Recitation Exam 6

Graded

Student

Ivan Wang

Total Points

1 / 6 pts

Question 1

(PS4) - Question 1

1 / 1 pt

✔ **– 0 pts** (M) Mastery

Question 2

(PS5) - Question 2

0 / 1 pt

✔ **– 1 pt** (P) Progressing

✔ **– 0 pts** **x-terms:** Missing the powers of $(x-1)$ in the power series.

Question 3

(PS6) - Question 3

0 / 1 pt

✔ **– 1 pt** (P) Progressing

✔ **– 0 pts** **r-value:** Incorrect r-value plugged into $\sum x^n$

✔ **– 0 pts** algebraic error

you didn't distribute the 2n+1 properly, review exponents before the final. Should be x^(6n+7) and 2^(2n+1) which if you'd really like to distribute is equivalent to 2*(4)^n. Think of 2x^3 as your new 'x' and use the form provided. This is all that's needed for this problem. no need to taylor expand directly.

1. incorrect exponent on the x
2. incorrect exponent on the 2

Question 4

(AD1) - Question 4

0 / 1 pt

✔ **– 1 pt** (P) Progressing

✔ **– 0 pts** **Intersection Points:** Incorrect / missing calculation of intersection points.

✔ **– 0 pts** **Integral:** Incorrect / missing calculation of definite integral.

Question 5

(AD2) - Question 5

0 / 1 pt

- ✔ **– 1 pt** (P) Progressing
- ✔ **– 0 pts** **Intersection Points:** Incorrect / missing calculation of intersection points.
- ✔ **– 0 pts** **Integral Setup:** Incorrect order of the subtraction when setting up the integral. Should be Outer Area minus Inner area.

3. You are rotating around a vertical line so you should leave the functions in y.
4. You did not square these

Question 6

(PC1) - Question 6

0 / 1 pt

- ✔ **– 1 pt** (P) Progressing

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MTH 142 — Recitation Exam #6

Directions

1. **You're going to do great!**
2. **Simplifying** - Unless otherwise stated, you do not need to simplify your answer.
3. Show all necessary work, unless otherwise indicated.
4. Use correct notation.

Academic Integrity

Take this exam with integrity. **Don't cheat.**

1. No calculators or electronic devices are allowed.
2. No other resources are allowed during the exam (this means notes, formula sheets, people, websites, etc.)

Any academic integrity violation will result in **at least** a 0 on this exam.

Grading

Each question will be graded on the M/P/U scale described in the course syllabus.

Mastery (M): All necessary work is shown, your answer is correct, and correct mathematical notation is used. (Small non-calculus mistakes that do not significantly detract from the solution may be okay.)

Progressing (P): Any question earning this score **can be attempted again** during the follow-up exam. This gives you another opportunity to demonstrate Mastery. Future attempts will not necessarily be the exact same question, but will assess the same learning outcome.

Unsatisfactory (U): Any question earning this score **cannot be attempted again.**

Q1: (PS4) Find a **power series representation** centered at $a = 0$ for the given function (using intergration and differentiation). Write the power series in sigma form: $\sum c_k x^k$

$$\frac{1}{(7-x)^2}$$

$\int \frac{1}{(7-x)^2}\,dx \Rightarrow \frac{1}{2}\cdot\frac{1}{(7-x)}$

Prototype

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

↓ Power series representation

$\frac{1}{7}\cdot\frac{1}{1-\frac{x}{7}}$

$\frac{1}{7}\cdot\frac{1}{1-\frac{x}{7}} = \frac{1}{7}\sum_{n=0}^{\infty}\left(\frac{x}{7}\right)^n \Rightarrow \frac{1}{7}\sum_{n=0}^{\infty}\left(\frac{1}{7}\right)^n (x)^n \Rightarrow \sum_{n=0}^{\infty}\left(\frac{1}{7}\right)^{n+1}(x)^n$

Differentiation

$\sum_{n=1}^{\infty}\left(\frac{1}{7}\right)^{n+1}\cdot n(x)^{n-1}$ radius of convergence $=$

Q2: (PS5) Find the **Taylor Series** centered at $a = 1$ for the function. Write out the first **four** terms of the series as your answer. (Do not write your answer in sigma notation.) $f(x) = \sin(4x)$

Taylor series: $\frac{f^n(a)}{n!}(x-a)$

$n=0$ $f(x) = \sin(4x) \rightarrow \sin(4)$

$n=1$ $f'(x) = 4\cos(4x) \rightarrow 4\cos(4)$

$n=2$ $f''(x) = (4)(4)(-\sin(4x)) \rightarrow (4)(4)(-\sin(4))$

$n=3$ $f'''(x) = (4)(4)(4)(-\cos(4x)) \rightarrow (4)(4)(4)(-\cos(4))$ $f^n(4) =$

$n=4$ $f''''(x) = (4)(4)(4)(4)(\sin(4x)) \rightarrow$

$= (4)(4)(4)(4)(\sin(4))$

~~$\sin(4) \quad 4\cos(4)$~~

Q3: (PS6) Use the Maclaurin Series for $\sin x$ given below, to find the **Maclaurin series** for $f(x)$. Write the power series in sigma form: $\sum c_k x^k$ $f(x) = x^4 \sin(2x^3)$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

