Recitation Exam 4

Graded

Student

Ivan Wang

Total Points

1 / 7 pts

Question 1

(S4) - Question 1

0 / 1 pt

- **– 1 pt** (P) Progressing
- **– 0 pts** **a-value:** Incorrect / missing a-value
- **– 0 pts** **r-value:** Incorrect / missing r-value
- **– 0 pts** Need to write the series in the form $\sum ar^n$

1. $|r| < 1$ implies convergence
   $|r| \geq 1$ implies divergence

2. $\sum_{n=0}^{\infty} 2^{2n+3} \cdot 5^{-n+1} =$
   $\sum_{n=0}^{\infty} 2^3 \cdot 2^{2n} \cdot 5^{-n} \cdot 5 =$
   $\sum_{n=0}^{\infty} 8(5)(2^2)^n(\frac{1}{5})^n =$
   $\sum_{n=0}^{\infty} 40 \cdot (\frac{4}{5})^n$

3. Convergent, Geometric series with $r = \frac{4}{5}$ and $|r| < 1$

4. Sum: $\dfrac{a}{1-r} = \dfrac{40}{1-\frac{4}{5}} = 200$

Question 2

(S5) - Question 2

1 / 1 pt

- **– 0 pts** (M) Mastery

Question 3

(ST1) - Question 3

0 / 1 pt

- **– 1 pt** (P) Progressing

5. Test is inconclusive, not the series

Question 4

(ST2) - Question 4

0 / 1 pt

- ✔ **− 1 pt** (P) Progressing
- ✔ **− 0 pts** **Function:** Need to state $f(x)$ is eventually positive, continuous, and decreasing
- ✔ **− 0 pts** **Decreasing:** Need to show $f(x)$ is decreasing using either inequalities or derivatives
- ✔ **− 0 pts** **Improper Integral and Limits:** Need to set up and calculate the improper integral using limits.
- ✔ **− 0 pts** **Improper Integral Value:** Incorrect / Missing / Incomplete calculation of the improper integral.

Question 5

(ST3) - Question 5

0 / 1 pt

- ✔ **− 1 pt** (P) Progressing

Question 6

(ST4) - Question 6

0 / 1 pt

- ✔ **− 1 pt** (P) Progressing
- ✔ **− 0 pts** **Comparable Series:** Incorrect / Missing / Incomplete **explanation** for why $\sum b_n$ converges or diverges
- ✔ **− 0 pts** **Limit Work:** Missing / Incomplete work for $\lim\limits_{n\to\infty} a_n/b_n$
- ✔ **− 0 pts** **Limit Value:** Incorrect / missing value for $\lim\limits_{n\to\infty} a_n/b_n$

6 Need to explain why this is convergent.

Question 7

(ST5) - Question 7

0 / 1 pt

- ✔ **− 1 pt** (P) Progressing
- ✔ **− 0 pts** **Conclusion:** Incorrect conclusion about series convergence / divergence
- ✔ **− 0 pts** Wrong test

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MTH 142 —– Recitation Exam #4

Directions

1. **You're going to do great!**
2. You do not need to simplify your answer, unless otherwise indicated.
3. Show all necessary work, unless otherwise indicated.
4. Use correct notation.

Academic Integrity

Take this exam with integrity. **Don't cheat.**

1. No calculators or electronic devices are allowed.
2. No other resources are allowed during the exam (this means notes, formula sheets, people, websites, etc.)

Any academic integrity violation will result in **at least** a 0 on this exam.

Grading

Each question will be graded on the M/P/U scale described in the course syllabus.

Mastery (M): All necessary work is shown, your answer is correct, and correct mathematical notation is used. (Small non-calculus mistakes that do not significantly detract from the solution may be okay.)

Progressing (P): Any question earning this score **can be attempted again** during the follow-up exam. This gives you another opportunity to demonstrate Mastery. Future attempts will not necessarily be the exact same question, but will assess the same learning outcome.

