Recitation Exam 2

Student

Ivan Wang

Total Points

0 / 5 pts

Question 1

(I6) - Question 1   **0** / 1 pt

✔ **− 1 pt** (P) Progressing

Question 2

(AI1) - Question 2   **0** / 1 pt

✔ **− 1 pt** (P) Progressing

✔ **− 0 pts** Incorrect substitution for x

✔ **− 0 pts** Incorrect / Missing differential

✔ **− 0 pts** Incorrect a value

Question 3

(AI2) - Question 3   **0** / 1 pt

✔ **− 1 pt** (P) Progressing

✔ **− 0 pts** Incorrect trig substitution

Question 4

(AI3) - Question 4   🚩 **0** / 1 pt

✔ **− 1 pt** (P) Progressing

① Factored numerator incorrectly

② Incorrectly plugged these values into the decomposition

Question 5

(AI4) - Question 5   **0** / 1 pt

- ✔ **− 1 pt** (P) Progressing

- ✔ **− 0 pts** Incorrect 2nd k-form antiderivative

- ✔ **− 0 pts** Incorrect antiderivative of $x/(x^2 + a^2)$ term

- ✔ **− 0 pts** Incorrect antiderivative of $1/(x^2 + a^2)$ term

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MTH 142 —— Recitation Exam #2

Directions

1. **You're going to do great!**

2. You do not need to simplify your answer, unless otherwise indicated.

3. Show all necessary work, unless otherwise indicated.

4. Use correct notation.

Academic Integrity

Take this exam with integrity. **Don't cheat.**

1. No calculators or electronic devices are allowed.

2. No other resources are allowed during the exam (this means notes, formula sheets, people, websites, etc.)

   Any academic integrity violation will result in **at least** a 0 on this exam.

Grading

Each question will be graded on the M/P/U scale described in the course syllabus.

Mastery (M): All necessary work is shown, your answer is correct, and correct mathematical notation is used. (Small non-calculus mistakes that do not significantly detract from the solution may be okay.)

Progressing (P): Any question earning this score **can be attempted again** during the follow-up exam. This gives you another opportunity to demonstrate Mastery. Future attempts will not necessarily be the exact same question, but will assess the same learning outcome.

Unsatisfactory (U): Any question earning this score **cannot be attempted again.**

Q1: (I6) Calculate $\int \tan^4 x \sec^4 x \, dx$

$\sec^2 x = \tan^2 x + 1 \quad 1 + \tan^2 x$

$\tan^2 x = \sec^2 x - 1$

$\int \tan^3 x \sec^4 x \tan x \, dx$

$\int \tan^2 x (\tan^2 x + 1)(\tan^2 x + 1) \tan x \, dx$

$\int \tan^4 \sec^2 x \sec x \, dx$

$\int (\sec^2 x - 1)(\sec^2 x - 1) \sec^3 x \sec x \, dx$

$\int \tan^3 x \sec^3 x (\tan x \sec x) \, dx$

$u = \sec x$
$du = \tan x \sec x \, dx$

$\int \tan^3 x \sec^3 x \, du$

$\int \tan^3 x \, u^3 \, du$

$\int \tan x \, dx = \ln|\sec^2 x|$

$= \ln|\sec^2 x|^3 + \frac{1}{4}\sec x + C$

Q2: (AI1) Use a trigonometric substitution to rewrite this integral as a trig integral. (Do not integrate.) $\int \frac{x^2}{\sqrt{9 + x^2}} \, dx$

$x = 9\tan x$
$dx = 9\sec x$
$\sqrt{9 + x^2} = 9\sec x$

$\int \frac{9\tan x}{9\sec x} \, 9\sec x$

$\int 9\tan x \, dx$

$x = 9\sin x$
$dx = 9\cos x$
$\sqrt{9 + x^2} = 9\cos x$

$\int \frac{9\sin x}{9\cos x} \, 9\cos x$

$\int 9\sin x \, dx$

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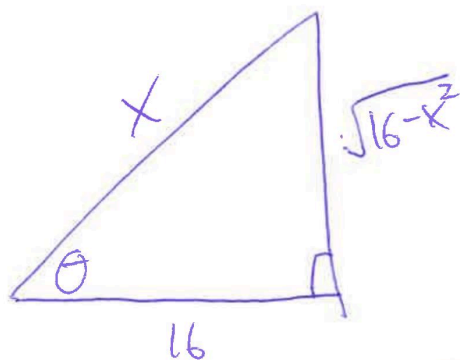**Q3: (AI2)** Use a trigonometric substitution to calculate the integral. (Do the integration for this one.) $\int \dfrac{x^5}{\sqrt{16-x^2}}\,dx$

$x = 16\sec x$

$dx = 16\sec x \tan x$

$\sqrt{16-x^2} = 16\tan x$

$\int \dfrac{(16\sec x)^5}{16\tan x} \cdot 16\sec x \tan x$

$\int 16^5 \sec^5 x \cdot \sec x \; dx$

$\int 16^5 \sec^6 x \; dx$

$\int 16^5 \cdot \left(\dfrac{x}{16}\right)^6 dx$

$= 16^5 \cdot \left(\dfrac{x}{16}\right)^6 + C$

x

$\sqrt{16-x^2}$

θ

16

$\tan x = \dfrac{\sqrt{16-x^2}}{16}$

$\sec x = \dfrac{x}{16}$

Q4: (AI3) Find the partial fraction decomposition of f, including coefficients. (Do not integrate.)

$$f(x) = \frac{14x + 7}{x^2 - 3x - 10} = f(x) = \frac{7(x+1)}{x^2-3x-10} = \frac{7(x+1)}{(x-5)(x+2)}$$

①

$$\left(\frac{7(x+1)}{(x-5)(x+2)} = \frac{A}{(x-5)} + \frac{B}{(x+2)} \right)(x-5)(x+2)$$

$$7(x+1) = A(x+2) + B(x-5)$$

$$x = -2 \quad A(0) + B(-7) = 1$$

$$x = 5 \quad A(8) + B(0) = 6 \quad ②$$

$$= \frac{1}{(x-5)} + \frac{6}{(x+2)}$$

$7x+7 = -14+7 = $
-7

$35+7 = 42$

Q5: (AI4) Calculate the integral. No work needs to be shown for this problem. You can just state your answer.

$$\int \left(\frac{1}{8x+5} + \frac{1}{(8x+5)^2} + \frac{7x+1}{x^2+36} \right) dx$$

$$\frac{1}{8}\ln|8x+5| + \frac{1}{2}\ln|8x+5| + \tan^{-1}(x^2+36) + C$$

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