n} \cdot \frac{(x-3)^{n+1}}{(x-3)^n} \cdot \frac{n^{1/3}}{(n+1)^{1/3}} \right|$

$\Rightarrow \left| 2 \cdot (x-3) \cdot \frac{n^{1/3}}{(n+1)^{1/3}} \right|$

$|2(x-3)| \cdot \left(\frac{n^{1/3}}{(n+1)^{1/3}}\right)$

$\lim_{n\to\infty} |2(x-3)| \cdot \left(\frac{n^{1/3}}{(n+1)^{1/3}}\right)$

$|2(x-3)| \lim_{n\to\infty} \frac{n^{1/3}}{(n+1)^{1/3}}$

$|2(x-3)| \left(\lim_{n\to\infty} \frac{n}{n+1}\right)^{1/3}$

$\lim_{n\to\infty} \frac{n}{n+1} \cdot \frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1$

If $L<1$ series converges
If $L>1$ series diverges
If $L=1$ test inconclusive

M

$|2(x-3)| < 1$

$\frac{-1}{2} < \frac{2(x-3)}{2} < \frac{1}{2} \longrightarrow -\frac{1}{2} < x-3 < \frac{1}{2}$

$-\frac{1}{2}+3 < x < \frac{1}{2}+3$

$-\frac{1}{2}+\frac{6}{2} < x < \frac{1}{2}+\frac{6}{2}$

$\frac{5}{2} < x < \frac{7}{2}$

ROC: $R = \frac{\left(\frac{7}{2}-\frac{5}{2}\right)}{2} = \frac{\left(\frac{2}{2}\right)}{2} = \frac{1}{2}$

PS2 $\sum_{n=1}^{\infty} (-1)^n \frac{2^n(x-3)^n}{n^{1/3}}$

Check endpoints

$x=\frac{5}{2} \rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{2^n(\frac{5}{2}-3)^n}{n^{1/3}} = (-1)^n \frac{2^n(-\frac{1}{3})^n}{n^{1/3}}$

$\frac{5}{2}-3 = (-\frac{1}{2})$

$x=\frac{7}{2} \rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{2^n(\frac{7}{2}-3)^n}{n^{1/3}} = \frac{(-1)^n 2^n(\frac{1}{3})^n}{n^{1/3}}$

P

PS3 $\frac{3x^2}{4+2x} \Rightarrow \frac{3x^2}{4-(-2x)}$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

P