Unsatisfactory (U): Any question earning this score **cannot be attempted again.**

Q1: (S4) Determine if the series is convergent or divergent and explain why. If convergent, find the sum.

$$\sum_{n=0}^{\infty} 2^{2n+3} \cdot 5^{-n+1}$$

$(2^3)^{2n} \cdot \left(\frac{5}{n}\right) \cdot 5$

$5 \cdot (8)^{2n} \cdot \frac{5}{n}$

$5 \cdot (64)^n \cdot \frac{5}{n}$

$(320)^n \cdot 5^{-n}$

$\sum_{n=0}^{\infty} 5(320)^1$

$r = \frac{a}{1-r}$

Series is divergent b/c $r > 1$.

Q2: (S5) Determine if the series is convergent or divergent and explain why.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$$ ~ p-series

$p > 1$ convergent

$p \leq 1$ divergent

since $p = \frac{1}{3}$, the series is divergent.

Q3: (ST1) Investigate this series using the **Test for Divergence.** Determine if the test tells us the series is convergent, divergent, or if the test is inconclusive.

$$\sum_{n=1}^{\infty} \frac{5n+1}{6n^4+2n}$$

Series is not p-series or geometric series.

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5n+1}{6n^4+2n} \cdot \frac{\left(\frac{1}{n^4}\right)}{\left(\frac{1}{n^4}\right)} \Rightarrow$

$\lim_{n\to\infty} \frac{\frac{5}{n^3} + \frac{1}{n^4}}{6 + \frac{2}{n^3}} = 0$

If $\lim_{n\to\infty} a_n \neq 0 \rightarrow$ divergent

If $\lim_{n\to\infty} a_n = 0 \rightarrow$ inconclusive

Since $\lim_{n\to\infty} \frac{5n+1}{6n^4+2n} = 0$, the series is inconclusive.

Q4: (ST2) Use the **Integral Test** to determine if the series is convergent or divergent. $\sum_{n=1}^{\infty} \frac{1}{(5n+2)^{7/3}}$

$\cancel{\lim_{b \to \infty} (5n+2)^{7/3} \Big|_1^{\infty}}$

$f' = \frac{1}{\frac{7}{3}(5n+2)^{4/3}}$

$\lim_{b \to \infty} \frac{1}{\frac{7}{3}(5n+2)^{4/3}} \Bigg|_1^{\infty}$

series convesent

Q5: (ST3) Use the **Comparison Test** to determine if the series is convergent or divergent. $\sum_{n=1}^{\infty} \frac{2\sqrt{n}}{7n^2+2}$

$a_n = \frac{2\sqrt{n}}{7n^2+2}$ $b_n = \frac{\sqrt{n}}{7n^2}$

$a_n \geq b_n$

Series converget

Q6: (ST4) Use the **Limit Comparison Test** to determine if the series is convergent or divergent. $\sum_{n=1}^{\infty} \frac{5n+1}{6n^4+2n}$

$a_n = \frac{5n+1}{6n^4+2n}$ $b_n = \frac{5n}{6n^4}$ 6

$\lim_{n \to \infty} \frac{5n+1}{6n^4+2n} \cdot \frac{\left(\frac{1}{n^4}\right)}{\left(\frac{1}{n^4}\right)}$

$\lim_{n \to \infty} \frac{\frac{5}{n^3}+\frac{1}{n^4}}{6+\frac{2}{n^3}} = 0$

$\lim_{n \to \infty} \frac{5n}{6n^4} \cdot \frac{\left(\frac{1}{n^4}\right)}{\left(\frac{1}{n^4}\right)} = \lim_{n \to \infty} \frac{\frac{5}{n^3}}{6} = 0$

series is conversent b/c $\lim_{n \to \infty} a_n$ and $\lim_{n \to \infty} b_n = 0$.

Q7: (ST5) Use the **Alternating Series Test** to determine if the series is convergent or divergent. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+5n}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+5n} = \left| \frac{(-1)^n}{(n+1)^2+5(n+1)} \cdot \frac{n^2+5n}{(-1)^{n-1}} \right|$$

$$= \frac{\cancel{(-1)^n} \cdot n^2+5n}{(n+1)^2+5(n+1) \cdot \cancel{(-1)^{n-1}}} = \frac{n^2+5n}{(n+1)^2+5(n+1)} = \infty$$

Series is diverget b/c the series DNE