

Recitation Exam 3

● Graded

Student

Ivan Wang

Total Points

4 / 5 pts

Question 1

(AI5) - Question 1

1 / 1 pt

✓ - 0 pts (M) Mastery

Question 2

(AI6) - Question 2

1 / 1 pt

✓ - 0 pts (M) Mastery

✓ - 0 pts Does not identify this as a p-form improper integral

💬 need to state p form integral

1 include this with your explanation

Question 3

(S1) - Question 3

1 / 1 pt

✓ - 0 pts (M) Mastery

Question 4

(S2) - Question 4

0 / 1 pt

✓ - 1 pt (P) Progressing

✓ - 0 pts Incorrect limit of 1st sequence

✓ - 0 pts Incorrect limit of 2nd sequence

Question 5

(S3) - Question 5

1 / 1 pt

✓ - 0 pts (M) Mastery

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MTH 142 — Recitation Exam #3

Directions

1. **You're going to do great!**
 2. You do not need to simplify your answer, unless otherwise indicated.
 3. Show all necessary work, unless otherwise indicated.
 4. Use correct notation.
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Academic Integrity

Take this exam with integrity. **Don't cheat.**

1. No calculators or electronic devices are allowed.
2. No other resources are allowed during the exam (this means notes, formula sheets, people, websites, etc.)

Any academic integrity violation will result in **at least** a 0 on this exam.

Grading

Each question will be graded on the M/P/U scale described in the course syllabus.

Mastery (M): All necessary work is shown, your answer is correct, and correct mathematical notation is used. (Small non-calculus mistakes that do not significantly detract from the solution may be okay.)

Progressing (P): Any question earning this score **can be attempted again** during the follow-up exam. This gives you another opportunity to demonstrate Mastery. Future attempts will not necessarily be the exact same question, but will assess the same learning outcome.

Unsatisfactory (U): Any question earning this score **cannot be attempted again**.

Q1: (AI5) Determine if the integral is convergent or divergent. If convergent, evaluate the integral.

$$\int_1^{\infty} 6e^{-2x} dx$$

$$\lim_{b \rightarrow \infty} \int_1^b 6e^{-2x} dx = \lim_{b \rightarrow \infty} \int_1^b 6 \cdot -\frac{1}{2} e^{-2x} dx = \lim_{b \rightarrow \infty} -3e^{-2x} \Big|_{x=1}^{x=b}$$

$$\lim_{b \rightarrow \infty} \left[-3 \cdot \frac{1}{e^{2b}} - \left(-3 \cdot \frac{1}{e^2} \right) \right]$$

k-form integral

$$= \lim_{b \rightarrow \infty} \left[0 + \frac{3}{e^2} \right]$$

$$= \frac{3}{e^2}$$

integral is convergent b/c the limit exist.

Q2: (AI6) State whether the integral is convergent or divergent and explain why. (Do not integrate.)

$$\int_1^{\infty} \frac{1}{\sqrt[3]{x}} dx$$

* $p > 1 \Rightarrow \text{convergent}$
 $p \leq 1 \Rightarrow \text{divergent}$

p-form

$$= x^{-\frac{1}{3}}$$

Integral is divergent b/c $p = \frac{1}{3}$, which is $p \leq 1 \rightarrow$ divergent.

Q3: (S1) Write out the first 4 terms of the sequence.
(Calculate each factorial term in your answer.)

$$\left\{ \frac{3^{n-1}}{n!} \right\}_{n=1}^{\infty}$$

$$n=1 \rightarrow \frac{3^{1-1}}{1!} \Rightarrow \frac{3^0}{1} = \frac{1}{1} = 1$$

$$n=2 \rightarrow \frac{3^{2-1}}{2!} \Rightarrow \frac{3}{2} = \frac{3}{2}$$

$$n=3 \rightarrow \frac{3^{3-1}}{3!} = \frac{9}{6} = \frac{3}{2}$$

$$n=4 \rightarrow \frac{3^{4-1}}{4!} = \frac{27}{24} = \frac{9}{8}$$

$$\left\{ 1, \frac{3}{2}, \frac{3}{2}, \frac{27}{24}, \dots \right\}$$

Q4: (S2) Determine if each sequence listed below is convergent or divergent. If convergent, find the value of the limit.

$$\left\{ \frac{6n^3 + 4\sqrt{n}}{5n^3 + 7n - 9} \right\}_{n=1}^{\infty}$$

$$\left\{ \frac{(-1)^n}{2^{n+3}} \right\}_{n=1}^{\infty}$$

$$\lim_{n \rightarrow \infty} \frac{6n^3 + 4\sqrt{n}}{5n^3 + 7n - 9} \cdot \left(\frac{\frac{1}{n^3}}{\frac{1}{n^3}} \right)$$

$$\lim_{n \rightarrow \infty} \frac{6 + \frac{4\sqrt{n}}{n^3}}{5 + \frac{7n}{n^3} - \frac{9}{n^3}} = \lim_{n \rightarrow \infty} \frac{6 + \frac{\infty}{\infty}}{5 + \frac{\infty}{\infty} - \frac{\infty}{\infty}} = \infty$$

Divergent b/c limit doesn't exist

$$\lim_{n \rightarrow \infty} \frac{(-1)^n}{2^{n+3}}$$

$$= \lim_{n \rightarrow \infty} \frac{(-1)^n}{2^{n+3}}$$

Divergent limit doesn't exist

Write out the first 3 terms of the sequence of
Q5: (S3) partial sums for the given series.
(Do not simplify.)

$$\sum_{n=1}^{\infty} \frac{4}{2n^2}$$

$$a_1 = \frac{4}{2(1)^2} = \frac{4}{2} = 2$$

$$a_2 = \frac{4}{2(2)^2} = \frac{4}{8} = \frac{1}{2}$$

$$a_3 = \frac{4}{2(3)^2} = \frac{4}{18} = \frac{4}{18}$$

$$\frac{4}{2n^2} = \frac{A}{2} + \frac{B}{n} + \frac{C}{n^2}$$

~~$4 = A(n^2) + B(n) + C$~~
 ~~$4 = 0 + 0 + C$~~
 ~~$C = 4$~~

$$S_3 = a_1 + a_2 + a_3$$

$$S_3 = 2 + \frac{1}{2} + \frac{4}{18}$$

Extra Space - Clearly indicate in the original answer space if there is any work you want graded here.