~~$\sin(2x^3)$~~

$\frac{x^3}{(3)!}$, $\frac{-x^3}{(3)!}$, $\frac{x^5}{(5)!}$, $\frac{-x^7}{(7)!}$

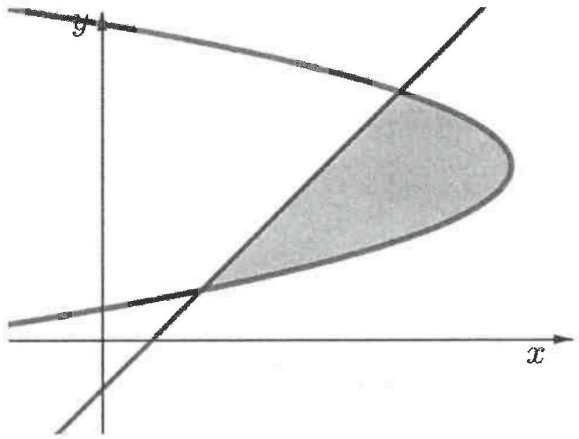
$n=0$ $n=1$ $n=2$ $n=3$

$$\sin(2x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{2x^{2n+4}}{(2n+1)!}$$

$$x^4 \sin(2x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{2x^{2n+8}}{(2n+1)!}$$

Q4: (AD1) Find the **area of** the region bounded by the given curves. You do not need to simplify your answer.

$y - x + 1 = 0$ $\quad x = -y^2 + 7y - 4$



$x = 1 + y$

$y = x - 1$

$-x = -1 - y$

$x + 4 = -y^2 + 7y$

$\int_a^b (f(b) - f(a))$

$-y(y-7)$

$\int_a^b (f(b) - f(a))\,dx$

Intercepts

$1 + y = -y^2 + 7y - 4$

$-y \qquad -y$

$-1 \qquad\qquad -1$

$0 = (y^2 - 6y - 5) - 1$

$y^2 + 6y + 5 = 0$

$(y+5)(y+1)$

$y = -5 \quad y = -1$

$x = y^2 - 7y + 4$

$1 + y = -y^2 + 7y - 4$

$-y \qquad\qquad +4$

$5 = -y^2 + 6y$

$-y^2 + 6y - 5 = 0$

$-1(-y^2 + 6y - 5) = 0$

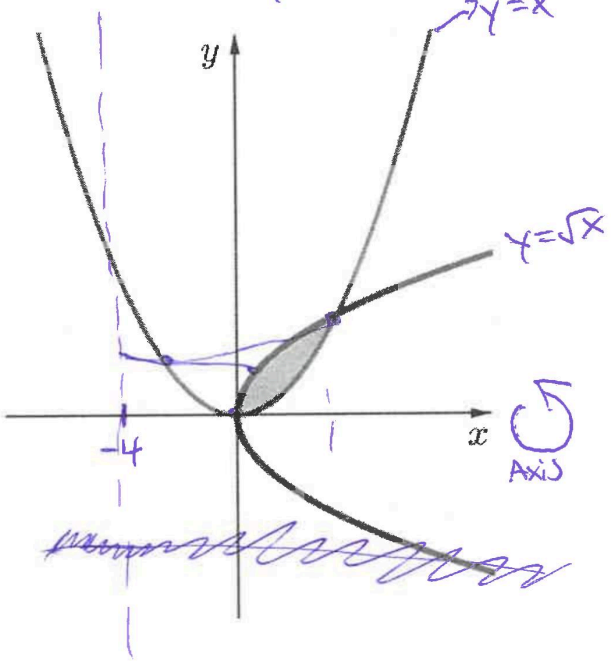
$y^2 - 6y + 5$

$(y-3)(y-2) \quad y = 3, 2$

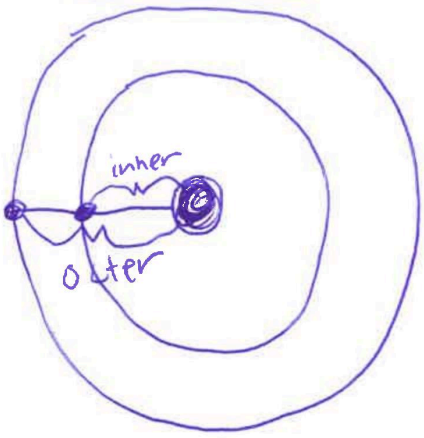
Q5: (AD2) Find the **volume** of the solid obtained by rotating the region bounded between the given curves about the given line. Sketch a typical cross section of the solid, and clearly state and label the outer radius and inner radius.

$x = y^2$ $y = x^2$ about the line: $x = -4$

$y = \sqrt{x}$ $x = \sqrt{y}$



Cross section



intercepts

$\sqrt{x} = x^2$

$x^{1/2} = x^2$

$x^{3/2} = 0$

$x = 1$

radii

$r_{out} = x + 4 \Rightarrow \sqrt{x} + 4$

$r_{in} = x + 4 \Rightarrow x^2 + 4$

$= \frac{2}{3} + 4 - \frac{1}{2} - 4 - 0$

Area: $A(x) = r_{out}^2 - r_{in}^2 = (\sqrt{x} + 4) - (x^2 + 4)$

$(x^{1/2} + 4) - (x^2 + 4)$

Volume: $\int_0^1 (x^{1/2} + 4) - (x^2 + 4)\, dx \Rightarrow \left(\frac{2}{3}x^{3/2} + 4x\right) - \left(\frac{1}{3}x^3 + 4x\right)\Big|_0^1$

$\frac{2}{3}x^{3/2} + 4x - \frac{1}{3}x^3 - 4x$

$= \frac{2}{3} + 4 - \frac{1}{3} - 4$

Q6: (PC1) The point P is given in Cartesian coordinates. Convert this into **polar coordinates** with $r \geq 0$ and $0 \leq \theta < 2\pi$ You can leave your answer in terms of an inverse trigonometric function.

$P(3\sqrt{3}, -3)$

Extra Space - Clearly indicate in the original answer space if there is any work you want graded here.